GLOBAL EDITION

Physics for Scientists and Engineers

A Strategic Approach with Modern Physics

FIFTH EDITION

Randall D. Knight

PHYSICS

For Scientists and Engineers | A Strategic Approach

WITH MODERN PHYSICS

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For Scientists and Engineers | A Strategic Approach | 5e

WITH MODERN PHYSICS

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RANDALL D. KNIGHT

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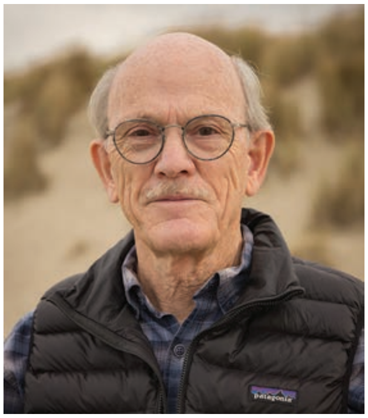
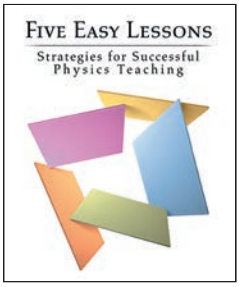


RANDY KNIGHT taught introductory physics for 32 years at Ohio State University and California Polytechnic State University, where he is Professor Emeritus of Physics. Professor Knight received a PhD in physics from the University of California, Berkeley, and was a post-doctoral fellow at the Harvard-Smithsonian Center for Astrophysics before joining the faculty at Ohio State University. His growing awareness of the importance of research in physics education led first to *Physics for Scientists and Engineers: A Strategic Approach* and later, with co-authors Brian Jones and Stuart Field, to *College Physics: A Strategic Approach* and the new *University Physics for the Life Sciences*. Professor Knight's research interests are in the fields of laser spectroscopy and environmental science. When he's not in front of a computer, you can find Randy hiking, traveling, playing the piano, or spending time with his wife Sally and their five cats.

Preface to the Instructor

This fifth edition of *Physics for Scientists and Engineers: A Strategic Approach* continues to build on the research-driven instructional techniques introduced in the first edition and the extensive feedback from thousands of users. From the beginning, the objectives have been:

- To produce a textbook that is more focused and coherent, less encyclopedic.
- To integrate proven results from physics education research into the classroom in a way that allows instructors to use a range of teaching styles.
- To provide a balance of quantitative reasoning and conceptual understanding, with special attention to concepts known to cause student difficulties.
- To develop students' problem-solving skills in a systematic manner.



A more complete explanation of these goals and the rationale behind them can be found in the Ready-To-Go Teaching Modules and in my paperback book, *Five Easy Lessons: Strategies for Successful Physics Teaching*. Please request a copy from your local Pearson sales representative if it is of interest to you (ISBN 978-0-805-38702-5).

What's New to This Edition

The fifth edition of *Physics for Scientists and Engineers* continues to utilize the best results from educational research and to tailor them for this course and its students. At the same time, the extensive feedback we've received from both instructors and students has led to many changes and improvements to the text, the figures, and the end-of-chapter problems. Changes include:

- The Chapter 6 section on drag has been expanded to include drag in a viscous fluid (Stokes' law). The Reynolds number is introduced as an indicator of whether drag is primarily viscous or primarily inertial.
- Chapter 14 on fluids now includes the flow of viscous fluids (Poiseuille's equation) and a discussion of turbulence.
- An optional Advanced Topic section on coupled oscillations and normal modes has been added to Chapter 15.
- Chapter 20 now includes an extensive quantitative section on entropy and its application.
- A vector review has been added to Chapter 22, the first electricity chapter, and the worked examples make extra effort to remind students how to work with vectors. Returning to vectors after not having used them extensively since mechanics is a stumbling block for many students.
- The number of applications illustrated with sidebar figures has been increased and now includes accelerometers, helicopter rotors, quartz oscillators, laser printers, and wireless chargers.
- There are more than 400 new or significantly revised end-of-chapter problems. Scores of other problems have been edited to improve clarity. Difficulty ratings have been recalibrated based on Mastering® Physics.
- Several substantial new Challenge Problems have been added to cover interesting and contemporary topics such as gravitational waves, normal modes of the carbon dioxide molecule, and Bose-Einstein condensates.
- New Ready-To-Go Teaching Modules are an easy-to-use online instructor's guide. These modules provide background information about topics and techniques that are known student stumbling blocks along with suggestions and assignments for use before, during, and after class.

Textbook Organization

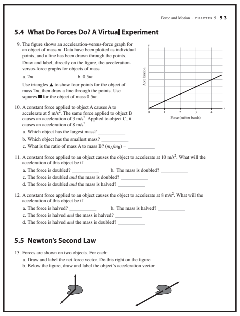
Physics for Scientists and Engineers is divided into eight parts: Part I: *Newton's Laws*, Part II: *Conservation Laws*, Part III: *Applications of Newtonian Mechanics*, Part IV: *Oscillations and Waves*, Part V: *Thermodynamics*, Part VI: *Electricity and Magnetism*, Part VII: *Optics*, and Part VIII: *Relativity and Quantum Mechanics*. Note that covering the parts in this order is by no means essential. Each topic is self-contained, and Parts III–VII can be rearranged to suit an instructor's needs. Part VII: *Optics* does need to follow Part IV: *Oscillations and Waves*; optics can be taught either before or after electricity and magnetism.

The complete 42-chapter version of *Physics for Scientists and Engineers* is intended for a three-semester course. A two-semester course typically covers 30–32 chapters with the judicious omission of a few sections.

There's a growing sentiment that quantum physics is becoming the province of engineers, not just physicists, and that even a two-semester course should include a reasonable introduction to quantum ideas. The Ready-To-Go Teaching Modules outline a couple of routes through the book that allow many of the quantum physics chapters to be included in a two-semester course. I've written the book with the hope that an increasing number of instructors will choose one of these routes.

The Student Workbook

A key component of *Physics for Scientists and Engineers: A Strategic Approach* is the accompanying *Student Workbook*. The workbook bridges the gap between textbook and homework problems by providing students the opportunity to learn and practice skills prior to using those skills in quantitative end-of-chapter problems, much as a musician practices technique separately from performance pieces. The workbook exercises, which are keyed to each section of the textbook, focus on developing specific skills, ranging from identifying forces and drawing free-body diagrams to interpreting wave functions.


Force and Motion · CHAPTER 5 5-3

5.4 What Do Forces Do? A Virtual Experiment

9. The figure shows an acceleration-versus-force graph for an object of mass m. Data have been plotted as individual points, and a line has been drawn through the points. Draw and label, directly on the figure, the acceleration-versus-force graphs for objects of mass
a. $2m$ b. $0.5m$
Use triangles ▲ to show four points for the object of mass $2m$, then draw a line through the points. Use squares ■ for the object of mass $0.5m$.

10. A constant force applied to object A causes A to accelerate at 5 m/s^2. The same force applied to object B causes an acceleration of 3 m/s^2. Applied to object C, it causes an acceleration of 8 m/s^2.
a. Which object has the largest mass? ________
b. Which object has the smallest mass? ________
c. What is the ratio of mass A to mass B? (m_A/m_B) = ________

11. A constant force applied to an object causes the object to accelerate at 10 m/s^2. What will the acceleration of this object be if
a. The force is doubled? ________ b. The mass is doubled? ________
c. The force is doubled *and* the mass is doubled? ________
d. The force is doubled *and* the mass is halved? ________

12. A constant force applied to an object causes the object to accelerate at 8 m/s^2. What will the acceleration of this object be if
a. The force is halved? ________ b. The mass is halved? ________
c. The force is halved *and* the mass is halved? ________
d. The force is halved *and* the mass is doubled? ________

5.5 Newton's Second Law

13. Forces are shown on two objects. For each:
a. Draw and label the net force vector. Do this right on the figure.
b. Below the figure, draw and label the object's acceleration vector.

The workbook exercises, which are generally qualitative and/or graphical, draw heavily upon the physics education research literature. The exercises deal with issues known to cause student difficulties and employ techniques that have proven to be effective at overcoming those difficulties. The workbook exercises can be used in class as part of an active-learning teaching strategy, in recitation sections, or as assigned homework.

Instructor Resources

A variety of resources are available to help instructors teach more effectively and efficiently. These can be downloaded from the Instructor Resources area of Mastering® Physics.

- **Ready-To-Go Teaching Modules** are an online instructor's guide. Each chapter contains background information on what is known from physics education research about student misconceptions and difficulties, suggested teaching strategies, suggested lecture demonstrations, and suggested pre- and post-class assignments.
- **Mastering® Physics** is Pearson's online homework system through which the instructor can assign pre-class reading quizzes, tutorials that help students solve a problem with hints and wrong-answer feedback, direct-measurement videos, and end-of-chapter questions and problems. Instructors can set up their own assignments or utilize pre-built assignments that have been designed with a balance of problem types and difficulties.
- **PowerPoint Lecture Slides** can be modified by the instructor but provide an excellent starting point for class presentations. The lecture slides include QuickCheck questions.
- **QuickCheck "Clicker Questions"** are conceptual questions, based on known student misconceptions, for in-class use with some form of personal response system. They are designed to be used as part of an active-learning teaching strategy. The Ready-To-Go teaching modules provide information on the effective use of QuickCheck questions.
- The **Instructor's Solution Manual** is available in both Word and PDF formats. We do require that solutions for student use be posted only on a secure course website.
- All of the textbook figures, key equations, Problem-Solving Strategies, Tactics Boxes, and more can be downloaded.
- The **TestGen Test Bank** contains over 2000 conceptual and multiple-choice questions. Test files are provided in both TestGen® and Word formats.

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Preface to the Student

From Me to You

The most incomprehensible thing about the universe is that it is comprehensible.
—Albert Einstein

The day I went into physics class it was death.
—Sylvia Plath, *The Bell Jar*

Let's have a little chat before we start. A rather one-sided chat, admittedly, because you can't respond, but that's OK. I've talked with many of your fellow students over the years, so I have a pretty good idea of what's on your mind.

What's your reaction to taking physics? Fear and loathing? Uncertainty? Excitement? All the above? Let's face it, physics has a bit of an image problem on campus. You've probably heard that it's difficult, maybe impossible unless you're an Einstein. Things that you've heard, your experiences in other science courses, and many other factors all color your *expectations* about what this course is going to be like.

It's true that there are many new ideas to be learned in physics and that the course, like college courses in general, is going to be much faster paced than science courses you had in high school. I think it's fair to say that it will be an *intense* course. But we can avoid many potential problems and difficulties if we can establish, here at the beginning, what this course is about and what is expected of you—and of me!

Just what is physics, anyway? Physics is a way of thinking about the physical aspects of nature. Physics is not better than art or biology or poetry or religion, which are also ways to think about nature; it's simply different. One of the things this course will emphasize is that physics is a human endeavor. The ideas presented in this book were not found in a cave or conveyed to us by aliens; they were discovered and developed by real people engaged in a struggle with real issues.

You might be surprised to hear that physics is not about "facts." Oh, not that facts are unimportant, but physics is far more focused on discovering *relationships* and *patterns* than on learning facts for their own sake.

For example, the colors of the rainbow appear both when white light passes through a prism and—as in this photo—when white light reflects from a thin film of oil on water. What does this pattern tell us about the nature of light?

Our emphasis on relationships and patterns means that there's not a lot of memorization when you study physics. Some—there are still definitions and equations to learn—but less than in many other courses. Our emphasis, instead, will be on thinking and reasoning. This is important to factor into your expectations for the course.

Perhaps most important of all, *physics is not math!* Physics is much broader. We're going to look for patterns and relationships in nature, develop the logic that relates different ideas, and search for the reasons *why* things happen as they do. In doing so, we're going to stress qualitative reasoning, pictorial and graphical reasoning, and reasoning by analogy. And yes, we will use math, but it's just one tool among many.

It will save you much frustration if you're aware of this physics–math distinction up front. Many of you, I know, want to find a formula and plug numbers into it—that is, to do a math problem. Maybe that worked in high school science courses, but it is *not* what this course expects of you. We'll certainly do many calculations, but the specific numbers are usually the last and least important step in the analysis.

As you study, you'll sometimes be baffled, puzzled, and confused. That's perfectly normal and to be expected. Making mistakes is OK too *if* you're willing to learn from the experience. No one is born knowing how to do physics any more than he or she is born knowing how to play the piano or shoot basketballs. The ability to do physics comes from practice, repetition, and struggling with the ideas until you "own" them and can apply them yourself in new situations. There's no way to make learning effortless, at least for anything worth learning, so expect to have some difficult moments ahead. But also expect to have some moments of excitement at the joy of discovery. There will be instants at which the pieces suddenly click into place and you *know* that you understand a powerful idea. There will be times when you'll surprise yourself by successfully working a difficult problem that you didn't think you could solve. My hope, as an author, is that the excitement and sense of adventure will far outweigh the difficulties and frustrations.

Getting the Most Out of Your Course

Many of you, I suspect, would like to know the "best" way to study for this course. There is no best way. People are different and what works for one student is less effective for another. But I do want to stress that *reading the text* is vitally important. The basic knowledge for this course is written down on these pages, and your instructor's *number-one expectation* is that you will read carefully to find and learn that knowledge.

Despite there being no best way to study, I will suggest *one* way that is successful for many students.

1. **Read each chapter *before* it is discussed in class.** I cannot stress too strongly how important this step is. Class attendance is much more effective if you are prepared. When you first read a chapter, focus on learning new vocabulary, definitions, and notation. There's a list of terms and notations at the end of each chapter. Learn them! You won't understand what's being discussed or how the ideas are being used if you don't know what the terms and symbols mean.

2. **Participate actively in class.** Take notes, ask and answer questions, and participate in discussion groups. There is ample scientific evidence that *active participation* is much more effective for learning science than passive listening.
3. **After class, go back for a careful re-reading of the chapter.** In your second reading, pay closer attention to the details and the worked examples. Look for the *logic* behind each example (I've highlighted this to make it clear), not just at what formula is being used. And use the textbook tools that are designed to help your learning, such as the problem-solving strategies, the chapter summaries, and the exercises in the *Student Workbook*.
4. **Finally, apply what you have learned to the homework problems at the end of each chapter.** I strongly encourage you to form a study group with two or three classmates. There's good evidence that students who study regularly with a group do better than the rugged individualists who try to go it alone.

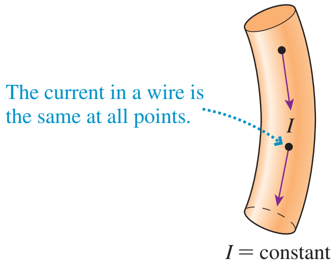
Did someone mention a workbook? The companion *Student Workbook* is a vital part of the course. Its questions and exercises ask you to reason *qualitatively,* to use graphical information, and to give explanations. It is through these exercises that you will learn what the concepts mean and will practice the reasoning skills appropriate to the chapter. You will then have acquired the baseline knowledge and confidence you need *before* turning to the end-of-chapter homework problems. In sports or in music, you would never think of performing before you practice, so why would you want to do so in physics? The workbook is where you practice and work on basic skills.

Many of you, I know, will be tempted to go straight to the homework problems and then thumb through the text looking for a formula that seems like it will work. That approach will not succeed in this course, and it's guaranteed to make you frustrated and discouraged. Very few homework problems are of the "plug and chug" variety where you simply put numbers into a formula. To work the homework problems successfully, you need a better study strategy—either the one outlined above or your own—that helps you learn the concepts and the relationships between the ideas.

Getting the Most Out of Your Textbook

Your textbook provides many features designed to help you learn the concepts of physics and solve problems more effectively.

- **TACTICS BOXES** give step-by-step procedures for particular skills, such as interpreting graphs or drawing special diagrams. Tactics Box steps are explicitly illustrated in subsequent worked examples, and these are often the starting point of a full *Problem-Solving Strategy*.
- **PROBLEM-SOLVING STRATEGIES** are provided for each broad class of problems—problems characteristic of a chapter or group of chapters. The strategies follow a consistent four-step approach to help you develop confidence and proficient problem-solving skills: **MODEL, VISUALIZE, SOLVE, REVIEW**.
- Worked **EXAMPLES** illustrate good problem-solving practices through the consistent use of the four-step problem-solving approach The worked examples are often very detailed and carefully lead you through the *reasoning* behind the solution as well as the numerical calculations.
- **STOP TO THINK** questions embedded in the chapter allow you to quickly assess whether you've understood the main idea of a section. A correct answer will give you confidence to move on to the next section. An incorrect answer will alert you to re-read the previous section.
- Blue annotations on figures help you better understand what the figure is showing. They will help you to interpret graphs; translate between graphs, math, and pictures; grasp difficult concepts through a visual analogy; and develop many other important skills.



- Schematic *Chapter Summaries* help you organize what you have learned into a hierarchy, from general principles (top) to applications (bottom). Side-by-side pictorial, graphical, textual, and mathematical representations are used to help you translate between these key representations.
- Each part of the book ends with a **KNOWLEDGE STRUCTURE** designed to help you see the forest rather than just the trees.

Now that you know more about what is expected of you, what can you expect of me? That's a little trickier because the book is already written! Nonetheless, the book was prepared on the basis of what I think my students throughout the years have expected—and wanted—from their physics textbook. Further, I've listened to the extensive feedback I have received from thousands of students like you, and their instructors, who used the first four editions of this book.

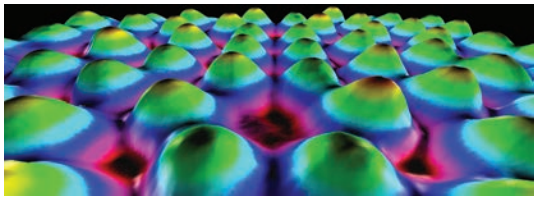
You should know that these course materials—the text and the workbook—are based on extensive research about how students learn physics and the challenges they face. The effectiveness of many of the exercises has been demonstrated through extensive class testing. I've written the book in an informal style that I hope you will find appealing and that will encourage you to do the reading. And, finally, I have endeavored to make clear not only that physics, as a technical body of knowledge, is relevant to your profession but also that physics is an exciting adventure of the human mind.

I hope you'll enjoy the time we're going to spend together.

Brief Contents

Detailed Contents

PART VIII Relativity and Quantum Physics



Useful Data

M_e	Mass of the earth	5.97×10^{24} kg	
R_e	Radius of the earth	6.37×10^{6} m	
g	Free-fall acceleration on earth	$9.80\ \mathrm{m/s^2}$	
G	Gravitational constant	$6.67 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$	
k_B	Boltzmann's constant	1.38×10^{-23} J/K	
R	Gas constant	8.31 J/mol K	
N_A	Avogadro's number	6.02×10^{23} particles/mol	
T_0	Absolute zero	−273°C	
σ	Stefan-Boltzmann constant	$5.67 \times 10^{-8}\ \mathrm{W/m^2\,K^4}$	
p_{atm}	Standard atmosphere	101,300 Pa	
v_{sound}	Speed of sound in air at 20°C	343 m/s	
m_p	Mass of the proton (and the neutron)	1.67×10^{-27} kg	
m_e	Mass of the electron	9.11×10^{-31} kg	
K	Coulomb's law constant ($1/4\pi\epsilon_0$)	$8.99 \times 10^{9}\ \mathrm{N\,m^2/C^2}$	
ϵ_0	Permittivity constant	$8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2}$	
μ_0	Permeability constant	1.26×10^{-6} T m/A	
e	Fundamental unit of charge	1.60×10^{-19} C	
c	Speed of light in vacuum	3.00×10^{8} m/s	
h	Planck's constant	6.63×10^{-34} J s	4.14×10^{-15} eV s
$\hbar$	Planck's constant	1.05×10^{-34} J s	6.58×10^{-16} eV s
a_B	Bohr radius	5.29×10^{-11} m	

Common Prefixes

Prefix	Meaning
femto-	10^{-15}
pico-	10^{-12}
nano-	10^{-9}
micro-	10^{-6}
milli-	10^{-3}
centi-	10^{-2}
kilo-	10^{3}
mega-	10^{6}
giga-	10^{9}
terra-	10^{12}

Conversion Factors

Length
1 in = 2.54 cm
1 mi = 1.609 km
1 m = 39.37 in
1 km = 0.621 mi

Velocity
1 mph = 0.447 m/s
1 m/s = 2.24 mph = 3.28 ft/s

Mass and energy
1 u = 1.661×10^{-27} kg
1 cal = 4.19 J
1 eV = 1.60×10^{-19} J

Time
1 day = 86,400 s
1 year = 3.16×10^{7} s

Pressure
1 atm = 101.3 kPa = 760 mm of Hg
1 atm = 14.7 lb/in^2

Rotation
1 rad = $180°/\pi = 57.3°$
1 rev = $360° = 2\pi$ rad
1 rev/s = 60 rpm

Mathematical Approximations

Binomial approximation: $(1 + x)^n \approx 1 + nx$ if $x \ll 1$
Small-angle approximation: $\sin\theta \approx \tan\theta \approx \theta$ and $\cos\theta \approx 1$ if $\theta \ll 1$ radian

Greek Letters Used in Physics

Alpha		α	Mu		μ
Beta		β	Pi		π
Gamma	Γ	γ	Rho		ρ
Delta	Δ	δ	Sigma	Σ	σ
Epsilon		ϵ	Tau		τ
Eta		η	Phi	Φ	ϕ
Theta	Θ	θ	Psi		ψ
Lambda		λ	Omega	Ω	ω

Problem-Solving Strategies and Model Boxes

PART

I

Newton's Laws

OVERVIEW

Why Things Move

Each of the seven parts of this book opens with an overview to give you a look ahead, a glimpse at where your journey will take you in the next few chapters. It's easy to lose sight of the big picture while you're busy negotiating the terrain of each chapter. In addition, each part closes with a Knowledge Structure to help you consolidate your knowledge. You might want to look ahead now to the Part I Knowledge Structure on page 230.

In Part I, the big picture, in a word, is *motion.*

- **How do we describe motion?** It is easy to say that an object moves, but it's not obvious how we should measure or characterize the motion if we want to analyze it mathematically. The mathematical description of motion is called *kinematics,* and it is the subject matter of Chapters 1 through 4.
- **How do we explain motion?** Why do objects have the particular motion they do? Why, when you toss a ball upward, does it go up and then come back down rather than keep going up? What "laws of nature" allow us to predict an object's motion? The explanation of motion in terms of its causes is called *dynamics,* and it is the topic of Chapters 5 through 8.

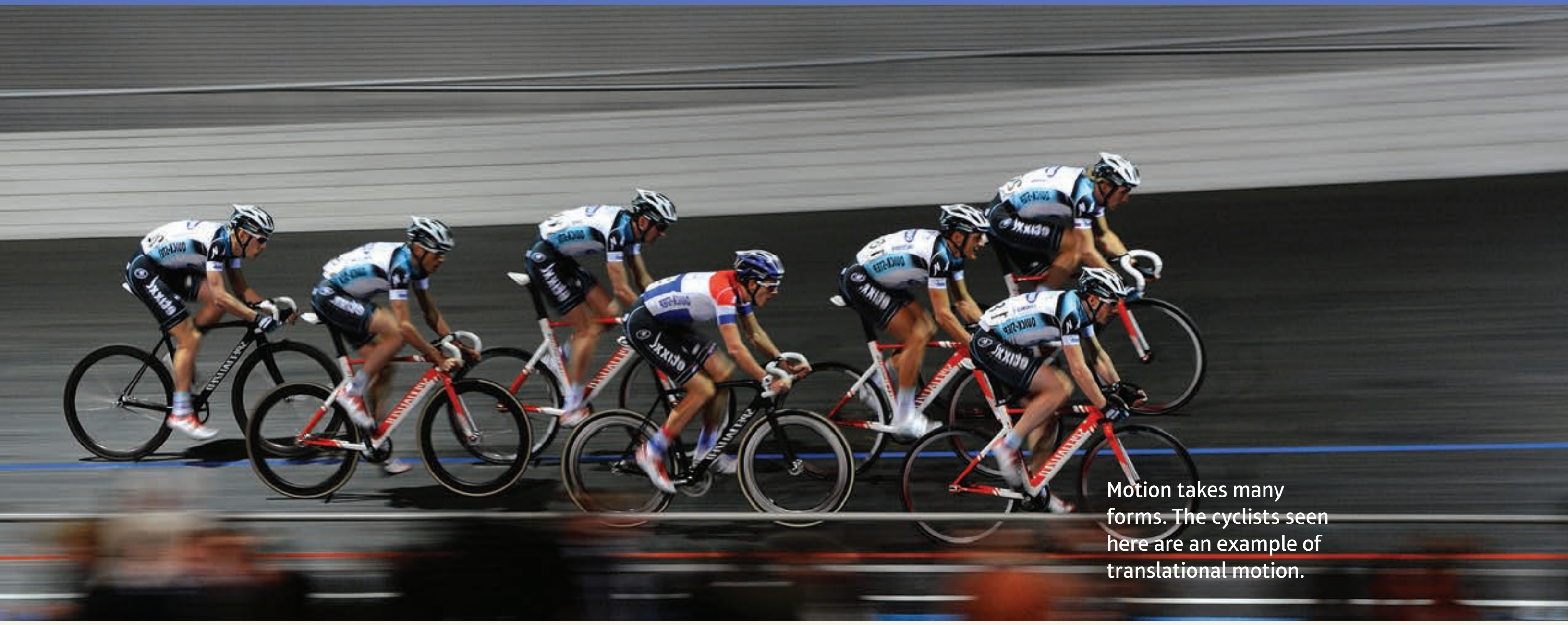
Two key ideas for answering these questions are *force* (the "cause") and *acceleration* (the "effect"). A variety of pictorial and graphical tools will be developed in Chapters 1 through 5 to help you develop an *intuition* for the connection between force and acceleration. You'll then put this knowledge to use in Chapters 5 through 8 as you analyze motion of increasing complexity.

Another important tool will be the use of *models.* Reality is extremely complicated. We would never be able to develop a science if we had to keep track of every little detail of every situation. A model is a simplified description of reality—much as a model airplane is a simplified version of a real airplane—used to reduce the complexity of a problem to the point where it can be analyzed and understood. We will introduce several important models of motion, paying close attention, especially in these earlier chapters, to where simplifying assumptions are being made, and why.

The *laws of motion* were discovered by Isaac Newton roughly 350 years ago, so the study of motion is hardly cutting-edge science. Nonetheless, it is still extremely important. Mechanics—the science of motion—is the basis for much of engineering and applied science, and many of the ideas introduced here will be needed later to understand things like the motion of waves and the motion of electrons through circuits. Newton's mechanics is the foundation of much of contemporary science, thus we will start at the beginning.

Motion can be slow and steady, or fast and sudden. This rocket, with its rapid acceleration, is responding to forces exerted on it by thrust, gravity, and the air.

1 Concepts of Motion



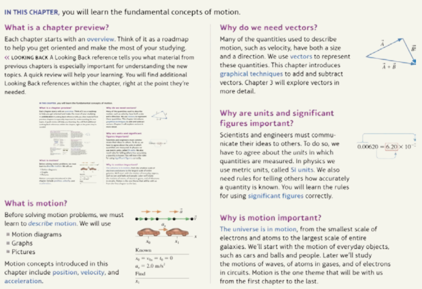
Motion takes many forms. The cyclists seen here are an example of translational motion.

IN THIS CHAPTER, you will learn the fundamental concepts of motion.

What is a chapter preview?

Each chapter starts with an **overview**. Think of it as a roadmap to help you get oriented and make the most of your studying.

« **LOOKING BACK** A Looking Back reference tells you what material from previous chapters is especially important for understanding the new topics. A quick review will help your learning. You will find additional Looking Back references within the chapter, right at the point they're needed.

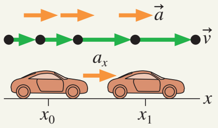


What is motion?

Before solving motion problems, we must learn to ***describe* motion**. We will use

- Motion diagrams
- Graphs
- Pictures

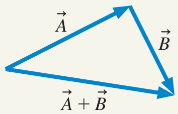
Motion concepts introduced in this chapter include **position**, **velocity**, and **acceleration**.



Known
$x_0 = v_{0x} = t_0 = 0$
$a_x = 2.0\ \mathrm{m/s^2}$
Find
x_1

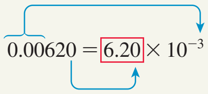
Why do we need vectors?

Many of the quantities used to describe motion, such as velocity, have both a size and a direction. We use **vectors** to represent these quantities. This chapter introduces **graphical techniques** to add and subtract vectors. Chapter 3 will explore vectors in more detail.



Why are units and significant figures important?

Scientists and engineers must communicate their ideas to others. To do so, we have to agree about the *units* in which quantities are measured. In physics we use metric units, called **SI units**. We also need rules for telling others how accurately a quantity is known. You will learn the rules for using **significant figures** correctly.



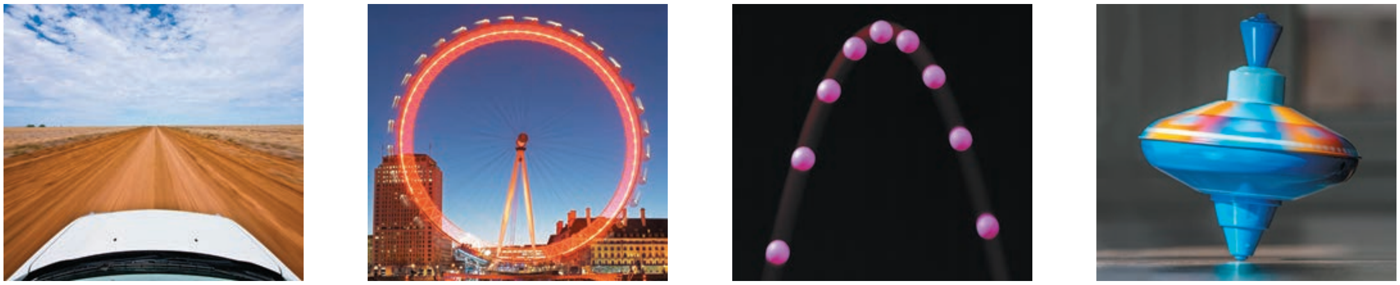
Why is motion important?

The universe is in motion, from the smallest scale of electrons and atoms to the largest scale of entire galaxies. We'll start with the motion of everyday objects, such as cars and balls and people. Later we'll study the motions of waves, of atoms in gases, and of electrons in circuits. Motion is the one theme that will be with us from the first chapter to the last.

1.1 Motion Diagrams

Motion is a theme that will appear in one form or another throughout this entire book. Although we all have intuition about motion, based on our experiences, some of the important aspects of motion turn out to be rather subtle. So rather than jumping immediately into a lot of mathematics and calculations, this first chapter focuses on *visualizing* motion and becoming familiar with the *concepts* needed to describe a moving object. Our goal is to lay the foundations for understanding motion.

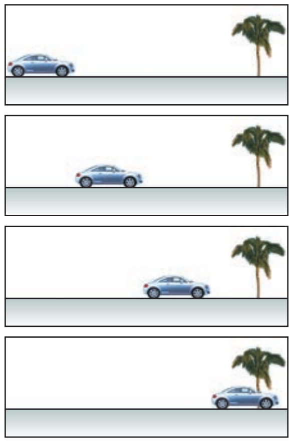
FIGURE 1.1 Four basic types of motion.



Linear motion **Circular motion** **Projectile motion** **Rotational motion**

To begin, let's define **motion** as the change of an object's position with time. FIGURE 1.1 shows four basic types of motion that we will study in this book. The first three—linear, circular, and projectile motion—in which the object moves through space are called **translational motion.** The path along which the object moves, whether straight or curved, is called the object's **trajectory.** Rotational motion is somewhat different because there's movement but the object as a whole doesn't change position. We'll defer rotational motion until later and, for now, focus on translational motion.

FIGURE 1.2 Four frames from a video.

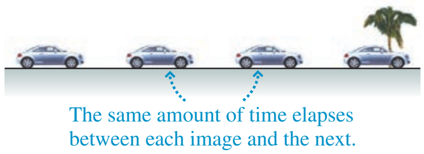


Making a Motion Diagram

An easy way to study motion is to make a video of a moving object. A video camera, as you probably know, takes images at a fixed rate, typically 30 every second. Each separate image is called a *frame*. As an example, FIGURE 1.2 shows four frames from a video of a car going past. Not surprisingly, the car is in a somewhat different position in each frame.

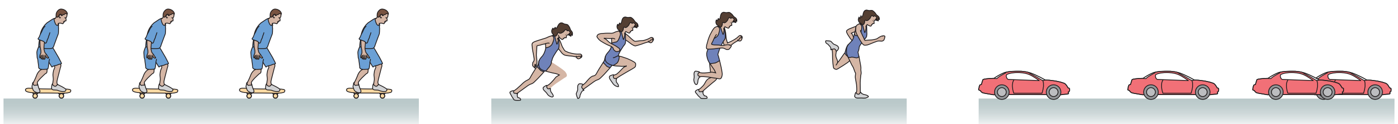
Suppose we edit the video by layering the frames on top of each other, creating the composite image shown in FIGURE 1.3. This edited image, showing an object's position at several *equally spaced instants of time,* is called a **motion diagram.** As the examples below show, we can define concepts such as constant speed, speeding up, and slowing down in terms of how an object appears in a motion diagram.

FIGURE 1.3 A motion diagram of the car shows all the frames simultaneously.



NOTE It's important to keep the camera in a *fixed position* as the object moves by. Don't "pan" it to track the moving object.

Examples of motion diagrams

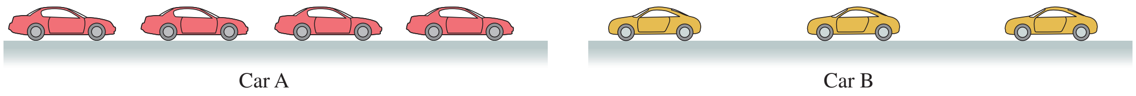


Images that are *equally spaced* indicate an object moving with *constant speed.*

An *increasing distance* between the images shows that the object is *speeding up.*

A *decreasing distance* between the images shows that the object is *slowing down.*

STOP TO THINK 1.1 Which car is going faster, A or B? Assume there are equal intervals of time between the frames of both videos.



NOTE Each chapter will have several *Stop to Think* questions. These questions are designed to see if you've understood the basic ideas that have been presented. The answers are given at the end of the book, but you should make a serious effort to think about these questions before turning to the answers.

1.2 Models and Modeling

We can model an airplane's takeoff as a particle (a descriptive model) undergoing constant acceleration (a descriptive model) in response to constant forces (an explanatory model).

The real world is messy and complicated. Our goal in physics is to brush aside many of the real-world details in order to discern patterns that occur over and over. For example, a swinging pendulum, a vibrating guitar string, a sound wave, and jiggling atoms in a crystal are all very different—yet perhaps not so different. Each is an example of a system moving back and forth around an equilibrium position. If we focus on understanding a very simple oscillating system, such as a mass on a spring, we'll automatically understand quite a bit about the many real-world manifestations of oscillations.

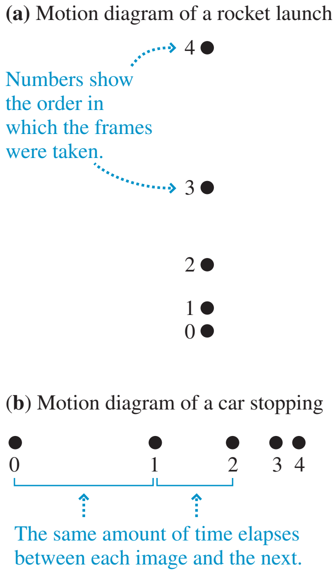
Stripping away the details to focus on essential features is a process called *modeling.* A **model** is a highly simplified picture of reality, but one that still captures the essence of what we want to study. Thus "mass on a spring" is a simple but realistic model of almost all oscillating systems.

Models allow us to make sense of complex situations by providing a framework for thinking about them. One could go so far as to say that developing and testing models is at the heart of the scientific process. Albert Einstein once said, "Physics should be as simple as possible—but not simpler." We want to find the simplest model that allows us to understand the phenomenon we're studying, but we can't make the model so simple that key aspects of the phenomenon get lost.

We'll develop and use many models throughout this textbook; they'll be one of our most important thinking tools. These models will be of two types:

- *Descriptive models:* What are the essential characteristics and properties of a phenomenon? How do we describe it in the simplest possible terms? For example, the mass-on-a-spring model of an oscillating system is a descriptive model.
- *Explanatory models:* Why do things happen as they do? Explanatory models, based on the laws of physics, have predictive power, allowing us to test—against experimental data—whether a model provides an adequate explanation of our observations.

FIGURE 1.4 Motion diagrams in which the object is modeled as a particle.



The Particle Model

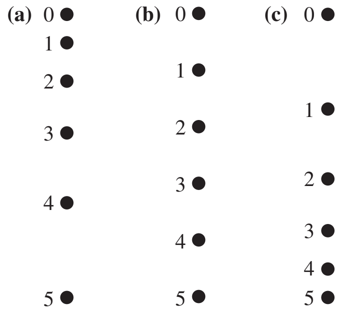
For many types of motion, such as that of balls, cars, and rockets, the motion of the object *as a whole* is not influenced by the details of the object's size and shape. All we really need to keep track of is the motion of a single point on the object, so we can treat the object *as if* all its mass were concentrated into this single point. An object that can be represented as a mass at a single point in space is called a **particle.** A particle has no size, no shape, and no distinction between top and bottom or between front and back.

If we model an object as a particle, we can represent the object in each frame of a motion diagram as a simple dot rather than having to draw a full picture. **FIGURE 1.4** shows how much simpler motion diagrams appear when the object is represented as a particle. Note that the dots have been numbered 0, 1, 2, . . . to tell the sequence in which the frames were taken.

Treating an object as a particle is, of course, a simplification of reality—but that's what modeling is all about. The **particle model** of motion is a simplification in which we treat a moving object as if all of its mass were concentrated at a single point. The particle model is an excellent approximation of reality for the translational motion of cars, planes, rockets, and similar objects.

Of course, not everything can be modeled as a particle; models have their limits. Consider, for example, a rotating gear. The center doesn't move at all while each tooth is moving in a different direction. We'll need to develop new models when we get to new types of motion, but the particle model will serve us well throughout Part I of this book.

STOP TO THINK 1.2 Three motion diagrams are shown. Which is a dust particle settling to the floor at constant speed, which is a ball dropped from the roof of a building, and which is a descending rocket slowing to make a soft landing on Mars?



1.3 Position, Time, and Displacement

To use a motion diagram, you would like to know *where* the object is (i.e., its *position*) and *when* the object was at that position (i.e., the *time*). Position measurements can be made by laying a coordinate-system grid over a motion diagram. You can then measure the (x, y) coordinates of each point in the motion diagram. Of course, the world does not come with a coordinate system attached. A coordinate system is an artificial grid that *you* place over a problem in order to analyze the motion. You place the origin of your coordinate system wherever you wish, and different observers of a moving object might all choose to use different origins.

Time, in a sense, is also a coordinate system, although you may never have thought of time this way. You can pick an arbitrary point in the motion and label it "$t = 0$ seconds." This is simply the instant you decide to start your clock or stopwatch, so it is the origin of your time coordinate. Different observers might choose to start their clocks at different moments. A video frame labeled "$t = 4$ seconds" was taken 4 seconds after you started your clock.

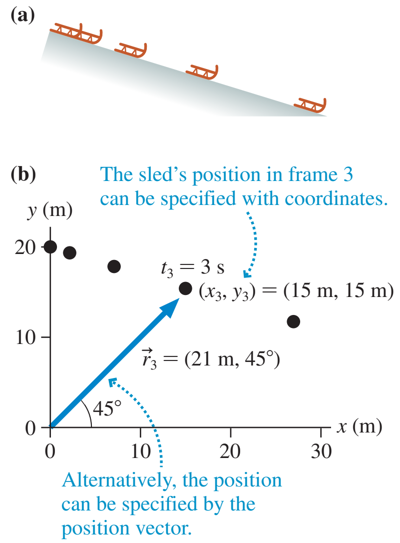
We typically choose $t = 0$ to represent the "beginning" of a problem, but the object may have been moving before then. Those earlier instants would be measured as negative times, just as objects on the x-axis to the left of the origin have negative values of position. Negative numbers are not to be avoided; they simply locate an event in space or time *relative to an origin*.

To illustrate, **FIGURE 1.5a** shows a sled sliding down a snow-covered hill. **FIGURE 1.5b** is a motion diagram for the sled, over which we've drawn an xy-coordinate system. You can see that the sled's position is $(x_3, y_3) = (15\text{ m}, 15\text{ m})$ at time $t_3 = 3$ s. Notice how we've used subscripts to indicate the time and the object's position in a specific frame of the motion diagram.

NOTE The frame at $t = 0$ s is frame 0. That is why the fourth frame is labeled 3.

Another way to locate the sled is to draw its **position vector:** an arrow from the origin to the point representing the sled. The position vector is given the symbol $\vec{r}$. Figure 1.5b shows the position vector $\vec{r}_3 = (21\text{ m}, 45°)$. The position vector $\vec{r}$ does not tell us anything different than the coordinates (x, y). It simply provides the information in an alternative form.

FIGURE 1.5 Motion diagram of a sled with frames made every 1 s.



Scalars and Vectors

Some physical quantities, such as time, mass, and temperature, can be described completely by a single number with a unit. For example, the mass of an object is 6 kg and its temperature is 30°C. A single number (with a unit) that describes a physical quantity is called a **scalar.** A scalar can be positive, negative, or zero.

Many other quantities, however, have a directional aspect and cannot be described by a single number. To describe the motion of a car, for example, you must specify not only how fast it is moving, but also the *direction* in which it is moving. A quantity having both a *size* (the "How far?" or "How fast?") and a *direction* (the "Which way?") is called a **vector.** The size or length of a vector is called its *magnitude*. Vectors will be studied thoroughly in Chapter 3, so all we need for now is a little basic information.

We indicate a vector by drawing an arrow over the letter that represents the quantity. Thus $\vec{r}$ and $\vec{A}$ are symbols for vectors, whereas r and A, without the arrows, are symbols for scalars. In handwritten work you must draw arrows over all symbols that represent vectors. This may seem strange until you get used to it, but it is very important because we will often use both r and $\vec{r}$, or both A and $\vec{A}$, in the same problem, and they mean different things! Note that the arrow over the symbol always points to the right, regardless of which direction the actual vector points. Thus we write $\vec{r}$ or $\vec{A}$, never $\overleftarrow{r}$ or $\overleftarrow{A}$.

Displacement

We said that motion is the change in an object's position with time, but how do we show a change of position? A motion diagram is the perfect tool. **FIGURE 1.6** is the motion diagram of a sled sliding down a snow-covered hill. To show how the sled's position changes between, say, $t_3 = 3$ s and $t_4 = 4$ s, we draw a vector arrow between the two dots of the motion diagram. This vector is the sled's **displacement,** which is given the symbol $\Delta\vec{r}$. The Greek letter delta (Δ) is used in math and science to indicate the *change* in a quantity. In this case, as we'll show, the displacement $\Delta\vec{r}$ is the change in an object's position.

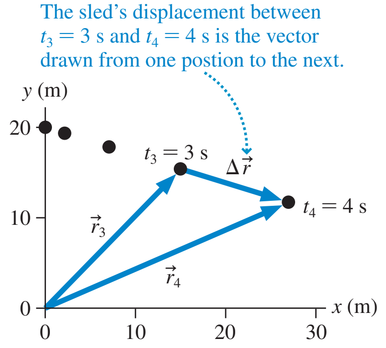


FIGURE 1.6 The sled undergoes a displacement $\Delta\vec{r}$ from position $\vec{r}_3$ to position $\vec{r}_4$.

NOTE $\Delta\vec{r}$ is a *single* symbol. It shows "from here to there." You cannot cancel out or remove the Δ.

Notice how the sled's position vector $\vec{r}_4$ is a combination of its early position $\vec{r}_3$ with the displacement vector $\Delta\vec{r}$. In fact, $\vec{r}_4$ is the *vector sum* of the vectors $\vec{r}_3$ and $\Delta\vec{r}$. This is written

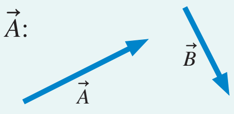
$$\vec{r}_4 = \vec{r}_3 + \Delta\vec{r} \tag{1.1}$$

Here we're adding vector quantities, not numbers, and vector addition differs from "regular" addition. We'll explore vector addition more thoroughly in Chapter 3, but for now you can add two vectors $\vec{A}$ and $\vec{B}$ with the three-step procedure of « **TACTICS BOX 1.1**.

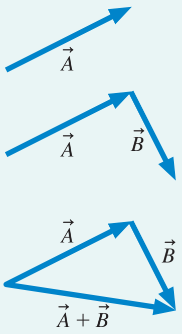
TACTICS BOX 1.1

Vector addition

To add $\vec{B}$ to $\vec{A}$:



1. Draw $\vec{A}$.
2. Place the tail of $\vec{B}$ at the tip of $\vec{A}$.
3. Draw an arrow from the tail of $\vec{A}$ to the tip of $\vec{B}$. This is vector $\vec{A} + \vec{B}$.



If you examine Figure 1.6, you'll see that the steps of Tactics Box 1.1 are exactly how $\vec{r}_3$ and $\Delta\vec{r}$ are added to give $\vec{r}_4$.

NOTE A vector is not tied to a particular location on the page. You can move a vector around as long as you don't change its length or the direction it points. Vector $\vec{B}$ is not changed by sliding it to where its tail is at the tip of $\vec{A}$.

Equation 1.1 told us that $\vec{r}_4 = \vec{r}_3 + \Delta\vec{r}$. This is easily rearranged to give a more precise definition of displacement: **The displacement $\Delta\vec{r}$ of an object as it moves from one position $\vec{r}_a$ to a different position $\vec{r}_b$ is**

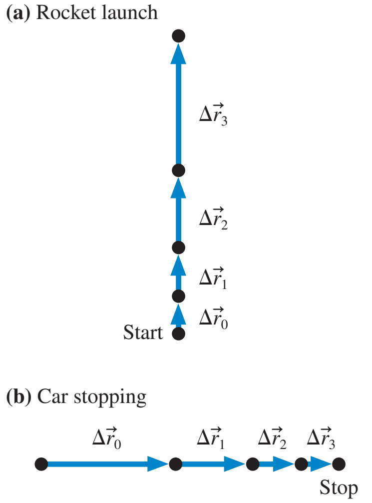
$$\Delta\vec{r} = \vec{r}_b - \vec{r}_a \tag{1.2}$$

That is, displacement is the change (i.e., the difference) in position. **Graphically, $\Delta\vec{r}$ is a vector arrow drawn from position $\vec{r}_a$ to position $\vec{r}_b$.**

Motion Diagrams with Displacement Vectors

The first step in analyzing a motion diagram is to determine all of the displacement vectors, which are simply the arrows connecting each dot to the next. Label each arrow with a *vector* symbol $\Delta\vec{r}_n$, starting with $n = 0$. FIGURE 1.7 shows the motion diagrams of Figure 1.4 redrawn to include the displacement vectors.

FIGURE 1.7 Motion diagrams with the displacement vectors.



NOTE When an object either starts from rest or ends at rest, the initial or final dots are *as close together* as you can draw the displacement vector arrow connecting them. In addition, just to be clear, you should write "Start" or "Stop" beside the initial or final dot. It is important to distinguish stopping from merely slowing down.

Now we can conclude, more precisely than before, that, as time proceeds:

- An object is speeding up if its displacement vectors are increasing in length.
- An object is slowing down if its displacement vectors are decreasing in length.

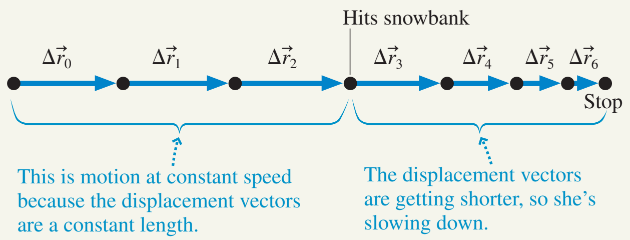
EXAMPLE 1.1 ■ Headfirst into the snow

Alice is sliding along a smooth, icy road on her sled when she suddenly runs headfirst into a large, very soft snowbank that gradually brings her to a halt. Draw a motion diagram for Alice. Show and label all displacement vectors.

MODEL The details of Alice and the sled—their size, shape, color, and so on—are not relevant to understanding their overall motion. So we can model Alice and the sled as one particle.

VISUALIZE FIGURE 1.8 shows a motion diagram. The problem statement suggests that the sled's speed is very nearly constant until it hits the snowbank. Thus the displacement vectors are of equal length as Alice slides along the icy road. She begins slowing when she hits the snowbank, so the displacement vectors then get shorter until the sled stops. We're told that her stop is gradual, so we want the vector lengths to get shorter gradually rather than suddenly.

FIGURE 1.8 The motion diagram of Alice and the sled.



A stopwatch is used to measure a time interval.

Time Interval

It's also useful to consider a *change* in time. For example, the clock readings of two frames of a video might be t_1 and t_2. The specific values are arbitrary because they are timed relative to an arbitrary instant that you chose to call $t = 0$. But the **time interval** $\Delta t = t_2 - t_1$ is *not* arbitrary. It represents the elapsed time for the object to move from one position to the next.

The time interval $\Delta t = t_b - t_a$ measures the elapsed time as an object moves from position $\vec{r}_a$ at time t_a to position $\vec{r}_b$ at time t_b. The value of Δt is independent of the specific clock used to measure the times.

To summarize the main idea of this section, we have added coordinate systems and clocks to our motion diagrams in order to measure *when* each frame was exposed and *where* the object was located at that time. Different observers of the motion may choose different coordinate systems and different clocks. However, all observers find the *same* values for the displacements $\Delta\vec{r}$ and the time intervals Δt because these are independent of the specific coordinate system used to measure them.

1.4 Velocity

The victory goes to the runner with the highest average speed.

It's no surprise that, during a given time interval, a speeding bullet travels farther than a speeding snail. To extend our study of motion so that we can compare the bullet to the snail, we need a way to measure how fast or how slowly an object moves.

One quantity that measures an object's fastness or slowness is its **average speed,** defined as the ratio

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time interval spent traveling}} = \frac{d}{\Delta t} \tag{1.3}$$

If you drive 15 miles (mi) in 30 minutes $\left(\frac{1}{2}\text{ h}\right)$, your average speed is

$$\text{average speed} = \frac{15\text{ mi}}{\frac{1}{2}\text{ h}} = 30\text{ mph} \tag{1.4}$$

Although the concept of speed is widely used in our day-to-day lives, it is not a sufficient basis for a science of motion. To see why, imagine you're trying to land a jet plane on an aircraft carrier. It matters a great deal to you whether the aircraft carrier is moving at 20 mph (miles per hour) to the north or 20 mph to the east. Simply knowing that the ship's speed is 20 mph is not enough information!

It's the displacement $\Delta\vec{r}$, a vector quantity, that tells us not only the distance traveled by a moving object, but also the *direction* of motion. Consequently, a more useful ratio than $d/\Delta t$ is the ratio $\Delta\vec{r}/\Delta t$. In addition to measuring how fast an object moves, this ratio is a vector that points in the direction of motion.

It is convenient to give this ratio a name. We call it the **average velocity,** and it has the symbol $\vec{v}_{\text{avg}}$. **The average velocity of an object during the time interval Δt, in which the object undergoes a displacement $\Delta\vec{r}$, is the vector**

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \tag{1.5}$$

An object's average velocity vector points in the same direction as the displacement vector $\Delta\vec{r}$. This is the direction of motion.

> **NOTE** In everyday language we do not make a distinction between speed and velocity, but in physics *the distinction is very important.* In particular, speed is simply "How fast?" whereas velocity is "How fast, and in which direction?" As we go along we will be giving other words more precise meanings in physics than they have in everyday language.

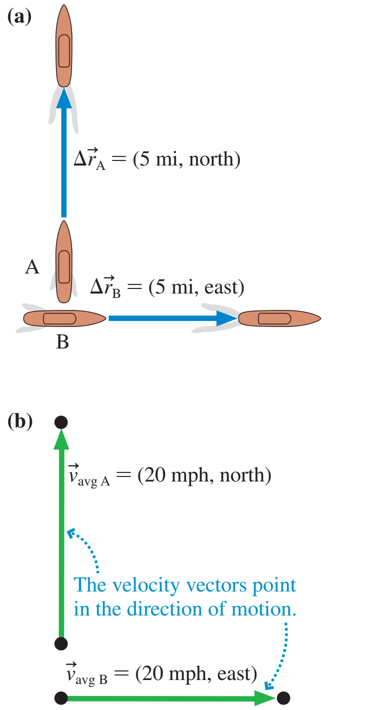
As an example, **FIGURE 1.9a** shows two ships that move 5 miles in 15 minutes. Using Equation 1.5 with $\Delta t = 0.25$ h, we find

$$\vec{v}_{\text{avg A}} = (20 \text{ mph, north})$$
$$\vec{v}_{\text{avg B}} = (20 \text{ mph, east}) \tag{1.6}$$

Both ships have a speed of 20 mph, but their velocities differ. Notice how the velocity *vectors* in **FIGURE 1.9b** point in the direction of motion.

NOTE Our goal in this chapter is to *visualize* motion with motion diagrams. Strictly speaking, the vector we have defined in Equation 1.5, and the vector we will show on motion diagrams, is the *average* velocity $\vec{v}_{\text{avg}}$. But to allow the motion diagram to be a useful tool, we will drop the subscript and refer to the average velocity as simply $\vec{v}$. Our definitions and symbols, which somewhat blur the distinction between average and instantaneous quantities, are adequate for visualization purposes, but they're not the final word. We will refine these definitions in Chapter 2, where our goal will be to develop the mathematics of motion.

FIGURE 1.9 The displacement vectors and velocities of ships A and B.



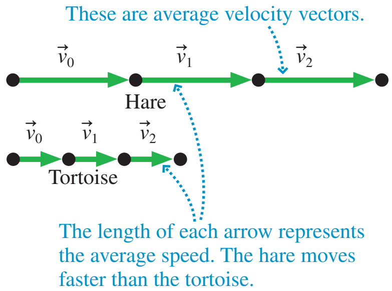
Motion Diagrams with Velocity Vectors

The velocity vector points in the same direction as the displacement $\Delta\vec{r}$, and the length of $\vec{v}$ is directly proportional to the length of $\Delta\vec{r}$. Consequently, the vectors connecting each dot of a motion diagram to the next, which we previously labeled as displacements, could equally well be identified as velocity vectors.

This idea is illustrated in **FIGURE 1.10,** which shows four frames from the motion diagram of a tortoise racing a hare. The vectors connecting the dots are now labeled as velocity vectors $\vec{v}$. **The length of a velocity vector represents the average speed with which the object moves between the two points.** Longer velocity vectors indicate faster motion. You can see that the hare moves faster than the tortoise.

Notice that the hare's velocity vectors do not change; each has the same length and direction. We say the hare is moving with *constant velocity*. The tortoise is also moving with its own constant velocity.

FIGURE 1.10 Motion diagram of the tortoise racing the hare.



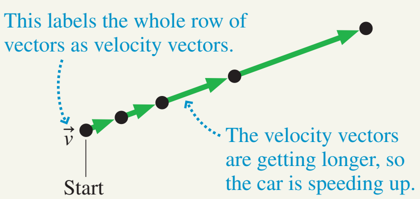
EXAMPLE 1.2 ■ Accelerating up a hill

The light turns green and a car accelerates, starting from rest, up a 20° hill. Draw a motion diagram showing the car's velocity.

MODEL Use the particle model to represent the car as a dot.

VISUALIZE The car's motion takes place along a straight line, but the line is neither horizontal nor vertical. A motion diagram should show the object moving with the correct orientation—in this case, at an angle of 20°. **FIGURE 1.11** shows several frames of the motion diagram, where we see the car speeding up. The car starts from rest, so the first arrow is drawn as short as possible and the first dot is labeled "Start." The displacement vectors have been drawn from each dot to the next, but then they are identified and labeled as average velocity vectors $\vec{v}$.

FIGURE 1.11 Motion diagram of a car accelerating up a hill.



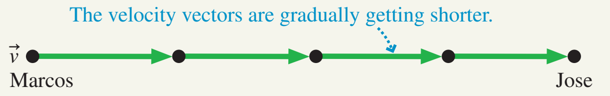
EXAMPLE 1.3 ■ A rolling soccer ball

Marcos kicks a soccer ball. It rolls along the ground until stopped by Jose. Draw a motion diagram of the ball.

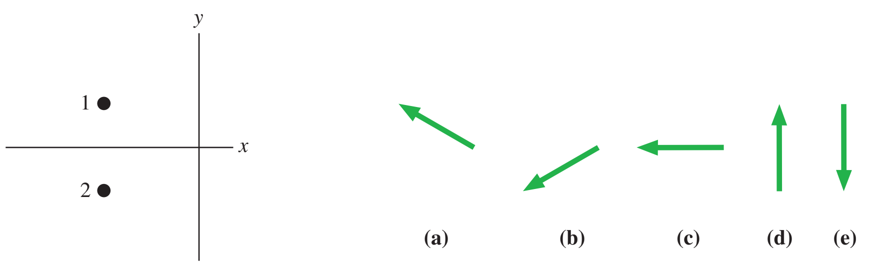
MODEL This example is typical of how many problems in science and engineering are worded. The problem does not give a clear statement of where the motion begins or ends. Are we interested in the motion of the ball just during the time it is rolling between Marcos and Jose? What about the motion *as* Marcos kicks it (ball rapidly speeding up) or *as* Jose stops it (ball rapidly slowing down)? The point is that *you* will often be called on to make a *reasonable interpretation* of a problem statement. In this problem, the details of kicking and stopping the ball are complex. The motion of the ball across the ground is easier to describe, and it's a motion you might expect to learn about in a physics class. So our *interpretation* is that the motion diagram should start as the ball leaves Marcos's foot (ball already moving) and should end the instant it touches Jose's foot (ball still moving). In between, the ball will slow down a little. We will model the ball as a particle.

VISUALIZE With this interpretation in mind, **FIGURE 1.12** shows the motion diagram of the ball. Notice how, in contrast to the car of Figure 1.11, the ball is already moving as the motion diagram video begins. As before, the average velocity vectors are found by connecting the dots. You can see that the average velocity vectors get shorter as the ball slows. Each $\vec{v}$ is different, so this is *not* constant-velocity motion.

FIGURE 1.12 Motion diagram of a soccer ball rolling from Marcos to Jose.



STOP TO THINK 1.3 A particle moves from position 1 to position 2 during the time interval Δt. Which vector shows the particle's average velocity?



1.5 Linear Acceleration

Position, time, and velocity are important concepts, and at first glance they might appear to be sufficient to describe motion. But that is not the case. Sometimes an object's velocity is constant, as it was in Figure 1.10. More often, an object's velocity changes as it moves, as in Figures 1.11 and 1.12. We need one more motion concept to describe a *change* in the velocity.

Because velocity is a vector, it can change in two possible ways:

1. The magnitude can change, indicating a change in speed; or
2. The direction can change, indicating that the object has changed direction.

We will concentrate for now on the first case, a change in speed. The car accelerating up a hill in Figure 1.11 was an example in which the magnitude of the velocity vector changed but not the direction. We'll return to the second case in Chapter 4.

When we wanted to measure changes in position, the ratio $\Delta\vec{r}/\Delta t$ was useful. This ratio is the *rate of change of position*. By analogy, consider an object whose velocity changes from $\vec{v}_a$ to $\vec{v}_b$ during the time interval Δt. Just as $\Delta\vec{r} = \vec{r}_b - \vec{r}_a$ is the change of position, the quantity $\Delta\vec{v} = \vec{v}_b - \vec{v}_a$ is the change of velocity. The ratio $\Delta\vec{v}/\Delta t$ is then the *rate of change of velocity*. It has a large magnitude for objects that speed up quickly and a small magnitude for objects that speed up slowly.

The ratio $\Delta\vec{v}/\Delta t$ is called the **average acceleration,** and its symbol is $\vec{a}_{\text{avg}}$. **The average acceleration of an object during the time interval Δt, in which the object's velocity changes by $\Delta\vec{v}$, is the vector**

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} \tag{1.7}$$

The average acceleration vector points in the same direction as the vector $\Delta\vec{v}$.

Acceleration is a fairly abstract concept. Yet it is essential to develop a good intuition about acceleration because it will be a key concept for understanding why objects move as they do. Motion diagrams will be an important tool for developing that intuition.

The Audi TT accelerates from 0 to 60 mph in 6 s.

NOTE As we did with velocity, we will drop the subscript and refer to the average acceleration as simply $\vec{a}$. This is adequate for visualization purposes, but not the final word. We will refine the definition of acceleration in Chapter 2.

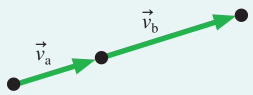
Finding the Acceleration Vectors on a Motion Diagram

Perhaps the most important use of a motion diagram is to determine the acceleration vector $\vec{a}$ at each point in the motion. From its definition in Equation 1.7, we see that **$\vec{a}$ points in the same direction as $\Delta\vec{v}$,** the change of velocity, so we need to find the direction of $\Delta\vec{v}$. To do so, we rewrite the definition $\Delta\vec{v} = \vec{v}_b - \vec{v}_a$ as $\vec{v}_b = \vec{v}_a + \Delta\vec{v}$. This is now a vector addition problem: What vector must be added to $\vec{v}_a$ to turn it into $\vec{v}_b$? Tactics Box 1.2 shows how to do this.

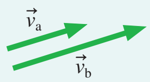
TACTICS BOX 1.2

Finding the acceleration vector

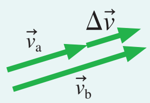
To find the acceleration as the velocity changes from $\vec{v}_a$ to $\vec{v}_b$, we must determine the *change* of velocity $\Delta\vec{v} = \vec{v}_b - \vec{v}_a$.



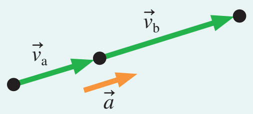
❶ Draw velocity vectors $\vec{v}_a$ and $\vec{v}_b$ with their tails together.



❷ Draw the vector from the tip of $\vec{v}_a$ to the tip of $\vec{v}_b$. This is $\Delta\vec{v}$ because $\vec{v}_b = \vec{v}_a + \Delta\vec{v}$.



❸ Return to the original motion diagram. Draw a vector at the middle dot in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration at the midpoint between $\vec{v}_a$ and $\vec{v}_b$.



Exercises 21–24

Many Tactics Boxes will refer you to exercises in the *Student Workbook* where you can practice the new skill.

Notice that the acceleration vector goes beside the middle dot, not beside the velocity vectors. This is because each acceleration vector is determined by the *difference* between the *two* velocity vectors on either side of a dot. The length of $\vec{a}$ does not have to be the exact length of $\Delta\vec{v}$; it is the direction of $\vec{a}$ that is most important.

The procedure of « **TACTICS BOX 1.2** can be repeated to find $\vec{a}$ at each point in the motion diagram. Note that we cannot determine $\vec{a}$ at the first and last points because we have only one velocity vector and can't find $\Delta\vec{v}$.

The Complete Motion Diagram

You've now seen two *Tactics Boxes*. Tactics Boxes to help you accomplish specific tasks will appear in nearly every chapter in this book. We'll also, where appropriate, provide *Problem-Solving Strategies*.

PROBLEM-SOLVING STRATEGY 1.1

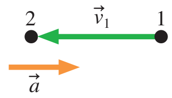
Motion diagrams

MODEL Determine whether it is appropriate to model the moving object as a particle. Make simplifying assumptions when interpreting the problem statement.

VISUALIZE A complete motion diagram consists of:

- The position of the object in each frame of the video, shown as a dot. Use five or six dots to make the motion clear but without overcrowding the picture. The motion should change gradually from one dot to the next, not drastically. More complex motions will need more dots.
- The average velocity vectors, found by connecting each dot in the motion diagram to the next with a vector arrow. There is *one* velocity vector linking each *two* position dots. Label the row of velocity vectors $\vec{v}$.
- The average acceleration vectors, found using Tactics Box 1.2. There is *one* acceleration vector linking each *two* velocity vectors. Each acceleration vector is drawn at the dot between the two velocity vectors it links. Use $\vec{0}$ to indicate a point at which the acceleration is zero. Label the row of acceleration vectors $\vec{a}$.

STOP TO THINK 1.4 A particle undergoes acceleration $\vec{a}$ while moving from point 1 to point 2. Which of the choices shows the most likely velocity vector $\vec{v}_2$ as the particle leaves point 2?



Examples of Motion Diagrams

Let's look at some examples of the full strategy for drawing motion diagrams.

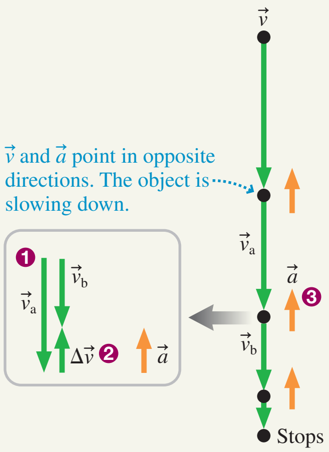
EXAMPLE 1.4 ■ The first astronauts land on Mars

A spaceship carrying the first astronauts to Mars descends safely to the surface. Draw a motion diagram for the last few seconds of the descent.

MODEL The spaceship is small in comparison with the distance traveled, and the spaceship does not change size or shape, so it's reasonable to model the spaceship as a particle. We'll assume that its motion in the last few seconds is straight down. The problem ends as the spacecraft touches the surface.

VISUALIZE **FIGURE 1.13** shows a complete motion diagram as the spaceship descends and slows, using its rockets, until it comes to rest on the surface. Notice how the dots get closer together as it slows. The inset uses the steps of Tactics Box 1.2 (numbered circles) to show how the acceleration vector $\vec{a}$ is determined at one point. All the other acceleration vectors will be similar because for each pair of velocity vectors the earlier one is longer than the later one.

FIGURE 1.13 Motion diagram of a spaceship landing on Mars.



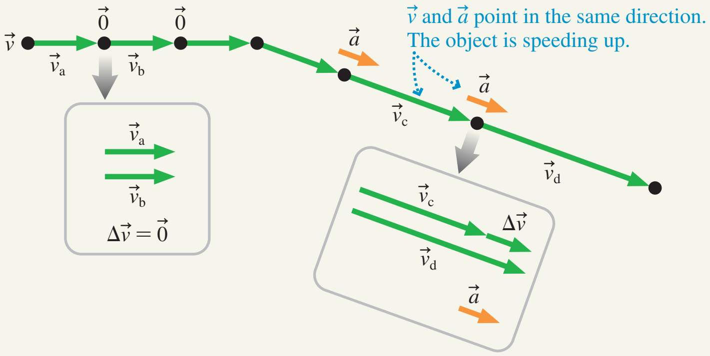
EXAMPLE 1.5 ■ Skiing through the woods

A skier glides along smooth, horizontal snow at constant speed, then speeds up going down a hill. Draw the skier's motion diagram.

MODEL Model the skier as a particle. It's reasonable to assume that the downhill slope is a straight line. Although the motion as a whole is not linear, we can treat the skier's motion as two separate linear motions.

VISUALIZE **FIGURE 1.14** shows a complete motion diagram of the skier. The dots are equally spaced for the horizontal motion, indicating constant speed; then the dots get farther apart as the skier speeds up going down the hill. The insets show how the average acceleration vector $\vec{a}$ is determined for the horizontal motion and along the slope. All the other acceleration vectors along the slope will be similar to the one shown because each velocity vector is longer than the preceding one. Notice that we've explicitly written $\vec{0}$ for the acceleration beside the dots where the velocity is constant. The acceleration at the point where the direction changes will be considered in Chapter 4.

FIGURE 1.14 Motion diagram of a skier.



Notice something interesting in Figures 1.13 and 1.14. Where the object is speeding up, the acceleration and velocity vectors point in the *same direction*. Where the object is slowing down, the acceleration and velocity vectors point in *opposite directions*. These results are always true for motion in a straight line. **For motion along a line:**

- **An object is speeding up if and only if $\vec{v}$ and $\vec{a}$ point in the same direction.**
- **An object is slowing down if and only if $\vec{v}$ and $\vec{a}$ point in opposite directions.**
- **An object's velocity is constant if and only if $\vec{a} = \vec{0}$.**

NOTE In everyday language, we use the word *accelerate* to mean "speed up" and the word *decelerate* to mean "slow down." But speeding up and slowing down are both changes in the velocity and consequently, by our definition, *both* are accelerations. In physics, *acceleration* refers to changing the velocity, no matter what the change is, and not just to speeding up.

EXAMPLE 1.6 ■ Tossing a ball

Draw the motion diagram of a ball tossed straight up in the air.

MODEL This problem calls for some interpretation. Should we include the toss itself, or only the motion after the ball is released? What about catching it? It appears that this problem is really concerned with the ball's motion through the air. Consequently, we begin the motion diagram at the instant that the tosser releases the ball and end the diagram at the instant the ball touches his hand. We will consider neither the toss nor the catch. And, of course, we will model the ball as a particle.

VISUALIZE We have a slight difficulty here because the ball retraces its route as it falls. A literal motion diagram would show the upward motion and downward motion on top of each other, leading to confusion. We can avoid this difficulty by horizontally separating the upward motion and downward motion diagrams. This will not affect our conclusions because it does not change any of the vectors. **FIGURE 1.15** shows the motion diagram drawn this way. Notice that the very top dot is shown twice—as the end point of the upward motion and the beginning point of the downward motion.

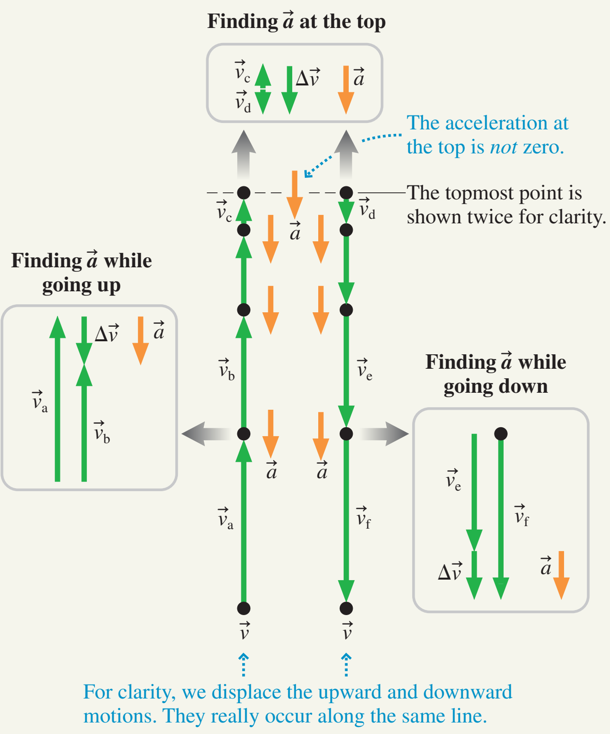
The ball slows down as it rises. You've learned that the acceleration vectors point opposite the velocity vectors for an object that is slowing down along a line, and they are shown accordingly. Similarly, $\vec{a}$ and $\vec{v}$ point in the same direction as the falling ball speeds up. Notice something interesting: The acceleration vectors point downward both while the ball is rising *and* while it is falling. Both "speeding up" and "slowing down" occur with the *same* acceleration vector. This is an important conclusion, one worth pausing to think about.

Now look at the top point on the ball's trajectory. The velocity vectors point upward but are getting shorter as the ball approaches the top. As the ball starts to fall, the velocity vectors point downward and are getting longer. There must be a moment—just an instant as $\vec{v}$ switches from pointing up to pointing down—when the velocity is zero. Indeed, the ball's velocity *is* zero for an instant at the precise top of the motion!

But what about the acceleration at the top? The inset shows how the average acceleration is determined from the last upward velocity before the top point and the first downward velocity. We find that the acceleration at the top is pointing downward, just as it does elsewhere in the motion.

Many people expect the acceleration to be zero at the highest point. But the velocity at the top point *is* changing—from up to down. If the velocity is changing, there *must* be an acceleration. A downward-pointing acceleration vector is needed to turn the velocity vector from up to down. Another way to think about this is to note that zero acceleration would mean no change of velocity. When the ball reached zero velocity at the top, it would hang there and not fall if the acceleration were also zero!

FIGURE 1.15 Motion diagram of a ball tossed straight up in the air.



1.6 Motion in One Dimension

An object's motion can be described in terms of three fundamental quantities: its position $\vec{r}$, velocity $\vec{v}$, and acceleration $\vec{a}$. These are vectors, but for motion in one dimension, the vectors are restricted to point only "forward" or "backward." Consequently, we can describe one-dimensional motion with the simpler quantities x, v_x, and a_x (or y, v_y, and a_y). However, we need to give each of these quantities an explicit *sign*, positive or negative, to indicate whether the position, velocity, or acceleration vector points forward or backward.

Determining the Signs of Position, Velocity, and Acceleration

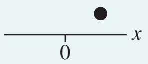
Position, velocity, and acceleration are measured with respect to a coordinate system, a grid or axis that *you* impose on a problem to analyze the motion. We will find it convenient to use an x-axis to describe both horizontal motion and motion along an inclined plane. A y-axis will be used for vertical motion. A coordinate axis has two essential features:

1. An origin to define zero; and
2. An x or y label (with units) at the positive end of the axis.

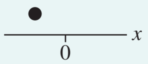
NOTE In this textbook, we will follow the convention that **the positive end of an x-axis is to the right and the positive end of a y-axis is up.** The signs of position, velocity, and acceleration are based on this convention.

TACTICS BOX 1.3

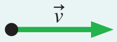
Determining the sign of the position, velocity, and acceleration

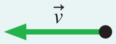


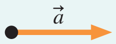
$x > 0$ Position to right of origin.

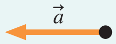


$x < 0$ Position to left of origin.

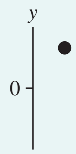

$v_x > 0$ Direction of motion is to the right.


$v_x < 0$ Direction of motion is to the left.

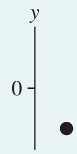

$a_x > 0$ Acceleration vector points to the right.


$a_x < 0$ Acceleration vector points to the left.

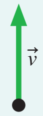
- The sign of position (x or y) tells us *where* an object is.
- The sign of velocity (v_x or v_y) tells us *which direction* the object is moving.
- The sign of acceleration (a_x or a_y) tells us which way the acceleration vector points, *not* whether the object is speeding up or slowing down.

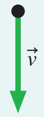


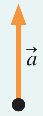
$y > 0$
Position above origin.

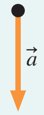


$y < 0$
Position below origin.


$v_y > 0$
Direction of motion is up.


$v_y < 0$
Direction of motion is down.


$a_y > 0$
Acceleration vector points up.


$a_y < 0$
Acceleration vector points down.

Exercises 30–31

Acceleration is where things get a bit tricky. A natural tendency is to think that a positive value of a_x or a_y describes an object that is speeding up while a negative value describes an object that is slowing down (decelerating). However, this interpretation *does not work*.

Acceleration is defined as $\vec{a}_{avg} = \Delta\vec{v}/\Delta t$. The direction of $\vec{a}$ can be determined by using a motion diagram to find the direction of $\Delta\vec{v}$. The one-dimensional acceleration a_x (or a_y) is then positive if the vector $\vec{a}$ points to the right (or up), negative if $\vec{a}$ points to the left (or down).

FIGURE 1.16 One of these objects is speeding up, the other slowing down, but they both have a positive acceleration a_x.

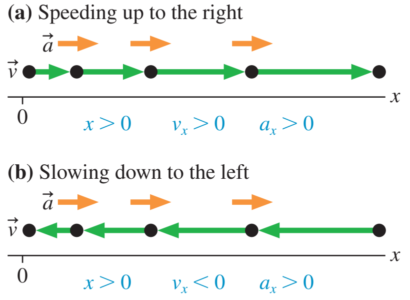


FIGURE 1.16 shows that this method for determining the sign of a does not conform to the simple idea of speeding up and slowing down. The object in Figure 1.16a has a positive acceleration $(a_x > 0)$ not because it is speeding up but because the vector $\vec{a}$ points in the positive direction. Compare this with the motion diagram of Figure 1.16b. Here the object is slowing down, but it still has a positive acceleration $(a_x > 0)$ because $\vec{a}$ points to the right.

In the previous section, we found that an object is speeding up if $\vec{v}$ and $\vec{a}$ point in the same direction, slowing down if they point in opposite directions. For one-dimensional motion this rule becomes:

- An object is speeding up if and only if v_x and a_x have the same sign.
- An object is slowing down if and only if v_x and a_x have opposite signs.
- An object's velocity is constant if and only if $a_x = 0$.

Notice how the first two of these rules are at work in Figure 1.16.

Position-versus-Time Graphs

FIGURE 1.17 is a motion diagram, made at 1 frame per minute, of a student walking to school. You can see that she leaves home at a time we choose to call $t = 0$ min and makes steady progress for a while. Beginning at $t = 3$ min there is a period where the distance traveled during each time interval becomes less—perhaps she slowed down to speak with a friend. Then she picks up the pace, and the distances within each interval are longer.

FIGURE 1.17 The motion diagram of a student walking to school and a coordinate axis for making measurements.

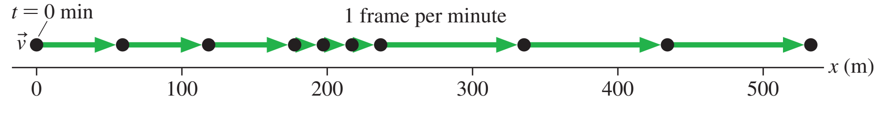


TABLE 1.1 Measured positions of a student walking to school

Time t (min)	Position x (m)	Time t (min)	Position x (m)
0	0	5	220
1	60	6	240
2	120	7	340
3	180	8	440
4	200	9	540

Figure 1.17 includes a coordinate axis, and you can see that every dot in a motion diagram occurs at a specific position. **TABLE 1.1** shows the student's positions at different times as measured along this axis. For example, she is at position $x = 120$ m at $t = 2$ min.

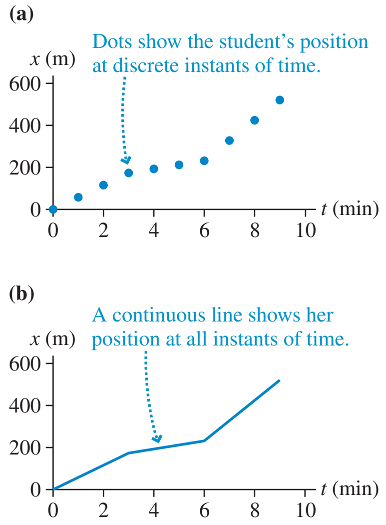
The motion diagram is one way to represent the student's motion. Another is to make a graph of the measurements in Table 1.1. **FIGURE 1.18a** is a graph of x versus t for the student. The motion diagram tells us only where the student is at a few discrete points of time, so this graph of the data shows only points, no lines.

NOTE A graph of "a versus b" means that a is graphed on the vertical axis and b on the horizontal axis. Saying "graph a versus b" is really a shorthand way of saying "graph a as a function of b."

However, common sense tells us the following. First, the student was *somewhere specific* at all times. That is, there was never a time when she failed to have a well-defined position, nor could she occupy two positions at one time. Second, the student moved *continuously* through all intervening points of space. She could not go from $x = 100$ m to $x = 200$ m without passing through every point in between. It is thus quite reasonable to believe that her motion can be shown as a continuous line passing through the measured points, as shown in **FIGURE 1.18b**. A continuous line or curve showing an object's position as a function of time is called a **position-versus-time graph** or, sometimes, just a *position graph*.

NOTE A graph is *not* a "picture" of the motion. The student is walking along a straight line, but the graph itself is not a straight line. Further, we've graphed her position on the vertical axis even though her motion is horizontal. Graphs are *abstract representations* of motion. We will place significant emphasis on the process of interpreting graphs, and many of the exercises and problems will give you a chance to practice these skills.

FIGURE 1.18 Position graphs of the student's motion.



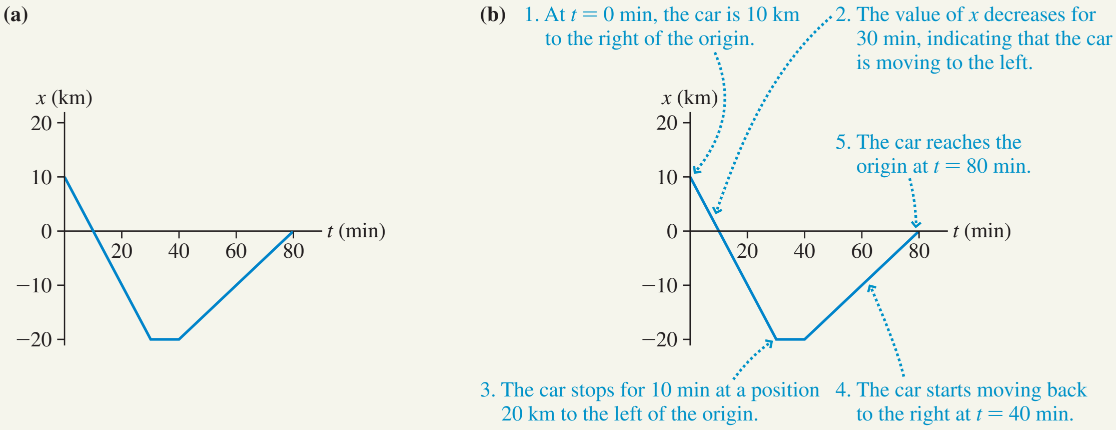
EXAMPLE 1.7 ■ Interpreting a position graph

The graph in **FIGURE 1.19a** represents the motion of a car along a straight road. Describe the motion of the car.

MODEL We'll model the car as a particle with a precise position at each instant.

VISUALIZE As **FIGURE 1.19b** shows, the graph represents a car that travels to the left for 30 minutes, stops for 10 minutes, then travels back to the right for 40 minutes.

FIGURE 1.19 Position-versus-time graph of a car.



1.7 Solving Problems in Physics

Physics is not mathematics. Math problems are clearly stated, such as "What is $2 + 2$?" Physics is about the world around us, and to describe that world we must use language. Now, language is wonderful—we couldn't communicate without it—but language can sometimes be imprecise or ambiguous.

The challenge when reading a physics problem is to translate the words into symbols that can be manipulated, calculated, and graphed. **The translation from words to symbols is the heart of problem solving in physics.** This is the point where ambiguous words and phrases must be clarified, where the imprecise must be made precise, and where you arrive at an understanding of exactly what the question is asking.

Using Symbols

Symbols are a language that allows us to talk with precision about the relationships in a problem. As with any language, we all need to agree to use words or symbols in the same way if we want to communicate with each other. Many of the ways we use symbols in science and engineering are somewhat arbitrary, often reflecting historical roots. Nonetheless, practicing scientists and engineers have come to agree on how to use the language of symbols. Learning this language is part of learning physics.

We will use subscripts on symbols, such as x_3, to designate a particular point in the problem. Scientists usually label the starting point of the problem with the subscript "0," not the subscript "1" that you might expect. When using subscripts, make sure that all symbols referring to the same point in the problem have the *same numerical subscript*. To have the same point in a problem characterized by position x_1 but velocity v_{2x} is guaranteed to lead to confusion!

Drawing Pictures

You may have been told that the first step in solving a physics problem is to "draw a picture," but perhaps you didn't know why, or what to draw. The purpose of drawing a picture is to aid you in the words-to-symbols translation. Complex problems have far more information than you can keep in your head at one time. Think of a picture as a "memory extension," helping you organize and keep track of vital information.

Although any picture is better than none, there really is a *method* for drawing pictures that will help you be a better problem solver. It is called the **pictorial representation** of the problem. We'll add other pictorial representations as we go along, but the following procedure is appropriate for motion problems.

TACTICS BOX 1.4

Drawing a pictorial representation

1. **Draw a motion diagram.** The motion diagram develops your intuition for the motion.
2. **Establish a coordinate system.** Select your axes and origin to match the motion. For one-dimensional motion, you want either the x-axis or the y-axis parallel to the motion. The coordinate system determines whether the signs of v and a are positive or negative.
3. **Sketch the situation.** Not just any sketch. Show the object at the *beginning* of the motion, at the *end,* and at any point where the character of the motion changes. Show the object, not just a dot, but very simple drawings are adequate.
4. **Define symbols.** Use the sketch to define symbols representing quantities such as position, velocity, acceleration, and time. *Every* variable used later in the mathematical solution should be defined on the sketch. Some will have known values, others are initially unknown, but all should be given symbolic names.
5. **List known information.** Make a table of the quantities whose values you can determine from the problem statement or that can be found quickly with simple geometry or unit conversions. Some quantities are implied by the problem, rather than explicitly given. Others are determined by your choice of coordinate system.
6. **Identify the desired unknowns.** What quantity or quantities will allow you to answer the question? These should have been defined as symbols in step 4. Don't list every unknown, only the one or two needed to answer the question.

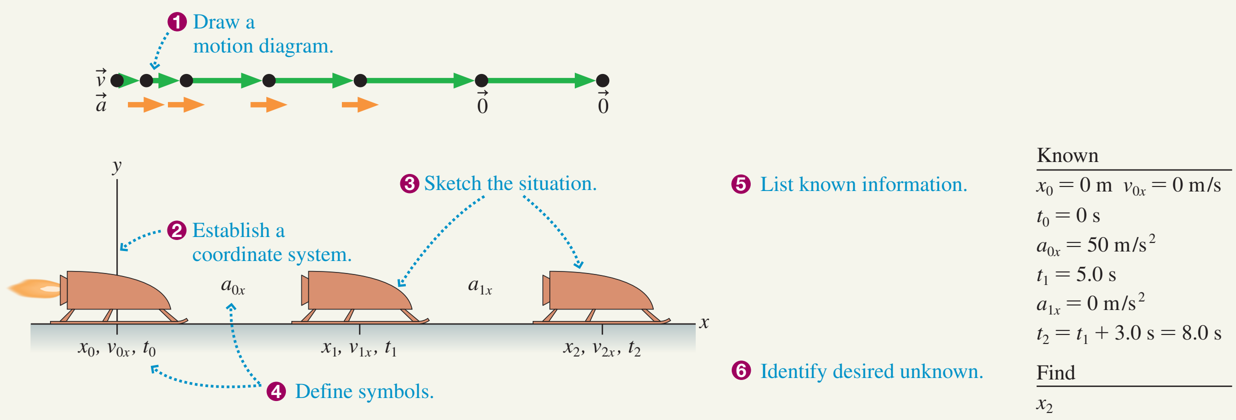
It's not an overstatement to say that a well-done pictorial representation of the problem will take you halfway to the solution. The following example illustrates how to construct a pictorial representation for a problem that is typical of problems you will see in the next few chapters.

EXAMPLE 1.8 ■ Drawing a pictorial representation

Draw a pictorial representation for the following problem: A rocket sled accelerates horizontally at 50 m/s^2 for 5.0 s, then coasts for 3.0 s. What is the total distance traveled?

VISUALIZE **FIGURE 1.20** is the pictorial representation. The motion diagram shows an acceleration phase followed by a coasting phase. Because the motion is horizontal, the appropriate coordinate system is an x-axis. We've chosen to place the origin at the starting point. The motion has a beginning, an end, and a point where the motion changes from accelerating to coasting, and these are the three sled positions sketched in the figure. The quantities x, v_x, and t are needed at each of three *points,* so these have been defined on the sketch and distinguished by subscripts. Accelerations are associated with *intervals* between the points, so only two accelerations are defined. Values for three quantities are given in the problem statement, although we need to use the motion diagram, where we find that $\vec{a}$ points to the right, to know that $a_{0x} = +50$ m/s^2 rather than -50 m/s^2. The values $x_0 = 0$ m and $t_0 = 0$ s are choices we made when setting up the coordinate system. The value $v_{0x} = 0$ m/s is part of our *interpretation* of the problem. Finally, we identify x_2 as the quantity that will answer the question. We now understand quite a bit about the problem and would be ready to start a quantitative analysis.

FIGURE 1.20 A pictorial representation.



We didn't *solve* the problem; that is not the purpose of the pictorial representation. The pictorial representation is a systematic way to go about interpreting a problem and getting ready for a mathematical solution. Although this is a simple problem, and you probably know how to solve it if you've taken physics before, you will soon be faced with much more challenging problems. Learning good problem-solving skills at the beginning, while the problems are easy, will make them second nature later when you really need them.

Representations

A picture is one way to *represent* your knowledge of a situation. You could also represent your knowledge using words, graphs, or equations. Each **representation of knowledge** gives us a different perspective on the problem. The more tools you have for thinking about a complex problem, the more likely you are to solve it.

There are four representations of knowledge that we will use over and over:

1. The *verbal* representation. A problem statement, in words, is a verbal representation of knowledge. So is an explanation that you write.
2. The *pictorial* representation. The pictorial representation, which we've just presented, is the most literal depiction of the situation.
3. The *graphical* representation. We will make extensive use of graphs.
4. The *mathematical* representation. Equations that can be used to find the numerical values of specific quantities are the mathematical representation.

NOTE The mathematical representation is only one of many. Much of physics is more about thinking and reasoning than it is about solving equations.

A new building requires careful planning. The architect's visualization and drawings have to be complete before the detailed procedures of construction get under way. The same is true for solving problems in physics.

A Problem-Solving Strategy

One of the goals of this textbook is to help you learn a *strategy* for solving physics problems. The purpose of a strategy is to guide you in the right direction with minimal wasted effort. The four-part problem-solving strategy—**Model, Visualize, Solve, Review**—is based on using different representations of knowledge. You will see this problem-solving strategy used consistently in the worked examples throughout this textbook, and you should endeavor to apply it to your own problem solving.

GENERAL PROBLEM-SOLVING STRATEGY

MODEL It's impossible to treat every detail of a situation. Simplify the situation with a model that captures the essential features. For example, the object in a mechanics problem is often represented as a particle.

VISUALIZE This is where expert problem solvers put most of their effort.

- Draw a *pictorial representation.* This helps you visualize important aspects of the physics and assess the information you are given. It starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these representations; they need not be done in any particular order.

SOLVE Only after modeling and visualizing are complete is it time to develop a *mathematical representation* with specific equations that must be solved. All symbols used here should have been defined in the pictorial representation.

REVIEW Is your result believable? Does it have proper units? Does it make sense?

Throughout this textbook we will emphasize the first two steps. They are the *physics* of the problem, as opposed to the mathematics of solving the resulting equations. This is not to say that those mathematical operations are always easy—in many cases they are not. But our primary goal is to understand the physics.

Textbook illustrations are obviously more sophisticated than what you would draw on your own paper. To show you a figure very much like what *you* should draw, the final example of this section is in a "pencil sketch" style. We will include one or more pencil-sketch examples in nearly every chapter to illustrate exactly what a good problem solver would draw.

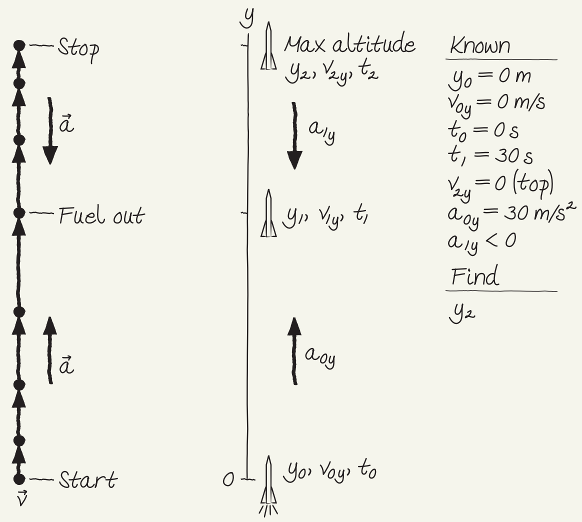
EXAMPLE 1.9 ■ Launching a weather rocket

Use the first two steps of the problem-solving strategy to analyze the following problem: A small rocket, such as those used for meteorological measurements of the atmosphere, is launched vertically with an acceleration of 30 m/s^2. It runs out of fuel after 30 s. What is its maximum altitude?

MODEL We need to do some interpretation. Common sense tells us that the rocket does not stop the instant it runs out of fuel. Instead, it continues upward, while slowing, until it reaches its maximum altitude. This second half of the motion, after running out of fuel, is like the ball that was tossed upward in the first half of Example 1.6. Because the problem does not ask about the rocket's descent, we conclude that the problem ends at the point of maximum altitude. We'll model the rocket as a particle.

VISUALIZE **FIGURE 1.21** shows the pictorial representation in pencil-sketch style. The rocket is speeding up during the first half of the motion, so $\vec{a}_0$ points upward, in the positive y-direction. Thus the initial acceleration is $a_{0y} = 30$ m/s^2. During the second half, as the rocket slows, $\vec{a}_1$ points downward. Thus a_{1y} is a negative number.

FIGURE 1.21 Pictorial representation for the rocket.



This information is included with the known information. Although the velocity v_{2y} wasn't given in the problem statement, it must—just like for the ball in Example 1.6—be zero at the very top of the trajectory. Last, we have identified y_2 as the desired unknown. This, of course, is not the only unknown in the problem, but it is the one we are specifically asked to find.

REVIEW If you've had a previous physics class, you may be tempted to assign a_{1y} the value $-9.8\ \text{m/s}^2$, the free-fall acceleration. However, that would be true only if there is no air resistance on the rocket. We will need to consider the *forces* acting on the rocket during the second half of its motion before we can determine a value for a_{1y}. For now, all that we can safely conclude is that a_{1y} is negative.

Our task in this chapter is not to *solve* problems—all that in due time—but to focus on what is happening in a problem. In other words, to make the translation from words to symbols in preparation for subsequent mathematical analysis. Modeling and the pictorial representation will be our most important tools.

1.8 Units and Significant Figures

Science is based upon experimental measurements, and measurements require *units*. The system of units used in science is called *le Système Internationale d'Unités*. These are commonly referred to as **SI units.** In casual speaking we often refer to *metric units*.

All of the quantities needed to understand motion can be expressed in terms of the three basic SI units shown in **TABLE 1.2**. Other quantities can be expressed as a combination of these basic units. Velocity, expressed in meters per second or m/s, is a ratio of the length unit to the time unit.

TABLE 1.2 **The basic SI units**

Quantity	Unit	Abbreviation
time	second	s
length	meter	m
mass	kilogram	kg

Time

The standard of time prior to 1960 was based on the *mean solar day*. As time-keeping accuracy and astronomical observations improved, it became apparent that the earth's rotation is not perfectly steady. Meanwhile, physicists had been developing a device called an *atomic clock*. This instrument is able to measure, with incredibly high precision, the frequency of radio waves absorbed by atoms as they move between two closely spaced energy levels. This frequency can be reproduced with great accuracy at many laboratories around the world. Consequently, the SI unit of time—the second—was redefined in 1967 as follows:

> One *second* is the time required for 9,192,631,770 oscillations of the radio wave absorbed by the cesium-133 atom. The abbreviation for second is the letter s.

An atomic clock at the National Institute of Standards and Technology is the primary standard of time.

Several radio stations around the world broadcast a signal whose frequency is linked directly to the atomic clocks. This signal is the time standard, and any time-measuring equipment you use was calibrated from this time standard.

Length

The SI unit of length—the meter—was originally defined as one ten-millionth of the distance from the north pole to the equator along a line passing through Paris. There are obvious practical difficulties with implementing this definition, and it was later abandoned in favor of the distance between two scratches on a platinum-iridium bar stored in a special vault in Paris. The present definition, agreed to in 1983, is as follows:

> One *meter* is the distance traveled by light in vacuum during 1/299,792,458 of a second. The abbreviation for meter is the letter m.

This is equivalent to defining the speed of light to be exactly 299,792,458 m/s. Laser technology is used in various national laboratories to implement this definition and to calibrate secondary standards that are easier to use. These standards ultimately make their way to your ruler or to a meter stick. It is worth keeping in mind that any measuring device you use is only as accurate as the care with which it was calibrated.

Mass

For 130 years, the kilogram was defined as the mass of a polished platinum-iridium cylinder stored in a vault in Paris. By the 1990s, this was the only SI unit still defined by a manufactured object rather than by natural phenomena. That changed in 2019 with a new definition of the kilogram, although one that is rather hard to understand:

> One *kilogram* is defined by fixing the value of the *Planck constant*—a quantity that appears in quantum physics—to be $6.626\,070\,15 \times 10^{-34}\ \text{kg m}^2/\text{s}$. The abbreviation for kilogram is kg.

This obscure definition is implemented using a device called a *Kibble balance* in which an electromagnet is used to balance the weight of a test mass, and the required electric current is measured using quantum standards that depend on the Planck constant. Despite the prefix *kilo,* it is the kilogram, not the gram, that is the SI unit.

Using Prefixes

We will have many occasions to use lengths, times, and masses that are either much less or much greater than the standards of 1 meter, 1 second, and 1 kilogram. We will do so by using *prefixes* to denote various powers of 10. **TABLE 1.3** lists the common prefixes that will be used frequently throughout this book. Memorize it! Few things in science are learned by rote memory, but this list is one of them. A more extensive list of prefixes is shown inside the front cover of the book.

TABLE 1.3 **Common prefixes**

Prefix	Power of 10	Abbreviation
giga-	10^9	G
mega-	10^6	M
kilo-	10^3	k
centi-	10^{-2}	c
milli-	10^{-3}	m
micro-	10^{-6}	μ
nano-	10^{-9}	n

Although prefixes make it easier to talk about quantities, the SI units are seconds, meters, and kilograms. Quantities given with prefixed units must be converted to SI units before any calculations are done. Unit conversions are best done at the very beginning of a problem, as part of the pictorial representation.

Unit Conversions

Although SI units are our standard, we cannot entirely forget that the United States still uses English units. Thus it remains important to be able to convert back and forth between SI units and English units. **TABLE 1.4** shows several frequently used conversions, and these are worth memorizing if you do not already know them. While the English system was originally based on the length of the king's foot, it is interesting to note that today the conversion 1 in = 2.54 cm is the *definition* of the inch. In other words, the English system for lengths is now based on the meter!

TABLE 1.4 **Useful unit conversions**

1 in = 2.54 cm
1 mi = 1.609 km
1 mph = 0.447 m/s
1 m = 39.37 in
1 km = 0.621 mi
1 m/s = 2.24 mph

There are various techniques for doing unit conversions. One effective method is to write the conversion factor as a ratio equal to one. For example, using information in Tables 1.3 and 1.4, we have

$$\frac{10^{-6}\text{ m}}{1\ \mu\text{m}} = 1 \qquad \text{and} \qquad \frac{2.54\text{ cm}}{1\text{ in}} = 1$$

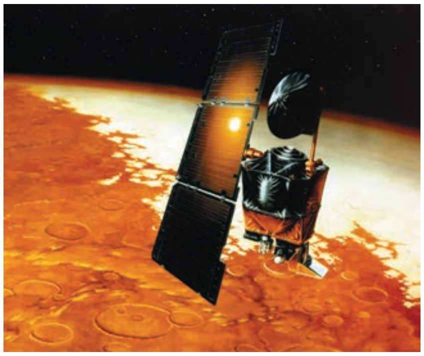
Because multiplying any expression by 1 does not change its value, these ratios are easily used for conversions. To convert 3.5 μm to meters we compute

$$3.5\ \cancel{\mu\text{m}} \times \frac{10^{-6}\text{ m}}{1\ \cancel{\mu\text{m}}} = 3.5 \times 10^{-6}\text{ m}$$

Similarly, the conversion of 2 feet to meters is

$$2.00\ \cancel{\text{ft}} \times \frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}} \times \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}} \times \frac{10^{-2}\text{ m}}{1\ \cancel{\text{cm}}} = 0.610\text{ m}$$

Notice how units in the numerator and in the denominator cancel until only the desired units remain at the end. You can continue this process of multiplying by 1 as many times as necessary to complete all the conversions.



In 1999, the $125-million Mars Climate Orbiter burned up in the Martian atmosphere instead of entering a safe orbit. The problem was faulty units! The engineering team supplied data in English units, but the navigation team assumed that the data were in metric units.

Assessment

As we get further into problem solving, you will need to decide whether or not the answer to a problem "makes sense." To determine this, at least until you have more experience with SI units, you may need to convert from SI units back to the English units in which you think. But this conversion does not need to be very accurate. For example, if you are working a problem about automobile speeds and reach an answer of 35 m/s, all you really want to know is whether or not this is a realistic speed for a car. That requires a "quick and dirty" conversion, not a conversion of great accuracy.

TABLE 1.5 shows several approximate conversion factors that can be used to assess the answer to a problem. Using 1 m/s $\approx$ 2 mph, you find that 35 m/s is roughly 70 mph, a reasonable speed for a car. But an answer of 350 m/s, which you might get after making a calculation error, would be an unreasonable 700 mph. Practice with these will allow you to develop intuition for metric units.

NOTE These approximate conversion factors are accurate to only one significant figure. This is sufficient to assess the answer to a problem, but do *not* use the conversion factors from Table 1.5 for converting English units to SI units at the start of a problem. Use Table 1.4.

TABLE 1.5 Approximate conversion factors. Use these for assessment, not in problem solving.

1 cm $\approx \frac{1}{2}$ in
10 cm $\approx$ 4 in
1 m $\approx$ 1 yard
1 m $\approx$ 3 feet
1 km $\approx$ 0.6 mile
1 m/s $\approx$ 2 mph

Significant Figures

It is necessary to say a few words about a perennial source of difficulty: significant figures. Mathematics is a subject where numbers and relationships can be as precise as desired, but physics deals with a real world of ambiguity. It is important in science and engineering to state clearly what you know about a situation—no less and, especially, no more. Numbers provide one way to specify your knowledge.

If you report that a length has a value of 6.2 m, the implication is that the actual value falls between 6.15 m and 6.25 m and thus rounds to 6.2 m. If that is the case, then reporting a value of simply 6 m is saying less than you know; you are withholding information. On the other hand, to report the number as 6.213 m is wrong. Any person reviewing your work—perhaps a client who hired you—would interpret the number 6.213 m as meaning that the actual length falls between 6.2125 m and 6.2135 m, thus rounding to 6.213 m. In this case, you are claiming to have knowledge and information that you do not really possess.

The way to state your knowledge precisely is through the proper use of **significant figures.** You can think of a significant figure as being a digit that is reliably known. A number such as 6.2 m has *two* significant figures because the next decimal place—the one-hundredths—is not reliably known. As **FIGURE 1.22** shows, the best way to determine how many significant figures a number has is to write it in scientific notation.

FIGURE 1.22 Determining significant figures.

Leading zeros locate the decimal point. They are not significant.

$$0.00620 = \boxed{6.20} \times 10^{-3}$$

A trailing zero after the decimal place is reliably known. It is significant.

The number of significant figures is the number of digits when written in scientific notation.

- The number of significant figures $\neq$ the number of decimal places.
- Changing units shifts the decimal point but does not change the number of significant figures.

What about numbers like 320 m and 20 kg? Whole numbers with trailing zeros are ambiguous unless written in scientific notation. Even so, writing 2.0×10^1 kg is tedious, and few practicing scientists or engineers would do so. In this textbook, we'll adopt the rule that **whole numbers always have at least two significant figures,** even if one of those is a trailing zero. By this rule, 320 m, 20 kg, and 8000 s each have two significant figures, but 8050 s would have three.

Calculations with numbers follow the "weakest link" rule. The saying, which you probably know, is that "a chain is only as strong as its weakest link." If nine out of ten links in a chain can support a 1000 pound weight, that strength is meaningless if the tenth link can support only 200 pounds. Nine out of the ten numbers used in a calculation might be known with a precision of 0.01%; but if the tenth number is poorly known, with a precision of only 10%, then the result of the calculation cannot possibly be more precise than 10%.

TACTICS BOX 1.5

Using significant figures

1. When multiplying or dividing several numbers, or taking roots, the number of significant figures in the answer should match the number of significant figures of the *least* precisely known number used in the calculation.
2. When adding or subtracting several numbers, the number of decimal places in the answer should match the *smallest* number of decimal places of any number used in the calculation.
3. Exact numbers are perfectly known and do not affect the number of significant figures an answer should have. Examples of exact numbers are the 2 and the π in the formula $C = 2\pi r$ for the circumference of a circle.
4. It is acceptable to keep one or two extra digits during intermediate steps of a calculation, to minimize rounding error, as long as the final answer is reported with the proper number of significant figures.
5. For examples and problems in this textbook, **the appropriate number of significant figures for the answer is determined by the data provided.** Whole numbers with trailing zeros, such as 20 kg, are interpreted as having at least two significant figures.

Exercises 38–39

NOTE Be careful! Many calculators have a default setting that shows two decimal places, such as 5.23. This is dangerous. If you need to calculate 5.23/58.5, your calculator will show 0.09 and it is all too easy to write that down as an answer. By doing so, you have reduced a calculation of two numbers having three significant figures to an answer with only one significant figure. The proper result of this division is 0.0894 or 8.94×10^{-2}. You will avoid this error if you keep your calculator set to display numbers in *scientific notation* with two decimal places.

EXAMPLE 1.10 ■ Using significant figures

An object consists of two pieces. The mass of one piece has been measured to be 6.47 kg. The volume of the second piece, which is made of aluminum, has been measured to be $4.44 \times 10^{-4}\ \text{m}^3$. A handbook lists the density of aluminum as $2.7 \times 10^3\ \text{kg/m}^3$. What is the total mass of the object?

SOLVE First, calculate the mass of the second piece:

$$m = (4.44 \times 10^{-4}\ \text{m}^3)(2.7 \times 10^3\ \text{kg/m}^3)$$
$$= 1.199\ \text{kg} = 1.2\ \text{kg}$$

The number of significant figures of a product must match that of the *least* precisely known number, which is the two-significant-figure density of aluminum. Now add the two masses:

$$\begin{array}{r} 6.47\ \text{kg} \\ +\ 1.2\ \ \ \text{kg} \\ \hline 7.7\ \ \ \text{kg} \end{array}$$

The sum is 7.67 kg, but the hundredths place is not reliable because the second mass has no reliable information about this digit. Thus we must round to the one decimal place of the 1.2 kg. The best we can say, with reliability, is that the total mass is 7.7 kg.

Proper use of significant figures is part of the "culture" of science and engineering. We will frequently emphasize these "cultural issues" because you must learn to speak the same language as the natives if you wish to communicate effectively. Most students know the rules of significant figures, having learned them in high school, but many fail to apply them. It is important to understand the reasons for significant figures and to get in the habit of using them properly.

Orders of Magnitude and Estimating

Precise calculations are appropriate when we have precise data, but there are many times when a very rough estimate is sufficient. Suppose you see a rock fall off a cliff and would like to know how fast it was going when it hit the ground. By doing a mental comparison with the speeds of familiar objects, such as cars and bicycles, you might judge that the rock was traveling at "about" 20 mph.

This is a one-significant-figure estimate. With some luck, you can distinguish 20 mph from either 10 mph or 30 mph, but you certainly cannot distinguish 20 mph from 21 mph. A one-significant-figure estimate or calculation, such as this, is called an **order-of-magnitude estimate.** An order-of-magnitude estimate is indicated by the symbol $\sim$, which indicates even less precision than the "approximately equal" symbol $\approx$. You would say that the speed of the rock is $v \sim 20$ mph.

A useful skill is to make reliable estimates on the basis of known information, simple reasoning, and common sense. This is a skill that is acquired by practice. Many chapters in this book will have homework problems that ask you to make order-of-magnitude estimates. The following example is a typical estimation problem.

TABLES 1.6 and **1.7** have information that will be useful for doing estimates.

TABLE 1.6 Some approximate lengths

	Length (m)
Altitude of jet planes	10,000
Distance across campus	1000
Length of a football field	100
Length of a classroom	10
Length of your arm	1
Width of a textbook	0.1
Length of a fingernail	0.01

TABLE 1.7 Some approximate masses

	Mass (kg)
Small car	1000
Large human	100
Medium-size dog	10
Science textbook	1
Apple	0.1
Pencil	0.01
Raisin	0.001

EXAMPLE 1.11 ■ **Estimating a sprinter's speed**

Estimate the speed with which an Olympic sprinter crosses the finish line of the 100 m dash.

SOLVE We do need one piece of information, but it is a widely known piece of sports trivia. That is, world-class sprinters run the 100 m dash in about 10 s. Their *average* speed is $v_{\text{avg}} \approx (100\ \text{m})/(10\ \text{s}) \approx 10\ \text{m/s}$. But that's only average. They go slower than average at the beginning, and they cross the finish line at a speed faster than average. How much faster? Twice as fast, 20 m/s, would be $\approx$40 mph. Sprinters don't seem like they're running as fast as a 40 mph car, so this probably is too fast. Let's *estimate* that their final speed is 50% faster than the average. Thus they cross the finish line at $v \sim 15\ \text{m/s}$.

STOP TO THINK 1.5 Rank in order, from the most to the least, the number of significant figures in the following numbers. For example, if b has more than c, c has the same number as a, and a has more than d, you could give your answer as b > c = a > d.

a. 82 b. 0.0052 c. 0.430 d. 4.321×10^{-10}

Summary

The goal of Chapter 1 has been to learn the fundamental concepts of motion.

General Strategy

Problem Solving

MODEL Make simplifying assumptions.

VISUALIZE Use:

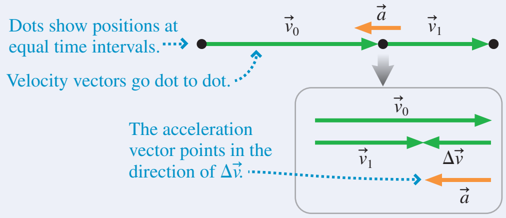
- **Pictorial representation**
- **Graphical representation**

SOLVE Use a **mathematical representation** to find numerical answers.

REVIEW Does the answer have the proper units and correct significant figures? Does it make sense?

Motion Diagrams

- Help visualize motion.
- Provide a tool for finding acceleration vectors.



▶ These are the *average* velocity and acceleration vectors.

Important Concepts

The **particle model** represents a moving object as if all its mass were concentrated at a single point.

Position locates an object with respect to a chosen coordinate system. Change in position is called **displacement**.

Velocity is the rate of change of the position vector $\vec{r}$.

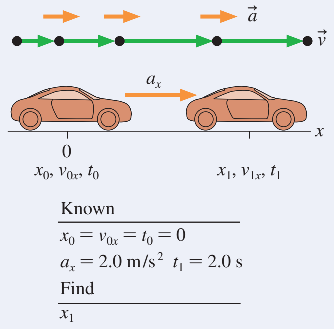
Acceleration is the rate of change of the velocity vector $\vec{v}$.

An object has an acceleration if it

- Changes speed and/or
- Changes direction.

Pictorial Representation

1. Draw a motion diagram.
2. Establish coordinates.
3. Sketch the situation.
4. Define symbols.
5. List knowns.
6. Identify desired unknown.



Applications

For motion along a line:

- Speeding up: $\vec{v}$ and $\vec{a}$ point in the same direction, v_x and a_x have the same sign.
- Slowing down: $\vec{v}$ and $\vec{a}$ point in opposite directions, v_x and a_x have opposite signs.
- Constant speed: $\vec{a} = \vec{0}$, $a_x = 0$.

Acceleration a_x is positive if $\vec{a}$ ptoints right, negative if $\vec{a}$ points left. The sign of a_x does *not* imply speeding up or slowing down.

Significant figures are reliably known digits. The number of significant figures for:

- Multiplication, division, powers is set by the value with the fewest significant figures.
- Addition, subtraction is set by the value with the smallest number of decimal places.

The appropriate number of significant figures in a calculation is determined by the data provided.

Terms and Notation

motion
translational motion
trajectory
motion diagram
model
particle
particle model
position vector, $\vec{r}$
scalar
vector
displacement, $\Delta\vec{r}$
time interval, Δt
average speed
average velocity, $\vec{v}$
average acceleration, $\vec{a}$
position-versus-time graph
pictorial representation
representation of knowledge
SI units
significant figures
order-of-magnitude estimate

CONCEPTUAL QUESTIONS

1. How many significant figures does each of the following numbers have?
   a. 9.90 b. 0.99 c. 0.099 d. 99
2. How many significant figures does each of the following numbers have?
   a. 0.0044 b. 4.40×10^{-4} c. 440 d. 2.90
3. Is the particle in **FIGURE Q1.3** speeding up? Slowing down? Or can you tell? Explain.

FIGURE Q1.3

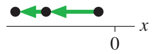
4. Does the object represented in **FIGURE Q1.4** have a positive or negative value of a_x? Explain.
5. Does the object represented in **FIGURE Q1.5** have a positive or negative value of a_y? Explain.

FIGURE Q1.4

FIGURE Q1.5

6. Determine the signs (positive, negative, or zero) of the position, velocity, and acceleration for the particle in **FIGURE Q1.6**.

FIGURE Q1.6



7. Determine the signs (positive, negative, or zero) of the position, velocity, and acceleration for the particle in **FIGURE Q1.7**.

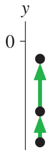


FIGURE Q1.7

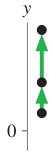


FIGURE Q1.8

8. Determine the signs (positive, negative, or zero) of the position, velocity, and acceleration for the particle in **FIGURE Q1.8**.

EXERCISES AND PROBLEMS

Exercises

Section 1.1 Motion Diagrams

1. A jet plane lands on the deck of an aircraft carrier and quickly comes to a halt. Draw a basic motion diagram, using the images from the video, from the time the jet touches down until it stops.
2. You are watching a jet ski race. A racer speeds up from rest to 70 mph in 10 s, then continues at a constant speed. Draw a basic motion diagram of the jet ski, using images from the video, from its start until 10 s after reaching top speed.
3. A rocket is launched straight up. Draw a basic motion diagram, using the images from the video, from the moment of liftoff until the rocket is at an altitude of 500 m.

Section 1.2 Models and Modeling

4. a. Write a paragraph describing the particle model. What is it, and why is it important?
   b. Give two examples of situations, different from those described in the text, for which the particle model is appropriate.
   c. Give an example of a situation, different from those described in the text, for which it would be inappropriate.

Section 1.3 Position, Time, and Displacement

Section 1.4 Velocity

5. A baseball player starts running to the left to catch the ball as soon as the hit is made. Use the particle model to draw a motion diagram showing the position and average velocity vectors of the player during the first few seconds of the run.
6. You drop a soccer ball from your third-story balcony. Use the particle model to draw a motion diagram showing the ball's position and average velocity vectors from the time you release the ball until the instant it touches the ground.
7. A car skids to a halt to avoid hitting an object in the road. Use the particle model to draw a motion diagram showing the car's position and its average velocity from the time the skid begins until the car stops.

Section 1.5 Linear Acceleration

8. a. **FIGURE EX1.8** shows the first three points of a motion diagram. Is the object's average speed between points 1 and 2 greater than, less than, or equal to its average speed between points 0 and 1? Explain how you can tell.
   b. Use Tactics Box 1.2 to find the average acceleration vector at point 1. Draw the completed motion diagram, showing the velocity vectors and acceleration vector.

FIGURE EX1.8

FIGURE EX1.9

9. **FIGURE EX1.9** shows five points of a motion diagram. Use Tactics Box 1.2 to find the average acceleration vectors at points 1, 2, and 3. Draw the completed motion diagram showing velocity vectors and acceleration vectors.

10. || FIGURE EX1.10 shows two dots of a motion diagram and vector $\vec{v}_2$. Copy this figure, then add dot 4 and the next velocity vector $\vec{v}_3$ if the acceleration vector $\vec{a}$ at dot 3 (a) points right and (b) points left.

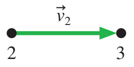


FIGURE EX 1.10

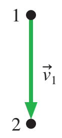


FIGURE EX1.11

11. || FIGURE EX1.11 shows two dots of a motion diagram and vector $\vec{v}_1$. Copy this figure, then add dot 3 and the next velocity vector $\vec{v}_2$ if the acceleration vector $\vec{a}$ at dot 2 (a) points up and (b) points down.
12. | A car travels to the left at a steady speed for a few seconds, then brakes for a stop sign. Draw a complete motion diagram of the car.
13. | A speed skater accelerates from rest and then keeps skating at a constant speed. Draw a complete motion diagram of the skater.
14. | A bowling ball rolls up an incline and then onto a smooth, level surface. Draw a complete motion diagram of the bowling ball. Don't try to find the acceleration vector at the point where the motion changes direction; that's an issue for Chapter 4.
15. | You use a long rubber band to launch a paper wad straight up. Draw a complete motion diagram of the paper wad from the moment you release the stretched rubber band until the paper wad reaches its highest point.
16. | A roof tile falls straight down from a two-story building. It lands in a swimming pool and settles gently to the bottom. Draw a complete motion diagram of the tile.
17. | Your roommate drops a tennis ball from a third-story balcony. It hits the sidewalk and bounces as high as the second story. Draw a complete motion diagram of the tennis ball from the time it is released until it reaches the maximum height on its bounce. Be sure to determine and show the acceleration at the lowest point.

Section 1.6 Motion in One Dimension

18. || FIGURE EX1.18 shows the motion diagram of a drag racer. The camera took one frame every 2 s.

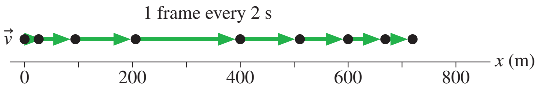


FIGURE EX1.18

a. Measure the x-value of the racer at each dot. List your data in a table similar to Table 1.1, showing each position and the time at which it occurred.
b. Make a position-versus-time graph for the drag racer. Because you have data only at certain instants, your graph should consist of dots that are not connected together.

19. | Write a short description of the motion of a real object for which FIGURE EX1.19 would be a realistic position-versus-time graph.

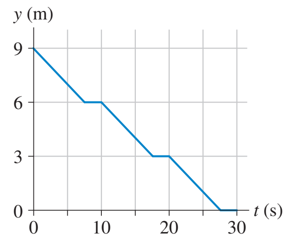


FIGURE EX 1.19

20. | Write a short description of the motion of a real object for which FIGURE EX1.20 would be a realistic position-versus-time graph.

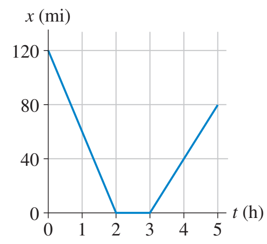


FIGURE EX1.20

Section 1.7 Solving Problems in Physics

21. || Draw a pictorial representation for the following problem. Do *not* solve the problem. What acceleration does a rocket need to reach a speed of 200 m/s at a height of 1.0 km?
22. || Draw a pictorial representation for the following problem. Do *not* solve the problem. The light turns green, and a bicyclist starts forward with an acceleration of 1.5 m/s^2. How far must she travel to reach a speed of 7.5 m/s?

Section 1.8 Units and Significant Figures

23. | How many significant figures are there in each of the following values?
a. 8.263×10^{-1} b. 0.0414
c. 75.0 d. 0.07×10^{8}
24. || Convert the following to basic SI units or a combination of basic SI units:
a. 8.0 in b. 66 ft/s
c. 60 mph d. 14 in^2
25. | Convert the following to basic SI units or a combination of basic SI units:
a. 87 in b. 7.89×10^{6} yr
c. 48 ft/day d. 1.7×10^{3} mi^2
26. || Using the approximate conversion factors in Table 1.5, convert the following to SI units *without* using your calculator.
a. 20 ft b. 60 mi
c. 60 mph d. 8 in
27. | Using the approximate conversion factors in Table 1.5, convert the following SI units to English units *without* using your calculator.
a. 50 cm b. 15 km
c. 35 m/s d. 3 m

28. | Perform the following calculations with the correct number of significant figures.
   a. 159.31×204.6 b. $5.1125 + 0.67 + 3.2$
   c. $7.662 - 7.425$ d. $16.5/3.45$
29. | Compute the following numbers, applying the significant figure rules adopted in this textbook.
   a. 33.3×25.4 b. $33.3 - 25.4$
   c. $\sqrt{33.3}$ d. $333.3 \div 25.4$
30. | Estimate (don't measure!) the length of a typical car. Give your answer in both feet and meters. Briefly describe how you arrived at this estimate.
31. | Estimate the height of a telephone pole. Give your answer in both feet and meters. Briefly describe how you arrived at this estimate.
32. || BIO Estimate the average speed with which the hair on your head grows. Give your answer in both m/s and μm/hour. Briefly describe how you arrived at this estimate.
33. | BIO Motor neurons in mammals transmit signals from the brain to skeletal muscles at approximately 25 m/s. Estimate how long in ms it takes a signal to get from your brain to your hand.

Problems

For Problems 34 through 43, draw a complete pictorial representation. **Do *not* solve these problems or do any mathematics.**

34. | A jet plane is cruising at 300 m/s when suddenly the pilot turns the engines up to full throttle. After traveling 4.0 km, the jet is moving with a speed of 400 m/s. What is the jet's acceleration as it speeds up?
35. | A Porsche accelerates from a stoplight at 5.0 m/s^2 for five seconds, then coasts for three more seconds. How far has it traveled?
36. | Sam is recklessly driving 60 mph in a 30 mph speed zone when he suddenly sees the police. He steps on the brakes and slows to 30 mph in three seconds, looking nonchalant as he passes the officer. How far does he travel while braking?
37. | A car starts from rest at a stop sign. It accelerates at 4.0 m/s^2 for 6.0 s, coasts for 2.0 s, and then slows at a rate of 2.5 m/s^2 for the next stop sign. How far apart are the stop signs?
38. | Santa loses his footing and slides down a frictionless, snowy roof that is tilted at an angle of 30°. If Santa slides 10 m before reaching the edge, what is his speed as he leaves the roof?
39. | A speed skater moving across frictionless ice at 8.0 m/s hits a 5.0-m-wide patch of rough ice. She slows steadily, then continues on at 6.0 m/s. What is her acceleration on the rough ice?
40. | A motorist is traveling at 20 m/s. He is 60 m from a stoplight when he sees it turn yellow. His reaction time, before stepping on the brake, is 0.50 s. What steady deceleration while braking will bring him to a stop right at the light?
41. | A car traveling at 30 m/s runs out of gas while traveling up a 10° slope. How far up the hill will the car coast before starting to roll back down?
42. || A Porsche challenges a Honda to a 400 m race. Because the Porsche's acceleration of 3.5 m/s^2 is greater than the Honda's 3.0 m/s^2, the Honda gets a 1.0 s head start. Who wins?
43. || David is driving a steady 30 m/s when he passes Tina, who is sitting in her car at rest. Tina begins to accelerate at a steady 2.0 m/s^2 at the instant when David passes. How far does Tina drive before passing David?

Problems 44 through 48 show a motion diagram. For each of these problems, write a one or two sentence "story" about a *real object* that has this motion diagram. Your stories should talk about people or objects by name and say what they are doing. Problems 34 through 43 are examples of motion short stories.

44. |

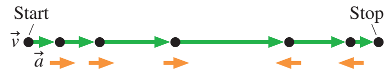


FIGURE P1.44

45. |

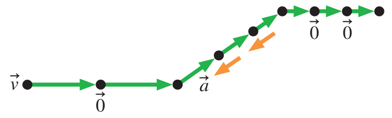


FIGURE P1.45

46. |

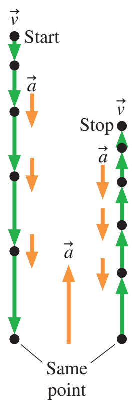


FIGURE P1.46

47. |

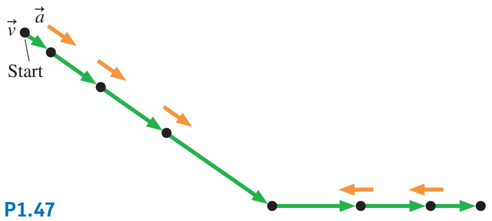


FIGURE P1.47

48. |

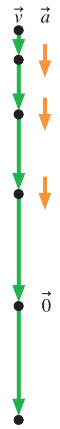


FIGURE P1.48

Problems 49 through 52 show a partial motion diagram. For each:

a. Complete the motion diagram by adding acceleration vectors.
b. Write a physics *problem* for which this is the correct motion diagram. Be imaginative! Don't forget to include enough information to make the problem complete and to state clearly what is to be found.
c. Draw a pictorial representation for your problem.

49. 

FIGURE P1.49

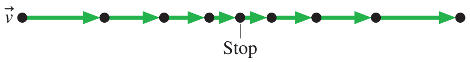
50. 

FIGURE P1.50

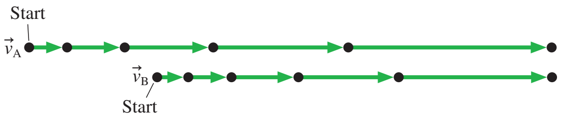
51. 

FIGURE P1.51

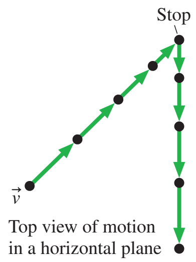
52. 

FIGURE P1.52

53. | As an architect, you are designing a new house. A window has a height between 140 cm and 150 cm and a width between 74 cm and 70 cm. What are the smallest and largest areas that the window could be?

54. | A regulation soccer field for international play is a rectangle with a length between 100 m and 110 m and a width between 64 m and 75 m. What are the smallest and largest areas that the field could be?

55. || A 5.8-cm-diameter cylinder has a length of 15.5 cm. What is the cylinder's volume in basic SI units?

56. || An intravenous saline drip has 4.5 g of sodium chloride per liter of water. By definition, 1 mL = 1 cm^3. Express the salt concentration in kg/m^3.
BIO

57. || The quantity called *mass density* is the mass per unit volume of a substance. What are the mass densities in basic SI units of the following objects?
a. A 245 cm^3 solid with a mass of 0.0159 kg
b. 82 cm^3 of a liquid with a mass of 59 g

58. | **FIGURE P1.58** shows a motion diagram of a car traveling down a street. The camera took one frame every 10 s. A distance scale is provided.

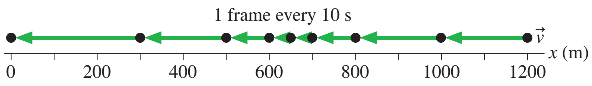


FIGURE P1.58

a. Measure the *x*-value of the car at each dot. Place your data in a table, similar to Table 1.1, showing each position and the instant of time at which it occurred.
b. Make a position-versus-time graph for the car. Because you have data only at certain instants of time, your graph should consist of dots that are not connected together.

59. | Write a short description of a real object for which **FIGURE P1.59** would be a realistic position-versus-time graph.

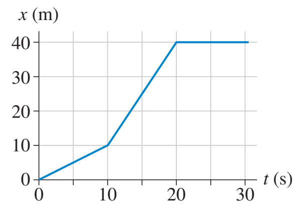


FIGURE P1.59

60. | Write a short description of a real object for which **FIGURE P1.60** would be a realistic position-versus-time graph.

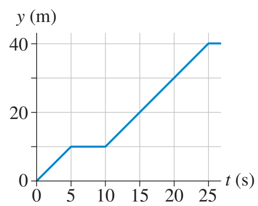


FIGURE P1.60

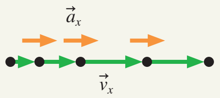
2 Kinematics in One Dimension

This Japanese “bullet train” accelerates slowly but steadily until reaching a speed of 300 km/h.

IN THIS CHAPTER, you will learn to solve problems about motion along a straight line.

What is kinematics?

Kinematics is the mathematical description of motion. We begin with motion along a straight line. Our primary tools will be an object’s **position**, **velocity**, and **acceleration**.

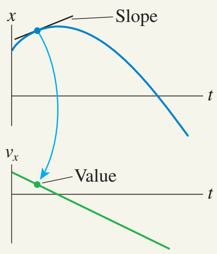


« **LOOKING BACK** Sections 1.4–1.6 Velocity, acceleration, and Tactics Box 1.3 about signs

How are graphs used in kinematics?

Graphs are a very important visual representation of motion, and learning to “think graphically” is one of our goals. We’ll work with graphs showing how position, velocity, and acceleration **change with time**. These graphs are related to each other:

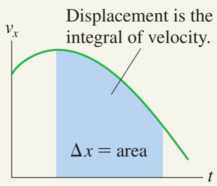
- Velocity is the slope of the position graph.
- Acceleration is the slope of the velocity graph.



How is calculus used in kinematics?

Motion is change, and calculus is the mathematical tool for describing a quantity’s **rate of change**. We’ll find that

- Velocity is the **time derivative** of position.
- Acceleration is the time derivative of velocity.



What are models?

A **model** is a simplified description of a situation that focuses on essential features while ignoring many details. Models allow us to make sense of complex situations by seeing them as variations on a common theme, all with the **same underlying physics**.

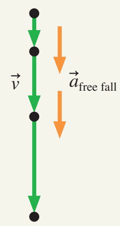
MODEL 2.1

Look for model boxes like this throughout the book.

- Key figures
- Key equations
- Model limitations

What is free fall?

Free fall is motion under the influence of gravity only. Free fall is not literally “falling” because it also applies to objects thrown straight up and to projectiles. Surprisingly, all objects in free fall, *regardless of their mass*, have the same acceleration. Motion on a frictionless **inclined plane** is closely related to free-fall motion.



How will I use kinematics?

The equations of motion that you learn in this chapter will be used throughout the entire book. In Part I, we’ll see how an object’s motion is related to forces acting on the object. We’ll later apply these **kinematic equations** to the motion of waves and to the motion of charged particles in electric and magnetic fields.

2.1 Uniform Motion

The simplest possible motion is motion along a straight line at a constant, unvarying speed. We call this **uniform motion.** Because velocity is the combination of speed and direction, **uniform motion is motion with constant velocity.**

FIGURE 2.1 shows the motion diagram of an object in uniform motion. For example, this might be you riding your bicycle along a straight line at a perfectly steady 5 m/s (≈ 10 mph). Notice how all the displacements are exactly the same; this is a characteristic of uniform motion.

If we make a position-versus-time graph—remember that position is graphed on the *vertical* axis—it's a straight line. In fact, an alternative definition is that **an object's motion is uniform if and only if its position-versus-time graph is a straight line.**

« SECTION 1.4 defined an object's **average velocity** as $\Delta\vec{r}/\Delta t$. For one-dimensional motion, this is simply $\Delta x/\Delta t$ (for horizontal motion) or $\Delta y/\Delta t$ (for vertical motion). Recall that Δx is the object's displacement during the time interval Δt. You can see in Figure 2.1 that Δx and Δt are, respectively, the "rise" and "run" of the position graph. Because rise over run is the slope of a line,

$$v_{\text{avg}} \equiv \frac{\Delta x}{\Delta t} \text{ or } \frac{\Delta y}{\Delta t} = \text{slope of the position-versus-time graph} \qquad (2.1)$$

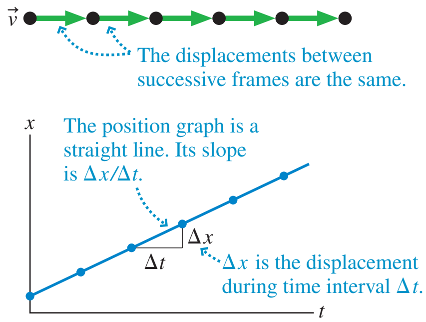
That is, **the average velocity is the slope of the position-versus-time graph.** Velocity has units of "length per time," such as "miles per hour." The SI units of velocity are meters per second, abbreviated m/s.

NOTE The symbol $\equiv$ in Equation 2.1 stands for "is defined as." This is a stronger statement than the two sides simply being equal.

The constant slope of a straight-line graph is another way to see that the velocity is constant for uniform motion. There's no real need to specify "average" for a velocity that doesn't change, so we will drop the subscript and refer to the average velocity as v_x or v_y.

An object's **speed** v is how fast it's going, independent of direction. This is simply $v = |v_x|$ or $v = |v_y|$, the magnitude or absolute value of the object's velocity. Although we will use speed from time to time, our mathematical analysis of motion is based on velocity, not speed. The subscript in v_x or v_y is an essential part of the notation, reminding us that, even in one dimension, the velocity is a vector.

FIGURE 2.1 Motion diagram and position graph for uniform motion.



EXAMPLE 2.1 ■ Relating a velocity graph to a position graph

FIGURE 2.2 on the next page is the position-versus-time graph of a car.

a. Draw the car's velocity-versus-time graph.

b. Describe the car's motion.

MODEL Model the car as a particle, with a well-defined position at each instant of time.

VISUALIZE Figure 2.2 is the graphical representation.

SOLVE a. The car's position-versus-time graph is a sequence of three straight lines. Each of these straight lines represents uniform motion at a constant velocity. We can determine the car's velocity during each interval of time by measuring the slope of the line.

The position graph starts out sloping downward—a negative slope. Although the car moves a distance of 4.0 m during the first 2.0 s, its *displacement* is

$$\Delta x = x_{\text{at } 2.0\text{ s}} - x_{\text{at } 0.0\text{ s}} = -4.0\text{ m} - 0.0\text{ m} = -4.0\text{ m}$$

The time interval for this displacement is $\Delta t = 2.0$ s, so the velocity during this interval is

$$v_x = \frac{\Delta x}{\Delta t} = \frac{-4.0\text{ m}}{2.0\text{ s}} = -2.0\text{ m/s}$$

The car's position does not change from $t = 2$ s to $t = 4$ s ($\Delta x = 0$), so $v_x = 0$. Finally, the displacement between $t = 4$ s and $t = 6$ s is $\Delta x = 10.0$ m. Thus the velocity during this interval is

$$v_x = \frac{10.0\text{ m}}{2.0\text{ s}} = 5.0\text{ m/s}$$

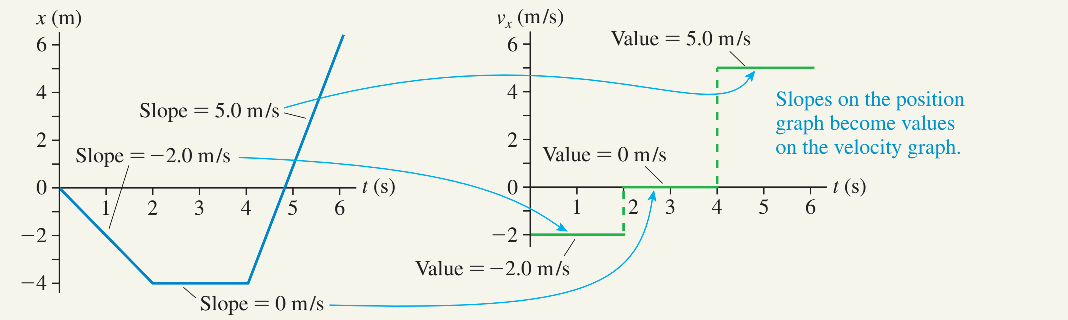
These velocities are shown on the velocity-versus-time graph of FIGURE 2.3 on the next page.

b. The car backs up for 2 s at 2.0 m/s, sits at rest for 2 s, then drives forward at 5.0 m/s for at least 2 s. We can't tell from the graph what happens for $t > 6$ s.

REVIEW The velocity graph and the position graph look completely different. The *value* of the velocity graph at any instant of time equals the *slope* of the position graph.

Continued

FIGURE 2.2 Position-versus-time graph. **FIGURE 2.3** The corresponding velocity-versus-time graph.



Example 2.1 brought out several points that are worth emphasizing.

TACTICS BOX 2.1

Interpreting position-versus-time graphs

1. Steeper slopes correspond to faster speeds.
2. Negative slopes correspond to negative velocities and, hence, to motion to the left (or down).
3. The slope is a ratio of intervals, $\Delta x / \Delta t$, not a ratio of coordinates. That is, the slope is *not* simply x / t.

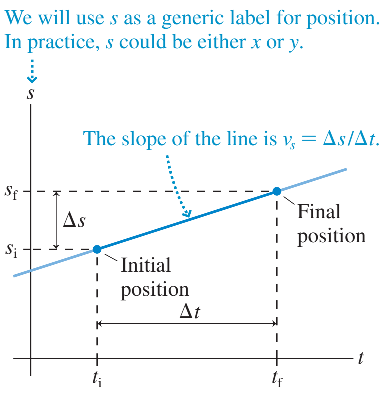
Exercises 1–3

NOTE We are distinguishing between the *actual* slope and the *physically meaningful* slope. If you were to use a ruler to measure the rise and the run of the graph, you could compute the actual slope of the line as drawn on the page. That is not the slope to which we are referring when we equate the velocity with the slope of the line. Instead, we find the *physically meaningful* slope by measuring the rise and run using the scales along the axes. The "rise" Δx is some number of meters; the "run" Δt is some number of seconds. The physically meaningful rise and run include units, and the ratio of these units gives the units of the slope.

The Mathematics of Uniform Motion

The physics of the motion is the same regardless of whether an object moves along the x-axis, the y-axis, or any other straight line. Consequently, **it will be convenient to write equations for a "generic axis" that we will call the s-axis.** The position of an object will be represented by the symbol s and its velocity by v_s.

FIGURE 2.4 The velocity is found from the slope of the position-versus-time graph.



NOTE In a specific problem you should use either x or y rather than s.

Consider an object in uniform motion along the s-axis with the linear position-versus-time graph shown in **FIGURE 2.4**. The object's **initial position** is s_i at time t_i. The term *initial position,* designated with subscript i, refers to the starting point of our analysis or the starting point in a problem; the object may or may not have been in motion prior to t_i. At a later time t_f, the ending point of our analysis, the object's **final position,** denoted by f, is s_f.

The object's velocity v_s along the s-axis can be determined by finding the slope of the graph:

$$v_s = \frac{\text{rise}}{\text{run}} = \frac{\Delta s}{\Delta t} = \frac{s_f - s_i}{t_f - t_i} \tag{2.2}$$

Equation 2.2 is easily rearranged to give

$$s_f = s_i + v_s \, \Delta t \qquad \text{(uniform motion)} \qquad (2.3)$$

Equation 2.3 tells us that the object's position increases linearly as the elapsed time Δt increases—exactly as we see in the straight-line position graph.

The Uniform-Motion Model

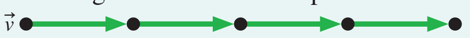
Chapter 1 introduced a *model* as a simplified picture of reality, but one that still captures the essence of what we want to study. When it comes to motion, few real objects move with a precisely constant velocity. Even so, there are many cases in which it is quite reasonable to model their motion as being uniform. That is, uniform motion is a very good approximation of their actual, but more complex, motion. The **uniform-motion model** is a coherent set of representations—words, pictures, graphs, and equations—that allows us to explain an object's motion and to predict where the object will be at a future instant of time.

MODEL 2.1

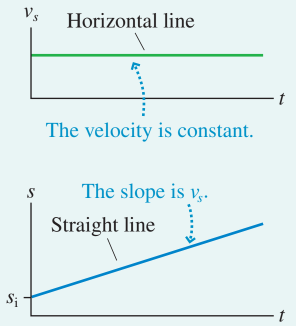
Uniform motion

For motion with constant velocity.

- Model the object as a particle moving in a straight line at constant speed:



- Mathematically:
  - $v_s = \Delta s / \Delta t$
  - $s_f = s_i + v_s \, \Delta t$
- Limitations: Model fails if the particle has a significant change of speed or direction.



Exercise 4

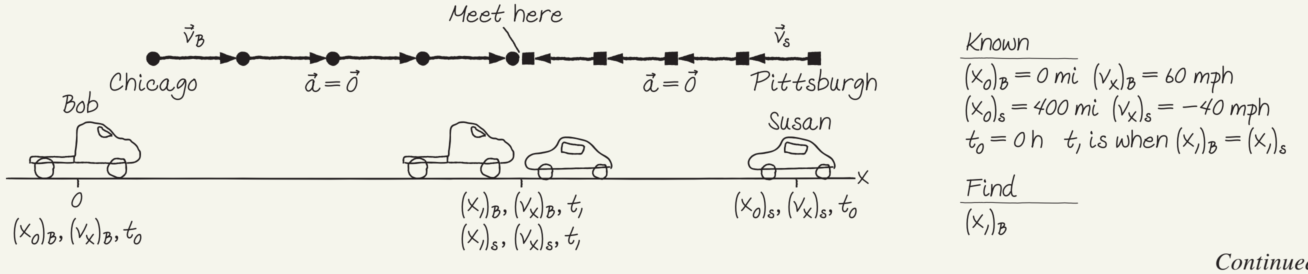
EXAMPLE 2.2 ■ Lunch in Cleveland?

Bob leaves home in Chicago at 9:00 A.M. and drives east at 60 mph. Susan, 400 miles to the east in Pittsburgh, leaves at the same time and travels west at 40 mph. Where will they meet for lunch?

MODEL Here is a problem where, for the first time, we can really put all four aspects of our problem-solving strategy into play. To begin, we'll model Bob's and Susan's cars as being in uniform motion. Their real motion is certainly more complex, but over a long drive it's reasonable to approximate their motion as constant speed along a straight line.

VISUALIZE FIGURE 2.5 shows the pictorial representation. The equal spacings of the dots in the motion diagram indicate that the motion is uniform. In evaluating the given information, we

FIGURE 2.5 Pictorial representation for Example 2.2.



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recognize that the starting time of 9:00 A.M. is not relevant to the problem. Consequently, the initial time is chosen as simply $t_0 = 0$ h. Bob and Susan are traveling in opposite directions, hence one of the velocities must be a negative number. We have chosen a coordinate system in which Bob starts at the origin and moves to the right (east) while Susan is moving to the left (west). Thus Susan has the negative velocity. Notice how we've assigned position, velocity, and time symbols to each point in the motion. Pay special attention to how subscripts are used to distinguish different points in the problem and to distinguish Bob's symbols from Susan's.

One purpose of the pictorial representation is to establish what we need to find. Bob and Susan meet when they have the same position at the same time t_1. Thus we want to find $(x_1)_B$ at the time when $(x_1)_B = (x_1)_S$. Notice that $(x_1)_B$ and $(x_1)_S$ are Bob's and Susan's *positions,* which are equal when they meet, not the distances they have traveled.

SOLVE The goal of the mathematical representation is to proceed from the pictorial representation to a mathematical solution of the problem. We can begin by using Equation 2.3 to find Bob's and Susan's positions at time t_1 when they meet:

$$(x_1)_B = (x_0)_B + (v_x)_B(t_1 - t_0) = (v_x)_B t_1$$

$$(x_1)_S = (x_0)_S + (v_x)_S(t_1 - t_0) = (x_0)_S + (v_x)_S t_1$$

Notice two things. First, we started by writing the *full* statement of Equation 2.3. Only then did we simplify by dropping those terms known to be zero. You're less likely to make accidental errors if you follow this procedure. Second, we replaced the generic symbol s with the specific horizontal-position symbol x, and we replaced the generic subscripts i and f with the specific symbols 0 and 1 that we defined in the pictorial representation. This is also good problem-solving technique.

The condition that Bob and Susan meet is

$$(x_1)_B = (x_1)_S$$

By equating the right-hand sides of the above equations, we get

$$(v_x)_B t_1 = (x_0)_S + (v_x)_S t_1$$

Solving for t_1 we find that they meet at time

$$t_1 = \frac{(x_0)_S}{(v_x)_B - (v_x)_S} = \frac{400 \text{ miles}}{60 \text{ mph} - (-40) \text{ mph}} = 4.0 \text{ hours}$$

Finally, inserting this time back into the equation for $(x_1)_B$ gives

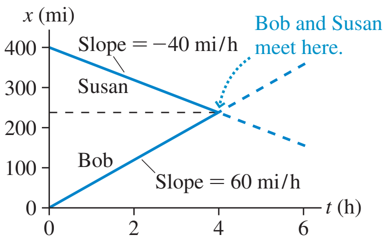
$$(x_1)_B = \left(60 \frac{\text{miles}}{\text{hour}}\right) \times (4.0 \text{ hours}) = 240 \text{ miles}$$

As noted in Chapter 1, this textbook will assume that all data are good to at least two significant figures, even when one of those is a trailing zero. So 400 miles, 60 mph, and 40 mph each have two significant figures, and consequently we've calculated results to two significant figures.

While 240 miles is a number, it is not yet the answer to the question. The phrase "240 miles" by itself does not say anything meaningful. Because this is the value of Bob's *position,* and Bob was driving east, the answer to the question is, "They meet 240 miles east of Chicago."

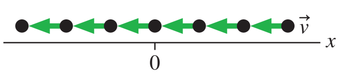
REVIEW Before stopping, we should check whether or not this answer seems reasonable. We certainly expected an answer between 0 miles and 400 miles. We also know that Bob is driving faster than Susan, so we expect that their meeting point will be *more* than halfway from Chicago to Pittsburgh. Our review tells us that 240 miles is a reasonable answer.

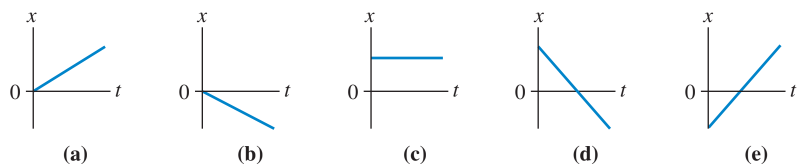
FIGURE 2.6 Position-versus-time graphs for Bob and Susan.



It is instructive to look at this example from a graphical perspective. **FIGURE 2.6** shows position-versus-time graphs for Bob and Susan. Notice the negative slope for Susan's graph, indicating her negative velocity. The point of interest is the intersection of the two lines; this is where Bob and Susan have the same position at the same time. Our method of solution, in which we equated $(x_1)_B$ and $(x_1)_S$, is really just solving the mathematical problem of finding the intersection of two lines. This procedure is useful for many problems in which there are two moving objects.

STOP TO THINK 2.1 Which position-versus-time graph represents the motion shown in the motion diagram?





2.2 Instantaneous Velocity

Uniform motion is simple, but objects rarely travel for long with a constant velocity. Far more common is a velocity that changes with time. For example, **FIGURE 2.7** shows the motion diagram and position graph of a car speeding up after the light turns green. Notice how the velocity vectors increase in length, causing the graph to curve upward as the car's displacements get larger and larger.

If you were to watch the car's speedometer, you would see it increase from 0 mph to 10 mph to 20 mph and so on. At any instant of time, the speedometer tells you how fast the car is going *at that instant*. If we include directional information, we can define an object's **instantaneous velocity**—speed and direction—as its velocity at a single instant of time.

For uniform motion, the slope of the straight-line position graph is the object's velocity. **FIGURE 2.8** shows that there's a similar connection between instantaneous velocity and the slope of a curved position graph.

FIGURE 2.7 Motion diagram and position graph of a car speeding up.

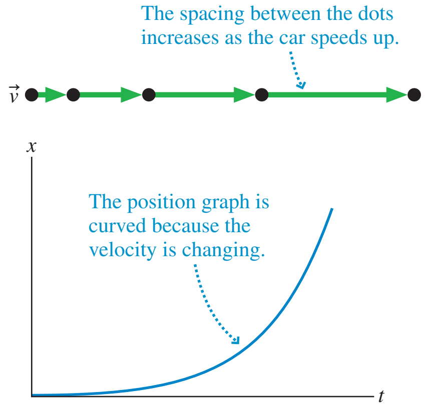
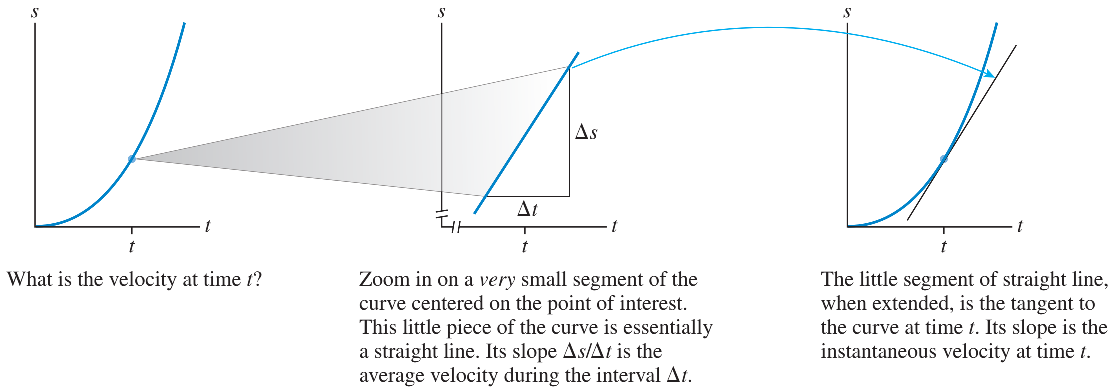


FIGURE 2.8 Instantaneous velocity at time t is the slope of the tangent to the curve at that instant.



What we see graphically is that the average velocity $v_{avg} = \Delta s/\Delta t$ becomes a better and better approximation to the instantaneous velocity v_s as the time interval Δt over which the average is taken gets smaller and smaller. We can state this idea mathematically in terms of the limit $\Delta t \rightarrow 0$:

$$v_s \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(instantaneous velocity)} \qquad (2.4)$$

As Δt continues to get smaller, the average velocity $v_{avg} = \Delta s/\Delta t$ reaches a constant or *limiting* value. That is, **the instantaneous velocity at time t is the average velocity during a time interval Δt, centered on t, as Δt approaches zero.** In calculus, this limit is called *the derivative of s with respect to t*, and it is denoted ds/dt.

Graphically, $\Delta s/\Delta t$ is the slope of a straight line. As Δt gets smaller (i.e., more and more magnification), the straight line becomes a better and better approximation of the curve *at that one point*. In the limit $\Delta t \rightarrow 0$, the straight line is tangent to the curve. As Figure 2.8 shows, **the instantaneous velocity at time t is the slope of the line that is tangent to the position-versus-time graph at time t.** That is,

$$v_s = \text{slope of the position-versus-time graph at time } t \qquad (2.5)$$

The steeper the slope, the larger the magnitude of the velocity.

EXAMPLE 2.3 ■ Finding velocity from position graphically

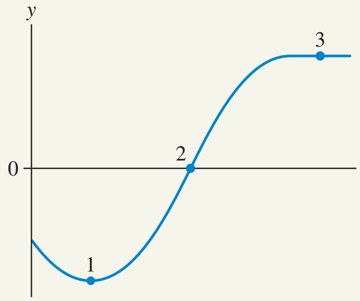
FIGURE 2.9 shows the position-versus-time graph of an elevator.

a. At which labeled point or points does the elevator have the least velocity?

b. At which point or points does the elevator have maximum velocity?

c. Sketch an approximate velocity-versus-time graph for the elevator.

FIGURE 2.9 Position-versus-time graph.



MODEL Model the elevator as a particle.

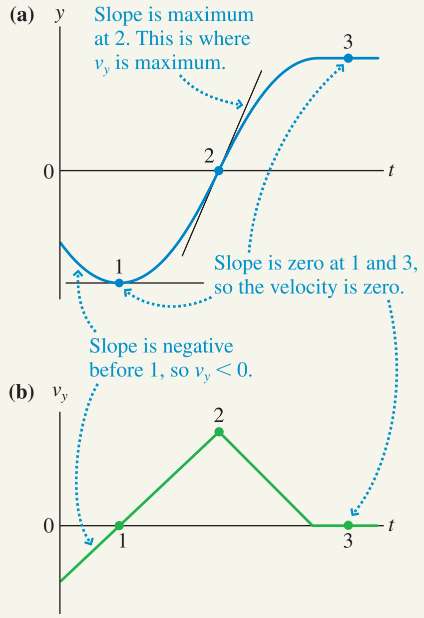
VISUALIZE Figure 2.9 is the graphical representation.

SOLVE a. At any instant, an object's velocity is the slope of its position graph. **FIGURE 2.10a** shows that the elevator has the least velocity—no velocity at all!—at points 1 and 3 where the slope is zero. At point 1, the velocity is only instantaneously zero. At point 3, the elevator has actually stopped and remains at rest.

b. The elevator has maximum velocity at 2, the point of steepest slope.

c. Although we cannot find an exact velocity-versus-time graph, we can see that the slope, and hence v_y, is initially negative, becomes zero at point 1, rises to a maximum value at point 2, decreases back to zero a little before point 3, then remains at zero thereafter.

FIGURE 2.10 The velocity-versus-time graph is found from the slope of the position graph.



Thus **FIGURE 2.10b** shows, at least approximately, the elevator's velocity-versus-time graph.

REVIEW Once again, the shape of the velocity graph bears no resemblance to the shape of the position graph. You must transfer *slope* information from the position graph to *value* information on the velocity graph.

A Little Calculus: Derivatives

Scientists and engineers must use calculus to calculate the orbits of satellites.

Calculus—invented simultaneously in England by Newton and in Germany by Leibniz—is designed to deal with instantaneous quantities. In other words, it provides us with the tools for evaluating limits such as the one in Equation 2.4.

The notation ds/dt is called *the derivative of s with respect to t*, and Equation 2.4 defines it as the limiting value of a ratio. As Figure 2.8 showed, ds/dt can be interpreted graphically as the slope of the line that is tangent to the position graph.

The most common functions we will use in Parts I and II of this book are powers and polynomials. Consider the function $u(t) = ct^n$, where c and n are constants. The symbol u is a "dummy name" to represent any function of time, such as $x(t)$ or $y(t)$. The following result is proven in calculus:

$$\text{The derivative of } u = ct^n \text{ is } \frac{du}{dt} = nct^{n-1} \tag{2.6}$$

For example, suppose the position of a particle as a function of time is $s(t) = 2t^2$ m, where t is in s. We can find the particle's velocity $v_s = ds/dt$ by using Equation 2.6 with $c = 2$ and $n = 2$ to calculate

$$v_s = \frac{ds}{dt} = 2 \cdot 2t^{2-1} = 4t$$

This is an expression for the particle's velocity as a function of time.

FIGURE 2.11 shows the particle's position and velocity graphs. It is critically important to understand the relationship between these two graphs. The *value* of the velocity graph at any instant of time, which we can read directly off the vertical axis, is the *slope* of the position graph at that same time. This is illustrated at $t = 3$ s.

A value that doesn't change with time, such as the position of an object at rest, can be represented by the function $u = c =$ constant. That is, the exponent of t^n is $n = 0$. You can see from Equation 2.6 that the derivative of a constant is zero. That is,

$$\frac{du}{dt} = 0 \text{ if } u = c = \text{constant} \tag{2.7}$$

This makes sense. The graph of the function $u = c$ is simply a horizontal line. The slope of a horizontal line—which is what the derivative du/dt measures—is zero.

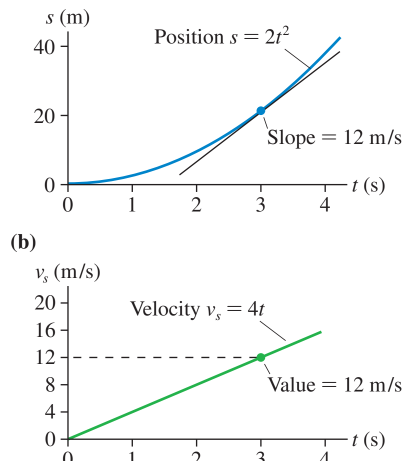
The only other information we need about derivatives for now is how to evaluate the derivative of the sum of two functions. Let u and w be two separate functions of time. You will learn in calculus that

$$\frac{d}{dt}(u + w) = \frac{du}{dt} + \frac{dw}{dt} \tag{2.8}$$

That is, the derivative of a sum is the sum of the derivatives.

NOTE You may have learned in calculus to take the derivative dy/dx, where y is a function of x. The derivatives we use in physics are the same; only the notation is different. We're interested in how quantities change with time, so our derivatives are with respect to t instead of x.

FIGURE 2.11 Position-versus-time graph and the corresponding velocity-versus-time graph.



EXAMPLE 2.4 ■ Using calculus to find the velocity

A particle's position is given by the function $x(t) = (-t^3 + 3t)$ m, where t is in s.

a. What are the particle's position and velocity at $t = 2$ s?

b. Draw graphs of x and v_x during the interval $-3\text{ s} \le t \le 3\text{ s}$.

c. Draw a motion diagram to illustrate this motion.

SOLVE a. We can compute the position directly from the function x:

$$x(\text{at } t = 2\text{ s}) = -(2)^3 + (3)(2) = -8 + 6 = -2\text{ m}$$

The velocity is $v_x = dx/dt$. The function for x is the sum of two polynomials, so

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-t^3 + 3t) = \frac{d}{dt}(-t^3) + \frac{d}{dt}(3t)$$

The first derivative is a power with $c = -1$ and $n = 3$; the second has $c = 3$ and $n = 1$. Using Equation 2.6, we have

$$v_x = (-3t^2 + 3)\text{ m/s}$$

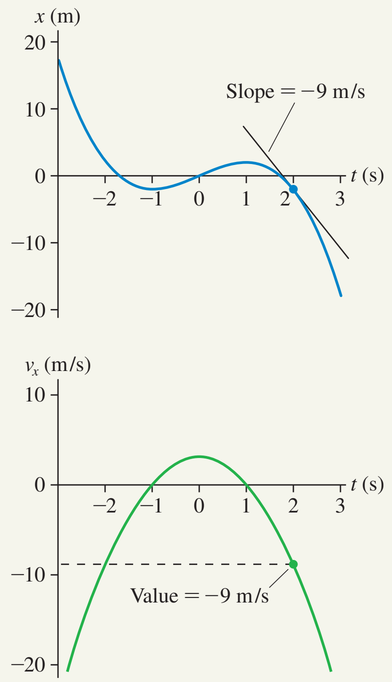
where t is in s. Evaluating the velocity at $t = 2$ s gives

$$v_x(\text{at } t = 2\text{ s}) = -3(2)^2 + 3 = -9\text{ m/s}$$

The negative sign indicates that the particle, at this instant of time, is moving to the *left* at a speed of 9 m/s.

b. **FIGURE 2.12** shows the position graph and the velocity graph. You can make graphs like these with a graphing calculator or graphing software. The slope of the position-versus-time graph at $t = 2$ s is -9 m/s; this becomes the *value* that is graphed for the velocity at $t = 2$ s.

FIGURE 2.12 Position and velocity graphs.



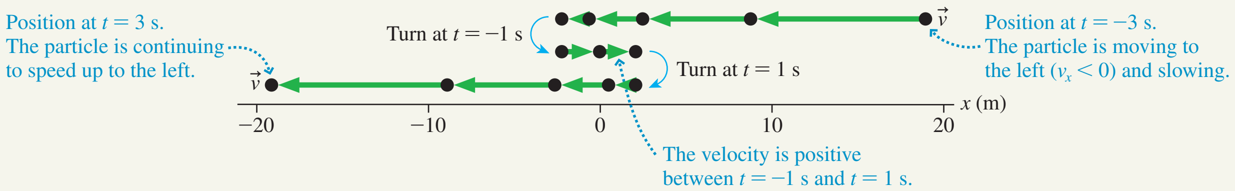
Continued

c. Finally, we can interpret the graphs in Figure 2.12 to draw the motion diagram shown in **FIGURE 2.13**.

- The particle is initially to the right of the origin ($x > 0$ at $t = -3$ s) but moving to the left ($v_x < 0$). Its *speed* is slowing ($v = |v_x|$ is decreasing), so the velocity vector arrows are getting shorter.
- The particle passes the origin $x = 0$ m at $t \approx -1.5$ s, but it is still moving to the left.
- The position reaches a minimum at $t = -1$ s; the particle is as far left as it is going. The velocity is *instantaneously* $v_x = 0$ m/s as the particle reverses direction.
- The particle moves back to the right between $t = -1$ s and $t = 1$ s ($v_x > 0$).
- The particle turns around again at $t = 1$ s and begins moving back to the left ($v_x < 0$). It keeps speeding up, then disappears off to the left.

A point in the motion where a particle reverses direction is called a **turning point.** It is a point where the velocity is instantaneously zero while the position is a maximum or minimum. This particle has two turning points, at $t = -1$ s and again at $t = +1$ s. We will see many other examples of turning points.

FIGURE 2.13 Motion diagram for Example 2.4.



STOP TO THINK 2.2 Which velocity-versus-time graph goes with the position-versus-time graph on the left?

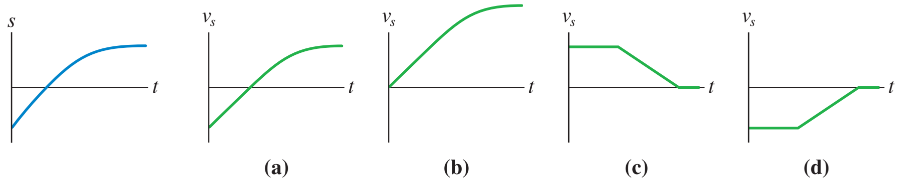
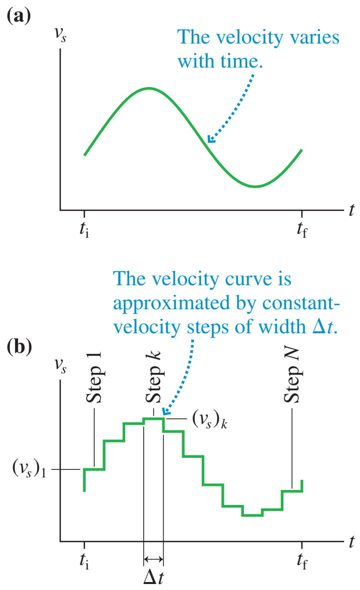


FIGURE 2.14 Approximating a velocity-versus-time graph with a series of constant-velocity steps.



2.3 Finding Position from Velocity

Equation 2.4 allows us to find the instantaneous velocity v_s if we know the position s as a function of time. But what about the reverse problem? Can we use the object's velocity to calculate its position at some future time t? Equation 2.3, $s_f = s_i + v_s \Delta t$, does this for the case of uniform motion with a constant velocity. We need to find a more general expression that is valid when v_s is not constant.

FIGURE 2.14a is a velocity-versus-time graph for an object whose velocity varies with time. Suppose we know the object's position to be s_i at an initial time t_i. Our goal is to find its final position s_f at a later time t_f.

Because we know how to handle constant velocities, using Equation 2.3, let's *approximate* the velocity function of Figure 2.14a as a series of constant-velocity steps of width Δt. This is illustrated in **FIGURE 2.14b**. During the first step, from time t_i to time $t_i + \Delta t$, the velocity has the constant value $(v_s)_1$. The velocity during step k has the constant value $(v_s)_k$. Although the approximation shown in the figure is rather rough, with only 11 steps, we can easily imagine that it could be made as accurate as desired by having more and more ever-narrower steps.

The velocity during each step is constant (uniform motion), so we can apply Equation 2.3 to each step. The object's displacement Δs_1 during the first step is simply $\Delta s_1 = (v_s)_1 \Delta t$. The displacement during the second step $\Delta s_2 = (v_s)_2 \Delta t$, and during step k the displacement is $\Delta s_k = (v_s)_k \Delta t$.

The total displacement of the object between t_i and t_f can be approximated as the sum of all the individual displacements during each of the N constant-velocity steps. That is,

$$\Delta s = s_f - s_i \approx \Delta s_1 + \Delta s_2 + \cdots + \Delta s_N = \sum_{k=1}^{N} (v_s)_k \Delta t \tag{2.9}$$

where Σ (Greek sigma) is the symbol for summation. With a simple rearrangement, the particle's final position is

$$s_f \approx s_i + \sum_{k=1}^{N} (v_s)_k \Delta t \tag{2.10}$$

Our goal was to use the object's velocity to find its final position s_f. Equation 2.10 nearly reaches that goal, but Equation 2.10 is only approximate because the constant-velocity steps are only an approximation of the true velocity graph. But if we now let $\Delta t \to 0$, each step's width approaches zero while the total number of steps N approaches infinity. In this limit, the series of steps becomes a perfect replica of the velocity-versus-time graph and Equation 2.10 becomes exact. Thus

$$s_f = s_i + \lim_{\Delta t \to 0} \sum_{k=1}^{N} (v_s)_k \Delta t = s_i + \int_{t_i}^{t_f} v_s \, dt \tag{2.11}$$

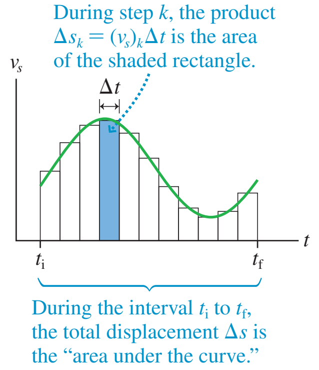
The expression on the right is read, "the integral of $v_s\,dt$ from t_i to t_f." Equation 2.11 is the result that we were seeking. It allows us to predict an object's position s_f at a future time t_f.

We can give Equation 2.11 an important geometric interpretation. **FIGURE 2.15** shows step k in the approximation of the velocity graph as a tall, thin rectangle of height $(v_s)_k$ and width Δt. The product $\Delta s_k = (v_s)_k \Delta t$ is the area (base $\times$ height) of this small rectangle. The sum in Equation 2.11 adds up all of these rectangular areas to give the total area enclosed between the t-axis and the tops of the steps. The limit of this sum as $\Delta t \to 0$ is the total area enclosed between the t-axis and the velocity curve. This is called the "area under the curve." Thus a graphical interpretation of Equation 2.11 is

$$s_f = s_i + \text{area under the velocity curve } v_s \text{ between } t_i \text{ and } t_f \tag{2.12}$$

NOTE Wait a minute! The displacement $\Delta s = s_f - s_i$ is a length. How can a length equal an area? Recall earlier, when we found that the velocity is the slope of the position graph, we made a distinction between the *actual* slope and the *physically meaningful* slope? The same distinction applies here. We need to measure the quantities we are using, v_s and Δt, by referring to the scales on the axes. Δt is some number of seconds while v_s is some number of meters per second. When these are multiplied together, the *physically meaningful* area has units of meters.

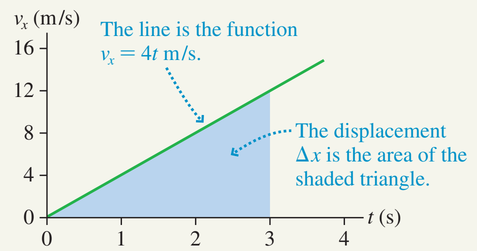
FIGURE 2.15 The total displacement Δs is the "area under the curve."



EXAMPLE 2.5 ■ The displacement during a drag race

FIGURE 2.16 shows the velocity-versus-time graph of a drag racer. How far does the racer move during the first 3.0 s?

FIGURE 2.16 Velocity-versus-time graph for Example 2.5.



MODEL Model the drag racer as a particle with a well-defined position at all times.

VISUALIZE Figure 2.16 is the graphical representation.

SOLVE The question "How far?" indicates that we need to find a displacement Δx rather than a position x. According to Equation 2.12, the car's displacement $\Delta x = x_f - x_i$ between $t = 0$ s and $t = 3$ s is the area under the curve from $t = 0$ s to $t = 3$ s. The curve in this case is an angled line, so the area is that of a triangle:

$$\Delta x = \text{area of triangle between } t = 0 \text{ s and } t = 3 \text{ s}$$
$$= \tfrac{1}{2} \times \text{base} \times \text{height}$$
$$= \tfrac{1}{2} \times 3 \text{ s} \times 12 \text{ m/s} = 18 \text{ m}$$

The drag racer moves 18 m during the first 3 seconds.

REVIEW The "area" is a product of s with m/s, so Δx has the proper units of m.

EXAMPLE 2.6 ■ Finding the turning point

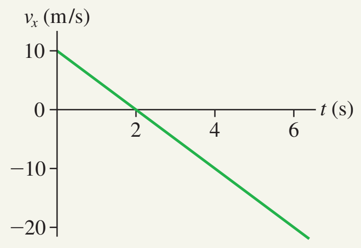
FIGURE 2.17 is the velocity graph for a particle that starts at $x_i = 30$ m at time $t_i = 0$ s.

a. Draw a motion diagram for the particle.

b. Where is the particle's turning point?

c. At what time does the particle reach the origin?

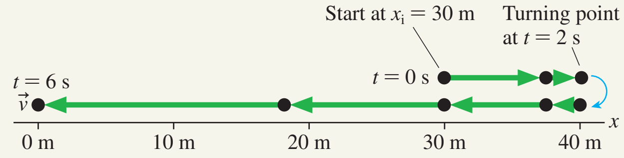
FIGURE 2.17 Velocity-versus-time graph for the particle of Example 2.6.



VISUALIZE The particle is initially 30 m to the right of the origin and moving *to the right* $(v_x > 0)$ with a speed of 10 m/s. But v_x is decreasing, so the particle is slowing down. At $t = 2$ s the velocity, just for an instant, is zero before becoming negative. This is the turning point. The velocity is negative for $t > 2$ s, so the particle has reversed direction and moves back toward the origin. At some later time, which we want to find, the particle will pass $x = 0$ m.

SOLVE a. **FIGURE 2.18** shows the motion diagram. The distance scale will be established in parts b and c but is shown here for convenience.

FIGURE 2.18 Motion diagram for the particle whose velocity graph was shown in Figure 2.17.



b. The particle reaches the turning point at $t = 2$ s. To learn *where* it is at that time we need to find the displacement during the first two seconds. We can do this by finding the area under the curve between $t = 0$ s and $t = 2$ s:

$$\begin{aligned} x(\text{at } t = 2\text{ s}) &= x_i + \text{area under the curve between 0 s and 2 s} \\ &= 30\text{ m} + \tfrac{1}{2}(2\text{ s} - 0\text{ s})(10\text{ m/s} - 0\text{ m/s}) \\ &= 40\text{ m} \end{aligned}$$

The turning point is at $x = 40$ m.

c. The particle needs to move $\Delta x = -40$ m to get from the turning point to the origin. That is, the area under the curve from $t = 2$ s to the desired time t needs to be -40 m. Because the curve is below the axis, with negative values of v_x, the area to the right of $t = 2$ s is a *negative* area. With a bit of geometry, you will find that the triangle with a base extending from $t = 2$ s to $t = 6$ s has an area of -40 m. Thus the particle reaches the origin at $t = 6$ s.

A Little More Calculus: Integrals

Taking the derivative of a function is equivalent to finding the slope of a graph of the function. Similarly, evaluating an integral is equivalent to finding the area under a graph of the function. The graphical method is very important for building intuition about motion but is limited in its practical application. Just as derivatives of standard functions can be evaluated and tabulated, so can integrals.

The integral in Equation 2.11 is called a *definite integral* because there are two definite boundaries to the area we want to find. These boundaries are called the lower (t_i) and upper (t_f) *limits of integration*. For the important function $u(t) = ct^n$, the essential result from calculus is that

$$\int_{t_i}^{t_f} u\,dt = \int_{t_i}^{t_f} ct^n\,dt = \frac{ct^{n+1}}{n+1}\Bigg|_{t_i}^{t_f} = \frac{ct_f^{n+1}}{n+1} - \frac{ct_i^{n+1}}{n+1} \qquad (n \neq -1) \tag{2.13}$$

The vertical bar in the third step with subscript t_i and superscript t_f is a shorthand notation from calculus that means—as seen in the last step—the integral evaluated at the upper limit t_f *minus* the integral evaluated at the lower limit t_i. You also need to know that for two functions u and w,

$$\int_{t_i}^{t_f} (u + w)\,dt = \int_{t_i}^{t_f} u\,dt + \int_{t_i}^{t_f} w\,dt \tag{2.14}$$

That is, the integral of a sum is equal to the sum of the integrals.

EXAMPLE 2.7 ■ Using calculus to find the position

Use calculus to solve Example 2.6.

SOLVE Figure 2.17 is a linear graph. Its "y-intercept" is seen to be 10 m/s and its slope is -5 (m/s)/s. Thus the velocity can be described by the equation

$$v_x = (10 - 5t) \text{ m/s}$$

where t is in s. We can find the position x at time t by using Equation 2.11:

$$x = x_i + \int_0^t v_x\,dt = 30 \text{ m} + \int_0^t (10 - 5t)\,dt$$
$$= 30 \text{ m} + \int_0^t 10\,dt - \int_0^t 5t\,dt$$

We used Equation 2.14 for the integral of a sum to get the final expression. The first integral is a function of the form $u = ct^n$ with $c = 10$ and $n = 0$; the second is of the form $u = ct^n$ with $c = 5$ and $n = 1$. Using Equation 2.13, we have

$$\int_0^t 10\,dt = 10t\Big|_0^t = 10 \cdot t - 10 \cdot 0 = 10t \text{ m}$$

and
$$\int_0^t 5t\,dt = \tfrac{5}{2}t^2\Big|_0^t = \tfrac{5}{2} \cdot t^2 - \tfrac{5}{2} \cdot 0^2 = \tfrac{5}{2}t^2 \text{ m}$$

Combining the pieces gives

$$x = \left(30 + 10t - \tfrac{5}{2}t^2\right) \text{ m}$$

This is a general result for the position at *any* time t.

The particle's turning point occurs at $t = 2$ s, and its position at that time is

$$x(\text{at } t = 2 \text{ s}) = 30 + (10)(2) - \tfrac{5}{2}(2)^2 = 40 \text{ m}$$

The time at which the particle reaches the origin is found by setting $x = 0$ m:

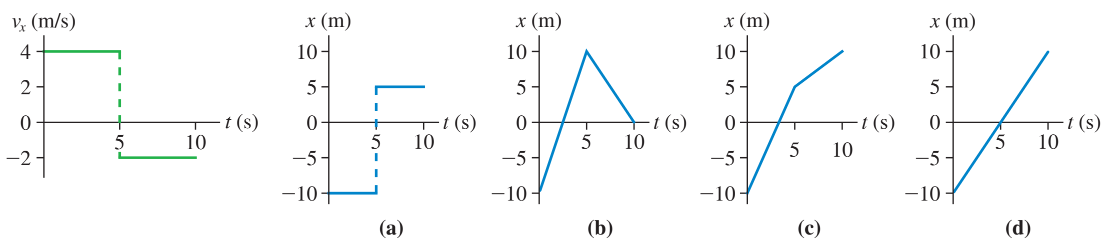
$$30 + 10t - \tfrac{5}{2}t^2 = 0$$

This quadratic equation has two solutions: $t = -2$ s or $t = 6$ s.

When we solve a quadratic equation, we cannot just arbitrarily select the root we want. Instead, we must decide which is the *meaningful* root. Here the negative root refers to a time before the problem began, so the meaningful one is the positive root, $t = 6$ s.

REVIEW The results agree with the answers we found previously from a graphical solution.

STOP TO THINK 2.3 Which position-versus-time graph goes with the velocity-versus-time graph on the left? The particle's position at $t_i = 0$ s is $x_i = -10$ m.



2.4 Motion with Constant Acceleration

We need one more major concept to describe one-dimensional motion: acceleration. Acceleration, as we noted in Chapter 1, is a rather abstract concept. Nonetheless, acceleration is the linchpin of mechanics. We will see very shortly that Newton's laws relate the acceleration of an object to the forces that are exerted on it.

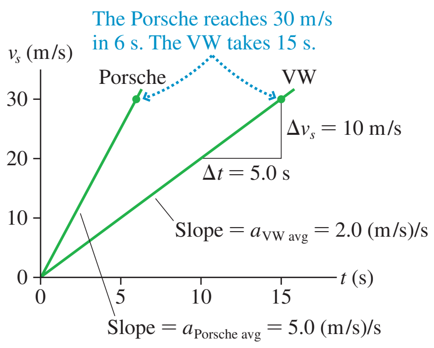
Let's conduct a race between a Volkswagen Beetle and a Porsche to see which can reach a speed of 30 m/s ($\approx$60 mph) in the shortest time. Both cars are equipped with computers that will record the speedometer reading 10 times each second. This gives a nearly continuous record of the *instantaneous* velocity of each car. **TABLE 2.1** shows some of the data. The velocity-versus-time graphs, based on these data, are shown in **FIGURE 2.19** on the next page.

How can we describe the difference in performance of the two cars? It is not that one has a different velocity from the other; both achieve every velocity between 0 and 30 m/s. The distinction is how long it took each to *change* its velocity from 0 to 30 m/s. The Porsche changed velocity quickly, in 6.0 s, while the VW needed 15 s to make

TABLE 2.1 Velocities of a Porsche and a Volkswagen Beetle

t (s)	v_{Porsche} (m/s)	v_{VW} (m/s)
0.0	0.0	0.0
0.1	0.5	0.2
0.2	1.0	0.4
0.3	1.5	0.6
⋮	⋮	⋮

FIGURE 2.19 Velocity-versus-time graphs for the Porsche and the VW Beetle.



the same velocity change. Because the Porsche had a velocity change $\Delta v_s = 30$ m/s during a time interval $\Delta t = 6.0$ s, the *rate* at which its velocity changed was

$$\text{rate of velocity change} = \frac{\Delta v_s}{\Delta t} = \frac{30\text{ m/s}}{6.0\text{ s}} = 5.0\text{ (m/s)/s} \tag{2.15}$$

Notice the units. They are units of "velocity per second." A rate of velocity change of 5.0 "meters per second per second" means that the velocity increases by 5.0 m/s during the first second, by another 5.0 m/s during the next second, and so on. In fact, the velocity will increase by 5.0 m/s during any second in which it is changing at the rate of 5.0 (m/s)/s.

Chapter 1 introduced *acceleration* as "the rate of change of velocity." That is, acceleration measures how quickly or slowly an object's velocity changes. In parallel with our treatment of velocity, let's define the **average acceleration** a_{avg} during the time interval Δt to be

$$a_{\text{avg}} \equiv \frac{\Delta v_s}{\Delta t} \quad \text{(average acceleration)} \tag{2.16}$$

Equations 2.15 and 2.16 show that the Porsche had the rather large acceleration of 5.0 (m/s)/s.

Because Δv_s and Δt are the "rise" and "run" of a velocity-versus-time graph, we see that a_{avg} can be interpreted graphically as the *slope* of a straight-line velocity-versus-time graph. In other words,

$$a_{\text{avg}} = \text{slope of the velocity-versus-time graph} \tag{2.17}$$

Figure 2.19 uses this idea to show that the VW's average acceleration is

$$a_{\text{VW avg}} = \frac{\Delta v_s}{\Delta t} = \frac{10\text{ m/s}}{5.0\text{ s}} = 2.0\text{ (m/s)/s}$$

This is less than the acceleration of the Porsche, as expected.

An object whose velocity-versus-time graph is a straight-line graph has a steady and unchanging acceleration. There's no need to specify "average" if the acceleration is constant, so we'll use the symbol a_s as we discuss motion along the s-axis with constant acceleration.

Signs and Units

An important aspect of acceleration is its *sign*. Acceleration $\vec{a}$, like position $\vec{r}$ and velocity $\vec{v}$, is a vector. For motion in one dimension, the sign of a_x (or a_y) is positive if the vector $\vec{a}$ points to the right (or up), negative if it points to the left (or down). This was illustrated in « **FIGURE 1.18** and the very important « **TACTICS BOX 1.3**, which you may wish to review. It's particularly important to emphasize that **positive and negative values of a_s do *not* correspond to "speeding up" and "slowing down."**

EXAMPLE 2.8 ■ Relating acceleration to velocity

a. A bicyclist has a velocity of 6 m/s and a constant acceleration of 2 (m/s)/s. What is her velocity 1 s later? 2 s later?

b. A bicyclist has a velocity of −6 m/s and a constant acceleration of 2 (m/s)/s. What is his velocity 1 s later? 2 s later?

SOLVE

a. An acceleration of 2 (m/s)/s *means* that the velocity increases by 2 m/s every 1 s. If the bicyclist's initial velocity is 6 m/s, then 1 s later her velocity will be 8 m/s. After 2 s, which is 1 additional second later, it will increase by another 2 m/s to 10 m/s. After 3 s it will be 12 m/s. Here a positive a_x is causing the bicyclist to speed up.

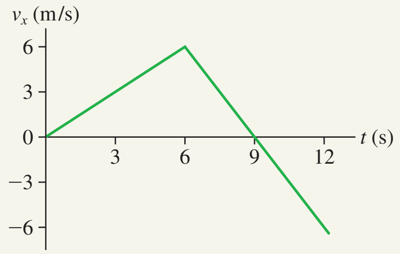
b. If the bicyclist's initial velocity is a *negative* −6 m/s but the acceleration is a positive +2 (m/s)/s, then 1 s later his velocity will be −4 m/s. After 2 s it will be −2 m/s, and so on. In this case, a positive a_x is causing the object to *slow down* (decreasing speed v). This agrees with the rule from Tactics Box 1.3: An object is slowing down if and only if v_x and a_x have opposite signs.

NOTE It is customary to abbreviate the acceleration units (m/s)/s as m/s^2. For example, the bicyclists in Example 2.8 had an acceleration of 2 m/s^2. We will use this notation, but keep in mind the *meaning* of the notation as "(meters per second) per second."

EXAMPLE 2.9 ■ Running the court

A basketball player starts at the left end of the court and moves with the velocity shown in FIGURE 2.20. Draw a motion diagram and an acceleration-versus-time graph for the basketball player.

FIGURE 2.20 Velocity-versus-time graph for the basketball player of Example 2.9.



VISUALIZE The velocity is positive (motion to the right) and increasing for the first 6 s, so the velocity arrows in the motion diagram are to the right and getting longer. From $t = 6$ s to 9 s the motion is still to the right (v_x is still positive), but the arrows are getting shorter because v_x is decreasing. There's a turning point at $t = 9$ s, when $v_x = 0$ m/s, and after that the motion is to the left (v_x is negative) and getting faster. The motion diagram of FIGURE 2.21a shows the velocity and the acceleration vectors.

SOLVE Acceleration is the slope of the velocity graph. For the first 6 s, the slope has the constant value

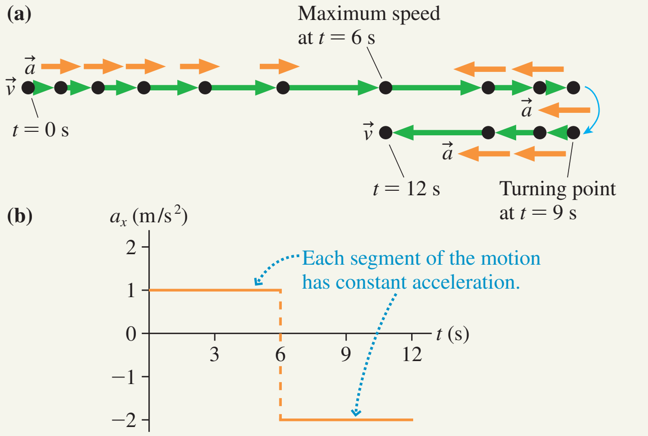
$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{6.0 \text{ m/s}}{6.0 \text{ s}} = 1.0 \text{ m/s}^2$$

The velocity then decreases by 12 m/s during the 6 s interval from $t = 6$ s to $t = 12$ s, so

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{-12 \text{ m/s}}{6.0 \text{ s}} = -2.0 \text{ m/s}^2$$

The acceleration graph for these 12 s is shown in FIGURE 2.21b. Notice that there is no change in the acceleration at $t = 9$ s, the turning point.

FIGURE 2.21 Motion diagram and acceleration graph for Example 2.9.



REVIEW The *sign* of a_x does *not* tell us whether the object is speeding up or slowing down. The basketball player is slowing down from $t = 6$ s to $t = 9$ s, then speeding up from $t = 9$ s to $t = 12$ s. Nonetheless, his acceleration is negative during this entire interval because his acceleration vector, as seen in the motion diagram, always points to the left.

The Kinematic Equations of Constant Acceleration

Consider an object whose acceleration a_s remains constant during the time interval $\Delta t = t_f - t_i$. At the beginning of this interval, at time t_i, the object has initial velocity v_{is} and initial position s_i. Note that t_i is often zero, but it does not have to be. We would like to predict the object's final position s_f and final velocity v_{fs} at time t_f.

The object's velocity is changing because the object is accelerating. FIGURE 2.22a shows the acceleration-versus-time graph, a horizontal line between t_i and t_f. It is not hard to find the object's velocity v_{fs} at a later time t_f. By definition,

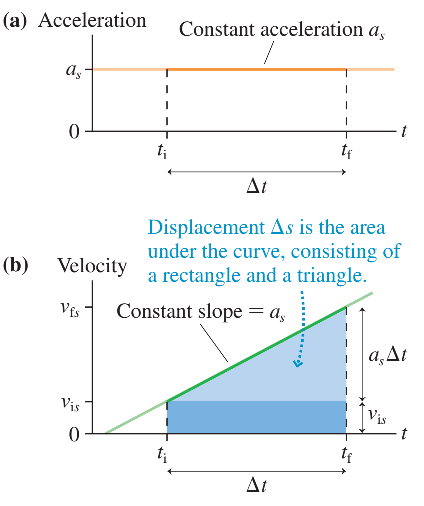
$$a_s = \frac{\Delta v_s}{\Delta t} = \frac{v_{fs} - v_{is}}{\Delta t} \tag{2.18}$$

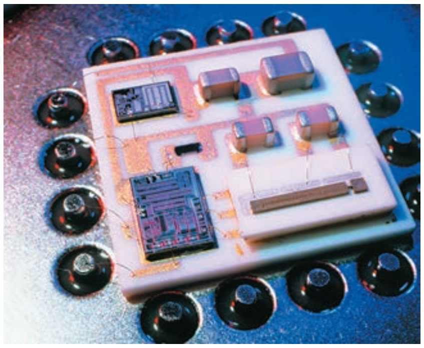
which is easily rearranged to give

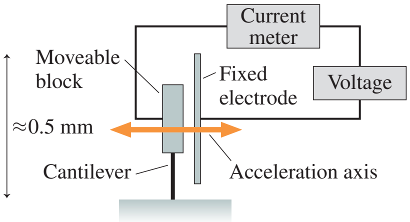
$$v_{fs} = v_{is} + a_s \Delta t \tag{2.19}$$

The velocity-versus-time graph, shown in FIGURE 2.22b, is a straight line that starts at v_{is} and has slope a_s.

FIGURE 2.22 Acceleration and velocity graphs for constant acceleration.







Your phone contains a miniature *accelerometer*, smaller than a millimeter, built into an integrated-circuit chip like the one shown here. These little accelerometers—the long bar on the lower right—have a tiny block of metal attached to a thin cantilever that acts like a spring. The block and a nearby electrode form what's called a *capacitor*, an electronic device you'll study in Chapter 26. Acceleration causes the block to sway slightly toward or away from the electrode, thus changing a current that is continuously monitored and used to infer the acceleration along that axis. Miniature accelerometers are used in navigation systems, robotics, medical devices, and even the activity tracker you wear while exercising. Most devices have three independent sensors, one for each axis. A continuous record of acceleration can be numerically integrated to determine velocity and position changes.

As you learned in the last section, the object's final position is

$$s_f = s_i + \text{area under the velocity curve } v_s \text{ between } t_i \text{ and } t_f \tag{2.20}$$

The shaded area in Figure 2.22b can be subdivided into a rectangle of area $v_{is}\,\Delta t$ and a triangle of area $\frac{1}{2}(a_s\,\Delta t)(\Delta t) = \frac{1}{2}a_s(\Delta t)^2$. Adding these gives

$$s_f = s_i + v_{is}\,\Delta t + \tfrac{1}{2}a_s(\Delta t)^2 \tag{2.21}$$

where $\Delta t = t_f - t_i$ is the elapsed time. The quadratic dependence on Δt causes the position-versus-time graph for constant-acceleration motion to have a parabolic shape, as shown in Model 2.2.

Equations 2.19 and 2.21 are two of the basic kinematic equations for motion with *constant* acceleration. They allow us to predict an object's position and velocity at a future instant of time. We need one more equation to complete our set, a direct relation between position and velocity. First use Equation 2.19 to write $\Delta t = (v_{fs} - v_{is})/a_s$. Substitute this into Equation 2.21, giving

$$s_f = s_i + v_{is}\left(\frac{v_{fs} - v_{is}}{a_s}\right) + \tfrac{1}{2}a_s\left(\frac{v_{fs} - v_{is}}{a_s}\right)^2 \tag{2.22}$$

With a bit of algebra, this is rearranged to read

$$v_{fs}^{\,2} = v_{is}^{\,2} + 2a_s\,\Delta s \tag{2.23}$$

where $\Delta s = s_f - s_i$ is the *displacement* (not the distance!). Equation 2.23 is the last of the three kinematic equations for motion with constant acceleration.

The Constant-Acceleration Model

Few objects with changing velocity have a perfectly constant acceleration, but it is often reasonable to model their acceleration as being constant. We do so by utilizing the **constant-acceleration model.** Once again, a model is a set of words, pictures, graphs, and equations that allows us to explain and predict an object's motion.

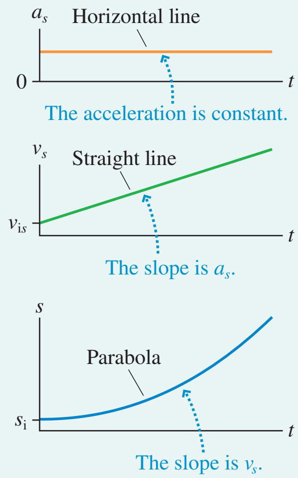
MODEL 2.2

Constant acceleration

For motion with constant acceleration.

- Model the object as a particle moving in a straight line with constant acceleration.

- Mathematically:
  - $v_{fs} = v_{is} + a_s\,\Delta t$
  - $s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$
  - $v_{fs}^{\,2} = v_{is}^{\,2} + 2a_s\,\Delta s$
- Limitations: Model fails if the particle's acceleration changes.



Exercise 16

In this text, we'll usually model runners, cars, planes, and rockets as having constant acceleration. Their actual acceleration is often more complicated (for example, a car's acceleration gradually decreases rather than remaining constant until full speed is reached), but the mathematical complexity of dealing with realistic accelerations would detract from the physics we're trying to learn.

The constant-acceleration model is the basis for a problem-solving strategy.

PROBLEM-SOLVING STRATEGY 2.1

Kinematics with constant acceleration

MODEL Model the object as having constant acceleration.

VISUALIZE Use different representations of the information in the problem.

- Draw a *pictorial representation*. This helps you assess the information you are given and starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these two representations as needed.

SOLVE The mathematical representation is based on the three kinematic equations:

$$v_{fs} = v_{is} + a_s \Delta t$$
$$s_f = s_i + v_{is}\,\Delta t + \tfrac{1}{2} a_s (\Delta t)^2$$
$$v_{fs}^{\;2} = v_{is}^{\;2} + 2a_s\,\Delta s$$

- Use x or y, as appropriate to the problem, rather than the generic s.
- Replace i and f with numerical subscripts defined in the pictorial representation.

REVIEW Check that your result has the correct units and significant figures, is reasonable, and answers the question.

NOTE You are strongly encouraged to solve problems on the Dynamics Worksheets found at the back of the Student Workbook. These worksheets will help you use the Problem-Solving Strategy and develop good problem-solving skills.

EXAMPLE 2.10 ■ The motion of a rocket sled

A rocket sled's engines fire for 5.0 s, boosting the sled to a speed of 250 m/s. The sled then deploys a braking parachute, slowing by 3.0 m/s per second until it stops. What is the total distance traveled?

MODEL We're not given the sled's initial acceleration, while the rockets are firing, but rocket sleds are aerodynamically shaped to minimize air resistance and so it seems reasonable to model the sled as a particle undergoing constant acceleration.

VISUALIZE FIGURE 2.23 shows the pictorial representation. We've made the reasonable assumptions that the sled starts from rest and that the braking parachute is deployed just as the rocket burn ends. There are three points of interest in this problem: the start, the change from propulsion to braking, and the stop. Each of these points has been assigned a position, velocity, and time. Notice that we've replaced the generic subscripts i and f of the kinematic equations with the numerical subscripts 0, 1, and 2. Accelerations are associated not with specific points in the motion but with the intervals between the points, so acceleration a_{0x} is the acceleration between points 0 and 1 while acceleration a_{1x} is the acceleration between points 1 and 2. The acceleration vector $\vec{a}_1$ points to the left, so a_{1x} is negative. The sled stops at the end point, so $v_{2x} = 0$ m/s.

SOLVE We know how long the rocket burn lasts and the velocity at the end of the burn. Because we're modeling the sled as having uniform acceleration, we can use the first kinematic equation of Problem-Solving Strategy 2.1 to write

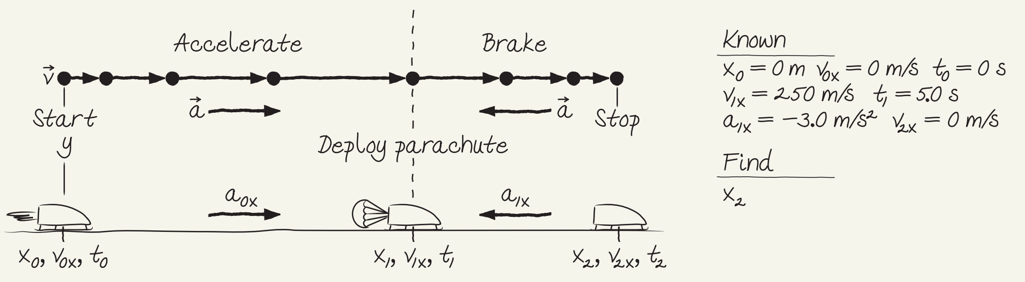
$$v_{1x} = v_{0x} + a_{0x}(t_1 - t_0) = a_{0x}\,t_1$$

We started with the complete equation, then simplified by noting which terms were zero. Solving for the boost-phase acceleration, we have

$$a_{0x} = \frac{v_{1x}}{t_1} = \frac{250\text{ m/s}}{5.0\text{ s}} = 50\text{ m/s}^2$$

Notice that we worked algebraically until the last step—a hallmark of good problem-solving technique that minimizes the chances of

FIGURE 2.23 Pictorial representation of the rocket sled.



Continued

calculation errors. Also, in accord with the significant figure rules of Chapter 1, 50 m/s^2 is considered to have two significant figures.

Now we have enough information to find out how far the sled travels while the rockets are firing. The second kinematic equation of Problem-Solving Strategy 2.1 is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_{0x}(t_1 - t_0)^2 = \tfrac{1}{2}a_{0x}t_1^2$$
$$= \tfrac{1}{2}(50\text{ m/s}^2)(5.0\text{ s})^2 = 625\text{ m}$$

The braking phase is a little different because we don't know how long it lasts. But we do know both the initial and final velocities, so we can use the third kinematic equation of Problem-Solving Strategy 2.1:

$$v_{2x}{}^2 = v_{1x}{}^2 + 2a_{1x}\Delta x = v_{1x}{}^2 + 2a_{1x}(x_2 - x_1)$$

Notice that Δx is *not* x_2; it's the displacement $(x_2 - x_1)$ during the braking phase. We can now solve for x_2:

$$x_2 = x_1 + \frac{v_{2x}{}^2 - v_{1x}{}^2}{2a_{1x}}$$
$$= 625\text{ m} + \frac{0 - (250\text{ m/s})^2}{2(-3.0\text{ m/s}^2)} = 11{,}000\text{ m}$$

We kept three significant figures for x_1 at an intermediate stage of the calculation but rounded to two significant figures at the end.

REVIEW The total distance is 11 km $\approx$ 7 mi. That's large but believable. Using the approximate conversion factor 1 m/s $\approx$ 2 mph from Table 1.5, we see that the top speed is $\approx$ 500 mph. It will take a long distance for the sled to gradually stop from such a high speed.

EXAMPLE 2.11 ■ A two-car race

Fred is driving his Volkswagen Beetle at a steady 20 m/s when he passes Betty sitting at rest in her Porsche. Betty instantly begins accelerating at 5.0 m/s^2. How far does Betty have to drive to overtake Fred?

MODEL Model the VW as a particle in uniform motion and the Porsche as a particle with constant acceleration.

VISUALIZE FIGURE 2.24 is the pictorial representation. Fred's motion diagram is one of uniform motion, while Betty's shows uniform acceleration. Fred is ahead in frames 1, 2, and 3, but Betty catches up with him in frame 4. The coordinate system shows the cars with the same position at the start and at the end—but with the important difference that Betty's Porsche has an acceleration while Fred's VW does not.

SOLVE This problem is similar to Example 2.2, in which Bob and Susan met for lunch. As we did there, we want to find Betty's position $(x_1)_B$ at the instant t_1 when $(x_1)_B = (x_1)_F$. We know, from the models of uniform motion and uniform acceleration, that Fred's position graph is a straight line but Betty's is a parabola. The position graphs in Figure 2.24 show that we're solving for the intersection point of the line and the parabola.

Fred's and Betty's positions at t_1 are

$$(x_1)_F = (x_0)_F + (v_{0x})_F(t_1 - t_0) = (v_{0x})_F t_1$$
$$(x_1)_B = (x_0)_B + (v_{0x})_B(t_1 - t_0) + \tfrac{1}{2}(a_{0x})_B(t_1 - t_0)^2 = \tfrac{1}{2}(a_{0x})_B t_1^2$$

By equating these,

$$(v_{0x})_F t_1 = \tfrac{1}{2}(a_{0x})_B t_1^2$$

we can solve for the time when Betty passes Fred:

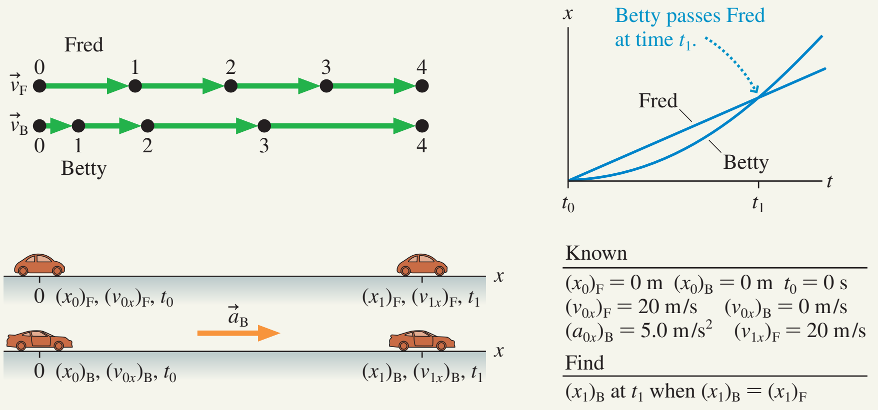
$$t_1\left[\tfrac{1}{2}(a_{0x})_B t_1 - (v_{0x})_F\right] = 0$$
$$t_1 = \begin{cases} 0\text{ s} \\ 2(v_{0x})_F/(a_{0x})_B = 8.0\text{ s} \end{cases}$$

Interestingly, there are two solutions. That's not surprising, when you think about it, because the line and the parabola of the position graphs have *two* intersection points: when Fred first passes Betty, and 8.0 s later when Betty passes Fred. We're interested in only the second of these points. We can now use either of the distance equations to find $(x_1)_B = (x_1)_F = 160$ m. Betty has to drive 160 m to overtake Fred.

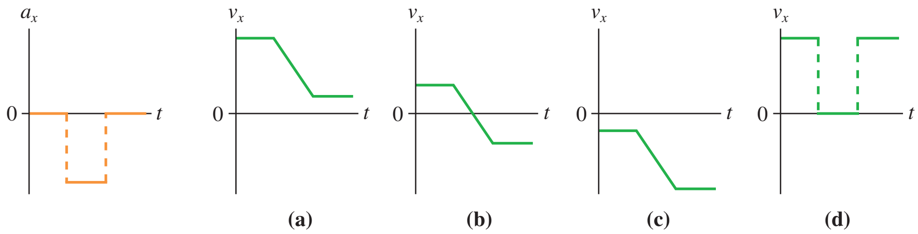
REVIEW 160 m $\approx$ 160 yards. Because Betty starts from rest while Fred is moving at 20 m/s $\approx$ 40 mph, needing 160 yards to catch him seems reasonable.

NOTE The purpose of the Review step is not to prove that an answer must be right but to rule out answers that, with a little thought, are clearly wrong.

FIGURE 2.24 Pictorial representation for Example 2.11.



STOP TO THINK 2.4 Which velocity-versus-time graph or graphs go with the acceleration-versus-time graph on the left? The particle is initially moving to the right.

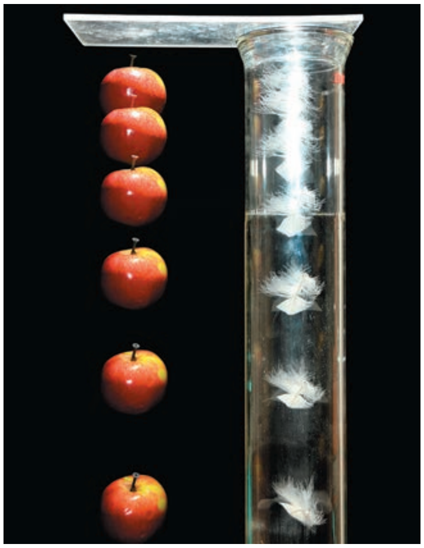


2.5 Free Fall

The motion of an object moving under the influence of gravity only, and no other forces, is called **free fall.** Strictly speaking, free fall occurs only in a vacuum, where there is no air resistance. Fortunately, the effect of air resistance is small for "heavy objects," so we'll make only a very slight error in treating these objects *as if* they were in free fall. For very light objects, such as a feather, or for objects that fall through very large distances and gain very high speeds, the effect of air resistance is *not* negligible. Motion with air resistance is a problem we will study in Chapter 6. Until then, we will restrict our attention to "heavy objects" and will make the reasonable assumption that falling objects are in free fall.

Galileo, in the 17th century, was the first to make detailed measurements of falling objects. The story of Galileo dropping different weights from the leaning bell tower at the cathedral in Pisa is well known, although historians cannot confirm its truth. Based on his measurements, wherever they took place, Galileo developed a *model* for motion in the absence of air resistance:

- Two objects dropped from the same height will, if air resistance can be neglected, hit the ground at the same time and with the same speed.
- Consequently, **any two objects in free fall, regardless of their mass, have the same acceleration $\vec{a}_{\text{free fall}}$.**



In a vacuum, the apple and feather fall at the same rate and hit the ground at the same time.

FIGURE 2.25a shows the motion diagram of an object that was released from rest and falls freely. FIGURE 2.25b shows the object's velocity graph. The motion diagram and graph are identical for a falling pebble and a falling boulder. The fact that the velocity graph is a straight line tells us the motion is one of constant acceleration, and $a_{\text{free fall}}$ is found from the slope of the graph. Careful measurements show that the value of $\vec{a}_{\text{free fall}}$ varies ever so slightly at different places on the earth, due to the slightly nonspherical shape of the earth and to the fact that the earth is rotating. A global average, at sea level, is

$$\vec{a}_{\text{free fall}} = (9.80 \text{ m/s}^2, \text{ vertically downward}) \tag{2.24}$$

Vertically downward means along a line toward the center of the earth.

The length, or magnitude, of $\vec{a}_{\text{free fall}}$ is known as the **free-fall acceleration,** and it has the special symbol g:

$$g = 9.80 \text{ m/s}^2 \text{ (free-fall acceleration)}$$

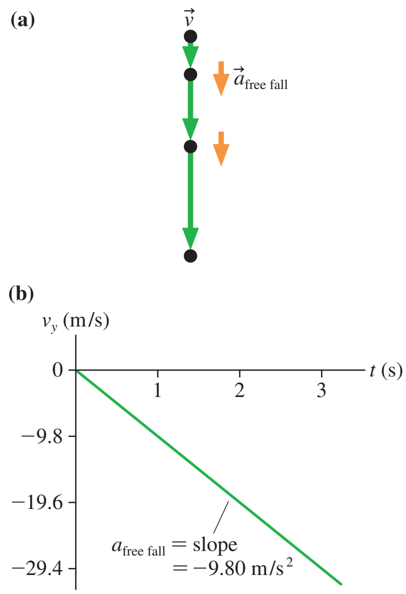
Several points about free fall are worthy of note:

- g, by definition, is *always* positive. **There will never be a problem that will use a negative value for g.** But, you say, objects fall when you release them rather than rise, so how can g be positive?
- g is *not* the acceleration $a_{\text{free fall}}$, but simply its magnitude. Because we've chosen the y-axis to point vertically upward, the downward acceleration vector $\vec{a}_{\text{free fall}}$ has the vertical component

$$a_y = a_{\text{free fall}} = -g \tag{2.25}$$

It is a_y that is negative, not g.

FIGURE 2.25 Motion of an object in free fall.



- We can model free fall as motion with constant acceleration, with $a_y = -g$.
- g is not called "gravity." Gravity is a force, not an acceleration. The symbol g recognizes the influence of gravity, but g is *the free-fall acceleration.* You may also see g called *the acceleration due to gravity*.
- $g = 9.80\ \text{m/s}^2$ only on earth. Other planets have different values of g. You will learn in Chapter 13 how to determine g for other planets.

NOTE Despite the name, free fall is not restricted to objects that are literally falling. Any object moving under the influence of gravity only, and no other forces, is in free fall. This includes objects falling straight down, objects that have been tossed or shot straight up, and projectile motion.

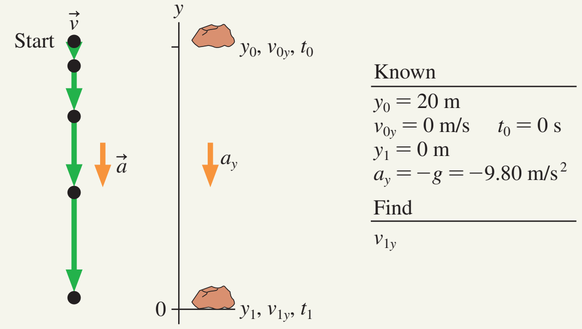
EXAMPLE 2.12 ■ A falling rock

A rock is dropped from the top of a 20-m-tall building. What is its impact velocity?

MODEL A rock is fairly heavy, and air resistance is probably not a serious concern in a fall of only 20 m. It seems reasonable to model the rock's motion as free fall: constant acceleration with $a_y = a_{\text{free fall}} = -g$.

VISUALIZE FIGURE 2.26 shows the pictorial representation. We have placed the origin at the ground, which makes $y_0 = 20\ \text{m}$. Although the rock falls 20 m, it is important to notice that the *displacement* is $\Delta y = y_1 - y_0 = -20\ \text{m}$.

FIGURE 2.26 Pictorial representation of a falling rock.



SOLVE In this problem we know the displacement but not the time, which suggests that we use the third kinematic equation from Problem-Solving Strategy 2.1:

$$v_{1y}^2 = v_{0y}^2 + 2a_y\,\Delta y = -2g\,\Delta y$$

We started by writing the general equation, then noted that $v_{0y} = 0\ \text{m/s}$ and substituted $a_y = -g$. Solving for v_{1y}:

$$v_{1y} = \sqrt{-2g\Delta y} = \sqrt{-2(9.8\ \text{m/s}^2)(-20\ \text{m})} = \pm 20\ \text{m/s}$$

A common error would be to say, "The rock fell 20 m, so $\Delta y = 20\ \text{m}$." That would have you trying to take the square root of a negative number. As noted above, Δy is a *displacement,* not a distance, and in this case $\Delta y = -20\ \text{m}$.

The $\pm$ sign indicates that there are two mathematical solutions; therefore, we have to use physical reasoning to choose between them. The rock does hit with a *speed* of 20 m/s, but the question asks for the impact *velocity*. The velocity vector points down, so the sign of v_{1y} is negative. Thus the impact velocity is $-20\ \text{m/s}$.

REVIEW Is the answer reasonable? Well, 20 m is about 60 feet, or about the height of a five- or six-story building. Using $1\ \text{m/s} \approx 2\ \text{mph}$, we see that $20\ \text{m/s} \approx 40\ \text{mph}$. That seems quite reasonable for the speed of an object after falling five or six stories. If we had misplaced a decimal point, though, and found 2.0 m/s, we would be suspicious that this was much too small after converting it to $\approx 4\ \text{mph}$.

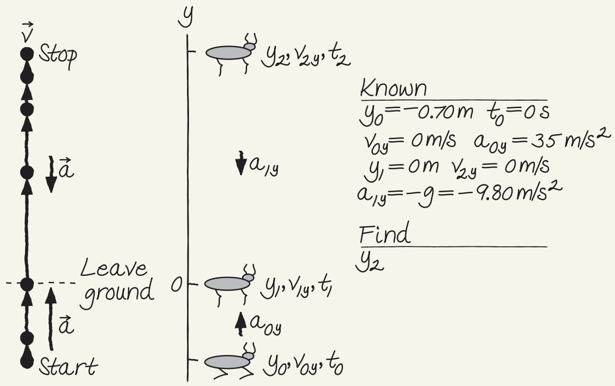
EXAMPLE 2.13 ■ Finding the height of a leap

The springbok, an antelope found in Africa, gets its name from its remarkable jumping ability. When startled, a springbok will leap straight up into the air—a maneuver called a "pronk." A springbok goes into a crouch to perform a pronk. It then extends its legs forcefully, accelerating at $35\ \text{m/s}^2$ for 0.70 m as its legs straighten. Legs fully extended, it leaves the ground and rises into the air. How high does it go?

MODEL The springbok is changing shape as it leaps, so can we reasonably model it as a particle? We can if we focus on the *body* of the springbok, treating the expanding legs like external springs. Initially, the body of the springbok is driven upward by its legs. We'll model this as a particle—the body—undergoing constant acceleration. Once the springbok's feet leave the ground, we'll model the motion of the springbok's body as a particle in free fall.

VISUALIZE **FIGURE 2.27** shows the pictorial representation. This is a problem with a beginning point, an end point, and a point in between where the nature of the motion changes. We've identified these points with subscripts 0, 1, and 2. The motion from 0 to 1 is a rapid upward acceleration until the springbok's feet leave the ground at 1. Even though the springbok is moving upward from 1 to 2, this is free-fall motion because the springbok is now moving under the influence of gravity only.

FIGURE 2.27 Pictorial representation of a startled springbok.



How do we put "How high?" into symbols? The clue is that the very top point of the trajectory is a *turning point,* and we've seen that the instantaneous velocity at a turning point is $v_{2y} = 0$. This was not explicitly stated but is part of our interpretation of the problem.

SOLVE For the first part of the motion, pushing off, we know a displacement but not a time interval. We can use

$$v_{1y}{}^2 = v_{0y}{}^2 + 2a_{0y}\,\Delta y = 2(35\text{ m/s}^2)(0.70\text{ m}) = 49\text{ m}^2/\text{s}^2$$

$$v_{1y} = \sqrt{49\text{ m}^2/\text{s}^2} = 7.0\text{ m/s}$$

The springbok leaves the ground with a velocity of 7.0 m/s. This is the starting point for the problem of a projectile launched straight up from the ground. One possible solution is to use the velocity equation to find how long it takes to reach maximum height, then the position equation to calculate the maximum height. But that takes two separate calculations. It is easier to make another use of the velocity-displacement equation:

$$v_{2y}{}^2 = 0 = v_{1y}{}^2 + 2a_{1y}\,\Delta y = v_{1y}{}^2 - 2g(y_2 - y_1)$$

where now the acceleration is $a_{1y} = -g$. Using $y_1 = 0$, we can solve for y_2, the height of the leap:

$$y_2 = \frac{v_{1y}{}^2}{2g} = \frac{(7.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 2.5\text{ m}$$

REVIEW 2.5 m is a bit over 8 feet, a remarkable vertical jump. But these animals are known for their jumping ability, so the answer seems reasonable. Note that it is especially important in a multipart problem like this to use numerical subscripts to distinguish different points in the motion.

2.6 Motion on an Inclined Plane

FIGURE 2.28a shows a problem closely related to free fall: that of motion down a straight, but frictionless, inclined plane, such as a skier going down a slope on frictionless snow. What is the object's acceleration? Although we're not yet prepared to give a rigorous derivation, we can deduce the acceleration with a plausibility argument.

FIGURE 2.28b shows the free-fall acceleration $\vec{a}_{\text{free fall}}$ the object would have if the incline suddenly vanished. The free-fall acceleration points straight down. This vector can be broken into two pieces: a vector $\vec{a}_{\parallel}$ that is parallel to the incline and a vector $\vec{a}_{\perp}$ that is perpendicular to the incline. The surface of the incline somehow "blocks" $\vec{a}_{\perp}$, through a process we will examine in Chapter 6, but $\vec{a}_{\parallel}$ is unhindered. It is this piece of $\vec{a}_{\text{free fall}}$, parallel to the incline, that accelerates the object.

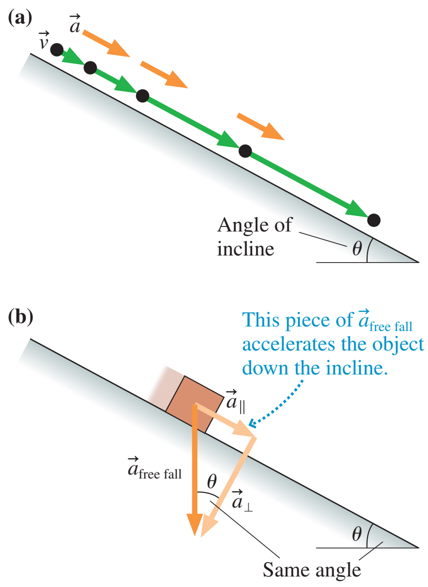
By definition, the length, or magnitude, of $\vec{a}_{\text{free fall}}$ is g. Vector $\vec{a}_{\parallel}$ is opposite angle θ (Greek theta), so the length, or magnitude, of $\vec{a}_{\parallel}$ must be $g \sin\theta$. Consequently, the one-dimensional acceleration along the incline is

$$a_s = \pm g\sin\theta \tag{2.26}$$

The correct sign depends on the direction in which the ramp is tilted. Examples will illustrate.

Equation 2.26 makes sense. Suppose the plane is perfectly horizontal. If you place an object on a horizontal surface, you expect it to stay at rest with no acceleration. Equation 2.26 gives $a_s = 0$ when $\theta = 0°$, in agreement with our expectations. Now suppose you tilt the plane until it becomes vertical, at $\theta = 90°$. Without friction, an object would simply fall, in free fall, parallel to the vertical surface. Equation 2.26 gives $a_s = -g = a_{\text{free fall}}$ when $\theta = 90°$, again in agreement with our expectations. Equation 2.26 gives the correct result in these *limiting cases.*

FIGURE 2.28 Acceleration on an inclined plane.



EXAMPLE 2.14 ■ Measuring acceleration

In the laboratory, a 2.00-m-long track has been inclined as shown in FIGURE 2.29. Your task is to measure the acceleration of a cart on the ramp and to compare your result with what you might have expected. You have available five "photogates" that measure the cart's speed as it passes through. You place a gate every 30 cm from a line you mark near the top of the track as the starting line. One run generates the data shown in the table. The first entry isn't a photogate, but it is a valid data point because you know the cart's speed is zero at the point where you release it.

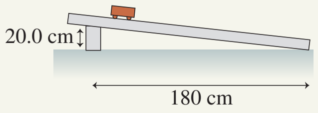


FIGURE 2.29 The experimental setup.

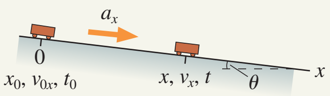
Distance (cm)	Speed (m/s)
0	0.00
30	0.75
60	1.15
90	1.38
120	1.56
150	1.76

NOTE Physics is an experimental science. Our knowledge of the universe is grounded in observations and measurements. Consequently, some examples and homework problems throughout this book will be based on data. Data-based homework problems require the use of a spreadsheet, graphing software, or a graphing calculator in which you can "fit" data with a straight line.

MODEL Model the cart as a particle.

VISUALIZE FIGURE 2.30 shows the pictorial representation. The track and axis are tilted at angle $\theta = \tan^{-1}(20.0\text{ cm}/180\text{ cm}) = 6.34°$. This is motion on an inclined plane, so you might expect the cart's acceleration to be $a_x = g\sin\theta = 1.08\text{ m/s}^2$.

FIGURE 2.30 The pictorial representation of the cart on the track.



Known
$x_0 = 0$ m $\quad v_{0x} = 0$ m/s
$t_0 = 0$ s $\quad \theta = 6.34°$

Find
a_x

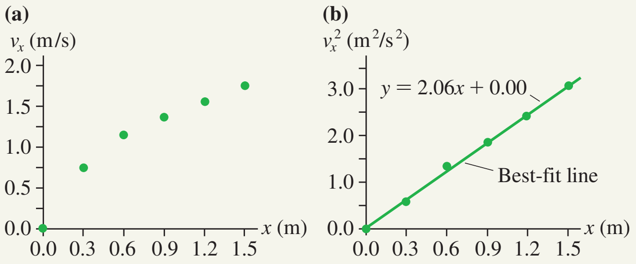
SOLVE In analyzing data, we want to use *all* the data. Further, we almost always want to use graphs when we have a series of measurements. We might start by graphing speed versus distance traveled. This is shown in FIGURE 2.31a, where we've converted distances to meters. As expected, speed increases with distance, but the graph isn't linear and that makes it hard to analyze.

Rather than proceeding by trial and error, let's be guided by theory. *If* the cart has constant acceleration—which we don't yet know and need to confirm—the third kinematic equation tells us that velocity and displacement should be related by

$$v_x^2 = v_{0x}^2 + 2a_x\Delta x = 2a_x x$$

The last step was based on starting from rest ($v_{0x} = 0$) at the origin ($\Delta x = x - x_0 = x$).

FIGURE 2.31 Graphs of velocity and of velocity squared. The equation of the best-fit line is given as $y =$ because that is how it would be shown in a spreadsheet.



Rather than graphing v_x versus x, suppose we graph v_x^2 versus x. If we let $y = v_x^2$, the kinematic equation reads

$$y = 2a_x x$$

This is in the form of a linear equation: $y = mx + b$, where m is the slope and b is the y-intercept. In this case, $m = 2a_x$ and $b = 0$. So if the cart really does have constant acceleration, a graph of v_x^2 versus x should be linear with a y-intercept of zero. This is a prediction that we can test.

Thus our analysis has three steps:

1. Graph v_x^2 versus x. If the graph is a straight line with a y-intercept of zero (or very close to zero), then we can conclude that the cart has constant acceleration on the ramp. If not, the acceleration is *not* constant and we cannot use the kinematic equations for constant acceleration.
2. If the graph has the correct shape, we can determine its slope m.
3. Because kinematics predicts $m = 2a_x$, the acceleration must be $a_x = m/2$.

FIGURE 2.31b is the graph of v_x^2 versus x. It does turn out to be a straight line with a y-intercept of zero, and this is the evidence we need that the cart has a constant acceleration on the ramp. To proceed, we want to determine the slope by finding the straight line that is the "best fit" to the data. This is a statistical technique, justified in a statistics class, but one that is implemented in spreadsheets and graphing calculators. The solid line in Figure 2.31b is the best-fit line for this data, and its equation is shown. We see that the slope is $m = 2.06\text{ m/s}^2$. **Slopes have units**, and the units come not from the fitting procedure but by looking at the axes of the graph. Here the vertical axis is velocity squared, with units of m^2/s^2, while the horizontal axis is position, measured in m. Thus the slope, rise over run, has units of m/s^2.

Finally, we can determine that the cart's acceleration was

$$a_x = \frac{m}{2} = 1.03\text{ m/s}^2$$

This is about 5% less than the 1.08 m/s^2 we expected. Two possibilities come to mind. Perhaps the distances used to find the tilt angle weren't measured accurately. Or, more likely, the cart rolls with a small bit of friction. The predicted acceleration $a_x = g\sin\theta$ is for a *frictionless* inclined plane; any friction would decrease the acceleration.

REVIEW The acceleration is just slightly less than predicted for a frictionless incline, so the result is reasonable.

Thinking Graphically

A good way to solidify your intuitive understanding of motion is to consider the problem of a hard, smooth ball rolling on a smooth track. The track is made up of several straight segments connected together. Each segment may be either horizontal or inclined. Your task is to analyze the ball's motion graphically.

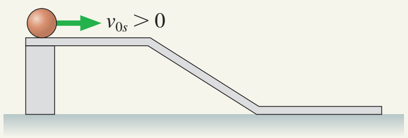
There are a small number of rules to follow:

1. Assume that the ball passes smoothly from one segment of the track to the next, with no abrupt change of speed and without ever leaving the track.
2. The graphs have no numbers, but they should show the correct *relationships.* For example, the position graph should be steeper in regions of higher speed.
3. The position s is the position measured *along* the track. Similarly, v_s and a_s are the velocity and acceleration parallel to the track.

EXAMPLE 2.15 ■ From track to graphs

Draw position, velocity, and acceleration graphs for the ball on the smooth track of FIGURE 2.32.

FIGURE 2.32 A ball rolling along a track.

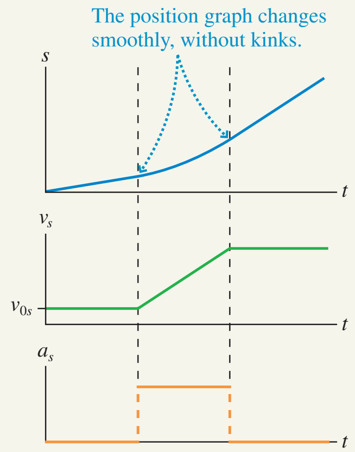


VISUALIZE It is often easiest to begin with the velocity. There is no acceleration on the horizontal surface ($a_s = 0$ if $\theta = 0°$), so the velocity remains constant at v_{0s} until the ball reaches the slope. The slope is an inclined plane where the ball has constant acceleration. The velocity increases linearly with time during constant-acceleration motion. The ball returns to constant-velocity motion after reaching the bottom horizontal segment. The middle graph of FIGURE 2.33 shows the velocity.

We can easily draw the acceleration graph. The acceleration is zero while the ball is on the horizontal segments and has a constant positive value on the slope. These accelerations are consistent with the slope of the velocity graph: zero slope, then positive slope, then a return to zero. The acceleration cannot *really* change instantly from zero to a nonzero value, but the change can be so quick that we do not see it on the time scale of the graph. That is what the vertical dashed lines imply.

Finally, we need to find the position-versus-time graph. The position increases linearly with time during the first segment at constant velocity. It also does so during the third segment of motion, but with a steeper slope to indicate a faster velocity. In between, while the acceleration is nonzero but constant, the position graph has a *parabolic* shape. Notice that the parabolic section blends *smoothly* into the straight lines on either side. An abrupt change of slope (a "kink") would indicate an abrupt change in velocity and would violate rule 1.

FIGURE 2.33 Motion graphs for the ball in Example 2.15.

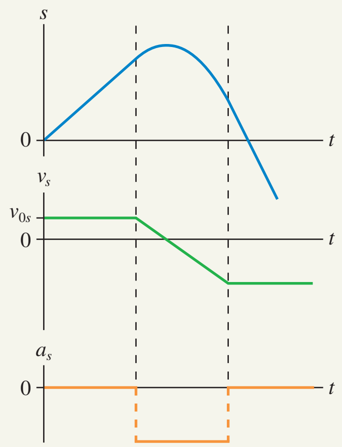


EXAMPLE 2.16 ■ From graphs to track

FIGURE 2.34 shows a set of motion graphs for a ball moving on a track. Draw a picture of the track and describe the ball's initial condition. Each segment of the track is *straight,* but the segments may be tilted.

VISUALIZE The ball starts with initial velocity $v_{0s} > 0$ and maintains this velocity for awhile; there's no acceleration. Thus the ball must start out rolling to the right on a horizontal track. At the end of the motion, the ball is again rolling on a horizontal track (no acceleration, constant velocity), but it's rolling to the *left* because v_s is negative. Further, the final speed ($|v_s|$) is greater than the initial speed. The middle section of the graph shows us what happens. The ball starts slowing with constant acceleration (rolling uphill), reaches a turning point (s is maximum, $v_s = 0$), then speeds up in the opposite direction (rolling downhill). This is still a negative acceleration because the ball is speeding up in the negative

FIGURE 2.34 Motion graphs of a ball rolling on a track of unknown shape.



Continued

s-direction. It must roll farther downhill than it had rolled uphill before reaching a horizontal section of track. **FIGURE 2.35** shows the track and the initial conditions that are responsible for the graphs of Figure 2.34.

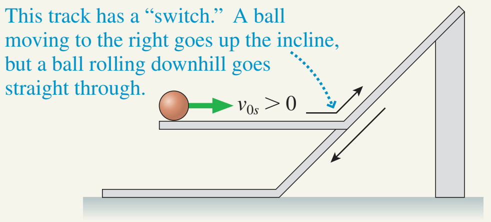
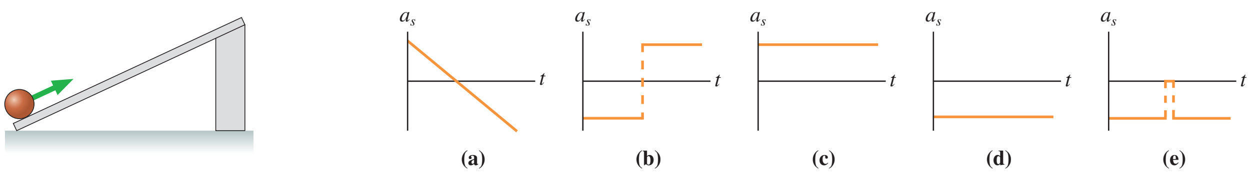


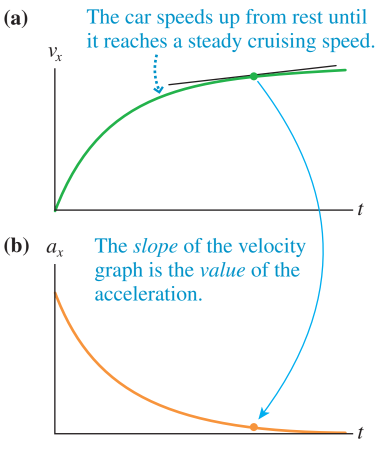
FIGURE 2.35 Track responsible for the motion graphs of Figure 2.34.

STOP TO THINK 2.5 The ball rolls up the ramp, then back down. Which is the correct acceleration graph?



2.7 ADVANCED TOPIC Instantaneous Acceleration

FIGURE 2.36 Velocity and acceleration graphs of a car leaving a stop sign.



Although the constant-acceleration model is very useful, real moving objects only rarely have constant acceleration. For example, **FIGURE 2.36a** is a realistic velocity-versus-time graph for a car leaving a stop sign. The graph is not a straight line, so this is *not* motion with constant acceleration.

We can define an instantaneous acceleration much as we defined the instantaneous velocity. The instantaneous velocity at time t is the slope of the position-versus-time graph at that time or, mathematically, the derivative of the position with respect to time. By analogy: The **instantaneous acceleration a_s is the slope of the line that is tangent to the velocity-versus-time curve at time t.** Mathematically, this is

$$a_s = \frac{dv_s}{dt} = \text{slope of the velocity-versus-time graph at time } t \qquad (2.27)$$

FIGURE 2.36b applies this idea by showing the car's acceleration graph. At each instant of time, the *value* of the car's acceleration is the *slope* of its velocity graph. The initially steep slope indicates a large initial acceleration. The acceleration decreases to zero as the car reaches cruising speed.

The reverse problem—to find the velocity v_s if we know the acceleration a_s at all instants of time—is also important. Again, with analogy to velocity and position, we have

$$v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s\, dt \qquad (2.28)$$

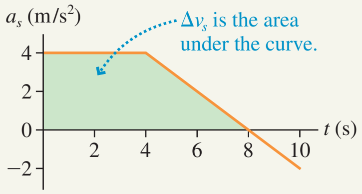
The graphical interpretation of Equation 2.28 is

$$v_{fs} = v_{is} + \text{area under the acceleration curve } a_s \text{ between } t_i \text{ and } t_f \qquad (2.29)$$

EXAMPLE 2.17 ■ Finding velocity from acceleration

FIGURE 2.37 shows the acceleration graph for a particle with an initial velocity of 10 m/s. What is the particle's velocity at $t = 8$ s?

FIGURE 2.37 Acceleration graph for Example 2.17.



MODEL We're told this is the motion of a particle.

VISUALIZE Figure 2.37 is a graphical representation of the motion.

SOLVE The change in velocity is found as the area under the acceleration curve:

$$v_{fs} = v_{is} + \text{area under the acceleration curve } a_s \text{ between } t_i \text{ and } t_f$$

The area under the curve between $t_i = 0$ s and $t_f = 8$ s can be subdivided into a rectangle $(0\text{ s} \le t \le 4\text{ s})$ and a triangle $(4\text{ s} \le t \le 8\text{ s})$. These areas are easily computed. Thus

$$\begin{aligned} v_s(\text{at } t = 8\text{ s}) &= 10\text{ m/s} + (4\text{ m/s}^2)(4\text{ s}) \\ &\quad + \tfrac{1}{2}(4\text{ m/s}^2)(4\text{ s}) \\ &= 34\text{ m/s} \end{aligned}$$

EXAMPLE 2.18 ■ A realistic car acceleration

Starting from rest, a car takes T seconds to reach its cruising speed v_{max}. A plausible expression for the velocity as a function of time is

$$v_x(t) = \begin{cases} v_{max}\left(\dfrac{2t}{T} - \dfrac{t^2}{T^2}\right) & t \le T \\ v_{max} & t \ge T \end{cases}$$

a. Demonstrate that this is a plausible function by drawing velocity and acceleration graphs.

b. Find an expression for the distance traveled at time T in terms of T and the maximum acceleration a_{max}.

c. What are the maximum acceleration and the distance traveled for a car that reaches a cruising speed of 15 m/s in 8.0 s?

MODEL Model the car as a particle.

VISUALIZE **FIGURE 2.38a** shows the velocity graph. It's an inverted parabola that reaches v_{max} at time T and then holds that value. From the slope, we see that the acceleration should start at a maximum value a_{max}, steadily decrease until T, and be zero for $t > T$.

SOLVE a. We can find an expression for a_x by taking the derivative of v_x. Starting with $t \le T$, and using Equation 2.6 for the derivatives of polynomials, we find

$$a_x = \frac{dv_x}{dt} = v_{max}\left(\frac{2}{T} - \frac{2t}{T^2}\right) = \frac{2v_{max}}{T}\left(1 - \frac{t}{T}\right) = a_{max}\left(1 - \frac{t}{T}\right)$$

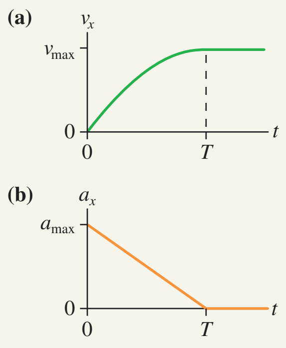
where $a_{max} = 2v_{max}/T$. For $t \ge T$, $a_x = 0$. Altogether,

$$a_x(t) = \begin{cases} a_{max}\left(1 - \dfrac{t}{T}\right) & t \le T \\ 0 & t \ge T \end{cases}$$

This expression for the acceleration is graphed in **FIGURE 2.38b**. The acceleration decreases linearly from a_{max} to 0 as the car accelerates from rest to its cruising speed.

b. To find the position as a function of time, we need to integrate the velocity (Equation 2.11) using Equation 2.13 for the integrals of polynomials. At time T, when cruising speed is reached,

FIGURE 2.38 Velocity and acceleration graphs for Example 2.18.



$$\begin{aligned} x_T = x_0 + \int_0^T v_x\,dt &= 0 + \frac{2v_{max}}{T}\int_0^T t\,dt - \frac{v_{max}}{T^2}\int_0^T t^2\,dt \\ &= \frac{2v_{max}}{T}\frac{t^2}{2}\bigg|_0^T - \frac{v_{max}}{T^2}\frac{t^3}{3}\bigg|_0^T \\ &= v_{max}T - \tfrac{1}{3}v_{max}T = \tfrac{2}{3}v_{max}T \end{aligned}$$

Recalling that $a_{max} = 2v_{max}/T$, we can write the distance traveled as

$$x_T = \tfrac{2}{3}v_{max}T = \tfrac{1}{3}\left(\frac{2v_{max}}{T}\right)T^2 = \tfrac{1}{3}a_{max}T^2$$

If the acceleration stayed constant, the distance would be $\frac{1}{2}aT^2$. We have found a similar expression but, because the acceleration is steadily decreasing, with a smaller fraction in front.

c. With $v_{max} = 15$ m/s and $T = 8.0$ s, realistic values for city driving, we find

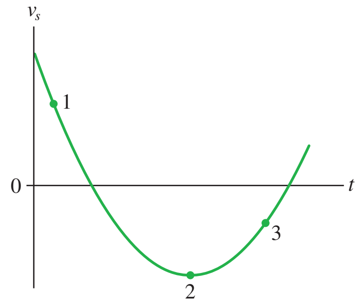
$$a_{max} = \frac{2v_{max}}{T} = \frac{2(15\text{ m/s})}{8.0\text{ s}} = 3.75\text{ m/s}^2$$

$$x_T = \tfrac{1}{3}a_{max}T^2 = \tfrac{1}{3}(3.75\text{ m/s}^2)(8.0\text{ s})^2 = 80\text{ m}$$

REVIEW 80 m in 8.0 s to reach a cruising speed of 15 m/s $\approx$ 30 mph is very reasonable. This gives us good reason to believe that a car's initial acceleration is $\approx \frac{1}{3}g$.

STOP TO THINK 2.6 Rank in order, from most positive to least positive, the accelerations at points 1 to 3.

a. $a_1 > a_2 > a_3$
b. $a_3 > a_1 > a_2$
c. $a_3 > a_2 > a_1$
d. $a_2 > a_1 > a_3$



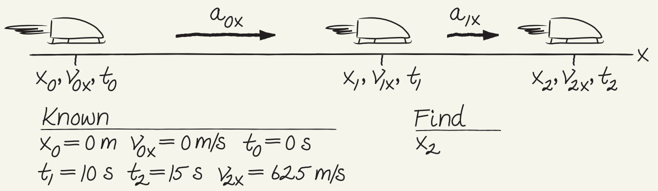
CHAPTER 2 CHALLENGE EXAMPLE Rocketing along

A rocket sled accelerates along a long, horizontal rail. Starting from rest, two rockets burn for 10 s, providing a constant acceleration. One rocket then burns out, halving the acceleration, but the other burns for an additional 5 s to boost the sled's speed to 625 m/s. How far has the sled traveled when the second rocket burns out?

MODEL Model the rocket sled as a particle with constant acceleration.

VISUALIZE **FIGURE 2.39** shows the pictorial representation. This is a two-part problem with a beginning, an end (the second rocket burns out), and a point in between where the motion changes (the first rocket burns out).

FIGURE 2.39 The pictorial representation of the rocket sled.



SOLVE The difficulty with this problem is that there's not enough information to completely analyze either the first or the second part of the motion. A successful solution will require combining information about both parts of the motion, and that can be done only by working algebraically, not worrying about numbers until the end of the problem. A well-drawn pictorial representation and clearly defined symbols are essential.

The first part of the motion, with both rockets firing, has acceleration a_{0x}. The sled's position and velocity when the first rocket burns out are

$$x_1 = x_0 + v_{0x}\,\Delta t + \tfrac{1}{2}a_{0x}(\Delta t)^2 = \tfrac{1}{2}a_{0x}t_1^{\,2}$$

$$v_{1x} = v_{0x} + a_{0x}\,\Delta t = a_{0x}t_1$$

where we simplified as much as possible by knowing that the sled started from rest at the origin at $t_0 = 0$ s. We can't compute numerical values, but these are valid algebraic expressions that we can carry over to the second part of the motion.

From t_1 to t_2, the acceleration is a smaller a_{1x}. The velocity when the second rocket burns out is

$$v_{2x} = v_{1x} + a_{1x}\,\Delta t = a_{0x}t_1 + a_{1x}(t_2 - t_1)$$

where for v_{1x} we used the algebraic result from the first part of the motion. Now we have enough information to complete the solution. We know that the acceleration is halved when the first rocket burns out, so $a_{1x} = \frac{1}{2}a_{0x}$. Thus

$$v_{2x} = 625\text{ m/s} = a_{0x}(10\text{ s}) + \tfrac{1}{2}a_{0x}(5\text{ s}) = (12.5\text{ s})a_{0x}$$

Solving, we find $a_{0x} = 50\text{ m/s}^2$.

With the acceleration now known, we can calculate the position and velocity when the first rocket burns out:

$$x_1 = \tfrac{1}{2}a_{0x}t_1^{\,2} = \tfrac{1}{2}(50\text{ m/s}^2)(10\text{ s})^2 = 2500\text{ m}$$

$$v_{1x} = a_{0x}t_1 = (50\text{ m/s}^2)(10\text{ s}) = 500\text{ m/s}$$

Finally, the position when the second rocket burns out is

$$x_2 = x_1 + v_{1x}\,\Delta t + \tfrac{1}{2}a_{1x}(\Delta t)^2$$
$$= 2500\text{ m} + (500\text{ m/s})(5\text{ s}) + \tfrac{1}{2}(25\text{ m/s}^2)(5\text{ s})^2 = 5300\text{ m}$$

The sled has traveled 5300 m when it reaches 625 m/s at the burnout of the second rocket.

REVIEW 5300 m is 5.3 km, or roughly 3 miles. That's a long way to travel in 15 s! But the sled reaches incredibly high speeds. At the final speed of 625 m/s, over 1200 mph, the sled would travel nearly 10 km in 15 s. So 5.3 km in 15 s for the accelerating sled seems reasonable.

Summary

The goal of Chapter 2 has been to learn to solve problems about motion along a straight line.

General Principles

Kinematics describes motion in terms of position, velocity, and acceleration.

General kinematic relationships are given **mathematically** by:

Instantaneous velocity $v_s = ds/dt$ = slope of position graph

Instantaneous acceleration $a_s = dv_s/dt$ = slope of velocity graph

Final position $s_f = s_i + \int_{t_i}^{t_f} v_s\,dt = s_i + \begin{cases}\text{area under the velocity}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$

Final velocity $v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s\,dt = v_{is} + \begin{cases}\text{area under the acceleration}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$

Solving Kinematics Problems

MODEL Uniform motion or constant acceleration.

VISUALIZE Draw a pictorial representation.

SOLVE

- Uniform motion $s_f = s_i + v_s\,\Delta t$
- Constant acceleration $v_{fs} = v_{is} + a_s\,\Delta t$

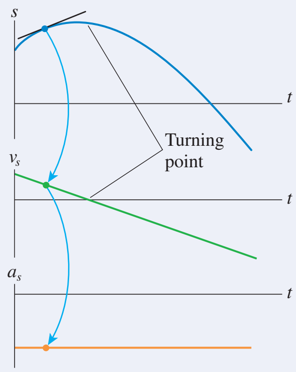
$$s_f = s_i + v_{is}\,\Delta t + \tfrac{1}{2}a_s(\Delta t)^2$$
$$v_{fs}^{\,2} = v_{is}^{\,2} + 2a_s\,\Delta s$$

REVIEW Is the result reasonable?

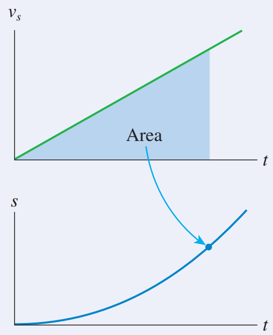
Important Concepts

Position, velocity, and acceleration are related graphically.

- The slope of the position-versus-time graph is the value on the velocity graph.
- The slope of the velocity graph is the value on the acceleration graph.
- s is a maximum or minimum at a turning point, and $v_s = 0$.



- Displacement is the area under the velocity curve.



Applications

The sign of v_s indicates the direction of motion.

- $v_s > 0$ is motion to the right or up.
- $v_s < 0$ is motion to the left or down.

The sign of a_s indicates which way $\vec{a}$ points, *not* whether the object is speeding up or slowing down.

- $a_s > 0$ if $\vec{a}$ points to the right or up.
- $a_s < 0$ if $\vec{a}$ points to the left or down.
- The direction of $\vec{a}$ is found with a motion diagram.

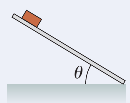
An object is speeding up if and only if v_s and a_s have the same sign.

An object is slowing down if and only if v_s and a_s have opposite signs.

Free fall is constant-acceleration motion with

$$a_y = -g = -9.80 \text{ m/s}^2$$

Motion on an inclined plane has $a_s = \pm g\sin\theta$. The sign depends on the direction of the tilt.



Terms and Notation

kinematics
uniform motion
average velocity, v_{avg}
speed, v
initial position, s_i
final position, s_f
uniform-motion model
instantaneous velocity, v_s
turning point
average acceleration, a_{avg}
constant-acceleration model
free fall
free-fall acceleration, g
instantaneous acceleration, a_s

CONCEPTUAL QUESTIONS

For Questions 1 through 3, interpret the position graph given in each figure by writing a very short "story" of what is happening. Be creative! Have characters and situations! Simply saying that "a car moves 100 meters to the right" doesn't qualify as a story. Your stories should make *specific reference* to information you obtain from the graph, such as distance moved or time elapsed.

1.

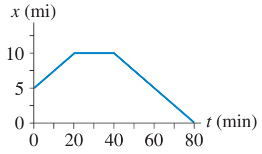


FIGURE Q2.1

2.

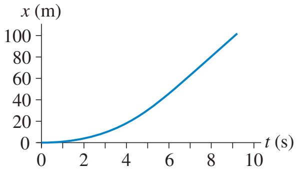


FIGURE Q2.2

3.

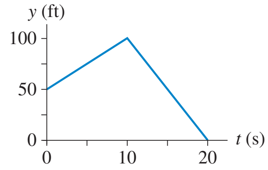


FIGURE Q2.3

4. FIGURE Q2.4 shows a position-versus-time graph for the motion of objects A and B as they move along the same axis.
   a. At the instant $t = 1$ s, is the speed of A greater than, less than, or equal to the speed of B? Explain.
   b. Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.

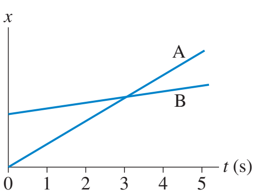


FIGURE Q2.4

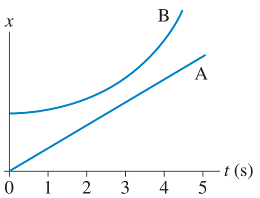


FIGURE Q2.5

5. FIGURE Q2.5 shows a position-versus-time graph for the motion of objects A and B as they move along the same axis.
   a. At the instant $t = 1$ s, is the speed of A greater than, less than, or equal to the speed of B? Explain.
   b. Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.
6. FIGURE Q2.6 shows the position-versus-time graph for a moving object. At which numbered point or points:
   a. Is the object *moving* the slowest?
   b. Is the object moving the fastest?
   c. Is the object at rest?
   d. Is the object moving to the left?

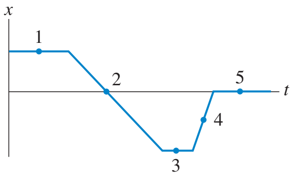


FIGURE Q2.6

7. FIGURE Q2.7 shows the position-versus-time graph for a moving object. At which numbered point or points:
   a. Is the object moving the fastest?
   b. Is the object moving to the left?
   c. Is the object speeding up?
   d. Is the object turning around?

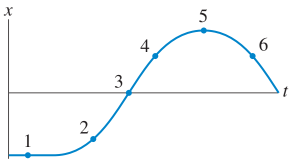


FIGURE Q2.7

8. FIGURE Q2.8 shows six frames from the motion diagrams of two moving cars, A and B.
   a. Do the two cars ever have the same position at one instant of time? If so, in which frame number (or numbers)?
   b. Do the two cars ever have the same velocity at one instant of time? If so, between which two frames?

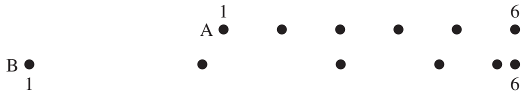


FIGURE Q2.8

9. You want to pass on a note to your friend who is traveling by a bus that does not stop in front of your house. You start jogging toward the bus the moment you see it at a distance. As the bus crosses you, do you think you can pass the note to your friend's outstretched hand?
10. When a space shuttle lands on a runway, it immediately deploys parachutes to reduce its tremendous speed. At this point, do the velocity and acceleration of the shuttle have the same direction? Explain.
11. Give an example of a motion
    a. where there is a positive acceleration, yet zero velocity.
    b. with zero acceleration but positive velocity.
12. You travel by car at a constant 90 km/h for 90 km. Then, due to heavy traffic, you need to reduce your speed to 50 km/h for another 100 km. What is your car's average speed for the 190-km trip?
13. A rock is *thrown* (not dropped) straight down from a bridge into the river below. At each of the following instants, is the magnitude of the rock's acceleration greater than g, equal to g, less than g, or 0? Explain.
    a. Immediately after being released.
    b. Just before hitting the water.
14. FIGURE Q2.14 shows the velocity-versus-time graph for a moving object. At which numbered point or points:
    a. Is the object speeding up?
    b. Is the object slowing down?
    c. Is the object moving to the left?
    d. Is the object moving to the right?

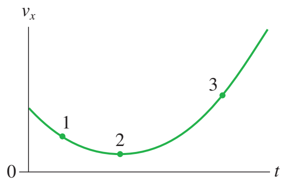


FIGURE Q2.14

EXERCISES AND PROBLEMS

Exercises

Section 2.1 Uniform Motion

1. || Larry leaves home at 9:05 and runs at constant speed to the lamppost seen in FIGURE EX2.1. He reaches the lamppost at 9:07, immediately turns, and runs to the tree. Larry arrives at the tree at 9:10.
   a. What is Larry's average velocity, in m/min, during each of these two intervals?
   b. What is Larry's average velocity for the entire run?

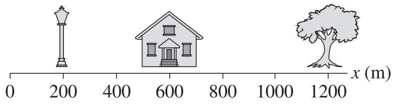


FIGURE EX2.1

2. || Julie drives 120 miles to her grandmother's house. She covers half the distance at 40 mph and the other half at 60 mph. On her return trip, she drives half the time at 40 mph and the rest at 60 mph.
   a. What is Julie's average speed on the way to her grandmother's house?
   b. What is her average speed on the return trip?
3. || Alan leaves London at 8:00 A.M. to drive to Leeds, 200 mi away. He travels at a steady 50 mph. Beth leaves London at 8:45 A.M. and drives a steady 60 mph.
   a. Who gets to Leeds first?
   b. How long does the first to arrive have to wait for the second?
4. || FIGURE EX2.4 is the position-versus-time graph of a bicycle. What is the bicycle's velocity at (a) $t = 5$ s, (b) $t = 15$ s, and (c) $t = 30$ s?

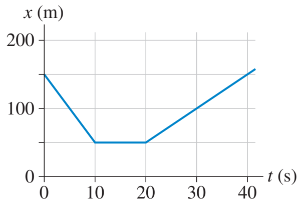


FIGURE EX2.4

Section 2.2 Instantaneous Velocity

Section 2.3 Finding Position from Velocity

5. | FIGURE EX2.5 shows the position graph of a particle.
   a. Draw the particle's velocity graph for the interval $0\text{ s} \le t \le 4\text{ s}$.
   b. Does this particle have a turning point or points? If so, at what time or times?

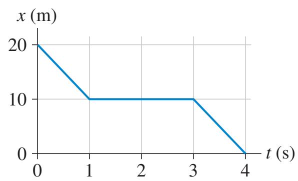


FIGURE EX2.5

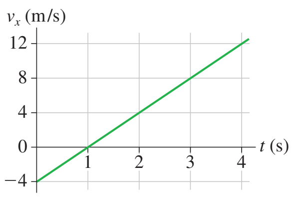


FIGURE EX2.6

6. || A particle starts from $x_0 = 10$ m at $t_0 = 0$ s and moves with the velocity graph shown in FIGURE EX2.6.
   a. Does this particle have a turning point? If so, at what time?
   b. What is the object's position at $t = 2$ s and 4 s?

7. | FIGURE EX2.7 shows the velocity graph for a particle having initial position $x_0 = 0$ m at $t_0 = 0$ s. At what time or times is the particle found at $x = 35$ m?

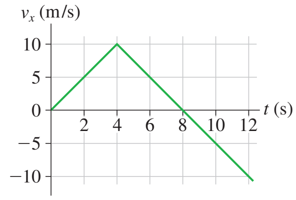


FIGURE EX2.7

8. || BIO FIGURE EX2.8 is a somewhat idealized graph of the velocity of blood in the ascending aorta during one beat of the heart. Approximately how far, in cm, does the blood move during one beat?

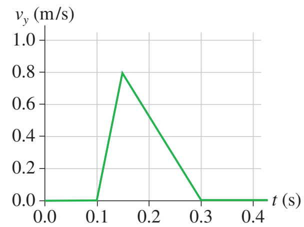


FIGURE EX2.8

Section 2.4 Motion with Constant Acceleration

9. | FIGURE EX2.9 shows the velocity graph of a particle. Draw the particle's acceleration graph for the interval $0\text{ s} \le t \le 4\text{ s}$.

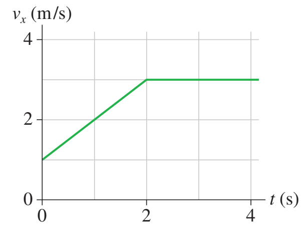


FIGURE EX2.9

10. || FIGURE EX2.10 shows the velocity graph of a particle moving along the x-axis. Its initial position is $x_0 = 2.0$ m at $t_0 = 0$ s. At $t = 2.0$ s, what are the particle's (a) position, (b) velocity, and (c) acceleration?

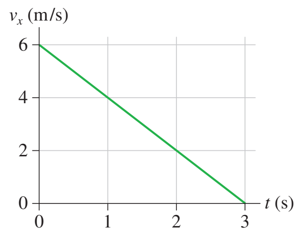


FIGURE EX2.10

11. || BIO FIGURE EX2.8 showed the velocity graph of blood in the aorta. What is the blood's acceleration during each phase of the motion, speeding up and slowing down?

12. | **FIGURE EX2.12** shows the velocity-versus-time graph for a particle moving along the x-axis. Its initial position is $x_0 = 2.0$ m at $t_0 = 0$ s.
   a. What are the particle's position, velocity, and acceleration at $t = 1.0$ s?
   b. What are the particle's position, velocity, and acceleration at $t = 3.0$ s?

FIGURE EX2.12

13. || a. What constant acceleration, in SI units, must a car have to go from zero to 60 mph in 4.9 s?
   b. How far has the car traveled when it reaches 60 mph? Give your answer both in SI units and in feet.
14. || A jet plane is cruising at 280 m/s when suddenly the pilot turns the engines to full throttle. After traveling 4.0 km, the jet moves with a speed of 380 m/s. What is the jet's acceleration, assuming it to be a constant acceleration?
15. || It has been proposed that a very small probe could be sent to a nearby star system by using a powerful laser beam, fired from an earth-orbiting satellite, to push on a lightweight "solar sail." Very high speeds could be reached in the vacuum of space by a fairly modest acceleration that continues for a long interval of time.
   a. Write an expression for the constant acceleration a_x an object needs to reach velocity v_{max} in time t_{push}, starting from rest.
   b. Write an expression in terms of v_{max} and t_{push} for the distance d the object travels during this time.
   c. For the mission to be feasible, the probe needs to reach 10% of the speed of light after being pushed for 1.0 year. The probe would then coast the rest of the way. What constant acceleration is needed? Note that the speed of light and much other useful data needed to solve problems are given inside the front and back covers of the book.
   d. What fraction of a light year will the probe have traveled at the end of the year? A light year (ly) is the distance traveled by light in 1 year.
16. || When you sneeze, the air in your lungs accelerates from rest
BIO to 150 km/h in approximately 0.50 s. What is the magnitude of the acceleration of the air in m/s^2?
17. || A speed skater moving to the left across frictionless ice at 7.0 m/s hits a 5.0-m-wide patch of rough ice. She slows steadily, then continues on at 4.0 m/s. What is her acceleration on the rough ice?
18. || A Porsche challenges a Honda to a 400 m race. Because the Porsche's acceleration of 3.5 m/s^2 is larger than the Honda's 3.0 m/s^2, the Honda gets a 1.0 s head start. Who wins? By how many seconds?
19. || A Lamborghini Aventador S can go from 0 to 60 mph in 2.7 s. Assume the acceleration is constant.
   a. What is the magnitude of the acceleration?
   b. How far has the car traveled when it reaches 60 mph?

Section 2.5 Free Fall

20. | Ball bearings are made by letting spherical drops of molten metal fall inside a tall tower—called a "shot tower"—and solidify as they fall.
   a. If a bearing needs 5 seconds to solidify enough for impact, how high must the tower be?
   b. What is the bearing's impact velocity?
21. || A rock is tossed straight up from ground level with a speed of 20 m/s. When it returns, it falls into a hole 10 m deep.
   a. What is the rock's speed as it hits the bottom of the hole?
   b. How long is the rock in the air, from the instant it is released until it hits the bottom of the hole?
22. || A student standing on the ground throws a ball straight up. The ball leaves the student's hand with a speed of 15 m/s when the hand is 2.0 m above the ground. How long is the ball in the air before it hits the ground? (The student moves her hand out of the way.)
23. || A science project involves dropping a watermelon from the Empire State Building to the sidewalk below, from a height of 350 m. It so happens that Superman is flying by at the instant the watermelon is dropped. He is headed straight down at a speed of 40 m/s. How fast is the watermelon falling when it passes Superman?
24. ||| When jumping, a flea accelerates at an astounding 1000 m/s^2,
BIO but over only the very short distance of 0.50 mm. If a flea jumps straight up, and if air resistance is neglected (a rather poor approximation in this situation), how high does the flea go?
25. ||| A rock is dropped from the top of a tall building. The rock's displacement in the last second before it hits the ground is 45% of the entire distance it falls. How tall is the building?

Section 2.6 Motion on an Inclined Plane

26. || A car traveling at 30 m/s runs out of gas while traveling up a 10° slope. How far up the hill will it coast before starting to roll back down?
27. || A skier is gliding along at 3.0 m/s on horizontal, frictionless snow. He suddenly starts down a 10° incline. His speed at the bottom is 15 m/s.
   a. What is the length of the incline?
   b. How long does it take him to reach the bottom?
28. || Santa loses his footing and slides down a frictionless, snowy roof that is tilted at an angle of 30°. If Santa slides 10 m before reaching the edge, what is his speed as he leaves the roof?
29. || A bicycle coasting at 7.0 m/s comes to a 6.0-m-long, 1.0-m-high ramp. What is the bicycle's speed as it leaves the top of the ramp?
30. || A snowboarder glides down a 50-m-long, 15° hill. She then glides horizontally for 10 m before reaching a 25° upward slope. Assume the snow is frictionless.
   a. What is her speed at the bottom of the hill?
   b. How far can she travel up the 25° slope?

Section 2.7 Instantaneous Acceleration

31. || **FIGURE EX2.31** shows the acceleration-versus-time graph of a particle moving along the x-axis. Its initial velocity is $v_{0x} =$ 8.0 m/s at $t_0 = 0$ s. What is the particle's velocity at $t = 4.0$ s?

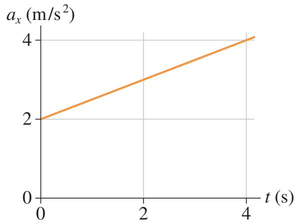


FIGURE EX2.31

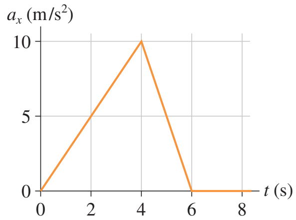


FIGURE EX2.32

32. || **FIGURE EX2.32** shows the acceleration graph for a particle that starts from rest at $t = 0$ s. What is the particle's velocity at $t = 6$ s?

33. | A particle moving along the x-axis has its position described CALC by the function $x = (3.00t^3 - 3.00t + 5.00)$ m, where t is time (in seconds). At $t = 2.00$, what is
a. the position of the particle?
b. its velocity?
c. its acceleration?

34. | A particle moving along the x-axis has its velocity described CALC by the function $v_x = 2t^2$ m/s, where t is in s. Its initial position is $x_0 = 1$ m at $t_0 = 0$ s. At $t = 1$ s what are the particle's (a) position, (b) velocity, and (c) acceleration?

35. || The vertical position of a particle is given by the function CALC $y = (t^2 - 4t + 2)$ m, where t is in s.
a. At what time does the particle have a turning point in its motion?
b. What is the particle's position at that time?

36. || The position of a particle is given by the function CALC $x = (2t^3 - 6t^2 + 12)$ m, where t is in s.
a. At what time does the particle reach its minimum velocity? What is $(v_x)_{\min}$?
b. At what time is the acceleration zero?

Problems

37. || Particles A, B, and C move along the x-axis. Particle C has an initial velocity of 10 m/s. In FIGURE P2.37, the graph for A is a position-versus-time graph; the graph for B is a velocity-versus-time graph; the graph for C is an acceleration-versus-time graph. Find each particle's velocity at $t = 7.0$ s.

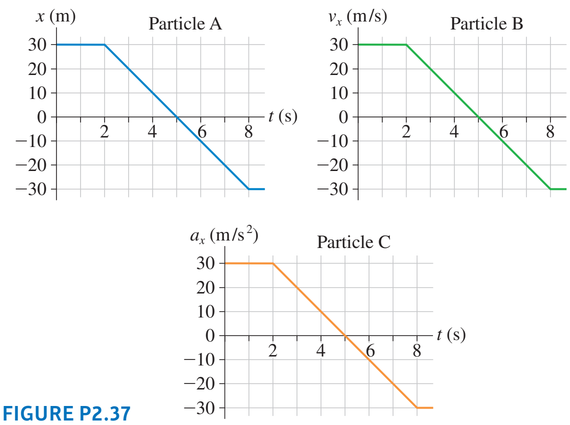


FIGURE P2.37

38. | A block is suspended from a spring, pulled down, and released. The block's position-versus-time graph is shown in FIGURE P2.38.
a. At what times is the velocity zero? At what times is the velocity most positive? Most negative?
b. Draw a reasonable velocity-versus-time graph.

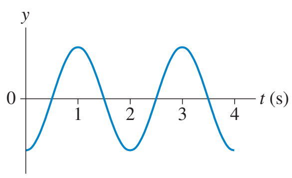


FIGURE P2.38

39. || A particle's velocity is described by the function CALC $v_x = (t^2 - 10t + 21)$ m/s, where t is in s.
a. At what times does the particle reach its turning points?
b. What is the particle's acceleration at each of the turning points?

40. ||| A particle's velocity is described by the function $v_x = kt^2$ m/s, CALC where k is a constant and t is in s. The particle's position at $t_0 = 0$ s is $x_0 = -9.0$ m. At $t_1 = 3.0$ s, the particle is at $x_1 = 9.0$ m. Determine the value of the constant k. Be sure to include the proper units.

41. || A particle's acceleration is described by the function CALC $a_x = (10 - t)$ m/s^2, where t is in s. Its initial conditions are $x_0 = 0$ m and $v_{0x} = 0$ m/s at $t = 0$ s.
a. At what time is the velocity again zero?
b. What is the particle's position at that time?

42. || A particle's velocity is given by the function CALC $v_x = (2.0 \text{ m/s}) \sin(\pi t)$, where t is in s.
a. What is the first time after $t = 0$ s when the particle reaches a turning point?
b. What is the particle's acceleration at that time?

43. || A ball rolls along the smooth track shown in FIGURE P2.43. Each segment of the track is straight, and the ball passes smoothly from one segment to the next without changing speed or leaving the track. Draw three vertically stacked graphs showing position, velocity, and acceleration versus time. Each graph should have the same time axis, and the proportions of the graph should be qualitatively correct. Assume that the ball has enough speed to reach the top.

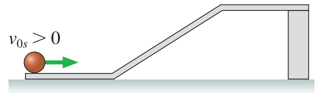


FIGURE P2.43

44. || Draw position, velocity, and acceleration graphs for the ball shown in FIGURE P2.44. See Problem 43 for more information.

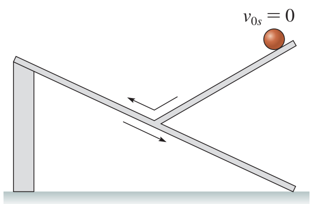


FIGURE P2.44

45. || FIGURE P2.45 shows a set of kinematic graphs for a ball rolling on a track. All segments of the track are straight lines, but some may be tilted. Draw a picture of the track and also indicate the ball's initial condition.

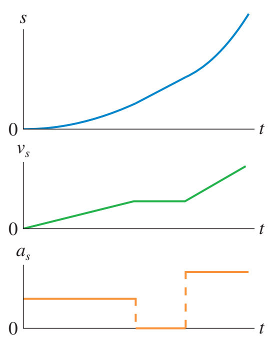


FIGURE P2.45

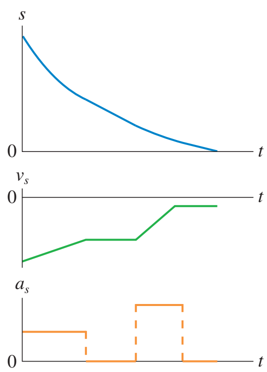


FIGURE P2.46

46. || FIGURE P2.46 shows a set of kinematic graphs for a ball rolling on a track. All segments of the track are straight lines, but some may be tilted. Draw a picture of the track and also indicate the ball's initial condition.

47. || You are driving to the grocery store at 20 m/s. You are 110 m from an intersection when the traffic light turns red. Assume that your reaction time is 0.50 s and that your car brakes with constant acceleration. What magnitude braking acceleration will bring you to a stop exactly at the intersection?

48. | The takeoff speed for an Airbus A320 jetliner is 80 m/s. Velocity data measured during takeoff are as shown.

t (s)	v_x (m/s)
0	0
10	23
20	46
30	69

a. Is the jetliner's acceleration constant during takeoff? Explain.
b. At what time do the wheels leave the ground?
c. For safety reasons, in case of an aborted takeoff, the runway must be three times the takeoff distance. Can an A320 take off safely on a 2.5-mi-long runway?

49. || You're driving down the highway late one night at 20 m/s when a deer steps onto the road 35 m in front of you. Your reaction time before stepping on the brakes is 0.50 s, and the maximum deceleration of your car is 10 m/s^2.
a. How much distance is between you and the deer when you come to a stop?
b. What is the maximum speed you could have and still not hit the deer?

50. || The Smooth Company has proposed transporting people between Paris and Amsterdam, a distance of 430 km, through an underground tube from which the air has been removed to eliminate air drag. Small pods carrying four passengers would accelerate at 2.5 m/s^2 until reaching a speed of 180 m/s. Later, they would brake at 1.5 m/s^2. A launch of one pod per minute would transport 240 passengers per hour, roughly equivalent to one jet plane per hour.
a. What would be the trip time in minutes from Paris to Amsterdam?
b. How far apart would two adjacent pods be on the constant-speed segment of the journey?

51. || A car starts from rest at a stop sign. It accelerates at 4.0 m/s^2 for 6.0 s, coasts for 2.0 s, and then slows down at a rate of 3.0 m/s^2 for the next stop sign. How far apart are the stop signs?

52. || A cheetah spots a Thomson's gazelle, its preferred prey, and
BIO leaps into action, quickly accelerating to its top speed of 30 m/s, the highest of any land animal. However, a cheetah can maintain this extreme speed for only 15 s before having to let up. The cheetah is 170 m from the gazelle as it reaches top speed, and the gazelle sees the cheetah at just this instant. With negligible reaction time, the gazelle heads directly away from the cheetah, accelerating at 4.6 m/s^2 for 5.0 s, then running at constant speed. Does the gazelle escape? If so, by what distance is the gazelle in front when the cheetah gives up?

53. || a. Find an expression for the minimum stopping distance d_{stop} of a car traveling at speed v_0 if the driver's reaction time is T_{react} and the magnitude of the acceleration during maximum braking is a constant a_{brake}.
b. A car traveling at 30 m/s can stop in a distance of 60 m, including the distance traveled during the driver's reaction time of 0.50 s. What is the minimum stopping distance for the same car traveling at 40 m/s?

54. || Bob is driving the getaway car after the big bank robbery. He's going 50 m/s when his headlights suddenly reveal a nail strip that the cops have placed across the road 150 m in front of him. If Bob can stop in time, he can throw the car into reverse and escape. But if he crosses the nail strip, all his tires will go flat and he will be caught. Bob's reaction time before he can hit the brakes is 0.60 s, and his car's maximum deceleration is 10 m/s^2. Does Bob stop before or after the nail strip? By what distance?

55. ||| A 1000 kg weather rocket is launched straight up. The rocket motor provides a constant acceleration for 16 s, then the motor stops. The rocket altitude 20 s after launch is 5100 m. You can ignore any effects of air resistance. What was the rocket's acceleration during the first 16 s?

56. || A 200 kg weather rocket is loaded with 100 kg of fuel and fired straight up. It accelerates upward at 30 m/s^2 for 30 s, then runs out of fuel. Ignore any air resistance effects.
a. What is the rocket's maximum altitude?
b. How long is the rocket in the air before hitting the ground?

57. || A lead ball is dropped into a lake from a diving board 5.0 m above the water. After entering the water, it sinks to the bottom with a constant velocity equal to the velocity with which it hit the water. The ball reaches the bottom 3.0 s after it is released. How deep is the lake?

58. || A hotel elevator ascends 200 m with a maximum speed of 5.0 m/s. Its acceleration and deceleration both have a magnitude of 1.0 m/s^2.
a. How far does the elevator move while accelerating to full speed from rest?
b. How long does it take to make the complete trip from bottom to top?

59. || Your car's anti-lock brake system is designed to keep the wheels from "locking" and starting to skid. The deceleration of a skidding car is less than that of a car that has the maximum braking without skidding—a topic we'll explore in Chapter 6. In one test, a car equipped with anti-lock brakes was able to decelerate at 7.0 m/s^2. while the same car without anti-lock brakes decelerated at 4.8 m/s^2 while skidding. In an emergency stop at a highway speed of 30 m/s, how much additional stopping distance would be needed by the skidding car compared to the car with anti-lock brakes?

60. || You are 9.0 m from the door of your bus, behind the bus, when it pulls away with an acceleration of 1.0 m/s^2. You instantly start running toward the still-open door at 4.5 m/s.
a. How long does it take for you to reach the open door and jump in?
b. What is the maximum time you can wait before starting to run and still catch the bus?

61. || Ann and Carol are driving their cars along the same straight road. Carol is located at $x = 2.4$ mi at $t = 0$ h and drives at a steady 36 mph. Ann, who is traveling in the same direction, is located at $x = 0.0$ mi at $t = 0.50$ h and drives at a steady 50 mph.
a. At what time does Ann overtake Carol?
b. What is their position at this instant?
c. Draw a position-versus-time graph showing the motion of both Ann and Carol.

62. || A steel ball rolls across a 30-cm-wide felt pad, starting from one edge. The ball's speed has dropped to half after traveling 20 cm. Will the ball stop on the felt pad or roll off?

63. || A very slippery block of ice slides down a smooth ramp tilted at angle θ. The ice is released from rest at vertical height h above the bottom of the ramp. Find an expression for the speed of the ice at the bottom.

64. || **FIGURE P2.64** shows a fixed vertical disk of radius R. A thin, frictionless rod is attached to the bottom point of the disk and to a point on the edge, making angle ϕ (Greek phi) with the vertical. Find an expression for the time it takes a bead to slide from the top end of the rod to the bottom.

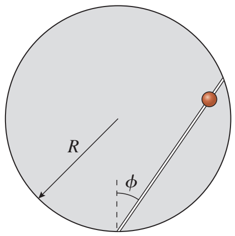


FIGURE P2.64

65. || A skateboarder starts up a 3.0-m-long ramp at 4.0 m/s. What is the maximum height of the ramp for which the skateboarder goes off the end rather than rolling back down?

66. || A motorist is driving at 20 m/s when she sees that a traffic light 200 m ahead has just turned red. She knows that this light stays red for 15 s, and she wants to reach the light just as it turns green again. It takes her 1.0 s to step on the brakes and begin slowing. What is her speed as she reaches the light at the instant it turns green?

67. || Nicole throws a ball straight up. Chad watches the ball from a window 5.0 m above the point where Nicole released it. The ball passes Chad on the way up, and it has a speed of 10 m/s as it passes him on the way back down. How fast did Nicole throw the ball?

68. || David is driving a steady 30 m/s when he passes Tina, who is sitting in her car at rest. Tina begins to accelerate at a steady 2.0 m/s^2 at the instant when David passes.
   a. How far does Tina drive before passing David?
   b. What is her speed as she passes him?

69. ||| If a Tesla Model S P100D in "Ludicrous mode" is pushed to
CALC its limit, the first 3.0 s of acceleration can be modeled as

$$a_x = \begin{cases} (35\text{ m/s}^3)t & 0\text{ s} \le t \le 0.40\text{ s} \\ 14.6\text{ m/s}^2 - (1.5\text{ m/s}^3)t & 0.40\text{ s} \le t \le 3.0\text{ s} \end{cases}$$

   a. How long does it take to accelerate to 60 mph? Your answer, which seems impossibly short, is confirmed by track tests.
   b. What acceleration would be needed to achieve the same speed in the same time at constant acceleration? Give your answer as a multiple of g.

70. ||| I was driving along at 20 m/s, trying to change a CD and not watching where I was going. When I looked up, I found myself 45 m from a railroad crossing. And wouldn't you know it, a train moving at 30 m/s was only 60 m from the crossing. In a split second, I realized that the train was going to beat me to the crossing and that I didn't have enough distance to stop. My only hope was to accelerate enough to cross the tracks before the train arrived. If my reaction time before starting to accelerate was 0.50 s, what minimum acceleration did my car need for me to be here today writing these words?

71. || As an astronaut visiting Planet X, you're assigned to measure the free-fall acceleration. Getting out your meter stick and stop watch, you time the fall of a heavy ball from several heights. Your data are as follows:

Height (m)	Fall time (s)
0.0	0.00
1.0	0.54
2.0	0.72
3.0	0.91
4.0	1.01
5.0	1.17

Analyze these data to determine the free-fall acceleration on Planet X. Your analysis method should involve fitting a straight line to an appropriate graph, similar to the analysis in Example 2.14.

72. ||| A ball is launched straight up at speed v_0. The second half of the total distance to the highest point is traveled during the final 1.0 s. How long does it take the ball to reach its maximum height?

73. || When a 1984 Alfa Romeo Spider sports car accelerates at
CALC the maximum possible rate, its motion during the first 20 s is extremely well modeled by the simple equation

$$v_x^2 = \frac{2P}{m}t$$

where $P = 3.6 \times 10^4$ watts is the car's power output, $m = 1200$ kg is its mass, and v_x is in m/s. That is, the square of the car's velocity increases linearly with time.
   a. Find an algebraic expression in terms of P, m, and t for the car's acceleration at time t.
   b. What is the car's speed at $t = 2$ s and $t = 10$ s?
   c. Evaluate the acceleration at $t = 2$ s and $t = 10$ s.

74. || Masses A and B in **FIGURE P2.74** slide on frictionless wires. They are connected by a pivoting rigid rod of length L. Prove that $v_{Bx} = -v_{Ay}\tan\theta$.

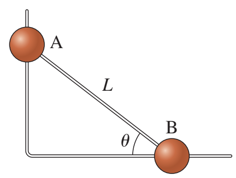


FIGURE P2.74

In Problems 75 through 78, you are given the kinematic equation or equations that are used to solve a problem. For each of these, you are to:
   a. Write a *realistic* problem for which this is the correct equation(s). Be sure that the answer your problem requests is consistent with the equation(s) given.
   b. Draw the pictorial representation for your problem.
   c. Finish the solution of the problem.

75. $64\text{ m} = 0\text{ m} + (32\text{ m/s})(4\text{ s} - 0\text{ s}) + \frac{1}{2}a_x(4\text{ s} - 0\text{ s})^2$

76. $(10\text{ m/s})^2 = v_{0y}{}^2 - 2(9.8\text{ m/s}^2)(10\text{ m} - 0\text{ m})$

77. $(0\text{ m/s})^2 = (5\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(\sin 10°)(x_1 - 0\text{ m})$

78. $v_{1x} = 0\text{ m/s} + (20\text{ m/s}^2)(5\text{ s} - 0\text{ s})$
$x_1 = 0\text{ m} + (0\text{ m/s})(5\text{ s} - 0\text{ s}) + \frac{1}{2}(20\text{ m/s}^2)(5\text{ s} - 0\text{ s})^2$
$x_2 = x_1 + v_{1x}(10\text{ s} - 5\text{ s})$

Challenge Problems

79. ||| Water drops fall from the edge of a roof at a steady rate. A fifth drop starts to fall just as the first drop hits the ground. At this instant, the second and third drops are exactly at the bottom and top edges of a 1.00-m-tall window. How high is the edge of the roof?

80. III A rocket is launched straight up with constant acceleration. Four seconds after liftoff, a bolt falls off the side of the rocket. The bolt hits the ground 6.0 s later. What was the rocket's acceleration?

81. III A good model for the acceleration of a car trying to reach top speed in the least amount of time is $a_x = a_0 - kv_x$, where a_0 is the initial acceleration and k is a constant.
CALC
   a. Find an expression for k in terms of a_0 and the car's top speed v_{max}.
   b. Find an expression for the car's velocity as a function of time.
   c. A MINI Cooper S has an initial acceleration of 4.0 m/s^2 and a top speed of 60 m/s. At maximum acceleration, how long does it take the car to reach 95% of its top speed?

82. III Careful measurements have been made of Olympic sprinters in the 100 meter dash. A quite realistic model is that the sprinter's velocity is given by
CALC

$$v_x = a(1 - e^{-bt})$$

where t is in s, v_x is in m/s, and the constants a and b are characteristic of the sprinter. Sprinter Carl Lewis's run at the 1987 World Championships is modeled with $a = 11.81$ m/s and $b = 0.6887$ s^{-1}.
   a. What was Lewis's acceleration at $t = 0$ s, 2.00 s, and 4.00 s?
   b. Find an expression for the distance traveled at time t.
   c. Your expression from part b is a transcendental equation, meaning that you can't solve it for t. However, it's not hard to use trial and error to find the time needed to travel a specific distance. To the nearest 0.01 s, find the time Lewis needed to sprint 100.0 m. His official time was 0.01 s more than your answer, showing that this model is very good, but not perfect.

83. III A sprinter can accelerate with constant acceleration for 4.0 s before reaching top speed. He can run the 100 meter dash in 10.0 s. What is his speed as he crosses the finish line?

84. III A rubber ball is shot straight up from the ground with speed v_0. Simultaneously, a second rubber ball at height h directly above the first ball is dropped from rest.
   a. At what height above the ground do the balls collide? Your answer will be an *algebraic expression* in terms of h, v_0, and g.
   b. What is the maximum value of h for which a collision occurs before the first ball falls back to the ground?
   c. For what value of h does the collision occur at the instant when the first ball is at its highest point?

85. III The Starship Enterprise returns from warp drive to ordinary space with a forward speed of 50 km/s. To the crew's great surprise, a Klingon ship is 100 km directly ahead, traveling in the same direction at a mere 20 km/s. Without evasive action, the Enterprise will overtake and collide with the Klingons in just slightly over 3.0 s. The Enterprise's computers react instantly to brake the ship. What magnitude acceleration does the Enterprise need to just barely avoid a collision with the Klingon ship? Assume the acceleration is constant.

Hint: Draw a position-versus-time graph showing the motions of both the Enterprise and the Klingon ship. Let $x_0 = 0$ km be the location of the Enterprise as it returns from warp drive. How do you show graphically the situation in which the collision is "barely avoided"? Once you decide what it looks like graphically, express that situation mathematically.

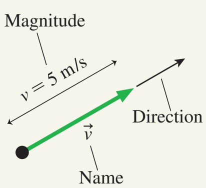
3 Vectors and Coordinate Systems

Wind has both a speed and a direction, hence the motion of the wind is described by a vector.

IN THIS CHAPTER, you will learn how vectors are represented and used.

What is a vector?

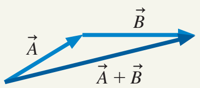
A **vector** is a quantity with both a size—its **magnitude**—and a **direction**. Vectors you'll meet in the next few chapters include position, displacement, velocity, acceleration, force, and momentum.



« LOOKING BACK **Tactics Box 1.1** on vector addition

How are vectors added and subtracted?

Vectors are **added** "tip to tail." The order of addition does not matter. To **subtract** vectors, **turn the subtraction into addition** by writing $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$. The vector $-\vec{B}$ is the same length as $\vec{B}$ but points in the opposite direction.

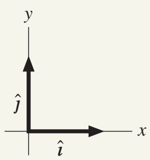


What are unit vectors?

Unit vectors define what we *mean* by the **$+x$- and $+y$-directions** in space.

- A unit vector has magnitude 1.
- A unit vector has no units.

Unit vectors simply point.

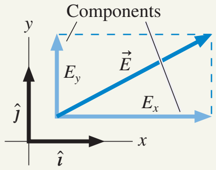


What are components?

Components of vectors are the pieces of vectors parallel to the coordinate axes—in the directions of the unit vectors. We write

$$\vec{E} = E_x\hat{\imath} + E_y\hat{j}$$

Components simplify vector math.



How are components used?

Components let us do **vector math** with algebra, which is easier and more precise than adding and subtracting vectors using geometry and trigonometry. Multiplying a vector by a number simply multiplies all of the vector's components by that number.

$$\vec{C} = 2\vec{A} + 3\vec{B}$$

means

$$\begin{cases} C_x = 2A_x + 3B_x \\ C_y = 2A_y + 3B_y \end{cases}$$

How will I use vectors?

Vectors appear everywhere in physics and engineering—from velocities to electric fields and from forces to fluid flows. The tools and techniques you learn in this chapter will be used throughout your studies and your professional career.

3.1 Scalars and Vectors

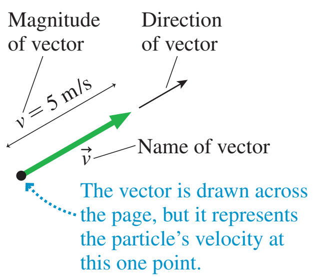
A quantity that is fully described by a single number (with units) is called a **scalar.** Mass, temperature, volume, and energy are all scalars. We will often use an algebraic symbol to represent a scalar quantity. Thus m will represent mass, T temperature, V volume, E energy, and so on.

Our universe has three dimensions, so some quantities also need a direction for a full description. If you ask someone for directions to the post office, the reply "Go three blocks" will not be very helpful. A full description might be, "Go three blocks south." A quantity having both a size and a direction is called a **vector.**

The mathematical term for the length, or size, of a vector is **magnitude,** so we can also say that a vector is **a quantity having a magnitude and a direction.**

FIGURE 3.1 shows that the *geometric representation* of a vector is an arrow, with the tail of the arrow (not its tip!) placed at the point where the measurement is made. An arrow makes a natural representation of a vector because it inherently has both a length and a direction. As you've already seen, we label vectors by drawing a small arrow over the letter that represents the vector: $\vec{r}$ for position, $\vec{v}$ for velocity, $\vec{a}$ for acceleration.

FIGURE 3.1 The velocity vector $\vec{v}$ has both a magnitude and a direction.



NOTE Although the vector arrow is drawn across the page, from its tail to its tip, this does *not* indicate that the vector "stretches" across this distance. Instead, the vector arrow tells us the value of the vector quantity only at the one point where the tail of the vector is placed.

The magnitude of a vector can be written using absolute value signs or, more frequently, as the letter without the arrow. For example, the magnitude of the velocity vector in Figure 3.1 is $v = |\vec{v}| = 5$ m/s. This is the object's *speed.* The magnitude of the acceleration vector $\vec{a}$ is written a. **The magnitude of a vector is a scalar.** Note that magnitude of a vector cannot be a negative number; it must be positive or zero, with appropriate units.

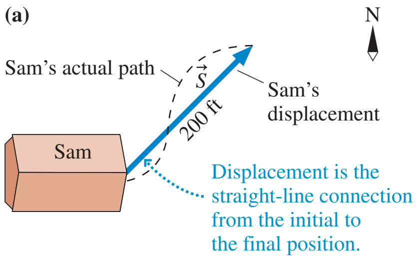
It is important to get in the habit of using the arrow symbol for vectors. If you omit the vector arrow from the velocity vector $\vec{v}$ and write only v, then you're referring only to the object's speed, not its velocity. The symbols $\vec{r}$ and r, or $\vec{v}$ and v, do *not* represent the same thing.

3.2 Using Vectors

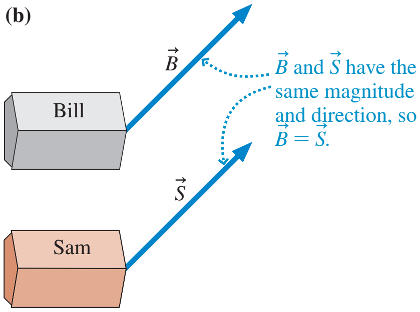
Suppose Sam starts from his front door, walks across the street, and ends up 200 ft to the northeast of where he started. Sam's displacement, which we will label $\vec{S}$, is shown in FIGURE 3.2a. The displacement vector is a *straight-line connection* from his initial to his final position, not necessarily his actual path.

FIGURE 3.2 Displacement vectors.

(a)



(b)



To describe a vector we must specify both its magnitude and its direction. We can write Sam's displacement as $\vec{S} = (200\text{ ft, northeast})$. The magnitude of Sam's displacement is $S = |\vec{S}| = 200$ ft, the distance between his initial and final points.

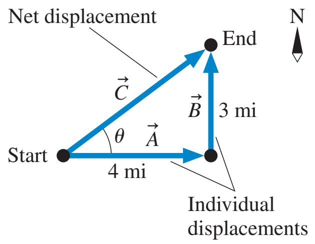
Sam's next-door neighbor Bill also walks 200 ft to the northeast, starting from his own front door. Bill's displacement $\vec{B} = (200\text{ ft, northeast})$ has the same magnitude and direction as Sam's displacement $\vec{S}$. Because vectors are defined by their magnitude and direction, **two vectors are equal if they have the same magnitude and direction.** Thus the two displacements in FIGURE 3.2b are equal to each other, and we can write $\vec{B} = \vec{S}$.

NOTE A vector is unchanged if you move it to a different point on the page as long as you don't change its length or the direction it points.

Vector Addition

If you earn \$50 on Saturday and \$60 on Sunday, your *net* income for the weekend is the sum of \$50 and \$60. With numbers, the word *net* implies addition. The same is true with vectors. For example, **FIGURE 3.3** shows the displacement of a hiker who first hikes 4 miles to the east, then 3 miles to the north. The first leg of the hike is described by the displacement $\vec{A} = (4\text{ mi, east})$. The second leg of the hike has displacement $\vec{B} = (3\text{ mi, north})$. Vector $\vec{C}$ is the *net displacement* because it describes the net result of the hiker's first having displacement $\vec{A}$, then displacement $\vec{B}$.

FIGURE 3.3 The net displacement $\vec{C}$ resulting from two displacements $\vec{A}$ and $\vec{B}$.



The net displacement $\vec{C}$ is an initial displacement $\vec{A}$ *plus* a second displacement $\vec{B}$, or

$$\vec{C} = \vec{A} + \vec{B} \tag{3.1}$$

The sum of two vectors is called the **resultant vector.** It's not hard to show that vector addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. That is, you can add vectors in any order you wish.

« **TACTICS BOX 1.1** on page 28 showed the three-step procedure for adding two vectors, and it's highly recommended that you turn back for a quick review. This tip-to-tail method for adding vectors, which is used to find $\vec{C} = \vec{A} + \vec{B}$ in Figure 3.3, is called **graphical addition.** Any two vectors of the same type—two velocity vectors or two force vectors—can be added in exactly the same way.

The graphical method for adding vectors is straightforward, but we need to do a little geometry to come up with a complete description of the resultant vector $\vec{C}$. Vector $\vec{C}$ of Figure 3.3 is defined by its magnitude C and by its direction. Because the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ form a right triangle, the magnitude, or length, of $\vec{C}$ is given by the Pythagorean theorem:

$$C = \sqrt{A^2 + B^2} = \sqrt{(4\text{ mi})^2 + (3\text{ mi})^2} = 5\text{ mi} \tag{3.2}$$

Notice that Equation 3.2 uses the magnitudes A and B of the vectors $\vec{A}$ and $\vec{B}$. The angle θ, which is used in Figure 3.3 to describe the direction of $\vec{C}$, is easily found for a right triangle:

$$\theta = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{3\text{ mi}}{4\text{ mi}}\right) = 37° \tag{3.3}$$

Altogether, the hiker's net displacement is $\vec{C} = \vec{A} + \vec{B} = (5\text{ mi, }37°\text{ north of east})$.

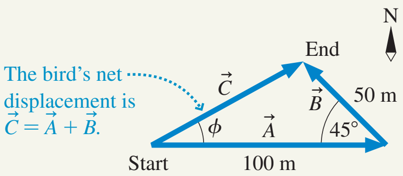
NOTE Vector mathematics makes extensive use of geometry and trigonometry. Appendix A, at the end of this book, contains a brief review of these topics.

EXAMPLE 3.1 ■ Using graphical addition to find a displacement

A bird flies 100 m due east from a tree, then 50 m northwest (that is, 45° north of west). What is the bird's net displacement?

VISUALIZE **FIGURE 3.4** shows the two individual displacements, which we've called $\vec{A}$ and $\vec{B}$. The net displacement is the vector sum $\vec{C} = \vec{A} + \vec{B}$, which is found graphically.

FIGURE 3.4 The bird's net displacement is $\vec{C} = \vec{A} + \vec{B}$



SOLVE The two displacements are $\vec{A} = (100\text{ m, east})$ and $\vec{B} = (50\text{ m, northwest})$. The net displacement $\vec{C} = \vec{A} + \vec{B}$ is found by drawing a vector from the initial to the final position. But describing $\vec{C}$ is a bit trickier than the example of the hiker because $\vec{A}$ and $\vec{B}$ are not at right angles. First, we can find the magnitude of $\vec{C}$ by using the law of cosines from trigonometry:

$$\begin{aligned} C^2 &= A^2 + B^2 - 2AB\cos 45° \\ &= (100\text{ m})^2 + (50\text{ m})^2 - 2(100\text{ m})(50\text{ m})\cos 45° \\ &= 5430\text{ m}^2 \end{aligned}$$

Thus $C = \sqrt{5430\text{ m}^2} = 74\text{ m}$. Then a second use of the law of cosines can determine angle ϕ (the Greek letter phi):

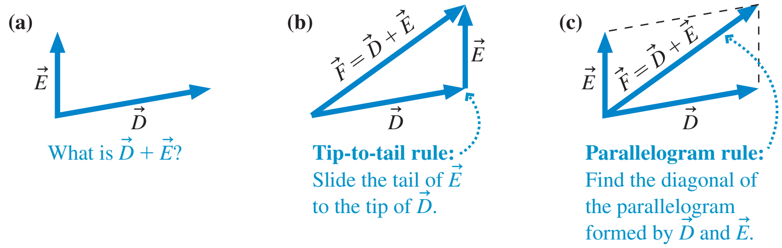
$$B^2 = A^2 + C^2 - 2AC\cos\phi$$

$$\phi = \cos^{-1}\left[\frac{A^2 + C^2 - B^2}{2AC}\right] = 29°$$

The bird's net displacement is

$$\vec{C} = (74\text{ m, }29°\text{ north of east})$$

It is often convenient to draw two vectors with their tails together, as shown in **FIGURE 3.5a**. To evaluate $\vec{D}+\vec{E}$, you could move vector $\vec{E}$ over to where its tail is on the tip of $\vec{D}$, then use the tip-to-tail rule of graphical addition. That gives vector $\vec{F}=\vec{D}+\vec{E}$ in **FIGURE 3.5b**. Alternatively, **FIGURE 3.5c** shows that the vector sum $\vec{D}+\vec{E}$ can be found as the diagonal of the parallelogram defined by $\vec{D}$ and $\vec{E}$. This method for vector addition is called the *parallelogram rule* of vector addition.



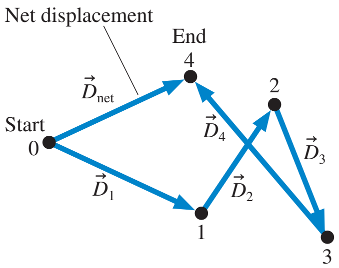
▶ **FIGURE 3.5** Two vectors can be added using the tip-to-tail rule or the parallelogram rule.

Vector addition is easily extended to more than two vectors. **FIGURE 3.6** shows the path of a hiker moving from initial position 0 to position 1, then position 2, then position 3, and finally arriving at position 4. These four segments are described by displacement vectors $\vec{D}_1$, $\vec{D}_2$, $\vec{D}_3$, and $\vec{D}_4$. The hiker's *net* displacement, an arrow from position 0 to position 4, is the vector $\vec{D}_{\text{net}}$. In this case,

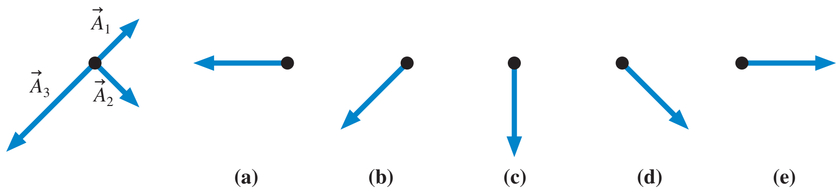
$$\vec{D}_{\text{net}}=\vec{D}_1+\vec{D}_2+\vec{D}_3+\vec{D}_4 \tag{3.4}$$

The vector sum is found by using the tip-to-tail method three times in succession.

FIGURE 3.6 The net displacement after four individual displacements.



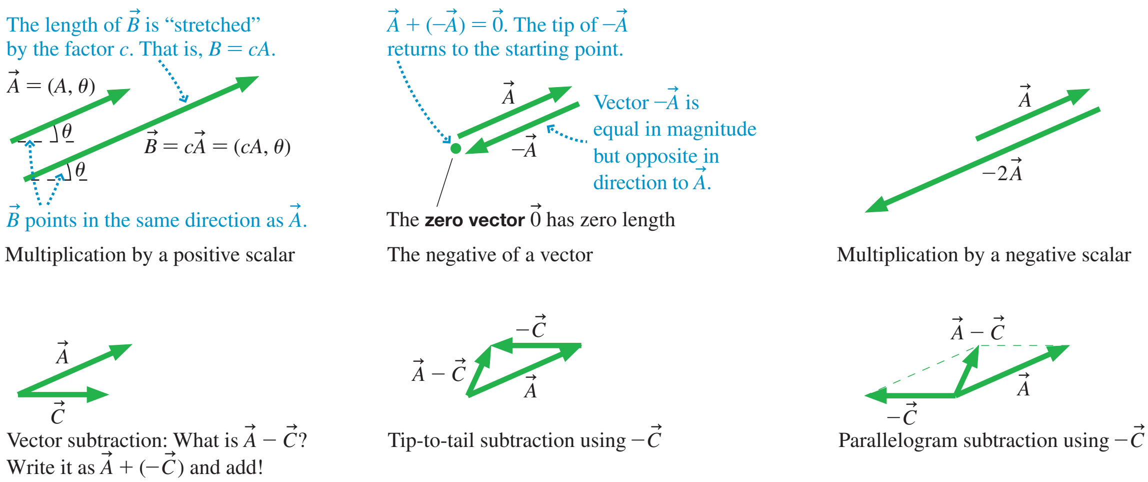
STOP TO THINK 3.1: Which figure shows $\vec{A}_1+\vec{A}_2+\vec{A}_3$?



More Vector Mathematics

In addition to adding vectors, we will need to subtract vectors, multiply vectors by scalars, and understand how to interpret the negative of a vector. These operations are illustrated in **FIGURE 3.7**.

FIGURE 3.7 Working with vectors.



EXAMPLE 3.2 ■ **Velocity and displacement**

Carolyn drives her car north at 30 km/h for 1 hour, east at 60 km/h for 2 hours, then north at 50 km/h for 1 hour. What is Carolyn's net displacement?

SOLVE Chapter 1 defined average velocity as

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t}$$

so the displacement $\Delta\vec{r}$ during the time interval Δt is $\Delta\vec{r} = (\Delta t)\,\vec{v}$. This is multiplication of the vector $\vec{v}$ by the scalar Δt. Carolyn's velocity during the first hour is $\vec{v}_1 = (30 \text{ km/h, north})$, so her displacement during this interval is

$$\Delta\vec{r}_1 = (1 \text{ hour})(30 \text{ km/h, north}) = (30 \text{ km, north})$$

Similarly,

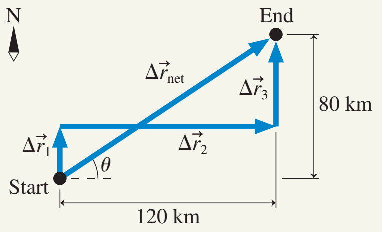
$$\Delta\vec{r}_2 = (2 \text{ hours})(60 \text{ km/h, east}) = (120 \text{ km, east})$$

$$\Delta\vec{r}_3 = (1 \text{ hour})(50 \text{ km/h, north}) = (50 \text{ km, north})$$

In this case, multiplication by a scalar changes not only the length of the vector but also its units, from km/h to km. The direction, however, is unchanged. Carolyn's net displacement is

$$\Delta\vec{r}_{\text{net}} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$$

FIGURE 3.8 The net displacement is the vector sum $\Delta\vec{r}_{\text{net}} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$.



This addition of the three vectors is shown in FIGURE 3.8, using the tip-to-tail method. $\Delta\vec{r}_{\text{net}}$ stretches from Carolyn's initial position to her final position. The magnitude of her net displacement is found using the Pythagorean theorem:

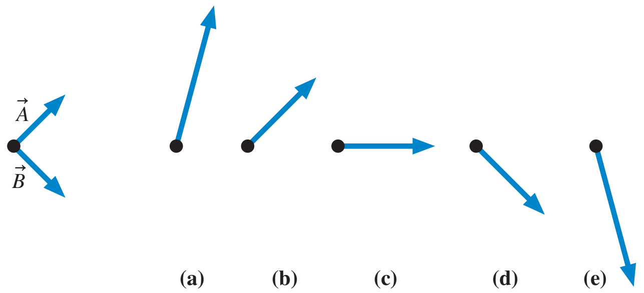
$$r_{\text{net}} = \sqrt{(120 \text{ km})^2 + (80 \text{ km})^2} = 144 \text{ km}$$

The direction of $\Delta\vec{r}_{\text{net}}$ is described by angle θ, which is

$$\theta = \tan^{-1}\left(\frac{80 \text{ km}}{120 \text{ km}}\right) = 34°$$

Thus Carolyn's net displacement is $\Delta\vec{r}_{\text{net}} = (144 \text{ km, } 34° \text{ north of east})$.

STOP TO THINK 3.2: Which figure shows $2\vec{A} - \vec{B}$



3.3 Coordinate Systems and Vector Components

Vectors do not require a coordinate system. We can add and subtract vectors graphically, and we will do so frequently to clarify our understanding of a situation. But the graphical addition of vectors is not an especially good way to find quantitative results. In this section we will introduce a *coordinate representation* of vectors that will be the basis of an easier method for doing vector calculations.

A GPS uses satellite signals to find your position in the earth's coordinate system with amazing accuracy.

Coordinate Systems

The world does not come with a coordinate system attached to it. A coordinate system is an artificially imposed grid that you place on a problem in order to make quantitative measurements. You are free to choose:

- Where to place the origin, and
- How to orient the axes.

Different problem solvers may choose to use different coordinate systems; that is perfectly acceptable. However, some coordinate systems will make a problem easier to solve. Part of our goal is to learn how to choose an appropriate coordinate system for each problem.

FIGURE 3.9 A conventional xy-coordinate system and the quadrants of the xy-plane.

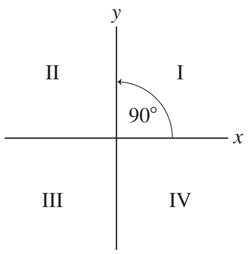


FIGURE 3.9 shows the xy-coordinate system we will use in this book. The placement of the axes is not entirely arbitrary: the positive y-axis is always located 90° *counterclockwise* (ccw) from the positive x-axis. Figure 3.9 also identifies the four **quadrants** of the coordinate system, I through IV.

Coordinate axes have a positive end and a negative end, separated by zero at the origin where the two axes cross. When you draw a coordinate system, it is important to label the axes. This is done by placing x and y labels at the *positive* ends of the axes, as in Figure 3.9. The purpose of the labels is twofold:

- To identify which axis is which, and
- To identify the positive ends of the axes.

This will be important when you need to determine whether the quantities in a problem should be assigned positive or negative values. **This textbook will follow the convention that the positive direction of the x-axis is to the right and the positive direction of the y-axis is up.**

Component Vectors

FIGURE 3.10 Component vectors $\vec{A}_x$ and $\vec{A}_y$ are drawn parallel to the coordinate axes such that $\vec{A} = \vec{A}_x + \vec{A}_y$.

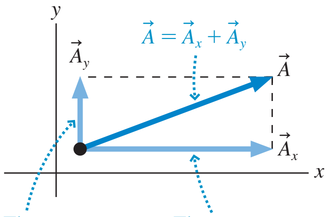


FIGURE 3.10 shows a vector $\vec{A}$ and an xy-coordinate system that we've chosen. Once the directions of the axes are known, we can define two new vectors *parallel to the axes* that we call the **component vectors** of $\vec{A}$. You can see, using the parallelogram rule, that $\vec{A}$ is the vector sum of the two component vectors:

$$\vec{A} = \vec{A}_x + \vec{A}_y \tag{3.5}$$

In essence, we have broken vector $\vec{A}$ into two perpendicular vectors that are parallel to the coordinate axes. This process is called the **decomposition** of vector $\vec{A}$ into its component vectors.

NOTE It is not necessary for the tail of $\vec{A}$ to be at the origin. All we need to know is the *orientation* of the coordinate system so that we can draw $\vec{A}_x$ and $\vec{A}_y$ parallel to the axes.

Components

You learned in Chapters 1 and 2 to give the kinematic variable v_x a positive sign if the velocity vector $\vec{v}$ points toward the positive end of the x-axis, a negative sign if $\vec{v}$ points in the negative x-direction. We need to extend this idea to vectors in general.

Suppose vector $\vec{A}$ has been decomposed into component vectors $\vec{A}_x$ and $\vec{A}_y$ parallel to the coordinate axes. We can describe each component vector with a single number called the **component.** The *x-component* and *y-component* of vector $\vec{A}$, denoted A_x and A_y, are determined as follows:

TACTICS BOX 3.1

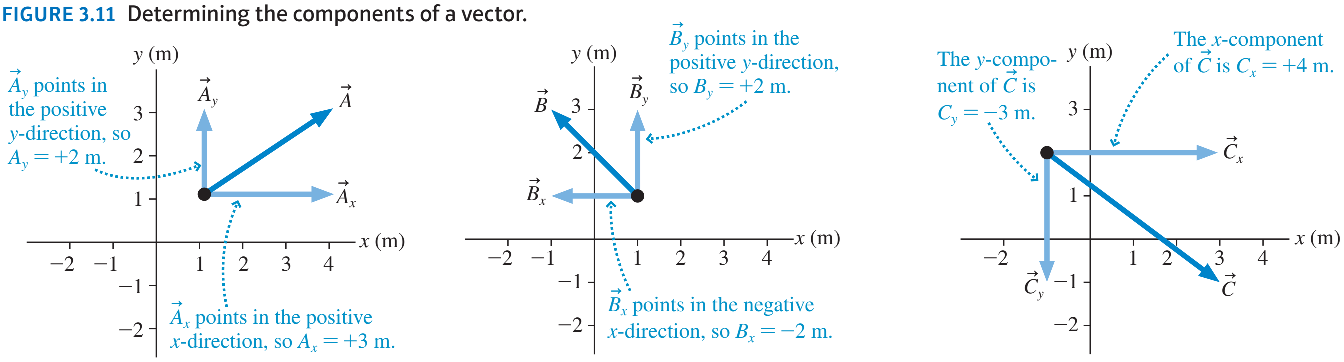
Determining the components of a vector

1. The absolute value $|A_x|$ of the x-component A_x is the magnitude of the component vector $\vec{A}_x$.
2. The sign of A_x is positive if $\vec{A}_x$ points in the positive x-direction (right), negative if $\vec{A}_x$ points in the negative x-direction (left).
3. The y-component A_y is determined similarly.

Exercises 10–18

In other words, the component A_x tells us two things: how big $\vec{A}_x$ is and, with its sign, which end of the axis $\vec{A}_x$ points toward. FIGURE 3.11 shows three examples of determining the components of a vector.

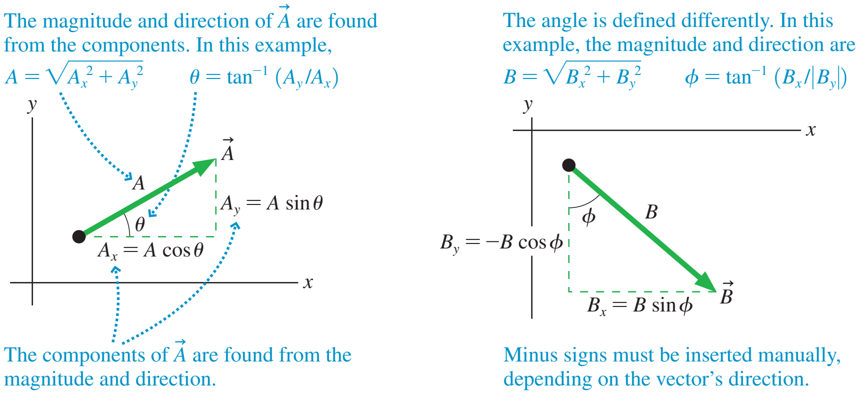
FIGURE 3.11 Determining the components of a vector.



NOTE Beware of the somewhat confusing terminology. $\vec{A}_x$ and $\vec{A}_y$ are called *component vectors,* whereas A_x and A_y are simply called *components*. The components A_x and A_y are just numbers (with units), so make sure you do *not* put arrow symbols over the components.

We will frequently need to decompose a vector into its components. We will also need to "reassemble" a vector from its components. In other words, we need to move back and forth between the geometric and the component representations of a vector. **FIGURE 3.12** shows how this is done.

FIGURE 3.12 Moving between the geometric representation and the component representation.



Each decomposition requires that you pay close attention to the direction in which the vector points and the angles that are defined.

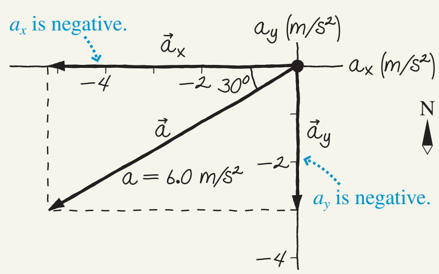
- If a component vector points left (or down), you must *manually* insert a minus sign in front of the component, as was done for B_y in Figure 3.12.
- The role of sines and cosines can be reversed, depending upon which angle is used to define the direction. Compare A_x and B_x.
- The angle used to define direction is almost always between 0° and 90°, so you must take the inverse tangent of a positive number. Use absolute values of the components, as was done to find angle ϕ (Greek phi) in Figure 3.12.

EXAMPLE 3.3 ■ Finding the components of an acceleration vector

Seen from above, a hummingbird's acceleration is (6.0 m/s^2, 30° south of west). Find the x- and y-components of the acceleration vector $\vec{a}$.

VISUALIZE It's important to *draw* vectors. **FIGURE 3.13** establishes a map-like coordinate system with the x-axis pointing east and the y-axis north. Vector $\vec{a}$ is then decomposed into components parallel to the axes. Notice that the axes are "acceleration axes" with units of acceleration, not xy-axes, because we're measuring an acceleration vector.

FIGURE 3.13 Decomposition of $\vec{a}$.



Continued

SOLVE The acceleration vector points to the left (negative x-direction) and down (negative y-direction), so the components a_x and a_y are both negative:

$$a_x = -a\cos 30° = -(6.0\text{ m/s}^2)\cos 30° = -5.2\text{ m/s}^2$$

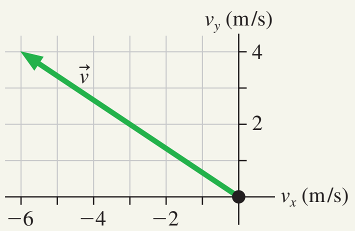
$$a_y = -a\sin 30° = -(6.0\text{ m/s}^2)\sin 30° = -3.0\text{ m/s}^2$$

REVIEW The units of a_x and a_y are the same as the units of vector $\vec{a}$. Notice that we had to insert the minus signs manually by observing that the vector points left and down.

EXAMPLE 3.4 ■ Finding the direction of motion

FIGURE 3.14 shows a car's velocity vector $\vec{v}$. Determine the car's speed and direction of motion.

FIGURE 3.14 The velocity vector $\vec{v}$ of Example 3.4.

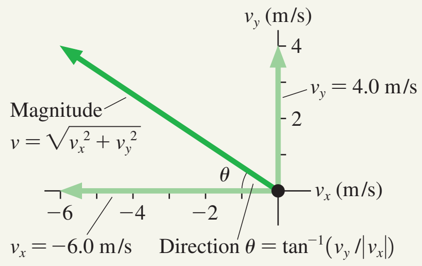


VISUALIZE **FIGURE 3.15** shows the components v_x and v_y and defines an angle θ with which we can specify the direction of motion.

SOLVE We can read the components of $\vec{v}$ directly from the axes: $v_x = -6.0$ m/s and $v_y = 4.0$ m/s. Notice that v_x is negative. This is enough information to find the car's speed v, which is the magnitude of $\vec{v}$:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-6.0\text{ m/s})^2 + (4.0\text{ m/s})^2} = 7.2\text{ m/s}$$

FIGURE 3.15 Decomposition of $\vec{v}$.



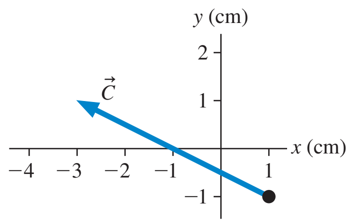
From trigonometry, angle θ is

$$\theta = \tan^{-1}\left(\frac{v_y}{|v_x|}\right) = \tan^{-1}\left(\frac{4.0\text{ m/s}}{6.0\text{ m/s}}\right) = 34°$$

The absolute value signs are necessary because v_x is a negative number. The velocity vector $\vec{v}$ can be written in terms of the speed and the direction of motion as

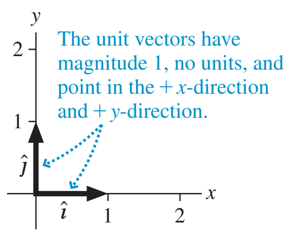
$$\vec{v} = (7.2\text{ m/s, }34°\text{ above the negative }x\text{-axis})$$

STOP TO THINK 3.3: What are the x- and y-components C_x and C_y of vector $\vec{C}$?



3.4 Unit Vectors and Vector Algebra

FIGURE 3.16 The unit vectors $\hat{\imath}$ and $\hat{\jmath}$.



The vectors (1, $+x$-direction) and (1, $+y$-direction), shown in **FIGURE 3.16**, have some interesting and useful properties. Each has a magnitude of 1, has no units, and is parallel to a coordinate axis. A vector with these properties is called a **unit vector.** These unit vectors have the special symbols

$$\hat{\imath} \equiv (1\text{, positive }x\text{-direction})$$

$$\hat{\jmath} \equiv (1\text{, positive }y\text{-direction})$$

The notation $\hat{\imath}$ (read "i hat") and $\hat{\jmath}$ (read "j hat") indicates a unit vector with a magnitude of 1. Recall that the symbol $\equiv$ means "is defined as."

Unit vectors establish the directions of the positive axes of the coordinate system. Our choice of a coordinate system may be arbitrary, but once we decide to place a coordinate system on a problem we need something to tell us "That direction is the positive x-direction." This is what the unit vectors do.

The unit vectors provide a useful way to write component vectors. The component vector $\vec{A}_x$ is the piece of vector $\vec{A}$ that is parallel to the x-axis. Similarly, $\vec{A}_y$ is parallel to the y-axis. Because, by definition, the vector $\hat{\imath}$ points along the x-axis and $\hat{\jmath}$ points along the y-axis, we can write

$$\vec{A}_x = A_x\hat{\imath} \qquad \vec{A}_y = A_y\hat{\jmath} \tag{3.6}$$

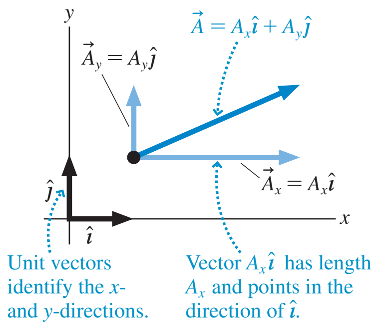
Equations 3.6 separate each component vector into a length and a direction. The full decomposition of vector $\vec{A}$ can then be written

$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{\jmath} \tag{3.7}$$

FIGURE 3.17 shows how the unit vectors and the components fit together to form vector $\vec{A}$.

NOTE In three dimensions, the unit vector along the $+z$-direction is called $\hat{k}$, and to describe vector $\vec{A}$ we would include an additional component vector $\vec{A}_z = A_z\hat{k}$.

FIGURE 3.17 The decomposition of vector $\vec{A}$ is $A_x\hat{\imath} + A_y\hat{\jmath}$.

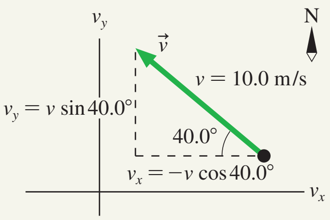


EXAMPLE 3.5 ■ Run rabbit run!

A rabbit, escaping a fox, runs 40.0° north of west at 10.0 m/s. A coordinate system is established with the positive x-axis to the east and the positive y-axis to the north. Write the rabbit's velocity in terms of components and unit vectors.

VISUALIZE **FIGURE 3.18** shows the rabbit's velocity vector and the coordinate axes. We're showing a velocity vector, so the axes are labeled v_x and v_y rather than x and y.

FIGURE 3.18 The velocity vector $\vec{v}$ is decomposed into components v_x and v_y.



SOLVE 10.0 m/s is the rabbit's *speed,* not its velocity. The velocity, which includes directional information, is

$$\vec{v} = (10.0 \text{ m/s}, 40.0° \text{ north of west})$$

Vector $\vec{v}$ points to the left and up, so the components v_x and v_y are negative and positive, respectively. The components are

$$v_x = -(10.0 \text{ m/s}) \cos 40.0° = -7.66 \text{ m/s}$$

$$v_y = +(10.0 \text{ m/s}) \sin 40.0° = 6.43 \text{ m/s}$$

With v_x and v_y now known, the rabbit's velocity vector is

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (-7.66\hat{\imath} + 6.43\hat{\jmath}) \text{ m/s}$$

Notice that we've pulled the units to the end, rather than writing them with each component.

REVIEW Notice that the minus sign for v_x was inserted manually. **Signs don't occur automatically; you have to set them after checking the vector's direction.**

Vector Math

You learned in Section 3.2 how to add vectors graphically, but it can be a tedious problem in geometry and trigonometry to find precise values for the magnitude and direction of the resultant. The addition and subtraction of vectors become much easier if we use components and unit vectors.

To see this, let's evaluate the vector sum $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. To begin, write this sum in terms of the components of each vector:

$$\begin{aligned}\vec{D} = D_x\hat{\imath} + D_y\hat{\jmath} &= \vec{A} + \vec{B} + \vec{C} \\ &= (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath}) + (C_x\hat{\imath} + C_y\hat{\jmath})\end{aligned} \tag{3.8}$$

We can group together all the x-components and all the y-components on the right side, in which case Equation 3.8 is

$$(D_x)\hat{\imath} + (D_y)\hat{\jmath} = (A_x + B_x + C_x)\hat{\imath} + (A_y + B_y + C_y)\hat{\jmath} \tag{3.9}$$

Comparing the x- and y-components on the left and right sides of Equation 3.9, we find:

$$\begin{aligned}D_x &= A_x + B_x + C_x \\ D_y &= A_y + B_y + C_y\end{aligned} \tag{3.10}$$

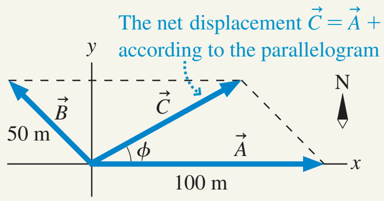
Stated in words, Equation 3.10 says that we can perform vector addition by adding the x-components of the individual vectors to give the x-component of the resultant and by adding the y-components of the individual vectors to give the y-component of the resultant. This method of vector addition is called **algebraic addition.**

EXAMPLE 3.6 ■ Using algebraic addition to find a displacement

Example 3.1 was about a bird that flew 100 m to the east, then 50 m to the northwest. Use the algebraic addition of vectors to find the bird's net displacement.

VISUALIZE **FIGURE 3.19** shows displacement vectors $\vec{A} = (100 \text{ m, east})$ and $\vec{B} = (50 \text{ m, northwest})$. We draw vectors tip-to-tail to add them graphically, but it's usually easier to draw them all from the origin if we are going to use algebraic addition.

FIGURE 3.19 The net displacement is $\vec{C} = \vec{A} + \vec{B}$.



SOLVE To add the vectors algebraically we must know their components. From the figure these are seen to be

$$\vec{A} = 100\,\hat{\imath} \text{ m}$$
$$\vec{B} = (-50 \cos 45°\,\hat{\imath} + 50 \sin 45°\hat{\jmath}) \text{ m} = (-35.3\hat{\imath} + 35.3\hat{\jmath}) \text{ m}$$

Notice that vector quantities must include units. Also notice, as you would expect from the figure, that $\vec{C}$ has a negative x-component. Adding $\vec{A}$ and $\vec{B}$ by components gives

$$\vec{C} = \vec{A} + \vec{B} = 100\hat{\imath} \text{ m} + (-35.3\hat{\imath} + 35.3\hat{\jmath}) \text{ m}$$
$$= (100 \text{ m} - 35.3 \text{ m})\hat{\imath} + (35.3 \text{ m})\hat{\jmath} = (64.7\hat{\imath} + 35.3\hat{\jmath}) \text{ m}$$

This would be a perfectly acceptable answer for many purposes. However, we need to calculate the magnitude and direction of $\vec{C}$ if we want to compare this result to our earlier answer. The magnitude of $\vec{C}$ is

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(64.7 \text{ m})^2 + (35.3 \text{ m})^2} = 74 \text{ m}$$

The angle ϕ, as defined in Figure 3.19, is

$$\phi = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{35.3 \text{ m}}{64.7 \text{ m}}\right) = 29°$$

Thus $\vec{C} = (74 \text{ m, } 29° \text{ north of east})$, in perfect agreement with Example 3.1.

Vector subtraction and the multiplication of a vector by a scalar, using components, are very much like vector addition. To find $\vec{R} = \vec{P} - \vec{Q}$ we would compute

$$R_x = P_x - Q_x$$
$$R_y = P_y - Q_y \tag{3.11}$$

Similarly, $\vec{T} = c\vec{S}$ would be

$$T_x = cS_x$$
$$T_y = cS_y \tag{3.12}$$

In other words, a vector equation is interpreted as meaning: Equate the x-components on both sides of the equals sign, then equate the y-components, and then the z-components. Vector notation allows us to write these three equations in a compact form.

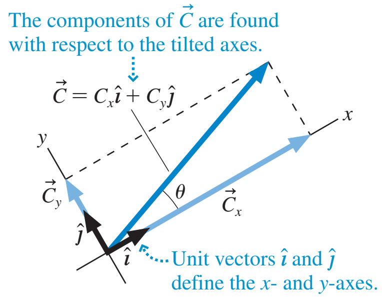
Tilted Axes and Arbitrary Directions

As we've noted, the coordinate system is entirely your choice. It is a grid that you impose on the problem in a manner that will make the problem easiest to solve. As you saw in Chapter 2, it is often convenient to tilt the axes of the coordinate system, such as those shown in **FIGURE 3.20**. The axes are perpendicular, and the y-axis is oriented correctly with respect to the x-axis, so this is a legitimate coordinate system. There is no requirement that the x-axis has to be horizontal.

Finding components with tilted axes is no harder than what we have done so far. Vector $\vec{C}$ in Figure 3.20 can be decomposed into $\vec{C} = C_x\hat{\imath} + C_y\hat{\jmath}$, where $C_x = C\cos\theta$ and $C_y = C\sin\theta$. Note that the unit vectors $\hat{\imath}$ and $\hat{\jmath}$ correspond to the *axes,* not to "horizontal" and "vertical," so they are also tilted.

Tilted axes are useful if you need to determine component vectors "parallel to" and "perpendicular to" an arbitrary line or surface. This is illustrated in the following example.

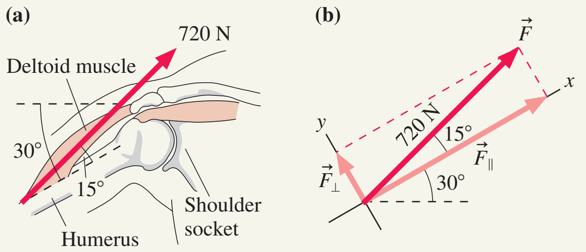
FIGURE 3.20 A coordinate system with tilted axes.



EXAMPLE 3.7 ■ Muscle and bone

The deltoid—the rounded muscle across the top of your upper arm—allows you to lift your arm away from your side. It does so by pulling on an attachment point on the humerus, the upper arm bone, at an angle of 15° with respect to the humerus. If you hold your arm at an angle 30° below horizontal, the deltoid must pull with a force of 720 N to support the weight of your arm, as shown in **FIGURE 3.21a**. (You'll learn in Chapter 5 that force is a vector quantity measured in units of *newtons*, abbreviated N.) What are the components of the muscle force parallel to and perpendicular to the bone?

FIGURE 3.21 Finding the components of force parallel and perpendicular to the humerus.



VISUALIZE **FIGURE 3.21b** shows a tilted coordinate system with the x-axis parallel to the humerus. The force $\vec{F}$ is shown 15° from the x-axis. The component of force parallel to the bone, which we can denote $F_\parallel$, is equivalent to the x-component: $F_\parallel = F_x$. Similarly, the component of force perpendicular to the bone is $F_\perp = F_y$.

SOLVE From the geometry of Figure 3.21b, we see that

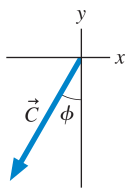
$$F_\parallel = F\cos 15° = (720\text{ N})\cos 15° = 695\text{ N}$$

$$F_\perp = F\sin 15° = (720\text{ N})\sin 15° = 186\text{ N}$$

REVIEW The muscle pulls nearly parallel to the bone, so we expected $F_\parallel \approx 720$ N and $F_\perp \ll F_\parallel$. Thus our results seem reasonable.

STOP TO THINK 3.4: Angle ϕ that specifies the direction of $\vec{C}$ is given by

a. $\tan^{-1}(|C_x|/C_y)$ b. $\tan^{-1}(C_x/|C_y|)$
c. $\tan^{-1}(|C_x|/|C_y|)$ d. $\tan^{-1}(|C_y|/C_x)$
e. $\tan^{-1}(C_y/|C_x|)$ f. $\tan^{-1}(|C_y|/|C_x|)$



CHAPTER 3 CHALLENGE EXAMPLE Finding the net force

FIGURE 3.22 shows three forces acting at one point. What is the net force $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$?

VISUALIZE Figure 3.22 shows the forces and a tilted coordinate system.

SOLVE The vector equation $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ is really two simultaneous equations:

$$(F_{net})_x = F_{1x} + F_{2x} + F_{3x}$$

$$(F_{net})_y = F_{1y} + F_{2y} + F_{3y}$$

The components of the forces are determined with respect to the axes. Thus

$$F_{1x} = F_1\cos 45° = (50\text{ N})\cos 45° = 35\text{ N}$$

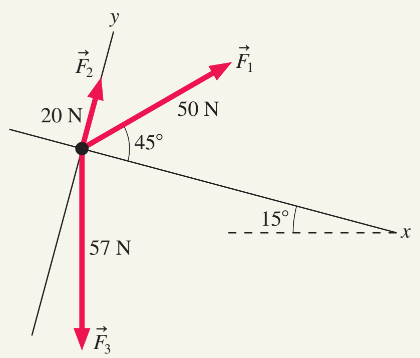
$$F_{1y} = F_1\sin 45° = (50\text{ N})\sin 45° = 35\text{ N}$$

$\vec{F}_2$ is easier. It is pointing along the y-axis, so $F_{2x} = 0$ N and $F_{2y} = 20$ N. To find the components of $\vec{F}_3$, we need to recognize—because $\vec{F}_3$ points straight down—that the angle between $\vec{F}_3$ and the x-axis is 75°. Thus

$$F_{3x} = F_3\cos 75° = (57\text{ N})\cos 75° = 15\text{ N}$$

$$F_{3y} = -F_3\sin 75° = -(57\text{ N})\sin 75° = -55\text{ N}$$

FIGURE 3.22 Three forces.



The minus sign in F_{3y} is critical, and it appears not from some formula but because we recognized—from the figure—that the y-component of $\vec{F}_3$ points in the $-y$-direction. Combining the pieces, we have

$$(F_{net})_x = 35\text{ N} + 0\text{ N} + 15\text{ N} = 50\text{ N}$$

$$(F_{net})_y = 35\text{ N} + 20\text{ N} + (-55\text{ N}) = 0\text{ N}$$

Thus the net force is $\vec{F}_{net} = 50\hat{\imath}$ N. It points along the x-axis of the tilted coordinate system.

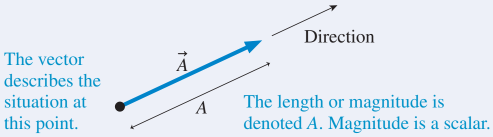
REVIEW Notice that all work was done with reference to the axes of the coordinate system, not with respect to vertical or horizontal.

Summary

The goals of Chapter 3 have been to learn how vectors are represented and used.

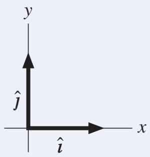
Important Concepts

A **vector** is a quantity described by both a magnitude and a direction.



Unit Vectors

Unit vectors have magnitude 1 and no units. Unit vectors $\hat{\imath}$ and $\hat{\jmath}$ define the directions of the x- and y-axes.



Using Vectors

Components

The component vectors are parallel to the x- and y-axes:

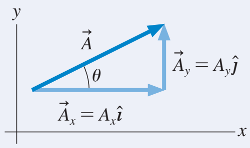
$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{\jmath}$$

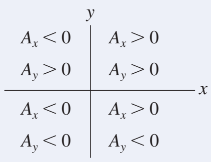
In the figure at the right, for example:

$$A_x = A\cos\theta \qquad A = \sqrt{A_x^2 + A_y^2}$$

$$A_y = A\sin\theta \qquad \theta = \tan^{-1}(A_y/A_x)$$

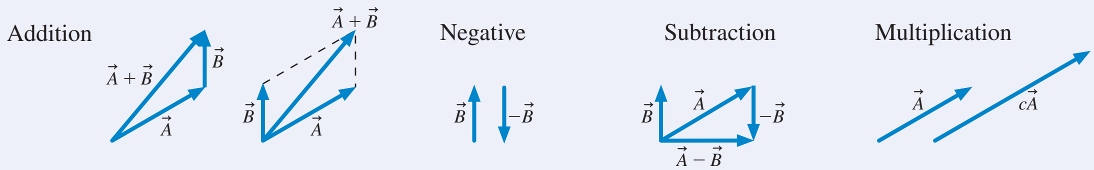
▶ Minus signs need to be included if the vector points down or left.





The components A_x and A_y are the magnitudes of the component vectors $\vec{A}_x$ and $\vec{A}_y$ *and* a plus or minus sign to show whether the component vector points toward the positive end or the negative end of the axis.

Working Graphically



Working Algebraically

Vector calculations are done component by component: $\vec{C} = 2\vec{A}_y + \vec{B}$ means $\begin{cases} C_x = 2A_x + B_x \\ C_y = 2A_y + B_y \end{cases}$

The magnitude of $\vec{C}$ is then $C = \sqrt{C_x^2 + C_y^2}$ and its direction is found using $\tan^{-1}$.

Terms and Notation

scalar	resultant vector	quadrants	component
vector	graphical addition	component vector	unit vector, $\hat{\imath}$ or $\hat{\jmath}$
magnitude	zero vector, $\vec{0}$	decomposition	algebraic addition

CONCEPTUAL QUESTIONS

1. Can the magnitude of the displacement vector be more than the distance traveled? Less than the distance traveled? Explain.
2. If $\vec{C} = \vec{A} + \vec{B}$, can $C = A + B$? Can $C > A + B$? For each, show how or explain why not.
3. If $\vec{C} = \vec{A} + \vec{B}$, can $C = 0$? Can $C < 0$? For each, show how or explain why not.
4. Is it possible to add a scalar to a vector? If so, demonstrate. If not, explain why not.
5. How would you define the *zero vector* $\vec{0}$?
6. Two vectors have lengths of 4 units each. What is the range of possible lengths obtainable for the vector representing the sum of the two?
7. Can a vector have zero magnitude if one of its components is nonzero? Explain.
8. Two vectors of unequal magnitudes can never add up to a zero vector. Does this hold true for three unequal vectors? Explain with an example.
9. Are the following statements true or false? Explain your answer.
   a. The magnitude of a vector can be different in different coordinate systems.
   b. The direction of a vector can be different in different coordinate systems.
   c. The components of a vector can be different in different coordinate systems.

EXERCISES AND PROBLEMS

Exercises

Section 3.1 Scalars and Vectors

Section 3.2 Using Vectors

1. | Trace the vectors in **FIGURE EX3.1** onto your paper. Then find (a) $\vec{A} + \vec{B}$, and (b) $\vec{A} - \vec{B}$.

FIGURE EX3.1

FIGURE EX3.2

2. | Trace the vectors in **FIGURE EX3.2** onto your paper. Then find (a) $\vec{A} + \vec{B}$, and (b) $\vec{A} - \vec{B}$.

Section 3.3 Coordinate Systems and Vector Components

3. || a. What are the x- and y-components of vector $\vec{E}$ shown in **FIGURE EX3.3** in terms of the angle θ and the magnitude E?
   b. For the same vector, what are the x- and y-components in terms of the angle ϕ and the magnitude E?

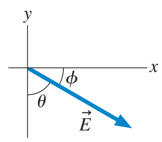


FIGURE EX3.3

4. || A velocity vector 40° below the positive x-axis has a y-component of -10 m/s. What is the value of its x-component?
5. | A position vector in the first quadrant has an x-component of 10 m and a magnitude of 12 m. What is the value of its y-component?
6. | Draw each of the following vectors. Then find its x- and y-components.
   a. $\vec{a} = (3.5\text{ m/s}^2$, negative x-direction)
   b. $\vec{v} = (440$ m/s, 30° below the positive x-axis)
   c. $\vec{r} = (12$ m, 40° above the positive x-axis)
7. || Draw each of the following vectors. Then find its x- and y-components.
   a. $\vec{v} = (7.5$ m/s, 30° clockwise from the positive y-axis)
   b. $\vec{a} = (1.5\text{ m/s}^2$, 30° above the negative x-axis)
   c. $\vec{F} = (50.0$ N, 36.9° counterclockwise from the positive y-axis)
8. || Let $\vec{C} = (3.15$ m, 15° above the negative x-axis) and $\vec{D} = (25.6$ m, 30° to the right of the negative y-axis). Find the x- and y-components of each vector.
9. || A runner is training for an upcoming marathon by running around a 100-m-diameter circular track at constant speed. Let a coordinate system have its origin at the center of the circle with the x-axis pointing east and the y-axis north. The runner starts at $(x, y) = (50\text{ m}, 0\text{ m})$ and runs 2.5 times around the track in a clockwise direction. What is his displacement vector? Give your answer as a magnitude and direction.

Section 3.4 Unit Vectors and Vector Algebra

10. | Draw each of the following vectors, label an angle that specifies the vector's direction, and then find the vector's magnitude and direction.
    a. $\vec{A} = 3.0\hat{\imath} + 7.0\hat{\jmath}$
    b. $\vec{a} = (-2.0\hat{\imath} + 4.5\hat{\jmath})\text{ m/s}^2$
    c. $\vec{v} = (14\hat{\imath} - 11\hat{\jmath})$ m/s
    d. $\vec{r} = (-2.2\hat{\imath} - 3.3\hat{\jmath})$ m
11. | Draw each of the following vectors, label an angle that specifies the vector's direction, then find its magnitude and direction.
    a. $\vec{B} = -4.0\hat{\imath} + 4.0\hat{\jmath}$
    b. $\vec{r} = (-2.0\hat{\imath} - 1.0\hat{\jmath})$ cm
    c. $\vec{v} = (-10\hat{\imath} - 100\hat{\jmath})$ m/s
    d. $\vec{a} = (20\hat{\imath} + 10\hat{\jmath})\text{ m/s}^2$
12. | Let $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$, and $\vec{C} = \vec{A} + \vec{B}$.
    a. Write vector $\vec{C}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.
    c. What are the magnitude and direction of vector $\vec{C}$?
13. | Let $\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$, $\vec{B} = 2\hat{\imath} - 4\hat{\jmath}$, and $\vec{C} = \vec{A} + \vec{B}$.
    a. Write vector $\vec{C}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.
    c. What are the magnitude and direction of vector $\vec{C}$?
14. | Let $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$, and $\vec{E} = 2\vec{A} + 3\vec{B}$.
    a. Write vector $\vec{E}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{E}$.
    c. What are the magnitude and direction of vector $\vec{E}$?
15. | Let $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$, and $\vec{D} = \vec{A} - \vec{B}$.
    a. Write vector $\vec{D}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{D}$.
    c. What are the magnitude and direction of vector $\vec{D}$?

16. | Let $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$, and $\vec{F} = \vec{A} - 4\vec{B}$.
   a. Write vector $\vec{F}$ in component form.
   b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{F}$.
   c. What are the magnitude and direction of vector $\vec{F}$?

17. | Let $\vec{E} = 4\hat{\imath} + 5\hat{\jmath}$ and $\vec{F} = 2\hat{\imath} - 3\hat{\jmath}$. Find the magnitude of
   a. $\vec{E}$ and $\vec{F}$ b. $\vec{E} + \vec{F}$ c. $-\vec{E} - 2\vec{F}$

18. || For the three vectors shown in FIGURE EX3.18, $\vec{A} + \vec{B} + \vec{C} = 1\hat{\jmath}$. What is vector $\vec{B}$.
   a. Write $\vec{B}$ in component form.
   b. Write $\vec{B}$ as a magnitude and a direction.

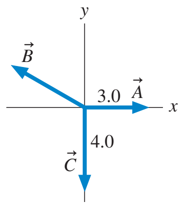


FIGURE EX3.18

19. || FIGURE EX3.19 shows vectors $\vec{A}$ and $\vec{B}$. What is $\vec{C} = \vec{A} + \vec{B}$? Write your answer in component form using unit vectors.

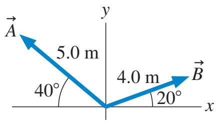


FIGURE EX3.19

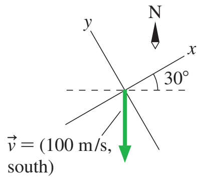


FIGURE EX3.20

20. | What are the x- and y-components of the velocity vector shown in FIGURE EX3.20?

21. ||| Let $\vec{B} = (5.0\text{ m}, 30°$ counterclockwise from vertically up). Find the x- and y-components of $\vec{B}$ in each of the two coordinate systems shown in FIGURE EX3.21.

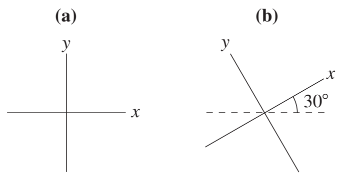


FIGURE EX3.21

Problems

22. || Let $\vec{A} = (3.0\text{ m}, 20°$ south of east), $\vec{B} = (2.0\text{ m, north})$, and $\vec{C} = (5.0\text{ m}, 70°$ south of west).
   a. Draw and label $\vec{A}$, $\vec{B}$, and $\vec{C}$ with their tails at the origin. Use a coordinate system with the x-axis to the east.
   b. Write $\vec{A}$, $\vec{B}$, and $\vec{C}$ in component form, using unit vectors.
   c. Find the magnitude and the direction of $\vec{D} = \vec{A} + \vec{B} + \vec{C}$.

23. || The position of a particle as a function of time is given by
CALC $\vec{r} = (3.0\hat{\imath} + 8.0\hat{\jmath})t^2$ m, where t is in seconds.
   a. What is the particle's distance from the origin at $t = 0$, 2, and 5 s?
   b. Find an expression for the particle's velocity $\vec{v}$ as a function of time.
   c. What is the particle's speed at $t = 0$, 2, and 5 s?

24. | a. What is the angle ϕ between vectors $\vec{E}$ and $\vec{F}$ in FIGURE P3.24?
   b. Use geometry and trigonometry to determine the magnitude and direction of $\vec{G} = \vec{E} + \vec{F}$.
   c. Use components to determine the magnitude and direction of $\vec{G} = \vec{E} + \vec{F}$.

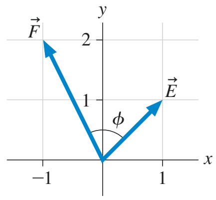


FIGURE P3.24

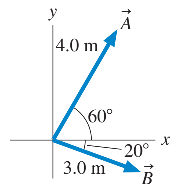


FIGURE P3.25

25. ||| FIGURE P3.25 shows vectors $\vec{A}$ and $\vec{B}$. Find vector $\vec{C}$ such that $\vec{A} + \vec{B} + \vec{C} = \vec{0}$. Write your answer in component form.

26. || FIGURE P3.26 shows vectors $\vec{A}$ and $\vec{B}$. Find $\vec{D} = 2\vec{A} + \vec{B}$. Write your answer in component form.

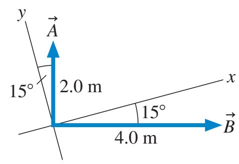


FIGURE P3.26

27. || Find a vector that points in the same direction as the vector $(\hat{\imath} + \hat{\jmath})$ and whose magnitude is 1.

28. || The minute hand on a watch is 2.0 cm in length. What is the displacement vector of the tip of the minute hand in each case? Use a coordinate system in which the y-axis points toward the 12 on the watch face.
   a. From 8:00 to 8:20 A.M.
   b. From 8:00 to 9:00 A.M.

29. || While vacationing in the mountains you do some hiking. In the morning, your displacement is $\vec{S}_{\text{morning}} = (2000\text{ m, east}) + (3000\text{ m, north}) + (200\text{ m, vertical})$. Continuing on after lunch, your displacement is $\vec{S}_{\text{afternoon}} = (1500\text{ m, west}) + (2000\text{ m, north}) - (300\text{ m, vertical})$.
   a. At the end of the hike, how much higher or lower are you compared to your starting point?
   b. What is the magnitude of your net displacement for the day?

30. || Lucia drives with velocity $\vec{v}_1 = (25\hat{\imath} - 35\hat{\jmath})$ mph for 1.0 h, then $\vec{v}_2 = (30\hat{\imath} + 40\hat{\jmath})$ mph for 2.0 h. What is Lucia's displacement? Write your answer in component form using unit vectors.

31. || Ruth sets out to visit her friend Ward, who lives 50 mi north and 100 mi east of her. She starts by driving east, but after 30 mi she comes to a detour that takes her 15 mi south before going east again. She then drives east for 8 mi and runs out of gas, so Ward flies there in his small plane to get her. What is Ward's displacement vector? Give your answer (a) in component form, using a coordinate system in which the y-axis points north, and (b) as a magnitude and direction.

32. | A cannon tilted upward at 30° fires a cannonball with a speed of 100 m/s. What is the component of the cannonball's velocity parallel to the ground?

33. || A cannonball leaves the barrel with velocity $\vec{v} = (65\hat{\imath} + 75\hat{\jmath})$ m/s. At what angle is the barrel tilted above horizontal?

34. | You are fixing the roof of your house when the head of your hammer breaks loose and slides down. The roof makes an angle of 30° with the horizontal, and the head is moving at 3.5 m/s when it reaches the edge. What are the horizontal and vertical components of the head's velocity just as it leaves the roof?

35. | Jack and Jill ran up the hill at 4 m/s. The horizontal component of Jill's velocity vector was 3.5 m/s.
   a. What was the angle of the hill?
   b. What was the vertical component of Jill's velocity?

36. | Kami is walking through the airport with her two-wheeled suitcase. The suitcase handle is tilted 40° from vertical, and Kami pulls parallel to the handle with a force of 120 N. (Force is measured in *newtons,* abbreviated N.) What are the horizontal and vertical components of her applied force?

37. | A pine cone falls straight down from a pine tree growing on a 20° slope. The pine cone hits the ground with a speed of 10 m/s. What is the component of the pine cone's impact velocity (a) parallel to the ground and (b) perpendicular to the ground?

38. | A jet plane taking off from an aircraft carrier has acceleration $\vec{a} = (14\ \text{m/s}^2, 21°\ \text{above horizontal})$. What are the horizontal and vertical components of the jet's acceleration?

39. | Your neighbor Paul has rented a truck with a loading ramp. The ramp is tilted upward at 25°, and Paul is pulling a large crate up the ramp with a rope that angles 10° above the ramp. If Paul pulls with a force of 550 N, what are the horizontal and vertical components of his force? (Force is measured in *newtons,* abbreviated N.)

40. || Tom is climbing a 3.0-m-long ladder that leans against a vertical wall, contacting the wall 2.5 m above the ground. His weight of 680 N is a vector pointing vertically downward. (Weight is measured in *newtons,* abbreviated N.) What are the components of Tom's weight parallel and perpendicular to the ladder?

41. || The treasure map in **FIGURE P3.41** gives the following directions to the buried treasure: "Start at the old oak tree, walk due north for 500 paces, then due east for 100 paces. Dig." But when you arrive, you find an angry dragon just north of the tree. To avoid the dragon, you set off along the yellow brick road at an angle 60° east of north. After walking 300 paces you see an opening through the woods. In which direction should you walk, as an angle west of north, and how far, to reach the treasure?

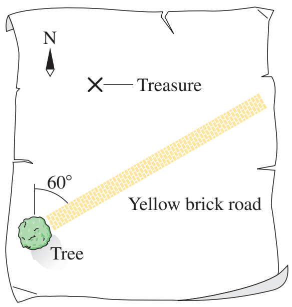


FIGURE P3.41

42. ||| The bacterium *E. coli* is a single-cell organism that lives in the gut of healthy animals, including humans. When grown in a uniform medium in the laboratory, these bacteria swim along zig-zag paths at a constant speed of 20 μm/s. **FIGURE P3.42** shows the trajectory of an *E. coli* as it moves from point A to point E. What are the magnitude and direction of the bacterium's average velocity for the entire trip?
BIO

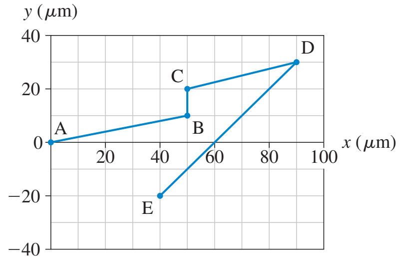


FIGURE P3.42

43. || **FIGURE P3.43** shows three ropes tied together in a knot. One of your friends pulls on a rope with 3.0 units of force and another pulls on a second rope with 5.0 units of force. How hard and in what direction must you pull on the third rope to keep the knot from moving? Give the direction as an angle below the negative x-axis.

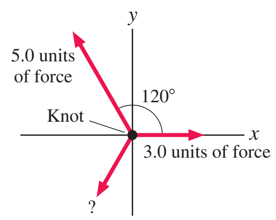


FIGURE P3.43

44. || A crate, seen from above, is pulled with three ropes that have the tensions shown in **FIGURE P3.44**. Tension is a vector directed along the rope, measured in *newtons* (abbreviated N). Suppose the three ropes are replaced with a single rope that has exactly the same effect on the crate. What is the tension in this rope? Write your answer in component form using unit vectors.

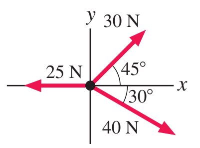


FIGURE P3.44

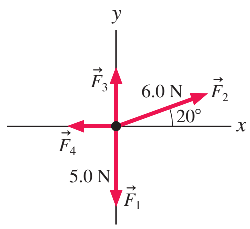


FIGURE P3.45

45. || Four forces are exerted on the object shown in **FIGURE P3.45**. (Forces are measured in *newtons,* abbreviated N.) The *net force* on the object is $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 4.0\hat{\imath}$ N. What are (a) $\vec{F}_3$ and (b) $\vec{F}_4$? Give your answers in component form.

46. || **FIGURE P3.46** shows four electric charges located at the corners of a rectangle. Like charges, you will recall, repel each other while opposite charges attract. Charge B exerts a repulsive force (directly *away from* B) on charge A of 3.0 N. Charge C exerts an attractive force (directly *toward* C) on charge A of 6.0 N. Finally, charge D exerts an attractive force of 2.0 N on charge A. Assuming that forces are vectors, what are the magnitude and direction of the net force $\vec{F}_{net}$ exerted on charge A?

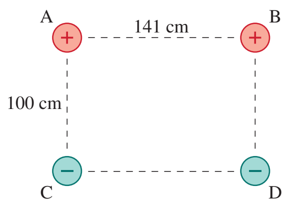


FIGURE P3.46

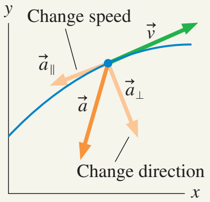
4 Kinematics in Two Dimensions

This motocross bike follows the parabolic trajectory of projectile motion.

IN THIS CHAPTER, you will learn how to solve problems about motion in a plane.

How do objects accelerate in two dimensions?

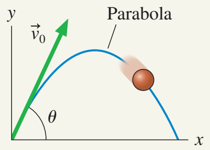
An object accelerates when it **changes velocity**. In two dimensions, velocity can change by **changing magnitude** (speed) or by **changing direction**. These are represented by acceleration components tangent to and perpendicular to an object's **trajectory**.



« **LOOKING BACK** Section 1.5 Finding acceleration vectors on a motion diagram

What is projectile motion?

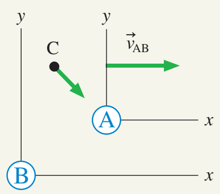
Projectile motion is two-dimensional free-fall motion under the influence of only gravity. Projectile motion follows a **parabolic trajectory**. It has uniform motion in the horizontal direction and $a_y = -g$ in the vertical direction.



« **LOOKING BACK** Section 2.5 Free fall

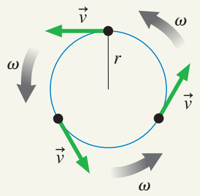
What is relative motion?

Coordinate systems that move **relative** to each other are called **reference frames.** If object C has velocity $\vec{v}_{CA}$ relative to a reference frame A, and if A moves with velocity $\vec{v}_{AB}$ relative to another reference frame B, then the velocity of C in reference frame B is $\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$.



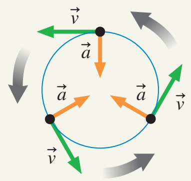
What is circular motion?

An object moving in a circle (or rotating) has an *angular displacement* instead of a linear displacement. Circular motion is described by **angular velocity** ω (analogous to velocity v_s) and **angular acceleration** α (analogous to acceleration a_s). We'll study both uniform and accelerated circular motion.



What is centripetal acceleration?

An object in **circular motion** is always changing direction. The acceleration of changing direction—called **centripetal acceleration**—points to the center of the circle. All circular motion has a centripetal acceleration. An object also has a *tangential acceleration* if it is changing speed.



Where is two-dimensional motion used?

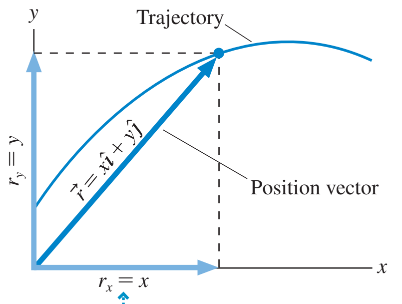
Linear motion allowed us to introduce the concepts of motion, but most **real motion** takes place in two or even three dimensions. Balls move along curved trajectories, cars turn corners, planets orbit the sun, and electrons spiral in the earth's magnetic field. Where is two-dimensional motion used? Everywhere!

4.1 Motion in Two Dimensions

Motion diagrams are an important tool for visualizing motion, and we'll continue to use them, but we also need to develop a mathematical description of motion in two dimensions. For convenience, we'll say that any two-dimensional motion is in the xy-plane regardless of whether the plane of motion is horizontal or vertical.

FIGURE 4.1 shows a particle moving along a curved path—its *trajectory*—in the xy-plane. We can locate the particle in terms of its position vector $\vec{r} = x\hat{\imath} + y\hat{\jmath}$.

FIGURE 4.1 A particle moving along a trajectory in the xy-plane.



The x- and y-components of $\vec{r}$ are simply x and y.

NOTE In Chapter 2 we made extensive use of position-versus-time graphs, either x versus t or y versus t. Figure 4.1, like many of the graphs we'll use in this chapter, is a graph of y versus x. In other words, it's an actual *picture* of the trajectory, not an abstract representation of the motion.

FIGURE 4.2a shows the particle moving from position $\vec{r}_1$ at time t_1 to position $\vec{r}_2$ at a later time t_2. The average velocity—pointing in the direction of the displacement $\Delta\vec{r}$—is

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} \tag{4.1}$$

You learned in Chapter 2 that the instantaneous velocity is the limit of $\vec{v}_{avg}$ as $\Delta t \to 0$. As Δt decreases, point 2 moves closer to point 1 until, as **FIGURE 4.2b** shows, the displacement vector becomes tangent to the curve. Consequently, **the instantaneous velocity vector $\vec{v}$ is tangent to the trajectory.**

Mathematically, the limit of Equation 4.1 gives

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} \tag{4.2}$$

We can also write the velocity vector in terms of its x- and y-components as

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} \tag{4.3}$$

Comparing Equations 4.2 and 4.3, you can see that the velocity vector $\vec{v}$ has x- and y-components

$$v_x = \frac{dx}{dt} \quad \text{and} \quad v_y = \frac{dy}{dt} \tag{4.4}$$

That is, the x-component v_x of the velocity vector is the rate dx/dt at which the particle's x-coordinate is changing. The y-component is similar.

FIGURE 4.2c illustrates another important feature of the velocity vector. If the vector's angle θ is measured from the positive x-direction, the velocity vector components are

$$\begin{aligned} v_x &= v\cos\theta \\ v_y &= v\sin\theta \end{aligned} \tag{4.5}$$

where

$$v = \sqrt{v_x^2 + v_y^2} \tag{4.6}$$

is the particle's *speed* at that point. Speed is always a positive number (or zero), whereas the components are *signed* quantities (i.e., they can be positive or negative) to convey information about the direction of the velocity vector. Conversely, we can use the two velocity components to determine the direction of motion:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) \tag{4.7}$$

NOTE In Chapter 2, you learned that the *value* of the velocity is the *slope* of the position-versus-time graph. Now we see that the *direction* of the velocity vector $\vec{v}$ is the *tangent* to the y-versus-x graph of the trajectory. **FIGURE 4.3**, on the next page, reminds you that these two graphs use different interpretations of the tangent lines. The tangent to the trajectory does not tell us anything about how fast the particle is moving.

FIGURE 4.2 The instantaneous velocity vector is tangent to the trajectory.

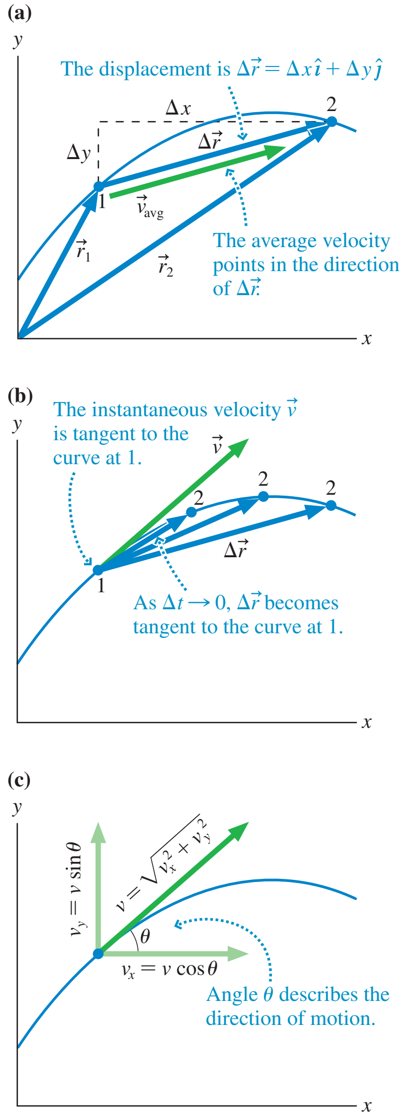
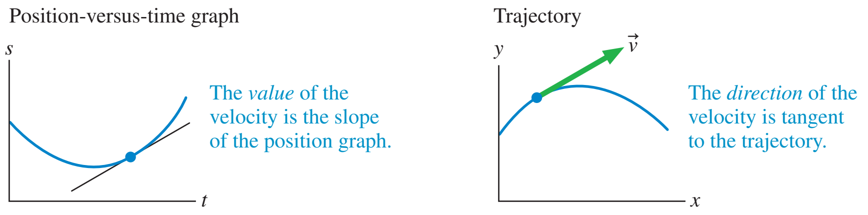


FIGURE 4.3 Two different uses of tangent lines.



EXAMPLE 4.1 ■ Finding velocity

A sports car's position on a winding road is given by

$$\vec{r} = (6.0t - 0.10t^2)\hat{\imath} + (8.0t - 0.00095t^3)\hat{\jmath}$$

where the y-axis points north, t is in s, and r is in m. What are the car's speed and direction at $t = 120$ s?

MODEL Model the car as a particle.

SOLVE Velocity is the derivative of position, so

$$v_x = \frac{dx}{dt} = 6.0 - 2(0.10t)$$

$$v_y = \frac{dy}{dt} = 8.0 - 3(0.00095t^2)$$

Written as a vector, the velocity is

$$\vec{v} = (6.0 - 0.20t)\hat{\imath} + (8.0 - 0.00285t^2)\hat{\jmath}$$

where t is in s and v is in m/s. At $t = 120$ s, we can calculate $\vec{v} = (-18\hat{\imath} - 33\hat{\jmath})$ m/s. The car's speed at this instant is

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}} = \sqrt{(-18\text{ m/s})^2 + (-33\text{ m/s})^2} = 38\text{ m/s}$$

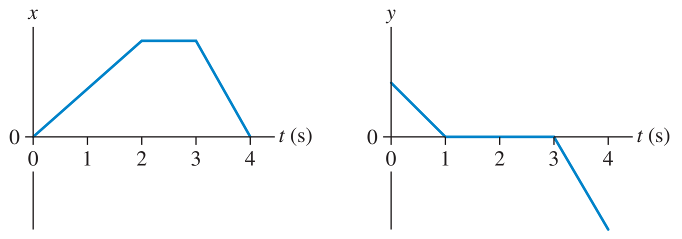
Both components of the velocity vector are negative, so the direction of motion is to the left (west) and down (south). The angle below the negative x-axis is

$$\theta = \tan^{-1}\left(\frac{|-33\text{ m/s}|}{|-18\text{ m/s}|}\right) = 61°$$

So, at this instant, the car is headed 61° south of west at a speed of 38 m/s.

STOP TO THINK 4.1 During which time interval or intervals is the particle described by these position graphs at rest? More than one may be correct.

a. 0–1 s
b. 1–2 s
c. 2–3 s
d. 3–4 s



Acceleration Graphically

In « **SECTION 1.5** we defined the *average acceleration* $\vec{a}_{avg}$ of a moving object to be

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} \tag{4.8}$$

From its definition, we see that **$\vec{a}$ points in the same direction as $\Delta\vec{v}$,** the change of velocity. As an object moves, its velocity vector can change in two possible ways:

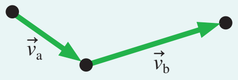
1. The magnitude of $\vec{v}$ can change, indicating a change in speed, or
2. The direction of $\vec{v}$ can change, indicating that the object has changed direction.

Chapters 1 and 2 considered only the acceleration of changing speed. The acceleration of changing direction can be determined by finding the direction of $\Delta\vec{v}$. If an object changes from velocity $\vec{v}_a$ to velocity $\vec{v}_b$, its change of velocity $\Delta\vec{v} = \vec{v}_b - \vec{v}_a$ can be written as $\vec{v}_b = \vec{v}_a + \Delta\vec{v}$. Thus $\Delta\vec{v}$ is the vector that must be added to $\vec{v}_a$ to get $\vec{v}_b$. Tactics Box 4.1 shows how to use graphical vector addition to find the acceleration vector.

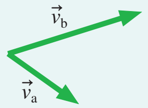
TACTICS BOX 4.1

Finding the acceleration vector

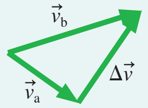
To find the acceleration between velocity $\vec{v}_a$ and velocity $\vec{v}_b$:



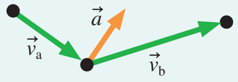
❶ Draw velocity vectors $\vec{v}_a$ and $\vec{v}_b$ with their tails together.



❷ Draw the vector from the tip of $\vec{v}_a$ to the tip of $\vec{v}_b$. This is $\Delta\vec{v}$ because $\vec{v}_b = \vec{v}_a + \Delta\vec{v}$.



❸ Return to the original motion diagram. Draw a vector at the middle dot in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration between $\vec{v}_a$ and $\vec{v}_b$.



Exercises 1–4

Our everyday use of the word "accelerate" means "speed up." The mathematical definition of acceleration—the rate of change of velocity—also includes slowing down, as you learned in Chapter 2, as well as changing direction. All these are motions that change the velocity.

EXAMPLE 4.2 ■ Through the valley

A ball rolls down a long hill, through the valley, and back up the other side. Draw a complete motion diagram of the ball.

MODEL Model the ball as a particle.

VISUALIZE **FIGURE 4.4** is the motion diagram. Where the particle moves along a *straight line,* it speeds up if $\vec{a}$ and $\vec{v}$ point in the same direction and slows down if $\vec{a}$ and $\vec{v}$ point in opposite directions. This important idea was the basis for the one-dimensional kinematics we developed in Chapter 2. When the direction of $\vec{v}$ changes, as it does when the ball goes through the valley, we need to use Tactics Box 4.1 to find the direction of $\Delta\vec{v}$ and thus of $\vec{a}$. The procedure is shown at one point in the motion diagram. Notice that $\vec{a}$ is perpendicular to the trajectory at the bottom point where only the direction, not the speed, is changing. We'll return to this idea when we discuss circular motion.

FIGURE 4.4 The motion diagram of the ball of Example 4.2.

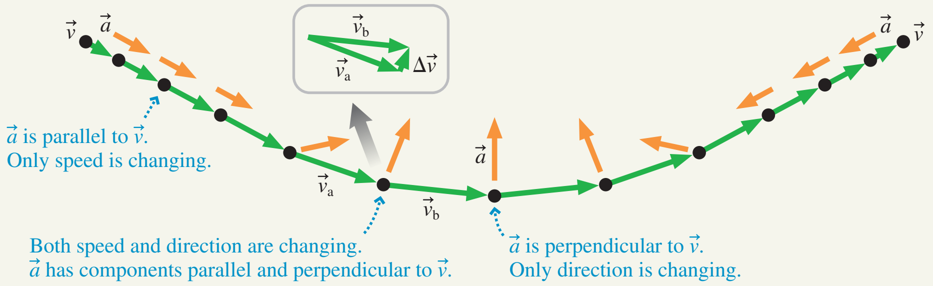


FIGURE 4.5 Analyzing the acceleration vector.

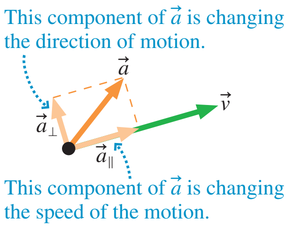
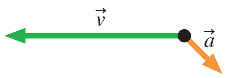


FIGURE 4.5 shows that an object's acceleration vector can be decomposed into a component parallel to the velocity—that is, parallel to the direction of motion—and a component perpendicular to the velocity. **$\vec{a}_\parallel$ is the piece of the acceleration that causes the object to change speed,** speeding up if $\vec{a}_\parallel$ points in the same direction as $\vec{v}$, slowing down if they point in opposite directions. **$\vec{a}_\perp$ is the piece of the acceleration that causes the object to change direction.** An object changing direction *always* has a component of acceleration perpendicular to the direction of motion.

Looking back at Example 4.2, we see that $\vec{a}$ is parallel to $\vec{v}$ on the straight portions of the hill where only speed is changing. At the very bottom, where the ball's direction is changing but not its speed, $\vec{a}$ is perpendicular to $\vec{v}$. The acceleration is angled with respect to velocity—having both parallel and perpendicular components—at those points where both speed and direction are changing.

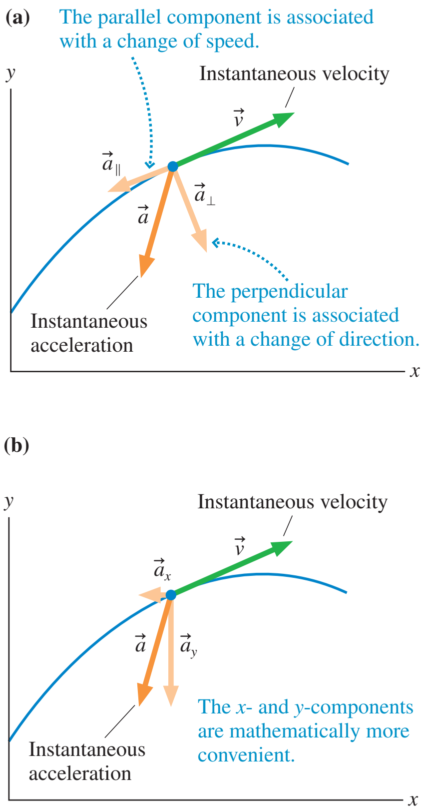
STOP TO THINK 4.2 This acceleration will cause the particle to



a. Speed up and curve upward.
b. Speed up and curve downward.
c. Slow down and curve upward.
d. Slow down and curve downward.
e. Move to the right and down.
f. Reverse direction.

Acceleration Mathematically

FIGURE 4.6 The instantaneous acceleration $\vec{a}$.



In Tactics Box 4.1, the average acceleration is found from two velocity vectors separated by the time interval Δt. If we let Δt get smaller and smaller, the two velocity vectors get closer and closer. In the limit $\Delta t \to 0$, we have the instantaneous acceleration $\vec{a}$ at the same point on the trajectory (and the same instant of time) as the instantaneous velocity $\vec{v}$. This is shown in **FIGURE 4.6**.

By definition, the acceleration vector $\vec{a}$ is the rate at which the velocity $\vec{v}$ is changing at that instant. To show this, Figure 4.6a decomposes $\vec{a}$ into components $\vec{a}_\parallel$ and $\vec{a}_\perp$ that are parallel and perpendicular to the trajectory. As we just showed, $\vec{a}_\parallel$ is associated with a change of speed, and $\vec{a}_\perp$ is associated with a change of direction. Both kinds of changes are accelerations. Notice that $\vec{a}_\perp$ always points toward the "inside" of the curve because that is the direction in which $\vec{v}$ is changing.

Although the parallel and perpendicular components of $\vec{a}$ convey important ideas about acceleration, it's often more practical to write $\vec{a}$ in terms of the x- and y-components shown in Figure 4.6b. Because $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$, we find

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} \tag{4.9}$$

from which we see that

$$a_x = \frac{dv_x}{dt} \quad \text{and} \quad a_y = \frac{dv_y}{dt} \tag{4.10}$$

That is, the x-component of $\vec{a}$ is the rate dv_x/dt at which the x-component of velocity is changing.

NOTE Figures 4.6a and Figure 4.6b show the *same* acceleration vector; all that differs is how we've chosen to decompose it. For motion with constant acceleration, which includes projectile motion, the decomposition into x- and y-components is most convenient. But we'll find that the parallel and perpendicular components are especially suited to an analysis of circular motion.

Constant Acceleration

If the acceleration $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$ is constant, then the two components a_x and a_y are both constant. In this case, everything you learned about constant-acceleration kinematics in « **SECTION 2.4** carries over to two-dimensional motion.

Consider a particle that moves with constant acceleration from an initial position $\vec{r}_i = x_i\hat{\imath} + y_i\hat{\jmath}$, starting with initial velocity $\vec{v}_i = v_{ix}\hat{\imath} + v_{iy}\hat{\jmath}$. Its position and velocity at a final point f are

$$x_f = x_i + v_{ix}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \qquad y_f = y_i + v_{iy}\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$v_{fx} = v_{ix} + a_x\,\Delta t \qquad v_{fy} = v_{iy} + a_y\,\Delta t \tag{4.11}$$

There are *many* quantities to keep track of in two-dimensional kinematics, making the pictorial representation all the more important as a problem-solving tool.

NOTE For constant acceleration, the x-component of the motion and the y-component of the motion are independent of each other. However, they remain connected through the fact that Δt must be the same for both.

EXAMPLE 4.3 ■ Plotting a spacecraft trajectory

In the distant future, a small spacecraft is drifting "north" through the galaxy at 680 m/s when it receives a command to return to the starship. The pilot rotates the spacecraft until the nose is pointed 25° north of east, then engages the ion engine. The spacecraft accelerates at 75 m/s^2. Plot the spacecraft's trajectory for the first 20 s.

MODEL Model the spacecraft as a particle with constant acceleration.

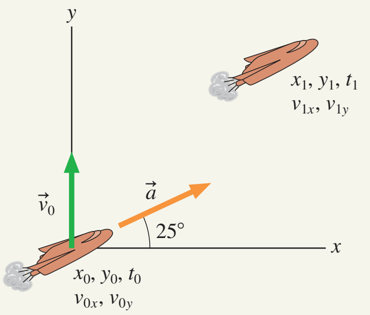
VISUALIZE FIGURE 4.7 shows a pictorial representation in which the y-axis points north and the spacecraft starts at the origin. Notice that each point in the motion is labeled with *two* positions (x and y), *two* velocity components (v_x and v_y), and the time t. This will be our standard labeling scheme for trajectory problems.

SOLVE The acceleration vector has both x- and y-components; their values have been calculated in the pictorial representation. But it is a *constant* acceleration, so we can write

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2$$
$$= 34.0t_1^{\,2}\text{ m}$$
$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2$$
$$= 680t_1 + 15.8t_1^{\,2}\text{ m}$$

where t_1 is in s. Graphing software produces the trajectory shown in FIGURE 4.8. The trajectory is a parabola, which is characteristic of two-dimensional motion with constant acceleration.

FIGURE 4.7 Pictorial representation of the spacecraft.



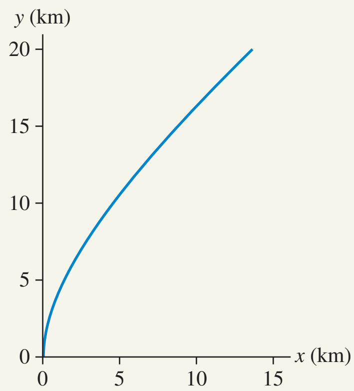
Known

$x_0 = y_0 = 0$ m $\quad v_{0x} = 0$ m/s $\quad v_{0y} = 680$ m/s
$a_x = (75\text{ m/s}^2)\cos 25° = 68.0\text{ m/s}^2$
$a_y = (75\text{ m/s}^2)\sin 25° = 31.6\text{ m/s}^2$
$t_0 = 0$ s $\quad t_1 = 0$ s to 20 s

Find

x_1 and y_1

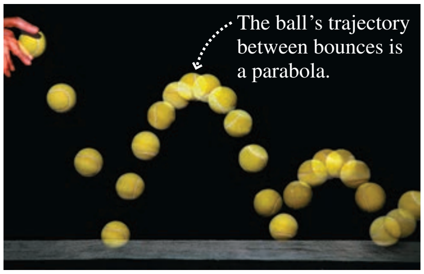
FIGURE 4.8 The spacecraft trajectory.



4.2 Projectile Motion

Baseballs and tennis balls flying through the air, Olympic divers, and daredevils shot from cannons all exhibit what we call *projectile motion*. A **projectile is an object that moves in two dimensions under the influence of only gravity.** Projectile motion is an extension of the free-fall motion we studied in Chapter 2. We will continue to neglect the influence of air resistance, leading to results that are a good approximation of reality for relatively heavy objects moving relatively slowly over relatively short distances. As we'll see, projectiles in two dimensions follow a *parabolic trajectory* like the one seen in FIGURE 4.9.

FIGURE 4.9 A parabolic trajectory.



The start of a projectile's motion, be it thrown by hand or shot from a gun, is called the *launch*, and the angle θ of the initial velocity $\vec{v}_0$ above the horizontal (i.e., above

FIGURE 4.10 A projectile launched with initial velocity $\vec{v}_0$.

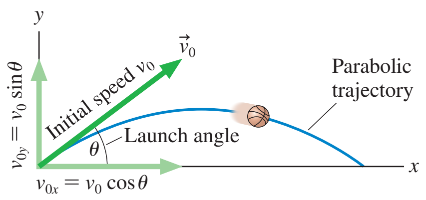
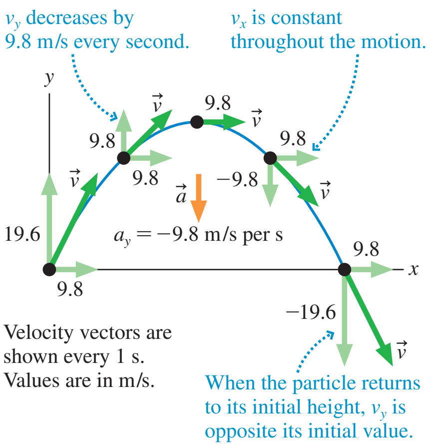


FIGURE 4.11 The velocity and acceleration vectors of a projectile.



the x-axis) is called the **launch angle.** **FIGURE 4.10** illustrates the relationship between the initial velocity vector $\vec{v}_0$ and the initial values of the components v_{0x} and v_{0y}. You can see that

$$
\begin{aligned}
v_{0x} &= v_0 \cos\theta \\
v_{0y} &= v_0 \sin\theta
\end{aligned}
\qquad (4.12)
$$

where v_0 is the initial speed.

NOTE A projectile launched at an angle *below* the horizontal (such as a ball thrown downward from the roof of a building) has *negative* values for θ and v_{0y}. However, the *speed* v_0 is always positive.

Gravity acts downward, and we know that objects released from rest fall straight down, not sideways. Hence a projectile has no horizontal acceleration, while its vertical acceleration is simply that of free fall. Thus

$$
\begin{aligned}
a_x &= 0 \\
a_y &= -g
\end{aligned}
\qquad \text{(projectile motion)} \qquad (4.13)
$$

In other words, **the vertical component of acceleration a_y is just the familiar $-g$ of free fall, while the horizontal component a_x is zero. Projectiles are in free fall.**

To see how these conditions influence the motion, **FIGURE 4.11** shows a projectile launched from $(x_0, y_0) = (0\text{ m}, 0\text{ m})$ with an initial velocity $\vec{v}_0 = (9.8\hat{\imath} + 19.6\hat{\jmath})$ m/s. The value of v_x never changes because there's no horizontal acceleration, but v_y decreases by 9.8 m/s every second. This is what it *means* to accelerate at $a_y = -9.8\text{ m/s}^2 = (-9.8\text{ m/s})$ per second.

You can see from Figure 4.11 that **projectile motion is made up of two independent motions:** uniform motion at constant velocity in the horizontal direction and free-fall motion in the vertical direction. The kinematic equations that describe these two motions are simply Equations 4.11 with $a_x = 0$ and $a_y = -g$.

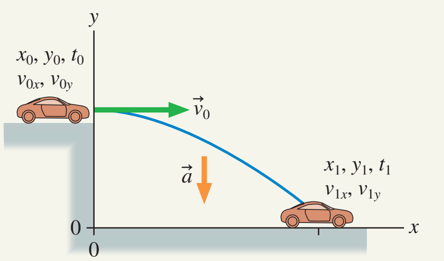
EXAMPLE 4.4 ■ Don't try this at home!

A stunt man drives a car off a 10.0-m-high cliff at a speed of 20.0 m/s. How far does the car land from the base of the cliff?

MODEL Model the car as a particle in free fall. Assume that the car is moving horizontally as it leaves the cliff.

VISUALIZE The pictorial representation, shown in **FIGURE 4.12**, is *very* important because the number of quantities to keep track of is quite large. We have chosen to put the origin at the base of the cliff. The assumption that the car is moving horizontally as it leaves the cliff leads to $v_{0x} = v_0$ and $v_{0y} = 0$ m/s.

FIGURE 4.12 Pictorial representation for the car of Example 4.4.



Known	Find
$x_0 = 0$ m $\quad v_{0y} = 0$ m/s $\quad t_0 = 0$ s	x_1
$y_0 = 10.0$ m $\quad v_{0x} = v_0 = 20.0$ m/s	
$a_x = 0\text{ m/s}^2 \quad a_y = -g \quad y_1 = 0$ m	

SOLVE Each point on the trajectory has x- and y-components of position, velocity, and acceleration but only *one* value of time. The time needed to move horizontally to x_1 is the *same* time needed to fall vertically through distance y_0. **Although the horizontal and vertical motions are independent, they are connected through the time t.** This is a critical observation for solving projectile motion problems. The kinematics equations with $a_x = 0$ and $a_y = -g$ are

$$
\begin{aligned}
x_1 &= x_0 + v_{0x}(t_1 - t_0) = v_0 t_1 \\
y_1 = 0 &= y_0 + v_{0y}(t_1 - t_0) - \tfrac{1}{2}g(t_1 - t_0)^2 = y_0 - \tfrac{1}{2}g t_1^2
\end{aligned}
$$

We can use the vertical equation to determine the time t_1 needed to fall distance y_0:

$$t_1 = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(10.0\text{ m})}{9.80\text{ m/s}^2}} = 1.43\text{ s}$$

We then insert this expression for t into the horizontal equation to find the distance traveled:

$$x_1 = v_0 t_1 = (20.0\text{ m/s})(1.43\text{ s}) = 28.6\text{ m}$$

REVIEW The cliff height is ≈ 33 ft and the initial speed is $v_0 \approx 40$ mph. Traveling $x_1 = 29\text{ m} \approx 95$ ft before hitting the ground seems reasonable.

The x- and y-equations of Example 4.4 are parametric equations. It's not hard to eliminate t and write an expression for y as a function of x. From the x_1 equation, $t_1 = x_1/v_0$. Substituting this into the y_1 equation, we find

$$y = y_0 - \frac{g}{2v_0^{\,2}}x^2 \tag{4.14}$$

The graph of $y = cx^2$ is a parabola, so Equation 4.14 represents an inverted parabola that starts from height y_0. This proves, as we asserted previously, that a projectile follows a parabolic trajectory.

Reasoning About Projectile Motion

Suppose a heavy ball is launched exactly horizontally at height h above a horizontal field. At the exact instant that the ball is launched, a second ball is simply dropped from height h. Which ball hits the ground first?

It may seem hard to believe, but—if air resistance is neglected—the balls hit the ground *simultaneously*. They do so because the horizontal and vertical components of projectile motion are independent of each other. The initial horizontal velocity of the first ball has *no* influence over its vertical motion. Neither ball has any initial motion in the vertical direction, so both fall distance h in the same amount of time. You can see this in **FIGURE 4.13**.

FIGURE 4.13 A projectile launched horizontally falls in the same time as a projectile that is released from rest.

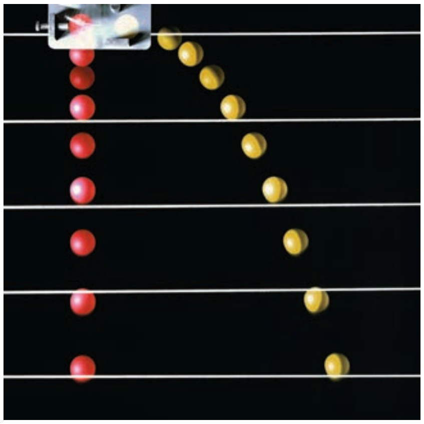


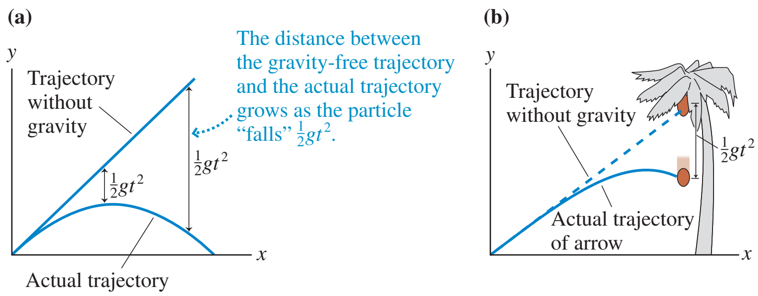
FIGURE 4.14a shows a useful way to think about the trajectory of a projectile. Without gravity, a projectile would follow a straight line. Because of gravity, the particle at time t has "fallen" a distance $\frac{1}{2}gt^2$ below this line. The separation grows as $\frac{1}{2}gt^2$, giving the trajectory its parabolic shape.

Use this idea to think about the following "classic" problem in physics:

> A hungry bow-and-arrow hunter in the jungle wants to shoot down a coconut that is hanging from the branch of a tree. He points his arrow directly at the coconut, but as luck would have it, the coconut falls from the branch at the *exact* instant the hunter releases the string. Does the arrow hit the coconut?

You might think that the arrow will miss the falling coconut, but it doesn't. Although the arrow travels very fast, it follows a slightly curved parabolic trajectory, not a straight line. Had the coconut stayed on the tree, the arrow would have curved under its target as gravity caused it to fall a distance $\frac{1}{2}gt^2$ below the straight line. But $\frac{1}{2}gt^2$ is also the distance the coconut falls while the arrow is in flight. Thus, as **FIGURE 4.14b** shows, the arrow and the coconut fall the same distance and meet at the same point!

FIGURE 4.14 A projectile follows a parabolic trajectory because it "falls" a distance $\frac{1}{2}gt^2$ below a straight-line trajectory.



The Projectile Motion Model

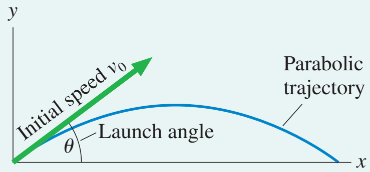
Projectile motion is an ideal that's rarely achieved by real objects. Nonetheless, the **projectile motion model** is another important simplification of reality that we can add to our growing list of models.

MODEL 4.1

Projectile motion

For motion under the influence of only gravity.

- Model the object as a particle launched with speed v_0 at angle θ:



A projectile follows a parabolic trajectory.

- Mathematically:
  - **Uniform motion** in the horizontal direction with $v_x = v_0 \cos\theta$.
  - **Constant acceleration** in the vertical direction with $a_y = -g$.
  - Same Δt for both motions.
- Limitations: Model fails if air resistance is significant.

Exercise 9

PROBLEM-SOLVING STRATEGY 4.1

Projectile motion problems

MODEL Is it reasonable to ignore air resistance? If so, use the projectile motion model.

VISUALIZE Establish a coordinate system with the x-axis horizontal and the y-axis vertical. Define symbols and identify what the problem is trying to find. For a launch at angle θ, the initial velocity components are $v_{ix} = v_0 \cos\theta$ and $v_{iy} = v_0 \sin\theta$.

SOLVE The acceleration is known: $a_x = 0$ and $a_y = -g$. Thus the problem is one of two-dimensional kinematics. The kinematic equations are

Horizontal	Vertical
$x_f = x_i + v_{ix}\,\Delta t$	$y_f = y_i + v_{iy}\,\Delta t - \frac{1}{2}g(\Delta t)^2$
$v_{fx} = v_{ix} = \text{constant}$	$v_{fy} = v_{iy} - g\,\Delta t$

Δt is the same for the horizontal and vertical components of the motion. Find Δt from one component, then use that value for the other component.

REVIEW Check that your result has correct units and significant figures, is reasonable, and answers the question.

EXAMPLE 4.5 ■ Jumping frog contest

Frogs, with their long, strong legs, are excellent jumpers. And thanks to the good folks of Calaveras County, California, who have a jumping frog contest every year in honor of a Mark Twain story, we have very good data on how far a determined frog can jump.

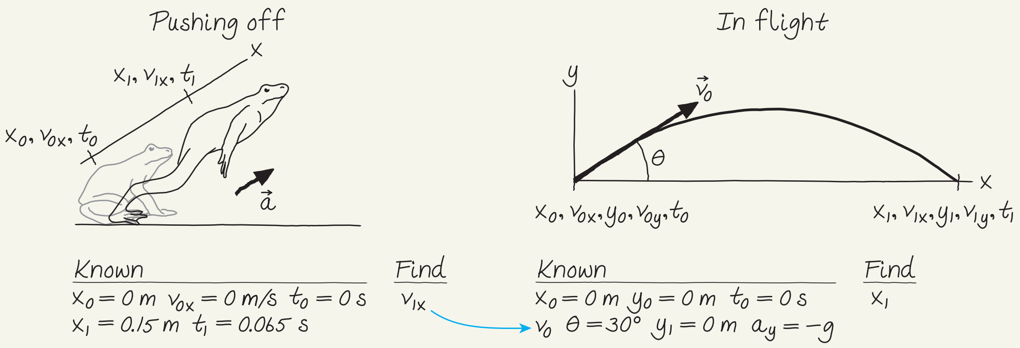
High-speed cameras show that a good jumper goes into a crouch, then rapidly extends his legs by typically 15 cm during a 65 ms push off, leaving the ground at a 30° angle. How far does this frog leap?

MODEL Model the push off as linear motion with uniform acceleration. A bullfrog is fairly heavy and dense, so ignore air resistance and model the leap as projectile motion.

VISUALIZE This is a two-part problem: linear acceleration followed by projectile motion. A key observation is that **the final velocity for pushing off the ground becomes the initial velocity of the projectile motion.** FIGURE 4.15 shows a separate pictorial representation for each part. Notice that we've used different coordinate systems for the two parts; coordinate systems are our choice, and for each part of the motion we've chosen the coordinate system that makes the problem easiest to solve.

SOLVE While pushing off, the frog travels $15\text{ cm} = 0.15\text{ m}$ in $65\text{ ms} = 0.065\text{ s}$. We could find his speed at the end of pushing off if we knew the acceleration. Because the initial velocity is zero,

FIGURE 4.15 Pictorial representations of the jumping frog.



we can find the acceleration from the position-acceleration-time kinematic equation:

$$x_1 = x_0 + v_{0x}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 = \tfrac{1}{2}a_x(\Delta t)^2$$

$$a_x = \frac{2x_1}{(\Delta t)^2} = \frac{2(0.15\text{ m})}{(0.065\text{ s})^2} = 71\text{ m/s}^2$$

This is a substantial acceleration, but it doesn't last long. At the end of the 65 ms push off, the frog's velocity is

$$v_{1x} = v_{0x} + a_x\,\Delta t = (71\text{ m/s}^2)(0.065\text{ s}) = 4.62\text{ m/s}$$

We'll keep an extra significant figure here to avoid round-off error in the second half of the problem.

The end of the push off is the beginning of the projectile motion, so the second part of the problem is to find the distance of a projectile launched with velocity $\vec{v}_0 = (4.62\text{ m/s}, 30°)$. The initial x- and y-components of the launch velocity are

$$v_{0x} = v_0\cos\theta \qquad v_{0y} = v_0\sin\theta$$

The kinematic equations of projectile motion, with $a_x = 0$ and $a_y = -g$, are

$$x_1 = x_0 + v_{0x}\,\Delta t = (v_0\cos\theta)\Delta t \qquad y_1 = y_0 + v_{0y}\,\Delta t - \tfrac{1}{2}g(\Delta t)^2 = (v_0\sin\theta)\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

We can find the time of flight from the vertical equation by setting $y_1 = 0$:

$$0 = (v_0\sin\theta)\Delta t - \tfrac{1}{2}g(\Delta t)^2 = (v_0\sin\theta - \tfrac{1}{2}g\,\Delta t)\Delta t$$

and thus

$$\Delta t = 0 \quad\text{or}\quad \Delta t = \frac{2v_0\sin\theta}{g}$$

Both are legitimate solutions. The first corresponds to the instant when $y = 0$ at the launch, the second to when $y = 0$ as the frog hits the ground. Clearly, we want the second solution. Substituting this expression for Δt into the equation for x_1 gives

$$x_1 = (v_0\cos\theta)\frac{2v_0\sin\theta}{g} = \frac{2v_0^2\sin\theta\cos\theta}{g}$$

We can simplify this result with the trigonometric identity $2\sin\theta\cos\theta = \sin(2\theta)$. Thus the distance traveled by the frog is

$$x_1 = \frac{v_0^2\sin(2\theta)}{g}$$

Using $v_0 = 4.62$ m/s and $\theta = 30°$, we find that the frog leaps a distance of 1.9 m.

REVIEW 1.9 m is about 6 feet, or about 10 times the frog's body length. That's pretty amazing, but true. Jumps of 2.2 m have been recorded in the lab. And the Calaveras County record holder, Rosie the Ribeter, covered 6.5 m—21 feet—in three jumps!

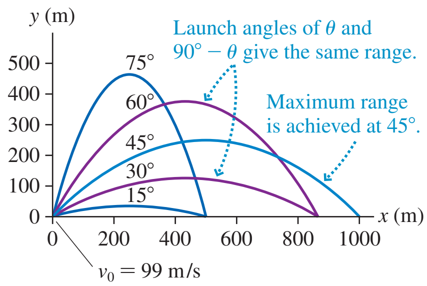
The distance a projectile travels is called its *range*. As Example 4.5 found, a projectile that lands at the same elevation from which it was launched has

$$\text{range} = \frac{v_0^2\sin(2\theta)}{g} \tag{4.15}$$

The maximum range occurs for $\theta = 45°$, where $\sin(2\theta) = 1$. But there's more that we can learn from this equation. Because $\sin(180° - x) = \sin x$, it follows that $\sin(2(90° - \theta)) = \sin(2\theta)$. Consequently, a projectile launched either at angle θ *or* at angle $(90° - \theta)$ will travel the same distance *over level ground*. **FIGURE 4.16** shows several trajectories of projectiles launched with the same initial speed.

NOTE Equation 4.15 is *not* a general result. It applies *only* in situations where the projectile lands at the same elevation from which it was fired.

FIGURE 4.16 Trajectories of a projectile launched at different angles with a speed of 99 m/s.



STOP TO THINK 4.3 A 50 g marble rolls off a table and hits 2 m from the base of the table. A 100 g marble rolls off the same table with the same speed. It lands at distance

a. Less than 1 m.
b. 1 m.
c. Between 1 m and 2 m.
d. 2 m.
e. Between 2 m and 4 m.
f. 4 m.

4.3 Relative Motion

FIGURE 4.17 Velocities in Amy's reference frame.

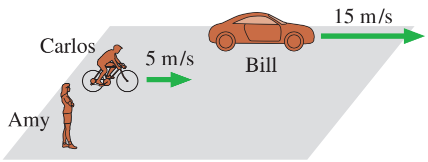


FIGURE 4.17 shows Amy and Bill watching Carlos on his bicycle. According to Amy, Carlos's velocity is $v_x = 5$ m/s. Bill sees the bicycle receding in his rearview mirror, in the *negative* x-direction, getting 10 m farther away from him every second. According to Bill, Carlos's velocity is $v_x = -10$ m/s. Which is Carlos's *true* velocity?

Velocity is not a concept that can be true or false. Carlos's velocity *relative to Amy* is $(v_x)_{CA} = 5$ m/s, where the subscript notation means "C relative to A." Similarly, Carlos's velocity *relative to Bill* is $(v_x)_{CB} = -10$ m/s. These are both valid descriptions of Carlos's motion.

It's not hard to see how to combine the velocities for one-dimensional motion:

The first subscript is the same on both sides.
The last subscript is the same on both sides.

$$(v_x)_{CB} = (v_x)_{CA} + (v_x)_{AB} \tag{4.16}$$

The inner subscripts "cancel."

We'll justify this relationship later in this section and then extend it to two-dimensional motion.

Equation 4.16 tells us that the velocity of C relative to B is the velocity of C relative to A *plus* the velocity of A relative to B. Note that

$$(v_x)_{AB} = -(v_x)_{BA} \tag{4.17}$$

because if B is moving to the right relative to A, then A is moving to the left relative to B. In Figure 4.17, Bill is moving to the right relative to Amy with $(v_x)_{BA} = 15$ m/s, so $(v_x)_{AB} = -15$ m/s. Knowing that Carlos's velocity relative to Amy is 5 m/s, we find that Carlos's velocity relative to Bill is, as expected, $(v_x)_{CB} = (v_x)_{CA} + (v_x)_{AB} = 5\text{ m/s} + (-15)\text{ m/s} = -10$ m/s.

EXAMPLE 4.6 ■ A speeding bullet

The police are chasing a bank robber. While driving at 50 m/s, they fire a bullet to shoot out a tire of his car. The police gun shoots bullets at 300 m/s. What is the bullet's speed as measured by a TV camera crew parked beside the road?

MODEL Assume that all motion is in the positive x-direction. The bullet is the object that is observed from both the police car and the ground.

SOLVE The bullet B's velocity relative to the gun G is $(v_x)_{BG} = 300$ m/s. The gun, inside the car, is traveling relative to the TV crew C at $(v_x)_{GC} = 50$ m/s. We can combine these values to find that the bullet's velocity relative to the TV crew on the ground is

$$(v_x)_{BC} = (v_x)_{BG} + (v_x)_{GC} = 300\text{ m/s} + 50\text{ m/s} = 350\text{ m/s}$$

REVIEW It should be no surprise in this simple situation that we simply add the velocities.

Reference Frames

A coordinate system in which an experimenter (possibly with the assistance of helpers) makes position and time measurements of physical events is called a **reference frame.** In Figure 4.17, Amy and Bill each had their own reference frame (where they were at rest) in which they measured Carlos's velocity.

More generally, **FIGURE 4.18** shows two reference frames, A and B, and an object C. It is assumed that the reference frames are moving with respect to each other. At this instant of time, the position vector of C in reference frame A is $\vec{r}_{CA}$, meaning "the position of C relative to the origin of frame A." Similarly, $\vec{r}_{CB}$ is the position vector of C in reference frame B. Using vector addition, you can see that

$$\vec{r}_{CB} = \vec{r}_{CA} + \vec{r}_{AB} \tag{4.18}$$

where $\vec{r}_{AB}$ locates the origin of A relative to the origin of B.

In general, object C is moving relative to both reference frames. To find its velocity in each reference frame, take the time derivative of Equation 4.18:

$$\frac{d\vec{r}_{CB}}{dt} = \frac{d\vec{r}_{CA}}{dt} + \frac{d\vec{r}_{AB}}{dt} \tag{4.19}$$

By definition, $d\vec{r}/dt$ is a velocity. The first derivative in Equation 4.19 is $\vec{v}_{CB}$, the velocity of C relative to B. Similarly, the second is the velocity of C relative to A, $\vec{v}_{CA}$. The last derivative is slightly different because it doesn't refer to object C. Instead, this is the velocity $\vec{v}_{AB}$ of reference frame A relative to reference frame B. As we noted in one dimension, $\vec{v}_{AB} = -\vec{v}_{BA}$.

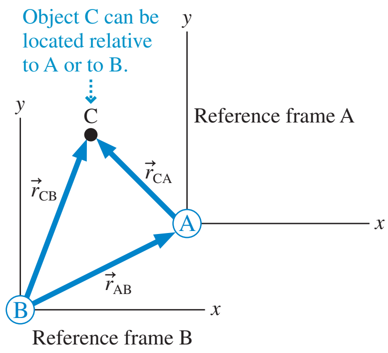
Writing Equation 4.19 in terms of velocities, we have

$$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB} \tag{4.20}$$

This relationship between velocities in different reference frames was recognized by Galileo in his pioneering studies of motion, hence it is known as the **Galilean transformation of velocity.** If you know an object's velocity in one reference frame, you can *transform* it into the velocity that would be measured in a different reference frame. Just as in one dimension, the velocity of C relative to B is the velocity of C relative to A plus the velocity of A relative to B, *but* you must add the velocities as vectors for two-dimensional motion.

As we've seen, the Galilean velocity transformation is pretty much common sense for one-dimensional motion. The real usefulness appears when an object travels in a *medium* moving with respect to the earth. For example, a boat moves relative to the water. What is the boat's net motion if the water is a flowing river? Airplanes fly relative to the air, but the air at high altitudes often flows at high speed. Navigation of boats and planes requires knowing both the motion of the vessel in the medium and the motion of the medium relative to the earth.

FIGURE 4.18 Two reference frames.

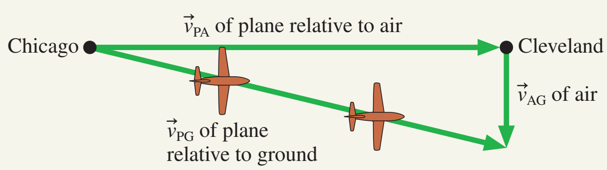


EXAMPLE 4.7 ■ Flying to Cleveland I

Cleveland is 300 miles east of Chicago. A plane leaves Chicago flying due east at 500 mph. The pilot forgot to check the weather and doesn't know that the wind is blowing to the south at 50 mph. What is the plane's ground speed? Where is the plane 0.60 h later, when the pilot expects to land in Cleveland?

MODEL Establish a coordinate system with the x-axis pointing east and the y-axis north. The plane P flies in the air, so its velocity relative to the air A is $\vec{v}_{PA} = 500\hat{\imath}$ mph. Meanwhile, the air is moving relative to the ground G at $\vec{v}_{AG} = -50\hat{\jmath}$ mph.

FIGURE 4.19 The wind causes a plane flying due east in the air to move to the southeast relative to the ground.



SOLVE The velocity equation $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$ is a vector-addition equation. **FIGURE 4.19** shows graphically what happens. Although the nose of the plane points east, the wind carries the plane in a direction somewhat south of east. The plane's velocity relative to the ground is

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG} = (500\hat{\imath} - 50\hat{\jmath}) \text{ mph}$$

The plane's ground speed is

$$v = \sqrt{(v_x)_{PG}{}^2 + (v_y)_{PG}{}^2} = 502 \text{ mph}$$

After flying for 0.60 h at this velocity, the plane's location (relative to Chicago) is

$$x = (v_x)_{PG}t = (500 \text{ mph})(0.60 \text{ h}) = 300 \text{ mi}$$

$$y = (v_y)_{PG}t = (-50 \text{ mph})(0.60 \text{ h}) = -30 \text{ mi}$$

The plane is 30 mi due south of Cleveland! Although the pilot thought he was flying to the east, his actual heading has been $\tan^{-1}(50 \text{ mph}/500 \text{ mph}) = \tan^{-1}(0.10) = 5.71°$ south of east.

EXAMPLE 4.8 ■ Flying to Cleveland II

A wiser pilot flying from Chicago to Cleveland on the same day plots a course that will take her directly to Cleveland. In which direction does she fly the plane? How long does it take to reach Cleveland?

MODEL Establish a coordinate system with the x-axis pointing east and the y-axis north. The air is moving relative to the ground at $\vec{v}_{AG} = -50\hat{\jmath}$ mph.

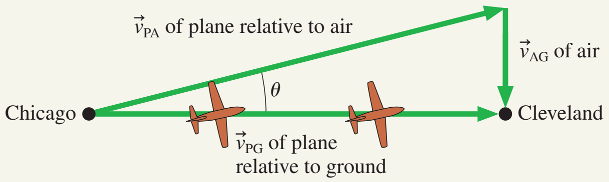
SOLVE The objective of navigation is to move between two points on the earth's surface. The wiser pilot, who knows that the wind will affect her plane, draws the vector picture of **FIGURE 4.20**. She sees that she'll need $(v_y)_{PG} = 0$ in order to fly due east to Cleveland. This will require turning the nose of the plane at an angle θ north of east, making $\vec{v}_{PA} = (500\cos\theta\,\hat{\imath} + 500\sin\theta\,\hat{\jmath})$ mph.

The velocity equation is $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$. The desired heading is found from setting the y-component of this equation to zero:

$$(v_y)_{PG} = (v_y)_{PA} + (v_y)_{AG} = (500\sin\theta - 50)\text{ mph} = 0\text{ mph}$$

$$\theta = \sin^{-1}\left(\frac{50\text{ mph}}{500\text{ mph}}\right) = 5.74°$$

FIGURE 4.20 To travel due east in a south wind, a pilot has to point the plane somewhat to the northeast.



The plane's velocity relative to the ground is then $\vec{v}_{PG} = 500\cos 5.74°\,\hat{\imath}$ mph $= 497\,\hat{\imath}$ mph. This is slightly slower than the speed relative to the air. The time needed to fly to Cleveland at this speed is

$$t = \frac{300\text{ mi}}{497\text{ mph}} = 0.604\text{ h}$$

It takes 0.004 h = 14 s longer to reach Cleveland than it would on a day without wind.

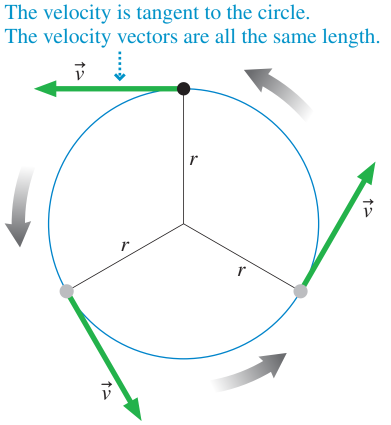
REVIEW A boat crossing a river or an ocean current faces the same difficulties. These are exactly the kinds of calculations performed by pilots of boats and planes as part of navigation.

STOP TO THINK 4.4 A plane traveling horizontally to the right at 100 m/s flies past a helicopter that is going straight up at 20 m/s. From the helicopter's perspective, the plane's direction and speed are

a. Right and up, less than 100 m/s.
b. Right and up, 100 m/s.
c. Right and up, more than 100 m/s.
d. Right and down, less than 100 m/s.
e. Right and down, 100 m/s.
f. Right and down, more than 100 m/s.

4.4 Uniform Circular Motion

FIGURE 4.21 A particle in uniform circular motion.



Projectile motion is one important example of motion in a plane. Another quite different type of motion in a plane is circular motion. **FIGURE 4.21** shows a particle moving around a circle of radius r. The particle might be a satellite in an orbit, a ball on the end of a string, or even just a dot painted on the side of a rotating wheel.

To begin the study of circular motion, consider a particle that moves at *constant speed* around a circle of radius r. This is called **uniform circular motion.** Regardless of what the particle represents, its velocity vector $\vec{v}$ is always tangent to the circle. The particle's speed v is constant, so vector $\vec{v}$ is always the same length.

The time interval it takes the particle to go around the circle once, completing one revolution (abbreviated rev), is called the **period** of the motion. Period is represented by the symbol T. It's easy to relate the particle's period T to its speed v. For a particle moving with constant speed, speed is simply distance/time. In one period, the particle moves once around a circle of radius r and travels the circumference $2\pi r$. Thus

$$v = \frac{1\text{ circumference}}{1\text{ period}} = \frac{2\pi r}{T} \tag{4.21}$$

EXAMPLE 4.9 ■ A rotating crankshaft

A 4.0-cm-diameter crankshaft turns at 2400 rpm (revolutions per minute). What is the speed of a point on the surface of the crankshaft?

SOLVE We need to determine the time it takes the crankshaft to make 1 rev. First, we convert 2400 rpm to revolutions per second:

$$\frac{2400\text{ rev}}{1\text{ min}} \times \frac{1\text{ min}}{60\text{ s}} = 40\text{ rev/s}$$

If the crankshaft turns 40 times in 1 s, the time for 1 rev is

$$T = \frac{1}{40}\text{ s} = 0.025\text{ s}$$

Thus the speed of a point on the surface, where $r = 2.0\text{ cm} = 0.020\text{ m}$, is

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.020\text{ m})}{0.025\text{ s}} = 5.0\text{ m/s}$$

Angular Position

Rather than using xy-coordinates, it will be more convenient to describe the position of a particle in circular motion by its distance r from the center of the circle and its angle θ from the positive x-axis. This is shown in **FIGURE 4.22**. The angle θ is the **angular position** of the particle.

We can distinguish a position above the x-axis from a position that is an equal angle below the x-axis by *defining* θ to be positive when measured *counterclockwise* (ccw) from the positive x-axis. An angle measured clockwise (cw) from the positive x-axis has a negative value. "Clockwise" and "counterclockwise" in circular motion are analogous, respectively, to "left of the origin" and "right of the origin" in linear motion, which we associated with negative and positive values of x. A particle 30° below the positive x-axis is equally well described by either $\theta = -30°$ or $\theta = +330°$. We could also describe this particle by $\theta = \frac{11}{12}$ rev, where *revolutions* are another way to measure the angle.

Although degrees and revolutions are widely used measures of angle, mathematicians and scientists usually find it more useful to measure the angle θ in Figure 4.22 by using the **arc length** s that the particle travels along the edge of a circle of radius r. We define the angular unit of **radians** such that

$$\theta(\text{radians}) \equiv \frac{s}{r} \qquad (4.22)$$

The radian, which is abbreviated rad, is the SI unit of angle. An angle of 1 rad has an arc length s exactly equal to the radius r.

The arc length completely around a circle is the circle's circumference $2\pi r$. Thus the angle of a full circle is

$$\theta_{\text{full circle}} = \frac{2\pi r}{r} = 2\pi\text{ rad}$$

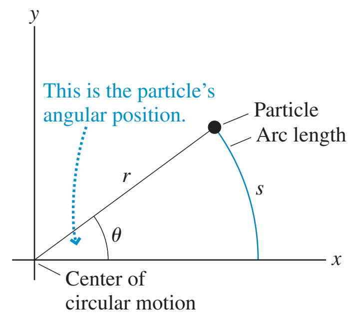
This relationship is the basis for the well-known conversion factors

$$1\text{ rev} = 360° = 2\pi\text{ rad}$$

As a simple example of converting between radians and degrees, let's convert an angle of 1 rad to degrees:

$$1\text{ rad} = 1\text{ rad} \times \frac{360°}{2\pi\text{ rad}} = 57.3°$$

FIGURE 4.22 A particle's position is described by distance r and angle θ.



Circular motion is one of the most common types of motion.

Thus a rough approximation is 1 rad $\approx 60°$. We will often specify angles in degrees, but keep in mind that the SI unit is the radian.

An important consequence of Equation 4.22 is that the arc length spanning angle θ is

$$s = r\theta \qquad (\text{with } \theta \text{ in rad}) \tag{4.23}$$

This is a result that we will use often, but it is valid *only* if θ is measured in radians and not in degrees. This very simple relationship between angle and arc length is one of the primary motivations for using radians.

NOTE Units of angle are often troublesome. Unlike the kilogram or the second, for which we have standards, the radian is a *defined* unit. It's really just a *name* to remind us that we're dealing with an angle. Consequently, the radian unit sometimes appears or disappears without warning. This seems rather mysterious until you get used to it. This textbook will call your attention to such behavior the first few times it occurs. With a little practice, you'll soon learn when the rad unit is needed and when it's not.

Angular Velocity

FIGURE 4.23 A particle moves with angular velocity ω.

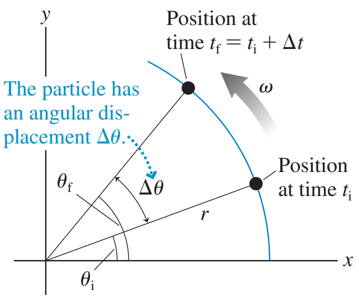


FIGURE 4.23 shows a particle moving in a circle from an initial angular position θ_i at time t_i to a final angular position θ_f at a later time t_f. The change $\Delta\theta = \theta_f - \theta_i$ is called the **angular displacement.** We can measure the particle's circular motion in terms of the rate of change of θ, just as we measured the particle's linear motion in terms of the rate of change of its position s.

In analogy with linear motion, let's define the *average angular velocity* to be

$$\text{average angular velocity} \equiv \frac{\Delta\theta}{\Delta t} \tag{4.24}$$

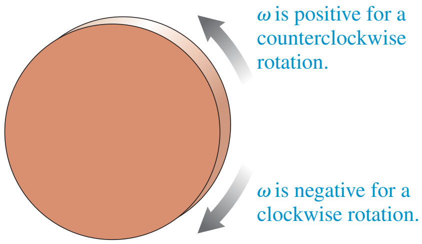
As the time interval Δt becomes very small, $\Delta t \rightarrow 0$, we arrive at the definition of the instantaneous **angular velocity:**

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \qquad (\text{angular velocity}) \tag{4.25}$$

The symbol ω is a lowercase Greek omega, *not* an ordinary w. The SI unit of angular velocity is rad/s, but °/s, rev/s, and rev/min are also common units. Revolutions per minute is abbreviated rpm.

Angular velocity is the *rate* at which a particle's angular position is changing as it moves around a circle. A particle that starts from $\theta = 0$ rad with an angular velocity of 0.5 rad/s will be at angle $\theta = 0.5$ rad after 1 s, at $\theta = 1.0$ rad after 2 s, at $\theta = 1.5$ rad after 3 s, and so on. Its angular position is increasing at the *rate* of 0.5 radian per second. **A particle moves with uniform circular motion if and only if its angular velocity ω is constant and unchanging.**

FIGURE 4.24 Positive and negative angular velocities.



Angular velocity, like the velocity v_s of one-dimensional motion, can be positive or negative. The signs shown in FIGURE 4.24 are based on the fact that θ was defined to be positive for a counterclockwise rotation. Because the definition $\omega = d\theta/dt$ for circular motion parallels the definition $v_s = ds/dt$ for linear motion, the graphical relationships we found between v_s and s in Chapter 2 apply equally well to ω and θ:

$$\begin{aligned} \omega &= \text{slope of the } \theta\text{-versus-}t \text{ graph at time } t \\ \theta_f &= \theta_i + \text{area under the } \omega\text{-versus-}t \text{ curve between } t_i \text{ and } t_f \\ &= \theta_i + \omega\,\Delta t \end{aligned} \tag{4.26}$$

You will see many more instances where circular motion is analogous to linear motion with angular variables replacing linear variables. Thus much of what you learned about linear kinematics carries over to circular motion.

EXAMPLE 4.10 ■ A graphical representation of circular motion

FIGURE 4.25 shows the angular position of a painted dot on the edge of a rotating wheel. Describe the wheel's motion and draw an ω-versus-t graph.

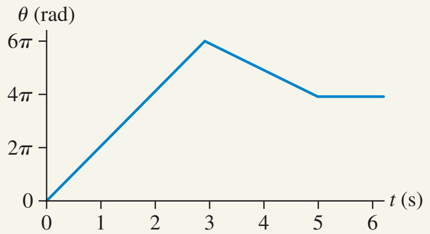


FIGURE 4.25 Angular position graph for the wheel of Example 4.10.

SOLVE Although circular motion seems to "start over" every revolution (every 2π rad), the angular position θ continues to increase. $\theta = 6\pi$ rad corresponds to three revolutions. This wheel makes 3 ccw rev (because θ is getting more positive) in 3 s, immediately reverses direction and makes 1 cw rev in 2 s, then stops at $t = 5$ s and holds the position $\theta = 4\pi$ rad. The angular velocity is found by measuring the slope of the graph:

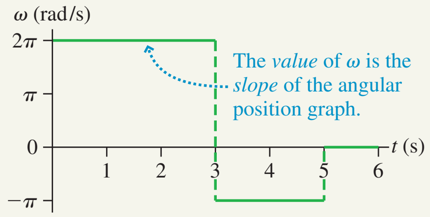
$$t = 0\text{–}3\text{ s} \qquad \text{slope} = \Delta\theta/\Delta t = 6\pi \text{ rad}/3\text{ s} = 2\pi \text{ rad/s}$$

$$t = 3\text{–}5\text{ s} \qquad \text{slope} = \Delta\theta/\Delta t = -2\pi \text{ rad}/2\text{ s} = -\pi \text{ rad/s}$$

$$t > 5\text{ s} \qquad \text{slope} = \Delta\theta/\Delta t = 0 \text{ rad/s}$$

These results are shown as an ω-versus-t graph in **FIGURE 4.26**. For the first 3 s, the motion is uniform circular motion with $\omega = 2\pi$ rad/s. The wheel then changes to a different uniform circular motion with $\omega = -\pi$ rad/s for 2 s, then stops.

FIGURE 4.26 ω-versus-t graph for the wheel of Example 4.10.



NOTE In physics, we nearly always want to give results as numerical values. Example 4.9 had a π in the equation, but we used its numerical value to compute $v = 5.0$ m/s. However, angles in radians are an exception to this rule. It's okay to leave a π in the value of θ or ω, and we have done so in Example 4.10.

Not surprisingly, the angular velocity ω is closely related to the period and speed of the motion. As a particle goes around a circle one time, its angular displacement is $\Delta\theta = 2\pi$ rad during the interval $\Delta t = T$. Thus, using the definition of angular velocity, we find

$$|\omega| = \frac{2\pi \text{ rad}}{T} \quad \text{or} \quad T = \frac{2\pi \text{ rad}}{|\omega|} \tag{4.27}$$

The period alone gives only the absolute value of $|\omega|$, which is the *angular speed*. You need to know the direction of motion to determine the sign of ω.

EXAMPLE 4.11 ■ At the roulette wheel

A small steel roulette ball rolls ccw around the inside of a 30-cm-diameter roulette wheel. The ball completes 2.0 rev in 1.20 s.

a. What is the ball's angular velocity?

b. What is the ball's position at $t = 2.0$ s? Assume $\theta_i = 0$.

MODEL Model the ball as a particle in uniform circular motion.

SOLVE a. The period of the ball's motion, the time for 1 rev, is $T = 0.60$ s. Angular velocity is positive for ccw motion, so

$$\omega = \frac{2\pi \text{ rad}}{T} = \frac{2\pi \text{ rad}}{0.60 \text{ s}} = 10.47 \text{ rad/s}$$

b. The ball starts at $\theta_i = 0$ rad. After $\Delta t = 2.0$ s, its position is

$$\theta_f = 0 \text{ rad} + (10.47 \text{ rad/s})(2.0 \text{ s}) = 20.94 \text{ rad}$$

where we've kept an extra significant figure to avoid round-off error. Although this is a mathematically acceptable answer, an observer would say that the ball is always located somewhere between 0° and 360°. Thus it is common practice to subtract an integer number of 2π rad, representing the completed revolutions. Because $20.94/2\pi = 3.333$, we can write

$$\begin{aligned}\theta_f &= 20.94 \text{ rad} = 3.333 \times 2\pi \text{ rad}\\ &= 3 \times 2\pi \text{ rad} + 0.333 \times 2\pi \text{ rad}\\ &= 3 \times 2\pi \text{ rad} + 2.09 \text{ rad}\end{aligned}$$

In other words, at $t = 2.0$ s the ball has completed 3 rev and is 2.09 rad = 120° into its fourth revolution. An observer would say that the ball's position is $\theta_f = 120°$.

As Figure 4.21 showed, the velocity vector $\vec{v}$ is always tangent to the circle. In other words, the velocity vector has only a *tangential component,* which we will designate v_t. The tangential velocity is positive for ccw motion, negative for cw motion.

Combining $v = 2\pi r/T$ for the speed with $\omega = 2\pi/T$ for the angular velocity—but keeping the sign of ω to indicate the direction of motion—we see that the tangential velocity and the angular velocity are related by

$$v_t = \omega r \qquad (\text{with } \omega \text{ in rad/s}) \tag{4.28}$$

Because v_t is the only nonzero component of $\vec{v}$, the particle's speed is $v = |v_t| = |\omega|r$. We'll sometimes write this as $v = \omega r$ if there's no ambiguity about the sign of ω.

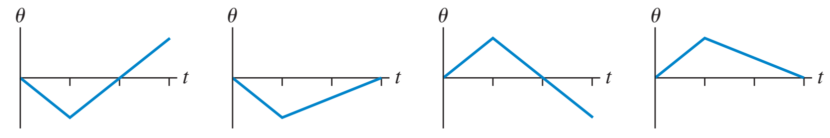
NOTE While it may be convenient in some problems to measure ω in rev/s or rpm, you must convert to SI units of rad/s before using Equation 4.28.

As a simple example, a particle moving cw at 2.0 m/s in a circle of radius 40 cm has angular velocity

$$\omega = \frac{v_t}{r} = \frac{-2.0 \text{ m/s}}{0.40 \text{ m}} = -5.0 \text{ rad/s}$$

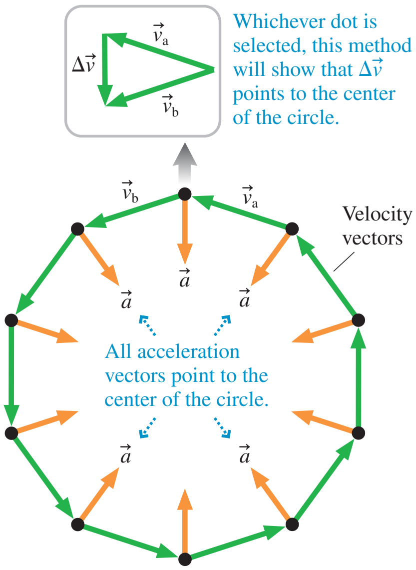
where v_t and ω are negative because the motion is clockwise. Notice the units. Velocity divided by distance has units of s^{-1}. But because the division, in this case, gives us an angular quantity, we've inserted the *dimensionless* unit rad to give ω the appropriate units of rad/s.

STOP TO THINK 4.5 A particle moves cw around a circle at constant speed for 2.0 s. It then reverses direction and moves ccw at half the original speed until it has traveled through the same angle. Which is the particle's angle-versus-time graph?



(a) (b) (c) (d)

FIGURE 4.27 Using Tactics Box 4.1 to find Maria's acceleration on the Ferris wheel.



Maria's acceleration is an acceleration of changing direction, not of changing speed.

4.5 Centripetal Acceleration

FIGURE 4.27 shows a motion diagram of Maria riding a Ferris wheel at the amusement park. Maria has constant speed but *not* constant velocity because her velocity vector is changing direction. She may not be speeding up, but Maria *is* accelerating because her velocity is changing. The inset to Figure 4.27 applies the rules of Tactics Box 4.1 to find that—at every point—**Maria's acceleration vector points toward the center of the circle.** This is an acceleration due to changing direction rather than changing speed. Because the instantaneous velocity is tangent to the circle, $\vec{v}$ and $\vec{a}$ are perpendicular to each other at all points on the circle.

The acceleration of uniform circular motion is called **centripetal acceleration,** a term from a Greek root meaning "center seeking." Centripetal acceleration is not a new type of acceleration; all we are doing is *naming* an acceleration that corresponds to a particular type of motion. The magnitude of the centripetal acceleration is constant because each successive $\Delta\vec{v}$ in the motion diagram has the same length.

The motion diagram tells us the direction of $\vec{a}$, but it doesn't give us a value for the magnitude a. To complete our description of uniform circular motion, we need to find a quantitative relationship between a and the particle's speed v. **FIGURE 4.28** shows the velocity $\vec{v}_a$ at one instant of motion and the velocity $\vec{v}_b$ an infinitesimal amount of time dt later. During this small interval of time, the particle has moved through the infinitesimal angle $d\theta$ and traveled distance $ds = r\,d\theta$.

By definition, the acceleration is $\vec{a} = d\vec{v}/dt$. We can see from the inset to Figure 4.28 that $d\vec{v}$ points toward the center of the circle—that is, $\vec{a}$ is a centripetal acceleration. To find the magnitude of $\vec{a}$, we can see from the isosceles triangle of velocity vectors that, if $d\theta$ is in radians,

$$dv = |d\vec{v}| = v_t\,d\theta \tag{4.29}$$

For uniform circular motion at constant speed, $v_t = ds/dt = r\,d\theta/dt$ and thus the time to rotate through angle $d\theta$ is

$$dt = \frac{r\,d\theta}{v_t} \tag{4.30}$$

Combining Equations 4.29 and 4.30, we see that the acceleration has magnitude

$$a = |\vec{a}| = \frac{|d\vec{v}|}{dt} = \frac{v\,d\theta}{r\,d\theta/v_t} = \frac{v_t^2}{r}$$

In vector notation, we can write

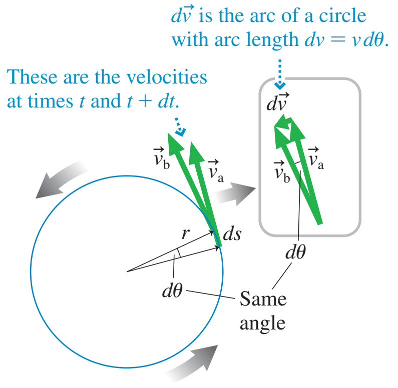
$$\vec{a} = \left(\frac{v_t^2}{r},\ \text{toward center of circle}\right) \qquad \text{(centripetal acceleration)} \tag{4.31}$$

Using Equation 4.28, $v_t = \omega r$, we can also express the magnitude of the centripetal acceleration in terms of the angular velocity ω as

$$a = \omega^2 r \tag{4.32}$$

NOTE Centripetal acceleration is not a constant acceleration. The magnitude of the centripetal acceleration is constant during uniform circular motion, but the direction of $\vec{a}$ is continuously changing. **Thus the constant-acceleration kinematics equations of Chapter 2 do *not* apply to circular motion.**

FIGURE 4.28 Finding the acceleration of circular motion.



The Uniform Circular Motion Model

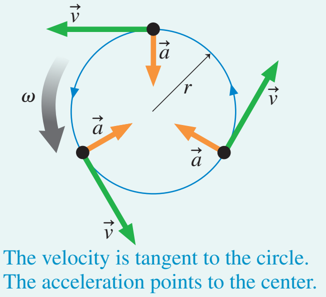
The **uniform circular motion model** is especially important because it applies not only to particles moving in circles but also to the uniform rotation of solid objects.

MODEL 4.2

Uniform circular motion

For motion with constant angular velocity ω.

- Applies to a particle moving along a circular trajectory at constant speed or to points on a solid object rotating at a steady rate.
- Mathematically:
  - The tangential velocity is $v_t = \omega r$.
  - The centripetal acceleration is v_t^2/r or $\omega^2 r$.
  - ω and v_t are positive for ccw rotation, negative for cw rotation.
- Limitations: Model fails if rotation isn't steady.



Exercise 20

EXAMPLE 4.12 ■ **The acceleration of a Ferris wheel**

A typical carnival Ferris wheel has a radius of 9.0 m and rotates 2.0 times per minute. What speed and acceleration do the riders experience?

MODEL Model the rider as a particle in uniform circular motion.

SOLVE The period is $T = \frac{1}{2}\text{ min} = 30\text{ s}$. From Equation 4.21, a rider's speed is

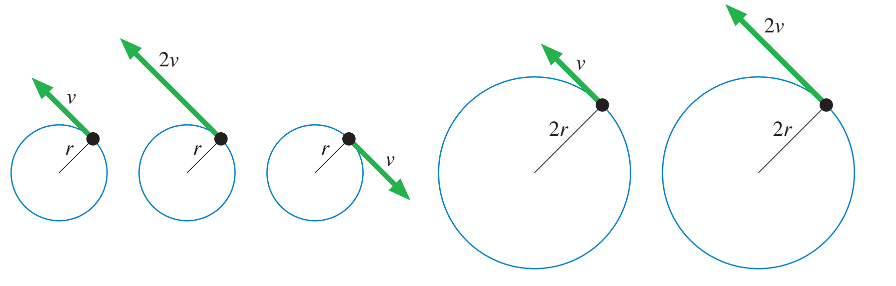
$$v_t = \frac{2\pi r}{T} = \frac{2\pi(9.0\text{ m})}{30\text{ s}} = 1.88\text{ m/s}$$

Consequently, the centripetal acceleration has magnitude

$$a = \frac{v_t^2}{r} = \frac{(1.88\text{ m/s})^2}{9.0\text{ m}} = 0.39\text{ m/s}^2$$

REVIEW This was not intended to be a profound problem, merely to illustrate how centripetal acceleration is computed. The acceleration is enough to be noticed and make the ride interesting, but not enough to be scary.

STOP TO THINK 4.6 Rank in order, from largest to smallest, the centripetal accelerations a_A to a_E of particles A to E.

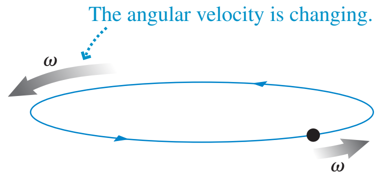


4.6 Nonuniform Circular Motion

A roller coaster car doing a loop-the-loop slows down as it goes up one side, speeds up as it comes back down the other. The ball in a roulette wheel gradually slows until it stops. Circular motion with a changing speed is called **nonuniform circular motion.** As you'll see, nonuniform circular motion is analogous to accelerated linear motion.

FIGURE 4.29 shows a point speeding up as it moves around a circle. This might be a car speeding up around a curve or simply a point on a solid object that is rotating faster and faster. The key feature of the motion is a *changing angular velocity*. For linear motion, we defined acceleration as $a_x = dv_x/dt$. By analogy, let's define the **angular acceleration** α (Greek alpha) of a rotating object, or a point on the object, to be

FIGURE 4.29 Circular motion with a changing angular velocity.



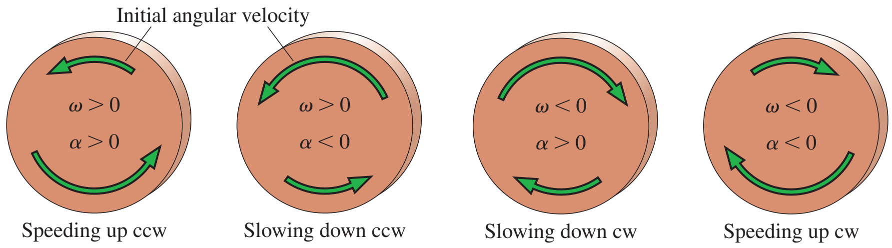
$$\alpha \equiv \frac{d\omega}{dt} \quad \text{(angular acceleration)} \tag{4.33}$$

Angular acceleration is the *rate* at which the angular velocity ω changes, just as linear acceleration is the rate at which the linear velocity v_x changes. The units of angular acceleration are rad/s^2.

For linear acceleration, you learned that a_x and v_x have the same sign when an object is speeding up, opposite signs when it is slowing down. The same rule applies to circular and rotational motion: ω and α have the same sign when the rotation is speeding up, opposite signs if it is slowing down. These ideas are illustrated in **FIGURE 4.30**.

NOTE Be careful with the sign of α. You learned in Chapter 2 that positive and negative values of the acceleration can't be interpreted as simply "speeding up" and "slowing down." Similarly, positive and negative values of angular acceleration can't be interpreted as a rotation that is speeding up or slowing down.

FIGURE 4.30 The signs of angular velocity and acceleration. The rotation is speeding up if ω and α have the same sign, slowing down if they have opposite signs.



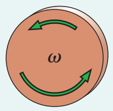
Angular position, angular velocity, and angular acceleration are defined exactly the same as linear position, velocity, and acceleration—simply starting with an angular rather than a linear measurement of position. Consequently, **the graphical interpretation and the kinematic equations of circular/rotational motion with constant angular acceleration are exactly the same as for linear motion with constant acceleration.** This is shown in the **constant angular acceleration model** below. All the problem-solving techniques you learned in Chapter 2 for linear motion carry over to circular and rotational motion.

MODEL 4.3

Constant angular acceleration

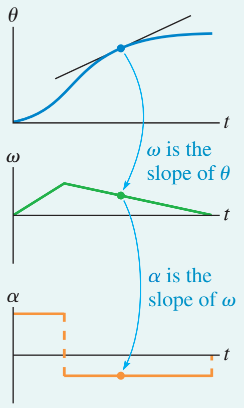
For motion with constant angular acceleration α.

- Applies to particles with circular trajectories and to rotating solid objects.



- Mathematically: The graphs and equations for this circular/rotational motion are analogous to linear motion with constant acceleration.
  - Analogs: $s \rightarrow \theta \quad v_s \rightarrow \omega \quad a_s \rightarrow \alpha$

Rotational kinematics	Linear kinematics
$\omega_f = \omega_i + \alpha\,\Delta t$	$v_{fs} = v_{is} + a_s\,\Delta t$
$\theta_f = \theta_i + \omega_i\,\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f{}^2 = \omega_i{}^2 + 2\alpha\,\Delta\theta$	$v_{fs}{}^2 = v_{is}{}^2 + 2a_s\,\Delta s$

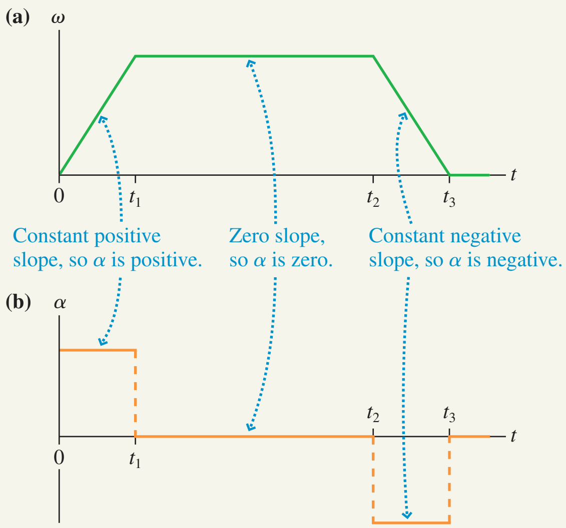


EXAMPLE 4.13 ■ **A rotating wheel**

FIGURE 4.31a is a graph of angular velocity versus time for a rotating wheel. Describe the motion and draw a graph of angular acceleration versus time.

SOLVE This is a wheel that starts from rest, gradually speeds up *counterclockwise* until reaching top speed at t_1, maintains a constant angular velocity until t_2, then gradually slows down until stopping at t_3. The motion is always ccw because ω is always positive. The angular acceleration graph of **FIGURE 4.31b** is based on the fact that α is the slope of the ω-versus-t graph.

Conversely, the initial linear increase of ω can be seen as the increasing area under the α-versus-t graph as t increases from 0 to t_1. The angular velocity doesn't change from t_1 to t_2 when the area under the α-versus-t is zero.



▶ **FIGURE 4.31** ω-versus-t graph and the corresponding α-versus-t graph for a rotating wheel.

EXAMPLE 4.14 ■ A slowing fan

A ceiling fan spinning at 60 rpm coasts to a stop 25 s after being turned off. How many revolutions does it make while stopping?

MODEL Model the fan as a rotating object with constant angular acceleration.

SOLVE We don't know which direction the fan is rotating, but the fact that the rotation is slowing tells us that ω and α have opposite signs. We'll assume that ω is positive. We need to convert the initial angular velocity to SI units:

$$\omega_i = 60\,\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 6.28\text{ rad/s}$$

We can use the first rotational kinematics equation in Model 4.3 to find the angular acceleration:

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0\text{ rad/s} - 6.28\text{ rad/s}}{25\text{ s}} = -0.25\text{ rad/s}^2$$

Then, from the second rotational kinematic equation, the angular displacement during these 25 s is

$$\begin{aligned}\Delta\theta &= \omega_i\,\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2\\ &= (6.28\text{ rad/s})(25\text{ s}) + \tfrac{1}{2}(-0.25\text{ rad/s}^2)(25\text{ s})^2\\ &= 78.9\text{ rad} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = 13\text{ rev}\end{aligned}$$

The kinematic equation returns an angle in rad, but the question asks for revolutions, so the last step was a unit conversion.

REVIEW Turning through 13 rev in 25 s while stopping seems reasonable. Notice that the problem is solved just like the linear kinematics problems you learned to solve in Chapter 2.

Tangential Acceleration

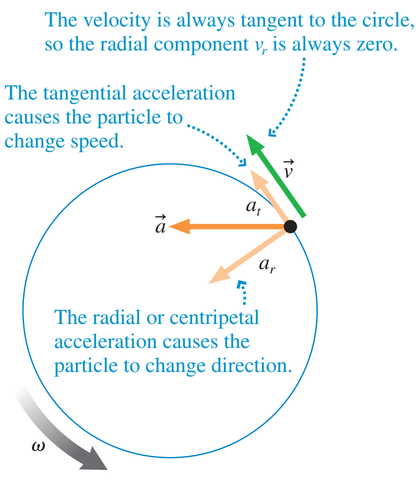


FIGURE 4.32 Acceleration in nonuniform circular motion.

FIGURE 4.32 shows a particle in nonuniform circular motion. *Any* circular motion, whether uniform or nonuniform, has a centripetal acceleration because the particle is changing direction; this was the acceleration component $\vec{a}_\perp$ of Figure 4.6. As a vector component, the centripetal acceleration, which points radially toward the center of the circle, is the **radial acceleration** a_r. The expression $a_r = v_t^2/r = \omega^2 r$ is still valid in nonuniform circular motion.

For a particle to speed up or slow down as it moves around a circle, it needs—in addition to the centripetal acceleration—an acceleration parallel to the trajectory or, equivalently, parallel to $\vec{v}$. This is the acceleration component $\vec{a}_\parallel$ associated with changing speed. We'll call this the **tangential acceleration** a_t because, like the velocity v_t, it is always tangent to the circle. Because of the tangential acceleration, **the acceleration vector $\vec{a}$ of a particle in nonuniform circular motion does not point toward the center of the circle**. It points "ahead" of center for a particle that is speeding up, as in Figure 4.32, but it would point "behind" center for a particle slowing down. You can see from Figure 4.32 that the magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_t^2} \tag{4.34}$$

If a_t is constant, then the arc length s traveled by the particle around the circle and the tangential velocity v_t are found from constant-acceleration kinematics:

$$\begin{aligned}s_f &= s_i + v_{it}\,\Delta t + \tfrac{1}{2}a_t(\Delta t)^2\\ v_{ft} &= v_{it} + a_t\,\Delta t\end{aligned} \tag{4.35}$$

Because tangential acceleration is the rate at which the tangential velocity changes, $a_t = dv_t/dt$, and we already know that the tangential velocity is related to the angular velocity by $v_t = \omega r$, it follows that

$$a_t = \frac{dv_t}{dt} = \frac{d(\omega r)}{dt} = \frac{d\omega}{dt}r = \alpha r \tag{4.36}$$

Thus $v_t = \omega r$ and $a_t = \alpha r$ are analogous equations for the tangential velocity and acceleration. In Example 4.14, where we found the fan to have angular acceleration $\alpha = -0.25\text{ rad/s}^2$, a blade tip 65 cm from the center would have tangential acceleration

$$a_t = \alpha r = (-0.25\text{ rad/s}^2)(0.65\text{ m}) = -0.16\text{ m/s}^2$$

EXAMPLE 4.15 ■ Analyzing rotational data

You've been assigned the task of measuring the start-up characteristics of a large industrial motor. After several seconds, when the motor has reached full speed, you know that the angular acceleration will be zero, but you hypothesize that the angular acceleration may be constant during the first couple of seconds as the motor speed increases. To find out, you attach a shaft encoder to the 3.0-cm-diameter axle. A shaft encoder is a device that converts the angular position of a shaft or axle to a signal that can be read by a computer. After setting the computer program to read four values a second, you start the motor and acquire the following data:

Time (s)	Angle (°)	Time (s)	Angle (°)
0.00	0	1.00	267
0.25	16	1.25	428
0.50	69	1.50	620
0.75	161		

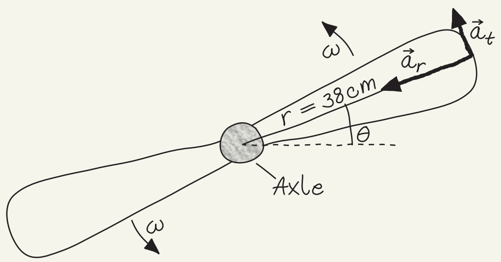
a. Do the data support your hypothesis of a constant angular acceleration? If so, what is the angular acceleration? If not, is the angular acceleration increasing or decreasing with time?

b. A 76-cm-diameter blade is attached to the motor shaft. At what time does the acceleration of the tip of the blade reach 10 m/s^2?

MODEL The axle is rotating with nonuniform circular motion. Model the tip of the blade as a particle.

VISUALIZE FIGURE 4.33 shows that the blade tip has both a tangential and a radial acceleration.

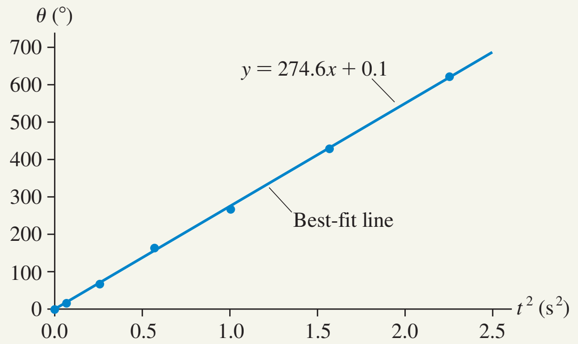
FIGURE 4.33 Pictorial representation of the axle and blade.



SOLVE a. *If* the motor starts up with constant angular acceleration, with $\theta_i = 0$ and $\omega_i = 0$ rad/s, the angle-time equation of rotational kinematics is $\theta = \frac{1}{2}\alpha t^2$. This can be written as a linear equation $y = mx + b$ if we let $\theta = y$ and $t^2 = x$. That is, constant angular acceleration predicts that a graph of θ versus t^2 should be a straight line with slope $m = \frac{1}{2}\alpha$ and y-intercept $b = 0$. We can test this.

FIGURE 4.34 is the graph of θ versus t^2, and it confirms our hypothesis that the motor starts up with constant angular acceleration. The best-fit line, found using a spreadsheet, gives a

FIGURE 4.34 Graph of θ versus t^2 for the motor shaft.



slope of 274.6°/s^2. The units come not from the spreadsheet but by looking at the units of rise (°) over run (s^2 because we're graphing t^2 on the x-axis). Thus the angular acceleration is

$$\alpha = 2m = 549.2°/\text{s}^2 \times \frac{\pi \text{ rad}}{180°} = 9.6 \text{ rad/s}^2$$

where we used 180° = π rad to convert to SI units of rad/s^2.

b. The magnitude of the linear acceleration is

$$a = \sqrt{a_r^2 + a_t^2}$$

The tangential acceleration of the blade tip is

$$a_t = \alpha r = (9.6 \text{ rad/s}^2)(0.38 \text{ m}) = 3.65 \text{ m/s}^2$$

We were careful to use the blade's radius, not its diameter, and we kept an extra significant figure to avoid round-off error. The radial (centripetal) acceleration increases as the rotation speed increases, and the total acceleration reaches 10 m/s^2 when

$$a_r = \sqrt{a^2 - a_t^2} = \sqrt{(10 \text{ m/s}^2)^2 - (3.65 \text{ m/s}^2)^2} = 9.31 \text{ m/s}^2$$

Radial acceleration is $a_r = \omega^2 r$, so the corresponding angular velocity is

$$\omega = \sqrt{\frac{a_r}{r}} = \sqrt{\frac{9.31 \text{ m/s}^2}{0.38 \text{ m}}} = 4.95 \text{ rad/s}$$

For constant angular acceleration, $\omega = \alpha t$, so this angular velocity is achieved at

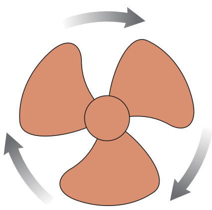
$$t = \frac{\omega}{\alpha} = \frac{4.95 \text{ rad/s}}{9.6 \text{ rad/s}^2} = 0.52 \text{ s}$$

Thus it takes 0.52 s for the acceleration of the blade tip to reach 10 m/s^2.

REVIEW The acceleration at the tip of a long blade is likely to be large. It seems plausible that the acceleration would reach 10 m/s^2 in ≈0.5 s.

STOP TO THINK 4.7 The fan blade is slowing down. What are the signs of ω and α?

a. ω is positive and α is positive.
b. ω is positive and α is negative.
c. ω is negative and α is positive.
d. ω is negative and α is negative.



CHAPTER 4 CHALLENGE EXAMPLE **Hit the target!**

Amanda is riding on a 20.0-m-diameter Ferris wheel. The bottom of the wheel is at ground level. As Amanda goes over the top, she throws a 120 g ball forward, parallel to the ground, at a speed of 7.00 m/s. What angular speed, in rpm, must the Ferris wheel have for the ball to hit a target on the ground 20.0 m from the bottom of the wheel?

MODEL Model the ball as a particle. It first undergoes uniform circular motion with constant angular velocity. We will ignore air resistance and model the subsequent motion as projectile motion.

VISUALIZE **FIGURE 4.35** is a pictorial representation. We've established a coordinate system with the origin at the base of the Ferris wheel. We don't know which direction the Ferris wheel rotates, so we've assumed a clockwise rotation. This is a two-part problem in which the tangential velocity of the rotating ball combines with Amanda's throwing speed to give the initial velocity of the projectile motion. Each point in the projectile motion requires two components of position, two components of velocity, and the time.

SOLVE Amanda and the ball are initially moving in uniform circular motion with speed $v = |\omega|R$. We need the absolute value signs because ω is negative for the clockwise rotation we used in the pictorial representation. Velocity vectors are tangent to the circle, so Amanda's velocity at the top point, the instant she throws the ball, is $\vec{v}_{AG} = |\omega|R\,\hat{\imath}$. The notation $\vec{v}_{AG}$ indicates that this is Amanda's velocity relative to the ground. Amanda's throwing speed allows us to infer that the ball's velocity *relative to Amanda* is $\vec{v}_{BA} = v_{throw}\hat{\imath}$. We can add these velocities, using the Galilean transformation of velocity, to find that the ball's velocity relative to the ground, just as Amanda releases it, is

$$\vec{v}_{BG} = \vec{v}_{BA} + \vec{v}_{AG} = (v_{throw} + |\omega|R)\,\hat{\imath}$$

Thus the projectile is launched from the top of the Ferris wheel with $v_{0x} = v_{throw} + |\omega|R$ and $v_{0y} = 0$.

The vertical motion, with zero initial velocity, is simply the motion of an object dropped from height $y_0 = 2R = 20.0$ m. We can find the time it takes to fall to the ground from

$$y_1 = 0\text{ m} = y_0 + v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2 = 2R - \tfrac{1}{2}gt_1{}^2$$

Thus the ball hits the ground at time

$$t_1 = \sqrt{\frac{4R}{g}} = \sqrt{\frac{40.0\text{ m}}{9.80\text{ m/s}^2}} = 2.02\text{ s}$$

During this time, the ball travels horizontally with constant velocity v_{0x} to

$$x_1 = x_0 + v_{0x}\Delta t = (v_{throw} + |\omega|R)t_1$$

Amanda is trying to hit a target at $x_1 = 20.0$ m, and she will succeed if the Ferris wheel's angular speed is

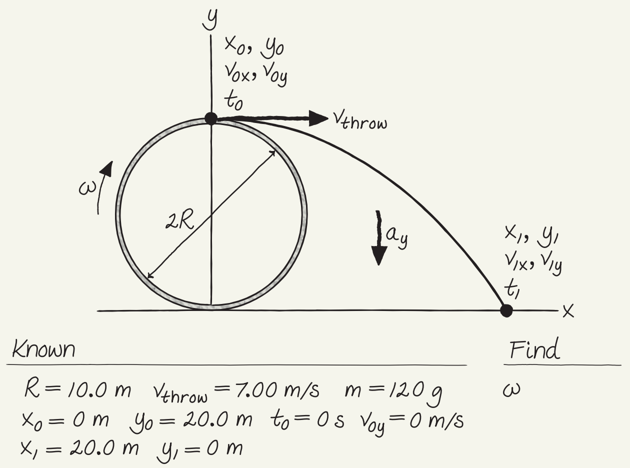
$$|\omega| = \frac{x_1/t_1 - v_{throw}}{R} = \frac{(20.0\text{ m})/(2.02\text{ s}) - 7.00\text{ m/s}}{10.0\text{ m}}$$
$$= 0.290\text{ rad/s}$$

The SI units of angular speed are rad/s, but the question asks for an answer in rpm. Thus we need to convert the units:

$$|\omega| = 0.290\,\frac{\text{rad}}{\text{s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 2.77\text{ rpm}$$

REVIEW 2.77 revolutions every minute, or a revolution every 22 s, seems appropriate for a fairly small carnival-size Ferris wheel. Remember that the purpose of a review is not to prove that the answer is correct but to rule out answers that are obviously incorrect. Notice that we did not need to know the ball's mass. Real-world problems don't come neatly packaged with exactly the information we need and nothing else, so part of becoming a better problem solver is learning to judge which information is relevant. Some homework problems will help you develop this skill by providing details that aren't necessary.

FIGURE 4.35 Pictorial representation of the motion of the ball.



Summary

The goal of Chapter 4 has been to learn how to solve problems about motion in a plane.

General Principles

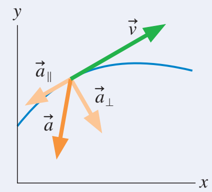
The instantaneous velocity

$$\vec{v} = d\vec{r}/dt$$

is a vector tangent to the trajectory.

The instantaneous acceleration is

$$\vec{a} = d\vec{v}/dt$$



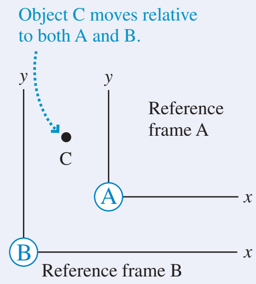
$\vec{a}_{\parallel}$, the component of $\vec{a}$ parallel to $\vec{v}$, is responsible for change of *speed*. $\vec{a}_{\perp}$, the component of $\vec{a}$ perpendicular to $\vec{v}$, is responsible for change of *direction*.

Relative Motion

If object C moves relative to reference frame A with velocity $\vec{v}_{CA}$, then it moves relative to a different reference frame B with velocity

$$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$$

where $\vec{v}_{AB}$ is the velocity of A relative to B. This is the Galilean transformation of velocity.



Important Concepts

Uniform Circular Motion

Angular velocity $\omega = d\theta/dt$.

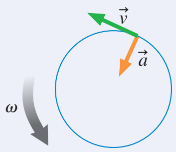
v_t and ω are constant:

$$v_t = \omega r$$

The centripetal acceleration points toward the center of the circle with magnitude

$$a = |\vec{a}| = \frac{v_t^2}{r} = \omega^2 r$$

It changes the particle's direction but not its speed.



Nonuniform Circular Motion

Angular acceleration $\alpha = d\omega/dt$.

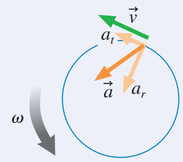
The radial acceleration

$$a_r = \frac{v_t^2}{r} = \omega^2 r$$

changes the particle's direction. The tangential component

$$a_t = \alpha r$$

changes the particle's speed.



Applications

Kinematics in two dimensions

If $\vec{a}$ is constant, then the x- and y-components of motion are independent of each other.

$$x_f = x_i + v_{ix}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$y_f = y_i + v_{iy}\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

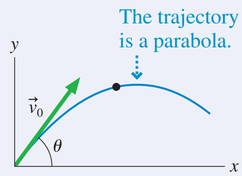
$$v_{fx} = v_{ix} + a_x\,\Delta t$$

$$v_{fy} = v_{iy} + a_y\,\Delta t$$

Projectile motion is motion under the influence of only gravity.

MODEL Model as a particle launched with speed v_0 at angle θ.

VISUALIZE Use coordinates with the x-axis horizontal and the y-axis vertical.



SOLVE The horizontal motion is uniform with $v_x = v_0\cos\theta$. The vertical motion is free fall with $a_y = -g$. The x and y kinematic equations have the *same* value for Δt.

Circular motion kinematics

Period $T = \dfrac{2\pi r}{v} = \dfrac{2\pi}{\omega}$

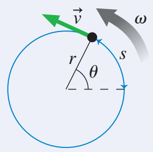
Angular position $\theta = \dfrac{s}{r}$

Constant angular acceleration

$$\omega_f = \omega_i + \alpha\,\Delta t$$

$$\theta_f = \theta_i + \omega_i\,\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

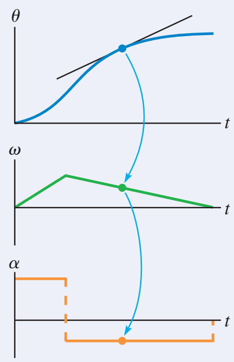
$$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$$



Circular motion graphs and kinematics are analogous to linear motion with constant acceleration.

Angle, angular velocity, and angular acceleration are related graphically.

- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.



Terms and Notation

projectile
launch angle, θ
projectile motion model
reference frame
Galilean transformation of velocity
uniform circular motion
period, T
angular position, θ
arc length, s
radians
angular displacement, $\Delta\theta$
angular velocity, ω
centripetal acceleration
uniform circular motion model
nonuniform circular motion
angular acceleration, α
constant angular acceleration model
radial acceleration, a_r
tangential acceleration, a_t

CONCEPTUAL QUESTIONS

1. a. At this instant, is the particle in **FIGURE Q4.1** speeding up, slowing down, or traveling at constant speed?
   b. Is this particle curving to the right, curving to the left, or traveling straight?

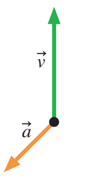


FIGURE Q4.1

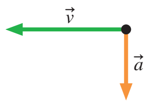


FIGURE Q4.2

2. a. At this instant, is the particle in **FIGURE Q4.2** speeding up, slowing down, or traveling at constant speed?
   b. Is this particle curving upward, curving downward, or traveling straight?
3. Three cricket balls are thrown from a tall tower—the first one is released from rest; the second one is thrown with a horizontal velocity of 7 m/s eastward; and the last one is thrown with a horizontal velocity of 10 m/s westward. Which of the balls will be the first to touch the ground?
4. A projectile is launched at an angle of 45°.
   a. Is there any point on the trajectory where $\vec{v}$ and $\vec{a}$ are parallel to each other? If so, where?
   b. Is there any point where $\vec{v}$ and $\vec{a}$ are perpendicular to one other? If so, where?
5. For a projectile, which of the following quantities are constant during the flight: $x, y, r, v_x, v_y, v, a_x, a_y$? Which of these quantities are zero throughout the flight?
6. A cart that is rolling at constant velocity on a level table fires a ball straight up.
   a. When the ball comes back down, will it land in front of the launching tube, behind the launching tube, or directly in the tube? Explain.
   b. Will your answer change if the cart is accelerating in the forward direction? If so, how?
7. A rock is thrown from a bridge at an angle 45° below the horizontal. Is the magnitude of acceleration, immediately after the rock is released, greater than, less than, or equal to g? Explain.
8. Anita is running to the right at 5 m/s in **FIGURE Q4.8**. Balls 1 and 2 are thrown toward her by friends standing on the ground. According to Anita, both balls are approaching her at 10 m/s. Which ball was thrown at a faster speed? Or were they thrown with the same speed? Explain.

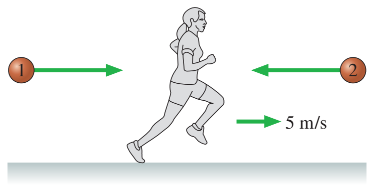


FIGURE Q4.8

9. An electromagnet on the ceiling of an airplane holds a steel ball. When a button is pushed, the magnet releases the ball. First, the button is pushed while the plane is parked on the ground. The point where the ball hits the floor is marked with an X. Next, the experiment is repeated while the plane is flying horizontally at a steady speed of 620 mph. Does the ball land in front of the X (toward the nose of the plane), on the X, or behind the X (toward the tail of the plane)? Explain.
10. Zack is driving past his house in **FIGURE Q4.10**. He wants to toss his physics book out the window and have it land in his driveway. If he lets go of the book exactly as he passes the end of the driveway, should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain.

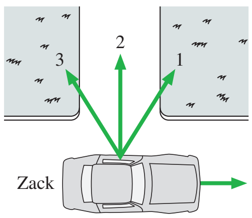


FIGURE Q4.10

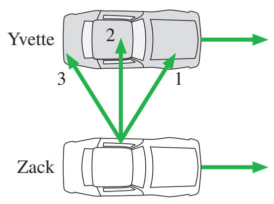


FIGURE Q4.11

11. In **FIGURE Q4.11**, Yvette and Zack are driving down the freeway side by side with their windows down. Zack wants to toss his physics book out the window and have it land in Yvette's front seat. Ignoring air resistance, should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain.

12. You tie a cricket ball with a string and hang it from a tall pole. The ball is then struck with a cricket bat. Ignoring the mass of the string, what should be the direction of the acceleration if it attains a constant speed along a circular path centering the pole? Which force is responsible for this acceleration?

13. FIGURE Q4.13 shows three points on a steadily rotating wheel.
   a. Rank in order, from largest to smallest, the angular velocities ω_1, ω_2, and ω_3 of these points. Explain.
   b. Rank in order, from largest to smallest, the speeds v_1, v_2, and v_3 of these points. Explain.

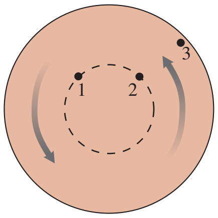


FIGURE Q4.13

14. FIGURE Q4.14 shows four rotating wheels. For each, determine the signs ($+$ or $-$) of ω and α.

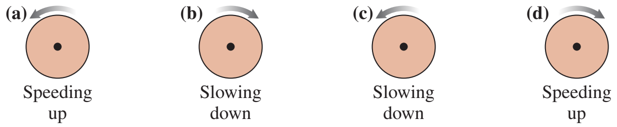


FIGURE Q4.14

15. FIGURE Q4.15 shows a pendulum at one end point of its arc.
   a. At this point, is ω positive, negative, or zero? Explain.
   b. At this point, is α positive, negative, or zero? Explain.

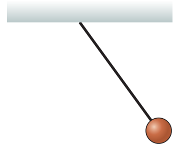


FIGURE Q4.15

EXERCISES AND PROBLEMS

Exercises

Section 4.1 Motion in Two Dimensions

Problems 1 and 2 show a partial motion diagram. For each:
   a. Complete the motion diagram by adding acceleration vectors.
   b. Write a physics *problem* for which this is the correct motion diagram. Be imaginative! Don't forget to include enough information to make the problem complete and to state clearly what is to be found.

1. |

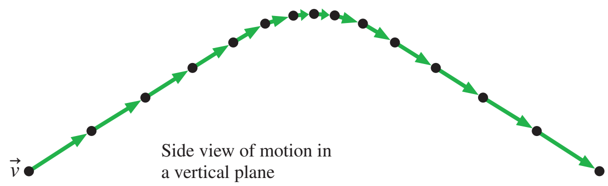


FIGURE EX4.1

2. |

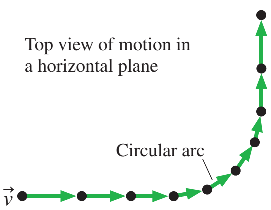
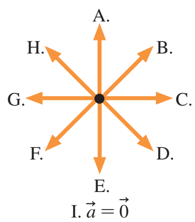


FIGURE EX4.2

Answer Problems 3 and 4 by choosing one of the eight labeled acceleration vectors or selecting option I: $\vec{a} = \vec{0}$.



3. || At this instant, the particle has steady speed and is curving to the right. What is the direction of its acceleration?

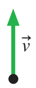


FIGURE EX4.3

4. || At this instant, the particle is speeding up and curving upward. What is the direction of its acceleration?

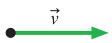


FIGURE EX4.4

5. | a. At this moment, is the particle in FIGURE EX4.5 speeding up, slowing down, or moving at constant speed?
   b. Is this particle curving upward, curving downward, or moving in a straight line?

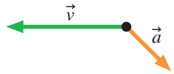


FIGURE EX4.5

6. || A rocket-powered hockey puck moves on a horizontal frictionless table. FIGURE EX4.6 shows graphs of v_x and v_y, the x- and y-components of the puck's velocity. The puck starts at the origin.
   a. In which direction is the puck moving at $t = 2$ s? Give your answer as an angle from the x-axis.
   b. How far from the origin is the puck at $t = 5$ s?

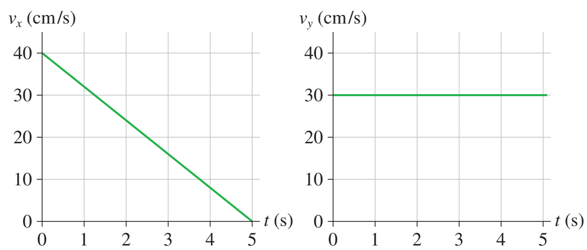


FIGURE EX4.6

7. || A rocket-powered hockey puck moves on a horizontal frictionless table. FIGURE EX4.7 shows graphs of v_x and v_y, the x- and y-components of the puck's velocity. The puck starts at the origin. What is the magnitude of the puck's acceleration at $t = 5$ s?

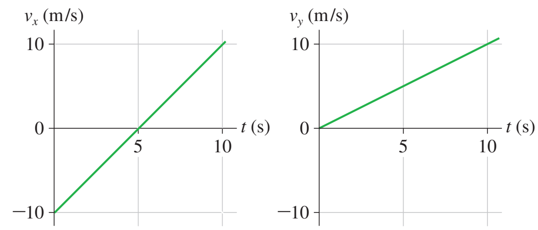


FIGURE EX4.7

8. || A particle moving in the xy-plane has velocity $\vec{v} = (2t\hat{\imath} + (3 - t^2)\hat{\jmath})$ m/s, where t is in s. What is the particle's acceleration vector at $t = 4$ s?
CALC

9. || A particle's trajectory is described by $x = \left(\frac{1}{6}t^3 - t^2\right)$m and $y = \left(\frac{1}{6}t^2 - t\right)$m, where t is in s.
CALC
   a. What are the particle's position and speed at $t = 0$ s and $t = 6$ s?
   b. What is the particle's direction of motion, measured as an angle from the x-axis, at $t = 0$ s and $t = 6$ s?

10. || You have a remote-controlled car that has been programmed to have velocity $\vec{v} = (-3t\hat{\imath} + 2t^2\hat{\jmath})$ m/s, where t is in s. At $t = 0$ s, the car is at $\vec{r}_0 = (3.0\hat{\imath} + 2.0\hat{\jmath})$ m. What are the car's (a) position vector and (b) acceleration vector at $t = 2.0$ s?
CALC

Section 4.2 Projectile Motion

11. || A ball thrown horizontally at 20 m/s travels a horizontal distance of 40 m before hitting the ground. From what height was the ball thrown?

12. || A supply plane needs to drop a package of food to scientists working on a glacier in Greenland. The plane flies 80 m above the glacier at a speed of 100 m/s. How far short of the target should it drop the package?

13. | A physics student on Planet Exidor throws a ball, and it follows the parabolic trajectory shown in FIGURE EX4.13. The ball's position is shown at 1 s intervals until $t = 3$ s. At $t = 1$ s, the ball's velocity is $\vec{v} = (2.0\,\hat{\imath} + 2.0\,\hat{\jmath})$ m/s.
   a. Determine the ball's velocity at $t = 0$ s, 2 s, and 3 s.
   b. What is the value of g on Planet Exidor?
   c. What was the ball's launch angle?

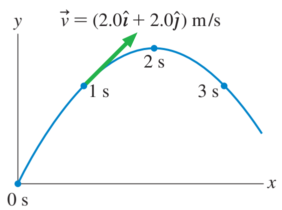


FIGURE EX4.13

14. || In the Olympic shotput event, an athlete throws the shot with an initial speed of 12.0 m/s at a 40.0° angle from the horizontal. The shot leaves her hand at a height of 1.80 m above the ground. How far does the shot travel?

15. | A rifle is aimed horizontally at a target 40 m away. The bullet hits 1 cm below the target.
   a. What was the bullet's flight time?
   b. What was the bullet's speed as it left the barrel?

16. || A friend of yours is a baseball player and wants to determine his pitching speed. You have him stand on a ledge and throw the ball horizontally from an elevation of 6 m above the ground. The ball lands 40 m away. What is his pitching speed?

17. || On the Apollo 14 mission to the moon, astronaut Alan Shepard hit a golf ball with a 6 iron. The free-fall acceleration on the moon is 1/6 of its value on earth. Suppose he hit the ball with a speed of 25 m/s at an angle 30° above the horizontal.
   a. How much farther did the ball travel on the moon than it would have on earth?
   b. For how much more time was the ball in flight?

Section 4.3 Relative Motion

18. || A boat takes 3.0 hours to travel 24 km down a river, then 4.0 hours to return. How fast is the river flowing?

19. | Mary needs to row her boat across a river 100 m wide that is flowing to the east at a speed of 2 m/s. Mary can row with a speed of 4 m/s.
   a. If Mary points her boat due north, how far will she be from her intended landing spot when she reaches the opposite shore?
   b. What is her speed with respect to the shore?

20. || When the moving sidewalk at the airport is broken, as it often seems to be, it takes you 50 s to walk from your gate to baggage claim. When it is working and you stand on the moving sidewalk the entire way, without walking, it takes 75 s to travel the same distance. How long will it take you to travel from the gate to baggage claim if you walk while riding on the moving sidewalk?

21. | A kayaker needs to paddle north across an 80-m-wide harbor. The tide is going out, creating a current that flows to the east at 3 m/s. The kayaker can paddle with a speed of 4 m/s.
   a. In which direction should he paddle in order to travel straight across the harbor?
   b. How long will it take him to cross the harbor?

22. || Harmeet, driving west at 54 km/h, and Kenza, driving south at 72 km/h, are approaching an intersection. What is Kenza's speed relative to Harmeet's reference frame?

Section 4.4 Uniform Circular Motion

23. || FIGURE EX4.23 shows the angular-velocity-versus-time graph for a particle moving in a circle. How many revolutions does the object make during the first 4 s?

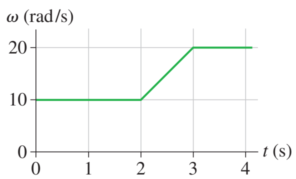


FIGURE EX4.23

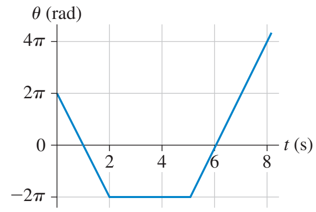


FIGURE EX4.24

24. | FIGURE EX4.24 shows the angular-position-versus-time graph for a particle moving in a circle. What is the particle's angular velocity at (a) $t = 1$ s, (b) $t = 4$ s, and (c) $t = 7$ s?

25. || FIGURE EX4.25 shows the angular-velocity-versus-time graph for a particle moving in a circle, starting from $\theta_0 = 0$ rad at $t = 0$ s. Draw the angular-position-versus-time graph. Include an appropriate scale on both axes.

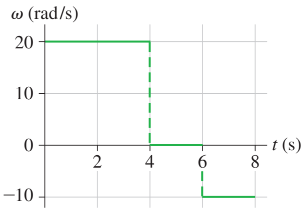


FIGURE EX4.25

26. | An old-fashioned single-play vinyl record rotates on a turntable at 72 rpm. What is
   a. the angular velocity in rad/s?
   b. the period of the motion?

27. || The earth's radius is about 4000 miles. Kampala, the capital of Uganda, and Singapore are both nearly on the equator. The distance between them is 5000 miles. The flight from Kampala to Singapore takes 9.0 hours. What is the plane's angular velocity with respect to the earth's surface? Give your answer in °/h.

28. || As the earth rotates, what is the speed of (a) a physics student in Kyoto, Japan, at latitude 35°, and (b) a physics student in Copenhagen, Denmark, at latitude 56°? Ignore the revolution of the earth around the sun. The radius of the earth is 6400 km.

29. || Mount Chimborazo is located on the equator and is the highest point above the center of the earth. The summit of Chimborazo is 6263 m above sea level. How much faster does a climber on top of the mountain move than a surfer at a nearby beach? The earth's radius is 6400 km.

30. | How fast must a plane fly along the earth's equator so that the sun stands still relative to the passengers? In which direction must the plane fly, east to west or west to east? Give your answer in both km/h and mph. The earth's radius is 6400 km.

Section 4.5 Centripetal Acceleration

31. | Peregrine falcons are known for their maneuvering ability. In a tight circular turn, a falcon can attain a centripetal acceleration 1.5 times the free-fall acceleration. What is the radius of the turn if the falcon is flying at 25 m/s?
BIO

32. | To withstand "g-forces" of up to 10 g's, caused by suddenly pulling out of a steep dive, fighter jet pilots train on a "human centrifuge." 10 g's is an acceleration of 98 m/s^2. If the length of the centrifuge arm is 12 m, at what speed is the rider moving when she experiences 10 g's?
BIO

33. | The radius of the earth's very nearly circular orbit around the sun is 1.5×10^{11} m. Find the magnitude of the earth's (a) velocity, (b) angular velocity, and (c) centripetal acceleration as it travels around the sun. Assume a year of 365 days.

34. | A speck of dust on a spinning DVD has a centripetal acceleration of 20 m/s^2.
    a. What is the acceleration of a different speck of dust that is twice as far from the center of the disk?
    b. What would be the acceleration of the first speck of dust if the disk's angular velocity was doubled?

35. || Your roommate is working on her bicycle and has the bike upside down. She spins the 70-cm-diameter wheel, and you notice that a pebble stuck in the tread goes by four times every second. What are the pebble's speed and acceleration?

Section 4.6 Nonuniform Circular Motion

36. | **FIGURE EX4.36** shows the angular velocity graph of the crankshaft in a car. What is the crankshaft's angular acceleration at (a) $t = 1$ s, (b) $t = 3$ s, and (c) $t = 5$ s?

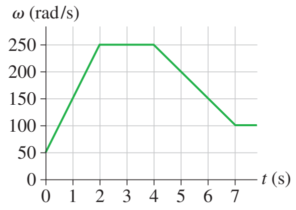


FIGURE EX4.36

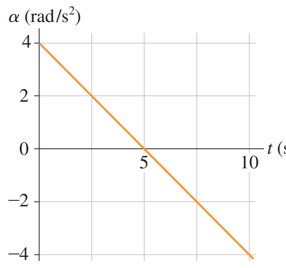


FIGURE EX4.37

37. | A turntable initially rotating at 20 rad/s experiences the angular acceleration shown in **FIGURE EX4.37**. What is the turntable's angular velocity at (a) $t = 5$ s and (b) $t = 10$ s?

38. || **FIGURE EX4.38** shows the angular-velocity-versus-time graph for a particle moving in a circle. How many revolutions does the object make during the first 4 s?

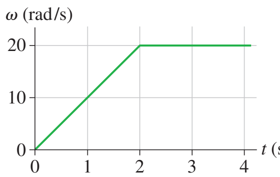


FIGURE EX4.38

39. || A wheel initially rotating at 60 rpm experiences the angular acceleration shown in **FIGURE EX4.39**. What is the wheel's angular velocity, in rpm, at $t = 3.0$ s?

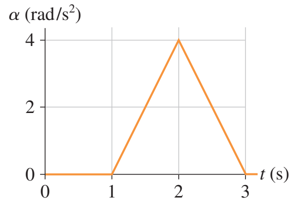


FIGURE EX4.39

40. || A 5.0-m-diameter merry-go-round is initially turning with a 4.0 s period. It slows down and stops in 20 s.
    a. Before slowing, what is the speed of a child on the rim?
    b. How many revolutions does the merry-go-round make as it stops?

41. || A bicycle wheel is rotating at 50 rpm when the cyclist begins to pedal harder, giving the wheel a constant angular acceleration of 0.50 rad/s^2.
    a. What is the wheel's angular velocity, in rpm, 10 s later?
    b. How many revolutions does the wheel make during this time?

42. || An electric fan goes from rest to 1800 rpm in 4.0 s. What is its angular acceleration?

43. || Starting from rest, a DVD steadily accelerates to 500 rpm in 1.0 s, rotates at this angular speed for 3.0 s, then steadily decelerates to a halt in 2.0 s. How many revolutions does it make?

Problems

44. ||| A spaceship maneuvering near Planet Zeta is located at $\vec{r} = (600\hat{\imath} - 400\hat{\jmath} + 200\hat{k}) \times 10^3$ km, relative to the planet, and traveling at $\vec{v} = 9500\hat{\imath}$ m/s. It turns on its thruster engine and accelerates with $\vec{a} = (40\hat{\imath} - 20\hat{k})$ m/s^2 for 35 min. What is the spaceship's position when the engine shuts off? Give your answer as a position vector measured in km.

45. ||| A particle moving in the xy-plane has velocity $\vec{v}_0 = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath}$ at $t = 0$. It undergoes acceleration $\vec{a} = bt\hat{\imath} - cv_y\hat{\jmath}$, where b and c are constants. Find an expression for the particle's velocity at a later time t.
CALC

46. || a. A projectile is launched with speed v_0 and angle θ. Derive an expression for the projectile's maximum height h.
    b. A baseball is hit with a speed of 33.6 m/s. Calculate its height and the distance traveled if it is hit at angles of 30.0°, 45.0°, and 60.0°.

47. || A projectile's horizontal range over level ground is $v_0^2 \sin 2\theta/g$. At what launch angle or angles will the projectile land at half of its maximum possible range?

48. || A projectile is launched from ground level at angle θ and speed v_0 into a headwind that causes a constant horizontal acceleration of magnitude a opposite the direction of motion.
CALC
    a. Find an expression in terms of a and g for the launch angle that gives maximum range.
    b. What is the angle for maximum range if a is 10% of g?

49. || A gray kangaroo can bound across level ground with each jump carrying it 10 m from the takeoff point. Typically the kangaroo leaves the ground at a 20° angle. If this is so:
BIO
    a. What is its takeoff speed?
    b. What is its maximum height above the ground?

50. || A ball is thrown toward a cliff of height h with a speed of 30 m/s and an angle of 60° above horizontal. It lands on the edge of the cliff 4.0 s later.
    a. How high is the cliff?
    b. What was the maximum height of the ball?
    c. What is the ball's impact speed?

51. || You are target shooting using a toy gun that fires a small ball at a speed of 15 m/s. When the gun is fired at an angle of 30° above horizontal, the ball hits the bull's-eye of a target at the same height as the gun. Then the target distance is halved. At what angle must you aim the gun to hit the bull's-eye in its new position? (Mathematically there are two solutions to this problem; the physically reasonable answer is the smaller of the two.)

52. || A tennis player hits a ball 2.0 m above the ground. The ball leaves his racquet with a speed of 20.0 m/s at an angle 5.0° above the horizontal. The horizontal distance to the net is 7.0 m, and the net is 1.0 m high. Does the ball clear the net? If so, by how much? If not, by how much does it miss?

53. || A snowboarder starts down a frictionless, 8.0-m-tall, 15° slope. The slope ends abruptly at the top of a 4.0-m-high wall that has level packed snow at its base. How far does the snowboarder land from the base of the wall?

54. || You are watching an archery tournament when you start wondering how fast an arrow is shot from the bow. Remembering your physics, you ask one of the archers to shoot an arrow parallel to the ground. You find the arrow stuck in the ground 60 m away, making a 3.0° angle with the ground. How fast was the arrow shot?

55. || You're 6.0 m from one wall of the house seen in **FIGURE P4.55**. You want to toss a ball to your friend who is 6.0 m from the opposite wall. The throw and catch each occur 1.0 m above the ground.
   a. What minimum speed will allow the ball to clear the roof?
   b. At what angle should you toss the ball?

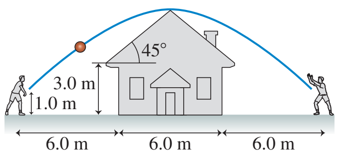


FIGURE P4.55

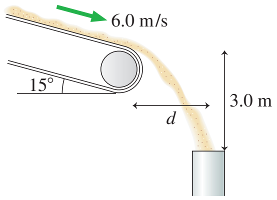


FIGURE P4.56

56. || Sand moves without slipping at 6.0 m/s down a conveyer that is tilted at 15°. The sand enters a pipe 3.0 m below the end of the conveyer belt, as shown in **FIGURE P4.56**. What is the horizontal distance d between the conveyer belt and the pipe?

57. || A stunt man drives a 1500 kg car at a speed of 20 m/s off a 30-m-high cliff. The road leading to the cliff is inclined upward at an angle of 20°.
   a. How far from the base of the cliff does the car land?
   b. What is the car's impact speed?

58. ||| BIO A javelin thrower standing at rest holds the center of the javelin behind her head, then accelerates it through a distance of 70 cm as she throws. She releases the 600 g javelin 2.0 m above the ground traveling at an angle of 30° above the horizontal. Top-rated javelin throwers do throw at about a 30° angle, not the 45° you might have expected, because the biomechanics of the arm allow them to throw the javelin much faster at 30° than they would be able to at 45°. In this throw, the javelin hits the ground 62 m away. What was the acceleration of the javelin during the throw? Assume that it has a constant acceleration.

59. || A cannonball is fired at 100 m/s from a barrel tilted upward at 25°.
   a. Find an expression for the cannonball's direction of travel, measured as an angle from horizontal, after traveling horizontal distance d.
   b. How far has the cannonball traveled horizontally when it reaches its maximum height?
   c. What is the angle after the cannonball travels 500 m?

60. || Ships A and B leave port together. For the next two hours, ship A travels at 20 mph in a direction 30° west of north while ship B travels 20° east of north at 25 mph.
   a. What is the distance between the two ships two hours after they depart?
   b. What is the speed of ship A as seen by ship B?

61. || While driving north at 10 m/s during a rainstorm, you notice that the rain makes an angle of 41° with the vertical. While driving back home moments later at the same speed but in the opposite direction, you see that the rain is falling straight down. From these observations, determine the speed and angle of the raindrops relative to the ground.

62. || BIO You are asked to consult for the city's research hospital, where a group of doctors is investigating the bombardment of cancer tumors with high-energy ions. As **FIGURE P4.62** shows, ions are fired directly toward the center of the tumor at speeds of 5.0×10^6 m/s. To cover the entire tumor area, the ions are deflected sideways by passing them between two charged metal plates that accelerate the ions perpendicular to the direction of their initial motion. The acceleration region is 5.0 cm long, and the ends of the acceleration plates are 1.5 m from the target. What sideways acceleration is required to deflect an ion 2.0 cm to one side?

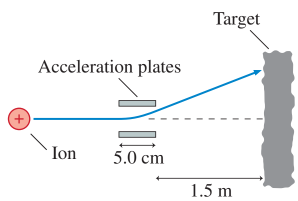


FIGURE P4.62

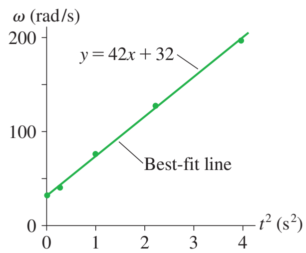


FIGURE P4.63

63. || CALC The angular velocity of a spinning gyroscope is measured every 0.5 s. The results and the best-fit line from a spreadsheet are shown in **FIGURE P4.63**.
   a. What is the gyroscope's initial angular velocity, at $t = 0$ s?
   b. What is the angular acceleration at $t = 2.0$ s?
   c. How many revolutions does the gyroscope make between $t = 0$ s and $t = 2.0$ s?

64. || A ball rolling on a circular track, starting from rest, has angular acceleration α. Find an expression, in terms of α, for the time at which the ball's acceleration vector $\vec{a}$ is 45° away from a radial line toward the center of the circle.

65. || A typical laboratory centrifuge rotates at 4000 rpm. Test tubes have to be placed into a centrifuge very carefully because of the very large accelerations.
   a. What is the acceleration at the end of a test tube that is 10 cm from the axis of rotation?
   b. For comparison, what is the magnitude of the acceleration a test tube would experience if dropped from a height of 1.0 m and stopped in a 1.0-ms-long encounter with a hard floor?

66. || Astronauts use a centrifuge to simulate the acceleration of a rocket launch. The centrifuge takes 30 s to speed up from rest to its top speed of 1 rotation every 1.3 s. The astronaut is strapped into a seat 6.0 m from the axis.
BIO
   a. What is the astronaut's tangential acceleration during the first 30 s?
   b. How many g's of acceleration does the astronaut experience when the device is rotating at top speed? Each 9.8 m/s^2 of acceleration is 1 g.

67. || A Ferris wheel of radius R speeds up with angular acceleration α starting from rest. Find expressions for the (a) velocity and (b) centripetal acceleration of a rider after the Ferris wheel has rotated through angle $\Delta\theta$.

68. || Communications satellites are placed in a circular orbit where they stay directly over a fixed point on the equator as the earth rotates. These are called *geosynchronous orbits.* The radius of the earth is 6.37×10^6 m, and the altitude of a geosynchronous orbit is 3.58×10^7 m ($\approx$22,000 miles). What are (a) the speed and (b) the magnitude of the acceleration of a satellite in a geosynchronous orbit?

69. || A computer hard disk 8.0 cm in diameter is initially at rest. A small dot is painted on the edge of the disk. The disk accelerates at 600 rad/s^2 for $\frac{1}{2}$ s, then coasts at a steady angular velocity for another $\frac{1}{2}$ s.
   a. What is the speed of the dot at $t = 1.0$ s?
   b. Through how many revolutions has the disk turned?

70. || A high-speed drill rotating ccw at 2400 rpm comes to a halt in 2.5 s.
   a. What is the magnitude of the drill's angular acceleration?
   b. How many revolutions does it make as it stops?

71. || Flywheels—rapidly rotating disks—are widely used in industry for storing energy. They are spun up slowly when extra energy is available, then decelerate quickly when needed to supply a boost of energy. A 20-cm-diameter rotor made of advanced materials can spin at 100,000 rpm.
   a. What is the speed of a point on the rim of this rotor?
   b. Suppose the rotor's angular velocity decreases by 40% over 30 s as it supplies energy. What is the magnitude of the rotor's angular acceleration? Assume that the angular acceleration is constant.
   c. How many revolutions does the rotor make during these 30 s?

72. || A 25 g steel ball is attached to the top of a 24-cm-diameter vertical wheel. Starting from rest, the wheel accelerates at 470 rad/s^2. The ball is released after $\frac{3}{4}$ of a revolution. How high does it go above the center of the wheel?

73. || The angular velocity of a process control motor is $\omega = \left(20 - \frac{1}{2}t^2\right)$ rad/s, where t is in seconds.
CALC
   a. At what time does the motor reverse direction?
   b. Through what angle does the motor turn between $t = 0$ s and the instant at which it reverses direction?

74. || A 6.0-cm-diameter gear rotates with angular velocity $\omega = \left(20 - \frac{1}{2}t^2\right)$ rad/s, where t is in seconds. At $t = 4.0$ s, what are:
CALC
   a. The gear's angular acceleration?
   b. The tangential acceleration of a tooth on the gear?

75. || A painted tooth on a spinning gear has angular position $\theta = (3.0\ \text{rad/s}^4)t^4$. What is the tooth's angular acceleration at the end of 16 revolutions?
CALC

76. ||| A car starts from rest on a curve with a radius of 120 m and accelerates tangentially at 1.0 m/s^2. Through what angle will the car have traveled when the magnitude of its total acceleration is 2.0 m/s^2?

77. ||| A long string is wrapped around a 6.0-cm-diameter cylinder, initially at rest, that is free to rotate on an axle. The string is then pulled with a constant acceleration of 1.5 m/s^2 until 1.0 m of string has been unwound. If the string unwinds without slipping, what is the cylinder's angular speed, in rpm, at this time?

In Problems 78 through 80 you are given the equations that are used to solve a problem. For each of these, you are to
   a. Write a realistic problem for which these are the correct equations. Be sure that the answer your problem requests is consistent with the equations given.
   b. Finish the solution of the problem, including a pictorial representation.

78. $100\ \text{m} = 0\ \text{m} + (50\cos\theta\ \text{m/s})t_1$
$0\ \text{m} = 0\ \text{m} + (50\sin\theta\ \text{m/s})t_1 - \frac{1}{2}(9.80\ \text{m/s}^2)t_1^2$

79. $v_x = -(6.0\cos 45°)\ \text{m/s} + 3.0\ \text{m/s}$
$v_y = (6.0\sin 45°)\ \text{m/s} + 0\ \text{m/s}$
$100\ \text{m} = v_y t_1,\ x_1 = v_x t_1$

80. $2.5\ \text{rad} = 0\ \text{rad} + \omega_i(10\ \text{s}) + \left((1.5\ \text{m/s}^2)/2(50\ \text{m})\right)(10\ \text{s})^2$
$\omega_f = \omega_i + \left((1.5\ \text{m/s}^2)/(50\ \text{m})\right)(10\ \text{s})$

Challenge Problems

81. ||| A skateboarder starts up a 1.0-m-high, 30° ramp at a speed of 7.0 m/s. The skateboard wheels roll without friction. At the top she leaves the ramp and sails through the air. How far from the end of the ramp does the skateboarder touch down?

82. ||| An archer standing on a 15° slope shoots an arrow 20° above the horizontal, as shown in **FIGURE CP4.82**. How far down the slope does the arrow hit if it is shot with a speed of 50 m/s from 1.75 m above the ground?

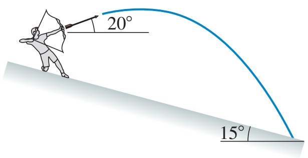


FIGURE CP4.82

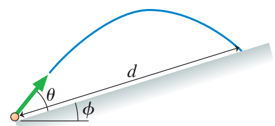


FIGURE CP4.83

83. ||| The cannon in **FIGURE CP4.83** fires a projectile at launch angle θ with respect to the slope, which is at angle ϕ. Find the launch angle that maximizes d.
CALC
Hint: Choosing the proper coordinate system is essential. There are two options.

84. ||| A cannon on a flat railroad car travels to the east with its barrel tilted 30° above horizontal. It fires a cannonball at 50 m/s. At $t = 0$ s, the car, starting from rest, begins to accelerate to the east at 2.0 m/s^2. At what time should the cannon be fired to hit a target on the tracks that is 400 m to the east of the car's initial position? Assume that the cannonball is fired from ground level.

||| A child in danger of drowning in a river is being carried downstream by a current that flows uniformly with a speed of 2.0 m/s. The child is 200 m from the shore and 1500 m upstream of the boat dock from which the rescue team sets out. If the boat speed is 8.0 m/s with respect to the water, at what angle from the shore must the boat travel in order to reach the child?

5 Force and Motion

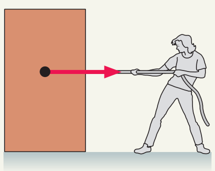

The motion of a sailboat is a response to the forces of wind and water.

IN THIS CHAPTER, you will learn about the connection between force and motion.

What is a force?

The fundamental concept of **mechanics** is **force**.

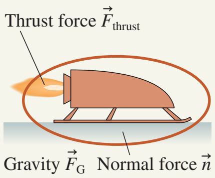
- A force is a **push** or a **pull**.
- A force acts on an object.
- A force requires an **agent**.
- A force is a **vector**.



How do we identify forces?

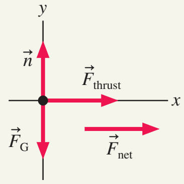
A force can be a **contact force** or a **long-range force**.

- Contact forces occur at points where the environment touches the object.
- Contact forces disappear the instant contact is lost. Forces have no memory.
- Long-range forces include gravity and magnetism.



How do we show forces?

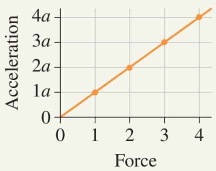
Forces can be displayed on a **free-body diagram**. You'll draw all forces—both pushes and pulls—as vectors with their tails on the particle. A well-drawn free-body diagram is an essential step in solving problems, as you'll see in the next chapter.



What do forces do?

A **net force** causes an object to ***accelerate*** with an acceleration directly proportional to the size of the force. This is **Newton's second law**, the most important statement in mechanics. For a particle of mass m,

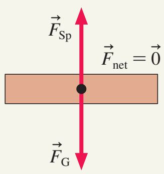
$$\vec{a} = \frac{1}{m}\vec{F}_{\text{net}}$$



« **LOOKING BACK** Sections 1.4, 2.4, and 3.2 Acceleration and vector addition

What is Newton's first law?

Newton's first law—an object at rest stays at rest and an object in motion continues moving at constant speed in a straight line if and only if the **net force** on the object is zero—helps us define what a force *is*. It is also the basis for identifying the reference frames—called **inertial reference frames**—in which Newton's laws are valid.



What good are forces?

Kinematics describes *how* an object moves. For the more important tasks of knowing *why* an object moves and being able to predict its position and orientation at a future time, we have to know the forces acting on the object. **Relating force to motion** is the subject of **dynamics,** and it is one of the most important underpinnings of all science and engineering.

5.1 Force

The two major issues that this chapter will examine are:

- What is a force?
- What is the connection between force and motion?

We begin with the first of these questions in the table below.

What is a force?

A force is a push or a pull.

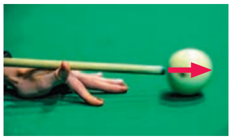
Our commonsense idea of a **force** is that it is a *push* or a *pull.* We will refine this idea as we go along, but it is an adequate starting point. Notice our careful choice of words: We refer to "*a* force," rather than simply "force." We want to think of a force as a very specific *action,* so that we can talk about a single force or perhaps about two or three individual forces that we can clearly distinguish. Hence the concrete idea of "a force" acting on an object.

A force acts on an object.

Implicit in our concept of force is that a **force acts on an object.** In other words, pushes and pulls are applied *to* something—an object. From the object's perspective, it has a force *exerted* on it. Forces do not exist in isolation from the object that experiences them.

A force requires an agent.

Every force has an **agent,** something that acts or exerts power. That is, a force has a specific, identifiable *cause.* As you throw a ball, it is your hand, while in contact with the ball, that is the agent or the cause of the force exerted on the ball. *If* a force is being exerted on an object, you must be able to identify a specific cause (i.e., the agent) of that force. Conversely, a force is not exerted on an object *unless* you can identify a specific cause or agent. Although this idea may seem to be stating the obvious, you will find it to be a powerful tool for avoiding some common misconceptions about what is and is not a force.



A force is a vector.

If you push an object, you can push either gently or very hard. Similarly, you can push either left or right, up or down. To quantify a push, we need to specify both a magnitude *and* a direction. It should thus come as no surprise that force is a vector. The general symbol for a force is the vector symbol $\vec{F}$. The size or strength of a force is its magnitude F.

A force can be either a contact force …

There are two basic classes of forces, depending on whether the agent touches the object or not. **Contact forces** are forces that act on an object by touching it at a point of contact. The bat must touch the ball to hit it. A string must be tied to an object to pull it. The majority of forces that we will examine are contact forces.

… or a long-range force.

Long-range forces are forces that act on an object without physical contact. Magnetism is an example of a long-range force. You have undoubtedly held a magnet over a paper clip and seen the paper clip leap up to the magnet. A coffee cup released from your hand is pulled to the earth by the long-range force of gravity.

NOTE In the particle model, objects cannot exert forces on themselves. A force on an object will always have an agent or cause external to the object. Now, there are certainly objects that have internal forces (think of all the forces inside the engine of your car!), but the particle model is not valid if you need to consider those internal forces. If you are going to treat your car as a particle and look only at the overall motion of the car as a whole, that motion will be a consequence of external forces acting on the car.

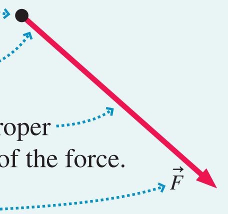
Force Vectors

We can use a simple diagram to visualize how forces are exerted on objects.

TACTICS BOX 5.1

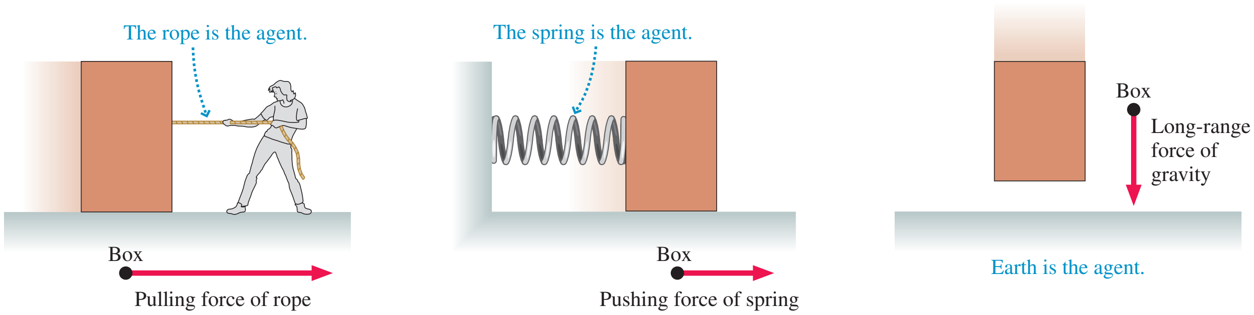
Drawing force vectors

1. Model the object as a particle.
2. Place the *tail* of the force vector on the particle.
3. Draw the force vector as an arrow pointing in the proper direction and with a length proportional to the size of the force.
4. Give the vector an appropriate label.



Step 2 may seem contrary to what a "push" should do, but recall that moving a vector does not change it as long as the length and angle do not change. The vector $\vec{F}$ is the same regardless of whether the tail or the tip is placed on the particle. **FIGURE 5.1** shows three examples of force vectors.

FIGURE 5.1 Three examples of forces and their vector representations.



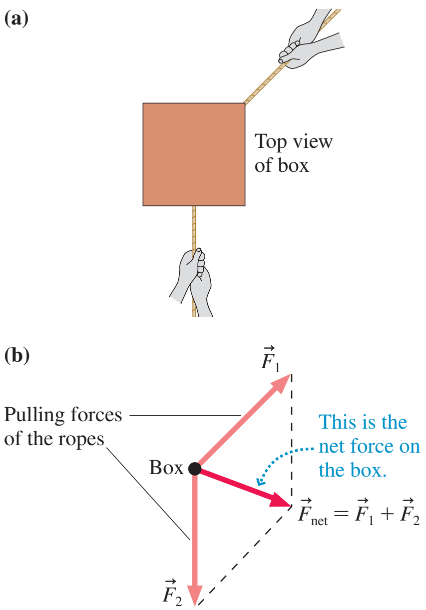
Combining Forces

FIGURE 5.2a shows a box being pulled by two ropes, each exerting a force on the box. How will the box respond? Experimentally, we find that when several forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3, \ldots$ are exerted on an object, they combine to form a **net force** given by the *vector* sum of *all* the forces:

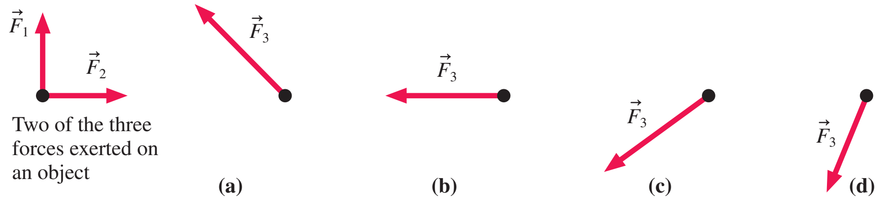
$$\vec{F}_{\text{net}} \equiv \sum_{i=1}^{N} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N \tag{5.1}$$

Recall that $\equiv$ is the symbol meaning "is defined as." Mathematically, this summation is called a **superposition of forces. FIGURE 5.2b** shows the net force on the box.

FIGURE 5.2 Two forces applied to a box.



STOP TO THINK 5.1 Two of the three forces exerted on an object are shown. The net force points to the left. Which is the missing third force?



5.2 A Short Catalog of Forces

There are many forces we will deal with over and over. This section will introduce you to some of them. Many of these forces have special symbols. As you learn the major forces, be sure to learn the symbol for each.

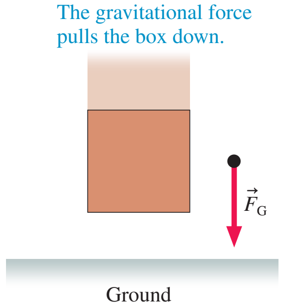
Gravity

Gravity—the only long-range force we will encounter in the next few chapters—keeps you in your chair and the planets in their orbits around the sun. We'll have a thorough look at gravity in Chapter 13. For now we'll concentrate on objects on or near the surface of the earth (or other planet).

The pull of a planet on an object on or near the surface is called the **gravitational force.** The agent for the gravitational force is the *entire planet*. Gravity acts on *all* objects, whether moving or at rest. The symbol for gravitational force is $\vec{F}_G$. **The gravitational force vector always points vertically downward,** as shown in FIGURE 5.3.

NOTE We often refer to "the weight" of an object. For an object at rest on the surface of a planet, its weight is simply the magnitude F_G of the gravitational force. However, weight and gravitational force are not the same thing, nor is weight the same as mass. We will briefly examine mass later in the chapter, and we'll explore the rather subtle connections among gravity, weight, and mass in Chapter 6.

FIGURE 5.3 Gravity.

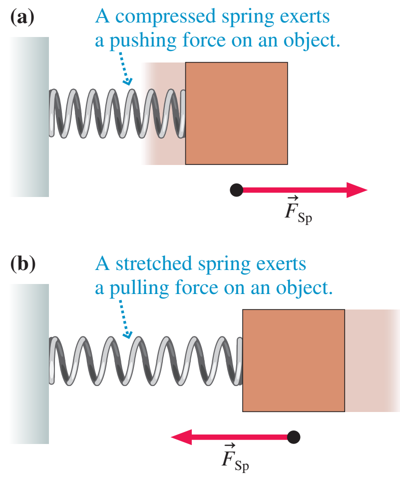


Spring Force

Springs exert one of the most common contact forces. **A spring can either push (when compressed) or pull (when stretched).** FIGURE 5.4 shows the **spring force,** for which we use the symbol $\vec{F}_{Sp}$. In both cases, pushing and pulling, the tail of the force vector is placed on the particle in the force diagram.

Although you may think of a spring as a metal coil that can be stretched or compressed, this is only one type of spring. Hold a ruler, or any other thin piece of wood or metal, by the ends and bend it slightly. It flexes. When you let go, it "springs" back to its original shape. This is just as much a spring as is a metal coil.

FIGURE 5.4 The spring force.

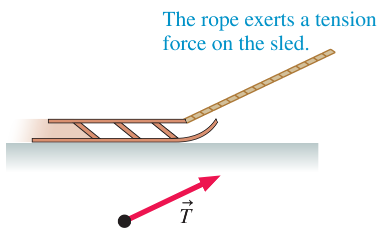


Tension Force

When a string or rope or wire pulls on an object, it exerts a contact force that we call the **tension force,** represented by a capital $\vec{T}$. **The direction of the tension force is always along the direction of the string or rope,** as you can see in FIGURE 5.5. The commonplace reference to "the tension" in a string is an informal expression for T, the size or magnitude of the tension force.

NOTE Tension is represented by the symbol T. This is logical, but there's a risk of confusing the tension T with the identical symbol T for the period of a particle in circular motion. The number of symbols used in science and engineering is so large that some letters are used several times to represent different quantities. The use of T is the first time we've run into this problem, but it won't be the last. You must be alert to the *context* of a symbol's use to deduce its meaning.

FIGURE 5.5 Tension.



We can obtain a deeper understanding of some forces and interactions with a picture of what's happening at the atomic level. You'll recall from chemistry that matter consists of *atoms* that are attracted to each other by *molecular bonds*. Although the details are complex, governed by quantum physics, we can often use a simple **ball-and-spring model** of a solid to get an idea of what's happening at the atomic level.