

# Acoustics and Noise Control

THIRD EDITION

R.J. Peters, B.J. Smith & Margaret Hollins



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Third edition

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# Preface

Although the underlying physical laws and principles of acoustics do not change there have been very many changes and additions to standards, laws and regulations, codes of practice relating to noise, and in noise measurement techniques and noise control technology since the last edition of *Acoustics and Noise Control* was published in 1995. Therefore much of the material relating to these aspects of the second edition is now severely out of date. The fact that, despite this, the book is still being used and found to be of value by many people has provided much of the inspiration for the creation of this third edition.

This new edition has been almost completely re-written and re-structured. [Chapters 1 to 6](#) have been completely reorganized and now contain some of the material formerly contained in the later [chapters 7, 8 and 9](#). Each chapter starts with a brief introduction indicating what is to be covered and making links with other chapters. An important feature of the original book, written many years ago by Brian Smith, was its student-centred approach, with many worked examples, and list of questions (with answers to numeric parts) at the end of each chapter. This approach has been retained. The excellent chapter on law written originally by Stephanie Owen has been comprehensively and expertly revised and updated by Margaret Hollins.

The book is an introductory text for those completely new to the subject but it also aims to serve the needs of the practitioner working in local government environmental health departments as well as at junior consultant level, and is a suitable introduction to more advanced texts, a list of which is given in the bibliography. Through worked examples the book illustrates the application of prediction and design calculations routinely used in noise control practice, with an emphasis on the assumptions and limitations that apply to these calculations,

and, new to this edition, with derivations added in appendices at end of chapters, for the benefit of readers who may wish to delve deeper into the theoretical background to the subject. A bibliography contains lists of more advanced texts, of reports, standards and codes of practice, and a complete list of formulae and equations is given in an appendix. The glossary was a much valued feature of previous editions and has been completely revised and the number of entries considerably increased.

This new edition covers much of the Institute of Acoustics Diploma syllabus, and those of various MSc courses in acoustics, as well as the acoustics and noise control components of degree and higher level technician courses in architecture, construction, engineering, environmental health, environmental science, health and safety and occupational hygiene. The chapter on law relating to noise will be of use to lawyers, barristers and other professionals dealing with noise cases, but also of interest to many non-expert readers with an interest in the legal framework relating to noise control in Europe and the UK.

Doing, seeing and listening will always enhance understanding and enjoyment of any subject and this is particularly so in the study of acoustics. Therefore an appendix provides a list of possible experiments, exercises and observations for the reader to carry out. A wide variety of audio and animated visual demonstrations are available via the internet. These can enhance explanations given in the text, and readers are referred to these and encouraged to investigate for themselves.

It is hoped that readers of this new edition will find it useful and informative.

*Bob Peters*  
*October 2010*

# Authors' acknowledgements

The authors wish to acknowledge the important contribution of Stephanie Owen, who wrote the law chapter in the second edition, which is the basis of the revision in this edition. They also wish to thank the Institute of Acoustics for giving permission to use material from the Institute's Distance Learning material and examination questions.

# Publisher's acknowledgements

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## Figures

Figure 1.16 from The Open Door Web Site IB Physics Waves Diffraction files, <http://www.saburchill.com/physics/chapters2/0008.htm>; Figure 1.22 from <http://hyperphysics.phy-astr.gsu.edu/hphys.html>; Figures 2.5, 2.8, 2.9, 2.10, 2.11, 2.12, 2.13, 2.17, 2.18 from Distance Learning notes for the General Principles of Acoustics Module of the Institute of Acoustics Diploma in Acoustics and Noise Control; Figure 2.16 from ISO9613-2 (1996) General Method of Calculation, Table A.1; Figure 3.9 from ISO 532:1975, Acoustics – Expression of the subjective magnitude of sound or noise, Part 2: Method for calculating loudness level; Figure 3.10 from ISO recommendation R507, 1966; Figure 5.15 from BS EN ISO 10534-2001; Figure 6.15 from BS8233; Figure 7.14 from BS7385:part 2 1993. Permission to reproduce extracts from British Standards is granted by the British Standards Institution (BSI). No other use of this material is permitted. British Standards can be obtained in PDF or hard copy formats from the BSI online shop: can be

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### Tables

[Table 1.1](#) in [Chapter 1](#) Appendix from BS EN ISO 266:1997 Preferred Frequencies; [Table 5.2](#) from BS8233; [Table 6.4](#) from ISO 717-2:2004; [Table 6.6](#) from BS EN 717-1: 1990; Table on page 186 from [Table B.2](#) of BS5228:2009 part 2; Table on page 190 from [Table 1](#) from BS6472; [Table 8.1](#) from BS EN 61672-1:2003; [Table 8.2](#) from BS EN 61672-1:2003, Permission to reproduce British Standards is granted by the British Standards Institution (BSI). No other use of this material is permitted. British Standards can be obtained in PDF or hard copy formats from the BSI online shop: <http://shop.bsigroup.com> or by contacting BSI Customer Services for hard copies only: Tel: +44 (0)20 8996 9001, Email: [cservices@bsigroup.com](mailto:cservices@bsigroup.com); [Table 3.2](#) from Guidelines for Community Noise, 1999, World Health Organization, <http://www.who.int/docstore/peh/noise/>

[guidelines2.html](#); [Table 5.3](#) from Building Bulletin 93 (BB93); Tables on page 159 and page 160, from Building Regulations Approved Document E, 2003, Crown Copyright material is reproduced with permission under the terms of the Click-Use License.

### Text

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# Terminology and notation

It is current practice in the literature on the subject to represent all decibel quantities by  $L$ , with appropriate subscripts such as  $L_p$ ,  $L_I$ ,  $L_{Aeq}$ ,  $L_{EP,d}$ ,  $L_{AE}$  (see Glossary for fuller list). Again in line with common usage, pascals (Pa) have been used instead of  $N/m^2$ .

## Symbols

A list of the main symbols used in the book is given below. The list is not exhaustive and other symbols are defined in the text.

$a$	acceleration
$A$	frequency weighting
$A$	attenuation, acoustic absorption, acceleration amplitude
$B$	bandwidth
$c$	velocity of sound
$C$	frequency weighting, spectral adaptation terms
$d$	depth or thickness
$D$	frequency weighting, source dimensions
$D$	level difference
$e$	the exponential number (= 2.718 . . .)
$f$	frequency
$F$	fast time weighting, force amplitude
$g$	gram
$g$	acceleration due to gravity
$h$	height, hour
Hz	hertz
$I$	impulse time weighting
$I$	sound intensity
$J$	joule
$j$	the complex operator, i.e. $\sqrt{-1}$
$k$	kilo (e.g. kg, km, kHz)
$k$	wave number, $k = 2\pi/\lambda$ , stiffness
$K$	elastic modulus
$l$	length
$L$	level in decibels
$m$	metres
$m$	mass, surface density, minutes
$n$	standing wave ratio
$N$	newtons, number of decibels, number of items, noys
$p$	sound pressure
$P$	atmospheric pressure

Pa	pascals
$Q$	dynamic magnification factor, directivity factor
$r$	distance, damping constant, reflection coefficient
$R$	sound reduction index/transmission loss, real part of a complex impedance, electrical resistance, universal gas constant
$R_C$	Room constant
$S$	slow time weighting, area ( $m^2$ ), Sones, microphone sensitivity
$t$	time, thickness, transmission coefficient
$T$	reverberation time, transmissibility, period, duration, temperature
$u$	acoustic particle velocity
$v$	vibration velocity
$V$	vibration velocity amplitude, vibration dose value, volume
$w$	vibration frequency weighting
W	watt
$x$	distance, displacement
$X$	displacement amplitude, reactance i.e. imaginary part of a complex impedance, static deflection
$z$	acoustic impedance
$Z$	frequency weighting
$\alpha$	acoustic absorption coefficient
$\gamma$	ratio of specific heats of a gas
$\delta$	logarithmic decrement (vibration), path difference (barriers)
$\Delta$	small increment
$\epsilon$	energy density
$\eta$	isolation efficiency
$\theta$	angle
$\lambda$	wavelength $\lambda = 2\pi/k$
$\xi$	damping ratio
$\pi$	ratio of circumference to diameter of a circle, pi = 3.142 . . .
$\rho$	density
$\sigma$	radiation efficiency
$\varphi$	phase difference
$\omega$	angular frequency, $\omega = 2\pi f$

Where the same symbol has more than one meaning, multiple representation has been retained to comply with common usage. The meaning will be clear from the context and is made clear in the text.

# Chapter 1 The nature and behaviour of sound

## 1.1 A qualitative picture of wave motion

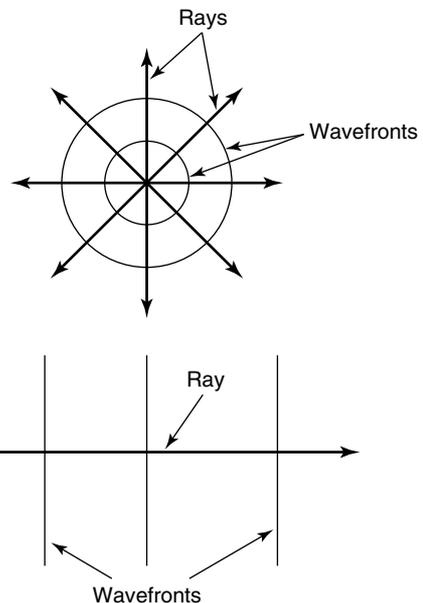
Acoustics is the science of sound, and sound is a wave motion. In a wave a change or disturbance in some physical property of a medium is transmitted through that medium. For example when a sound occurs in air the sound wave causes the particles in the air to move to and fro (i.e. to vibrate), and because the particles are elastically connected (air being an elastic medium) this vibration is transmitted through the air. The vibrating layers of air contain energy and so another feature of all waves is that they contain energy. The essential features of a medium which is able to transmit sound waves are that it must possess elasticity and inertia (mass); sound waves can travel through solids, liquids and gases but not through a vacuum. In any real medium there will always be some frictional processes at work so that some of the energy of the vibrating particles of the medium will be lost to the sound wave and turned into heat, a process known as sound absorption.

The two simplest types of sound waves are **spherical waves** and **plane waves** (see Figure 1.1), and it helps to understand them if we consider the analogy of waves on the surface of water. If we drop a small object such as a stone into water we see circular ripples travelling outwards. The invisible spherical sound waves in air are their three-dimensional counterparts. If the stretch of water is linear, e.g. a canal, and the stone is replaced by a long plank of wood we would see plane ripples move along the water surface.

The wavefront represents the leading edge of the wave, i.e. it tells us how far the wave has travelled and the rays, always perpendicular to the wavefronts, indicate the direction in which the wave is travelling.

These two forms of wave are idealized models of wave propagation and are useful because waves for sound sources can often approximate to one of these models. Sound from a loudspeaker tends to radiate equally in all directions at low frequencies (i.e. like spherical waves) but be much more directional (i.e. more like plane waves) at high frequencies.

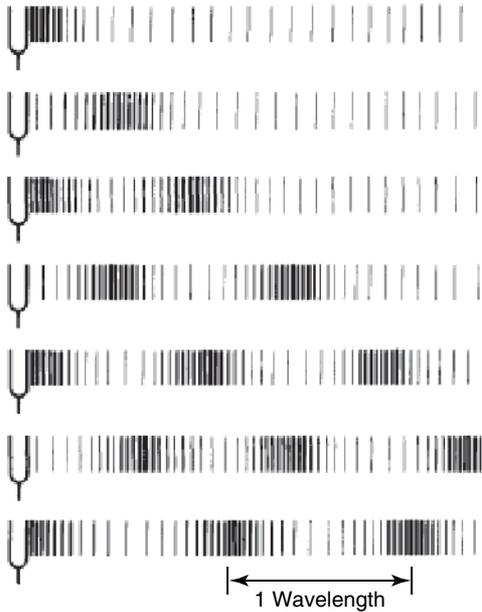
Plane waves travelling in one direction only are the simplest form of waves and can be used to explain frequency and wavelength.



**Figure 1.1** Sketch illustrating rays and wavefronts for spherical and plane waves

In a sound wave in air, as a result of the to and fro motion, sometimes the air particles are bunched together, causing a very slight increase in pressure in the atmospheric pressure (a compression) and sometimes causing them to be spaced further apart, causing a very slight reduction in pressure (a rarefaction). This is shown in Figure 1.2 where compressions and rarefactions from the vibrations of a tuning fork are shown travelling in one dimension (down a tube or pipe for example). These very small fluctuations in pressure in the tube constitute the sound pressure caused by the passage of the sound wave down the tube.

The disturbance caused by the sound waves could be described in terms of the vibrations of the air particles, either as a displacement, as a velocity or as an acceleration, and these alternatives will be discussed in more detail in Chapter 7 on vibration. However, since these movements cannot be seen, and since our human ears and our microphones respond to the changes in



**Figure 1.2** Propagation of a one-dimensional sound wave

pressure caused by sound waves it is more usual to measure and describe sound waves in terms of sound pressure, in pascals (Pa.).

The simplest form of plane wave occurs when the vibration of the air particles causes a sinusoidal variation in sound pressure with time and can be used to explain frequency and wavelength. The sound pressure in such a plane wave varies with both distance and time, as shown

in Figures 1.3 and 1.4. This sinusoidal variation of sound pressure with time represents a sound with a single frequency, called a pure tone.

## 1.2 Frequency and wavelength and sound speed

After a certain amount of time, called the period,  $T$ , of the motion, the cycle repeats itself (Figure 1.3). The frequency,  $f$ , of the vibration and of the wave is the number of cycles of the motion which occur in one second:

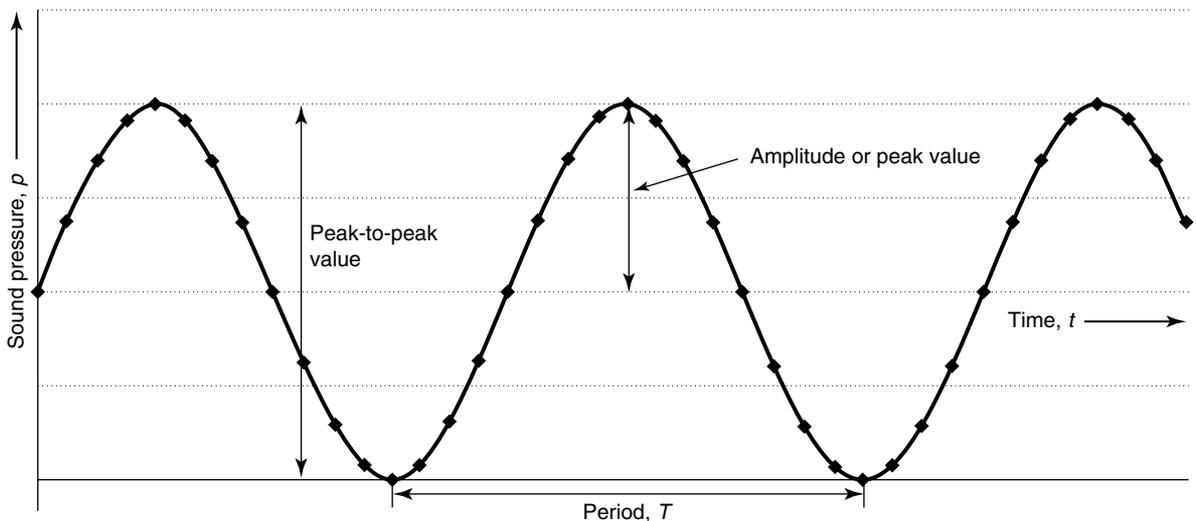
$$f = 1/T$$

Thus frequency,  $f$ , is measured in cycles per second or hertz (abbreviation Hz).

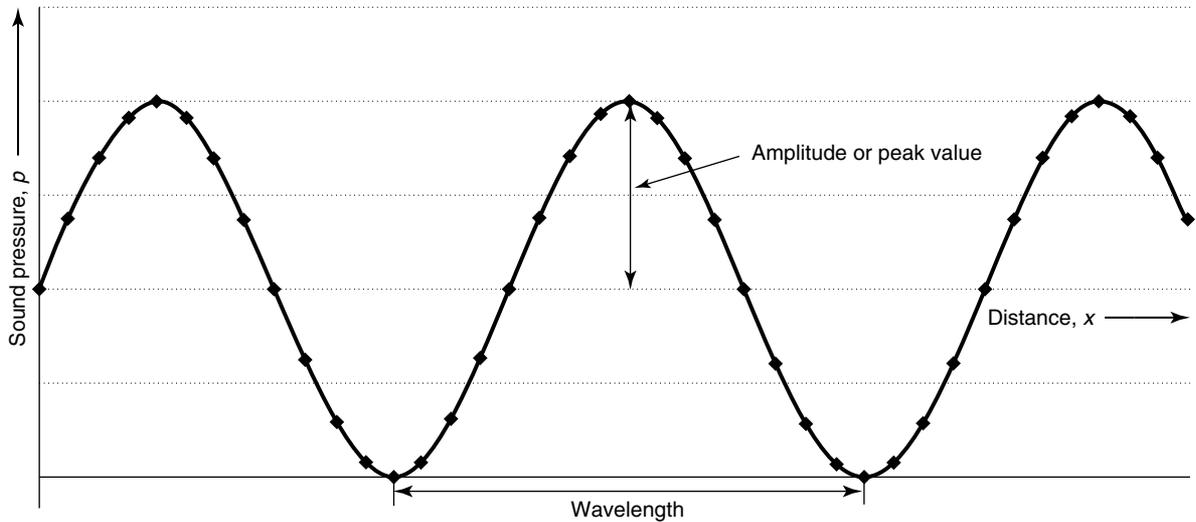
The wavelength,  $\lambda$ , is the minimum distance between points on the wave where the air particles are vibrating in step or in phase as shown in Figures 1.1 and 1.3.

### The relationship between sound speed, frequency and wavelength

In order for air particles, which are one wavelength apart, to be in phase, it must be the case that the wave travels one wavelength in the time that it takes for any one of the particles to complete one cycle of motion. Since the number of such cycles completed in one second corresponds to the frequency of the wave, and since the wave velocity is the distance travelled by the wave in one second, it follows that frequency,  $f$ , wavelength,  $\lambda$ ,



**Figure 1.3** Graph showing variation of sound pressure with time (at one position in space) for a pure tone



**Figure 1.4** Graph showing variation of sound pressure with position (at one moment of time) for a pure tone

and wave velocity,  $c$ , are related by the well known equation:

$$c = f\lambda$$

For sound waves in air the speed of sound ranges, approximately, between 330 and 340 metres per second, depending upon air temperature. Thus for a frequency of 100 Hz, at the lower end of the audio range the wavelength will be about 3.3 metres, whereas at the much higher frequency of 1000 Hz it is about 0.33 metres, i.e. the lower the frequency the greater the wavelength, and vice versa.

Note that the frequency of the wave is determined only by the source of the sound. The sound velocity depends on the medium through which the wave is travelling. These two factors then determine the wavelength in the medium, according to the equation  $c = f\lambda$ . If the sound moves from one medium to another, from air into water, for example, the frequency will remain the same in both media, but because of the difference in sound velocities the wavelengths in the two media will be different.

Can you sketch the curve for positions one quarter, one half and three quarters of a wavelength away?

Can you sketch the curve for positions one quarter, one half and three quarters of a cycle later?

### Example 1.1

A plane sound wave in air has a single frequency of 660 Hz. Taking the velocity of sound in air as 330 m/s, what is the phase difference:

- (a) between two points in the wave at distance of 0.125 metres apart, at the same moment?

- (b) at the same position, but separated by 0.0001 seconds?

### Solution

Wavelength = sound velocity/frequency =  $330/660 = 0.5$  metres.

- (a) In terms of phase, a distance of one wavelength, 0.5 metres, corresponds to a phase difference of 360 degrees. Therefore a distance of 0.125 metres corresponds to a phase difference of  $360 \times (0.125/0.5) = 90^\circ$ .
- (b) In terms of phase, a time period of one cycle of the vibration, i.e.  $1/660$  of a second = 0.00152 seconds, corresponds to a phase difference of 360 degrees. Therefore a time separation of 0.0001 seconds corresponds to a phase difference of  $360 \times (0.0001/0.00152) = 23.7^\circ$ .

## 1.3 A mathematical description of a plane progressive wave

The variation of sound pressure with time (Figure 1.3) may be represented by the equation:

$p = A\sin(\omega t)$ , where  $\omega = 2\pi f$  = angular frequency, and  $A$  = sound pressure amplitude.

The variation of sound pressure with distance (Figure 1.4) may be represented by the equation:

$p = A\sin(kx)$ , where  $k = 2\pi/\lambda$  = wave number, and  $A$  = sound pressure amplitude.

*Note:* These two equations may be combined to produce one equation which gives the sound pressure  $p$  for any position,  $x$ , at any time,  $t$ :

$$p = A \sin(\omega t - kx) \text{ for the case where } p = 0 \text{ at } x = 0 \text{ when } t = 0$$

$$p = B \cos(\omega t - kx) \text{ for the case where } p = B \text{ at } x = 0 \text{ when } t = 0$$

$$p = A \sin(\omega t - kx) + B \cos(\omega t - kx) \text{ generally}$$

### Example 1.2

A plane progressive wave travelling in the  $x$  direction is presented by the following equation which gives the instantaneous sound pressure at any distance  $x$  and any time,  $t$ :

$$p = 0.9 \sin(3142t - 9.25x) \text{ Pa}$$

Calculate the frequency, wavelength, peak sound pressure of the sound, and the speed of sound in the medium through which the sound is travelling.

#### Solution

By comparison with the general equation  $p = A \sin(\omega t - kx)$ :

$$A = \text{amplitude or peak sound pressure} = 0.9 \text{ Pa}$$

$$\omega = 2\pi f = 3142, \text{ therefore frequency } f = 3142/2\pi = 500 \text{ Hz}$$

$$k = 2\pi/\lambda = 9.25, \text{ therefore wavelength } \lambda = 2\pi/9.25 = 0.68 \text{ m}$$

$$\text{velocity of sound } c = f\lambda = 500 \times 0.68 = 340 \text{ m/s}$$

### 1.4 The audible range of sound pressures and frequencies

The audible range of sound pressures is from about  $2 \times 10^{-5}$  Pa (or  $20 \times 10^{-6}$  Pa, i.e.  $20 \mu\text{Pa}$ ) to about 20 Pa. The audible range of sound frequencies is from about 20 Hz to about 20,000 Hz (or 20 kHz). Acoustic waves with frequencies above the audible range are called ultrasonic and those with frequencies below are called infrasonic.

### 1.5 Sound pressure, sound power, sound intensity and acoustic impedance

The sound pressure is related to the motion of the particles in the medium which cause it, and is most easily related to the particle velocity. These two quantities are related by the specific acoustic impedance of the wave,  $z$ ,

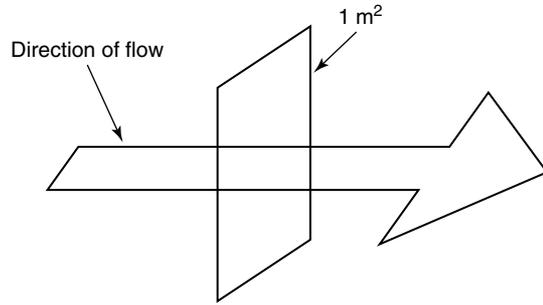


Figure 1.5 Illustration of the definition of sound intensity

which is the ratio of the acoustic pressure,  $p$  (measured in pascals (Pa)) and the acoustic particle velocity,  $v$  (measured in m/s).

Vibrating particles possess mechanical (potential and kinetic) energy, and the transmission of a disturbance through a medium involves the flow of energy. Sound power is the rate of flow of energy, i.e. energy transmitted per unit time is measured in watts (or joules (J) per second). The sound intensity at any point in the medium in any given direction is defined as the rate of flow of energy per unit area in that direction (Figure 1.5) and measured in  $\text{W/m}^2$ .

#### Exercise

If the sound intensity being transmitted through an open window of area  $0.5 \text{ m}^2$  is  $0.1 \text{ W/m}^2$  what is the sound power being transmitted, and how much sound energy will be transmitted through the window in 1 hour?

#### Answer

$0.05 \text{ W}$  and  $0.05 \times 3600 = 180 \text{ joules}$  or  $0.05 \times 10^{-3} \text{ kW hour}$ .

In any point in a sound wave the relationship between the acoustic pressure, the acoustic particle velocity and the sound intensity at that point are related by the following set of equations which also involve a quantity called the specific acoustic impedance,  $z$ , of the wave:

$$p = zv$$

$$I = pv$$

$$I = p^2/z$$

$$I = zv^2$$

For a wave travelling in one direction, called a plane wave, the specific acoustic impedance,  $z$ , depends only

upon the nature of the medium, and has the value of the product  $\rho c$  where  $\rho$  is the density of the medium, in  $\text{kg/m}^3$ , and  $c$  is the velocity of sound in the medium, in  $\text{m/s}$ . For air the value of  $\rho c$  varies with temperature and with atmospheric pressure but it typically varies between about 410 and 420  $\text{kg m/s}^2$  for typical atmospheric conditions. For water it is much larger,  $1.5 \times 10^6 \text{ kgm/s}^2$ . The quantity  $\rho c$  is also known as the characteristic acoustic impedance of the medium.

*Note:* In terms of the fundamental units  $\text{m}$ ,  $\text{kg}$  and  $\text{s}$  the units of specific acoustic impedance are  $\text{kgm/s}^2$  as above, but sometimes the quantity is expressed in terms of newtons (N) or pascals (Pa), as either  $\text{Nsm}^{-3}$  or as  $\text{Pa s/m}$ , and also as the rayl (after Lord Rayleigh, one of the pioneers of acoustics).

### Example 1.3

Calculate the acoustic particle velocity and acoustic intensity at a point in a plane wave where the sound pressure is 0.001 Pa. Take the value of  $\rho c$  as 415  $\text{kgm/s}^2$ .

#### Solution

Particle velocity,  $v = p/\rho c = 0.001/415$

$$= 2.41 \times 10^{-6} \text{ m/s}$$

Acoustic intensity,  $I = p^2/\rho c = (0.001)^2/415$

$$= 2.41 \times 10^{-9} \text{ W/m}^2$$

### Example 1.4

Calculate the sound pressure and the acoustic particle velocity at a point in a plane wave where the sound intensity is  $1 \times 10^{-6} \text{ W/m}^2$ . Take the value of  $\rho c$  as 415  $\text{kgm/s}^2$ .

#### Solution

$$p^2 = I\rho c = (1 \times 10^{-6})415 = 4.15 \times 10^{-4}$$

$$\text{therefore } p = \sqrt{(4.15 \times 10^{-4})} = 0.020 \text{ Pa}$$

$$V = I/p = 1 \times 10^{-6}/0.020 = 5.0 \times 10^{-5} \text{ m/s}$$

### Sound power

Sound sources have a tremendous range of sound powers, varying from about  $10^{-9} \text{ W}$  for the human voice when whispering, to millions of watts radiated by a space rocket during launching. The human voice radiates about  $20 \times 10^{-6} \text{ W}$  during conversation, and this could increase to about  $10^{-3} \text{ W}$  when shouting. A pneumatic drill used for road breaking radiates about 1 W, and a typical figure for the noise output of a jet airliner is about 50,000 W. Audible sounds can have a very wide range of intensities, from  $10^{-12} \text{ W/m}^2$ , i.e. a millionth of a millionth of a watt per square metre (the threshold of

hearing for the average person) to more than  $100 \text{ W/m}^2$  (approaching the threshold of pain) – a range of more than a million million to one. Sound intensity is a useful quantity because it can be related to the sound power of the noise source, and is one of the important factors in the subjectively assessed ‘loudness’ of the sound.

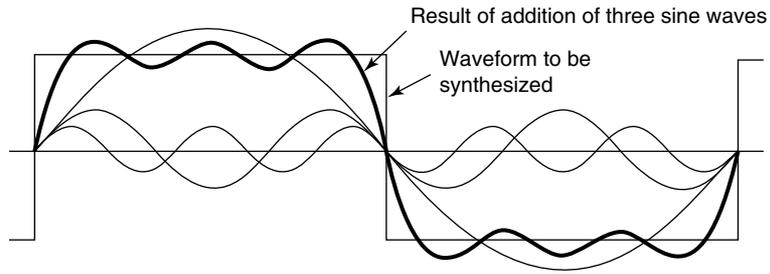
## 1.6 More complex waveforms – frequency analysis, peak and RMS values

The pure tone is the simplest sort of sound. It is produced, for example, by a tuning fork or by a loudspeaker fed with a sinusoidal voltage signal. The pure tone is a single frequency sound. Most sounds have a waveform which is more complicated than the simple sine wave and they can be considered to contain more than one frequency. The next simplest type of sound is that produced by a musical instrument, playing one single note. The waveform is harmonic, that is it repeats itself, but in a more complicated way than the sine wave of the pure tone waveform. The French mathematician Fourier showed that such a waveform could be ‘built up’ or ‘synthesized’ by combining together a number of simple sinusoidal waveforms. The frequencies of these components are the fundamental frequency (the repeating frequency of the complex waveform) and multiples of it, called harmonics. Figure 1.6 shows a harmonic waveform and illustrates how it may be produced from a fundamental and two harmonics.

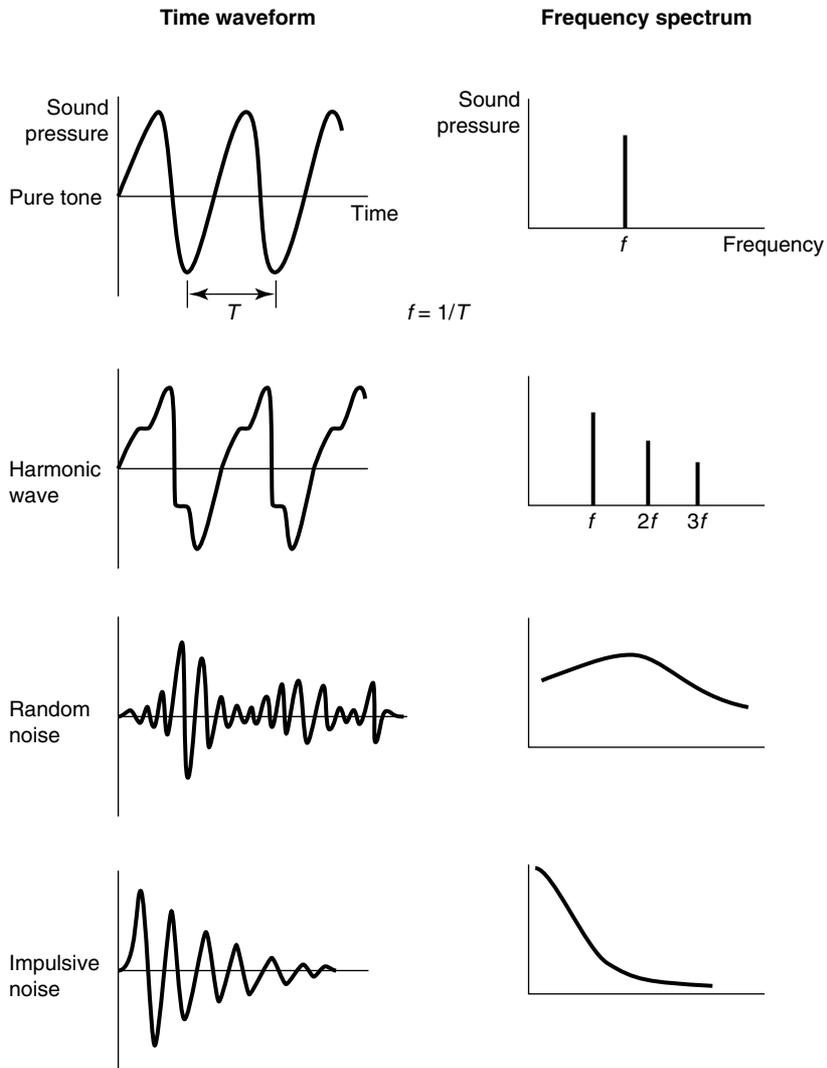
The reverse process to Fourier synthesis is Fourier analysis. This means the ‘breaking down’ or analysing of complex waveforms into their component frequencies. Mathematical techniques can be used to do this theoretically for relatively simple repeating waveforms such as in Figure 1.6, and more complicated waveforms can also be analysed numerically using computers. The analysis can also be performed using frequency analysis instrumentation attached to the sound measuring equipment. This consists of a series of electronic filters which allow only sounds within a particular range of frequencies to be measured.

### The frequency spectrum of sounds

A frequency analysis may be displayed on a graph called the frequency spectrum showing the amplitude of the different frequencies (or frequency bands). The frequency spectrum and the waveform can be considered as two alternative ways of describing a sound (see Figure 1.7). For a pure tone, the frequency spectrum is exceedingly simple – it consists of a single line indicating the amplitude of the single frequency. A harmonic waveform also has a relatively simple spectrum, consisting of a line



**Figure 1.6** Synthesis of a harmonic waveform, showing how a square wave can be built up from a series of sine waves (only the first three harmonics are shown)



**Figure 1.7** Some different types of waveform and their frequency spectra

spectrum of the fundamental and the harmonics. The spectrum envelope shows the relative amplitudes of the different harmonics and the fundamental, and thus the spectra of the same note played on two different musical instruments would be different.

It is worth mentioning two more types of waveform. Random waveforms describe sounds caused by processes which are random, and so never exactly repeat themselves. Hence the waveform is not repetitive. Examples are noise produced by traffic, by wind and by many sorts of machine. Even when it seems that the noise is produced by some repeating event such as the rotation of an engine, or of gear teeth etc., the noise often has a random component, since the machine process never repeats exactly from cycle to cycle due to slight changes in speed load and other conditions.

Transient waveforms die away to zero after the passage of a period of time. They arise from a source which provides only a short transient burst of acoustic energy such as a plucked violin string, or the impact of a hammer blow onto a rivet. The sound energy is dissipated by frictional processes. The simplest transient waveform resembles a sine wave but with an amplitude which decays with time. Examples are the noise produced by impulses such as those from punches, presses, hammering and mechanical handling of goods.

Frequency spectra can be assigned to transient and random sounds, but they are more complicated than periodic sounds. A random noise contains a little of all frequencies and so its frequency spectrum is a continuous curve, called a broadband spectrum. The frequency spectra of transient sounds are also complicated. In the case of the plucked violin string the spectrum will obviously contain the fundamental frequency and its harmonics. In the case of a repeated transient, such as the impacts between teeth in a gear mechanism, the repetition rate of the impacts and its harmonics will also be important.

Figure 1.7 shows some examples of different types of waveform and their frequency spectra.

### Frequency analysis and frequency spectra

In most cases noise has a broadband spectrum, i.e. it contains a mixture of all frequencies, but some more than others. If we wish to investigate the frequency content of the noise in more detail we have to split the frequency range into bands, and measure the sound pressure level in each band. This process is called frequency analysis, and the graph showing how the sound pressure level varies with the frequency of each band is called the frequency spectrum of the noise.

There are two ways of splitting the frequency range into bands: either on a constant bandwidth basis, or on a constant percentage bandwidth.

#### *Constant bandwidth*

In the constant bandwidth method each band has the same width, so that if for example the bandwidth is 100 Hz then one of the bands might be from 100 Hz to 200 Hz, and others might be, for example from 200 to 300 Hz, or from 1100 to 1200 Hz or from 10,000 to 10,100 Hz. The centre frequency of each band would be half way between its upper and lower cut-off points, so that for the band between 100 and 200 Hz the centre frequency would be 150 Hz.

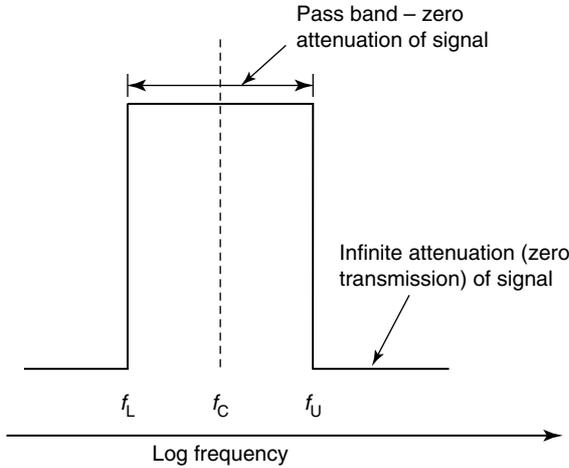
The constant bandwidth approach is usually only used for what is called ‘narrow band frequency analysis’ in which the main purpose is to identify with precision a particular frequency component in the spectrum, usually a pure tone which might be causing a disturbance (an annoying hum, whine or whistle), and to aid the diagnosis of the cause of such a component, e.g. to link it to a particular fan, motor or turbine via the rotational speed and number of rotors.

Typically the constant bandwidths might be 300 Hz, 100 Hz, 10 Hz, 3 Hz or 1 Hz, and they might be arranged in contiguous bands, e.g. 100 to 200, 200 to 300, 300 to 400 Hz etc. but it is also possible that the centre frequency will be continuously variable allowing a ‘frequency sweep’ to take place.

#### *Constant percentage bandwidth*

Covering the entire audio range (20 to 20,000 Hz) using constant bandwidth filters requires too many bands, and so, for convenience (but also because of the way the ear responds to broadband sound), constant percentage bandwidth frequency analysis is most frequently used for routine assessment of broad band noise. The most commonly used methods use octave and third octave bands, although one sixth, octave, one twelfth and one twenty fourth octaves are also available.

In musical terms an octave is a range for a particular frequency, say 256 Hz (which is ‘middle C’ on the piano) to double that frequency, i.e. 512 Hz. The internationally defined system of octave bands for sound measurement has a series of contiguous bands named by their nominal centre frequencies at 16 Hz, 31.5 Hz, 63 Hz, 125 Hz, 250 Hz, 500 Hz, 1000 Hz (or 1 kHz), 2 kHz, 4 kHz, 8 kHz and 16 kHz. In much noise measurement and assessment work the more limited range of seven bands, from 63 Hz to 4 kHz, is often used, with the 31.5 Hz and 8000 Hz bands sometimes being included.



**Figure 1.8** Characteristic features of a constant percentage bandwidth filter

In the one third octave system each octave is split into three bands. The standard range used for most building acoustics measurements consists of 16 bands, with nominal centre frequencies from 100 Hz to 3150 Hz: 100, 125, 160, 200, 250, 315, 400, 500, 630, 800, 1000, 1250, 1600, 2000, 2500 and 3150 Hz.

Sometimes an extended range, with five extra bands, is used, from 50 Hz to 5000 Hz. What are the five extra bands?<sup>1</sup>

**Log and linear frequency scales**

Unlike the constant bandwidth system the constant percentage bands get wider (in absolute terms) as the frequency increases, so that, for example the bandwidth of the 1000 Hz octave band is 10 times that of the 100 Hz band. When we plot an octave band spectrum, with sound pressure level in dB on a vertical scale, and octave band frequency on the horizontal scale we assign equal intervals of space to each octave along the horizontal axis. In doing this we are, in effect, plotting frequency on a logarithmic (log for short) frequency scale.

Using a logarithmic frequency scale the centre frequency  $f_C$  is half way between upper and lower cut-off frequencies (as shown in Figure 1.8); it is also known as the geometric mean of the upper ( $f_U$ ) and lower ( $f_L$ ) cut-off frequencies:

$$f_C = \sqrt{f_L \times f_U}$$

The exact centre frequencies of the standardized acoustic bands are slightly different from the nominal centre frequencies (except for the 1000 Hz band): for example the exact centre frequencies of the nominal 500, 2000, 800 and 1250 bands are 501.2, 1995.3 Hz, 794.3, 1258.9 Hz.

<sup>1</sup> Answer: 50 Hz, 63 Hz, 80 Hz, 4000 Hz and 5000 Hz.

The relationship between the upper, lower and centre frequencies of the various bands is defined by mathematical series based either on base 2 or on base 10.

For the base 10 series, considered be the more accurate of the two, the relationships between  $f_L$ ,  $f_C$  and  $f_U$  are:

For octaves:  $f_L = f_C/10^{0.15}$  and  $f_U = f_C \times 10^{0.15}$

And for one third octaves:

$$f_L = f_C/10^{0.05} \quad \text{and} \quad f_U = f_C \times 10^{0.05}$$

In the base 2 series the multiplying or dividing factor of  $10^{0.15}$  for octaves is replaced by  $2^{0.5}$  and for one third octaves the factor of  $10^{0.05}$  is replaced by  $(2)^{1/6}$ .

Since  $10^{0.15} = 1.4124$  and  $2^{0.5} = 1.4142$  the difference between the two methods is very small, and the same is true of the third octave multiplying and dividing factors, since  $10^{0.05} = 1.122018$  and  $2^{1/6} = 1.122462$ .

**Example 1.5**

For the 1000 Hz octave band:

$$f_L = 1000/1.4124 = 708.0 \text{ Hz} \quad \text{and}$$

$$f_U = 1000 \times 1.4124 = 1412.4 \text{ Hz}$$

And for the 2000 Hz one third octave band:

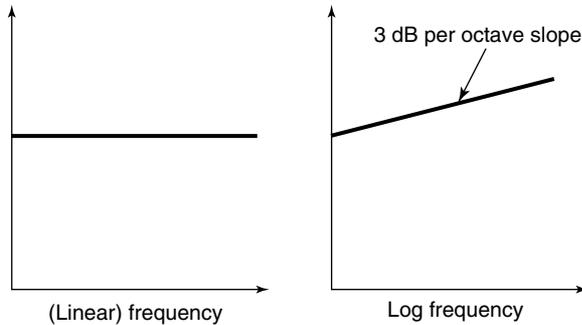
$$f_L = 2000/1.122018 = 1782.5 \text{ Hz} \quad \text{and}$$

$$f_U = 2000 \times 1.122018 = 2244.0 \text{ Hz}$$

The various frequencies for all the bands are specified in International Standard ISO 266:1997 *Acoustics – Preferred frequencies*. Appendix 1.1 gives a full list of octave and one third octave frequencies and bandwidths.

**Recognizing pure tones**

Sometimes a broadband noise from machinery contains a tonal component which is audibly recognizable as a whine, a hum or a whistle. The tonal quality makes the noise more annoying, and some noise assessment methods (such as BS 4142 for example) impose a penalty of 5 dB on such a noise (i.e. they rate it as being 5 dB higher than its measured value). It can therefore be important to recognize and agree on when a tone is present in a noise. In some cases identification and agreement can be reached simply by listening, but in less clear cut cases an objective measurement method is required. A fairly common ‘rule of thumb’ approach is to look at the third octave spectrum. A pure tone is indicated in any band which is several decibels higher than its immediate neighbouring bands. A 5 dB increase is a fairly strong indication of tonality and a 10 dB step between neighbouring bands is conclusive. There are more sophisticated techniques involving 1, 1/6, 1/12 and 1/24 octave bands,



**Figure 1.9** Characteristics of white noise

but they are beyond the scope of this chapter. Methods for the detection of pure tones in signals are described in ISO 1996-2:2007.

### White and pink noise

These are two special types of broadband noise that both have flat frequency spectra.

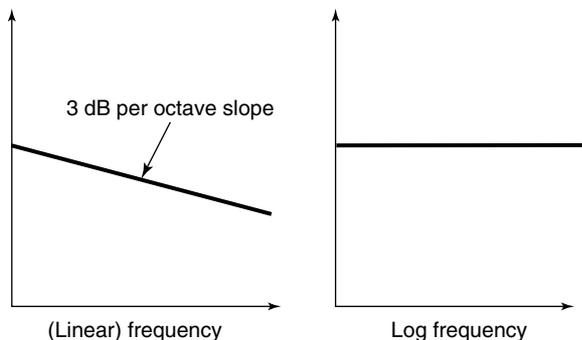
In the case of white noise it is the spectrum plotted on a linear frequency scale which would be flat (i.e. a horizontal line graph), because white noise is defined as having equal sound energy per constant bandwidth, or equal energy per Hz. If the white noise were plotted on a log frequency scale (e.g. measured in octaves or third octaves) then the spectrum would slope upwards at 3 dB increase per octave, because the bandwidth increases with frequency. (See [Figure 1.9](#).)

In the case of pink noise there is equal energy per percentage bandwidth, i.e. equal energy in each octave or each one third octave band.

Can you plot the corresponding graphs for pink noise? (See [Figure 1.10](#).)

### Frequency weightings

Sometimes the additional detail of a frequency spectrum is not needed, but it is required to describe the noise by a



**Figure 1.10** Characteristics of pink noise

single number which still in some way takes the broad spectrum characteristic of the noise into account, i.e. that it is predominantly high or low frequency in character. A number of single figure frequency weightings have been devised for this purpose, the best known being the A and C frequency weightings.

These are described in more detail in [section 1.10](#).

### The magnitude of sound pressures – RMS and peak values

If a pure tone was being played over the radio and the volume was turned up, the amplitude of the sound pressure would be increased – the sound would become louder. The amplitude is thus a convenient measure of the magnitude of the sound and can be related to its intensity and loudness, which will be discussed later.

With a more complicated waveform, however, it is not so easy. One might think that the magnitude of the peak pressure of the waveform would be the value which would be most useful. However, the sound pressure might be near to the peak value for only a small fraction of the duration of the sound, and might not be very closely related to the subjective impression of the sound. Perhaps an ‘average’ sound pressure would be a better measure of the ‘size’ of a sound? However, if we look at the sinusoidal waveform of a pure tone, we see that, taken over a complete cycle, the average sound pressure, including rarefactions and compressions, is zero. This is true for all waveforms, not just a pure tone. We need an ‘average’ which takes into account the magnitude of the sound pressure fluctuations but not their direction (positive and negative) so that the compressions and rarefactions do not average out. There are various possible ways of obtaining a ‘non-zero’ average sound pressure, but the one most commonly used is the root mean square (or RMS) sound pressure. This can best be described by looking at the waveform shown in [Figure 1.11](#). In effect the sound level meter first ‘squares’ the signal, that is multiplies it by itself. This has the effect of producing a pressure-squared waveform, which is always positive. (Remember that in algebra minus one times minus one gives plus one.) The next stage is to take the average (or mean value) of this pressure-squared waveform – called the ‘mean pressure squared’. Finally, by taking the square root of this value, we get back to a pressure – the root mean square pressure (strictly the square root of the mean pressure squared). The process is illustrated in [Figure 1.11](#).

Most sound level meters have electronic circuits which convert the microphone signal into an RMS value corresponding to the RMS sound pressure. The RMS pressure is used because it can be related to the average intensity of the sound and to the loudness of the sound.

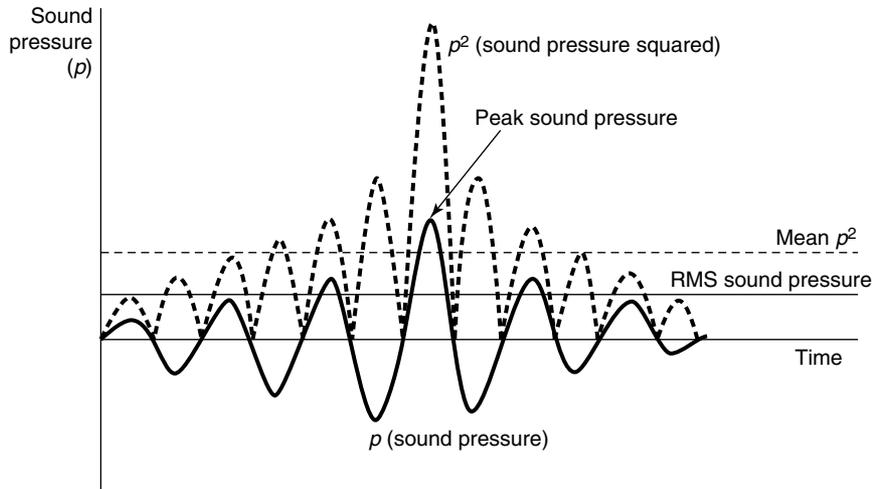


Figure 1.11 Illustrating peak and RMS values of a waveform

For a pure tone it can be shown that the peak pressure and the RMS pressure are simply related:

$$P_{\text{RMS}} = p_{\text{peak}}/\sqrt{2} = 0.707 \times p_{\text{peak}}$$

For more complex signals there is no simple relationship between the two.

There are occasions when it is important to measure the peak value of a complex sound waveform, or the peak to peak value, Examples could be the sound produced by impulsive noise, such as gunfire, explosions or punch presses. Some specialist impulse sound level meters can measure peak as well as RMS sound pressures.

In decibel terms the peak level of a pure tone is 3 dB higher than the RMS value, but the difference is higher for impulsive noise. The ratio of the peak value of a signal to its RMS value is called the **crest factor** of the signal. The more impulsive the signal the higher will be its crest factor.

### 1.7 The decibel scale

The decibel scale is used for comparing and measuring powers (electrical as well as acoustic), and related quantities such as sound intensity and sound pressure, so it is necessary to explain the relationships between these quantities.

However, although sound intensity is important as the basis of many prediction calculations, sound pressure is a more useful quantity in practical terms, and is the quantity which is always measured using the microphone of the sound level meter. The intensity ( $I$ ) of sound at a point is proportional to the square of the sound pressure ( $p$ ) at that point:

$$I \propto p^2$$

Thus for example if the sound pressure is doubled then the sound intensity increases fourfold. It is because of this important relationship that the RMS value of pressure is often measured, since the average value of ' $p^2$ ' will be proportional to the mean or average intensity of the sound over the measurement period.

#### The decibel scale

The decibel scale is a logarithmic scale for measuring or comparing energies or powers, or related quantities such as sound intensity.

If two sounds have intensities  $I_1$  and  $I_2$  then on a decibel scale  $I_2$  may be said to be  $N$  dB above  $I_1$ , where:

$$N = 10\log(I_2/I_1)$$

A similar scale may be used to compare the sound power outputs ( $W_2$  and  $W_1$ ) from two noise sources, i.e.

$$N = 10\log(W_2/W_1)$$

Sound pressure is related to the sound intensity as discussed above, and so two sounds with sound pressures  $p_1$  and  $p_2$  may also be compared on the decibel scale:

$$N = 10\log(I_2/I_1) = 10\log(p_2/p_1)^2 = 20\log(p_2/p_1)$$

Table 1.1 gives some examples of how the scale works.

Thus a 20 dB noise reduction, typically achieved by a single glazed window for example, corresponds to a 100-fold reduction in sound intensity, and the 50 dB sound insulation which is typically achieved by a masonry wall means that the wall only transmits one part in 100,000 of the sound energy incident upon it. The corresponding sound pressure ratios are the square root of those for

**Table 1.1** How the scale works

Intensity ratio $I_2/I_1$	Decibel difference $N$ dB
1	0 dB
2	3 dB
3	4.8, i.e. approx. 5 dB
4	6 dB
5	7 dB
6	7.8, i.e. approx. 8 dB
7	8.5 dB
8	9 dB
9	9.5 dB
$10 = 10^1$	10 dB
$100 = 10^2$	20 dB
$1000 = 10^3$	30 dB
$10,000 = 10^4$	40 dB
$100,000 = 10^5$	50 dB
$1,000,000 = 10^6$	60 dB

intensity, so that a 20 dB reduction is equivalent to a pressure ratio of 10:1 ( $= \sqrt{100}$ ). The table may be extended by adding or subtracting values from the right-hand column and multiplying or dividing the corresponding values from the left-hand column, or vice versa. Thus a decibel reduction of 26 dB ( $= 20 + 6$ ) corresponds to a sound intensity ratio of 400 ( $= 100 \times 4$ ), and a sound pressure ratio of 20 ( $= \sqrt{400}$ ). [Alternatively we could use arithmetic because if  $26 = 10\log(W_2/W_1)$  then  $W_2/W_1 = 10^{2.6} = 398$  (not exactly 400 because, more accurately, an intensity ratio of 400 gives  $10\log 400 = 26.02$  dB).]

### Reasons for using the decibel scale

The first reason is one of convenience, because by using a logarithmic scale the very large range of audible sound pressures (5 million:1, corresponding to an even larger range of sound intensities of 25 million million:1) is compressed into a much more manageable range of about 120 dB). The second reason is that the human response to sound is also logarithmic, with each tenfold increase (i.e. 10 dB) in sound intensity being judged, on average, to double the loudness of the sound, so that a 100-fold increase, i.e. 20 dB, would produce a fourfold increase in loudness and a 1000-fold increase (30 dB) will increase the loudness by a factor of 8. A 3 dB increase, which corresponds to a doubling of sound intensity, produces a small but noticeable subjective increase of loudness in typical situations, but a 1 dB increase is only just noticeable under the most favourable listening conditions. The third reason is again one of convenience, for it is easier to deal with decibel values which are added to or subtracted from each other (e.g. 20 dB + 30 dB = 50 dB) than with

very small ratios which would have to be multiplied or divided by each other (e.g.  $0.01 \times 0.001 = 0.00001$ ).

### Reference values: sound pressure level, sound intensity level and sound power level

The statement that machine A produces a noise level which is 10 dB higher than machine B uses the decibel scale in a relative way, without assigning an absolute value to either of the two levels.

The use of internationally agreed reference levels gives an absolute value to quantities measured on a decibel scale. The reference value of sound pressure,  $p_0$ , is  $2 \times 10^{-5}$  Pa, or  $20 \times 10^{-6}$  or 20 micropascals, which represents the threshold of hearing for the average young person with normal hearing and corresponds to 0 dB. Values measured on a decibel scale relative to this value are called sound pressure levels, and denoted by the symbols SPL or  $L_p$ . The reference value,  $W_0$ , for the sound power level scale ( $L_W$ ) is  $10^{-12}$  W, and for sound intensity level scale ( $L_I$ ) the reference value ( $I_0$ ) is  $10^{-12}$  W/m<sup>2</sup>. Thus:

$$L_p = 20\log(p/p_0)$$

$$L_I = 10\log(I/I_0)$$

$$L_W = 10\log(W/W_0)$$

These three different scales represent three different physical quantities (i.e. sound pressure, power and intensity) although in each case the measure is in dB. Which of the three is being referred to should be made clear by the context, but occasionally this may be made specific, or given emphasis by quoting the reference value, e.g. 120 dB re.  $10^{-12}$  W, which makes it clear that this a sound power level.

The relationship between the reference values  $I_0$  and  $p_0$  are such that for plane waves (when  $I = p^2/\rho c$ ) the sound intensity level has (approximately) the same numerical value as the sound pressure level. This is because, approximately,  $I_0 = p_0^2/\rho c$ , depending upon the value of  $\rho c$  used. Sound level meters, which measure sound pressure levels, may therefore be used to give an approximate indication of the magnitude of sound intensity levels, but unlike more specialist sound intensity meters, described in [Chapter 9](#), they cannot give any indication of the direction of flow of sound energy.

### Example 1.6

Calculate the sound pressure level at a point where the sound pressure is  $5.0 \times 10^{-3}$  Pa.

**Solution**

$$L_p = 20\log(p/p_0) = 20\log(5.0 \times 10^{-3}/2.0 \times 10^{-5})$$

$$= 48 \text{ dB re. } 2.0 \times 10^{-6} \text{ Pa}$$

**Example 1.7**

Calculate the sound intensity level at a point where the sound intensity is  $8.5 \times 10^{-7} \text{ W/m}^2$ .

**Solution**

$$L_I = 10\log(I/I_0) = 10\log(8.5 \times 10^{-7}/1 \times 10^{-12})$$

$$= 59.3 \text{ dB re. } 1 \times 10^{-12} \text{ W/m}^2$$

**Example 1.8**

Calculate the sound pressure, sound intensity and sound intensity level at a point in a plane wave at which the sound pressure level is 75 dB. Take the specific acoustic impedance of air as  $415 \text{ Nsm}^{-3}$ .

**Solution**

$$L_p = 20\log(p/p_0)$$

from which  $p = p_0 \times 10^{(L_p/20)} = 2.0 \times 10^{-5} \times 10^{(75/20)}$

$$= 0.112 \text{ Pa}$$

$$I = p^2/\rho c = (0.112)^2/415 = 3.0 \times 10^{-5} \text{ W/m}^2$$

$$L_I = 10\log(I/I_0) = 10\log(3.0 \times 10^{-5}/1 \times 10^{-12})$$

$$= 75 \text{ dB re. } 1 \times 10^{-12}$$

**Example 1.9**

Calculate the sound intensity, sound pressure and sound pressure level at a point in a plane wave at which the sound intensity level is 90 dB. Take the specific acoustic impedance of air as  $415 \text{ Nsm}^{-3}$ .

**Solution**

$$L_I = 10\log(I/I_0)$$

From which  $I = I_0 \times 10^{(L_I/10)} = 1 \times 10^{-12} \times 10^{9.0}$

$$= 1 \times 10^{-3} \text{ W/m}^2$$

$$I = p^2/\rho c, \text{ from which } p = \sqrt{I \times \rho c}$$

$$= \sqrt{(1 \times 10^{-3} \times 415)} = 0.64 \text{ Pa}$$

$$L_p = 20\log(0.64/2.0 \times 10^{-5})$$

$$= 90 \text{ dB re. } 2.0 \times 10^{-5} \text{ Pa}$$

**Combining sound pressure levels**

When more than one noise source is operating at once it becomes necessary to consider how the individual sound pressure levels combine. Since the decibel values are

**Table 1.2** Combining decibels

Add to the higher level	Difference between levels
3	0
3	1
2	2
2	3
1	4
1	5
1	6
1	7
1	8
1	9
0	10

based on logarithms we should not expect them to obey the rules of ordinary arithmetic. We learnt earlier that a doubling of sound energy, power or intensity corresponds to an increase of 3 dB, and so if two machines each individually produce a level of, say, 90 dB at a certain point, then when both are operating together we should expect the combined sound pressure level to increase to 93 dB, but certainly not to 180 dB!

Table 1.2 gives a method for combining levels in pairs, based on adding to the higher level a correction which depends upon the difference between the two levels. Although this is only an approximate method it should give results which are accurate to the nearest dB, which is satisfactory for most purposes.

**Example 1.10**

As an example of the use of Table 1.2 consider the combination of four decibel levels: 82 dB, 84 dB, 86 dB and 88 dB.

The levels are combined in pairs using Table 1.2. The first two levels in the series, 82 and 84, are combined to give 86 dB. This 'running total' of 86 dB is then combined with the next in the list, 86 dB, to give a new running total of 89 dB, which is then combined with the final value, 88 dB, to give a total combined level of 92 dB.

Note that according to Table 1.2 differences of 10 dB or more are negligible, so that the lower of the two levels may be ignored. Although the levels may be taken in any order, it is convenient to take them in ascending order, as in this example, so that lower values can be combined first, and so may make a significant contribution when combined with the higher levels.

The combined value,  $L_T$ , of several levels,  $L_1, L_2, L_3, \dots, L_N$  may also be calculated, accurately, using the formula:

$$L_T = 10\log[10^{L_1/10} + 10^{L_2/10} + 10^{L_3/10}$$

$$+ 10^{L_4/10} + \dots + 10^{L_N/10}]$$

Note that:

$$10^{L_1/10} = p_1^2/p_0^2 = (p_1/p_0)^2$$

and

$$10^{L_2/10} = p_2^2/p_0^2 = (p_2/p_0)^2 \text{ etc.}$$

Therefore in this formula each term inside the square brackets is related to the value of  $p^2$ , i.e. is related to the intensity of each of the component noise levels, and therefore the sum of the terms inside the square brackets relates to the total intensity of all the noises. In effect the formula is combining three steps into one calculation:

1. Turn each level back into a sound intensity.
2. Add (arithmetically) the intensities to find the total intensity.
3. Turn this total sound intensity back into a sound pressure level.

Applying this to Example 1.10:

$$L_T = 10\log[10^{8.2} + 10^{8.4} + 10^{8.6} + 10^{8.8}] = 91.6 \text{ dB}$$

which agrees with the earlier result of 92 dB using the chart method.

### Subtracting decibels

A similar approach to that for combining decibels may be used to 'subtract' a component sound level from a total level. A common use for this technique is to correct a measured noise level for the effects of background. The result,  $L_{A-B}$ , of subtracting level  $L_B$  from a higher level  $L_A$  is given by:

$$L_{A-B} = 10\log[10^{L_A/10} - 10^{L_B/10}]$$

As an example, suppose that the noise from a machine is measured (including the contribution of background noise) and found to be 87 dBA but when the machine is switched off the background noise alone is measured as 83 dBA. A more accurate value for the machine noise may be obtained by 'subtracting' the 83 dBA background noise from the combined level of 87 dBA; i.e.  $10\log[10^{8.7} - 10^{8.3}] = 84.8 \text{ dBA}$ . Note that if the measured noise level is more than 10 dB above background the correction is less than 0.5 dB and is usually considered to be negligible.

### Averaging sound pressure levels

Sometimes it is necessary to find the average value of a number of sound level measurements. A good example would be in building acoustics where in order to find a representative value of the sound level in a room a number of measurements are taken at different positions within the room, and an average value is calculated.

The appropriate average value is that which corresponds to the average sound intensity, or the average value of  $p^2$ . The average value,  $L_{\text{AVGE}}$ , of several levels,  $L_1, L_2, L_3, \dots, L_N$  may also be calculated using the formula:

$$L_{\text{AVGE}} = 10\log[(10^{L_1/10} + 10^{L_2/10} + 10^{L_3/10} + 10^{L_4/10} + \dots + 10^{L_N/10}) \times (1/N)]$$

In this formula the value inside the square brackets [...] corresponds to the average sound intensity.

### Example 1.11

Calculate the average of four decibel levels: 82 dB, 84 dB, 86 dB, 88 dB.

#### Solution

$$L_{\text{AVGE}} = 10\log[(10^{8.2} + 10^{8.4} + 10^{8.6} + 10^{8.8})/4] \\ = 85.6 \text{ dB}$$

Note that this average value, sometimes called the logarithmic average, is different from the arithmetic average of the four levels, which is 85.0 dB. In this case the difference is only 0.6 dB, but it will increase when the range of levels to be averaged is greater. The logarithmic average will always be higher than the arithmetic average.

#### Exercise

Compare the logarithmic and arithmetic averages of the following pairs of levels:

- (i) 80 dB and 90 dB
- (ii) 70 dB and 90 dB.

#### Answer

- (i) Logarithmic average = 87.4 dB; arithmetic average = 85 dB.
- (ii) Logarithmic average = 87.0 dB; arithmetic average = 80 dB.

In these two examples the lower level is making a negligible contribution to the total intensity and so the average of the two values is half that of the larger one, or in decibel terms 3 dB lower.

### Time weighted average sound levels

In the above example, the four different sound levels have been given equal weighting when calculating the average level, because they represent sound levels which are not varying with time but measured at different positions. In a different situation it might be the case that the sound level varies during different periods of the day. In this case it will be necessary to take into account the duration of each sound level when calculating the average over the entire period.

Suppose for example that over an eight-hour period at a particular location the sound levels in Example 1.11 had been measured over different periods, let us say 82 dB for 4 hours, 84 dB for 2 hours, 86 dB for 1.5 hours and 88 dB for 0.5 hours.

In this case the time weighted average level would be obtained by weighting each of the sound intensity values by the appropriate duration, and then dividing by the total duration (8 hours) as follows:

$$L_{\text{AVGE}} = 10 \log \left[ (4 \times 10^{8.2} + 2 \times 10^{8.4} + 1.5 \times 10^{8.6} + 0.5 \times 10^{8.8}) / 8 \right] = 84.1 \text{ dB}$$

This value is the **time average level**, also widely known as the **continuous equivalent noise level**,  $L_{\text{eq}}$ .  $L_{\text{eq}}$  is discussed in much more detail in Chapter 4, together with other methods of measuring time varying noise,

### Exercise

How will the time weighted average change if the durations of the 82 and 88 dB levels are interchanged, i.e. 82 dB for 0.5 hours and 88 dB for 4 hours (the other two components remaining the same)?

### Answer

86.7 dB.

## 1.8 Equal loudness contours and the A-weighting network, dBA

### A-weighted decibels – dBA

The vast majority of noise measurements are of A-weighted decibels, dBA. The A-weighting is the result of an electronic frequency weighting network in the sound level meter which attempts to build the human response to different frequencies into the reading indicated by a sound level meter, so that it will relate to the loudness of the noise. The relationship between the A-weighting scale and loudness is explained in more detail below.

### Loudness

Loudness is a measure of the subjective impression of the magnitude of a sound. It is mainly related not only to the intensity of the sound but also to its frequency. Intensity and frequency are measures of the physical characteristics of the noise, independent of human response, whereas loudness is a measure of human response. Frequency weighting networks were first introduced into sound level meters in the 1950s in an attempt to simulate the equal loudness contours which show how the loudness of pure tones is related to sound pressure level and frequency. The contours were developed from the results of experiments

in which listeners were asked to judge between the loudness of a reference tone at 1000 Hz, set at a constant level (e.g. 60 dB for the 60 dB contour), and a test tone, at another frequency. The level of the test tone is adjusted until the subject judges that the two tones are equally loud. The level and frequency are then plotted, as one point on the contour, and the experiment is then repeated at another frequency. The contours (Figure 1.12) show that human hearing is most sensitive to frequencies in the 1 to 4 kHz range, with a reduced response at the low and very high frequency ends of the spectrum. Thus, for example, a pure tone of say 60 dB at 1 kHz will sound louder than a tone of the same level at say 100 Hz, a measure of the difference being the increase in level of the lower frequency tone needed for it to sound equally as loud as the 1 kHz note (approximately 9 dB in this case).

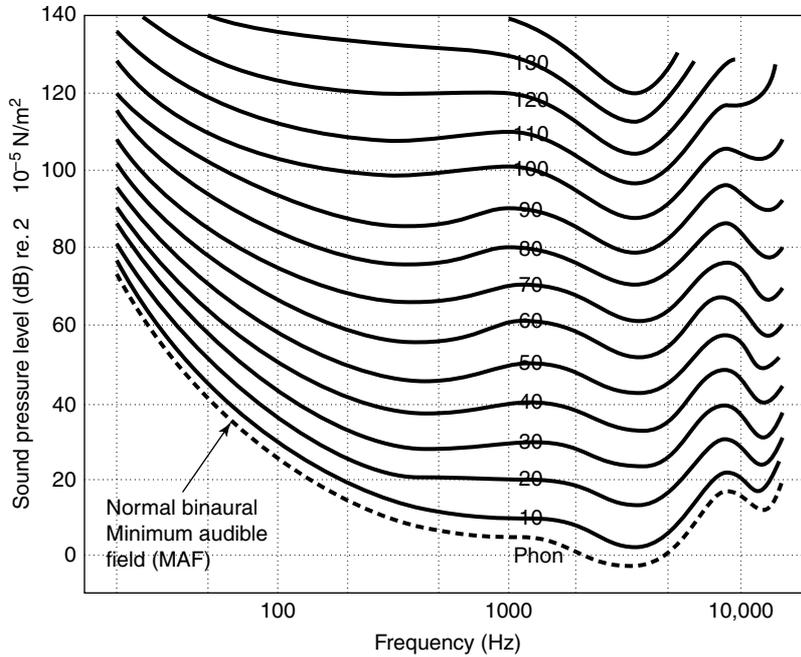
The shape of the equal loudness contours (Figure 1.12) indicate that human frequency response to sound varies with the sound level, with contours of higher level being ‘flatter’ than lower ones. Therefore three different frequency weightings, A, B and C, were originally devised, for use with sounds of low, medium and high levels, with a fourth weighting, D, being added later, specifically for use with aircraft noise (Figure 1.13).

Although the correlation of dBA with loudness is only approximate, and there are more accurate methods of determining loudness, the A-weighted SPL has, despite some limitations, become universally accepted as the simplest way of measuring a noise which does give some correlation with human response. The B- and D-weighting networks are no longer commonly used, but the C weighting is sometimes used, as explained below.

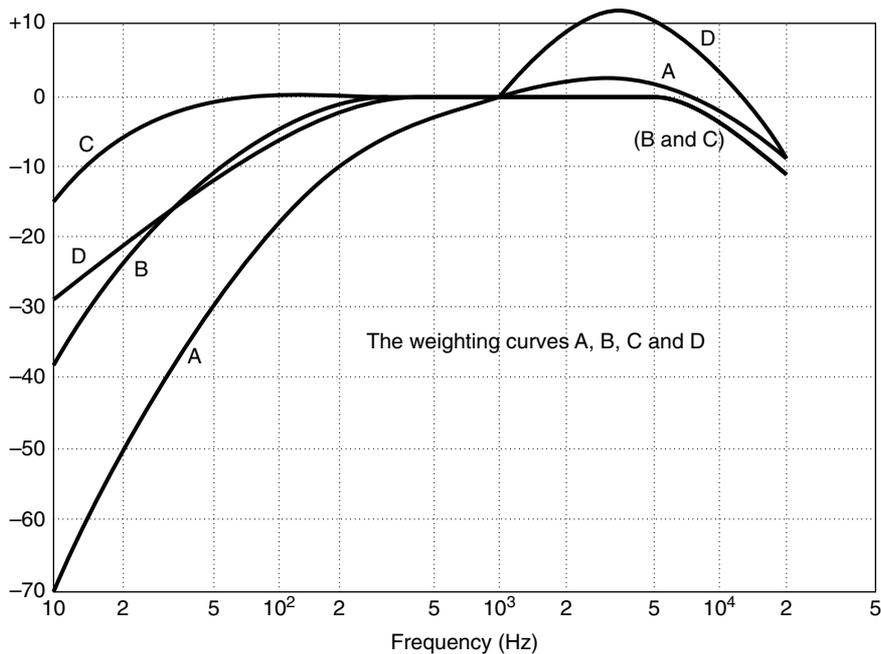
The value of the frequency weightings are specified by British and International Standard BS EN 61672-1: 2003 *Electroacoustics – Sound level meters – Part 1: Specification*.

### Low frequency noise – the C-weighting

The value of the A-weighting is largest at low frequencies, and one of the most common criticisms of the use of dBA is that it may undervalue the disturbing effects of low frequency noise, such as that from commercial and industrial fans, which sometimes only becomes noticeable at night-time, when other sources of noise have ceased to operate. A good indication of the low frequency content in a noise may be obtained by comparing the A-weighted and the unweighted values of sound pressure level. If the difference between these is small then the sound contains mainly medium and high frequencies, whereas a low frequency noise will have a dBA value which is well below the unweighted value. The C-weighting is the ‘flattest’ of the weightings and so dBC



**Figure 1.12** Equal loudness contours



**Figure 1.13** Frequency weighting curves

is sometimes used as an approximation to the un-weighted SPL, and the difference between dBA and dBC used as an indication of low frequency content of a noise. The C-weighting has also become increasingly used for measuring the peak sound pressure of impacts and

impulsive noise, for which the A-weighting is inappropriate because of its effect on the low frequency components of the noise.

In 2003 a new Z (for zero) frequency weighting was introduced into BS EN 61672-1 which defines

**Table 1.3** A and C octave band weighting (to nearest decibel)

Octave band (Hz)	A-weighting	C-weighting
63	-39.4	-1
125	-26.2	0
250	-16.1	0
500	-8.6	0
1000	0	0
2000	1.2	0
4000	1.0	-1
8000	-1.1	-3

**Table 1.4** Some typical sound levels in dBA

140 dBA	Threshold of pain
120 dBA	Jet aircraft at 100 m
110 dBA	
100 dBA	Road drill, loud disco
90 dBA	DIY drill (close to ear), lorry (roadside)
80 dBA	Traffic at a busy roadside
70 dBA	Hair dryer
60 dBA	Washing machine
50 dBA	TV in lounge
40 dBA	Quiet office
30 dBA	Bedroom at night
20 dBA	Broadcasting studio (background noise level)
10 dBA	
0 dBA	Threshold of hearing

performance of sound level meters. Prior to this different sound level meters had their own ways of measuring un-weighted sound pressure levels, variously described as ‘flat’ or ‘unweighted’ or ‘linear’ but these were not defined in any national or international standards – hence the growing use of the C-weighting.

The A-weighted sound pressure level may also be written as  $L_{pA}$  or as  $L_A$ .

Table 1.3 shows the A- and C-weightings, in octave bands, taken from BS EN 61672-1:2003.

Also shown is a table of typical dBA levels (Table 1.4) and the A-weighting curves and equal loudness contours (Figures 1.12 and 1.13).

The study of loudness and how it is measured and estimated is continued in more detail in Chapter 3.

**Calculation of dBA value from octave band sound pressure levels**

Octave band sound pressure levels may be A-weighted and combined to give the A-weighted sound pressure level, in dBA.

**Example 1.12**

Octave band frequency	Octave band SPL	A-weighting	A-weighted SPL
63	103	-26	77
125	96	-16	80
250	89	-9	80
500	82	-3	79
1000	84	0	84
2000	79	+1	80
4000	73	+1	74
8000	69	-1	68

The overall A-weighted level is obtained by combining the individual A-weighted band values, i.e. by combining the levels in the final column above:

$$L_A = 10\log[10^{7.7} + 10^{8.0} + 10^{8.0} + 10^{7.9} + 10^{8.4} + 10^{8.0} + 10^{7.4} + 10^{6.8}] = 88.5 \text{ dB}$$

**Attenuation, in dBA, produced by a noise control measure**

**Example 1.13**

The following information is given below, in octave bands (all in dB):

- Noise level at a particular reception point from machine A
- Noise level at a same reception point from machine B
- Attenuation produced by a particular type of noise enclosure
- The octave band A-weighting values.

Octave band	Machine A	Machine B	Attenuation	A-weighting
63	105	68	4	-26
125	107	79	9	-16
250	99	82	15	-9
500	94	87	21	-3
1000	91	92	24	0
2000	87	96	30	+1
4000	82	89	27	+1
8000	79	81	26	-1

**Exercise**

Calculate:

- the overall noise levels for machines A and B, in both dBA and dB(LIN)
- the noise levels in each case after the machines have been enclosed
- the attenuation, in dBA produced by the enclosure for each machine.

What conclusions can you draw about the difference between the dBA and dB(LIN) values, and between the amount of attenuation provided, in each case?

**Answer**

- Machine A without attenuation: 110 dB(LIN) and 98 dBA; after attenuation: 84 dBA.
- Machine B without attenuation: 99 dB(LIN) and 99 dBA; after attenuation: 72 dBA.
- Attenuation of enclosure: for machine A: 14 dBA, for machine B: 27 dBA

Machine A produces mostly low frequency noise, and therefore the dB(LIN) value is much higher than the dBA, whereas for machine B which produces mostly high frequency noise the two values are similar. Because the enclosure produces more attenuation at higher than at lower frequencies the dBA reduction it provides is greater for machine B. The conclusion is that it is not possible to give a single figure dBA reduction for an enclosure (or any other noise control device) because the reduction will depend on the spectrum of the noise being treated.

## 1.9 Types of elastic waves in solids and fluids

There are a wide variety of different types of elastic waves which can be transmitted through matter. In an unbounded, i.e. infinite, expanse of solid two types of waves are possible, **compressive waves** (sometimes called **P waves**) and **shear waves** (called **S-waves**). These two types of waves are important in transmitting vibration through the ground, e.g. from trains road traffic, construction and demolition activities, and are also responsible for transmitting shocks from earthquakes. In a compressive wave the to and fro motion of the particles of the medium are in the same direction as the direction of travel of the wave itself – such waves are called **longitudinal waves**. In the case of shear waves the particle vibration is perpendicular to the direction of wave travel – this type of wave is an example of a **transverse wave**.

If the solid is bounded, or finite, as in the case of a beam, rod or plate for example, then several other types of waves are possible, such as **flexural or bending waves**, or **torsional waves**. These types of waves are a combination of compressive and shear waves. Yet another type of elastic wave, called **surface waves**, can occur only on and close to the surfaces of solids and liquids. The ripples on the surface of a pond and waves on the ocean or at the seaside are obvious examples, but a type of surface wave called a **Rayleigh wave** is also important in transmitting vibration through the ground over relatively short distances, and

very high frequency ultrasonic surface waves in solids are important in the electronics and telecommunications industries. Bending, or flexural, waves in plates are examples of transverse waves, as are elastic waves on a wire or string, and all types of surface waves.

Shear forces applied to a fluid, i.e. a liquid or a gas, cause flow to occur. Thus unlike solids, liquids and gases cannot transmit shear waves, and so the only type of elastic waves that can travel through the bulk of a fluid are compressive waves, and therefore sound waves in air are of this type, and are longitudinal waves.

### The velocity of elastic waves in fluids and solids

An outcome of solving the wave equation for elastic waves travelling in an elastic medium is that the speed or velocity at which such waves travel through the medium is given by the formula:

$$C = \sqrt{(K/\rho)}$$

where  $K$  is the elastic modulus of the medium (in  $\text{N/m}^2$  or  $\text{Nm}^{-2}$ ), and  $\rho$  is the density of the medium (in  $\text{kg/m}^3$  or  $\text{kgm}^{-3}$ ).

$$\begin{aligned} \text{Elastic modulus} &= \text{stress/strain} \\ &= (\text{force per unit area}) / \\ &\quad (\text{fractional change in deformation}) \end{aligned}$$

Note that strain is a dimensionless quantity (i.e. length/length), and so the dimensions of the elastic modulus are the same as those of stress, or pressure, i.e.  $\text{N/m}^2$  or  $\text{Nm}^{-2}$ .

There are many different elastic moduli, for various types of elastic deformation, e.g. compression, shear, torsion, bending etc. and so a corresponding number of different types of elastic waves – compressional, shear, bending etc.

A solid is able to withstand both shear and compressional forces and so all possible types of waves may be transmitted. A fluid (liquid or gas) can only withstand compressive forces, but not shear forces.

Therefore the only type of elastic waves that can travel in a fluid are compressional waves – hence sound waves in water and air are compressional waves and are **longitudinal**. Solids can, in addition to compressive waves, also transmit shear and bending waves – these are **transverse waves**.

For compression waves in a fluid the appropriate elastic modulus is the **bulk modulus**, where the appropriate form of strain is the fractional change in volume of a fluid element subjected to a uniform fluid pressure (acting equally in all directions).

In a gas, such as air, there are in principle two possible values for the bulk modulus, depending upon whether or

not the heat flow changes caused by the compressions and rarefactions of the sound wave can take place quickly enough to follow (i.e. keep in step with) these pressure fluctuations, i.e. whether or not the changes are isothermal or adiabatic.

In fact the heat flow cannot keep pace with the pressure fluctuations, i.e. the compressions and rarefactions take place under adiabatic conditions, and for this situation the bulk modulus =  $\gamma P$ , where  $\gamma$  is a constant for a particular gas (1.4 for air) and  $P$  is the atmospheric pressure.

Hence speed of sound in air:

$$c = \sqrt{(\gamma P/\rho)}$$

From this it can be seen that at a given temperature the velocity of sound in gases with very low density, e.g. helium and hydrogen, is higher than for denser gases such as carbon dioxide. In fact, a change of a factor of 4 in the density would lead to a doubling of the velocity of sound. The velocity of sound for some different gases is shown below:

Gas	Sound velocity at 0°C, m/s
Oxygen	317.2
Air	331.0
Hydrogen	1269.0
Carbon dioxide	258.0

### Example 1.14

Calculate the velocity of sound at 20° Celsius.

#### Solution

At 20° Celsius the density of air =  $1.2 \text{ kgm}^{-3}$  and atmospheric pressure =  $1.01 \times 10^5 \text{ Pa}$ , or 101 kPa.

Therefore:

$$c = \sqrt{(1.4 \times 101,000/1.2)}$$

$$= 343.3 \text{ m/s or ms}^{-1}$$

For an ideal gas, pressure  $P$ , temperature  $T$  and volume  $V$  are related by the ideal gas equation  $PV = RT$  (where  $R$  = universal gas constant).

Therefore because of the factor  $P/\rho$  in the above formula, the speed of sound is independent of atmospheric pressure.

The speed of sound in a gas does, however, depend upon temperature according to:

$$C = \sqrt{(\gamma RT)}$$

And so the sound speed is proportional to the absolute temperature, in kelvins (K).

### Example 1.15

The speed of sound in air is  $343.3 \text{ ms}^{-1}$  at a temperature of 20° Celsius. What will it be at 0° Celsius?

#### Solution

$$0^\circ \text{ Celsius} = 273 + 0 = 273 \text{ kelvins}$$

$$20^\circ \text{ Celsius} = 273 + 20 = 293 \text{ kelvins}$$

Now since  $c$  is proportional to  $\sqrt{T}$ , if  $c_1$  and  $c_2$  are the sound speeds at temperatures  $T_1$  and  $T_2$  respectively then:

$$c_2/c_1 = \sqrt{(T_2/T_1)}$$

In this example:

$$c_1 = 343.3 \text{ ms}^{-1}, T_1 = 293 \text{ K} \quad \text{and} \quad T_2 = 273 \text{ K.}$$

Therefore at 0° Celsius:

$$c_2 = 343.3 \times \sqrt{(273/293)} = 331.4 \text{ ms}^{-1}.$$

### Dispersive and non-dispersive waves

The velocity of sound in air does not vary with the frequency of the sound – a fact of importance to theatre and concert goers – since it means that audience members in both the front and rear seats will all receive the same mixture of different frequencies issuing from the performers on stage. In this respect air is a non-dispersive medium for sound waves. Some types of elastic waves are, however, dispersive, most notably flexural or bending waves in solid plates in beams, where the wave speed increases with frequency. These types of waves are important for the transmission of sound in buildings and other structures and will be discussed again in [Chapter 6](#).

Note also that the speed of sound in air is independent of the amplitude of the sound wave, i.e. of the sound pressure level, for small amplitudes only. (This is called the linear acoustics approximation.)

### 1.10 Absorption and attenuation of sound

Sound absorption is a process whereby sound energy is lost from a sound wave and converted to heat as a result of some sort of frictional process. Since the energy content of sound waves is usually very small the actual temperature rises which result from sound absorption processes are usually negligible. The sound absorption can usually be thought of in terms of some frictional process which occurs between vibrating molecules which are transmitting the sound. Frictional processes also occur in vibrating bodies (e.g. vibrating panels) but in these cases the energy loss is usually referred to as ‘damping’. These two terms (damping and absorption) have much in common.

Absorption processes can occur within the sound transmitting medium, or at the interface with another medium, during reflection and scattering. Absorption which occurs during reflection at surfaces is discussed in detail in [Chapter 5](#). All of the sound absorption mechanisms which occur in air are frequency dependent, the amount of absorption being proportional to the square of the frequency of the sound.

One of the causes of sound absorption in air, molecular absorption, is due to the vibration and rotation of the oxygen and water vapour molecules in the air, and is therefore dependent on the relative humidity of the air as well as on the air temperature.

### Absorption and attenuation

It is important to distinguish between the terms absorption and attenuation. Attenuation simply means the reduction of sound level by whatever means, and not just by the process of absorption, i.e. conversion to heat through frictional processes. Attenuation may arise because of the sound spreading with distance, or as a result of the effect of scattering, diffraction, interference or refraction, and also as a result of sound insulation, isolation, the use of enclosures, barriers, silencers, as well as because of sound absorption.

### Diffraction, reflection, interference and refraction

All waves, including sound waves demonstrate the following behaviour: **diffraction**, **reflection**, **interference** and **refraction**. Some of this material will be revision of schooldays for some, and new for others. Alternative and additional explanations may be found in physics textbooks and there is also much helpful information available on many websites.

## 1.11 Diffraction

Diffraction is about the interaction between sound waves and solid objects. There are two different aspects to consider:

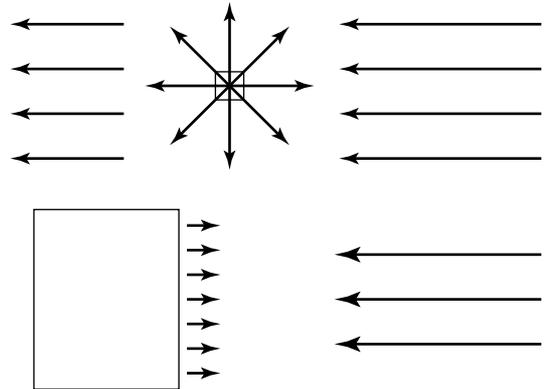
- What happens when a sound wave meets an obstacle in its path, i.e. to what extent is the sound scattered by, or bent around, or reflected by the object?
- What happens when the object is vibrating? How are the resulting sound waves radiated by the vibrating object? In other words, what is the resulting pattern of the sound radiated? How directional is the radiation?

Therefore diffraction of sound is important in determining:

- the effectiveness of noise barriers (limited by the extent to which sound ‘bends’ around the edges of the barrier)
- the directionality of noise sources, including the human voice
- the directionality of microphones, and of human hearing
- scattering of sound by objects, including by microphones, sound level meters, and by the human head and body.

Of overriding importance in all diffraction issues is the ratio of the size of the obstacle ( $D$ ) to the wavelength of the sound waves ( $\lambda$ ):  $D/\lambda$ .

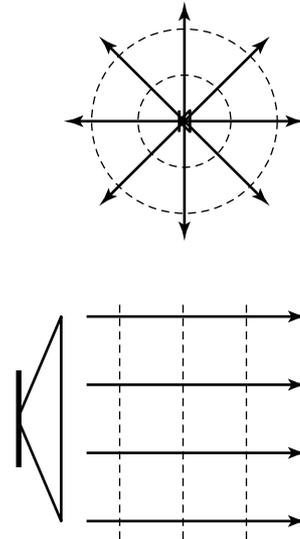
It is easiest to consider first of all the two extreme cases (see Figure 1.14):



(a) Waves meeting an object

Upper:  $\lambda \gg D$ , i.e. large  $\lambda$  (low frequency) waves and/or small object: the waves are scattered by the object (more or less equally in all directions) but otherwise the waves travel on undisturbed by the object.

Lower:  $D \gg \lambda$ , i.e. large object and small  $\lambda$  (high frequencies). The waves are reflected at the surface of the object. There is no bending of the waves around the object and there is a quiet shadow zone behind the object.



(b) Vibrating source radiating waves

Upper:  $\lambda \gg D$ , i.e. small loudspeaker radiating large  $\lambda$  (low frequency) waves: the waves are radiated equally in all directions.

Lower:  $D \gg \lambda$ , i.e. large loudspeaker radiating small  $\lambda$  (high frequencies). The sound radiation is highly directional (approximating to plane waves).

**Figure 1.14** The two extreme cases of diffraction

- If  $D/\lambda \gg 1$ , i.e. obstacle much larger than wavelength, then the object casts a sharp shadow, regions

‘behind’ the object are effectively shielded from the sound, and sound is reflected from the surface of the object facing the sound waves. This is what happens with light waves. Sound sources which are large compared to the wavelength they are radiating will radiate (approximately) plane waves travelling in one direction only.

- If  $D/\lambda \ll 1$ , i.e. obstacle much smaller than wavelength, then the waves are almost completely unaffected by the object. There may be some scattering of the waves, but there is little or no shielding effect. Sound sources which are small compared to the wavelength will radiate (approximately) spherical waves, i.e. equally in all directions.
- If  $D \approx 1$  the interaction between the object and the waves is much more complex, and the theory is the subject of many quite difficult equations in undergraduate and postgraduate physics and acoustics textbooks.

**The Huygens-Fresnel theory of secondary wavelets**

Why does diffraction occur? For example why does sound bend around corners? An insight can be gained by considering a theory of secondary wavelets, put forward by the Dutch physicist Huygens. He proposed that every point on a wavefront acts as a point source of new identical secondary wavelets radiating spherically in the direction the wave travel (see Figure 1.15a). (A later modification to this theory was proposed by Fresnel to explain why the wavelets did not radiate backwards towards the original source.) We can find out the position and shape of the new wavefront, a little while later, by combining the contributions from all the different secondary wavelets.

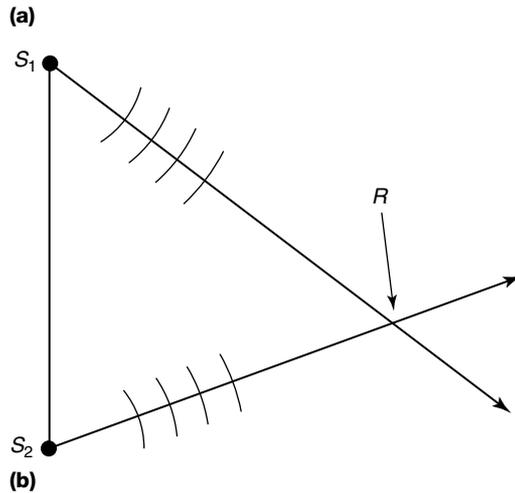
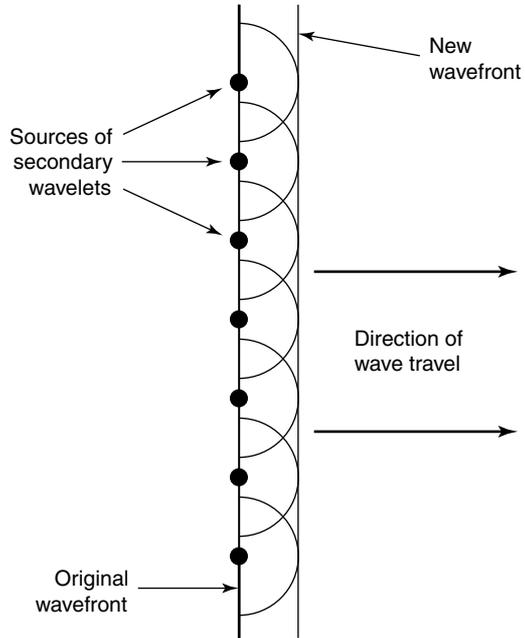
To explain this in a little more detail consider just two sources of secondary waves,  $S_1$  and  $S_2$  (see Figure 1.15b) radiating waves towards a receiver at position  $R$ . The waves leaving the two sources are identical in amplitude and are in phase. However, the sound pressures they produce ( $p_1$  and  $p_2$ ) when they arrive at  $R$  will be different both in amplitude (because of the inverse square law and spherical spreading) and in phase because they have travelled different distances to the receiver position.

If the difference in the two path lengths is  $d$  then the phase difference is given by:

$$\text{Phase difference, } \varphi = 2\pi d/\lambda$$

(on the basis that a path difference of one wavelength will result in a phase change of  $2\pi$ ).

The total sound pressure at  $R$  from the wavelets from the two sources will be, according to the principle of superposition,



**Figure 1.15** Illustrating the application of the Huygens-Fresnel theory

$$p_{\text{TOTAL}} = p_1 + p_2$$

This will be an equation rather like:

$$p_{\text{TOTAL}} = (A/r_1) \sin(\omega t - kr_1) + (A/r_2) \sin(\omega t - kr_2)$$

where  $A$  is the pressure amplitude of the wavelets at the sources,  $r_1$  and  $r_2$  their path lengths to the receive point  $R$ , and  $\omega$  the angular frequency ( $= 2\pi f$ ).

To find out the new wavefront we have to add in the contributions from all the other point sources on the existing wavefront (in theory an infinite number of them), and

then we have to repeat the exercise for all other (infinite number of) possible reception points – quite a lot of mathematics. If we were to do all this maths we would come up with the unsurprising result that the new wavefront is another plane wavefront, just a little bit further forward than the original one, and so on – a result we know intuitively by watching the movement of ripples on the surface of water. We could go through an exactly similar analysis to show that the wavefronts on the surface of a spherical wave will combine to form a new slightly advanced spherical wave, again a result that we know from everyday experience.

What this tells us is that in order to make a completely new plane wavefront contributions are needed from all of the entire original wavefront. In cases where the wave encounters an obstacle, or passes through an opening, it is the wavelets at the edge of the wavefront that spread out into the shadow area, causing the effect of the sound ‘spreading around the corner’ (see Figure 1.16).

## 1.12 Reflection

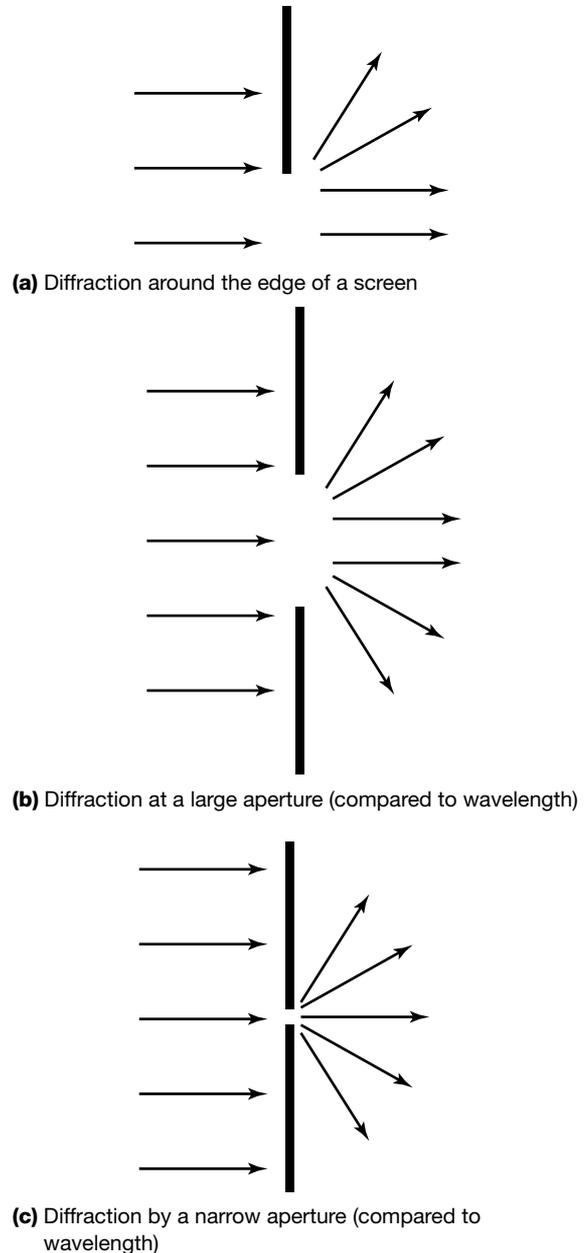
This is really a special case of diffraction. When sound waves meet a surface (of different acoustic impedance) which is large compared to the wavelength then sound is reflected (see Figure 1.17). If the reflecting surface is plane and smooth then **specular reflection** occurs, i.e. angle of incidence equals angle of reflection (as with light and a glass mirror, and balls on a billiard table). If the surface is rough then **diffuse reflection** occurs, with sound being scattered in all directions (as with light and frosted glass).

As with the optical case, concave surfaces can cause sound to be focused and convex surfaces cause sound to be dispersed.

A nearby wall or floor can reflect sound and increase sound pressure levels compared to positions at similar distances from the source but well away from reflecting surfaces. Very close to the surface (just a few centimetres) differences of up to 6 dB may be observed, because the incident and reflected waves may be considered to be coherent. At distances of about 1 m away, when the incident and reflected waves are considered to be incoherent (i.e. uncorrelated), increases of about 3 dB compared to the free field situation may be expected; and it is considered that sound levels further than about 3 m from the wall may not be significantly influenced by the reflections.

When we are holding a sound level meter, reflections from our body can affect the sound level at the microphone and so we should always hold the meter at arm’s length away from the body, or use a tripod, in order to minimize these effects.

For the same reasons (reflections from the head and body) readings from a dosimeter worn by an employee

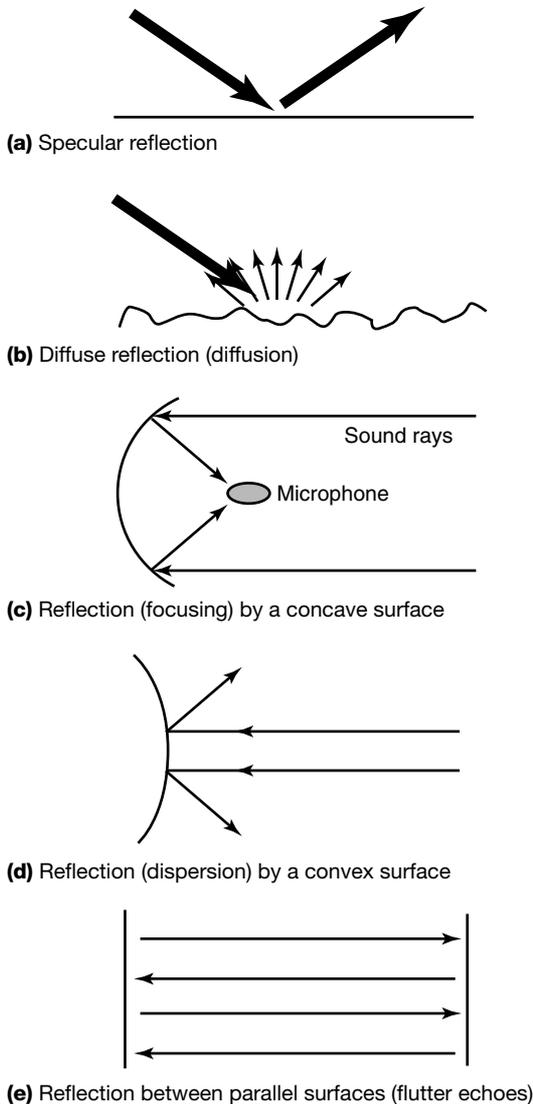


**Figure 1.16** Diffraction around edges and through openings

working close to a noisy machine may never quite agree with those from a sound level meter at similar distances from the machine: difference of about 2 dB are sometimes observed.

Note that smaller surfaces and obstacles will scatter (or diffract) the sound in a more complex manner.

An interesting example of the application of sound reflection, prior to the development of radar in



**Figure 1.17** Illustrating some aspects of reflection of sound waves from surfaces

the Second World War, was the use of large concrete ‘sound mirrors’ at Dungeness in Kent and other places along the UK coast, to try to give advance warning of the approach of enemy aircraft. A number of parabolic reflectors, tens of metres in diameter were constructed, with microphones placed at their focal points. (Further information is available on several websites in response to a search under ‘sound mirrors at Dungeness’ or similar.)

### Partial reflection

Sound waves are reflected when they arrive at an interface with a medium with a different acoustic impedance, but some sound energy will also be transmitted into the

second medium. The greater the change in specific acoustic impedance, the greater is the fraction of sound energy which is reflected. Air has a specific acoustic impedance of about  $415 \text{ kgm/s}^2$  at room temperature and water has a value of about 1.5 million  $\text{kgm/s}^2$ . Therefore, in view of this very large difference, we should expect that almost all of the sound energy would be reflected at an air–water interface, and only a small proportion of energy will be transmitted.

The fraction of sound energy,  $R$ , which is reflected at an interface between two media with acoustic impedances  $z_1$  and  $z_2$  is given by:

$$R = [(z_1 - z_2)/(z_1 + z_2)]^2$$

For an air–water interface the fraction of sound energy reflected will be 0.998899272. Therefore the fraction which is transmitted is:

$$(1 - 0.998899272)$$

In decibels this means that when a sound wave in air arrives at an interface with water the level of the sound wave transmitted into the water will be almost 30 dB below the level in air (and vice versa if the direction of wave travel is reversed). This is relevant to the role of the middle ear in facilitating the transmission of sound from the outer ear to the inner ear, as will be discussed in [Chapter 3](#).

### 1.13 Interference and standing waves

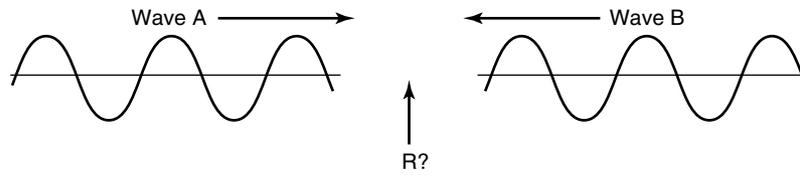
The phenomenon of interference describes what happens when two or more sound waves meet (see [Figure 1.18](#)).

The situation is governed by the **principle of superposition**, which states that the resulting total disturbance at any point is the algebraic sum of the disturbance caused by each of the waves at that point at that moment in time. (The ‘disturbance’ may be specified as an acoustic pressure, particle velocity, displacement or acceleration, and the term algebraic recognizes that these disturbances may be positive or negative.)

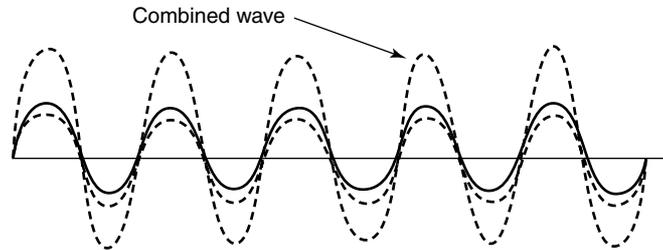
In order that waves may display interference they must be **coherent**, i.e. their waveforms must be identical in shape (i.e. in time profile), although they may be different in amplitude.

In the vast majority of situations, e.g. in the case of waves from two different noise sources, the waves will not have similar waveforms, and will not be coherent.

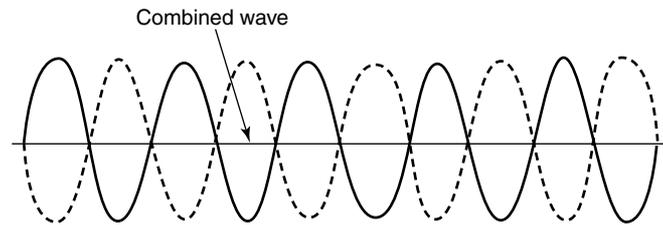
The most likely circumstance in which the waves would be coherent is if they are both pure tones derived from the same source so that they have the same sinusoidal waveform and the same frequency. If they also have the same or similar amplitudes it is possible that at a particular moment at a particular position the two waves could either cancel each other out (called **destructive**



(a) What happens when waves A and B meet at point R?



(b) Completely constructive interference, when waves A and B are in phase



(c) Completely destructive interference, when waves A and B are out of phase

**Figure 1.18** Illustrating the problem of interference

**interference**) or reinforce each other with doubled amplitude (**constructive interference**). This could happen, for example, at a point between two loudspeakers facing each other, with both being supplied by exactly the same pure tone signal. Under these circumstances, however, the net effect would simply be a 3 dB increase in level because as the waves from each source travel onwards the pattern of interference at a particular point would be continually changing from constructive to destructive and back again.

### Stationary or standing waves

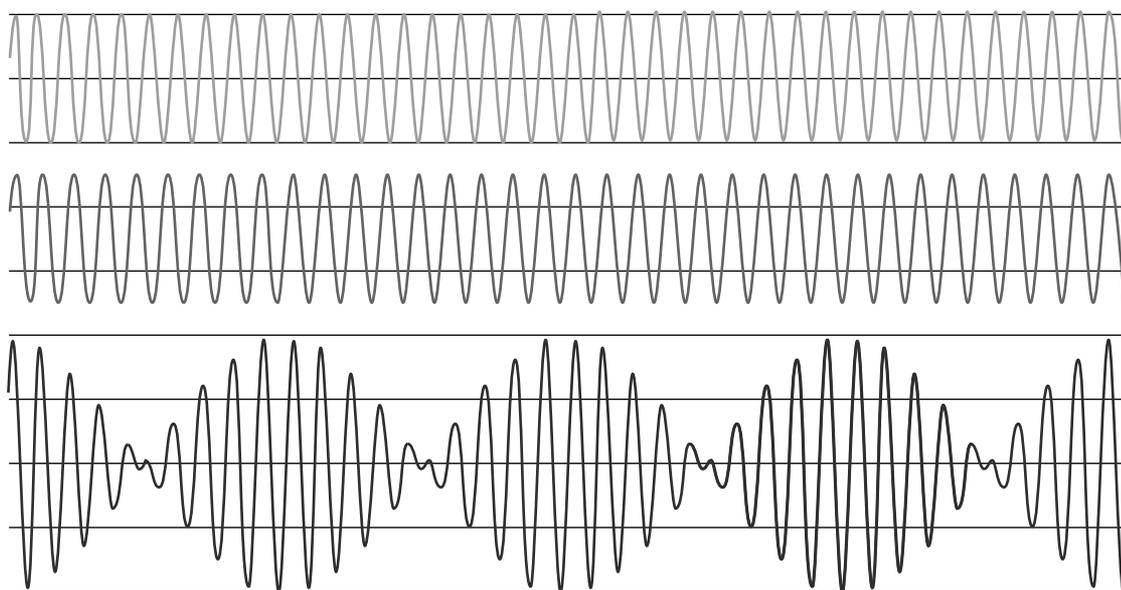
Reflection of sound produces waves which are coherent and which can interfere constructively or destructively with the original incident wave. Under certain circumstances **stationary** or **standing waves** are produced. These are steady patterns of interference characterized by large variations in amplitude with position, so that there will be alternately positions where the amplitude is a minimum, called **nodes**, as a result of destructive interference, and positions of maximum amplitude (called **antinodes**) resulting from a constructive interference.

### Beats

Beats are amplitude modulated pure tones where the amplitude of a higher frequency appears to be modulated by a lower frequency, for example a frequency of 100 Hz whose amplitude is modulated at a frequency 10 Hz. This can arise from a combination of two tones of slightly different frequency but equal or nearly equal similar amplitude. Consider for example two tones of 100 Hz and 90 Hz with equal amplitudes. They start off in phase but after five cycles they are out of phase, and cancel each other out (destructive interference), and after a further five cycles are back in phase, then out of phase five cycles later, and so on. This is shown in [Figure 1.19](#).

As shown earlier the sound pressure variation with time of two pure tones of the same amplitude  $A$  and frequencies  $f_1$  and  $f_2$  may be described by the equations  $p_1 = A \sin(2\pi f_1 t)$  and  $p_2 = A \sin(2\pi f_2 t)$ . Their combination,  $p_1 + p_2$ , is given by:

$$\begin{aligned} p_1 + p_2 &= A \sin(2\pi f_1 t) + A \sin(2\pi f_2 t) \\ &= 2A \cos[2\pi\{(f_1 - f_2)/2\}t] \sin[2\pi\{(f_1 + f_2)/2\}t] \end{aligned}$$



**Figure 1.19** Illustrating beats formed by combination of 100 Hz (top) and 90 Hz (centre) pure tone over 40 cycles of the 100 Hz tone

The expression on the second line, obtained using a standard trigonometric relationship, represents a pure tone of frequency  $(f_1 + f_2)$  modulated by a tone of frequency  $(f_1 - f_2)$ .

This shows that the beat frequency  $f_B$  is the difference between the two component frequencies,  $f_1$  and  $f_2$ :

$$f_B = f_1 - f_2$$

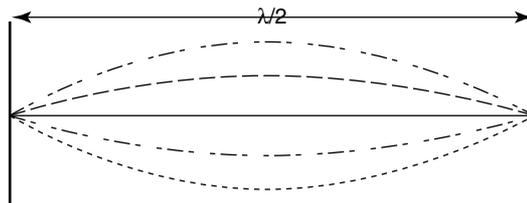
Beats can occur between different notes in some musical compositions and are used in the tuning of musical instruments. They may also be produced by rotating machines such as fans or motors which produce tones related to their rotation speed. If there are two such machines, nominally identical and rotating at the same speeds, beats can occur if in practice the two machines are running at slightly different speeds.

### One-dimensional standing waves

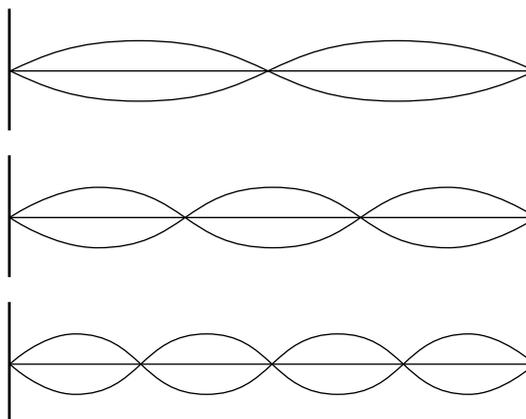
Standing waves in strings, pipes and rods are important in the operation of many musical instruments.

#### Standing waves in strings

The simplest case to consider is that of a string stretched between two fixed ends (see Figure 1.20). When a wave is created in the string it vibrates between the ends as a result of reflection. Interference will occur continuously



**(a)** Position of the string during one cycle of vibration performing its first mode at a frequency at which the length of the string equals one half a wavelength.



**(b)** Second, third and fourth modes of vibration of a string

**Figure 1.20** Mode shapes of standing waves in strings

between the waves travelling in the opposite directions, and in general these patterns of interference will be continually changing. But there are certain special frequencies at which steady, or standing, waves are set up. The lowest frequency, called the **fundamental**, or **first harmonic**, occurs at the frequency at which the length of the string is equal to one half of a wavelength. Under these circumstances every point on the string is vibrating in phase, but the amplitude increases from zero at the ends to a maximum in the middle. This is also called the first mode of vibration of the string, and, like every other mode, is characterized by its frequency and also by its mode shape which describes the shape of the vibrating string. (Note that you may also come across the alternative terms **eigenfunction** (mode shape) and **eigenfrequency** (mode frequency) in technical literature.)

The next mode of vibration occurs at a frequency for which the wavelength in the string will be one wavelength (or two half wavelengths). The mode shape for this mode contains two maxima (or antinodes)  $\frac{1}{4}$  and  $\frac{3}{4}$  along the length of the string, and three minima, or nodes, one at each end and one in the middle. The frequency of this mode, called the second harmonic, is twice that of that of the first mode.

Note that there will always be nodes at the two ends, because the string is fixed or clamped here, and so no motion is possible. This is an example of what is called a 'boundary condition' which acts as a constraint on what standing wave patterns (modes) are allowed for a string of this particular length and wave speed.

There are an infinite number of modes of vibration of such a string. The common link is that in each case the mode frequency is such that the length of the string is equal to a whole number of half wavelengths. For a string of length  $L$  along which waves travel at speed  $c$  and wavelength  $\lambda$  (remembering from earlier in the chapter that  $c = f\lambda$ ):

Mode number	Wavelength ( $\lambda$ )	Frequency ( $= c/\lambda$ )
1	$1(2L)$	$1(c/2L)$
2	$(1/2)(2L)$	$2(c/2L)$
3	$(1/3)(2L)$	$3(c/2L)$
4	$(1/4)(2L)$	$4(c/2L)$
$\vdots$	$\vdots$	$\vdots$
$n$	$(1/n)(2L)$	$n(c/2L)$

It can be seen that in this case the frequencies are in the ratio of 1:2:3: ... : $n$ , etc.

### Exercise

Compare the vibration (amplitudes and phases) at the same positions along a string: (a) when progressive

waves only are travelling along the string (assume for example that the string is very long so that there are no reflected waves); and (b) when there are standing waves (as described above) present. Assume that the vibration in both cases is of a single frequency (sinusoidal waves) and that there is no damping or attenuation of the waves.

### Answer

In the case of progressive waves the amplitude is the same at all positions on the string (because there is no attenuation) but there is a phase difference between the vibration cycles of adjacent parts of the string (such that the phase difference is  $180^\circ$ , or half a cycle) for positions half a wavelength apart and  $360^\circ$ , or one whole cycle, for positions one wavelength apart (see Figures 1.3 and 1.4 earlier).

For the standing waves all points on the string which lie in between two nodes always vibrate in phase with each other (but with different amplitudes), and they are out of phase, by half a cycle, with the points which are located between the adjacent pair of nodes. Unlike the progressive wave case the amplitude of vibration varies with position according to the mode shape from zero at nodes to a maximum at antinodes (see Figure 1.20).

When we pluck the string the resulting pattern of vibration will be made up of a combination of these different modes, and the frequency content of the resulting sound will be a mixture of these special natural frequencies, and of these frequencies only. In other words other frequencies are not allowed or do not occur.

### Why is this?

The answer is that it is only at these frequencies (i.e. when an exact number of half wavelengths fit in exactly between the ends of the string) that the pattern of interference that occurs at different points along the string is constant, leading to the observed standing wave patterns of vibration. At any other frequency the pattern of interference at any point on the string is constantly changing, and averaged over time so no interference effects are observed.

Although the note arising from the plucking of the string will always contain the same mixture of frequencies (the line frequency spectrum discussed briefly earlier in the chapter, see Figure 1.7) the exact nature of the mix, i.e. the relative amounts of different frequencies (and the shape of the frequency spectrum) will change, depending on how and where the note is plucked. In the case of the

same note played on different musical instruments such as a violin, guitar or piano for example, the mix of frequencies and the quality of the note reaching the ear (called the ‘timbre’) will also depend on other factors such as how effectively the different frequencies of vibration are converted to airborne sound, depending on the mechanical structure of the instrument. This is why the same musical note will have a different quality when played on different instruments.

### Exercise

If the string is held, or clamped, at its centre point (i.e. as well as at both ends) what difference will this make to the vibration of the string when it is plucked?

### Answer

The answer is that those modes which have a maximum amplitude at the centre will be suppressed, i.e. will not occur (because movement of the string at its centre point is now no longer allowed). Therefore the note will contain a mixture of only the even numbered harmonics (2, 4, 6 etc.) for which there is a node at the centre point of the string, and which are therefore unaffected by the central point being clamped.

What happens if instead of being plucked the string is subjected to a continuous vibration?

This obviously depends on the frequency or frequencies in the applied vibration. The string will respond very selectively producing large amplitudes that occur at its own natural frequencies and very little response at other frequencies. This is known as the phenomenon of resonance, and the natural frequencies are also known as the resonance frequencies of the string.

### Standing waves in pipes

Sound waves in a pipe are reflected at the end of the pipe if it is terminated by a rigid cap at the end, called a closed end. Reflection also occurs if the end of the pipe is open. This is because the acoustic impedance presented to the sound waves by the air inside the pipe is different from that of the open air just outside the end of the pipe. The change of impedance causes the waves to be reflected back down the pipe.

The difference in acoustic impedance depends on the diameter of the pipe compared to the wavelength of the sound, so that for a given wavelength the change in impedance (compared with open air), and therefore the fraction of sound reflected at the open end increases as the pipe diameter gets smaller.

### Exercise

How might you modify the end of the pipe in order to reduce reflection and increase the amount of sound radiated from the end of the pipe?

### Answer

The answer is to flare (i.e. gradually increase the diameter towards) the end of the pipe (like a trumpet).

### Change of phase on reflection

There is an important difference between the reflection at the open and closed ends of a pipe. At the closed end a sound wave is reflected without any change of phase, so that a compression is reflected as a compression (and a rarefaction as a rarefaction), but a change of phase of half a cycle occurs at the open end, where a compression will be reflected as a rarefaction, and vice versa.

### Acoustic particle displacement and acoustic pressure amplitudes

The standing waves in strings were described in terms of the amplitude of vibration of the string at various points along its length. Standing waves of sound in pipes may be described either in terms of the amplitudes of vibration of the air particles in the sound wave (i.e. similar to the string case) or in terms of the acoustic pressure at different positions along the pipe. These two quantities will be half a cycle out of phase with each other. This means that at a closed end of a pipe the particle displacement will always be a minimum (zero, as for the string) but the acoustic pressure amplitude will be a maximum, and conversely at an open end the particle movement will be a maximum but the acoustic pressure will be zero.

### The two cases (open–open and open–closed pipes)

There are two possible cases to consider (i.e. two sets of boundary conditions): pipes which are open at both ends, and pipes which are open at one end but closed at the other (see [Figure 1.21](#)).

For a pipe open at both ends the standing wave pattern must meet the boundary condition that the particle displacement is a maximum at both ends. The lowest frequency at which this can occur is when the length of the pipe is equal to one half wavelength, and the mode shape has a particle displacement minimum (node) half way along the pipe. The next mode is when the length of the pipe equals one wavelength (two half wavelengths) and the frequency is double that of the first mode. It can be seen that for the open–open pipe the natural frequencies are in the same 1:2:3 ratio as for the string, but the mode shapes are different.