

2nd Edition

## BASIC PHYSICAL CHEMISTRY FOR THE ATMOSPHERIC SCIENCES <br> Second Edition

Modern studies of atmospheres, oceans, and Earth and planetary systems require a good knowledge of basic chemical principles. This book provides a clear, concise grounding in these principles. Undergraduate and graduate students with little formal training in chemistry can work through the chapters, and the numerous exercises, within this book before accessing more advanced texts in atmospheric chemistry, geochemistry, and the environmental sciences.

Basic Physical Chemistry for the Atmospheric Sciences covers the fundamental concepts of chemical equilibria, chemical thermodynamics, chemical kinetics, solution chemistry, acid and base chemistry, oxidationreduction reactions, and photochemistry. This new edition has been updated and revised from the first edition.

In a companion volume entitled Introduction to Atmospheric Chemistry (2000, Cambridge University Press) Peter Hobbs provides an introduction to atmospheric chemistry itself, including its applications to air pollution, acid rain, the ozone hole, and climate change. Together these two books provide an ideal introduction to atmospheric chemistry for a variety of disciplines.

Peter V. Hobbs (University of Washington) is known internationally for his research on many aspects of the atmosphere: clouds, precipitation, aerosols, storms, atmospheric chemistry, and climate. He is the author of the definitive text Ice Physics, the author of Introduction to Atmospheric Chemistry (Cambridge University Press, 2000), co-author (with J. M. Wallace) of one of the most widely used textbooks in meteorology Atmospheric Sciences: An Introductory Survey, and editor of several other books. He has authored more than 300 scientific papers. Professor Hobbs has served on many national and international committees, including the Scientific Steering Committee of the International Global Atmospheric Chemistry Program, and international scientific bodies. He has been a visiting senior research scientist in England, France, Germany and Italy.

# BASIC PHYSICAL CHEMISTRY FOR THE ATMOSPHERIC SCIENCES SECOND EDITION <br> A Companion Text to <br> "Introduction to Atmospheric Chemistry" 

PETER V. HOBBS<br>University of Washington

## CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8ru, UK
Published in the United States of America by Cambridge University Press, New York
www.cambridge.org
Information on this title: www.cambridge.org/9780521780834

## © Cambridge University Press

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 1995
Second edition 2000
A catalogue record for this publication is available from the British Library
Library of Congress Cataloguing in Publication Data
Hobbs, Peter Victor,
Basic physical chemistry for the atmospheric sciences / Peter
V. Hobbs.
p. cm.

ISBN o-52I-78083-7. - ISBN 0-521-78567-7 (pbk)
I. Atmospheric sciences. 2. Atmospheric chemistry. 3. Chemistry,

Physical and theoretical. I. Title.
QC86I.2.H63 1995
551.5'il - dc20 94-19216 CIP
isbn 978-0-52I-78083-4 Hardback
ISBN 978-0-521-78567-9 Paperback
Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate. Information regarding prices, travel timetables, and other factual information given in this work is correct at the time of first printing but Cambridge University Press does not guarantee the accuracy of such information thereafter.

## Contents

Preface to first edition ..... ix
Preface to second edition ..... xi
1 Chemical equilibrium ..... 1
1.1 Some introductory concepts ..... 1
1.2 Equilibrium constants ..... 3
1.3 Reaction quotient ..... 8
1.4 LeChatelier's principle ..... 10
Exercises ..... 12
2 Chemical thermodynamics ..... 17
2.1 The first law of thermodynamics; enthalpy ..... 17
2.2 Enthalpies of reaction and formation ..... 21
2.3 Entropy and the second law of thermodynamics ..... 22
2.4 The third law of thermodynamics; absolute entropies ..... 26
2.5 Criteria for equilibrium and spontaneous transformation ..... 27
2.6 Standard free energy changes ..... 29
2.7 Free energy change and the equilibrium constant ..... 31
2.8 Chemical potential; homogeneous nucleation of water-vapor condensation ..... 34
Exercises ..... 38
3 Chemical kinetics ..... 43
3.1 Reactionrates ..... 43
3.2 Reactionmechanisms ..... 46
3.3 Reactionrates and equilibria ..... 50
3.4 Collision theory of gaseous reactions ..... 52
3.5 The effect of tem perature on reaction rates: the Arrhenius' relation ..... 55
3.6 Catalysis ..... 57
3.7 Half-life, residence time, and renewal time ..... 58
Exercises ..... 61
4 Solution chemistry and aqueous equilibria ..... 68
4.1 Definitions and types of solutions ..... 68
4.2 Solution concentrations ..... 68
4.3 Factors affecting solubility ..... 70
4.4 Colligative properties ..... 71
4.5 Aqueous solutions; electrolytes ..... 72
4.6 Aqueous equilibria ..... 73
4.7 Strong and weak electrolytes; ion-product constant for water ..... 78
Exercises ..... 80
5 Acids and bases ..... 83
5.1 Some definitions and concepts ..... 83
5.2 The nature of $\mathrm{H}^{-}(\mathrm{aq})$ ..... 86
5.3 The Brønsted-Lowry theory; conjugate acid-base pairs ..... 86
5.4 Strengths of acids and bases; acid-dissociation (or ionization) constant ..... 87
5.5 The Lewis theory ..... 88
5.6 The pH scale ..... 88
5.7 Polyprotic acids ..... 90
5.8 Hydrolysis ..... 90
5.9 Buffers ..... 93
5.10 Complex ions ..... 96
5.11 Mass balance and charge balance relations ..... 97
5.12 The pH of rainwater ..... 98
Exercises ..... 101
6 Oxidation-reduction reactions ..... 104
6.1 Some definitions ..... 104
6.2 Oxidation numbers ..... 105
6.3 Balancing oxidation-reduction reactions ..... 108
6.4 Half-reactions in electrochemical cells ..... 115
6.5 Strengths of oxidants and reductants; standard cell and half-cell potentials ..... 116
Contents ..... vii
6.6 Standard cell potentials and free-energy change ..... 124
6.7 The Nernst equation ..... 126
6.8 Redox potentials; Eh-pH diagrams ..... 128
6.9 Gram-equivalent weight and normality ..... 130
Exercises ..... 132
$7 \quad$ Photochemistry ..... 137
7.1 Some properties of electromagnetic waves ..... 137
7.2 Some photochemical terminology and principles ..... 139
7.3 Quantum yields ..... 141
7.4 Rate coefficients for photolysis ..... 143
7.5 Photostationary states ..... 145
7.6 Stratospheric ozone and photochemistry; depletion of stratospheric ozone ..... 146
Exercises ..... 154
Appendix I International system of units (SI) ..... 159
Appendix II Some useful numerical values ..... 160
Appendix III Atomic weights ..... 161
Appendix IV Equilibrium (or dissociation) constants forsome chemical reactions ..... 164
Appendix $V$ Some molar standard Gibbs free energies of formation, molar standard enthalpies (or heats) of formation, and molar absolute entropies at $25^{\circ} \mathrm{C}$ and 1 atmosphere ..... 169
Appendix VI Names, formulas, and charges of some common ions ..... 172
Appendix VII Answers to exercises and hints and solutions to selected exercises ..... 173
Index ..... 187

## Preface to first edition

A short account of the origins of this book will explain its purpose. In the 1970s I coauthored (with John M. Wallace) a textbook for senior undergraduates and first-year graduate students entitled Atmospheric Science: An Introductory Survey (Academic Press, 1977). At the time that text was written it was not considered necessary to include a chapter on atmospheric chemistry. By the early 1990s, when we began to think about a second edition of Atmospheric Science, the importance of atmospheric chemistry was such that it was inconceivable that such a book would not include a substantial chapter on this subject.

In the intervening years I had introduced a section on atmospheric chemistry into the survey course taken by all first-year graduate students in the Atmospheric Sciences Department at the University of Washington. I quickly discovered, however, that many of the students either had no previous instruction in chemistry or had long since forgotten what little they had known. I therefore wrote an (unpublished) primer on physical chemistry for these students; the present book grew out of that primer.

Reviewed herein are some of the fundamental concepts associated with chemical equilibrium, chemical thermodynamics, chemical kinetics, aqueous solutions, acid-base chemistry, oxidation-reduction reactions and photochemistry, all of which are essential to an understanding of atmospheric chemistry. The approach is primarily from the macroscopic viewpoint, which provides the tools needed by the pragmatist. A deeper understanding requires extensive treatment of the electronic structure of matter and chemical bonding, topics that are beyond the scope of this introductory text. This book can be used for either self-instruction, or as the basis for a short introductory class on chemistry, prior to courses in which chemistry is applied to one of the geosciences. In addition to
students (and I use this term in its broadest sense) of atmospheric sciences, I hope this book will be useful to others. It should be suitable, for example, as a precursor to undergraduate and graduate courses in which chemistry is applied to any of the geosciences and environmental sciences.

In keeping with the didactic approach of this book, and the view that any science is best learned by solving problems, I have provided solutions to 50 exercises in the text and posed 112 exercises for the student. Answers to all the quantitative problems, and hints and solutions to selected problems, are given in Appendix VII.

In preparing this book I benefited from the following texts, which are recommended to the reader: Chemistry: An Experimental Science edited by G. C. Pimentel (W. H. Freeman, 1963) gives a broad introduction to chemistry with emphasis on its experimental foundations; Chemistry: The Central Science by T. L. Brown and H. E. LeMay Jr. (Prentice-Hall Inc., 1981) and General Chemistry: Principles and Modern Applications by R. H. Petrucci (Macmillan Pub. Co., 1982) provide more extensive accounts of most of the topics discussed in the present book and deal with many other aspects of chemistry. Finally, for the student who wants to take the next step in chemistry beyond that presented here, University Chemistry by B. H. Mahan (Addison-Wesley, 1965) is highly recommended.

This book was started in 1984 when I was an Alexander von Humboldt Foundation Senior Scientist in Germany, and it was essentially finished in 1993 during a sabbatical at the Instituto FISBAT-CNR, Bologna, Italy. Thanks are due to both of these organizations for their generous support. It is also a pleasure to thank my colleagues Professors Dean Hegg and Conway Leovy, and many students, particularly John Herring and Cathy Cahill, who commented on various drafts of this book and made suggestions for its improvement. I am grateful to the National Science Foundation for supporting my own research in atmospheric chemistry over many years.

Any suggestions or corrections related to this book will be gratefully received.

Seattle
May 1994

## Preface to second edition

The success of the first edition of this book encouraged me to write a companion text entitled Introduction to Atmospheric Chemistry (Cambridge University Press, 2000). On the occasion of the publication of the latter text, the opportunity has been taken to issue a second edition of Basic Physical Chemistry for the Atmospheric Sciences.

In this second edition a number of minor (and a few major) errors have been corrected, and the text has been clarified in several places. I hope that, taken together, these two companion volumes will provide students, researchers, and even the interested layperson, with a sound introduction to the fascinating subject of atmospheric chemistry, which has emerged as a discipline in its own right in just the past few decades.

Comments on this book, which will be gratefully received, can be sent by e-mail to: phobbs@atmos.washington.edu. Current information on the book, including any errata, can be found on http://cargsun2.atmos. washington.edu/~debbie/HobbsWebPage/BasicChem/Info.html.

Peter V. Hobbs

## 1

## Chemical equilibrium

One of the major goals of chemistry is to predict what will happen when various substances come into contact. Will a chemical reaction occur, or will the substances just exist side by side? One way to approach this problem is through the concept of chemical equilibrium, which is the focus of this chapter.

### 1.1 Some introductory concepts

In a balanced equation for a chemical reaction, there are the same number of atoms of each element on the left side of the equation as there are on the right side. For example, the balanced equation for the chemical reaction representing photosynthesis is ${ }^{1, a}$

$$
\begin{equation*}
6 \mathrm{CO}_{2}(\mathrm{~g})+6 \mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightarrow \mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}(\mathrm{~s})+6 \mathrm{O}_{2}(\mathrm{~g}) \tag{1.1a}
\end{equation*}
$$

In a balanced chemical equation (which we will often call a reaction), the relative numbers of the molecules involved in the reaction are given by the numerical coefficients preceding the chemical symbol for the molecule. Thus, Reaction (1.1a) indicates that six molecules of carbon dioxide, $\mathrm{CO}_{2}(\mathrm{~g})$, react with six molecules of water, $\mathrm{H}_{2} \mathrm{O}(\mathrm{l})$, to form one molecule of glucose, $\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}(\mathrm{~s})$, and six molecules of oxygen, $\mathrm{O}_{2}(\mathrm{~g})$

> 6 molecules of $\mathrm{CO}_{2}(\mathrm{~g})+6$ molecules of $\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightarrow$
> 1 molecule of $\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}(\mathrm{~s})+6$ molecules of $\mathrm{O}_{2}(\mathrm{~g})$

Reaction (1.1a) does not necessarily mean that if six molecules of $\mathrm{CO}_{2}(\mathrm{~g})$ are mixed with six molecules of $\mathrm{H}_{2} \mathrm{O}(\mathrm{l})$ they will react completely

[^0]and produce one molecule of $\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}(\mathrm{~s})$ and six molecules of $\mathrm{O}_{2}(\mathrm{~g})$. Some chemical reactions proceed very quickly, others very slowly; and some never reach completion. However, what Reaction (1.1a) does tell us is that at any given instant in time the ratio of the numbers of molecules of $\mathrm{CO}_{2}(\mathrm{~g}), \mathrm{H}_{2} \mathrm{O}(\mathrm{l}), \mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}(\mathrm{~s})$, and $\mathrm{O}_{2}(\mathrm{~g})$ that have reacted is 6:6:1:6.

The relative masses of the various atoms are represented by their atomic weights (dimensionless) referenced to carbon-12 (i.e., a carbon atom containing six protons and six neutrons), where carbon-12 is arbitrarily assigned an exact atomic weight of 12 . Atomic weights are listed in Appendix III. Similarly, the relative masses of molecules are represented by their molecular weights (dimensionless), where the molecular weight is obtained by adding together the atomic weights of all the atoms in the molecule. For example, since the atomic weights of hydrogen and oxygen are 1.008 and 15.999 , respectively, the molecular weight of water $\left(\mathrm{H}_{2} \mathrm{O}\right)$ is $(2 \times 1.008)+15.999=18.015$.

One gram-molecular weight (abbreviation mole or mol) of any compound is a mass of that compound equal to its molecular weight in grams. Thus, 1 mole of water is 18.015 g of water. One mole of any compound contains the same number of molecules as one mole of any other compound. ${ }^{2}$ The number of molecules in 1 mole of any compound is $6.022 \times$ $10^{23}$, which is called Avogadro's number $\left(N_{\mathrm{A}}\right)$. Since the volume occupied by a gas depends on its temperature, pressure, and the number of molecules in the gas, at the same temperature and pressure 1 mole of the gas of any compound occupies the same volume as 1 mole of the gas of any other compound. At standard temperature and pressure (STP), which are defined as $0^{\circ} \mathrm{C}$ and $1 \mathrm{bar}\left(=10^{5} \mathrm{~Pa}\right),{ }^{3}$ the volume occupied by 1 mole of any gas is about 22.4 liters ( L ).

If we now multiply every term in the Relation (1.1b) by $N_{\mathrm{A}}$ we get

$$
\begin{aligned}
& 6 N_{\mathrm{A}} \text { molecules of } \mathrm{CO}_{2}(\mathrm{~g})+6 N_{\mathrm{A}} \text { molecules of } \mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightarrow \\
& 1 N_{\mathrm{A}} \text { molecule of } \mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}(\mathrm{~s})+6 N_{\mathrm{A}} \text { molecules of } \mathrm{O}_{2}(\mathrm{~g})
\end{aligned}
$$

or,

$$
\begin{align*}
& 6 \text { moles of } \mathrm{CO}_{2}(\mathrm{~g})+6 \text { moles of } \mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightarrow \\
& 1 \text { mole of } \mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}(\mathrm{~s})+6 \text { moles of } \mathrm{O}_{2}(\mathrm{~g}) \tag{1.1c}
\end{align*}
$$

Relations (1.1b) and (1.1c) demonstrate how we can move directly from a balanced chemical equation, such as Reaction (1.1a), to a statement about the relative numbers of molecules (1.1b) or the relative numbers of moles (1.1c) involved in the reaction.

Exercise 1.1. An important chemical reaction in atmospheric, earth, and ocean sciences is that of dissolved carbon dioxide with liquid water to form carbonic acid, $\mathrm{H}_{2} \mathrm{CO}_{3}(1)$,

$$
\begin{equation*}
\mathrm{CO}_{2}(\mathrm{~g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \rightarrow \mathrm{H}_{2} \mathrm{CO}_{3}(\mathrm{l}) \tag{1.2}
\end{equation*}
$$

Calculate the mass of carbonic acid that forms for every kilogram of carbon dioxide that reacts with liquid water.

Solution. From the balanced chemical equation (1.2) we see that for every mole of carbon dioxide that reacts with water one mole of carbonic acid is formed. Since the molecular weight of $\mathrm{CO}_{2}$ is 44.01 , the number of moles of $\mathrm{CO}_{2}$ in 1 kg is $1000 / 44.01=22.72$. Therefore, 22.72 moles of carbonic acid will form for every kilogram of $\mathrm{CO}_{2}$ that reacts with water. The molecular weight of carbonic acid is 62.02 ; therefore, the number of grams of carbonic acid in 22.72 moles is $(22.72 \times 62.02)=1409$. Therefore, for every kilogram of $\mathrm{CO}_{2}$ that reacts with water 1.409 kg of carbonic acid are formed.

### 1.2 Equilibrium constants

A vapor is in equilibrium with its liquid when the rate of condensation is equal to the rate of evaporation. An analogous state of equilibrium exists in a chemical system when the rate at which the reactants combine to form products is equal to the rate at which the products decompose to form the reactants. For example, a reaction that plays a role in tropospheric and stratospheric chemistry is

$$
\begin{equation*}
\mathrm{NO}_{2}(\mathrm{~g})+\mathrm{NO}_{3}(\mathrm{~g}) \rightarrow \mathrm{N}_{2} \mathrm{O}_{5}(\mathrm{~g}) \tag{1.3}
\end{equation*}
$$

However, some of the $\mathrm{N}_{2} \mathrm{O}_{5}$ molecules so formed break up again

$$
\begin{equation*}
\mathrm{N}_{2} \mathrm{O}_{5}(\mathrm{~g}) \rightarrow \mathrm{NO}_{2}(\mathrm{~g})+\mathrm{NO}_{3}(\mathrm{~g}) \tag{1.4}
\end{equation*}
$$

Reaction (1.3) is called the forward reaction and Reaction (1.4) the reverse reaction. Reactions (1.3) and (1.4) can be combined as follows

$$
\mathrm{NO}_{2}(\mathrm{~g})+\mathrm{NO}_{3}(\mathrm{~g}) \rightleftarrows \mathrm{N}_{2} \mathrm{O}_{5}(\mathrm{~g})
$$

At every temperature there exists partial pressures of the gases for which the forward and reverse reactions occur at the same rate; under these conditions, the system is said to be in chemical equilibrium.

A general chemical reaction can be represented by

$$
\begin{equation*}
a \mathrm{~A}+b \mathrm{~B}+\ldots \rightleftarrows g \mathrm{G}+h \mathrm{H}+\ldots \tag{1.5}
\end{equation*}
$$

where $\mathrm{A}, \mathrm{B}, \ldots$ and $\mathrm{G}, \mathrm{H}, \ldots$ represent the chemical reactants and products, respectively, and $a, b, \ldots$ and $g, h, \ldots$ their coefficients in the balanced chemical equation. If Reaction (1.5) is in chemical equilibrium, and if the reactants and products are ideal gases or are present as solutes in an ideal solution, ${ }^{4}$ then

$$
\begin{equation*}
\frac{[\mathrm{G}]^{\mathrm{g}}[\mathrm{H}]^{\mathrm{h}} \ldots}{[\mathrm{~A}]^{\mathrm{a}}[\mathrm{~B}]^{\mathrm{b}} \ldots}=K_{\mathrm{c}} \tag{1.6}
\end{equation*}
$$

where $[\mathrm{A}],[\mathrm{B}], \ldots$ and $[\mathrm{G}],[\mathrm{H}], \ldots$ represent the equilibrium concentrations of the reactants and products, and $K_{\mathrm{c}}$ is called the equilibrium constant for the forward reaction (or, simply, the equilibrium constant for the reaction). ${ }^{5}$ Equilibrium constants for some chemical reactions are given in Appendix IV. The value of $K_{\mathrm{c}}$ for a chemical reaction depends only on temperature (see Section 2.2), not on the concentrations of the chemical species or the volume or pressure of the system.
The concentrations in Eq. (1.6) may be expressed in molarity (M). For a gas the molarity is the number of moles of gas per liter of air; for a solution, it is the number of moles of solute per liter of solution. If any of the reactants or products are pure liquids or pure solids, their concentrations (i.e., densities) are essentially constant, compared to the large changes that are possible in the concentrations of the gases. Therefore, the concentrations of liquids and solids are incorporated into the value of $K_{\mathrm{c}}$. The practical consequence of this is that the concentration of any pure liquid or pure solid may be equated to unity in Eq. (1.6).

Exercise 1.2. At $2000^{\circ} \mathrm{C}$ the value of $K_{\mathrm{c}}$ for the reaction

$$
\begin{equation*}
\mathrm{N}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{NO}(\mathrm{~g}) \tag{1.7}
\end{equation*}
$$

is $1.0 \times 10^{-4}$. If the equilibrium concentrations of $\mathrm{O}_{2}(\mathrm{~g})$ and $\mathrm{NO}(\mathrm{g})$ are 50 M and 0.030 M , respectively, what is the equilibrium concentration of $\mathrm{N}_{2}(\mathrm{~g})$ ?

Solution. Application of Eq. (1.6) to Reaction (1.7) yields

$$
K_{\mathrm{c}}=\frac{[\mathrm{NO}(\mathrm{~g})]^{2}}{\left[\mathrm{~N}_{2}(\mathrm{~g})\right]\left[\mathrm{O}_{2}(\mathrm{~g})\right]}
$$

Therefore,

$$
1.0 \times 10^{-4}=\frac{(0.030)^{2}}{\left[\mathrm{~N}_{2}(\mathrm{~g})\right](50)}
$$

and,

$$
\left[\mathrm{N}_{2}(\mathrm{~g})\right]=0.18 \mathrm{M}
$$

At $25^{\circ} \mathrm{C}$ the value of $K_{c}$ for Reaction (1.7) is only $1 \times 10^{-30}$ ! This implies that the equilibrium concentration of $\mathrm{NO}(\mathrm{g})$ is very low at normal temperatures and that the equilibrium "lies to the left" of Reaction (1.7), favoring the reactants. Hence, in the troposphere, negligible quantities of $\mathrm{NO}(\mathrm{g})$ are produced by Reaction (1.7).

In the case of chemical reactions involving only gases, it is often more convenient to express the equilibrium constant for the reaction in terms of the partial pressures of the reactants and products instead of their molarities. However, before doing this we must review the ideal gas equation.

Laboratory experiments show that for a wide range of conditions the pressure $(p)$, volume $(V)$, and temperature $(T)$ of all gases follow closely the same relationship, which is called the ideal gas equation. In SI units (see Appendix I), the ideal gas equation can be written in the following forms. For mass $m$ (in kilograms) of a gas

$$
\begin{equation*}
p V=m R T \tag{1.8a}
\end{equation*}
$$

where $p$ is in pascals, $V$ in cubic meters, $T$ in $\mathrm{K}\left(\mathrm{K}={ }^{\circ} \mathrm{C}+273.15 \simeq{ }^{\circ} \mathrm{C}+\right.$ 273 ), and $R$ is the gas constant for 1 kg of a gas. The value of $R$ depends on the number of molecules in 1 kg of the gas, and therefore varies from one gas to another. Since $m / V=\rho$, where $\rho$ is the density of the gas,

$$
\begin{equation*}
p=R \rho T \tag{1.8b}
\end{equation*}
$$

For 1 kg of gas $(m=1)$, Eq. (1.8a) becomes

$$
\begin{equation*}
p \alpha=R T \tag{1.8c}
\end{equation*}
$$

where $\alpha$ is the specific volume of the gas (i.e., the volume occupied by 1 kg of the gas). One mole of any gas contains the same number of molecules $\left(N_{\mathrm{A}}\right)$. Therefore, the gas constant for 1 mole is the same for all gases and is called the universal gas constant $R^{*}\left(8.3143 \mathrm{~J} \mathrm{deg}^{-1} \mathrm{~mol}^{-1}\right)$. Therefore,

$$
\begin{equation*}
p V=n R^{*} T \tag{1.8d}
\end{equation*}
$$

where $n$ is the number of moles of the gas, which is given by

$$
n=\frac{1000 m}{M}
$$

where 1000 m is the number of grams of the gas and $M$ the molecular weight of the gas. Also,

$$
\begin{equation*}
R^{*}=M \frac{R}{1000} \tag{1.8e}
\end{equation*}
$$

where $R$ is divided by 1000 to obtain the gas constant for 1 g of gas. It can be seen from Eq. (1.8d) that at constant temperature and pressure the volume occupied by any gas is proportional to the number of moles (and therefore the number of molecules) in the gas. The gas constant for 1 molecule of any gas is also a universal constant, called the Boltzmann constant $k$. Since the gas constant for $N_{\mathrm{A}}$ molecules is $R^{*}$

$$
\begin{equation*}
k=\frac{R^{*}}{N_{\mathrm{A}}}=\frac{8.3143}{6.022 \times 10^{23}}=1.381 \times 10^{-23} \mathrm{~J} \mathrm{deg}^{-1} \mathrm{molecule}^{-1} \tag{1.8f}
\end{equation*}
$$

For a gas containing $n_{0}$ molecules per cubic meter, the gas equation can be written

$$
\begin{equation*}
p=n_{0} k T \tag{1.8~g}
\end{equation*}
$$

In chemistry, it is common because it is convenient, to depart from SI units in the gas equation and, instead, to express pressure in atmospheres and volume in liters ( $T$ is still in K ). In this case, for $n_{\mathrm{A}}$ moles of gas A with pressure $p_{\mathrm{A}}$ and volume $V_{\mathrm{A}}$ we can write the ideal gas equation as

$$
\begin{equation*}
p_{\mathrm{A}} V_{\mathrm{A}}=n_{\mathrm{A}} R_{\mathrm{c}}^{*} T \tag{1.8h}
\end{equation*}
$$

where $R_{\mathrm{c}}^{*}$ is the universal gas constant in "chemical units" (indicated by the subscript c); the value of $R_{\mathrm{c}}^{*}$ is 0.0821 L atm deg ${ }^{-1} \mathrm{~mol}^{-1}$. Since $n_{\mathrm{A}} / V_{\mathrm{A}}$ is the number of moles of the gas per liter, that is, the molarity [A] of the gas

$$
\begin{equation*}
[\mathrm{A}]=\frac{n_{\mathrm{A}}}{V_{\mathrm{A}}}=\frac{p_{\mathrm{A}}}{R_{\mathrm{c}}^{*} T} \tag{1.8i}
\end{equation*}
$$

Exercise 1.3. Carbon dioxide occupies about 354 parts per million by volume (ppmv) of air. How many $\mathrm{CO}_{2}$ molecules are there in $1 \mathrm{~m}^{3}$ of air at 1 atm and $0^{\circ} \mathrm{C}$ ?

Solution. Let us calculate first the number of molecules in $1 \mathrm{~m}^{3}$ of any gas at 1 atm and $0^{\circ} \mathrm{C}$ (which is called the Loschmidt number). This is given by $n_{0}$ in Eq. $(1.8 \mathrm{~g})$ with $p=1 \mathrm{~atm}=1013 \times 10^{2} \mathrm{~Pa}$, $T=273 \mathrm{~K}$ and $k=1.381 \times 10^{-23} \mathrm{~J} \mathrm{deg}^{-1} \mathrm{molecule}^{-1}$. Therefore,

Loschmidt number $=\frac{1013 \times 10^{2}}{\left(1.381 \times 10^{-23}\right) 273}=2.69 \times 10^{25}$ molecule $\mathrm{m}^{-3}$
Since, at the same temperature and pressure, the volumes occupied by gases are proportional to the numbers of molecules in the gases, we can write

Volume occupied by $\mathrm{CO}_{2}$ molecules in air
Volume occupied by air $=\frac{\text { Number of } \mathrm{CO}_{2} \text { molecules in } 1 \mathrm{~m}^{3} \text { of air }}{\text { Total number of molecules in } 1 \mathrm{~m}^{3} \text { of air }}$

Therefore,

$$
354 \times 10^{-6}=\frac{\text { Number of } \mathrm{CO}_{2} \text { molecules in } 1 \mathrm{~m}^{3} \text { of air }}{2.69 \times 10^{25}}
$$

Hence, the number of $\mathrm{CO}_{2}$ molecules in $1 \mathrm{~m}^{3}$ of air is ( $354 \times 10^{-6}$ ) $\times\left(2.69 \times 10^{25}\right)=9.52 \times 10^{21}$.

We can now derive an expression for the equilibrium constant for a chemical reaction involving only gases in terms of the partial pressures of the gases. From Eqs. (1.6) and (1.8i)

$$
K_{\mathrm{c}}=\frac{\left[p_{\mathrm{G}} / R_{\mathrm{c}}^{*} T\right]^{\mathrm{s}}\left[p_{\mathrm{H}} / R_{\mathrm{c}}^{*} T\right]^{\mathrm{h}} \ldots}{\left[p_{\mathrm{A}} / R_{\mathrm{c}}^{*} T\right]^{\mathrm{a}}\left[p_{\mathrm{B}} / R_{\mathrm{c}}^{*} T\right]^{\mathrm{b}} \ldots}=\frac{p_{\mathrm{G}}^{\mathrm{g}} p_{\mathrm{H}}^{\mathrm{h}} \cdots}{p_{\mathrm{A}}^{\mathrm{a}} p_{\mathrm{B}}^{\mathrm{b}} \ldots}\left(R_{\mathrm{c}}^{*} T\right)^{\mathrm{An}^{\mathrm{n}}}
$$

or,

$$
\begin{equation*}
K_{\mathrm{c}}=K_{\mathrm{p}}\left(R_{\mathrm{c}}^{*} T\right)^{\mathrm{An}} \tag{1.9a}
\end{equation*}
$$

where,

$$
\begin{equation*}
K_{\mathrm{p}}=\frac{p_{\mathrm{G}}^{\mathrm{g}} p_{\mathrm{H}}^{\mathrm{h}} \ldots}{p_{\mathrm{A}}^{\mathrm{a}} p_{\mathrm{B}}^{\mathrm{b}} \ldots} \tag{1.9b}
\end{equation*}
$$

and,

$$
\begin{equation*}
\Delta n=(a+b+\ldots)-(g+h+\ldots) \tag{1.9c}
\end{equation*}
$$

$K_{\mathrm{p}}$ is generally used as the equilibrium constant in problems involving gaseous reactions. As in the case of $K_{\mathrm{c}}$, terms for pure liquids and solids do not appear in the expression for $K_{\mathrm{p}}$, and the coefficients for these
terms are taken to be zero in the expression for $\Delta n$. Note that the units, as well as the numerical values, of $K_{\mathrm{c}}$ and $K_{\mathrm{p}}$ may differ. For example, for Reaction (1.3) the units of $K_{\mathrm{c}}$ are those of

$$
\begin{gathered}
\frac{\left[\mathrm{O}_{3}(\mathrm{~g})\right]}{[\mathrm{O}(\mathrm{~g})]\left[\mathrm{O}_{2}(\mathrm{~g})\right]} \text { or } \frac{\mathrm{M}}{(\mathrm{M})(\mathrm{M})}=\mathrm{M}^{-1}, \text { the units of } K_{\mathrm{p}} \text { are } \\
\frac{p_{\mathrm{O}_{3}}}{\left(p_{\mathrm{O}}\right)\left(p_{\mathrm{O}_{2}}\right)} \text { or } \frac{\mathrm{atm}}{(\mathrm{~atm})(\mathrm{atm})}=\mathrm{atm}^{-1}
\end{gathered}
$$

Nevertheless, it is common practice in chemistry not to indicate the units of equilibrium constants, with the understanding that when $K_{c}$ is used the concentrations are in molarity, and when $K_{\mathrm{p}}$ is used the partial pressures are in atmospheres.

Exercise 1.4. Ammonia, $\mathrm{NH}_{3}(\mathrm{~g})$, is produced commercially from the reaction of hydrogen, $\mathrm{H}_{2}(\mathrm{~g})$, and atmospheric nitrogen, $\mathrm{N}_{2}(\mathrm{~g})$, at high temperatures. If $\mathrm{H}_{2}(\mathrm{~g}), \mathrm{N}_{2}(\mathrm{~g})$, and $\mathrm{NH}_{3}(\mathrm{~g})$ attain equilibrium at $472^{\circ} \mathrm{C}$ when their concentrations are $0.12 \mathrm{M}, 0.04 \mathrm{M}$, and 0.003 M , respectively, calculate the values of $K_{\mathrm{c}}$ and $K_{\mathrm{p}}$ for the reaction at $472^{\circ} \mathrm{C}$.

Solution. The balanced chemical equation for the reaction is

$$
3 \mathrm{H}_{2}(\mathrm{~g})+\mathrm{N}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{NH}_{3}(\mathrm{~g})
$$

Hence, from Eq. (1.6)

$$
K_{\mathrm{c}}=\frac{\left[\mathrm{NH}_{3}(\mathrm{~g})\right]^{2}}{\left[\mathrm{H}_{2}(\mathrm{~g})\right]^{3}\left[\mathrm{~N}_{2}(\mathrm{~g})\right]}=\frac{(0.003)^{2}}{(0.12)^{3}(0.04)}=0.1
$$

From Eq. (1.9a)

$$
K_{\mathrm{c}}=K_{\mathrm{p}}\left(R_{\mathrm{c}}^{*} T\right)^{\mathrm{An}}
$$

where $K_{\mathrm{c}}=0.1, R_{\mathrm{c}}^{*}=0.0821 \mathrm{~L}$ atm $\mathrm{deg}^{-1} \mathrm{~mol}^{-1}, T=745 \mathrm{~K}$ and, from Eq. $(1.9 \mathrm{c}), \Delta n=(3+1)-(2)=2$. Therefore,

$$
K_{\mathrm{p}}=\frac{0.1}{(0.0821 \times 745)^{2}}=3 \times 10^{-5}
$$

### 1.3 Reaction quotient

If the general chemical reaction represented by Eq. (1.5) is not in equilibrium, we can still formulate a ratio of concentrations that has the same form as Eq. (1.6). This is called the reaction quotient, $Q$

$$
\begin{equation*}
Q=\frac{[\mathrm{G}]^{\mathrm{g}}[\mathrm{H}]^{\mathrm{h}} \ldots}{[\mathrm{~A}]^{\mathrm{a}}[\mathrm{~B}]^{\mathrm{b}} \ldots} \tag{1.10}
\end{equation*}
$$

Clearly, if $Q=K_{\mathrm{c}}$, the reaction is in chemical equilibrium. If $Q<K_{\mathrm{c}}$, the reaction is not in equilibrium, and it will proceed in the forward direction until $Q=K_{\mathrm{c}}$. If $Q>K_{\mathrm{c}}$, the reaction will proceed in the reverse direction until $Q=K_{\mathrm{c}}$.

Exercise 1.5. If 0.80 mole of $\mathrm{SO}_{2}(\mathrm{~g}), 0.30$ mole of $\mathrm{O}_{2}(\mathrm{~g})$, and 1.4 mole of $\mathrm{SO}_{3}(\mathrm{~g})$ simultaneously occupy a volume of 2 L at 1000 K , will the mixture be in equilibrium? If not, in what direction will it proceed to establish equilibrium? Consider only the species $\mathrm{SO}_{2}(\mathrm{~g}), \mathrm{O}_{2}(\mathrm{~g})$, and $\mathrm{SO}_{3}(\mathrm{~g})$ in the reaction

$$
\begin{equation*}
2 \mathrm{SO}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{SO}_{3}(\mathrm{~g}) \tag{1.11}
\end{equation*}
$$

with $K_{\mathrm{c}}=2.8 \times 10^{2}$.
Solution. The reaction quotient for Reaction (1.11) is

$$
Q=\frac{\left[\mathrm{SO}_{3}(\mathrm{~g})\right]^{2}}{\left[\mathrm{SO}_{2}(\mathrm{~g})\right]^{2}\left[\mathrm{O}_{2}(\mathrm{~g})\right]}
$$

To evaluate the initial value of $Q$ we must determine their initial molarities. These are: for $\mathrm{SO}_{2}=0.80 / 2=0.40 \mathrm{M}$, for $\mathrm{O}_{2}=0.30 / 2=$ 0.15 M , and for $\mathrm{SO}_{3}=1.4 / 2=0.70 \mathrm{M}$.

Hence,

$$
Q=\frac{(0.70)^{2}}{(0.40)^{2}(0.15)}=20
$$

Since this value of $Q$ is not equal to $K_{c}$ (namely, $2.8 \times 10^{2}$ ), the initial mixture is not in equilibrium. Moreover, since $Q<K_{\mathrm{c}}$, Reaction (1.11) will proceed in the forward direction.

Exercise 1.6. What are the equilibrium concentrations of $\mathrm{SO}_{2}(\mathrm{~g})$, $\mathrm{O}_{2}(\mathrm{~g})$, and $\mathrm{SO}_{3}(\mathrm{~g})$ in Exercise 1.5?

Solution. If $y$ moles (or $\frac{\mathrm{y}}{2} \mathrm{M}$ ) of $\mathrm{SO}_{3}(\mathrm{~g})$ are formed, it follows from Reaction (1.11) that $y$ moles (or $\frac{y}{2} \mathrm{M}$ ) of $\mathrm{SO}_{2}(\mathrm{~g})$ and $\frac{\mathrm{y}}{2}$ moles (or $\frac{y}{4} \mathrm{M}$ ) of $\mathrm{O}_{2}(\mathrm{~g})$ disappear. If this change establishes chemical equilibrium we have


[^0]:    ${ }^{\text {a }}$ Numerical superscripts in the text $(1,2, \ldots$, etc.) refer to Notes at the end of each chapter.

