

LECTURE NOTES IN LOGIC

RECURSION THEORY

JOSEPH R. SHOENFIELD



Recursion Theory

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JOSEPH R. SHOENFIELD works in the Department of Mathematics at Duke University, North Carolina.

LECTURE NOTES IN LOGIC

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LECTURE NOTES IN LOGIC 1

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Introduction

These notes originated in a one semester course given several times at Duke University. Students were generally graduate students in mathematics or computer science. They were expected to have a considerable degree of mathematical maturity. While there were no specific prerequisites, it was desirable that the student has some knowledge of elementary logic and computer programming. To appreciate some of the applications, it was desirable to have at least a slight acquaintance with some other mathematical topics, such as group theory and Borel sets.

My main object was to prepare the student for studying advanced books and journal articles in recursion theory. I therefore attempted to introduce as many as possible of the topics which are of interest in contemporary research without going deeply into any of them. One topic usually treated in the course which did not make its way into these notes is complexity theory; I hope that someone more competent than I will write an introduction to this topic suitable for serious mathematicians.

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