Geotechnical Safety and Risk 🚺



Editors: L.M. Zhang, Y. Wang, G. Wang and D.Q. Li



GEOTECHNICAL SAFETY AND RISK IV

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Preface

The 4th International Symposium on Geotechnical Safety and Risk (4th ISGSR) was organised by the Hong Kong University of Science and Technology under the auspices of the Geotechnical Safety Network (GEOSNet; Chair, Daniel Straub; Co-chair Limin Zhang), Technical Committee TC304 on Engineering Practice of Risk Assessment and Management (Chair, K.K. Phoon) and Technical Committee TC205 on Safety and Serviceability in Geotechnical Design (Chair, Brian Simpson) of the International Society of Soil Mechanics and Geotechnical Engineering (ISSMGE). The Symposium was also supported by Hong Kong Geotechnical Society, the Geotechnical Division of the Hong Kong Institution of Engineers, Chinese Institution of Soil Mechanics and Geotechnical Engineering, the Engineering Risk and Insurance Branch of China Civil Engineering Society, and American Society of Civil Engineers—Hong Kong Section.

The 4th ISGSR was a continuation of a series of symposiums and workshops on geotechnical risk and reliability starting with LSD2000 in Melbourne, Australia, IWS2002 in Tokyo and Kamakura, Japan, LSD2003 in Cambridge, USA, Georisk2004 in Bangalore, India, Taipei2006 in Taipei, 1st ISGSR in Shanghai, China in 2007, 2nd ISGSR in Gifu, Japan in 2009 and 3rd ISGSR in Munich, Germany in 2011.

Safety, reliability, and risk assessment and management have attracted growing interests of the geotechnical community in recent years due to the frequent occurrences of natural and man-made disasters and the needs for safe and cost-effective design, construction and operations of infrastructures. At the same time there is an increasing expectation of the general public that requires the engineering community to provide quantitative information concerning risks posed by geotechnical hazards. The 4th ISGSR provided an excellent opportunity to better understand the geotechnical safety and risk management issues in engineering practices and research. The proceedings contain seven invited keynotes and 69 accepted papers from 28 countries and regions. Each accepted paper in the conference proceedings was subject to review by two peers. These papers cover six themes: (1) geotechnical uncertainty and variability, (2) geohazards such as landslides, earthquakes and climate changes, (3) reliability and risk analysis, (4) reliabilitybased design and limit-state design in geotechnical engineering, (5) risk assessment and management in geotechnical engineering and infrastructural projects, and (6) practical applications.

One of the highlights of this symposium was the 3rd Wilson Tang Lecture. The lecture was inaugurated during the 2nd ISGSR in Gifu to recognize and honor the significant contributions of the late Professor Wilson Tang, who was one of the founding researchers in geotechnical reliability and risk. The first lecture was given by Prof. T. H. Wu of the Ohio State University and the second lecture by Prof. Y. Honjo of Gifu University. The 3rd lecture was given by Prof. Suzanne Lacasse of Norwegian Geotechnical Institute during the 4th ISGSR.

The credit for the proceedings goes to the authors and reviewers. The publication of the proceedings was financially supported by the National Basic Research Program of China (Grant No. 2011CB013500) and the National Natural Science Foundation of China's Oversea Collaborative Research Program (Grant No. 51129902).

Limin Zhang Chairman of the Regional Organising Committee August 2013, The Hong Kong University of Science and Technology, HKSAR This page intentionally left blank

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1 Wilson Tang lecture

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An homage to Wilson Tang: Reliability and risk in geotechnical practice—how Wilson led the way

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ABSTRACT: The paper is in homage to Professor Wilson Tang for his inspiration to fellow engineers in the area of geotechnical engineering. The role of statistics, probability and reliability in geotechnical engineering is first outlined. Examples of solutions based on Wilson Tang's pioneering work are presented: uncertainties in soil parameters; Bayesian updating applications; reliability of tailings dam; model uncertainty and calibration of safety factor. Two aspects of special interest to Wilson Tang are also briefly discussed: improving the cost-effectiveness of site investigations and the reliability of offshore structures.

1 INTRODUCTION

This paper is in homage to Professor Wilson Tang (1943–2012) for his inspiration to fellow engineers to pursue his pioneering work in the application of reliability and risk in geotechnical engineering. The paper illustrates how the work initiated by Wilson Tang led the way to further developments by his colleagues, research partners, friends and practitioners in the geotechnical profession. Case studies based on Wilson Tang's learnings are provided for several geotechnical applications.

Wilson Tang's work covered a wide range of expertise areas within statistics, probability and reliability. These include: characterization of soil properties and random field models, reliability methods, structural reliability-based design, Bayesian updating and decision-making. Wilson applied reliability concepts to, for example, site investigation and geotechnical anomaly characterization, the analysis of slopes and offshore structures, earthquake hazard, the analysis of foundation solutions, model uncertainty and the calibration of safety factors. Wilson's work also covered the reliability of landfill systems, accident hazard analysis and prediction, and road network reliability.

Wilson Tang was a graduate student of the second author, post-doctoral fellow at NGI, the external doctoral examiner for the third author, and an inspiration and friend to all four authors. This is only a random cross-section of three generations of engineers at NGI. His radius of influence is so much wider, as he touched the lives of many in so many ways.

Examples of Wilson Tang's lasting influence are the invited papers for the 2013 ISGSR.

The keynote speakers come from three continents, have very different backgrounds and different career profiles and are at different stages in their engineering profession. Yet, each of these recognized keynote lecturers has been influenced by Wilson's work, as witnessed by the list in Table 1.

After introductory comments on the role of statistics, probability and reliability in geotechnical engineering, the paper emphasizes four topics with solutions in large part developed thanks to the foundations laid by Wilson Tang:

- Uncertainties in soil parameters in practice.
- New applications for Bayesian updating.
- Reliability of containment facility.
- Model uncertainty and calibration of safety factors.

Table 1. Keynote contributions at ISGSR 2013.

Author	Title of keynote paper
Gilbert et al. (2013)	Advances in geotechnical risk and reliability for offshore applications
Griffiths et al. (2013)	Homogenization of geomaterials using the random finite element method
Huang et al. (2013)	Selecting optimal probability models for geotechnical reliability analysis
Juang et al. (2013)	Robust design of geotechnical barriers—A new design perspective
van Staveren (2013)	Integrated geo risk management: crossing boundaries
Wong (2013)	Is landslide risk quantifiable and manageable?

Furthermore, two additional aspects of special interest to Wilson Tang are briefly discussed: the use of probabilistic concepts to improve the costeffectiveness of site investigations and to estimate the reliability of offshore structures.

2 ROLE OF STATISTICS, PROBABILITY AND RELIABILITY IN GEOTECHNICAL ENGINEERING

Wilson Tang and his co-author A. H-S Ang firmly believed that the best and most effective way for engineers to learn the concepts of probability, statistics and risk was through applications of the principles to engineering problems. It was important for them to be able to show the usefulness of the method in physically meaningful terms.

The motivation for probabilistic and statistical decision theory is multi-fold: uncertainties are unavoidable, and they need to be considered and reduced where possible; the need for a systematic development of design criteria for engineering designs; and quantitative risk assessment offers a logical framework for decision-making and documentation of the steps towards the decisions. In light of uncertainties, the role of probability and statistics ranges from the description of the basic information to the development of formulations as basis for design and decision-making (Ang & Tang 2007). Especially in geotechnical engineering, our knowledge is imperfect.

As part of design and decision-making under uncertainty (Høeg 1996), the properties of inherently inhomogeneous and highly variable soil materials must be considered. Natural deposits typically have irregular layers of clay, silt, sand, gravel or a combination thereof. The soil properties that affect strength and compressibility often have a wide range of variation. The information comes from the local geology, and limited soil or rock sampling and limited coverage of the area of concern with *in situ* tests.

The calculated bearing capacity (stability) can vary widely according to the analysis parameters and the calculation method selected. The calculation will therefore involve some possibility of overestimating the actual resistance provided by the soil, or leading to unnecessarily high costs due to overly conservative design. There will therefore always be a finite probability that the forces on a structure founded on or in soil or rock can cause damage, or the total collapse, of the structure.

Statistics, probability, reliability and the decisions made on the basis of these concepts offer remarkable tools that can quantify the trade-off between cost and tolerable probability of nonperformance (failure) and risk (sentence slightly modified from Ang & Tang 2007). Such considerations, and as exemplified by Wilson Tang's long list of publications, can be extended to the entire chain of geotechnical design steps, from site investigation and soil testing, selection of design parameters to design calculations, reliability of a design method and selection of required safety factor(s).

The examples presented in this paper illustrate the role of statistics, probability and reliability in geo-engineering.

In the books "Probability concepts in engineering, planning and design" (Volume I and II 1975; 1984) and "Probability concepts in engineering— Emphasis on applications to civil and environmental engineering" (2nd ed. of Volume I–2007), Ang and Tang published two of the first books that made the probability concepts easily accessible to geotechnical engineers.

From an engineering standpoint, the Ang and Tang books, together with Benjamin & Cornell (1970) were instrumental in pointing the way for most users, including the authors of this paper. As a doctoral student at Stanford University, the young Wilson Tang greatly benefitted from the lectures and discussion with Professor Jack Benjamin.

Later books, especially Baecher & Christian (2003) and Fenton & Griffiths (2008) are of special relevance for geotechnical engineers. Vick (2002) and Jordaan (2008), for example, published books on decisions under uncertainty and continue the legacy of Wilson Tang. Yet, the first Ang and Tang's books have the far-reaching influence of being the pioneers for geotechnical engineers.

3 UNCERTAINTIES IN SOIL PARAMETERS IN PRACTICE

The terms 'aleatory' uncertainties (those associated with natural randomness) and 'epistemic' uncertainties (those associated with uncertainties in prediction and estimation) are known today. The terms 'aleatory' and 'epistemic' were first used by Hacking (1975) and Cornell, C.A (1982, Personal comm., Pau, France).

The importance of quantifying the variability in geotechnical design parameters is not adequately recognized in practice. Quantifying variability is a positive contribution as its consistent modelling and utilization lead, with limited additional effort, to more rational and economic designs. The modelling of soil variability belongs to one of two categories: (a) geostatistics, focusing on the interpolation of available data to estimate other values at the same location; and (b) reliability-based engineering, focusing on characterization for reliability/ risk assessment.

A soil variability analysis can include three steps, each with increasing level of complexity:

- statistical analysis of mean, variance (standard deviation) and probability density function;
- 2. analysis of spatial correlation describing the variation of the soil property in space; and
- 3. spatial averaging and variance reduction when averaging over a volume.

Integrated approaches making use of Monte Carlo simulation, finite element analysis and the results of high-level soil variability investigations have gained interest over recent years. These approaches allow enhanced modelling of the behavior of geotechnical systems, where spatial heterogeneity of soil properties invariably plays an important role.

Two aspects are described in more detail below: the statistical analysis of a random variable (level 1) and the analysis of spatially random variables (level 3). An example is then described for Troll clay offshore Norway.

3.1 *Level 1: Statistical analysis of random variable*

Following the precepts in Ang and Tang (1975), Figure 1 illustrates the phases leading to the probabilistic modelling of a random variable. The descriptive part includes the calculation of sample moments and visual inspection of data and histograms. The inferential part includes the selection of a probability density function type and the distribution parameters, and goodness-of-fit testing. The dashed lines indicate that the results of the descriptive analysis can be used in the inferential analysis; however, inference could also be performed without prior statistical description.

Different Probability Density Functions (PDF) have been used. The distributions are site- and parameter-specific. Based on cone penetration



Figure 1. Descriptive and inferential analysis of a random variable (Uzielli et al. 2006b).

data from artificial and natural deposits (Fig. 2), Popescu et al. (1998) observed that the distribution of soil strength in shallow layers were prevalently positively skewed, while for deeper soils the corresponding distributions tended to follow more symmetric distributions.

The resulting PDF's are in all cases close to the normal or lognormal PDF's. Lacasse & Nadim (1996) reviewed the probability distribution for several soil properties (Table 2).

3.2 Level 2: Spatial correlation analysis

Second-moment statistics alone are unable to describe the spatial variation of soil properties, whether measured in the laboratory or in-situ. Two sets of measurements may have similar secondmoment statistics (i.e. mean and standard deviation) and statistical distributions, but could display substantial differences in spatial distribution.

As part of spatial averaging effect and variance reduction due to spatial averaging, scale of fluctuation and the spatial coefficient of variation of inherent variability are descriptors of a random field. Uzielli et al (2006b) provided a review of the calculation methods for such parameters.

3.3 *Level 3: Modelling of spatially random variables*

The description of a random field through a mean, standard deviation, a scale of fluctuation and a spatial correlation function is useful to characterize a spatially variable soil property (e.g. Vanmarcke 1977, 1983; Elkateb et al. 2003; and Jaksa 2006).

If spatial variability of soil properties is included in an engineering model, stresses and/or displacements may change compared to the homogeneous case. A design that does not take spatial variability into account is biased towards the conservative side and therefore will lead to more costly solutions. One of the most important benefits of random field modelling is the capacity to simulate data series. By using sets of random field simulations and implementing the variability in non-linear finite element meshes, the Monte Carlo technique, for example, can be used to predict reliability of geotechnical systems with spatially variable properties.

Recent studies have focused on combining random fields, non-linear finite element analysis and Monte Carlo simulation to investigate the reliability of geotechnical systems including the variability of soil properties. The studies suggest that:

 When soils are modelled as spatially variable, the failure mechanisms seem different and more complex than in the case of deterministic properties.



Figure 2. Probability distributions for Cone Penetration (CPT) and Standard Penetration (SPT) Tests (Popescu et al 1998).

Table	2.	Probability	distributions	for	different	soil
proper	ties	(adapted from	m Lacasse & N	Vadii	n 1996).	

Soil property	Soil type	PDF
Cone resistance	Sand, clay	N/LN
Undrained shear	Clay (triaxial tests)	LN
strength	Clayey silt	Ν
Normalized undrained shear strength	Clay	N/LN
Plastic limit	Clay	Ν
Submerged unit weight	Clay, silt, sand	Ν
Friction angle	Sand	Ν
Void ratio, porosity	Clav. silt. sand	Ν
Overconsolidation ratio	Clay	N/LN

- There generally exists a critical correlation distance which corresponds to a minimum reliability.
- Phenomena governed by highly non-linear behavior laws are affected the most by spatial variations.

Variance reduction alone cannot convey a comprehensive picture of the implications of spatial variability on the behavior of a geotechnical system. Both statistical variability (i.e. secondmoment) and spatial variability (i.e. spatial correlation) of soil properties affect the reliability of geotechnical systems.

The number of studies making use of random field simulation, finite elements and Monte Carlo simulation is still limited. The importance of the results so far, however, should be a stimulus for the transposition of results to practice.

Popescu et al. (2005) investigated the differential settlements and bearing capacity of a rigid strip foundation on an overconsolidated clay layer. The undrained strength of the clay was modelled as a non-normal random field. The deformation modulus was assumed to be perfectly correlated to undrained shear strength. The settlements (uniform and differential settlements) were computed with non-linear finite elements in a Monte Carlo simulation framework. Anisotropy in spatial correlation was addressed, with the horizontal scale of fluctuation exceeding the vertical scale of fluctuation by a factor of 10.

Figure 3a shows the contours of maximum shear strain for a uniform soil deposit with undrained strength of 100 kPa and for a normalized vertical displacement at the center of foundation $\delta B = 0.1$. Different sample realizations of soil properties corresponded to fundamentally different failure surfaces. Figure 3b shows one sample realization where the spatial distribution of undrained strength is not symmetric with respect to the foundation. The configuration at failure, shown in Figure 3c, involves a rotation as well as vertical settlement. The repeated finite-element analysis allows an appreciation of the combined settlement and rotation of the footings, which could not be inferred from deterministic bearing capacity calculations (i.e. neglecting spatial variability). In general, the failure surfaces were not variations around the deterministic failure surface. There was also a significant reduction in the bearing capacity compared to the deterministic case. Figure 3d shows that the pressure required to induce a given settlement is always higher in the deterministic case.

Fenton and Griffiths (2005) investigated the reliability of shallow foundations against serviceability limit state failure, in the form of excessive and differential settlement, both for a single footing and for two footings. Figure 4 (left) shows cross-sections through finite element meshes of the single footing and the two footings. Figure 4 (right) provides the 3-D finite element mesh for the two-footings case. The elastic modulus of the soil was modelled as a lognormal random field with an isotropic correlation structure.

Fenton et al. (2005) investigated the failure and the reliability of a two-dimensional frictionless wall retaining a cohesionless drained backfill. Soil friction angle and unit weight are modelled as spatially variable properties using lognormal random fields with single exponential-type correlation structures. Figure 5 shows the active earth displacements for two realizations of the finite element mesh. The



Figure 3. Results of investigation on homogeneous and spatially random foundation soil (Popescu et al. 2005).



Figure 4. *Left:* Single footing and two footings founded on a spatially heterogeneous soil; *Right:* 3D finite element mesh of spatially heterogeneous soil volume supporting two footings (Fenton & Griffiths 2005).



Figure 5. Active earth displacements for two realizations with same correlation distance and coefficient of variation of the random field of soil friction angle (Fenton et al. 2005).

location and shape of the failure surface is strongly related to the presence of weaker soil zones (shown in lighter colors) and is, in both cases, markedly different from the shapes assumed in earth pressure theory.

Griffiths et al. (2013) brings new developments on soil variability and random finite element analysis.

3.4 Application to Troll clay

Uzielli et al. (2006a) did an uncertainty-based geotechnical characterization of the Troll clay, a site offshore Norway for the world's largest gravity structure. Second-moment statistics were obtained from laboratory and *in situ* tests. Bayesian updating combined the values of undrained shear strength resulting from triaxial compression tests and piezocone tests. Some of the results are presented herein.

The authors believe that the approach followed would have been close to what Wilson Tang would have done himself, if he had been asked to interpret and calculate the uncertainty in the Troll data.

The characterization consisted of four steps: (a) visual inspection of data by soil unit and preliminary second-moment data analysis; (b) identification of a deterministic trend function and decomposition; (c) identification of a suitable uncertainty model; and (d) quantification of the uncertainty (mean, variance, standard deviation or coefficient of variation). The Kendall's tau statistic test (Uzielli et al. 2006b) was run to check whether or not the data were statistically independent. A trend function was obtained by regression analysis (Ang & Tang 2007). An uncertainty model was used to merge the different uncertainty components to estimate the total uncertainty.

3.4.1 Laboratory data

The results of anisotropically Consolidated Undrained Triaxial Compression (CAUC) and constant volume (undrained) Direct Simple Shear (DSS) tests were used (Fig. 6). No outliers were evident from visual inspection. The data from depths 0 to 5 m were excluded for this analysis. The following model for the total Coefficient of Variation (COV_{tot}) was used:

$$COV_{tot}^2 = COV_{\omega}^2 + COV_m^2 + COV_{se}^2 \tag{1}$$

where COV_{ω} is the coefficient of variation of inherent variability, representative of aleatory uncertainty; COV_m is the coefficient of variation of measurement error; and COV_{SE} is the coefficient of variation of statistical estimation uncertainty.

Figures 7 (CAUC data) and 8 (DSS data) and Table 3 present the second-moment estimates of



Figure 6. Undrained shear strength versus depth from CAUC and DSS for Troll clay (Unit 1 and Unit 2).



Figure 7. CAUC undrained shear strength: (a) trends and standard deviations; (b) residuals of detrending (Uzielli et al. 2006a).



Figure 8. DSS undrained shear strength: (a) trends and standard deviations; (b) residuals of detrending (Uzielli et al. 2006a).

Table 3. Uncertainty components and total uncertainty for undrained shear strength (Uzielli et al. 2006a).

Lab test	CAUC		DSS	
Clay unit	Unit 1	Unit 2	Unit 1	Unit 2
μ_{t} (kPa)*	28.4	80.5	25.8	69.2
COV	0.09	0.06	0.21	0.11
COV	0.20	0.20	0.20	0.20
COV	0.02	0.01	0.04	0.02
COV_{tot}	0.22	0.21	0.29	0.23

 $*\mu_t$ is the mean value of the trend.

the uncertainty components and total uncertainty for the laboratory data. The data show a discontinuity at the interface between the two units, the higher undrained shear strength being in the upper unit. This was consistent with the results for the plasticity index.

The total uncertainty in Unit 2 is smaller than in Unit 1, due to the smaller aleatory uncertainties. Uzielli et al. (2006a) suggested that for both tests; the effect of measurement uncertainty is significant. Aleatory uncertainty is directly related to the selected trend function. Hence, it is important to report trends and testing method explicitly when presenting the results.

3.4.2 Piezocone measurements

Five piezocone soundings (CPTU) were available for the Unit 1 clay (Uzielli et al. 2006a). Figure 9 indicates the locations of the CPTU soundings from the 2005 site investigation and Figures 10 and 11 the measured cone resistance and pore pressure. The water depth was between 305 and 313 m. The profiles show a considerable regularity and smoothness. Despite the distances between the



Figure 9. Map showing piezocone locations at Troll site (2005).



Figure 10. Statistics of cone resistance from 5 CPTU tests: (a) measured data. (b) coefficients of variation, (c) mean and standard deviation (Uzielli et al. 2006a).



Figure 11. Statistics of pore pressure from 5 CPTU tests: (a) measured data, (b) coefficients of variation, (c) mean and standard deviation (Uzielli et al. 2006a).

sounding locations, there is considerable overlap between the soundings.

The lack of variability suggests homogeneity in the horizontal direction. Spatial correlation in the horizontal direction could not be investigated reliably due to the limited number of soundings and the considerable spacing between the soundings themselves.

At each measurement depth from the common zero-depth, the sample mean and standard deviation of the measurements from the 5 soundings were calculated.

The scatter in the data constituting each sample can be ascribed to the inherent variability of the penetrated soil and to measurement error. Phoon & Kulhawy (1999) suggested a coefficient of variation of 0.07 for the measurement error in cone resistance.

The total coefficient of variation for the cone resistance q_c and pore pressure u_2 at each measurement depth was obtained from the following model:

$$COV_{tot,D}^{2} = COV_{\xi,D}^{2} + COV_{m}^{2} + COV_{SE,D}^{2} + COV_{off}^{2}$$
(2)

in which $COV_{\xi D}$ is the coefficient of variation of the aleatory uncertainty; COV_m is the coefficient of variation of the cone resistance measurement error; COV_{SED} is the coefficient of variation of the statistical uncertainty and COV_{off} is an additional uncertainty term to account for the artificial offsetting of CPTU soundings at different water depths.

The Harr's (1987) "rule of thumb" guidelines with a value of 0.20 was used for COV_{off} . While aleatory uncertainty and statistical uncertainty were depth-dependent, the measurement error and the offset-related uncertainty were assumed were non-depth dependent (subscript 'D'). The depth factor was included because inherent variability is variable with depth and a different number of measurements may be available at greater depths (some soundings are deeper than others).

Figures 10 and 11 illustrate, for cone resistance and pore pressure measured by the CPTU, the average value at each measurement depth and the coefficients of variation for each component of total uncertainty and for the total uncertainty.

The undrained shear strength is usually derived from cone penetration test through the net cone resistance (e.g. Lunne et al. 1997) defined as:

$$q_{net} = q_c + (1 - a_c)u_2 - p_0 \tag{3}$$

in which q_c is the measured cone resistance; u_2 the pore pressure measured behind the cone; p_0 the total vertical overburden stress; and a_c the cone

area ratio. The cone area ratio for the cone used was $a_c = 0.75$. Uzielli et al. (2006a) estimated uncertainty in the undrained shear strength from piezocone tests with the first-order second moment (FOSM) approach (Ang & Tang 2007).

Figure 12 presents the profile of second-moment undrained shear strength derived from the CPTU data. The Coefficient of Variation (COV) varied from 21 to 26%, with an average of 24%. The COV is close to the COV from the laboratory CAUC data of 22%.

The classical statistical approach does not allow for the combination of subjective and observed data or the merging of data from different sources. However, Bayesian updating can be used to include different sets of data (see also Section 4 below).

Bayesian updating was done for the 17 CAUC measurements s_{uCAUC} . The second-moment parameters of each CAUC measurement of s_{uCAUC} were used as prior information. The second-moment parameters of the $s_{uCPTU-CAUC}$ values obtained from CPTU data at the same nominal depth of each CAUC measurement were taken as the likelihood function. The updated undrained shear strength was denoted s_{uB} . The details of the analysis are presented in Uzielli et al. (2006a).

Figure 12 compares the means of the prior, likelihood and updated (posterior) undrained shear strength. Table 4 lists the coefficients of variation obtained for each. For each data point, the standard deviation of the updated date (s_{uB}) was always smaller than that of s_{uCAUC} and $s_{uCPTU-CAUC}$. The COV of the posterior (updated) undrained shear strength is much lower than that of the likelihood. This is a general and beneficial result of Bayesian updating.



Figure 12. Bayesian updating CAUC undrained shear strength from CPTU data.

Undrained shear strength, s_u	Range	Ave COV _{17 points}
Prior COV $(s_{\mu CAUC})$	0.19-0.21	0.20
Likelihood COV	0.21 - 0.26	0.24
$(s_{uCPTU-CAUC})$ Posterior COV (s_{uB})	0.15-0.16	0.15

Table 4. COV's for the prior, likelihood and posterior undrained shear strength.

The results obtained for the Troll data should not be uncritically exported to data from other sites.

The components of uncertainty depend on trend functions which may be strictly site- or casespecific. Perhaps most importantly, geotechnical expertise and engineering judgment were found to be essential in every phase of the uncertainty characterization: in the preliminary examination of data; in the evaluation of second-moment statistics of the measured data; in the formulation of uncertainty models for each parameter and in the selection of appropriate transformation models to obtain parameters useful for design.

3.5 Summary

Wilson Tang published more than 30 papers on site characterization, uncertainties in soil properties and spatial variability. Newer design codes recognize uncertainties in soil properties and engineering models, and soil variability then assumes an increasingly important role in practice and research. Ang & Tang (2007) and Uzielli et al. (2006b) provided an overview of techniques for modelling the variability of soils and highlight the benefits and limitations of the approaches. A first step towards an uncertainty-based approach in geotechnical practice could be the wider reporting of data statistics. However, both the simple and more powerful modelling technique can yield unreliable results if the input data are insufficient in quantity and quality.

Research is on-going to simplify the use of variability-modelling techniques. Research efforts focus on advanced simulation techniques, enhanced capabilities of computing tools and use of sophisticated integrated methodologies to model with increasing realism the behavior of complex geotechnical systems. Geotechnical practice, on the contrary, still largely relies on deterministic approaches.

The gap between geotechnical research and practice should be narrowed: research should make the mathematical techniques more readily usable and practice should recognize the importance of addressing uncertainty and variability. There is a necessity to acquire additional competence regarding the statistical treatment of data. At the same time, a shift towards an uncertainty-based perspective is taking place in practice. In these two respects, the learnings from Wilson Tang, from his books and papers, are a most useful and effective source of information.

4 BAYESIAN UPDATING AND BAYESIAN NETWORKS

Wilson Tang showed a keen interest for Bayesian updating, and more so in the latter years of his career, with, among others, excellent oral contributions in Xian & Oslo in 2008 and the work summarized in Cheung & Tang (2005) and Zhang et al. (2009 a; b). Wilson Tang also had a close collaboration as Kwang-Hua Chair Professor at Tongji University in Shanghai. He was important in moving the application of Bayesian updating forward.

Wilson Tang said that the Bayesian method was "a natural tool for processing geotechnical information". He presented applications to obtain improved estimates of anomaly occurrence probability, anomaly size, pile capacity, model uncertainty, failure probability, liquefaction probability, slope stability, and even the value of added information from additional tests. Bayesian updating can be assimilated to "the past as a guidebook for the future", as for instance illustrated by Folayan et al. (1970) for settlement predictions. Bayesian work continues in Wilson's spirit, e.g. Liu & Nadim (2013) suggest a three-level framework for multi-risk assessment, the third level using the Bayesian approach.

Two examples of work that have pursued Wilson's ideas are presented below: the use of Bayesian networks for (1) the assessment of risk for earthquake-triggered landslides and (2) the stability assessment of talus slopes during road construction works.

4.1 *Earthquake-triggered landslides*

Strong earthquakes in mountainous regions usually trigger many landslides. Earthquake-triggered landslides represent some of the most common secondary disasters caused by earthquake in mountainous areas. In the Wenchuan earthquake of May 2008, more than 15,000 landslides were triggered in the steep mountain slopes (Huang 2008), causing over 20,000 fatalities (Yin et al. 2009) and destroying housing and settlements and irrigation channels (Tang et al. 2011). Landslide dams blocking natural rivers create a new hazard that can be devastating unless the impounded water may be released in a controlled manner.

The assessment of the risks associated with multi-hazards requires the consideration of

the interactions among the hazards and the vulnerabilities of the elements at risk. Zhang et al. (2013) did an assessment of the loss of lives due to sequential or concurrent landslides, rock fall and debris flows hazards. They proposed approaches to estimate the vulnerability factors for loss of life in multi-risk assessment. For sequential hazards, the method considers the gradual reduction of the elements at risk in earlier hazard events. For concurrent hazards, the method estimates the lower and higher bounds of the vulnerability. The occurrence of one or two hazards at an early time can cause redistributions of the elements at risk and change the risk profile under subsequent hazards.

In earthquake-triggered landslide risk assessment, complex interactions are present between the earthquake and landslide threats. The vulnerabilities of the elements at risk can also be correlated to the threats. To date, the risk assessment involving multiple hazards neglects possible cascade effects of multiple hazards (Marzocchi et al. 2012; Kappes et al. 2012).

A study separating the two hazards (earthquake and landslide) as single hazard processes might lead to a misjudgment of the risks associated with such cascading hazards. The assessment and mitigation of the risks require a multi-risk analysis approach that can account for the interactions among the threats and among the vulnerabilities to these threats.

The risk assessment for earthquake-triggered landslides using Bayesian network is illustrated with a sensitivity analysis identifying the most appropriate risk reduction strategy in a multihazard perspective. Nadim & Liu (2013a) looked at risk to the buildings exposed to the threat of earthquake-triggered landslides using Bayesian network.

4.1.1 Bayesian network for earthquake-triggered landslide risk assessment

Nadim & Liu (2013a) provided a brief review of Bayesian networks. Figure 13 presents graphically a simple Bayesian network with five nodes and five arcs. The nodes are: Magnitude (M), Distance (D), Seismic severity (S), Landslide severity (L), and Building damage (B). These nodes are connected



Figure 13. Simple Bayesian network (Nadim & Liu 2013a).

via the arcs: *M-S*, *D-S*, *S-L*, *S-B* and *L-B*. The user enters evidence, and the information propagates through the network. The probabilities in the network are updated when new information becomes available. The posterior probabilities and joint probabilities are calculated on the basis of Bayes' theorem (Ang & Tang 2007).

The network in Figure 14 estimates the risk to buildings under an earthquake-triggered landslide. One counts 11 nodes and 16 arcs. Each node has several discrete states (Table 5). Management includes options of 'no action', 'active' and 'passive' countermeasures, and 'warning systems' (a form of passive measure). Active measures, such as retaining walls and drainage result in lower probability of failure and reduced risk. Passive countermeasures, such as rock fall nets or protective sheds, reduce the vulnerability.

4.1.2 Quantifying the network

4.1.2.1 Seismic hazard

Figure 15 shows the distribution of the calculated distances to the seismic source taken as a line source. The annual probabilities as a function of the earthquake magnitude M_w (Fig. 16) used the recurrence relationship from Gutenberg & Richter



Figure 14. Decision making Bayesian network for earthquake-triggered landslide risk assessment (after Einstein et al. 2010).

Table 5.Nodes and their possible states in the Bayesiannetwork in Figure 2 (after Nadim & Liu 2013a).

Node	# of states	Possible discrete states
Magnitude (M _w)	6	4.0-4.5-5.0-5.5-
Distance (km)	6	22-25-28-31-34-37-40
PGA (g)	6	0-0.08-0.16-0.24- 0.32-0.40-0.48
Landslide	2	Happens; does not happen
Building damage	3	No damage; some damage; collapse
Alarm	2	On; off
Measure	2	Yes; no
Decision	4	Passive; active; no action; warning on
Cost measure, cost, utilities		-

(1994). The conditional probabilities of the Peak Ground Acceleration (PGA), given the magnitude and distance to epicenter, were calculated with the ground motion equation from Ambraseys et al. (2005) and a Monte Carlo simulation. Figure 17 illustrates the joint probabilities of the PGA inferred from the Bayesian network.

4.1.2.2 Landslide hazard

The approaches developed to assess the stability of slopes during earthquake fall into three analysis categories: (1) pseudo-static, (2) stress-deformation,



Figure 15. Discrete probabilities of distance to the seismic source (Nadim & Liu 2013a).



Figure 16. Discrete probabilities of earthquake magnitude (Nadim & Liu 2013a).



Figure 17. Discrete probabilities of peak ground acceleration (Nadim & Liu 2013a).

and (3) permanent displacement. The dynamic slope performance was modelled with the permanent displacement analysis by Newmark (1965). The critical acceleration of a landslide block was:

$$a_c = (FS - 1) \cdot g \cdot \sin\alpha \tag{4}$$

where *FS* is the static factor of safety; *g* the acceleration of gravity; and α the angle of the sliding surface. For an infinite slope, FS then becomes:

$$FS = c'/(\gamma z \cdot \sin\alpha \cdot \cos\alpha) + (1 - m\gamma_w/\gamma)\tan\varphi'/\tan\alpha \quad (5)$$

where *c'* and φ' are the effective cohesion and friction angle; *z* the depth of the failure surface; α the slope angle; γ the total unit weight of the soil; and γ_{v} the unit weight of water. Table 6 gives the properties used in the Nadim & Liu (2013a) study. The probability of slope failure (P_f) as a function of Newmark displacement (Jibson et al 2000) was estimated from:

$$P_{f} = 0.335 \cdot [1 - exp(-0.048 \cdot D_{n}^{1.565})]$$
(6)

where D_n is the Newmark displacement (cm).

The calculated probabilities of slope failure for different ranges of PGA are listed in Table 7. As mentioned above, countermeasures made to landslide can reduce risk. The probability of slope failure when active actions were used are also listed Table 7.

For a building subjected to a multi-hazard situation involving additive load effects (e.g. earthquake and landslide), the damage was increased. For the

Table 6. Soil and slope properties (Nadim & Liu 2013a).

Variable	Mean	Standard deviation
c' (N/m ²)	10,000	2,000
ϕ' (degree)	30	2
z (m)	2.5	0
α (degree)	35	0
γ (N/m ³)	27,500	0
$\gamma_{\rm m}$ (N/m ³)	10,000	0
M	0.4	0

Table 7. Computed probability of failure (Nadim & Liu 2013a).

PGA (10 ⁻² g)	0–8	8–16	16–24	24–32	32–40	40–48
$P_{f \ no \ action} \ P_{f, \ active \ actions}$	0.124	0.256	0.305	0.328	0.339	0.346
	0.025	0.03	0.035	0.04	0.045	0.05

other nodes, Nadim & Liu (2013a) adopted the Einstein et al (2010) probability approach and presented the results in tabular form. Table 8 gives an example for conditional probabilities of 'Building Damage'.

4.1.3 Results

Mitigation measures influence the outcome of multi-risk analyses. The results from the Bayesian network of the entire risk assessment and decision are shown in Figure 18 and compared to Einstein et al. (2010).

Different mitigation measures result in different utilities. The warning system, showing the lowest (negative) utility, is the most optimal mitigation measure. The expected losses for the four mitigation options increase due to the cascade probability triggered by the earthquake. Neglecting the cascade effect could therefore underestimate the risks.

The parameters in the analysis, e.g. the costs, the probability of slope failure or the reliability of the warning system, can vary. Sensitivity analyses were therefore conducted to assess the effects of these variations on the results.

Figure 19 shows the effect of changing the probability of landslide occurrence. In this graph, the best mitigation measure is the one having the less negative utility. For low failure probabilities (P[landslide] < 0.15), no action is preferable, as

Table 8. Conditional probabilities of 'Building Damage' for PGA = 0-0.08 g (Nadim & Liu 2013a, after Einstein et al. 2010).

	Measure	Passive		Active	
nodes	Landslide	Yes	No	Yes	No
Building	No damage	0.4	0.1	0.52	0.1
damage	Some damage	0.3	0.1	0.43	0.1
	Collapse	0.3	0.8	0.05	0.8



Figure 18. Losses for 'no action', 'active measures', 'passive measures' and 'warning system' (after Nadim & Liu 2013a).

expected; otherwise, active measures are preferred, except for probabilities between 0.15 to 0.25 where warnings system are slightly preferable to active measures or no action. This is only an example. The sensitivity of the decision to other factors needs to be similarly studied.

As a further application, one can assume that the average unit rebuilding cost for the "collapse" damage state is \notin 200,000, and the average repair costs for the "yielding" damage state as 50% (\notin 100,000) of the unit rebuilding cost (Nadim and Liu 2013b). Figure 20 presents comparative risk curves with and without the cascade effect. The mean expected loss increases for the same return period of the hazard(s) when the cascade effects are included.

The results are still a preliminary step in furthering the earthquake-triggered landslide risk and multi-hazard risk assessment. The approach follows the philosophy of the recent work carried out by Wilson Tang, and the decision-making principles described in the Ang and Tang Part II (1984) book.

4.2 Stability of talus

Liu (2011) did a similar Bayesian network study as part of his doctoral dissertation for talus



Figure 19. Sensitivity analysis of the risk as a function of the probability of slope failure for different mitigation actions—horizontal arrows indicate range where type of mitigation measure is the optimum (after Nadim & Liu 2013a).



Figure 20. Example of risk curve with and without cascade effect (Nadim & Liu 2013b).

landslides. He investigated the characteristics of talus landslides and the factors affecting talus stability from existing talus failures in China and implemented geotechnical engineering risk analysis with the observed failure mechanism of talus slopes.

Talus landslides are common during the construction of highways in mountainous regions and may lead to construction delay and cause fatalities and large economic and environmental losses. A talus is a slope formed by an accumulation of mainly rock debris or broken rock fragments at the base of mountain cliffs or valley shoulders (talus can also be called "screes"). Talus often have a concave upwards shape and the maximum inclination corresponds to the angle of repose of the mean debris size. The deformation and failure mechanism of talus slope is different from those of natural soil and rock slopes.

The study area is along the Shuifu-Maliuwan Highway, which is located in the area adjacent to the Yungui Plateau and Liangshan Mountain, in northeast of Yunnan Province in China. The area has high mountains, steep gorges prone to heavy erosion, rapidly moving rivers and saw-cuts. Many talus slides have occurred along this highway due to cuts and excavations. Figure 21 provides examples of some to the structural damage encountered.

The characteristics of and factors affecting failure of talus were studied from the analysis of typical talus slides. In addition, to evaluate the input parameters for the Bayesian network, the composition (grain size) and structure of talus material were analyzed in the laboratory and by *in situ* investigations. Extensive laboratory direct shear tests were also conducted to study the effects of rock content, rock shape, and soil properties on the shear resistance of the talus. Model tests investigated the effect of construction procedure on the deformation of talus slopes.

Building the Bayesian network of talus landslide risk is complex. The network was built by assembling relevant expert knowledge. The nodes were divided into three classes: hazard factor node, event node and loss node (Fig. 22). Each node was characterized by several discrete



Figure 21. Structural damage after talus slide: damaged bridge piers (left) and crack in retaining wall (right) (Liu 2011).

states (Table 9). The prior probabilities of the six root nodes (lithology, soil type, gravel content, vegetation type, slope angle and time of landslide) were quantified from a study of 51 talus landslide cases. The conditional probability of each node could be determined by expert knowledge and interrelationship of each information source.

The Bayesian network was constructed using logic relationships among triggering factors, vulnerability factors, and consequence factors. The nodes (factors) and arcs (inter-relationships) of the network were quantified with historical data, empirical models and experimental results. The risk value, given the probabilities of the root factors and vulnerabilities, were calculated based on the networked interrelationships.

In the application of Bayesian networks along the Shuifu-Maliuwan Highway adjacent to the Yungui Plateau and Liangshan Mountain in northeast of Yunnan Province in China, the following steps were used: (1) the lithology of rock, soil type, gravel content, slope angle and vegetation cover



Figure 22. Prior Bayesian network for assessment of talus landslide risk (Liu 2011).

Table 9. Examples of nodes and their states in the Bayesian network of Figure 21 (Liu 2011).

Node	State 1	State 2
Lithology	Sandstone	Mudstone
Soil type	Silty clay	Clay
Gravel content	≤50%	>50%
Veget. cover	Dry land	Mostly woods
Slope angle	≤30°	>30°
Travel length	≤60 m	>60 m
Volume	$\leq 10^{6} {\rm m}^{3}$	$>10^{6} \text{ m}^{3}$
Intensity	Weak	Strong
Time	06:00~18:00	18:00~0:00
Fatalities	Badly injured ≤ 3 or death ≤ 1	Badly injured > 3 or death > 1
Economic loss	<3% of investment	>3% of investment
Time overrun	≤30 days	>30 days

*States for marlite (3) and limestone (4) are not shown.

types were obtained through a geological survey; (2) to predict the scale of landslides, a stochastic model for generating the talus was developed based on Monte Carlo simulation and realized using the AutoCAD VBA program, accounting for gravel distribution, shape, position, size and content (Fig. 23 left). The finite element software ABAQUS was used to analyze the most likely slip surface and travel length (Fig. 23 right).

Using past failure and the spatial model developed, a decision-making Bayesian network was built to predict the potential economic loss, construction delay and time overrun and fatalities due to a talus slide.

Figure 24 presents the posterior probabilities of the Bayesian network together with the available evidence. The zones in red in the monitor windows indicate the parameters that have complete certainty due to the information acquired. If one compares Figures 22 and 24, the risk for losses and casualties for State 2, after updating, increased as expected for this specific talus landslide in light of



Figure 23. Bayesian network for estimating talus landslide risk: (left) stochastic model to generate talus; (right) slope failure calculation with ABAQUS software (Liu 2011).



Figure 24. Posterior (updated) Bayesian network for estimating specific talus landslide risk (Liu 2011).

the landslides that have occurred. The states of the fatalities and losses as well as the corresponding probabilities are listed in Table 10. Such Bayesian network could serve as an effective tool to manage talus landslide risk, provided that the information for the prior is available.

5 RISK OF TAILINGS DAM BREACH

In mid-career, Wilson Tang published 10 contributions on the safety of dams (e.g. Tang & Yen 1991; Cheng et al. 1993)). Although he did not work on this aspect in his later years, his colleagues at HKUST distinguished themselves in this area, perhaps also inspired by the work of Wilson (e.g. Xu & Zhang 2009).

The case study below is a hazard and risk analysis performed by NGI to estimate the probability of non-performance of a tailings management facility designed for gold mine development in Romania (www.gabrielresources.com/prj-rosia.htm). The analyses were to establish whether or not the dam would provide acceptable safety against release of tailings and toxic water, and whether or not additional hazard reducing measures were needed. The project lies within the existing Roşia Montană mining district north-east of the town of Abrud in the Apuseni Mountains of Transylvania.

The project should mitigate the consequences of the historic and future mining operations with the interception and containment of contaminated water currently entering the system, treatment of the contaminated waters and isolation and recovery of the waste rock piles within the project boundary. The operation of the project will generate tailings for approximately 17 years, producing tailings from the processing of a total of approximately 215 Mt of ore. The Tailings Management Facility (TMF) in the valley includes a Starter Dam as a first stage of the Completed Dam, a Secondary Containment Dam, a tailings delivery system, a reclaim water system and a waste rock stockpile (Fig. 25). The TMF is to provide the required design storage capacity for the life of the mine, plus an additional contingency capacity.

Table 10. Results of Bayesian updating of risk associated with talus landslide along the Shuifu-Maliuwan highway.

	Econom	ic loss*	Time ov	errun*	Casualti	es*
State	Prior	Posterior	Prior	Posterior	Prior	Posterior
1 2	60% 40%	45% 55%	52% 48%	37% 63%	38% 62%	32% 68%

*See Table 9 for definition.



Figure 25. Cross-section of tailings dam in Romania (Corser, P. 2009. Personal comm. MWH Americas Inc. Bucharest, Romania).

To establish whether the dam provides acceptable safety against "uncontrolled" release of tailings and water during its life, an event tree approach was used to do the hazard analyses. This technique identified potential failure mechanisms and followed how a series of events leading to non-performance of a dam might unfold. The probability of each scenario, given a triggering event, was quantified.

The event tree hazard analyses considered the dam at different stages of its life and estimated the probability of non-performance. A nonsatisfactory performance of the dam was defined as an uncontrolled release of tailings and water from the dam over a period of time. The release could be due to a breach of the dam or overtopping without breach of the dam. The analyses looked at critical scenarios, including all potential modes of non-performance under extreme triggers such as a rare, unusually strong earthquake and extreme rainfall in a 24-hour period.

5.1 Design considerations

The most significant requirements that influenced the probabilities in the hazard analyses include:

- Operational freeboard at all times of one meter above storage level for maximum reclaim pond and 2 PMP (probable maximum precipitation); the requirement leads to a storage volume capacity of two 1/10,000-yr rainfall within the same 24 hours.
- Gentle slopes for the Starter Dam (\approx 2H:1V upstream and \approx 2H:1V downstream).
- Gentle downstream slopes for the Completed Dam (3H:1V).
- Good quality rockfill for the Starter Dam construction and the Completed Dam.
- "Well drained" tailings beach at the upstream face of the dam, where equipment can move in for repairs, in case of movement or partial breach.
- Secondary Containment Dam (SCD) with about 50,000 m³ containment capacity after 16 years.
- Diversion channels along the sides of the valley to divert excess rainfall runoff away from the TMF pond to minimize the risk of overtopping.

- Emergency spillway to control any excess water released.
- Comprehensive geotechnical monitoring system for safety surveillance.
- Careful control of construction by owner and contractor/engineer.

5.2 Event tree analysis

To establish whether the dam provides acceptable safety against "uncontrolled" release of tailings and water during its life, an event tree analysis was done. A workshop was organized to develop the event trees and reach a consensus when quantifying the hazards. The analysis involved breaking down the complex system into its fundamental components, and determining the potential "failure" mechanisms leading to non-performance of the dam and the physical processes that could cause such mechanisms.

The key factors considered in the analyses included: dam configuration (Starter Dam, dam during construction and Completed Dam), and triggers, including earthquake shaking, extreme rainfall or snowmelt, natural terrain landslide in the valley or failure of the waste stockpile into the tailings reservoir.

Acts of war or sabotage, impact by meteorites or other extreme events of this type were not considered, as they would result in so low probabilities of non-performance that they are not realistic to consider.

The non-performance modes considered included:

- 1. Foundation failure, due to, e.g. excess pore pressures or weak layer in foundation leading to cracking, instability and breach of the dam.
- Dam slope instability downstream or upstream, due to e.g., construction pore pressure in core of Starter Dam, excessive pore pressures caused by static or earthquake loads or instability due to inertia forces.
- 3. Unravelling of downstream toe and slope, due to e.g. overtopping or excessive leakage through or under the dam. This can be caused by a slide into the reservoir, dam crest settlement due to deformations of the Starter Dam, piping, internal erosion and sinkhole formation, or excessive deformations (slumping) of the top vertical part of the Completed Dam during earthquake shaking.
- 4. Dam abutment failure followed by breach, due to e.g. slide close to and/or under part of the dam.
- 5. Liquefaction of the tailings.

Figure 26 presents some of the configurations and examples of the non-performance modes analyzed. Overtopping without breach of the dam, including under-capacity or damage of the



Figure 26. Examples of non-performance modes.

Secondary Containment Dam was also considered, not as a separate non-performance, but as one of the events in the sequence of events in the trees.

Different conditions can affect the probability of a hazard occurring or severity of a consequence, for example construction deficiencies or inadequate response of the field control team at the site when warning signals may appear. The analyses also looked into construction deficiencies, e.g. inadequate filters leading to uncontrolled internal erosion, inadequate drainage, very weak construction layers or zones in the embankment, inadequate types of material(s) in the embankment fill, or insufficient quality control and unforeseen construction schedule changes. These conditions were also integrated in the event trees as separate events during the course of the construction of the Starter Dam and Completed Dam.

5.3 Probability of non-performance

At the event tree workshop, the critical times in the life of the TMF were defined: during construction of the Starter Dam, during the downstream construction stages, during the centerline construction of the dam, and/or in the early years after the Completed Dam is built. A matrix of dam configuration versus time was prepared. The modes seen as most critical and susceptible to lead to the highest probabilities of non-performance were listed. As part of the mode screening, the following considerations were subjected to a consensus decision: extreme and critical precipitation (rainfall, flood and snowmelt), likelihood of failure of the waste stockpile, critical situations after construction of the dams, and geo-environmental considerations.

Event trees were developed for each dam configuration and trigger, with each non-performance mechanism looked at separately. In some cases, two non-performance mechanisms were

Table 11. Total probabilities of non-performance.

Configuration	P [non- performance]
Starter dam ($t = 1.5$ yr, internal erosion)	1.3×10^{-6} /yr
Completed dam ($t = 16$ yrs)	$1.3 \times 10^{-6}/yr$
Intermediate stage $(t = 4 \text{ yrs})$	$6.5 \times 10^{-7}/yr$
Intermediate stage (t = $9-12$ yrs)	$1.3 \times 10^{-6}/yr$

considered successively. The total probability of non-performance is the sum of all contributing probabilities to the non-performance for each of the dam configurations. Table 11 presents the total probabilities for each configuration of the dam (all triggers included). The probabilities were presented as a function of the release of tailings and water associated with the non-performance of the dam. The highest annual probability of non-performance was 10^{-6} .

The highest probabilities of non-performance were associated with earthquake shaking of the completed dam and the static liquefaction of the tailings at time 9 to 12 years after the start of construction. The non-performance scenarios would result in some material damage and some contamination, but only in the vicinity downstream of the dam. For the Starter Dam, no reasonable expected scenario lead to a significant release of tailings and water because of the limited quantity of water available and the reserve capacity provided (2 PMP's). Internal erosion may cause, with an annual probability of 10-6, a small escape of tailings and water. The escape would cause only modest contamination of the immediate vicinity downstream. Essentially all material released could be contained by the Secondary Containment Dam.

The analyses showed (1) no plausible events result in an annual probability of non-performance

greater than 10⁻⁶. The probabilities are lower than the values considered as acceptable criteria for dams and other containment structures around the world and lower than probabilities of nonperformance for most other engineered structures.

ICOLD (the International Commission on Large Dams) presented statistics of dam incidents where the mean probability of failure is between 10^{-4} and 10^{-5} per year (Londe 1993; ICOLD 1995; Foster et al. 2000; Høeg 2001). Peck (1980), based on work by Baecher et al. (1980a; b) who used the ICOLD database plus other data, reported that the probability of failure of dams in the United States and worldwide, was between 2 and 7×10^{-4} per year. Foster et al. (2000) reported annual probabilities of an accident due to downstream slope instability of 1 to 5×10^{-4} and an annual probability of failure of 1.5×10^{-5} .

Historical data are available for embankment dams that provide failure frequency per dam-year of operation. Figure 27 shows internal erosion failure frequencies for US dams. The annual probability of failure associated with internal erosion of earth dams is between 10^{-4} and 5×10^{-4} per year. Internal erosion failures tend to occur more frequently in the first 5 years reflecting first-filling failures. The data suggest significantly higher probabilities of failure than what was computed for the TMF at Roşia Montană.

For tailings dams, the probability of failure is significantly higher than the average annual probability of 10^{-4} and 10^{-5} reported above. Most of the tailings dams are dams entirely made of tailings, whereas the Roşia Montană TMF is made up of the Starter Dam (a regular type rockfill embankment dam), and when completed to top grade, has a downstream slope made of rockfill with gentle inclination of 1:3.

The probabilities of failure in Figure 27 are higher than the probability of non-performance computed for the TMF at Roşia Montană. The event tree analyses show that the probability of non-performance of the TMF is about 100 times lower than the probability of failure of containment dams, based on the performance observed for dams around the world.

The factors that contribute to the low probability of non-performance of the TMF include the use of good quality rock fill for the downstream shell



Figure 27. Annual probability of dam failure by internal erosion for different dams in the USA (Von Thun, 1985; Vick 2002).

of the dam, gentle downstream slopes for both the Starter and the Completed Dam, dam capacity to store extreme precipitation and/or snowmelt events, spillway to release excess water in a controlled manner, the safety monitoring and early warning of early signs of unexpected performance, and the proposed preparedness to remediate, given any unexpected behavior.

5.4 Environmental impact

The physical impact in terms of damage to the environment was also studied, if a breach in the dam should occur. The analysis suggested that the released tailings' volume would be limited, and would only flow 100 to 200 m (Fig. 28). Studies were also conducted to determine possible pollution of the river downstream. The levels of pollution may be above surface water discharge standards for a limited period of time and in the immediate neighborhood of the tailings dam, but only under the worst case conditions (low flow in the downstream river). However, monitoring, early warning and emergency procedures are to be implemented to contain damage to a minimum. The weather and flow conditions for this to occur combined with the probability of dam breach occurring at the same time resulted in the probability of occurrence would reduce to 10-7/year (Whitehead, P. 2009. Personal comm. Aquatic Environments Research Centre, Univ. of Reading, UK).

5.5 Risk assessment of dams in practice

The example illustrates that the event tree analysis is a systematic application of engineering judgment. Its application does not require the prior existence of extensive statistics or the application of complex mathematics. The process may provide meaningful and systematic estimates and outcomes on the basis of subjective probabilities (Vick 2002).

With increasing frequency, society demands that some form of risk analysis be carried out for activities involving risks imposed on the public. At the same time, society accepts or tolerates risks in terms of human life loss, damage to the environment and financial losses in a trade-off between extra safety and enhanced quality of life.

The role of the dam engineering profession is to explain the uncertainties involved in the



Figure 28. Physical impact of TMF dam breach at Roşia Montană (Corser, P. 2009. Personal comm. MWH Americas, Inc. Bucharest, Romania).

construction and operation of dams and to present the likelihood of incidents and failure in informative and meaningful terms. The conventional use of a factor of safety just does not do that, and concepts from probability theory and reliability analyses should be applied.

The key to making the risk analysis of dams effective begins with a detailed overview of all potential failure modes. If shortcuts are taken, the results could be misleading. Once the potential failure modes are understood, the screening process will identify the critical modes. A variety of tools are available for making the quantitative risk estimates. The event tree approach is useful and illustrative. It is recognized that risk estimates and risk assessment guidelines are only approximate, but they are useful for choosing among alternatives, comparing risk levels, and making decisions.

Høeg (2001) presented the basics of systematic risk analysis for dams. He concluded that after several years of optimism in the profession with developing and performing meaningful quantitative probabilistic risk analyses for dams, there now seems to be a trend towards increased use of the qualitative FMECA approach, or the Failure Modes, Effects and Criticality Analysis (BSI 1991). However, there is increased pressure on the decision-makers to quantify risk level so that it can be compared to acceptable or tolerable risk or public protection guidelines. In The Netherlands, the development in this direction is quite advanced and used in the safety evaluation and upgrading of dikes and storm surge barriers (Vrijling 2001).

Scott (2011) summarized the practice of risk assessment of dam safety of the US Bureau of Reclamation. Aging infrastructure, population growth downstream and limited resources render risk assessment of dam safety a reasonable and transparent method for risk management. Key to making the process effective is starting with a detailed analysis of the potential failure modes. Scott described a variety of tools available to do quantitative risk estimates. Such estimates and risk assessment guidelines are only approximate. In each case, it is essential to build the argumentation for the ability of the structure to withstand future loadings. If done diligently and openly, risk assessment is a very effective tool for managing the risks associated with containment facilities (Hartford & Baecher 2004).

6 MODEL UNCERTAINTY AND CALIBRATION OF SAFETY FACTORS

In an early paper, Høeg & Murarka (1974) studied the balanced, yet optimum, design of a gravity retaining wall by relating conventional factors of safety (load and resistance factors) to estimated probabilities of failure. Wilson Tang reviewed and commented on the manuscript, and he was really the first who worked systematically with model uncertainty. He wrote comprehensive reports for the American Petroleum Institute (e.g. Tang 1988) and several papers and discussions (e.g. Tang & Gilbert 1993b). His efforts were crowned in May 2013 with the induction in the Hall of Fame of the paper by Tang et al. (1990) on the performance reliability of offshore piles.

Wilson Tang was always concerned with two aspects: (1) the models used to quantify model uncertainty should duplicate as closely as possible the problem situation actually being calculated (Tang & Gilbert 1992), and (2) the profession should improve its ability to use experimental results to determine the uncertainties in its engineering models.

Today, work on this topic is still on-going. Lacasse et al. (2013 a; b; c) made a contribution which follows and expands on Wilson Tang's principle. The ultimate aim of the work was to obtain the appropriate factor of safety to use when designing offshore installations. To illustrate this, Gilbert et al. (2013) present at this conference the case of three actual offshore structures, presently under final design, where such calibration of the load and resistance safety factors was done. Only the approach and the conclusions are briefly reported herein.

The study was undertaken to document that the pile foundations were designed according to governing regulations. The goal was to make a recommendation on the appropriate resistance factor and minimum pile penetration depth to use for the design of the piles on an offshore jacket. The safety factors (load and resistance factors) for three case studies were calibrated for a target annual probability of failure, P_{e} of 10^{-4} .

The reliability analyses of the axial pile capacity methods included seven steps:

- Establish the mean, standard deviation and Probability Density Function (PDF) of the soil parameters. Include correlations among parameters.
- Establish the model uncertainty for the different pile capacity calculation methods used.
- Establish the effect of cyclic loading on the axial pile capacity and determine whether the piles in compression or in tension govern the design.
- Develop a model for the statistics of the static (permanent) and environmental loads on the top of the piles.
- Do deterministic analysis of the ultimate axial pile capacity, Q_{ult} .

- Do probabilistic analyses of axial pile capacity and obtain the PDF of the ultimate capacity, Q_{ult} .
- Calculate the annual probability of failure by combining the loads and the probabilistic description of Q_{ulr}
- Calibrate the load and resistance factors required for an annual $P_f = 10^{-4}$.

When doing an axial pile capacity analysis, the following aspects should be included: (1) a careful selection of the characteristic soil parameters used for design; (2) the effect of cyclic loading on the characteristic shear strength or ultimate pile capacity, for both piles loaded in compression and in tension; (3) the effect of gapping and/or erosion at the top of the piles on the axial pile capacity; and (4) a decision on whether or not to account for the effect of time after pile installation on the axial capacity.

The calibration analyses showed that:

- The calibration of the safety factors demonstrate that the annual probability of failure vary with the axial pile capacity calculation method.
- The values of model uncertainty used in the analyses have an overwhelming influence on the resulting annual probability of failure and therefore on the required resistance factor for a target annual probability of failure.
- The current state-of-the-art design still relies heavily on qualified engineering judgment to assess and ensure a consistent safety level.
- The resistance factors calibrated suggest that the newer CPT-methods of pile design are as reliable as the current API method.
- The findings on margin of safety and the definition of the characteristic shear strength have important implications for the design of piles offshore and can result in significant savings.

As illustrated in Gilbert et al. (2013), the pile length could be considerably reduced through the study of a safety level corresponding to an annual probability of failure of 10^{-4} . Table 12 reproduces the final results, comparing pile penetration depths. The first number is the penetration depth from the deterministic analyses with a resistance factor of 1.5 on the CPT-methods. The second number is the penetration depth ensuring that the annual probability of failure is less than 10^{-4} .

The significant reduction in the required pile penetration depth was possible because one could

Table 12. Pile penetration depth for design (Lacasse et al. 2013c).

Site A (clay)	Site B (sand)	Site C (clay and sand)
90 m to 75 m	51 m to 27 m	45 m to 38 m

demonstrate that the annual probability failure was less than the target P_f of 10⁻⁴/year for the piles originally designed with a resistance factor of 1.5. It was then possible to use a resistance factor of 1.3, as for the current API method, instead of the *a priori* resistance factor of 1.5 set for the newer CPT design methods. A load factor of 1.3 or 1.35 was used.

The analyses demonstrated the importance of how the characteristic shear strength parameters are defined. Lacasse et al (2013a) recommended that the characteristic strength be defined in specific terms, e.g. setting the characteristic shear strength for the deterministic design of axial pile capacity at a value equal to the mean minus $\frac{1}{2}$ or one standard deviation.

The importance of model uncertainty was pointed out early by Wilson Tang. This confirms the actuality of even his early papers. Here again, Wilson Tang led the way in his study of model uncertainty and calibration of safety factors in the early days of his career.

7 MORE OF WILSON TANG'S LEGACY

7.1 Cost-effectiveness of site investigation

Wilson Tang worked on the cost-effectiveness of site investigations, a central aspect of our profession. His contribution (Tang, 1987), published in the journal Structural Safety, may have passed unnoticed.

In general, more extensive site investigations and laboratory testing programs reduce the uncertainties in the soil characteristics and design parameters. At a certain point however, as Wilson Tang (1987) pointed out, the benefit obtained from further site investigations and testing may not yield sufficient added value (read: increase in the reliability of the performance) to the geotechnical system, and hence may not justify the additional cost (e.g. Folayan et al. 1970).

Soil investigations, in the way they are planned, represent a risk-based decision. The complexity of a soil characterization is based on the level of risk of a project. Lacasse & Nadim (1998; 1999) illustrated this graphically. A low risk project involves few hazards and has limited consequences. Simple *in situ* and laboratory testing and empirical correlations would be selected to document geotechnical feasibility. In a moderate risk project, there are concerns for hazards, and the consequences of non-performance are more serious than in the former case. Specific *in situ* tests and good quality soil samples are generally planned. For a high-risk project involving frequent hazards and potentially risk to life or substantial material or environmental damage, high quality *in situ* and laboratory tests are required, and higher costs are involved.

The decision-making process for selecting the appropriate soil investigation methods, although subconscious, is risk-based. It involves consideration of requirements, consequences and costs.

Uncertainty analysis can help optimize site investigations. The uncertainty in a geotechnical calculation is often related to the possible presence of an anomaly, e.g. boulders, soft clay pockets or drainage layer. Probability approaches can be used to establish the cost-effectiveness of additional site investigations to detect anomalies. Figure 29 presents an example where the presence of a drainage layer was determinant on the resulting post-construction building settlements. A settlement of less than 50 cm would mean an important reduction in costs. With drainage layer detectability for each boring of 50% or 80% (Fig. 28), and assuming a given drainage layer extent, 3 to 6 borings were required in this case to establish whether the drainage layer was present or not.

7.2 Reliability of offshore structures

Wilson Tang started working with offshore structures (Høeg & Tang 1977) when he came to NGI on the Guggenheim research fellowship. Wilson was very much indebted to the John Simon Guggenheim Memorial Foundation for making possible his research stay first at the Imperial College of Science and Technology in London and then at NGI in Oslo, Norway.

From there on, he continued his research and became a recognized figure in offshore circles, especially with respect to model uncertainties and the reliability of pile foundations offshore (e.g. Tang & Gilbert 1992; 1993a). Noteworthy are his studies for the American Petroleum Institute, which conclusions are still in use today.

Gilbert et al. (2013), at this conference, give an overview of the advances in geotechnical risk and reliability for offshore applications. The lessons learned from Wilson Tang are now also used to calculate the reliability of offshore wind energy turbines (Stuyts et al. 2013). Lacasse & Nadim (2007) also summarized the applications of statistics, reliability and risk in offshore geotechnical engineering based on the original work by Wilson Tang.

The methods for assessing hazards offshore can vary from approximate estimates to more complex calculations. Applications include piled foundations, jack-up structures, gravity foundations and underwater slopes. The applications demonstrate that probabilistic analyses complement the conventional deterministic safety factor and/ or deformation-based analyses, and contribute to achieving a safe and optimum design. The probabilistic approach adds value to the results with a modest additional effort. Engineering judgment is still necessary to achieve reliable results in both hazard and risk assessment.

8 SUMMARY AND CONCLUSIONS

Wilson Tang's work was an inspiration to move forward in the area of statistics, probability and reliability. He quickly saw the potential of these concepts in geotechnical engineering. He published his first book, together with Professor A.H-S Ang, one of the most useful and influential sources of information on the topic for geotechnical engineers, only six years after completing his PhD at Stanford University. With Ang and Tang's two volumes, one can find all the essential concepts and very many applications.

This paper presented only a few examples of how geotechnical engineers have taken the learnings of Wilson Tang and carried on with further applications in practice. The quantification of the natural and anthropogenic risks that can affect an area or engineering structures is today an essential component of a sustainable environment, land-use planning, and risk mitigation. To this development, Wilson Tang was a pioneer and before his time!

Wilson Tang was quick to see the importance and possible repercussions of using Bayesian updating in geotechnical engineering.



Figure 29. Cost reduction with increased number of borings (Lacasse & Nadim 1998 based on Tang 1987).

The advantages of the Bayesian approach include: (1) it is a probabilistic model instead of a deterministic model. The uncertainties in the parameters and their inter-relationships are represented by probabilities; (2) a large number of parameters and their inter-relationships can be considered in a systematic structure. The probabilities of one parameter can be updated via available information. The change in one parameter will influence the others in the network through their inter-relationships; and (3) physical mechanisms, previous studies, and statistical data can be accounted for. All three aspects are key to good geotechnical design.

The profession only gains by implementing, more systematically than before, probabilisticbased thinking and risk-based methodology. The geotechnical probabilistic approach still has major needs, including reducing uncertainty in the calculation model by obtaining and analyzing performance data of high quality, quantifying acceptable and tolerable hazard and risk levels, and convincing stakeholders of the value added in uncertaintybased analyses.

With the changes in climate and the occurrence of more extreme natural phenomena than before (e.g. storms and precipitation), one cannot only use data from existing experience to evaluate safety, but one should also include events and triggers that are not covered by e.g. 100- or 1000-year return periods. Another keyword is the importance of multidisciplinarity, meaning wider expertise teams than before when evaluating hazard and risk to society, and the need to document cost-effectiveness of different mitigation measures.

Bayesian updating and hazard and risk analysis are important and necessary. Hazard and risk assessment present an opportunity to look at the bigger picture and seek out designs that meet not just some arbitrary idea of acceptable/tolerable risk but an unknown risk. The engineer should concentrate on exploiting the good features of the approach. As contributor to the profession's goals of documentation, continuity, high-quality and innovation, and the ever increasing requirement of globalization, hazard and risk assessment and the management of risk serve as communication vehicle among geo-specialists and other sectors of expertise. The authors are convinced that Wilson Tang would have appreciated working on these emerging aspects and would have contributed with his usual innovation, elegance and wisdom.

After a special workshop on Reliability Methods for Risk Mitigation in Geotechnical Engineering at Irvine in 1992, Tang & Duncan (1994) concluded that probability methods should be part of the geotechnical engineer's toolbox. Although every successful geotechnical engineer has learned to cope with uncertainty by applying lessons learned by the profession over decades of practice, the probabilistic toolbox provides a complement to deterministic analyses, and should be used for several reasons, including (in Wilson Tang's own words):

- "(...) Society is demanding more explicit assessment of risk. (...) To work effectively with the public and (...) regulatory agencies, geotechnical engineers must have some knowledge of probability theory and probability methodologies, as well as traditional geotechnical expertise."
- "Probabilistic methods are useful as a basis for making economic decisions. [For example,] in areas such as dam rehabilitation, landslide hazards mitigation, environmental remediation, and infrastructure rehabilitation, effective allocation of funds relies on quantifying the trade-offs between benefits and risks (...). Probabilistic methods provide a quantitative basis through which the relative contribution of risk can be systemically analyzed and communicated. In this way, decisions can be made more rationally and justified more logically."
- "(...) There is a risk in risk analyses and probability analyses, if the analyses are performed improperly. This possibility can be minimized by expanding knowledge of probabilistic methods among geotechnical engineers and by expanding knowledge of geotechnical engineering practice among probability specialists."
- "(...) With a working knowledge of probability theory, geotechnical engineers will be better equipped to deal with the many uncertainties that pervade geotechnical engineering practice."

In closing, the authors wish to express their gratitude to Wilson, not only for his competence and invaluable scientific contribution, but also for his friendship, his kindness, thoughtfulness and help with articles, discussions, workshops and presentations, over many years. One example: when NGI decided in the early 80's to offer an internal education program on the practice of statistics and probability in geotechnical engineering, we chose Ang & Tang Part I (1975) as textbook. Hearing this, Wilson immediately sent NGI his book of worked out solutions to all the problems in the book, which turned out to be a godsend. We still use this booklet of solved examples!

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2 Keynote lectures

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Advances in geotechnical risk and reliability for offshore applications

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ABSTRACT: This paper describes recent advances in geotechnical reliability and risk for offshore applications. The topics addressed include spatial variability, model uncertainty, hazard characterization, reliability-based design, system reliability and risk management. Conclusions from the evolution of reliability and risk approaches include that practical implementation is key, that assessment is best considered in the context of decision making, and that collaboration of multiple disciplines and stakeholders is important to managing risk effectively.

1 INTRODUCTION

The offshore oil and gas industry has been a leader in considering risk and reliability explicitly in developing and implementing designs. This industry has constantly pushed further the frontiers for design and technology, with facilities being developed at present in 3,000 m of water. The consequences of a failure can be severe, and the costs associated with mitigating risks can be enormous. Therefore, there is a strong need to avoid both under-conservatism and over-conservatism.

One of the first reliability-based design guidance documents was developed for offshore facilities (API 1993a). A sampling of the work that led to implementing reliability-based approaches in offshore geotechnical practice includes Bea (1983), Lacasse & Goulois (1989), Wu et al. (1989), Tang et al. (1990), Nadim & Lacasse (1992) and Tang & Gilbert (1993). In addition, the offshore industry has had the opportunity to learn from experience as the performance of facilities subjected to extreme operation conditions has been observed.

The objective of this paper is to describe recent advances in geotechnical reliability and risk for offshore applications. The following areas are highlighted:

- 1. Accounting for spatial variability in geotechnical properties;
- 2. Characterizing model uncertainty in design methods;
- 3. Representing loads and hazards in geotechnical systems;
- 4. Implementing reliability-based design in practice;

- 5. Considering the reliability of systems as well as components; and
- 6. Including a wide variety of perspectives, consequences and hazards in managing risks.

These advances are inevitably motivated by practical needs in offshore applications. However, the advances are general and fundamental, and therefore relevant to a wide variety of geotechnical applications. Case histories are presented to illustrate the recent advances. The paper concludes with recommendations for continuing the development and application of reliability and risk approaches in geotechnical engineering.

2 SPATIAL VARIABILITY

Accounting for spatial variability in geotechnical properties poses a significant challenge offshore for the following reasons:

- 1. The locations for offshore developments are not readily accessible;
- The cost and time required to conduct offshore site investigations are orders of magnitude greater than for onshore site investigations¹; and
- 3. The facilities on the seafloor for a single development, including foundations for structures, wells, manifolds and valves and pipelines, can extend many kilometers.

^{1.} However, the costs of the geotechnical site investigations for offshore installations represent on y a very small fraction of the total development costs (less than 2%).



Figure 1. Coefficient of variation versus depth for point and depth-averaged values of design undrained shear strength in random field model (from Cheon & Gilbert 2013).

Consequently, it is not feasible to gather 100 percent knowledge of the geotechnical properties at the location of or along every foundation element.

Recent advances have been made in developing realistic models of spatial variability to account for it in designing foundations and optimizing site investigation programs (e.g., Keaveny et al., 1990, Gambino & Gilbert 1999 and Valdez-Llamas et al., 2003).

An example of a random field model for the design capacity of deep foundations is shown in Figures 1 to 4. The geologic setting is normally to slightly overconsolidated marine clays in 1,500 to 3,000 m deep water in the Gulf of Mexico. The random field model represents spatial variations in the design shear strength. The design strength is the strength selected by a designer for the purposes of foundation design based on all available laboratory and field test data and geologic information at a given location. The available data for this geologic setting included over 100 design profiles of undrained shear strength from site investigations with soil borings, jumbo piston cores, field vane tests and Cone Penetration Tests (CPT). These design profiles are located as close as 100's of meters to as far as 1,000's of kilometers from one another.

The three-dimensional random field model consists of two cross-correlated models for the design undrained shear strength: one for the design strength at a particular depth below the sea floor (to calculate end bearing) and one for the depth-averaged design strength from the sea floor to that depth (to calculated side shear). The model incorporates means and standard deviations that increase with depth, an anisotropic spatial correlation structure, and horizontal correlations that increase with depth. Details



Figure 2. Horizontal correlation distance² versus depth for point and depth-averaged values of design undrained shear strength in random field model (from Cheon & Gilbert 2013).

for this model and its calibration are provided in Cheon (2011) and Cheon & Gilbert (2013).

The model shows that the influence of the spatial variability relative to the mean decreases with depth (Fig. 1), possibly reflecting the increasing overburden stress damping variations in mineralogy or depositional history. The effect of spatial averaging in reducing variability for the depthaveraged strength increases with averaging length (Fig. 1).

The horizontal correlation distance² obtained was between 2 and 6 km, and is therefore hundreds of times greater than the vertical correlation distance (Fig. 2). Note that the correlation distance is much greater for the design undrained shear strength compared to that for individual measurements of undrained shear strength since the design profile implicitly averages out small-scale variations (either real or due to measurement methods) and reflects larger-scale variations. Both the horizontal and vertical correlation distances are greater for the depth-averaged versus the point strength (Fig. 2). The horizontal correlation structure is best modelled as anisotropic, with a longer horizontal correlation distance moving away from the continental shelf (in the direction of depositional flow) compared to moving along the continental shelf (Fig. 3).

This model of spatial variability can be used to support design decisions. An example application

Correlation distance was defined here as the separation distance at which the correlation coefficient is equal to 0.37 for an exponentially decreasing correlation coefficient with separation distance. This correlation distance is one-half the scale of fluctuation defined by Vanmarcke (1983).



Figure 3. Coordinate system describing horizontal distances along and off the continental shelf for horizontally anisotropic correlation model (from Cheon & Gilbert 2013).



Figure 4. Factored design axial capacity for 5.5-m diameter suction caisson foundation (from Cheon & Gilbert 2013).

is for the design of a suction caisson that will need to penetrate below the depth of an available design profile for strength obtained from a jumbo piston core. Figure 4 shows the factored axial capacity (i.e., the nominal design axial capacity reduced by the resistance factor for a Load and Resistance Factor Design check).

The available design profile at this location extends to a depth of 20 m. For a caisson longer than 20 m, there is additional uncertainty in the axial capacity due to spatial variability. The curve labeled "Accounting for Spatial Variability" in Figure 4 incorporates an additional partial resistance factor to provide the same level of reliability as if a design profile were available. If the factored design load is 10,000 kN, then the required caisson length is 27 m. If an additional site investigation was conducted to develop a design profile at this location below a depth of 20 m, then the expected value of this additional information is a reduction in required caisson length of about 2 m (obtained by comparing the curves labeled "Accounting for Spatial Variability" and "Neglecting Spatial Variability" in Figure 4). The expected cost savings can be compared against the cost of obtaining the additional information and can be determinant for the decision-making on whether or not to do additional site investigations.

An important point in Figure 4 is that the added conservatism required to account for spatial variability, a reduction in capacity less than ten percent, is small compared to a typical resistance factor of 0.8 or material factor of 1.25. Therefore in this geologic setting, the additional (aleatory) uncertainty due to not having site-specific geotechnical data is small compared to the (epistemic) uncertainty in selecting a design shear strength that represents the actual strength mobilized when the foundation is loaded.

3 MODEL UNCERTAINTY

Model uncertainty, which is defined as variations between the actual performance and that predicted by a design method, can be one of the largest sources of uncertainty in offshore geotechnical design. For example, the coefficient of variation for model uncertainty in the axial capacity of a pile foundation is typically greater than 0.2, while the coefficient of variation due to spatial variability is less than 0.2 (Fig. 1).

Recent advances have been made in better characterizing model uncertainty for offshore applications. One advance has been related to the axial capacity of driven piles in sand. Based on several large-scale load testing programs and additional data, several newly developed design methods could be verified (e.g., Randolph 2003, Jardine et al., 2005, Lehane et al., 2005, Clausen et al., 2005, Kolk et al., 2005, Schneider et al., 2008 and Lacasse et al., 2013c).

In addition to pile load tests, recent advances have been made by studying the performance of actual offshore structures loaded to or beyond their calculated capacities. Five major hurricanes moved through the oil and gas infrastructure in the Gulf of Mexico between 2004 and 2008. Figure 5 shows an example of new information on the predicted versus measured axial capacity of driven piles at large capacity in normally consolidated clays. The data point with the largest measured capacity is for a 1,220 mm diameter by 70 m long pile that failed in tension when a tripod jacket was loaded beyond its ultimate capacity in Hurricane Ike (2008). It



Figure 5. Comparison of measured with calculated axial capacity based on API current guidelines for driven piles in normally consolidated clays (adapted from Chen et al., 2013).



Figure 6. Comparison of measured and calculated pile system capacity for eight-pile jacket that survived Hurricane Ike (adapted from Gilbert et al., 2010).

is the largest published failure load to date for a driven pile in normally consolidated clay. The pile failed five years after installation under cyclic and rapid loading during a hurricane. It is notable because the predicted capacity, when a t-z analysis that accounts for strain-softening in side shear and an axial flexibility of the pile, matches very well with the most likely load at failure based on the hurricane hindcast.

Figure 6 shows an example of a pile system for an eight-leg jacket that survived Hurricane Ike. In this case, the piles are 920 to 1,070 mm in diameter, 52 m long, driven through layers of clay and sand, and tipped in sand. The estimated load in Hurricane Ike exceeded the calculated capacity of



Figure 7. Probability distributions for bias on calculated ratio of pile system capacity to pile system load (reserve strength ratio) (from Chen & Gilbert 2013).

the foundation system, represented by the "Base Case" interaction curve in Figure 6. However, the calculated capacity of the foundation system is potentially conservative because it assumes a nominal rather than an average yield strength for the steel piles; the lateral resistance of the soil was reduced to account for cyclic loading when the piles are pushed into undisturbed soil at ultimate failure of the entire system; and the effect of jacket leg stubs extending below the mudline was assumed as negligible. When more realistic assumptions are used to model the pile system, the calculated capacity is equal to or greater than the estimated hurricane load (Fig. 6).

Figure 7 shows how the performance of individual platforms (e.g., Figs. 5 and 6) can be used to update model uncertainty with Bayes' theorem. The bias is defined as a multiplicative correction factor on the calculated ratio of capacity to load, defined as the reserve strength ratio, where the capacity is calculated using the existing API design method and the load is calculated using the hurricane hindcast. Variations between the actual and calculated reserve strength ratio could occur due both to errors in the calculated capacity or in the load. The updated probability distribution for this ratio is shown in Figure 7 for individual platforms that survived or failed in a hurricane. In addition, the results from these individual platforms are combined together into an overall result, labeled "Updated—All Cases," by assuming independence between platform performances. The overall result indicates that while there is possibly a conservative bias in the calculated reserve strength ratio, there is also considerable uncertainty (Fig. 7). The results in Figure 7 should be used with caution because they are based on a small data set and subsequently treat similarly a variety of different failure mechanisms, including lateral and axial pile failures.



Figure 8. Comparison of measured axial capacity for driven piles in clay soils with estimated lower-bound capacity calculated assuming the side shear equal to the remolded undrained shear strength of the clay (adapted from Najjar 2005).



Figure 9. Probability distributions for load and capacity for a Tension Leg Platform foundation (adapted from Gilbert et al., 2010).

Another recent advance has been in refining models of the left-hand tail of capacity, which is the region of interest for reliability. Figure 8 shows an example of establishing a lower-bound on the axial capacity of a driven pile in clay based on the remolded undrained shear strength. This calculated lower-bound is less than the measured capacity in every load test. Figure 9 illustrates the physical significance of such a lower bound on the reliability of a Tension Leg Platform (TLP) foundation: the most probable point from a First Order Reliability Method (FORM) analysis, in which a conventional lognormal distribution is assumed for capacity, is well below the lower bound, which is unreasonable.

If a lower bound is incorporated into the probability distribution for capacity, then the reliability can be governed by this lower bound as opposed to



Figure 10. Effect of lower-bound on probability of failure for TLP foundation (adapted from Gilbert et al., 2010).

the mean or standard deviation (Najjar & Gilbert 2010). A lower bound on the capacity is particularly significant to the reliability in cases with relatively small uncertainty in the load or large factors of safety. In addition, a lower bound can be influential even when its exact value is uncertain. The application of this idea in practice is shown in Figure 10. The incorporation of a lower bound, which can be verified with pile driving monitoring during or after installation (i.e., a re-strike analysis), reduces the probability of failure for this foundation to within tolerable levels (Fig. 10).

4 HAZARD CHARACTERIZATION

The load or hazard is as important as the capacity in analyzing the reliability of a geotechnical system. In many cases, a thoughtful analysis of reliability can lead to advances in how the hazard is characterized.

Reliability-based design and the decision making processes in risk management often require an assessment of the failure probability during a reference time period, e.g., the annual failure probability or the failure probability during the lifetime of a project. The assessment of this probability requires a probabilistic description of the annual maximum environmental loads for foundation design, or a probabilistic description of frequency and intensity of trigger(s) for assessment of impact of geohazards on sea floor installations. Using this information, the probability of foundation failure or slope instability can be computed for all relevant scenarios and return periods in order to derive the annual or lifetime failure probability. However, including all possible scenarios can be



Figure 11. Results of probabilistic analyses of static undrained stability, prior to (black), updated (blue) and after the 3,000-year and 10,000-year earthquake (red) (from Nadim 2011).

time-consuming and impractical, and often only the few scenarios that contribute most to the failure probability are needed for a sound assessment.

Recent advances have been made in developing practical means to calculate the annual probability of earthquake-induced slope failure (Nadim 2002 and 2011). This work was supported by a number of joint-industry research projects and offshore geohazards studies in the North Sea, the Caspian Sea, the Black Sea, offshore Indonesia, and the Gulf of Mexico. The multi-step approach uses FORM, Monte Carlo simulation and Bayesian updating and is described in detail by Lacasse et al. (2013d). Nadim (2011) presents an example case study for a slightly overconsolidated clay slope in a moderately seismic area. Prior analyses showed that the earthquake events with return periods between 1,000 and 10,000 years contribute most to the annual probability of slope failure. The dynamic response analyses were therefore done for earthquake events with return periods of 3,000 and 10,000 years. Each of these events was represented by four sets of properly scaled acceleration time histories. Figure 11 shows the computed and the updated cumulative distribution functions for the static safety factor under undrained loading prior to the earthquake, and after the possible impact of 3,000-year and 10,000-year earthquake events.

To estimate the annual probability of slope failure, Nadim (2011) developed a simplified model similar to that suggested by Cornell (1996). The limit state function for the seismic resistance of the slope was defined as: G = Seismic resistance— Earthquake load = $A_{resist} - \varepsilon \cdot A_{max}$ where A_{max} is the annual peak ground acceleration representing the earthquake load, A_{resist} is the resistance of the slope to earthquake loading in terms of the peak ground acceleration causing slope failure, and ε describes the variability of the peak ground acceleration at a given return period.

The probability distribution of A_{max} was obtained from the site-specific Probabilistic Seismic Hazard Assessment (PSHA). A Pareto distribution provided a good fit for A_{max} with return periods greater than 100 years. The resistance parameter A_{resist} and the variability parameter ε were respectively assigned lognormal and normal distributions, and the parameters of the distribution functions were calibrated to match the conditional failure probabilities for the 3,000-year and the 10,000 year earthquake events (Fig. 11). With this limit state function, the annual probability of earthquake-triggered slope failure was computed using FORM to be $P_{fammal} = 4 \times 10^{-4}$.

In some situations, such as offshore geohazards studies, it can be extremely difficult to identify the trigger(s) for submarine slides and a reference time frame. One must then rely on the identification and dating of recent (in geological sense) slide events in the area. The dating results and other relevant geological evidence can then be used in a Bayesian framework to establish the annual probability of slope instability (e.g., Nadim 2002). In performing these analyses, it is very important to consider the relevancy of the conditions present in the historical record, such as the sea level, to the conditions that may be present during the reference time period of interest.

Hazard characterization has also provided insight into physical mechanisms. As an example, a recent advance was made in assessing the hazard for wave-induced mudslides in the Mississippi River delta. For most fixed facilities in shallow water, such as jacket platforms, the loads are governed by the wave height and not the wave period. Therefore, the hazard has conventionally been described by a wave height in combination with an associated wave period that corresponds to the strong (right-hand in the northern hemisphere) side of a hurricane.

However, the wave period is an important consideration for wave-induced mudslides. Figure 12 shows how the factor of safety for a slope failure is affected by the wave height and the wave period at one location in the Mississippi River Delta. Wave-induced mudslides occurred at this location in both Hurricane Ivan (2004) and Hurricane Katrina (2005). While the maximum wave height in Ivan was significantly smaller than that during Katrina, the factor of safety was smaller in Ivan because of a relatively large wave period (Fig. 12). The Delta was about 150 km to the left of the eye of Hurricane Ivan, meaning that it was on the weak side of the storm. However, the wave periods on the weak side were similar to those for the much larger wave heights on the strong side



Figure 12. Factor of safety for slope failure versus wave height and period at one location in Mississippi River Delta (from Gilbert et al., 2010).



Figure 13. Conditional probability distribution for wave period in the Mississippi River Delta given a maximum wave height for a hurricane in the Gulf of Mexico (adapted from Nodine et al., 2009).

of the storm. Therefore, both large wave heights on the strong side of a storm (i.e., Katrina in Fig. 12) and smaller wave heights with longer periods on the weak side of a storm (i.e., Ivan in Fig. 12) contribute to the risk for wave-induced mudslides.

Based on this experience, an updated hazard representation was developed for wave-induced mudslides in the Delta. The approach utilized the Theorem of Total Probability to account for the possibilities that the maximum wave height in the Delta corresponds to the strong side of a storm with the largest waves in the storm or to the weak side of a storm with larger wave heights outside of the Delta (Nodine et al., 2009). An example of the conditional probability for wave period given



Figure 14. Return period for wave-induced mudslides impacting exiting pipelines in Mississippi River Delta (from Nodine et al., 2009).

a maximum wave height in the Delta is shown in Figure 13: the most probable combination of wave height and period represents hurricanes with their strong side over the Delta, while the other combinations represent hurricanes with their weak side over the Delta.

An example result from using this hazard characterization in assessing the hazard of wave-induced mudslides in the Delta is shown in Figure 14. This map incorporates the wave hazard with the water depth, bottom slope, geotechnical properties and pipeline locations.

5 RELIABILITY-BASED DESIGN

A significant benefit of a reliability-based design approach is to promote designs that efficiently achieve target levels of reliability. Recent advances have been made in implementing this principle in practice.

Lacasse et al. (2013a, 2013b and 2013c) describe a case study concerning the reliability of axiallyloaded piles in sands. The API RP 2GEO (2011) and ISO 19902 (2007) guidelines included recently four CPT-methods for calculating the axial capacity of piles in sands. The design guidelines require that if newer methods are to be implemented in design, the same level of safety shall be documented for new methods as for existing methods.

Ensuring adequate reliability under severe loading is a necessary consideration, and the calculated safety margin depends on the uncertainty in the parameters used in the analyses and the model uncertainty. The design engineer attempts to compensate for the uncertainties by introducing appropriate (partial) "safety factor(s)" in design.

Table 1. Design methods considered in reliability analysis for axial capacity of driven piles.

Method	Methods in clay	Methods in sand
API	API-RP2 A 20th ed.1993	API-RP2 A 20th ed. 1993
NGI-05	Karlsrud et al. 2005	Clausen et al. 2005
ICP-05	Jardine et al. 1996; 2005	Jardine et al. 2005; API 2011; 2007
Fugro-96/05	Kolk and v.d.Velde 1996	Kolk et al. 2005
UWA-05	_	Lehane et al. 2005; Schneider et al. 2008



Figure 15. Simplified representation of reliability-based design calibration process (from Laasse et al., 2013a).

To evaluate the required resistance factor, Lacasse et al. (2013a, 2013b and 2013c) calculated the annual probability of failure for piles on offshore jackets designed with the API method and with the newer CPT-based methods. The goal was to make a recommendation on the appropriate resistance factor and minimum pile penetration depth to use for the design of the piles on an offshore jacket. Table 1 lists the axial pile capacity methods considered.

The reliability analyses of the axial pile capacity methods included a statistical analysis of the soil parameters; statistical analysis of the model uncertainty for the different pile capacity calculation methods used; statistical analysis of the static (permanent) and environmental loads on the top of the piles; deterministic analysis of the ultimate axial pile capacity, Q_{ult} ; probabilistic analyses of axial pile capacity to obtain the PDF of the ultimate capacity, Q_{ult} ; calculation of the annual probability of failure by combining the statistics of the loads and the probabilistic description of Q_{ult} ; and calibration of the safety factors (load and resistance factor) for each pile capacity design method, for a target annual probability of failure of 10^{-4} .

Three sites, where jackets are currently under design, were analyzed. For Jacket A, the soil conditions are characterized by mainly clay layers with intermittent thin sand and silt layers. For Jacket B, the soil consists of mainly dense to very dense sand layers, with rather thin clay layers in between. For Jacket C, the soil profile consists of alternating very dense sand and very stiff clay units. The parameters were estimated with statistical analyses of the soil data, combined with well-documented correlations and experience (bias factors).

An extended study of the model uncertainty was carried out for the different axial pile capacity calculation methods (Lacasse et al., 2013c). The model uncertainty was expressed as a bias (mean), standard deviation, coefficient of variation and Probability Density Function (PDF). The model uncertainty was obtained by comparing the predicted to the measured axial pile capacity from relevant and reliable pile model tests. The NGI database of "super pile" load tests NGI (2000; 2001) was used.

The calibration used (1) the results of the deterministic analyses giving the ultimate axial pile capacity with the characteristic strength parameters ($Q_{ult char}$); (2) the probabilistic analyses giving the PDF of the ultimate axial pile capacity ($Q_{ult mean}$); and (3) the results of the probabilistic analyses giving the annual probability of failure, P_{e}

Figure 15 is a simplification in two dimensions of the overlap of the probabilistic ultimate pile capacity (Q_{ult}) and probabilistic environmental load (P_{env}). The probability density function for the P_{env} was taken as the same for P_{f1} and P_{f2} in the calculations. The calibration of the resistance factor was coordinated with the definition of characteristic design load and the characteristic soil strength profile used for the calculation of axial pile capacity. The calibration details are described in Lacasse et al., 2013a.

Table 2 presents the results of the calibration of the resistance factor for the case study jackets to achieve an annual probability of failure of 10^{-4} . The resistance factor was obtained based on the axial pile capacity calculated with the characteristic undrained shear strength ($Q_{ult char}$). The load factors were maintained at the recommended values in the design guidance, although the load factor at the design point was smaller.

For a given pile length, the calibrated resistance factor varied with the pile design method. The factors reflect the varying influence of the uncertainty in the soil parameters and of the model uncertainties for the different methods. The results are generally consistent, where the axial pile capacity

Table 2. Calibrated resistance factors related to characteristic ultimate axial capacity.

Method	Site A (clay) 90-m pile	Site B (sand) 26-m pile	Site C (clay & sand) 40-m pile
NGI	1.23	1.35	1.20
ICP	1.52	1.45	1.32
Fugro	1.31	1.72	1.55
UWA	_	_	1.50
API	1.35	2.36	1.93

methods predicting higher axial pile capacity require a higher resistance factor to ensure that the annual probability of failure does not exceed 10^{-4} . The calibrated resistance factors apply to these case study jackets only, and cannot be transferred to other sites or structures without site-specific reliability studies.

These calibrated resistance factors allowed for a significant reduction in the required pile penetration depth because one could demonstrate that a target reliability could be achieved using lower resistance factors than the *a priori* values in the design guidance. The pile lengths could be reduced by 15 to 20 percent for Cases A and C and nearly 50 percent for Case B. A reliability analysis can therefore have important implications for the design of piles and result in significant savings.

This reliability study gave insight in the required resistance factor for different design methods of axial pile capacity to achieve the same annual P_f for a given pile penetration depth. The study is not meant to favor an approach. More case studies are needed on a variety of soil profiles to enable one to draw non site-specific recommendations on the resistance factor for each of the methods. The calibration analyses showed that:

- 1. The calibration of the safety factors demonstrates that the annual probability of failure varies with the axial pile capacity calculation method.
- The values of model uncertainty used in the analyses have an overwhelming influence on the resulting annual probability of failure and therefore on the required resistance factor for a target annual probability of failure.
- The current state-of-the-art design still relies heavily on qualified engineering judgment to assess and ensure a consistent safety level.
- The resistance factors calibrated show that the newer CPT-methods of pile design are as reliable as the current API method.
- 5. The selection of the characteristic shear strength was also a significant parameter that influences the calibrated resistance coefficient.



Figure 16. Interaction curves of pile system capacity exhibiting a robustness check (adapted from Chen et al., 2010).

6. The selection of the characteristic parameters to use in the deterministic analysis is often a source of uncertainty, and can be very subjective, varying from one engineer to the other. Lacasse et al. (2013a) provide recommendations for minimizing this variability.

6 SYSTEM RELIABILITY

Design checks are typically conducted on a component by component basis. However, the performance reliability of the entire system is generally of greatest interest in managing risk. Recent advances have been made in assessing system reliability for both fixed and floating offshore facilities.

Figure 16 shows a system robustness check for fixed jacket platforms. This idea was motivated by the performance of platforms in hurricanes in the Gulf of Mexico over the past decade. The check involves considering the capacity of the system when the lateral or axial capacity of any individual



Figure 17. Comparison of probabilities of failure in design life for different components in the most heavily loaded line of a mooring system (from Clukey et al., 2013).



Figure 18. Conditional probability of failure given failure of the most heavily-loaded line for a mooring system in a hurricane (from Clukey et al., 2013).

pile is reduced. For the three-leg jacket, the system capacity in overturning is essentially proportional to the axial capacity of the most heavily-loaded pile (Fig. 16b). For the six-leg jacket (Fig. 16a), the system capacity in overturning is less sensitive, reducing by about 10% for a 30% reduction in the axial capacity of the most heavily-loaded pile. For both cases, the system capacity in shear is much less sensitive to the lateral capacity of an individual pile. This proposed design check is to maintain a minimum system capacity when reducing the axial and lateral capacities of individual piles in order to achieve a consistent level of reliability for a wide variety of pile systems.

Figure 17 shows the results from reliability analyses for the mooring system of a floating production system located in three different water depths (Clukey et al., 2013). The probability of failure for the suction caisson foundation (anchor) is orders of magnitude smaller than those for the ropes and chains in the mooring line. In addition, the probability of failure for individual components depends on the water depth, with the smallest probabilities of failure associated with the deepest water because the uncertain environmental loads are smaller relative the certain pre-tension loads as the water depth increases.

Figure 18 shows how the redundancy in this mooring system is sensitive to whether a semi-taut or taut³ system is used. The redundancy in the taut system is greater than in the semi-taut system because the loads are re-distributed more evenly to the remaining lines when a single line fails (Fig. 18). Therefore, design checks based on single components in these mooring systems will not necessarily provide either a consistent or representative reliability with the system. This type of information is currently being considered in work to update the design guidance documents for mooring systems.

7 NEW TRENDS IN RISK MANAGEMENT

Disasters like the Maconda Well blowout, which caused the Deepwater Horizon oil spill in the Gulf of Mexico in May 2010, can catalyze moments of change in risk management aims, policy and practice. The population living along the coastline who might be affected by offshore accidents are demanding that their opinions are respected in the critical risk management decisions.

Quantitatively, risk is the expected consequence of an adverse event, where the consequences are obtained from the elements at risk and their vulnerability. Mitigation of risk can be accomplished by reducing the probability of the adverse event or by reducing the vulnerability and/or exposure of the elements at risk, or even by reducing both hazard and consequence (Fig. 19).

Designing participatory processes for stakeholder involvement the risk management decision making process is a new area of research. An example of this type of research was provided in the SafeLand Project (www.safeland-fp7.eu), a large collaborative project on landslide risk management within the European Commission's 7th Framework Programme. The SafeLand project developed and tested a public communication and participatory process for mitigating the risks of landslide in the highly at-risk community of Nocera Inferiore in southern Italy (SafeLand 2012). The pilot study demonstrated the potential

^{3.} In a semi-taut mooring system there is moderate catenary in the mooring lines, while in a taut mooring system there is small catenary in the mooring lines.



Figure 19. "Bow tie" diagram illustrating components in risk management (from Lacasse & Nadim 2009).

and challenges of public participation in decisions characterized by high personal stakes and intricate technical, economic and social considerations. It should prove useful in informing similar processes, as stakeholders in Europe increasingly demand a voice in choosing landslide mitigation measures.

The results of the pilot study in SafeLand showed that it is feasible to organize an expertinformed participatory process that respects and builds on conflicting citizen perspectives and interests, and demonstrates spheres of policy consensus as well as policy dissent. Increasingly public interventions to reduce the risk of landslides and other hazards are moving from "expert" decisions to include the public and other stakeholders in the decision process. Variations in the role of science and scientists, governance structures and interest groups, legislation, availability of economic and political instruments, social learning, facilitation of communication and trust, media intervention, access to information, and external pressures and shocks were some of the issues identified by the SafeLand research that impact the cognition and management of risk practice in a society.

Another new trend in risk management is stress testing. Stress testing is a procedure used to determine the stability of a system or entity. It involves testing the said system or entity to beyond its normal operational capacity, often to a breaking point, in order to observe its performance/reaction to a pre-defined internal or external effects (pressure/ force). Stress tests have been used for many years in air traffic safety, in particular for airplanes and helicopters. In recent years, stress testing has often been associated with methodologies to assess the vulnerability of a financial system or specific components of it, such as banks. A number of analytical tools have been developed in this area and have been frequently used since the late 1990's (e.g., Borio et al., 2012).

More recently, stress testing has been applied to the comprehensive safety and risk assessment of nuclear power plants, in particular in the aftermath of the 11th March 2011 East Japan earthquake



Figure 20. Stress testing as a tool to deal with residual or neglected risk for critical infrastructure (Nadim & Sparrevik, 2013).

and tsunami leading to the Fukushima Dai-ichi accident. Many aspects of the accident devastating the Fukushima Dai-ichi nuclear power plant are still uncertain. However, the accident highlighted three areas of potential weakness in the existing safety approaches:

- 1. Inadequacy of safety margins in the case of extreme external events, especially natural hazards.
- 2. Lack of robustness with respect to events that exceed the design basis.
- 3. Ineffectiveness of current emergency management under highly unfavourable conditions.

These issues were the focus of the stress tests imposed on all nuclear power plants in Europe in 2011 and 2012 (WENRA, 2011).

A stress test is an examination of the safety of a system under those particularly unfavourable scenarios that fall outside the design basis specified by the regulatory regime, by the operational institution or by the stakeholders. A stress test can test the system to assess its response to scenarios expected to be in the residual and neglected risk areas (Fig. 20). In this respect, stress testing is not a substitute for "conventional" risk or safety assessments, but it provides additional valuable insight under extreme situations. What stress tests and "conventional" risk or safety assessments have in common is that they both rely on a description of the system of interest, which helps to associate the state of the system and a set of consequences under any given or potential scenario.

Several multi-national research projects in Europe are starting up in 2014 to develop guidelines for stress testing of critical infrastructure under the action of natural hazards (Nadim & Sparrevik, 2013). The premises are that a critical infrastructure is designed to withstand the impact of natural hazards according to regulations in codes and standards or specifications from the owner and/or stakeholders. The regulations are often set through probabilistic evaluations with the objective of reducing risk to an acceptable level. This evaluated risk will be in accordance with what society will tolerate in terms of loss of life, environmental damages and the loss of assets through the definition of acceptance criteria that are incorporated into regulations.

The design rules that result from such regulations implicitly accept that there is a residual risk associated with rare, extreme events that is neglected because of the (objectively calculated or perceived) very low probability of occurrence. However, the Fukushima accident showed that as a consequence of this neglect, a system that is quite robust as long as events remain within its design basis can abruptly shift to complete failure when that threshold is passed. Stress tests can help detect such "cliff-edge effects" and identify ways to introduce some robustness in the system without any change in the acceptable level of risk.

Most risk evaluations are based on probability estimates using historical data, observations and/ or experience and engineering judgment, and consequence models that try to estimate the impact of unwanted future hazard situations. For natural hazards, historical data may in some cases be sparse or highly uncertain. There is also generally little experience with extreme events, because of their nature. Furthermore, simplified models of highly complex situations yield predictions of system response that contain significant uncertainty. The scarcity of data and model uncertainty may lead to optimistic evaluations that neglect the risks associated with extreme events. Stress testing provides a framework to address these neglected risks.

Stress tests have not yet been applied in offshore projects. However, the safety philosophy and premises for design of offshore structures are quite similar to those for onshore critical infrastructure. In the future, stress tests could complement the present risk assessment approaches for many offshore projects.

8 SUMMARY

This paper has described recent advances in geotechnical reliability and risk for offshore applications. The areas addressed include spatial variability, model uncertainty, hazard characterization, reliabilitybased design, system reliability and risk management. Case histories from real-world applications were described to illustrate the practical motivation for and usefulness of these advances.

The following conclusions are drawn from the evolution of reliability and risk approaches to their current state for offshore applications:

- 1. Applying the theory of reliability and risk in practice is critical to obtaining useful insights from the theory and to developing practical means to implement the theory. While the application of reliability and risk approaches has matured, each major practical application still involves a significant element of research and development to best suit that particular problem.
- 2. Assessing reliability and risk is most valuable if it is considered in the context of helping stakeholders make decisions. Opportunities in decision making exist both to mitigate risk as well as to reduce the cost required to achieve a target level of risk
- 3. Managing risk effectively requires the collaboration of multiple disciplines and the involvement of stakeholders at all stages of the process, from assessment to decision making.

While offshore applications have provided wonderful opportunities to advance these approaches, the results of these advances are relevant to a wide variety of geotechnical problems.

The future for reliability and risk approaches is bright. Public and private stakeholders will always welcome, seek and value help in making better and more defensible decisions. The following recommendations are offered to guide the continued advancement of these approaches:

- Develop means and methods to implement reliability and risk approaches that are as simple as possible while still capturing the important characteristics that describe hazards, consequences and the performance of engineered systems. Simplicity is important both to make implementation practical and to make the approaches as transparent as possible for the stakeholders.
- 2. Encourage the application of reliability and risk approaches in the earliest stages of project development when the greatest opportunities exist to impact decisions and to proactively plan to acquire valuable data for future decisions.
- 3. Continuously strive to update knowledge about hazards, consequences and performance based on historical information. Reliability and risk approaches provide the link between this information and the assessment and management of risk for future applications.
- 4. Increase awareness and understanding about reliability and risk approaches for technical professionals and colleagues in the other disciplines such as social sciences, as well as the general public.

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Homogenization of geomaterials using the random finite element method

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ABSTRACT: The homogenized stiffness of geomaterials that are highly variable at the micro-scale has long been of interest to geotechnical engineers. The purpose of this study is to investigate the influence of porosity and void size on the homogenized or effective properties of geomaterials. A Random Finite Element Method (RFEM) has been developed enabling the generation of spatially random voids of given porosity and size within a block of geomaterial. Following Monte-Carlo simulations, the mean and standard deviation of the effective property can be estimated leading to a probabilistic interpretation involving deformations. The probabilistic approach represents a rational methodology for guiding engineers in the risk management process. The influence of block size and the Representative Volume Elements (RVE) are discussed, in addition to the influence of anisotropy on the effective Young's modulus.

1 INTRODUCTION

The motivation of this work is to investigate the influence of porosity and void size on the stiffness of 3D geomaterials using a statistical approach. Even if the expected porosity of a site can be conservatively estimated, the location of the voids may be largely unknown such as in geological regions dominated by karstic deposits. This makes a statistical approach appealing. The work presented in this paper is developed from a study of 2D model homogenization of geomaterials containing voids by random fields and finite elements (Griffiths et al. 2012) and 3D random finite element methods (Fenton & Griffiths, 2005). The classic problem of homogenization of heterogeneous materials with variable micro-structure has long been of practical interest to engineers. In the current study, the influence of voids on effective elastic properties is investigated. The goal of homogenization is to predict the effective property of a heterogeneous material, where the effective value is defined as the property that would have led to the same response if the geomaterial had been homogeneous. A useful concept in this homogenization process is the Representative Volume Element (RVE). An RVE is an element of the heterogenous material that is large enough to represent the microstructure but small enough to achieve computational efficiency (e.g. Liu, 2005; Zeleniakiene et al. 2005).

Since the concept of the RVE was first introduced by Hill (1963), several theoretical models have been proposed for dealing with scale effects. Hazanov & Huet (1994) derived results involving mixed boundary conditions, which locate between the static and kinematic uniform boundary conditions for specimens smaller than the size of the RVE. Orthogonal mixed boundary conditions have also been proposed (e.g. Hazanov & Amieur, 1995; Havanov, 1998; Khisaeva & Ostoja-Starzewski, 2006). Numerical methods such as the Finite Element Method (FEM) have also been used to validate the RVE size of random heterogeneous materials. Kanit et al. (2003) used Monte-Carlo simulations to investigate RVE and effective properties, while Zohdi & Wriggers (2001) and Ostoja-Starzewski (2006) investigated the RVE size using a statistical computational approach. Although there are many models developed to investigate the effective properties of a material containing voids, there is no model that works for all problems