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**BERTRAND RUSSELL
MEMORIAL VOLUME**

EDITED BY GEORGE W ROBERTS



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MUIRHEAD LIBRARY OF PHILOSOPHY

An admirable statement of the aims of the Library of Philosophy was provided by the first editor, the late Professor J. H. Muirhead, in his description of the original programme printed in Erdmann's *History of Philosophy* under the date 1890. This was slightly modified in subsequent volumes to take the form of the following statement.

'The Muirhead Library of Philosophy was designed as a contribution to the History of Modern Philosophy under the heads: first of Different Schools of Thought – Sensationalist, Realist, Idealist, Intuitivist; secondly of different Subjects – Psychology, Ethics, Aesthetics, Political Philosophy, Theology. While much had been done in England in tracing the course of evolution in nature, history, economics, morals and religion, little had been done in tracing the development of thought on these subjects. Yet the "evolution of opinion is part of the whole evolution".'

'By the co-operation of different writers in carrying out this plan it was hoped that a thoroughness and completeness of treatment, otherwise unattainable, might be secured. It was believed also that from writers mainly British and American fuller consideration of English Philosophy than it had hitherto received might be looked for. In the earlier series of books containing, among others, Bosanquet's *History of Aesthetic*, Pfleiderer's *Rational Theology since Kant*, Albee's *History of English Utilitarianism*, Bonar's *Philosophy and Political Economy*, Brett's *History of Psychology*, Ritchie's *Natural Rights*, these objects were to a large extent effected.

'In the meantime original work of a high order was being produced both in England and America by such writers as Bradley, Stout, Bertrand Russell, Baldwin, Urban, Montague and others, and a new interest in foreign works, German, French and Italian, which had either become classical or were attracting public attention, had developed. The scope of the Library thus became extended into something more international, and it is entering on the fifth decade of its existence in the hope that it may contribute to that mutual understanding between countries which is so pressing a need of the present time.'

The need which Professor Muirhead stressed is no less pressing today, and few will deny that philosophy has much to do with enabling us to meet it, although no one, least of all Muirhead himself, would regard that as the sole, or even the main, object of philosophy. As Professor Muirhead continues to lend the distinction of his name to the Library of Philosophy it seemed not inappropriate to allow him to recall us to these aims in his own words. The emphasis on the history of thought also seemed to me very timely: and the number of important works promised for the Library in the very near future augurs well for the continued fulfilment, in this and other ways, of the expectations of the original editor.

H. D. LEWIS

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PREFACE

This volume attempts to assess some of the achievements of Bertrand Russell in philosophy, logic and mathematics, ethics and politics. The contributions range from those more closely concerned with his work and writings to those concerned with Russell's background, his relations to his contemporaries and successors, and his subjects in a way that sheds light on Russell's efforts and accomplishments, his attempts and failures and on the matters with which he was concerned. To say that these studies are independent-minded and critical of Russell, or that they range freely over the topics with which Russell's work and influence has engaged their authors, is to say nothing of them that could not have been said of Russell's own work. Here at any rate are some efforts to describe and come to terms with the transformations of the intellectual and practical scene effected, or affected, by one of the best philosophers and best human beings of our time.

I have imposed no thoroughgoing uniformity in terminology or notation on the contributors. Each has been left free to deal with his subject as he thought best. Some of the work commissioned for this volume has appeared elsewhere in the meantime with my permission. Acknowledgment of permission to reprint materials not written on commission for this volume is due to the appropriate editors and publishers for Gilbert Ryle's 'Bertrand Russell: 1872-1970', from *Proceedings of the Aristotelian Society*, vol. LXXI (1970-1; 77-84), for William C. Kneale's 'Russell's Paradox and Some Others', from *British Journal for the Philosophy of Science*, vol. XXII (1971; 321-38) and for Renford Bambrough's 'Foundations', from *Analysis*, vol. XXX (1970; 190-7). Copyright in G. N. A. Vesey's 'Self-acquaintance and the Meaning of 'I'' is reserved to him. Finally, special thanks are due to Joel I. Friedman for his assistance with this volume.

GEORGE W. ROBERTS

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BERTRAND RUSSELL: 1872–1970

Gilbert Ryle

We members of the Aristotelian Society are here tonight to say ‘Goodbye and thank you’ to that grand philosophical thinker, Bertrand Russell, who gave his first paper to this Society in 1896.¹ This is not an occasion for an exegetic commentary on the almost infinite variety of his thought, but rather one for concentrating our gratitudes on those three or four determining impulses by which his thinking has given to the philosophical thinking of all of us, quite irrespective of our particular opinions and specialities, much of its whole trajectory.

For what concerns us today and, I maintain, for what should chiefly concern the future historians of twentieth-century thought, it matters comparatively little whether a few or many of us accept, or whether a few or many of us reject, this or that Russellian doctrine. The fact that he *did* not found a school or capture disciples was due partly to the accidents of his career, but especially to certain admirable features of his thinking. Among these was his immunity from reverence in general and especially from reverence for himself. He would have found Russell-acolytes comical and Russell-echoes tedious. On the other hand, what matters immensely is that, not what we think but, so to speak, the very style of our philosophical thinking perpetuates, where we are ordinarily least conscious of it, a style of thinking that had not existed in philosophy before, say, 1900.

(1) In speaking, metaphorically, of the Russellian style of thinking, though I am not alluding primarily, I am alluding secondarily to one particular intellectual temper for which the credit – the great credit as I think – needs to be divided between William James and Russell. For in one respect James and Russell were quite unlike Mill, Sidgwick and Bradley, quite unlike Brentano, Meinong and Husserl, and quite unlike even Moore, namely in their combination of seriousness with humour. Hume and Bradley had wit, and Hume could play. But James and Russell found out for themselves and so taught us at our best how to pop doctrinal bubbles without drawing blood; how to be illuminatingly and unmaliciously naughty; and how, without being frivolous, to laugh off grave conceptual bosh. Stuffiness in diction and stuffiness in thought were not, of course, annihilated, but they were put on the defensive from the moment when James and Russell discovered that a joke can be the

beginning, though only the beginning, of a blessed release from a strangling theoretical millstone.

(2) Much more important was a new style of philosophical work that Russell, I think virtually single-handed, brought into the very tactics of philosophical thinking. Anticipated, I suggest, only by the unremembered Aristotle, Russell occasionally prescribed and often deliberately practised what can be called 'aporetic experimentation'. In his *Mind* article of 1905 'On Denoting', he says:

A logical theory may be tested by its capacity for dealing with puzzles, and it is a wholesome plan, in thinking about logic, to stock the mind with as many puzzles as possible, since these serve much the same purpose as is served by experiments in physical science. I shall therefore state three puzzles which a theory as to denoting ought to be able to solve; and I shall show later that my theory solves them.

In 1904, near the beginning of his first *Mind* article on 'Meinong's Theory of Complexes and Assumptions', he had praised Meinong for the excellence of his quasi-empirical method of psychological research. His 1908 article 'Mathematical Logic as based on the Theory of Types' opens with a list of seven selected contradictions demanding some common solution. Now of course other philosophers, indeed all other philosophers worthy of the name, always had resolutely and conscientiously tried to overcome theoretical difficulties. They knew that their theories were in jeopardy so long as hurdles remained uncleared or uncircumvented. Nearly all of them, too, had from time to time opposed error by putting up obstacles in the way of the erroneous views or the bad arguments of others. It is not criticism or self-criticism that Russell invented. What was, I think, new was Russell's heuristic policy of deliberately mobilising, stiffening and constructing his own hurdles against which to pit his own nascent speculations. Difficulties in the way of a theory are no longer obstacles to thought; they can be and should be constructed or collected as aids to thought. They can be the self-applied tests by which philosophical thinking may become a self-correcting undertaking. As in the laboratory a well-designed crucial experiment tests a physical or chemical hypothesis, so in logic and philosophy a well-designed conceptual puzzle may be the *experimentum crucis* of a speculation.

To us, in 1970, this heuristic policy is obviously right. The most modest discussion note in one of our philosophical journals presupposes that philosophical progress requires positive and planned operations of sifting the tares from the wheat of doctrines and of arguments. Criticism is now not hostility; self-criticism is now not surrender. But we should, I suggest, search eighteenth- and nineteenth-century philosophy in vain; and even search contemporary continental philosophy nearly in vain for

cases of a philosopher actively hunting for and designing conceptual hurdles to advance his own future progress.

In his *Principles of Mathematics*, chapter X, entitled 'The Contradiction', and in its second Appendix, Russell had launched himself on what was to prove to be that most arduous of his theoretical undertakings which culminated many years later in his history-making Theory of Types. Already, in 1903, he was marshalling a battery of heterogeneous paradoxes against which he would test the desiderated solution of the special paradox of self-membered classes. Each of these auxiliary paradoxes, whether superficial or fundamental, was to serve as a testing device, with its own special edges, of the theory-to-be of self-reference.

Two precautionary words. By 'aporetic experimentation' I do not mean tentativeness, diffidence or even undogmatism. Russell meant some of his conceptual experiments to yield not 'perhapses' but definite results. Next, in using the notion of *experimentation*, I am not, of course, referring to physical tests; and I am not supposing that it is the mission of conceptual experiments – if anything has this mission – to engender inductive generalisations.

Unlike Wittgenstein, Russell was not focally, but only peripherally concerned to fix the places in human knowledge of logic and philosophy. When, as in *Our Knowledge of the External World as a Field for Scientific Method in Philosophy*, he did try to do this, he adopted too easily the idea that philosophy could and should be disciplined into a science among sciences. It was not, however, by this sort of promised assimilation of philosophy to science that he taught us a new kind of dialectical craftsmanship, but by the examples that he set of planned puzzle-utilisation. Like Moore, Russell constantly preached Analysis; but what, when pioneering, he practised included this far more penetrating, because self-testing, method of inquiry.

(3) At the end of the ninth chapter of *The Problems of Philosophy* (1912) Russell wrote:

The world of universals, therefore, may also be described as the world of being. The world of being is unchangeable, rigid, exact, delightful to the mathematician, the logician, the builder of metaphysical systems, and all who love perfection more than life. The world of existence is fleeting, vague, without sharp boundaries, without any clear plan of arrangement, but it contains all thoughts and feelings, all the data of sense, and all physical objects, everything that can do either good or harm, everything that makes any difference to the value of life and the world. According to our temperaments, we shall prefer the contemplation of the one or the other. The one we do not prefer will probably seem to us a pale shadow of the one we prefer, and hardly worthy to be regarded as in any sense real. But the truth is that both have the same claim on our impartial attention, both are real, and

both are important to the metaphysician. Indeed no sooner have we distinguished the two worlds than it becomes necessary to consider their relations.

Here Russell declares, what his writings show, that he himself knew and loved the views from the Alpine heights where there dwelled Plato, Leibniz and Frege, but also knew and loved the valleys that were tilled by Hume, Mill and James. Russell was that rare being, a philosopher whose heart was divided between transcendentalism and naturalism. His mind had been formed in his youth both by John Stuart Mill and by pure mathematics.

Indeed Russell got much of the impetus and nearly all of the turbulence of his thinking from his being homesick for the peaks while he was in the plains, and homesick for the plains when he was on the heights. However drastic, his reductionisms had some reluctances in them; however uncompromising, his Platonisms were a little undevout. Neither transcendent being nor mundane occurring felt to him either quite real, or gravely unreal. When in the mood he could think flippantly of either.

His ice-breaking and Ockhamising article 'On Denoting' came out only two years later than his ice-breaking, Platonising *Principles of Mathematics*; and in his *Our Knowledge of the External World* (1914) the second chapter 'Logic as the Essence of Philosophy', which is Fregean in inspiration, is immediately succeeded by two chapters entirely in the vein of the phenomenalism of John Stuart Mill. His paper of 1919 'On Propositions', which is very largely in the idioms of Watson, James and Hume, succeeds by only a year his lectures on Logical Atomism, where he is talking as if in the hearing of Meinong, Whitehead and the youthful Wittgenstein.

In his very early Platonising days he submitted in the *Principles of Mathematics*, section 427, a list of terms or objects that possess being, though they lack existence, namely, 'Numbers, the Homeric gods, relations, chimeras and four-dimensional spaces . . . if they were not entities of a kind, we could make no propositions about them'. Though he wrote this with complete seriousness, yet we can surely detect in his list an accent of sly shockingness, as if he could already guess what it would be like to season this overhospitable platter of being with a pinch of salt; and even what it would be like one day, though not yet, to investigate the credentials of the argument 'if they were not entities of a kind, we could make no propositions about them'.

Conversely, however far he moved away from the Platonism of his youth, he never conceded to Mill's reductionism about the truths of mathematics anything more than the recognition that it really is one business of pure mathematics to be capable of being applied to what there is in the everyday world. In the Introduction to the 2nd edition (1937) of his *Principles of Mathematics* he rejects the formalism of Hilbert

for, apparently, excluding applications of mathematics to the real world; he allows, with regrets, that mathematical truths, with those of formal logic, being ‘formal’ truths, cannot, as he had once thought, be construed as describing transcendent entities. He allows too, again with regrets, that there is something in some way ‘linguistic’ about these formal truths. But not for a moment does he concede to Mill that these truths are merely high-grade inductive generalisations about things that exist and happen down here. None the less he would quite soon be developing a theory of perception and, therewith, a theory of physical objects which does not do very much more than bring up to date the phenomenalism of Mill’s *System of Logic*.

It is sometimes said that Russell merely oscillated, pendulum-like, between transcendentalism and naturalism, or between Platonism and empiricism. The truth, I suggest, is that, anyhow in his formative and creative years, we find him neither at rest in the valley nor at rest among the peaks, but mountaineering – trying to find a way from the valley back to the peaks, or a way from the peaks back to the valley. He had two homes. But where he toiled, and where he was alone, and where he was happy was on the mountainside.

(4) The last of the four determining impulses by which Russell directed the course of subsequent philosophy is this. Russell was not only a pioneer formal logician, but, like Aristotle and Frege, he was a logician-philosopher. He saw every advance in formal logic as, among other things, a potential source of new rigours in philosophy; and he saw every philosophical puzzle or tangle as a lock for which formal logic might already or might some day provide the key. It was due to him, as well as, in lesser degree, to Frege and Whitehead that some training in post-Aristotelian formal logic came fairly soon to be regarded as a *sine qua non* for the philosopher-to-be; and debates between philosophers on philosophical matters quickly began often merely to ape but sometimes to apply or employ the blackboard operations of the formal logician.

Naturally it was, at the start, the more dramatic innovations in Russellian logic that were adopted by philosophers. The new term–relation–term pattern of simple propositions was for a time expected to accomplish nearly all the philosophical tasks at which the subject–predicate pattern baulked. But even if not into this new pattern, still formalisation into some newly sponsored pattern or other was for a time expected to make short work of any surviving philosophical problems. But to say this is only to say that Russell, Whitehead and Frege made many philosophers enthusiasts for their new so-called Symbolic Logic – and enthusiasts are always impetuous. The remarkable thing is that these three – and Russell more than the other two – did fire this enthusiasm. Even outside the English-speaking world they fired it, partly through the mediation of Wittgenstein, as far away as Vienna; and without this mediation as far away as Poland.

Doubtless some of these zeals were ephemeral or factitious; doubtless, too, some of the Frege–Russell hopes for a monolithic Euclideanisation of mathematics were doomed to disappointment; and certainly we have long since forgotten the promise, if it was ever made, that philosophical problems would now receive their solutions by instant formalisation. None the less, philosophy in the English-speaking world has inherited from the *Principles of Mathematics* and *Principia Mathematica*, as well as from Frege’s logical writings, not only a respect for rigour, but a discipline in rigour, the absence of which from what, with reservations, I label ‘continental’ philosophy still makes cross-Channel discussion unrewarding.

However, I do not wish merely to acknowledge the huge effects of, especially, Russell’s logicising of philosophy. There was another massive legacy left by Russell, the logician-philosopher, which we can call the Theory of Types.

By 1903 Russell had found, and imparted to Frege, a contradiction in that notion of *class* which had been a central concept in the work of Cantor, as well as in Frege’s and Russell’s own definitions of *number*. With this contradiction the young Russell had associated a whole battery of partly similar antinomies, for all of which, it seemed, some general diagnosis and, hopefully, some general cure could be found. Either answer, ‘Yes’ or ‘No’, to the question ‘Is “I am now lying,” true?’ seems to establish the other; ‘Yes, if no; but no, if yes’. To the question ‘Is the class of classes that are not members of themselves a member of itself?’ the only answer again seems to be ‘Yes, if no; but no, if yes’. Russell came, in the long-postponed end, to the conclusion that for a specifiable reason these questions are unanswerable by ‘Yes’ or by ‘No’; they are improper questions. Epimenides’s assertion was a pseudo-assertion; an assertion cannot be a comment upon itself; and a given class C can only be nonsensically spoken of as one of the items that belongs, or even does *not* belong as a member to C.

Besides the sentences that convey standard propositions that are true or else false, there are grammatically passable sentences which are neither true nor false, but nonsense. It was some, but only a very few, nonsense-excluding rules that Russell, in his Theory of Types, tried to formulate and justify.

It is of some historical interest that the Vienna Circle misappropriated Russell’s notion of nonsense for its own special Augean purposes. But it is of huge historical importance that the whole *Tractatus Logico-Philosophicus* can be construed as a Procrustean essay in the theory of sense/nonsense. The *Philosophical Investigations* also is, in large measure, an inquiry into the rules of ‘grammar’ or ‘logical syntax’ of which patent or latent absurdities are in breach. In his lectures on Logical Atomism Russell showed how he had already been glad and proud to learn from the young Wittgenstein of 1912–3 some of the expansions, extensions and new applications of which his former Theory of Types had now become capable.

In these different, though doubtless internally connected ways, Russell taught us not to think his thoughts but how to move in our own philosophical thinking. In one way no one is now or will ever again be a Russellian; but in another way every one of us is now something of a Russellian. Perhaps we do not even read Russell very much; but in at least four radical ways what we say to philosophers and write for philosophers differs in intellectual method and intellectual temper from what we would have said and written in pre-Russell days and from what we would say and write today if we were – shall I say? – Ruritanians.

*Magdalen College
University of Oxford*

NOTE

- 1 This chapter was read at a meeting of the Aristotelian Society at 5-7 Tavistock Place, London WC1, on Monday, 7 December 1970, at 7.30 p.m.

PROPOSITIONS AND SENTENCES

Alan R. White

Russell's views both about the nature of analysis and about the analysis of belief were coloured – and, I think, vitiated – by his confusions about the notion of a proposition. He provides an interesting case history for the student of propositions since he was a prey to just those puzzles about their nature which have beset most investigators of the notion. Like them, he was torn between the reasons for identifying propositions and sentences and the reasons for distinguishing them. It is these reasons that I wish to discuss in the present chapter. Any attempt, however, to delineate his views is doubly difficult because he was, admirably, a changeable and, less admirably, a careless writer; careless both in that he neither took much care to be nor, apparently, cared much about being consistent even within the same piece of writing. The dating of the references shows his explicit changes of view; but they also show, I feel, that no consistent pattern of change can be found.

Russell, like other philosophers, introduced propositions to fill a variety of roles: (a) to be that which is asserted when it is asserted that *p* as contrasted with that which is questioned, commanded, etc. (e.g. 1918: 185; 1927: 271);¹ (b) more particularly to be that which is believed, doubted, desired, considered or in some other way regarded when it is believed, doubted, etc. that *p* (e.g. 1919: 285); (c) to be that which is true or false (e.g. 1903: xix; 1940: 76); (d) to be the meaning or significance of a sentence (e.g. 1940: 180); (e) to be what is related in some way to the facts (e.g. 1918: 182).

Having, for these reasons, introduced the notion of a proposition, he had to explain it. This he rightly tried to do, in the manner of most philosophers, by relating it to that form of words, e.g. the indicative sentence *p*, by which we assert something, express what we believe or doubt, or state what we take to be the facts. Sometimes Russell explicitly identified the proposition and the sentence which expressed it, sometimes he explicitly distinguished them and often, in one and the same passage, his implicit view on the question whether propositions and sentences are or are not to be distinguished is contrary to what by carelessness he explicitly states. When he did distinguish proposition and sentence he had various different views as to how they were related, for instance, that the proposition is a complex of constituents corresponding to the parts of the sentence which express it, that the prop-

osition is the meaning or significance of the sentence, that it is what is asserted by the sentence or that it is a class of sentences with the same meaning.

As early as 1903 (e.g. p. 42), as late as 1959 (e.g. p. 182), and often in the intervening period (e.g. 1904: 209; 1937: ix; 1940: 180) Russell clearly distinguished the proposition from its linguistic expression, the sentence. He spoke (1905: 43–5; 1918: 250–1) of ‘propositions in whose verbal expression denoting phrases occur’, of ‘the words in the statement of a proposition’, of propositions as what may be expressed in any language (e.g. 1940: 10) or asserted by a sentence (1940: 42) or of what is common between sentences in different languages which say the same thing (e.g. 1959: 182). In 1940, he suggested, perhaps *en passant*, a definition of a proposition as, ‘all the sentences (or “the class of all the sentences”’) having the same significance as a given sentence’ (1940: 10, 158). But, although a proposition might be either expressed by or even equivalent to each and every member of a collection, it is difficult to see how it could be the collection itself. Collections or classes of sentences are not what we express by a sentence, nor what we believe, nor what is true or false, nor what occurs on the top line of a page. Here, and elsewhere (e.g. 1919: 290; 1921: 188; 1924: 332; 1940: 22–3; 1944: 692; 1959: 153, 172; contrast 1940: 76; 1959: 145) Russell seems to have confused a class and its members with something and its instances. But to say that there are ten instances of the word ‘the’ on this page, is not to say that ‘the’ is a class of which these are members. ‘Dog’ does not signify a set of dogs in the way that ‘litter’ signifies a set of dogs (contrast 1919: 290; 1940: 22).

Sometimes Russell’s position became more ambiguous, for example, as signified by such a phrase as ‘a proposition in which symbolically a class occurs’ (e.g. 1918: 265). Sometimes he seems to have felt that it doesn’t really matter whether one speaks of propositions or sentences (1918: 186).

None of this, however, prevented him from speaking in the same way, even in the same writings, both of propositions and of sentences (e.g. 1905: 45, ‘propositions in which denoting phrases occur’ and ‘propositions in whose verbal expression denoting phrases occur’; 1918: 184–5, ‘the words in a statement of a proposition’ and ‘a proposition is a sentence in the indicative’, cf. 1903: 42–3). His first explicit equation of propositions and sentences seems to occur in 1918 when he stated that ‘a proposition is a sentence in the indicative’ (p. 185; cf. 1921: 240; 1924: 334; 1927: 271), and as such does not exist over and above such a sentence: a view which he sometimes expressed by saying that propositions are not real (1918: 214), not to be included in any inventory of the world; at other times in the extreme view that the word ‘proposition’ is ‘meaningless’ (1918: 263) because it does not stand for anything (1918: 289). In the following year he seemed to regard the identification of a proposition and a sentence, less dogmatically, as a plausible hypothesis rather than a definite conclusion (e.g. 1919: 289–90); though from 1918

to 1940 his usual manner of referring to propositions is as something composed of words or other symbols.² By 1940 he had moved back either to talking indifferently of propositions and sentences (e.g. p. 54) or to the admission of propositions as a class of sentences (e.g. p. 158) or as the significance of a sentence (p. 158), though he is still worried by the feeling that to admit that a sentence expresses a proposition is to admit the existence of an extra, mysterious thing called 'proposition' or 'the significance of a sentence'.

Russell's various views about the relation of a sentence and a proposition were naturally reflected in his views about the relation of a numeral and a number and of a symbol for a class and a class itself. But here, though he began his career with a firm belief in the independence of numbers from numerals and of classes from class symbols (e.g. 1903: 66–80), he soon reached the equally firm and lasting belief that numbers and classes did not exist over and above their numerals and class symbols (1918: 262–7; 1937: x–xi). This Ockhamite view was sometimes expressed as 'There are no such things as numbers', just as the parallel view about propositions was sometimes expressed as 'There are no such things as propositions'. It was, of course, also expressed in the view that 'numbers and classes are logical fictions' (e.g. 1918: 191, 270) or that numbers, like propositions, 'are not part of the ultimate constituents of our world' (1918: 270; 1937: ix). Russell's 'no-class' theory was probably not intended to mean that in ostensibly talking about classes we are really talking about class symbols, but that in using class symbols we are not talking about a constituent of the world called a 'class'. He possibly thought of a class symbol as, in this respect, analogous to the phrase 'the average man'. He would not have wanted to hold the absurd view that in ostensibly talking about the average man we are really talking about the phrase 'the average man'; but rather that in using the phrase 'the average man' we are not talking about a denizen of the world called 'the average man'. This plausible view, however, became assimilated to the view that numbers *are* the numerals that express them and that classes *are* the class symbols that express them (e.g. 1918: 253; 1937: ix–x). This would be like saying that 'the average man' is a phrase. In other words, the view that X is not something additional to Y was sometimes interpreted as X does not exist and sometimes as X is equivalent to Y.

A particular way in which Russell's ambivalence about the identity of propositions and sentences is reflected is in his corresponding ambivalence about what it is that plays certain roles, particularly that of being the bearer of truth or falsity, that of being the object of belief, that of being either molecular or atomic and that of having certain kinds of constituents. Let us consider each of these.

Truth and falsity were sometimes explicitly reserved for propositions (e.g. 1903: xix; 1918: 184; 1940: 76, 164), sometimes for sentences (1927: 245, 269, 272–3; 1940: 28, 60; 1948: 110), but were often explicitly or implicitly allowed to either propositions (1906: 48; 1918: 182, 208, 214;

1921: 273; 1927: 271; 1940: 9, 16; 1944: 14; 1948: 79, 133; 1959: 66, 111, 167) or sentences (1927: 270; 1940: 17; 1948: 75, 88, 100, 112, 115, 130; 1959: 186, 220) or both (1940: *passim*; 1948: 109). In his 1940 *An Inquiry into Meaning and Truth* especially, there are dozens of references for truth applied to propositions and dozens for its application to sentences. Even when Russell explicitly denies that propositions are real, he sometimes continues to wonder whether they can be true or false (e.g. 1918: 214); while, at others, insisting that 'really a proposition cannot be true or false because a proposition is nothing' (1918: 227).

Truth and falsity were also frequently attributed to beliefs (1906: 46; 1927: 182, 222, 227; 1919: 285; 1921: 232, 234; 1940: 304; 1948: 95, 119, 148). Indeed, it was said on occasion that beliefs are the primary bearers of truth or falsity (1910: 158; 1940: 203, 215, 223; 1948: 112, 148; 1959: 183; cf. 1927: 265; 1904: 204). But this is less of a difference than it sounds, since beliefs were at various times equated with propositions (1918: 182, 308) or sentences (1919: 308; 1927: 270) or both. Furthermore, on the one hand, a true belief is defined as 'a belief in a true proposition' (1918: 320), whereas, on the other, 'the relation of a sentence to the fact that makes it true or false' is said to be 'indirect through the belief expressed by the sentence' (1940: 199).

Part of Russell's reason – as we have just seen – for holding that truth and falsity apply to both sentences and propositions was the equation of both with beliefs. But he also frequently, though inconsistently, equated both sentences and propositions not merely with beliefs in a sense of what on occasion he called 'believings' (1918: 217; 1959: 118), but also with what is believed. Russell always held, for reasons I have given elsewhere,³ that when *A* believes that *p* what is believed is the proposition that *p* (e.g. 1904: 204, 218, 339–50, 509, 522–3; 1905: 52; 1906: 48; 1918: 187, 218; 1919: 285, 307; 1921: 233, 241, 272; 1927: 272; 1940: 18, 79, 119, 142, etc.); even when he held that belief is not a relation between the believer and a proposition (1918: 217, 224, 226–7) and even when the believer is an animal (e.g. 1940: 79, 179). Indeed, one definition of a 'proposition' is 'what is believed' or 'the content of a belief' (e.g. 1919: 308–9; 1921: 240). Naturally, therefore, his ambivalent view about propositions and sentences sometimes led him to say that what is believed when one believes that *p* is the sentence '*p*' (1921: 236, 245; 1927: 270; 1940: 179, 189, 199, 214; 1948: 98, 99, 101, 120, 125), since if there are no propositions, it cannot be the proposition that *p*. When, on the other hand, he distinguished propositions from sentences he asserted that 'what is believed is not the words "*p*" but what "*p*" signifies' (1940: 168, 255; 1948: 98–111, 146). Even when he held that 'what is believed' or 'the content of a belief' is a set of images (1921: 236, 241, 245), this could be reconciled with his view, since he also sometimes held that a proposition could be a set of images – what he called 'image-propositions' (1921: 275; 1940: 180).

Similarly Russell's distinction of molecular and atomic was expressed

as a distinction sometimes between propositions (e.g. 1918: 199, 209, 211, 216; 1940: 29, 42; 1948: 120), sometimes between sentences (e.g. 1940: 29, 32, 42, 90, etc.; 1948: 120) and sometimes between propositions containing different kinds of words (e.g. 1918: 207, 208).

Another major clue to Russell's view about the relation of a proposition and a sentence lies in his views about the kinds of constituent each has. Identity of kinds of constituent would, in the usual sense of 'constituent', allow identity of kinds of things which had the constituents, while difference in the former would imply difference in the latter.

Unfortunately, Russell seems to have held, at various times, two quite different views as to what it is for something to be a constituent of a proposition. On one view 'constituent' is used in the ordinary sense of that which is contained in or occurs in something, that of which the something is composed or in which it consists. That Russell often, perhaps most usually, used 'constituent' in this sense is clear not only from his explicit assertions, as when he says that a proposition 'contains the class as a constituent' (1903: 67, 49, 83-5; 1918: 230; 1905: 55; cf. 'fact' 1959: 152), or that a proposition contains or is composed of its constituents (1906: 48; 1912: 91), but also from the context in which 'constituent' is used (e.g. of words in 1918: 192, 196-7, 239; 1959: 182; of things other than words in 1903: 49, 67; 1904: 204, 345; 1905: 42, 56; 1912: 85, 91, 198; 1918: 238; 1919: 316, 345) and from the context in which some such words as 'occurs in', 'contains', 'consists in', 'is composed of', 'component' is used (e.g. for words in 1903: 43; 1905: 45, 47, 50; 1918: 192-7, 204, 207-8, 241, 247; 1919: 308-9; 1924: 334; 1940: 49, 56, 74, 327; 1944: 15, 694; 1959: 67, 84; for things other than words in 1903: 43, 46, 47, 53, 73, 85, 89; 1904: 209; 1912: 92; 1919: 315). Furthermore, it is in this sense that he uses 'constituent' when he speaks of a constituent of a sentence (e.g. 1940: 28), of the world (1918: 270), of a fact (1918: 217, 270; 1919: 286-7; 1948: 126; 1959: 152; cf. 'component' 1918: 182), of a belief (1918: 196) or of the content of a belief (1921: 235), or of time (1904: 213).

On the other view, to be a constituent of a proposition is to be what the proposition is about, 1903: 45, 56 (a term); 1903: 54; 1912: 85 (a person); 1918: 250-2, 262 (a thing); 1903: 43 (a name); 1903: 90; 1918: 246; 1940: 253 (a thing, not a name) – as likewise to be a constituent of a fact is to be what the fact is about (1919: 286) – *or* what the proposition mentions (1918: 262), *or* what is the subject of the proposition (e.g. 1903: 44; 1918: 252; 1924: 328), *or* what is signified, expressed or denoted by the words which express the proposition (1905: 45, 55; 1918: 250), *or* what corresponds to the words which express the proposition (1906: 48; 1918: 247-8; 1924: 328; 1937: ix-x). Such a view of 'constituent' is, perhaps, analogous to the sense in which a person can be said to be in or to appear in a book, newspaper or list in which he is mentioned.

It is quite clear from the references I have given that Russell at various

times throughout his life held both these views. Though he himself explicitly made some such distinction in 1919, it would be a mistake to suppose that such a date marks a clear line between an earlier and a later view.⁴ Russell is, unfortunately, a prime example of someone to whom the difference between what a man says he thinks and what he actually does think is pre-eminently applicable. The 'container' view, namely, that the constituents of a proposition are what the proposition is composed of, is clearly stated both before and after this date and for both verbal and non-verbal constituents. Similarly, the 'reference' or 'mention' view, namely, that the constituents of a proposition are what the proposition is about, is also clearly stated both before and after this date, usually for non-verbal items, but, on occasion, for verbal items.

It is also quite clear how these very different views restrict the sorts of things that could be constituents of something. Thus, if a proposition were a sentence, then a verbal item could either be contained in and not mentioned in it or mentioned in and not contained in it or both contained in and mentioned in it. A non-verbal item, however, could be mentioned in it, but it could not be contained in it. On the other hand, if a proposition were not a sentence, then non-verbal items could either be contained in and not mentioned in it (e.g. 1903: 53) or mentioned in and not contained in it (e.g. 1903: 53) or both mentioned in and contained in it. A verbal item, on the other hand, could be mentioned in but not contained in it. Hence, if verbal items are constituents of a proposition in the container sense, then a proposition must be a sentence, and if non-verbal items are constituents in this sense a proposition cannot be a sentence. In the reference sense, on the other hand, both the verbal and the non-verbal items could be constituents of a proposition, whether or not it were a sentence. Unfortunately, at various times – including times both before and after 1919 – Russell allowed both verbal and non-verbal items to be constituents of propositions in the 'container' sense and, therefore, allowed propositions both to be and not to be sentences. He also, quite properly, allowed both verbal and non-verbal items to be constituents of a proposition in the 'reference' sense, since the type of constituent, in this sense, does not determine the type of thing of which it is a constituent and does not, therefore, allow one to differentiate between propositions as sentences and propositions as other than sentences. Even here, however, he frequently quite inconsistently spoke in successive breaths both of a verbal item and of its corresponding non-verbal item being, in this sense, a constituent of the same proposition, despite other occasions on which he explicitly insisted that it was, for instance, Socrates and not the word 'Socrates' which was, in this sense, a constituent of the proposition 'Socrates loves Plato' (e.g. 1918: 238–9).

Finally, to make confusion worse confounded, Russell frequently wrote, even in the same few pages, in such a way as to assimilate the 'container' view and the 'reference' view of constituents; thus allowing that what is mentioned in a proposition not merely could also be

contained in it, but is necessarily contained in it. Thus, in 1903 (e.g. ch. 4) 'term' was invented as a technical word for the 'constituent' of a proposition, for whatever 'occurs in' a proposition, or whatever is the 'subject' of a proposition, for whatever the proposition is 'about' and for whatever the proposition 'contains'. In the same chapter words are both asserted and denied to be what occur in, are contained in, or are the subject of, a proposition, or are what the proposition is about. Nor is the assimilation of the container view and the reference view prevented by Russell's explicit insistence on the difference between the two, *either* in the 1905 form when he explained his allusion to 'denoting phrases occurring in propositions' but not 'standing for constituents of propositions' by treating the occurrence as occurrence in the verbal expression of the proposition (cf. 1918: 250; 1924: 328), *or* in the 1919 form when he insisted that the constituents of a proposition are the phrases which occur in the proposition and not anything referred to by such phrases, for the assimilation of the container and the reference view occurs in a random way at other dates (e.g. 1903: 43; 1918: 185).

The same ambiguities infect Russell's typical denial that a particular non-verbal item, such as a class, a number, Romulus or *or* could be a constituent of a proposition on the ground that such items do not exist and 'you cannot have a constituent of a proposition which is nothing at all' (1918: 242, 250, 253, 270; 1937: ix; cf. 1912: 139, 153). The relevance of such an argument is clear on the view (a) that a proposition is not a sentence and its constituents are what it contains, or (b) that a proposition is not a sentence and its constituents are what it mentions or (c) that a proposition is a sentence and its constituents are what it mentions; for on all three of these views non-verbal items are possible constituents of a proposition. The only view to which the argument seems irrelevant is (d) that a proposition is a sentence and its constituents are what it contains, for on such a view neither existent nor non-existent non-verbal items can be constituents of any proposition. Russell, however, might have overlooked this because he sometimes assimilated the view that numbers and classes do not exist to the view that they are the numerals or class symbols which express them. On the other hand, it is unlikely that in this context Russell was thinking either of view (d) or of view (c), for he usually stressed here the distinction between the proposition which wrongly appeared to have a number or a class as a constituent and the sentence which did have, for example, such a phrase as 'the number 2' or 'the class of all men' as a constituent. Nor is it likely that he was thinking of view (a), since he usually paraphrased his conclusion by saying that in such a case the proposition is not *about* a number or class and that neither of these is the *subject* of such-and-such a proposition. Furthermore, Russell subscribed to the very commonly held – but I think mistaken – opinion that what does not exist cannot be the subject of a proposition, or be what it is about or be mentioned or referred to by it. In such contexts it seems plausible, therefore, that Russell was thinking of

view (b), namely, that a proposition is something other than a sentence and its constituents are what are mentioned in it. Unfortunately, however, since, as we saw, he often assimilates the 'container' and the 'reference' view, it may be that he held both (b) and (a).

Though there are lots of problems about what it is for a proposition to mention or be about something, such problems do not make it difficult to understand what a proposition is. A sentence, too, can quite understandably mention or be about so-and-so. When, however, a proposition is said to contain non-verbal items, puzzles do arise about the nature of a proposition. Nor are they solved by supposing these non-verbal items to be psychological, e.g. images. For Russell did not attach importance to the difference between propositions as composed of words – what he sometimes called 'word-propositions' – and propositions as composed of images – 'image-propositions'. The important difference is between the view of propositions as composed of, having as constituents in the 'container' sense, words or images, and the view of propositions as composed of non-verbal items. What sort of a thing is a proposition if its constituents are, as Russell often said they were, entities such as material objects and people? An important, but not a very satisfactory, part of the answer to this question hinges on the relation of propositions to facts.

Russell, rightly distinguishing between words and the meanings of words, was undecided whether propositions were composed of words or of the meanings of words. He was further undecided what it was to be composed of the meaning of words, because he thought of a meaning of a word as something meant by, that is, referred to by, the word and he thought, at various times, that this might be a psychological item such as an image, a non-psychological item such as a person or a thing or a hybrid item called a 'concept'. Hence, in juggling with the three notions, *sentence*, *proposition* and *fact*, Russell, even within the same paper, sometimes identified the proposition with a sentence as something whose components are words and sometimes identified it with something whose components are meanings, either in the sense of concepts (e.g. 1903: 53, 73; 1948: 107) or of images (e.g. 1919: 308, 319; 1940: 180–2) or in the sense of things, for instance, material objects and people which are what is 'meant by' the words (1904: 204; 1905: 56; 1912: 91; 1918: 194, 224; 1919: 290 ff.) or the images (1919: 316; 1921: 207). Since he sometimes called what is composed of the things meant by words a 'fact', he therefore sometimes distinguished propositions from facts and the constituents of propositions from the constituents of facts (1918: 182 ff., 196–8; 1924: 335), and sometimes identified them (1904: 523; 1905: 45–8; 1918: 191, 248; 1919: 309). His denial in 1918 of the separate existence of propositions led him to give to facts many of the jobs which propositions, or at least true propositions, had hitherto done, especially that of containing the constituents corresponding to the words of the appropriate sentence (e.g. 1918: 191 ff., 248) and that of being the subject matter of analysis (e.g. 1918: 191, 198). On one occasion he

allowed that facts could be true or false (1918: 227). In 1919 he argued, supposedly under the influence of Wittgenstein, that a proposition, because it is something consisting of words or images, is one instance of a fact (1919: 309, 315–17; cf. 1921: 250; 1924: 332).

The difficulty about the identification of propositions is that, on the one hand, they seem inseparably linked to some means of (verbal) expression – hence, the temptation to identify them with sentences – while, on the other hand, they both lack the linguistic characteristics of sentences (e.g. being English, ungrammatical or misspelt) and possess non-linguistic characteristics inapplicable to sentences (being true, contradictory, unproved) – hence, the temptation to give them a separate existence of their own. Russell provides a salutary example of the difficulties inherent in either suggestion.

My own suggestion is that a proposition is the *logical role* which a sentence can, but need not, play. Meaningful sentences are capable of playing it, but meaningless ones are not. The same role can be played by various sentences, in the same or different languages – or even by things other than sentences – while the same sentence can play various roles or no role, just as Hamlet can be played by both Guinness and Gielgud, while both can also play Macbeth and Lear or can take a rest from acting. Furthermore, in uttering a sentence a man may play the role of making a statement just as in raising his arm he may be acting as a signal giver. Similarly the arabic numeral ‘9’ and the roman numeral ‘ix’ both play the role of the number nine; a piece of paper can play the role of a legal contract; a physical movement that of an action.

Although role players can exist without playing a role, roles do not exist independently of being played. The temporal and spatial location of a role is that of what plays the role. Hamlet is on that part of the stage where Gielgud is standing. A contract is in the filing cabinet in which is the piece of paper on which it is drawn up, a number is on that part of the gate where its numeral is, a proposition occurs on the line where its sentence is written, a signal comes from the spot where an arm is raised. More importantly, role players can be said to *be* the role they are playing and roles said to *be* what is playing the role. In the context of the role, Gielgud *is* Hamlet and Hamlet *is* Gielgud, the piece of paper in the filing cabinet *is* the contract and a contract *is* that piece of paper, raising my arm *is* signalling and signalling *is* raising my arm. Similarly that numeral on the gate *is* my number and my number *is* that numeral, the sentence in the footnote *is* the proposition that *p* and the proposition that *q* *is* the sentence underlined.

But though role players can be said to be what they play and vice versa, roles have characteristics which role players in themselves lack, while role players have characteristics which their roles lack. Hence, Hamlet, but not Gielgud, is a Shakespearian character; while Gielgud, but not Hamlet, is an Englishman. Similarly contracts, but not pieces of paper, are legal, binding and harsh, while pieces of paper, but not

contracts, are quarto, vellum and lined; numbers, but not numerals, are odd or even, while numerals, but not numbers, are arabic or roman; propositions, but not sentences, are true or false, while sentences, but not propositions, are English or French.

The impossibility of attributing all the characteristics of a role to its role player or of a role player to its role shows that the notion of a role is different from that of a role player, and hence the notion of a proposition different from that of a sentence, that of a number different from that of a numeral, that of a contract different from that of a piece of paper. But this difference between a notion of a role and that of a role player does not prevent the role player from *being* the role it plays. The notions of proposition and sentence, number and numeral, contract and piece of paper, signal and arm movement are different; yet sentences can *be* propositions, numerals numbers, pieces of paper contracts and arm movements signals. It is, I think, a misapplication of Leibniz's law to say that because we cannot attribute truth and falsity to the sentence '*p*' as we can to the proposition *p* and cannot argue from '*S* believes the proposition *p*' to '*S* believes the sentence "*p*"', therefore a sentence cannot be a proposition or vice versa.⁵ It is, on the other hand, a converse misapplication⁶ of the law to argue that because a proposition can be a sentence, therefore a sentence can be true or false and people can believe sentences.

The insistence that, though the role is logically different from that which plays the role, it does not exist without it, does not imply that it makes no sense to speak of roles that have never been played any more than of threats that have never been uttered or of generations yet unborn.

To discover what, if any, proposition a given sentence expresses is, therefore, to discover the logical – as contrasted with the linguistic – role of that sentence. Two sentences express the same proposition if they play the same logical role. In addition to its purely logical role, a sentence – and also the proposition expressed by it – can play a wide variety of what might, perhaps, be called 'illocutionary roles.' Thus, a sentence might play the role of, and hence be said to *be*, a premiss, conclusion, hypothesis, claim, assumption or objection. This is why, like sentences, objections can be deleted, assumptions put in the footnotes and claims sent through the post; though, unlike the sentences used to express them, they can also be unjustified, invalid and inconsistent, but not ungrammatical, inelegant or English.

There are, I think, traces in Russell of my suggested analysis of *proposition* – e.g. 1921: 241, 'A proposition is a series of words expressing the kind of thing that can be asserted or denied' – but his usual picture of a proposition caused him to mistake an inquiry into the logical role of a sentence for an inquiry into the structure of a supposed complex of constituents corresponding to the sentence. It led him, and other analysts of the time, to look for 'the form of the proposition' or even 'the form of the fact' instead of for the logical behaviour of a sentence. Thus, the

view that a sentence containing as grammatical subject the words 'The so-and-so' does not necessarily attribute a characteristic to anything became the view that the proposition expressed by these words does not contain a constituent named by the words 'The so-and-so' (e.g. 1905; 1924; cf. 1944: 14). When, on the other hand, Russell's sense of reality and his adherence to Ockham's razor led him to deny that there are any such things as propositions, he fell into the error of attributing the characteristics of propositions, e.g. the possibility of being true or false or believed, to the sentences which express them.

The realisation that it is certain characteristics of the behaviour of the verbal and not the structure of something non-verbal which analysts should seek, marks, I think, the transition from the thinking of Russell, Moore and the early Wittgenstein to the thinking of Ryle and the later Wittgenstein.⁷

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NOTES

- 1 The references, by date and page, are given at the end of this chapter.
- 2 Contrast A. J. Ayer, *Russell and Moore: The Analytical Heritage* (London, Macmillan, 1971: 81).
- 3 In Chapter 14, 'Belief as a Propositional Attitude', in this book.
- 4 For example, D. F. Pears, *Bertrand Russell and the British Tradition in Philosophy* (London, Collins, 1967: ch. 13).
- 5 For example, G. E. Moore, *Lectures on Philosophy* (London, George Allen & Unwin, 1966: 132–49).
- 6 For example, Russell, *passim*; or W. V. Quine, *Word and Object* (Cambridge, Mass., MIT Press, 1960: *passim*).
- 7 I am indebted to D. R. Cousin and P. T. Geach for comments on an earlier draft of this chapter.

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RUSSELL'S PARADOX AND SOME OTHERS

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(1) In a recent paper (1972¹) I argue (i) that a language which makes possible the characteristically human form of social life must allow for talk not only about its own sounds, but also about communication by means of those sounds, (ii) that failure to recognise this had led many philosophers into a dangerous confusion between sentences and propositions, (iii) that attempts to formulate logic as a theory of grammatically well-formed sentences involve neglect of the token-reflexive device and misunderstanding of the role of definite descriptions and (iv) that the paradox of the Liar holds no terrors for those who realise how the notion of truth is related to that of a proposition. My reason for concluding with an attempt to solve the old problem by means acceptable to a student of natural languages was, of course, a wish to counter Tarski's thesis that natural languages are all inconsistent through failure of their users to observe the distinction between language and metalanguage which he considers essential for solution of the Liar paradox. But it may be of interest to show that similar methods are sufficient for solution of Russell's paradox and some others that were formulated in the early years of this century during debates on the theories of Cantor and Frege.

In the Introduction to the first edition of *Principia Mathematica* Russell listed seven paradoxes, or apparent contradictions, for which he undertook to find solutions, namely:

- (a) The Liar paradox.
- (b) His own paradox of *the class of all classes that are not members of themselves*.
- (c) The corresponding paradox of *the relation between two relations when one does not have itself to the other*.
- (d) Burali-Forti's paradox of *the ordinal number of all ordinals*.
- (e) Berry's paradox of *the least integer not nameable in fewer than nineteen syllables*.
- (f) König's paradox of *the least undefinable ordinal*.
- (g) Richard's paradox of *the class of all decimals definable by a finite number of words*.

In each of these, he maintained, something is said about everything of a certain kind, and then from what is said there seems to be generated a

new case which both is and is not of the same kind as the case or cases covered by the original remark (Russell and Whitehead, 1910). By the time of the second edition of *Principia Mathematica* a new paradox of the same general character had become popular, namely:

(h) The paradox of Nelson and Grelling about the adjective '*heterological*'. But Russell had always held that there was no limit to the possibility of creating vicious-circle paradoxes by talk of illegitimate totalities, and he continued to maintain that all alike were to be solved by his theory of logical types.

In his essay of 1925 on 'The Foundations of Mathematics' Ramsey argued that of all the paradoxes listed above only those listed as (b), (c) and (d) could properly be classified with Cantor's worries as mathematical, or logical in that sense in which the mathematical had been identified with the logical (Ramsey, 1931: 20). All the others, he said, involved linguistic or epistemological notions, and it was significant that in Russell's treatment they alone required his distinction of orders within types. My own view is that all alike originate in misuses of language (how else could we fall into such contradictions?) and that none of them requires for its solution a restrictive theory of types, either simple or ramified. But I believe that (d), (f), and (g) are to be distinguished from the rest as involving essential use of terminology peculiar to the theory of sets, and I intend to consider here only the other five. Of these Berry's is the only one involving reference to numbers, but I think it is in some ways the simplest of all, since it requires for its solution no more than a recognition of the power of indefinite self-enrichment which belongs to all natural languages, and I shall therefore deal with it first.

(2) When we first read the phrase 'the least integer not nameable in fewer than nineteen syllables', we think that it must refer to some rather large number whose name in the common notation of 'hundred', 'thousand', 'million', etc. contains at least nineteen syllables. But then we realise that, if the phrase does indeed refer to such a number, it is itself a name of that number (in a large sense of the word 'name'), although it contains only eighteen syllables, and so we are bewildered for a moment. When, however, we recover from our shock, we naturally say 'The trick depends on using "nameable" in different ways', and we are right, though our explanation requires a little development to make it entirely satisfactory.

Just because natural languages contain words like 'name' and 'designation' they allow for talk about communication by use of language and so indirectly for a new style of talk about other things. Let us say for clarity that phrases like 'fifty-four' and 'one hundred and seventy-three' are primary designations of numbers. Then clearly with the vocabulary at our disposal we can make up secondary designations such as 'the least integer with a primary designation containing ninety syllables', and if the spirit moves we can go on to produce number designations of the

third and even higher levels. There is indeed no highest level of designation beyond which we may not progress, and no danger of paradox to be anticipated from addition of new levels, provided always we make clear in every utterance the level or levels with which we are concerned. In Berry's paradox, however, it is essential that the word 'nameable' is used without restriction to the first or any other level, and so his phrase 'the least integer not nameable in fewer than nineteen syllables' must be understood as though it were an abbreviation for 'the least integer not nameable *by a designation of any level* in fewer than nineteen syllables'. When, however, this has been conceded, it becomes clear that the phrase cannot designate any number at all not because of a fault in the supplementary words I have italicised, but simply because there is no limit to the developments of notation we may introduce, and so no limit to the magnitude of the integers that we may designate somehow or other in fewer than nineteen syllables. Thus if anyone maintains that among numbers which can be designated in fewer than nineteen syllables there must be a largest, we can easily refute him by stipulating first that the single letter 'l' is to be taken in the context as a short sign for his supposed upper bound and then, with this convention, writing 'l+1' as a sign which he should admit to stand for something larger.

According to Russell's ramified theory of types the simple solution I have just indicated is not acceptable because there is a vicious circle involved in the use of the phrase 'the least integer not nameable in fewer than nineteen syllables'. In the Introduction to *Principia Mathematica* he writes:

The word 'nameable' refers to the totality of names, and yet is allowed to occur in what professes to be one among names. Hence there can be no such thing as a totality of names, in the sense in which the paradox speaks of 'names'. It is easy to see that, in virtue of the hierarchy of functions, the theory of types renders a totality of 'names' impossible. We may, in fact, distinguish names of different orders as follows: (a) Elementary names will be such as are true 'proper names', i.e. conventional appellations not involving any description. (b) First-order names will be such as involve description by means of a first-order function; that is to say, if $\phi!x$ is a first-order function, 'the term which satisfies $\phi!x$ ' will be a first-order name, though there will not always be an object named by this name. (c) Second-order names will be such as involve a description by means of a second-order function; among such names will be those involving a reference to the totality of first-order names. And so we can proceed through a whole hierarchy. But at no stage can we give a meaning to the word 'nameable' unless we specify the order of names to be employed; and any name in which the phrase 'nameable by names of order n ' occurs is necessarily of a higher order than the n th. Thus, the paradox disappears (Russell and Whitehead, 1910: 63-4).

Essentially the same treatment of the paradox can be presented more simply by means of Tarski's distinction between a language and its metalanguage. According to this doctrine no language can provide for talk about the work of designation done by its own constituent symbols, nor yet for talk of the truth of its own propositional formulae. Any such word as 'designates' must be understood in relation to a particular language, i.e. as short for 'designates in L ', where L is the name of a language under consideration and not itself part of that language. Thus, in order to avoid confusion or contradiction, we must recognise a hierarchy of languages. At the bottom there will be a language L_0 in which we can talk of various things but not of anything linguistic except perhaps spoken sounds and written or printed shapes. Above this there will be a language L_1 in which we can talk of the work of L_0 , and in general above any language L_n another language L_{n+1} in which we can talk of the work of L_n . In an ideally simple scheme it may be assumed that each language L_n is contained in its metalanguage L_{n+1} , but it is essential that the symbols 'designates in L_n ' and 'designates in L_{n+1} ' should be distinguished and that no attempt be made to talk of designation without reference to a particular language, since the penalty for doing so is the kind of contradiction we find in Berry's paradox.

It cannot be denied that some signs presuppose others in the way indicated by Russell. No one, for example, can understand the mathematical notion of a limit unless he already understands the notion of series, and he may fall into absurdity if he assumes that the limit of a series is also a term in it. It is also beyond doubt that words like 'designate' involve at least implicit reference to a language, and that any one who ignores this fact does so at his logical peril. But there is a paradox already in Russell's insistence on the need for avoiding confusion of types, and again in Tarski's insistence on a strict distinction of every language from its metalanguage, namely that each philosopher violates his own rule when formulating it and cannot do otherwise. If Tarski wishes to tell us something of general importance about designation or truth, he must talk about all languages at once, including that which he is using, though he holds that any language which contains provision for talk about its own work is inconsistent. The objection is so obvious that it cannot be overlooked. That Tarski has nevertheless made a declaration against general talk about languages of all levels must be due, I think, to a belief that what he says in ordinary language is only an informal, unofficial introduction to the serious work of building a well-regulated hierarchy of languages fit for the work of science.

It is certainly necessary for scientists to formulate systems that may be called artificial languages, and it may perhaps be useful for certain purposes to construct a hierarchy of artificial languages in which each higher language is the metalanguage of that below it. But whatever is done in this way must be done by use of a natural language. No

formalised language of the sort that interests Tarski has been established, or could be established, in the direct way in which carterers used to teach their horses to behave at the orders 'Gee-up' and 'Whoa'. And the reason is that life presents no situations with which even a mathematical genius might conceivably learn to correlate formulae of abstract geometry or axiomatic set theory before he had learnt his mother tongue. There are indeed contexts especially appropriate for the production of scientific generalities; but they are linguistic contexts such as argument, and they are recognisable only by persons who have already acquired a full natural language, i.e. a system for communication that includes not only signs like 'Gee-up', 'Whoa', 'All gone', 'Nice pussy', and 'Stop kicking', but provision for talk about talk. In short, any distinctions of level that we may ever need to draw between artificial languages made for the purposes of science will be drawn with and within a natural language, since artificial languages of this kind are not, like Esperanto, capable of maintaining an independent existence.

The point may be illustrated most clearly by consideration of the various kinds of definition from which signs may acquire sense in scientific writings. When a new sign is introduced with an explicit definition for economy and perspicuity in the exposition of a formal system, the rule of substitutability by which it is introduced is not a formula of the artificial language to which the sign is added, but a formula of the metalanguage which has been used already in enunciation of rules of inference. Similarly when a set of axioms are said to furnish an implicit definition for a sign that has been used in them without explicit definition, what delimits the range of permissible interpretations for the sign is not the set of axioms, considered merely as a sequence of formulae in the basic language, but rather a declaration in a metalanguage of the way in which the formulae are to be received. And more important still, when a new mathematical sign is defined by abstraction, the linguistic level of the discourse by which it is introduced must be higher than that of the discourse in which it is to be used. When, for example, we use the Greek letter ω for the order type exhibited by the series of natural numbers, we talk at a higher level of language than that in which we do elementary arithmetic. But in explaining at the beginning of this development of mathematics how and why we propose to use ω we must already, it seems, be using language of a level still higher; and this is possibly only because such distinctions of levels are drawn inside one omniscient language with a power of indefinite self-enrichment. As soon as we have full mastery of a natural language we have an instrument adequate in essentials for the whole work of science; and when later we find it necessary to make a distinction of levels, we can do so without attempting the impossible feat of constructing a series of new linguistic instruments from the beginning. For the distinctions of importance in science and philosophy are not absolute distinctions of level in a single hierarchy such as that between 'Snow is white' and 'Tom said that Dick said that

Harry said that snow is white', where the second sentence (if admissible at all) belongs to the meta-meta-metalanguage, but relative distinctions, such as that between real numbers and rational numbers, which may be required in many different kinds of discourse.

If I am right, there is no good reason for banning talk about all number designations, and I do not think that anyone would have supposed for long that there was if the only problem he had in mind was that posed by Berry's paradox. But Poincaré, who was anxious to find arguments against Cantorism, had declared that Richard's paradox (itself a parody of Cantor's diagonal procedure) arose from trying to think of decimals as forming an actually complete totality instead of a potentially infinite supply, and Russell was anxious to show, in the interests of the doctrines he had inherited from Cantor and Frege, that none of the paradoxes recently formulated in arithmetical or set-theoretical terms depended solely on illegitimate use of the ideas of number and quantity. It was for this reason he wrote:

We shall begin by an enumeration of some of the more important and illustrative of the contradictions which have beset mathematical logic, and shall then show how they all embody vicious-circle fallacies, and are therefore all avoided by the theory of types (Russell and Whitehead, 1910: 60).

Clearly his ramified theory of types is sufficient for the purpose of eliminating all the paradoxes he had in mind, since they all involve a kind of reflexiveness banned by his theory. But that is not to say that the theory is necessary for the purpose and plainly true. On the contrary, it involves the sacrifice of a great deal that we should like to retain, as he himself came to realise; and I have argued that it commits him to an untenable theory of language. For a satisfactory solution of Berry's paradox, we have had to show that the trouble arises from a very special sort of mistake made possible by reflexive use of language but not essential to it. Similarly for an understanding of the other four paradoxes with which this chapter is concerned we must notice how reflexive talk may lead to nonsense in certain special circumstances, and it will be convenient to begin this task by examining once more the Liar paradox which has been supposed to prove the inconsistency of any language with pretensions to omniscience.

(3) In any fully human language it is possible for men to talk about talk and about what is said in talk. As might be expected, the most primitive way of indicating what a man has said is by direct quotation, that is to say, by production of another specimen of a pattern of which his utterance was a specimen. Sometimes when we speak in this way of what a man has *said* we are interested in the sounds of his utterance for their own sake; and when this is so, we are careful to indicate the pattern by means

of a new specimen, even though what we say about it is couched in another language. Thus if one Englishman asks another 'What did the Chinaman say?' and the second thinks the first is interested for some reason in the sounds made by the Chinaman, he will reply 'He said . . .' and then do his best to imitate those sounds. But very often when we talk about what has been said we are not interested in the sounds of an utterance for their own sake, but rather in the role of the utterance in communication; and when this is so, we do not hesitate to indicate what was said by producing a specimen of some other sound or sounds which can be used for making the same communication. Thus if one Englishman asks another 'What did the Chinaman say?' and the second, who is familiar with the Chinese language, thinks the first wants to know what the Chinaman communicated (or purported to communicate), he may reply in English, 'He said "Long live Chairman Mao!"'

Obviously there is an ambiguity in the verb 'say' (and similar verbs of other languages) which can only be removed by other elements in a context of use; and it is therefore not surprising that nouns by means of which we refer to what men say sometimes have similar ambiguities. In modern English the word 'sentence' is normally reserved for a form of words with the kind of completeness needed for making a successful communication. It was in this sense, for example, that our teachers used the word when they told us to translate into French the sentence 'My uncle has the pen of the gardener's wife'. But it is clear that the word once had a wider range of use, since it is just an English modification of a Latin word that covered the range of our words 'opinion', 'judgment', 'sentiment'. Similarly the word 'proposition', which is now used most commonly for talking of what utterances exemplify when they do the same work, apart from manifesting states of mind such as belief or curiosity, was sometimes defined in the past by the phrase 'verbal expression of a judgment' and has been used by some philosophers, in particular by Russell in his later writings, to cover a range of meanings almost as wide as that of the German *Satz*.

Unfortunately philosophers who are aware of a difference between the modern usages of 'sentence' and 'proposition' sometimes suppose that a proposition may be identified with the meaning of a sentence. But this is plainly unsatisfactory as an account of the relationship between the two notions. For just as utterances which are specimens of the same proposition may be specimens (or, as philosophers often say, tokens) of different sentences because they belong to different languages or involve different words from the same language, so utterances which are specimens of the same sentence (e.g. 'It is raining here') may be specimens of different propositions because they occur at different times and places or are spoken by different people. There are, it is true, some sentences so constructed that each corresponds to a single proposition in accordance with the customs of the language in which it is made. But these are the purely general sentences of science; and our ability to construct them

depends on our already having a language in which we can refer to individuals by means of the token-reflexive device. Without that device there could be no learning of language, and no use of language at any level of development. Those who suppose it possible in principle to communicate all thoughts by means of 'eternal' context-free sentences overlook the important fact that it is useless to know that you have an appointment for 1 January 1972 if you can never answer a question of the form 'What is the date *today*?'

For a proper understanding of the relation between sentences and propositions, it is necessary to start with the notion of communication. When once we have mastered the essentials of language, we can do many new things with words, including lying, getting married, conducting examinations and passing on orders that we dislike. But the basis of all these strange achievements is our understanding of the various ways of self-expression through sound which have become customary in our society. In particular, if we are to use words consciously for fulfilment of our purposes, we must be familiar with patterns of speech which normally manifest belief, curiosity and desire. These, however, all involve or presuppose presentation of possible states of affairs whose actuality the speaker assumes, questions or desires, as the case may be; and to say that an utterance is a specimen of a proposition is just to say that it presents a possible state of affairs for consideration. Sometimes philosophers (including myself) have confused propositions with possible states of affairs, perhaps because they have assumed too readily that sentences are the only kinds in which to classify utterances for the purposes of a theory of language. But it seems obvious on reflection that what a man desires when he makes a sincere request is not a proposition, but a state of affairs presentable by the specimens of a proposition; and the fact that we talk of believing propositions, rather than of believing states of affairs, can be explained satisfactorily by the consideration that 'belief' is primarily a word for trust in persons and their utterances. In order to remove any danger of confusion which there may be in writing of the proposition that Brutus murdered Caesar, we may, if we choose, follow the practice of G. E. Moore and write of *the proposition 'Brutus murdered Caesar'*. Inverted commas are not, as some philosophers suppose, a device for making a name of the symbol type of which they enclose a token, but simply a device for introduction of a specimen, and an utterance, as we have seen, can be a specimen of a proposition as well as of a sentence. If what we offer is not in fact a specimen of a proposition but something like a token of 'My uncle has the pen of the gardener's wife', said without reference to any uncle or any gardener, then, of course, we shall not succeed in referring to a proposition merely by introducing it with the words 'the proposition'. By contrast the notion of a sentence is a good deal more complicated.

In the first place, we do not call a phrase a sentence unless we think of it as containing all that is needed for manifestation of an attitude such as

belief, curiosity or desire. But provided it has this, it may be accounted a sentence even though it contains no separate provision for presentation of a possible state of affairs. Thus the one word 'Yes' is a sentence suitable for use in reply to an interrogative utterance which is itself a specimen of a proposition. It is possible, however, in many languages for a form of words to have some tokens which are specimens of sentences and others which are not. Thus an utterance of the English words 'it is raining' is a specimen of a sentence if it occurs in relative isolation, but not so if it occurs inside an utterance of the larger group 'if it is raining you had better take your umbrella'. In modern writing and printing the necessary isolation is produced by the capital letter with which a sentence begins and the full stop with which it ends, but in speech the distinction depends on distribution of pauses and contrasts of intonation. In those languages which have special subjunctive and conditional forms of the verb the possibility of using a form of words sometimes as a sentence and sometimes as a mere clause is less than in modern English; and so far as I can see, there might very well be an ordinary spoken language which resembled Frege's ideography in making no provision at all for such doubling of roles. It is a pity, therefore, that in quite recent times some English-speaking logicians have blurred an important distinction by using the word 'sentence' as though it were equivalent to the German *Satz* and therefore applicable to clauses.

Second, those sentences and clauses which may be called propositional phrases, because they contain grammatical provision for presentation of possible states of affairs, may nevertheless have tokens which are specimens of different propositions. This arises, as we have seen, from our dependence on the token-reflexive device and cannot be eliminated from natural human languages.

Third, sentences and other phrases which are propositional in the sense just explained may nevertheless have tokens which are not specimens of any propositions at all. This possibility arises from the fact that some of our partial utterances may have the syntactical forms of designations without in fact designating anything. When this happens the larger utterances in which they occur cannot be specimens of propositions even though they are tokens of grammatically well-formed sentences. An obvious example of such failure would be a present token of the sentence 'The king of France is bald'. Frege, Russell and Hilbert have made different attempts to eliminate such possibilities from the symbolic systems they have devised for mathematics, but in a language suitable for all purposes the risk is inevitable, because there can be no signs of the sort Russell called logically proper names.

With these distinctions in mind, let us examine the sentence 'What I am now saying is false'. Obviously it is well formed according to the rules of ordinary grammar, and it is a propositional phrase in the sense I have explained, but the solution of the Liar paradox consists in showing that no ordinary token of it can be a specimen of a proposition. Anyone who

finds an antinomy in an utterance of the sentence begins by assuming that the phrase 'what I am now saying' is used to refer to something which must of its nature be either true or false: that is essential to development of the argument. For reasons which I have tried to explain it is clear, however, that the Law of Bivalence holds neither of sentences nor of sentence tokens as such, but primarily of propositions and secondarily of their specimens. So in order to get properly started in the production of a paradox, we must assume that the phrase 'what I am now saying' is to be taken as short for 'the proposition of which my present utterance is a specimen'. But if we take it in this sense and draw the natural consequences, we find a very curious situation. The utterance as a whole cannot be a specimen of a proposition unless the opening part designates something; and the opening part cannot designate anything unless the utterance as a whole is a specimen of a proposition. That is to say, there is a vicious circle of preconditions, and so the utterance as a whole cannot be a specimen of any proposition. If anyone doubts that the circle is indeed vicious, he may perhaps be convinced by the reflection that the opening part cannot designate a proposition unless it is possible in principle to make another designation of the same proposition by putting the nominaliser 'that' in front of a specimen of the proposition. For from this it follows that, if the original utterance was a specimen of a proposition, another specimen can be produced by pronouncing a sentence of the form 'It is false that . . .', with the blank filled by a specimen of the proposition. But any such attempt leads only to the fatuous infinite progress, 'It is false that it is false that it is false that . . .'.

The puzzle depends on construction of a sentence in which the subject phrase purports to designate a proposition expressed by utterance of the sentence as a whole. In order to get the appearance of an antinomy, we must arrange for the sentence to be completed with the predicative phrase 'is false' or something equivalent. But the vicious circle of preconditions to which I have drawn attention is not due to choice of that or any other special phrase from among those suitable for talk of propositions. It could be found, for example, in an utterance of the sentence 'What I am now saying was asserted by Aristotle', if that was taken to refer to a proposition which it also expressed. And so we are entitled to conclude that there can be no completion which expresses a proposition already designated by utterance of the opening phrase. It might indeed be perfectly proper for a speaker to pronounce the sentence 'What I am now saying is false', if he did so in a parenthetical way, that is to say, for the purpose of warning his hearers against accepting some suggestion that he had been engaged in expounding at the moment he interrupted himself. But it seems clear beyond all doubt that expression of a proposition can never depend essentially on designation of the same proposition and this principle is sufficient for solution of the Liar paradox.

(4) At first consideration Russell's paradox of the class of all classes that

are not members of themselves seems far removed from the Liar paradox, and we have seen that writers who try to distinguish between mathematical and semantic antinomies count it as mathematical, while grouping the Liar with the inventions of Berry, Richard and Nelson. Russell himself did not hold this view, and it is interesting to notice that his first version of his paradox is concerned with predicates rather than with classes. In *The Principles of Mathematics* he introduces it as follows:

Among predicates, most of the ordinary instances cannot be predicated of themselves, though, by introducing negative predicates, it will be found that there are just as many instances of predicates which are predicable of themselves. One at least of these, namely predicability, or the property of being a predicate, is not negative; predicability, as is evident, is predicable, i.e. it is a predicate of itself. But the most common instances are negative; thus non-humanity is non-human, and so on. The predicates which are not predicable of themselves are, therefore, only a selection from among predicates, and it is natural to suppose that they form a class having a defining predicate. But if so, let us examine whether this defining predicate belongs to the class or not. If it belongs to the class, it is not predicable of itself, for that is the characteristic property of the class. But if it is not predicable of itself, then it does not belong to the class whose defining predicate it is, which is contrary to the hypothesis. On the other hand, if it does not belong to the class whose defining predicate it is, then it is not predicable of itself, i.e. it *is* one of those predicates that are not predicable of themselves, and therefore it does belong to the class whose defining predicate it is – again contrary to the hypothesis. Hence from either hypothesis we can deduce its contradictory (Russell, 1903: 79–80).

And a little later, when he tries to solve the paradox, he writes:

Let us assume that ‘not predicable of oneself’ is a predicate. Then to suppose that this predicate is, or that it is not, predicable of itself, is self-contradictory. The conclusion, in this case, seems obvious: ‘not predicable of oneself’ is not a predicate (ibid.: 102).

I believe that when he wrote the last of the sentences quoted above he was very near the truth, and that he failed to give a wholly satisfactory solution of the problem only because he did not consider carefully enough what was involved in his own use of the word ‘predicate’, though he had said earlier ‘No subtlety in distinguishing is likely to be excessive’ (ibid.: 80).

‘Predicate’ is a technical term of logic and grammar, but it has been used differently in the two sciences. For although both logicians and grammarians agree that a predicate is what is said of a subject, the

former think of what is said as something ascribed in a proposition, while the latter think of it as something pronounced in a sentence or clause. Since the Latin word *praedicatum* was introduced as a rendering of Aristotle's *kategoroumenon*, there can be no doubt that the logician's use is the older; and its primacy is confirmed by the fact that the correlative word 'subject' still seems more at home in its logical than in its grammatical role. If the name 'London' can be described as grammatical subject of the sentence 'London is a large city,' that is only because London, the town, is the subject of discourse to which we ascribe the predicate of being a large city when we pronounce the sentence. In the first of the passages I have quoted above Russell undoubtedly used the word 'predicate' in the traditional logical way, which accords well with his use of the word 'proposition' for what is essentially true or false (*ibid.*: ix). In what follows I shall maintain the same practice strictly; and when I have occasion to talk of what grammarians call a predicate, I shall therefore describe it as a predicative phrase, meaning by this not a phrase whose tokens are all specimens of a predicate, but one of a grammatical style appropriate for use in the expression of predicates.

Just as a token of a propositional phrase is a specimen of a proposition when it presents a state of affairs for consideration, so a token of a predicative phrase is a specimen of a predicate when it presents a property (in a large, non-Aristotelian sense of the word) for consideration in connection with a subject of discourse. At one point in the first of the passages quoted above Russell identifies a predicate with a property. This mistake is like the identification of a proposition with a possible state of affairs, but it has been even commoner among philosophers – indeed almost universal, probably because we can speak of *the predicate being-a-large-city* just as we speak of *the property being-a-large-city*. I recognise that by trying to distinguish them I lay myself open to a charge of multiplying entities beyond necessity, but I think it important to maintain that predicates, like propositions, are kinds to which utterances can belong and yet not phrases such as grammarians study. That we often indicate them by using words like those we use for talking of properties is perhaps no odder than the fact that a phrase such as 'the murder of Caesar' may be used in different contexts for a proposal, an event, and a fact. In order to avoid any danger of confusion we may, if we choose, talk of *the predicate 'is a large city'* just as we talk of *the proposition 'London is a large city'*. But it is important to realise that we may not always succeed in designating a predicate merely because we pronounce the words 'the predicate' before a predicative phrase. For whether or not a token of a predicative phrase is also a specimen of a predicate may depend on non-linguistic facts. Thus no use of the predicative phrase 'saw Merlin the magician' can ever serve to express a predicate, because there was no such person as Merlin; and no use of the predicative phrase 'saw his own wife' can express a predicate unless it is an application of the phrase to a married man. On the other hand, when

we succeed in designating a predicate by pronouncing the words 'the predicate' before a predicative phrase, *every* token of that phrase occurring in a predicative position in a token of a propositional phrase must also be a specimen of a predicate.

Obviously we cannot doubt that the sequence of words 'is not predicable of itself' is a well-formed predicative phrase according to ordinary rules of English grammar. And it seems good sense to say that a token of this phrase may be a specimen of a genuine predicate, as in the statement "The predicate "is human" is not predicable of itself. But Russell was right in thinking that the phrase is not used to express a predicate by anyone who produces the sentence 'The predicate "is not predicable of itself" is not predicable of itself', and also right in thinking that there is a similarity between this new paradox and that of the Liar, though he did not locate the source of the trouble correctly in either case. According to the theory of types, in which he systematised his views some years after the publication of *The Principles of Mathematics*, the sentence which I have just quoted does not express a proposition because no predicate can be affirmed or denied significantly of itself. I believe, on the contrary, that every genuine predicate must divide the whole universe of which it is a member and so be either true or false of itself. This thesis is more radical than that of Frege, who thinks that a concept, or predicate, cannot belong to the universe of objects it divides. But his exclusion of *concepts* in general from the realm of *objects* seems to be solely due to the fact that he allows no place in his symbolism for a distinction between the expression of concepts and the designation of them. If I am right, the fault of the paradoxical sentence cannot be its reflexiveness (since there is nothing wrong in the statement that being a predicate is a predicate) but must lie in some peculiarity of the phrase 'is predicable of itself'. In order to produce the appearance of an antinomy, we have to add the negative particle; but if, as Russell maintains, the paradoxical sentence does not express a proposition, the same must be true also of the sentence 'The predicate "is predicable of itself" is predicable of itself, and conversely. It will be sufficient therefore to examine the simpler positive sentence.

Whereas Russell thinks his paradox can be solved only by denying the significance of all reflexive predication, I wish to suggest that it is enough for his purpose to deny the significance of asking whether such reflexiveness is reflexive, and that, since this denial, unlike the general theory of types, is necessary for independent reasons, it is no real limitation of the possibilities of language. According to this view, Russell's paradox results, as Gödel once conjectured (Gödel, 1944: 150), from ignoring a very special rule something like that in arithmetic against dividing by zero. In order to test the thesis let us consider carefully what is involved in saying that anything is predicable of itself.

We have seen that the expression 'is true' goes properly with talk of propositions. In the same way the expression 'is predicable of' goes properly with talk of predicates, and the correspondence is very close

indeed when the second expression is used, as Russell uses it, with the special sense that Aristotle sometimes gave to *kategorēitai*², namely, 'is predicable *with truth* of' or more shortly 'is true of', 'is satisfied by'. For just as it is only of a proposition (as distinct from a propositional phrase) that we can say it must be either true or false, so it is only of a predicate (as distinct from a predicative phrase) that we can say it must divide the universe exhaustively by being either true or false of any given subject. And similarly, just as 'is true' undoes the nominalising work done by the prefix 'the proposition' when this appears before a specimen of a proposition, so 'is predicable of' undoes the nominalising work of the prefix 'the predicate', with the result that from 'The predicate "is a large city" is predicable of London' we can proceed immediately to 'London is a large city'. In Greek and the symbolism of formal logic, where predicate phrases often come before their grammatical subjects, this principle is even more obvious than in English, but we can easily see that it must hold for any language in which it is possible to designate predicates. Writing 'κ' as an abbreviation for Aristotle's *kategorēitai* and ' $\S x[F(x)]$ ' as an abbreviation for 'the predicate which is satisfied by x if and only if $F(x)$ ', we may put the rule of reduction shortly by saying that, except in contexts where designation of a predicate is required for faithful reporting, any proposition expressed in the form ' $\S x[F(x)]\kappa A$ ' can be expressed equally well in the form ' $F(A)$ ' without either ' $\S$ ' or ' κ '. To reject this would be to suggest that there might conceivably be a predicate which was expressible only by use of ' $\S$ ' and ' κ ' or their equivalents in other symbolism; and that is plainly absurd, since it goes against the fundamental principle of semantics that for propositions and predicates alike expression is logically prior to designation.

In our problem sentence the subject phrase, namely 'the predicate "is predicable of itself"', cannot designate anything unless there is one and the same predicate expressed by the predicative phrase 'is predicable of itself' whenever this occurs in predicative position in a token of a propositional phrase. In other words, 'itself' must be understood here as it is in 'The locomotive moves itself and so does the bus' rather than as it is in 'The locomotive moves itself and the carriages as well'. In a passage which I have quoted Russell slips into writing 'oneself' instead of 'itself', and I think that when he makes this curious change he probably intends us to take the grammatical subject with the sense which ' $\S x(x\kappa x)$ ' has in my symbolism and the whole sentence therefore with the sense of

$$\S x(x\kappa x) \kappa \S x(x\kappa x).$$

For unless we are prepared to render the English phraseology into logical symbolism by the device which logicians call identification of variables, we must immediately give up the claim that we are using the words 'is predicable of itself' to express a single predicate. But when we consider our new formula in detail, we find that it has a very curious property. Although it contains ' κ ' as its central sign, it can never be reduced to

anything simpler in accordance with the rule of reduction set out above. For if we try to apply the rule, putting ' $x\kappa x$ ' for ' $F(x)$ ' and ' $\S x(x\kappa x)$ ' for ' A ', we merely get again what we had at the beginning. It is true that when we first constructed the formula we intended it to be read with the articulation

$$[\S x(x\kappa x)] \kappa [\S x(x\kappa x)],$$

which corresponds best to the suggestion that reflexivity is reflexive. But the sense of ' κ ' requires us to consider it also with the articulation

$$\S x(x\kappa x) \kappa [\S x(x\kappa x)],$$

and once this reading has been admitted there can be no end to the process of reduction. In short, ' κ ' can never be eliminated from its central position, and so the phrase which precedes it cannot designate anything of the appropriate kind, namely, a predicate, just as the subject phrase of 'What I am now saying is true' cannot designate anything of the appropriate kind for its place, namely, a proposition, because there is no way of paraphrasing it without 'is true'.

Clearly there is nothing in all this to make us adopt Russell's theory of types and abandon such theses as

$$\S x(x \text{ is a predicate}) \kappa \S x(x \text{ is a predicate}).$$

For the peculiarity of the situation we have just discovered is to be explained by the peculiarity of the sign ' κ ' and its equivalents in natural languages. When we say with Aristotle ' $A\kappa B$ ', we think of ' κ ' primarily as a copula introduced for application of predicates after these have been designated. Admittedly it has the grammatical form of an ordinary relational sign and can be used with some sort of sense between genuine designations of any kind. If, for example, it occurs in a sentence such as 'London κ Washington', we can read it explicitly as '*is a predicate true of*' and say that the whole sentence expresses a false proposition. But in the use for which it is intended ' κ ' always follows a sign which purports to designate a predicate, that is a sign of the form ' $\S x[F(x)]$ ' or one which could in principle be replaced by something of that form, and this, as we have seen, implies that it can always be eliminated if the sign which it follows genuinely designates a predicate. Thus the thesis cited at the beginning of this paragraph can be restated more simply in the form

$$\S x(x \text{ is a predicate}) \text{ is a predicate,}$$

and in general every reflexive thesis expressible by use of ' κ ' can be expressed also in a fashion in which its predicate can be seen clearly to be the same as its subject and, therefore, different from the predicate of any reflexive thesis with a different subject. From this, however, it follows that ' $x\kappa x$ ' cannot properly be taken as the expression of a single predicate. Obviously it is a predicative phrase from which we can obtain an expression of a proposition by putting a genuine designation in each of