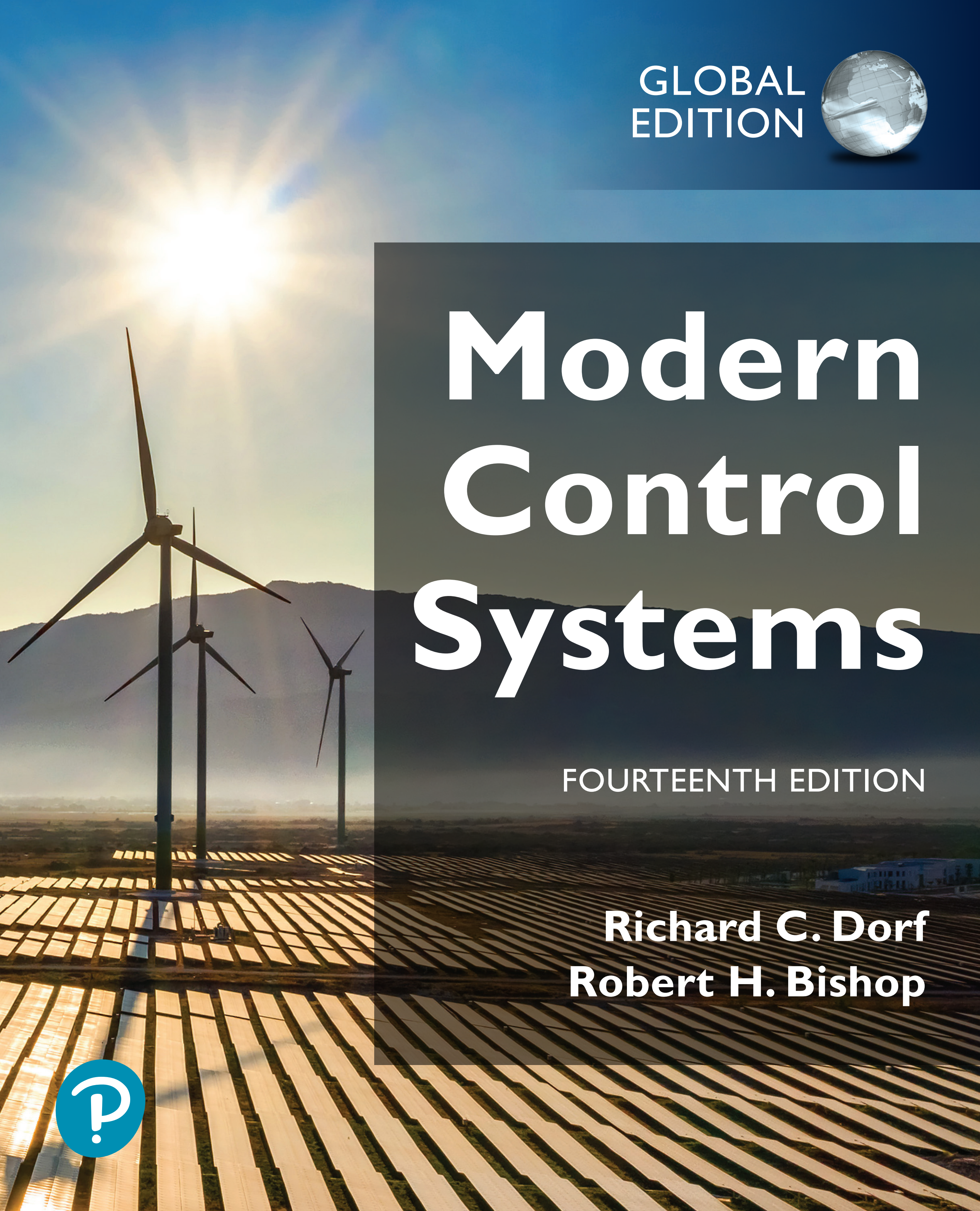
GLOBAL EDITION

Modern Control Systems

FOURTEENTH EDITION

Richard C. Dorf
Robert H. Bishop



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Richard C. Dorf
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Pearson Education Limited
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KAO Park
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Dedicated to the memory of

Professor Richard C. Dorf

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Preface

MODERN CONTROL SYSTEMS—THE BOOK

Global issues such as climate change, clean water, sustainability, pandemics, waste management, emissions reduction, and minimizing raw material and energy use have led many engineers to re-think existing approaches to engineering design. One outcome of the evolving design strategy is to consider *green engineering* and *human-centered design*. The goal of these approaches to engineering is to design products that minimize pollution, reduce the risk to human health, and improve the living environment. Applying the principles of green engineering and human-centered design highlights the power of feedback control systems as an enabling technology.

To reduce greenhouse gases and minimize pollution, it is necessary to improve both the quality and quantity of our environmental monitoring systems. One example is to use wireless measurements on mobile sensing platforms to measure the external environment. Another example is to monitor the quality of the delivered power to measure leading and lagging power, voltage variations, and waveform harmonics. Many green engineering systems and components require careful monitoring of current and voltages. For example, current transformers are used in various capacities for measuring and monitoring current within the power grid network of interconnected systems used to deliver electricity. Sensors are key components of any feedback control system because the measurements provide the required information as to the state of the system so the control system can take the appropriate action.

The role of control systems will continue to expand as the global issues facing us require ever increasing levels of automation and precision. In the book, we present key examples from green engineering such as wind turbine control and modeling of a photovoltaic generator for feedback control to achieve maximum power delivery as the sunlight varies over time.

The wind and sun are important sources of renewable energy around the world. Wind energy conversion to electric power is achieved by wind energy turbines connected to electric generators. The intermittency characteristic of the wind makes smart grid development essential to bring the energy to the power grid when it is available and to provide energy from other sources when the wind dies down or is disrupted. A smart grid can be viewed as a system comprised of hardware and software that routes power more reliably and efficiently to homes, businesses, schools, and other users of power in the presence of intermittency and other disturbances. The irregular character of wind direction and power also results in the need for reliable, steady electric energy by using control systems on the wind turbines themselves. The goal of these control devices is to reduce the effects of wind intermittency and the effect of wind direction change. Energy storage systems are also critical technologies for green engineering. We seek energy storage systems that are renewable, such as fuel cells. Active control can be a key element of effective renewable energy storage systems as well.

Another exciting development for control systems is the evolution of the Internet of Things—a network of physical objects embedded with electronics, software, sensors and connectivity. As envisioned, each of the millions of the devices on the network will possess an embedded computer with connectivity to the Internet. The ability to control these connected devices will be of great interest to control engineers. Indeed, control engineering is an exciting and a challenging field. By its very nature, control engineering is a multidisciplinary subject, and it has taken its place as a core course in the engineering curriculum. It is reasonable to expect different approaches to mastering and practicing the art of control engineering. Since the subject has a strong mathematical foundation, we might approach it from a strictly theoretical point of view, emphasizing theorems and proofs. On the other hand, since the ultimate objective is to implement controllers in real systems, we might take an ad hoc approach relying only on intuition and hands-on experience when designing feedback control systems. Our approach is to present a control engineering methodology that, while based on mathematical fundamentals, stresses physical system modeling and practical control system designs with realistic system specifications.

We believe that the most important and productive approach to learning is for each of us to rediscover and re-create anew the answers and methods of the past. Thus, the ideal is to present the student with a series of problems and questions and point to some of the answers that have been obtained over the past decades. The traditional method—to confront the student not with the problem but with the finished solution—is to deprive the student of all excitement, to shut off the creative impulse, to reduce the adventure of humankind to a dusty heap of theorems. The issue, then, is to present some of the unanswered and important problems that we continue to confront, for it may be asserted that what we have truly learned and understood, we discovered ourselves.

The purpose of this book is to present the structure of feedback control theory and to provide a sequence of exciting discoveries as we proceed through the text and problems. If this book is able to assist the student in discovering feedback control system theory and practice, it will have succeeded.

WHAT'S NEW IN THIS EDITION

This latest edition of *Modern Control Systems* incorporates the following key updates:

- Available as both an eText and print book.
- Video solutions for select problems throughout the text.
- Interactive figures added throughout the eText to enhance student learning.
- In the eText, interactive Skills Check multiple-choice questions at the end of each chapter.
- Over 20% new or updated problems. There are over 980 end-of-chapter exercises, problems, advanced problems, design problems, and computer problems.
- Expanded use of color for clarity of presentation.
- An updated companion website available at www.pearsonglobaleditions.com for students and faculty.

THE AUDIENCE

This text is designed for an introductory undergraduate course in control systems for engineering students. There is very little demarcation between the various engineering areas in control system practice; therefore, this text is written without any conscious bias toward one discipline. Thus, it is hoped that this book will be equally useful for all engineering disciplines and, perhaps, will assist in illustrating the utility of control engineering. The numerous problems and examples represent all fields, and the examples of the sociological, biological, ecological, and economic control systems are intended to provide the reader with an awareness of the general applicability of control theory to many facets of life. We believe that exposing students of one discipline to examples and problems from other disciplines will provide them with the ability to see beyond their own field of study. Many students pursue careers in engineering fields other than their own. We hope this introduction to control engineering will give students a broader understanding of control system design and analysis.

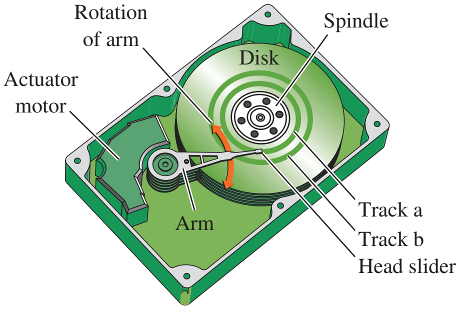
In its first thirteen editions, *Modern Control Systems* has been used in senior-level courses for engineering students at many colleges and universities globally. It also has been used in courses for engineering graduate students with no previous background in control engineering.

THE FOURTEENTH EDITION

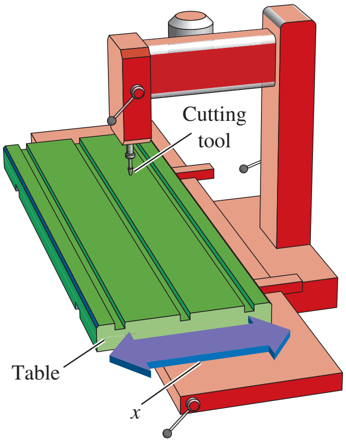
With the fourteenth edition, we have created an interactive e-textbook to fully use rich, digital content for *Modern Control Systems* to enhance the learning experience. This version contains embedded videos, dynamic graphs, live Skills Check quizzes, and active links to additional resources. The electronic version provides a powerful interactive experience that would be difficult, if not impossible, to achieve in a print book.

A companion website is also available to students and faculty using the fourteenth edition. The website contains many resources, including the m-files in the book, Laplace and z-Transform tables, written materials on matrix algebra and complex numbers, symbols, units, and conversion factors, and an introduction to MATLAB and to the LabVIEW MathScript RT Module. The MCS website is available at www.pearsonglobaleditions.com.

We continue the design emphasis that historically has characterized *Modern Control Systems*. Using the real-world engineering problems associated with designing a controller for a disk drive read system, we present the *Sequential Design Example*, which is considered sequentially in each chapter using the methods and concepts in that chapter. Disk drives are used in computers of all sizes and they represent an important application of control engineering. Various aspects of the design of controllers for the disk drive read system are considered in each chapter. For example, in Chapter 1 we identify the control goals, identify the variables to be controlled, write the control specifications, and establish the preliminary system configuration for the disk drive. Then, in Chapter 2, we obtain models of the process, sensors, and actuators. In the remaining chapters, we continue the design process, stressing the main points of the chapters.



In the same spirit as the *Sequential Design Example*, we present a design problem that we call the *Continuous Design Problem* to give students the opportunity to build upon a design problem from chapter to chapter. High-precision machinery places stringent demands on table slide systems. In the *Continuous Design Problem*, students apply the techniques and tools presented in each chapter to the development of a design solution that meets the specified requirements.



The computer-aided design and analysis component of the book continues to evolve and improve. Also, many of the solutions to various components of the *Sequential Design Example* utilize m-files with corresponding scripts included in the figures.

A Skills Check section is included at the end of each chapter. In each Skills Check section, we provide three sets of problems to test your knowledge of the chapter material. This includes True or False, Multiple Choice, and Word Match problems. To obtain direct feedback, you can check your answers with the answer key provided at the conclusion of the end-of-chapter problems.

PEDAGOGY

The book is organized around the concepts of control system theory as they have been developed in the frequency and time domains. An attempt has been made to make the selection of topics, as well as the systems discussed in the examples and problems, modern in the best sense. Therefore, this book includes discussions on robust control systems and system sensitivity, state variable models, controllability and observability, computer control systems, internal model control, robust PID controllers, and computer-aided design and analysis, to name a few. However, the classical topics of control theory that have proved to be so very useful in practice have been retained and expanded.

Building Basic Principles: From Classical to Modern. Our goal is to present a clear exposition of the basic principles of frequency and time-domain design techniques. The classical methods of control engineering are thoroughly covered: Laplace transforms and transfer functions; root locus design; Routh–Hurwitz stability analysis; frequency response methods, including Bode, Nyquist, and Nichols; steady-state error for standard test signals; second-order system approximations; and phase and gain margin and bandwidth. In addition, coverage of the state variable method is significant. Fundamental notions of controllability and observability for state variable models are discussed. Full state feedback design with Ackermann's formula for pole placement is presented, along with a discussion on the limitations of state variable feedback. Observers are introduced as a means to provide state estimates when the complete state is not measured.

Upon this strong foundation of basic principles, the book provides many opportunities to explore topics beyond the traditional. In the latter chapters, we present introductions into more advanced topics of robust control and digital control, as well as an entire chapter devoted to the design of feedback control systems with a focus on practical industrial lead and lag compensator structures. Problem solving is emphasized throughout the chapters. Each chapter (but the first) introduces the student to the notion of computer-aided design and analysis.

Progressive Development of Problem-Solving Skills. Reading the chapters, attending lectures and taking notes, and working through the illustrated examples are all part of the learning process. But the real test comes at the end of the chapter with the problems. The book takes the issue of problem solving seriously. In each chapter, there are five problem types:

- Exercises
- Problems
- Advanced Problems
- Design Problems
- Computer Problems

For example, the problem set for Mathematical Models of Systems, Chapter 2 includes 31 exercises, 51 problems, 9 advanced problems, 6 design problems, and

10 computer-based problems. The exercises permit the students to readily utilize the concepts and methods introduced in each chapter by solving relatively straightforward exercises before attempting the more complex problems. The problems require an extension of the concepts of the chapter to new situations. The advanced problems represent problems of increasing complexity. The design problems emphasize the design task; the computer-based problems give the student practice with problem solving using computers. In total, the book contains more than 980 problems. The abundance of problems of increasing complexity gives students confidence in their problem solving ability as they work their way from the exercises to the design and computer-based problems. An instructor's manual, available to all adopters of the text for course use, contains complete solutions to all end-of-chapter problems.

A set of m-files, the *Modern Control Systems Toolbox*, has been developed by the authors to supplement the text. The m-files contain the scripts from each computer-based example in the text. You may retrieve the m-files from the companion available at www.pearsonglobaleditions.com.

Design Emphasis without Compromising Basic Principles. The all-important topic of design of real-world, complex control systems is a major theme throughout the text. Emphasis on design for real-world applications addresses interest in design by ABET and industry.

The design process consists of seven main building blocks that we arrange into three groups:

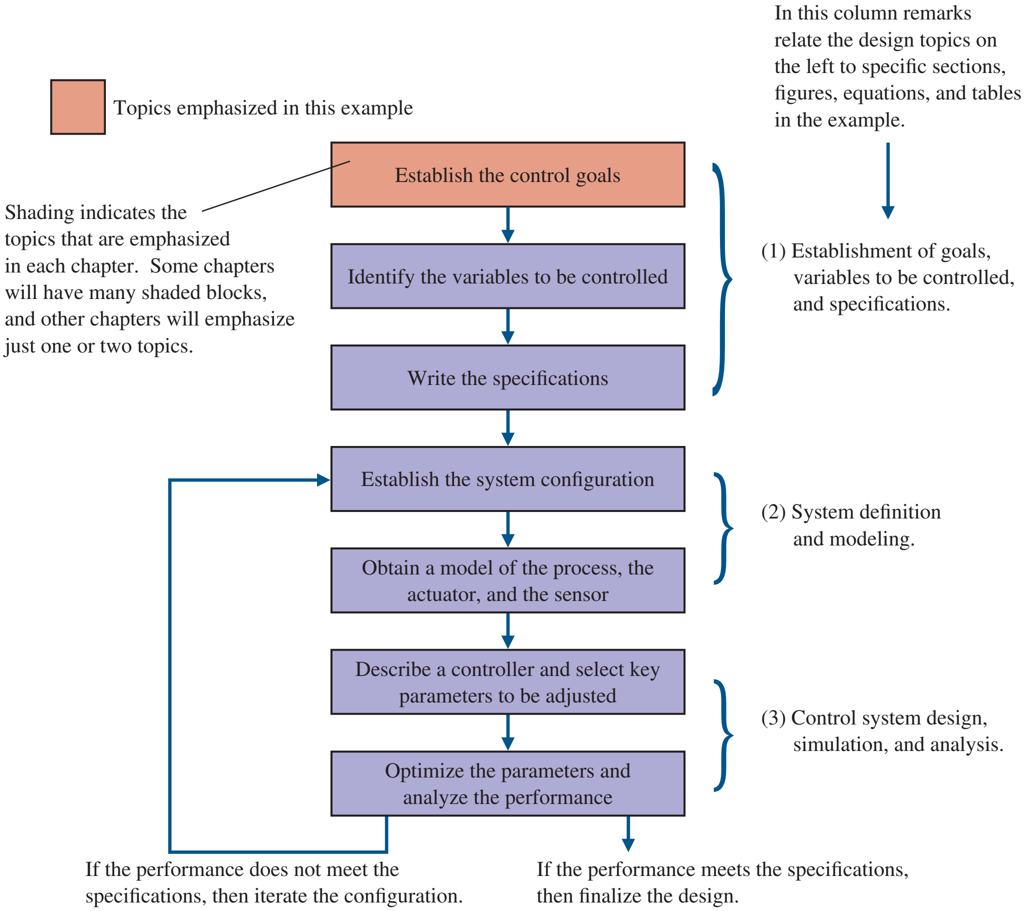
1. Establishment of goals and variables to be controlled, and definition of specifications (metrics) against which to measure performance
2. System definition and modeling
3. Control system design and integrated system simulation and analysis

In each chapter of this book, we highlight the connection between the design process and the main topics of that chapter. The objective is to demonstrate different aspects of the design process through illustrative examples.

Various aspects of the control system design process are illustrated in detail in many examples across all the chapters, including applications of control design in robotics, manufacturing, medicine, and transportation (ground, air, and space).

Each chapter includes a section to assist students in utilizing computer-aided design and analysis concepts and in reworking many of the design examples. Generally, m-files scripts are provided that can be used in the design and analyses of the feedback control systems. Each script is annotated with comment boxes that highlight important aspects of the script. The accompanying output of the script (generally a graph) also contains comment boxes pointing out significant elements. The scripts can also be utilized with modifications as the foundation for solving other related problems.

Learning Enhancement. Each chapter begins with a chapter preview describing the topics the student can expect to encounter. The chapters conclude with an end-of-chapter summary, skills check, as well as terms and concepts. These sections

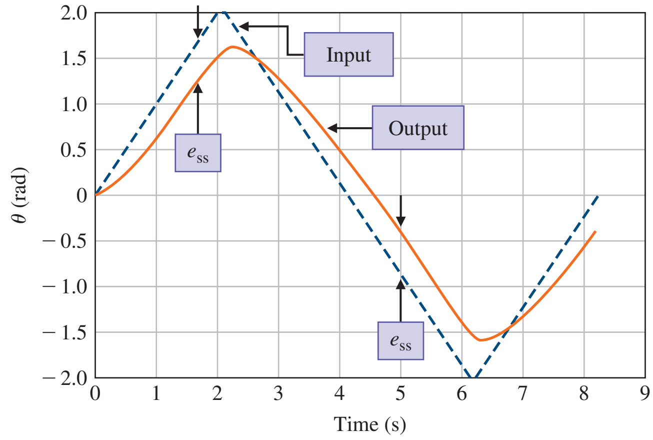


reinforce the important concepts introduced in the chapter and serve as a reference for later use.

Color is used to add emphasis when needed and to make the graphs and figures easier to interpret. For example, consider the computer control of a robot to spray-paint an automobile. We might ask the student to investigate the closed-loop system stability for various values of the controller gain K and to determine the response to a unit step disturbance, $T_d(s) = 1/s$, when the input $R(s) = 0$. The associated figure assists the student with (a) visualizing the problem, and (b) taking the next step to develop the transfer function model and to complete the analyses.

THE ORGANIZATION

Chapter 1 Introduction to Control Systems. Chapter 1 provides an introduction to the basic history of control theory and practice. The purpose of this chapter is to describe the general approach to designing and building a control system.



(a)

```
%Compute the response of the Mobile Robot Control
%System to a triangular wave input
%
numg=[10 20]; deng=[1 10 0]; sysg=tf(numg,deng);
[sys]=feedback(sysg, [1]);
t=[0:0.1:8.2]';
v1=[0:0.1:2]';v2=[2:-0.1:-2]';v3=[-2:0.1:0]';
u=[v1;v2;v3];
[y,T]=lsim(sys,u,t);
plot(T,y,t,u,'--'),
xlabel('Time (s)'), ylabel('\theta (rad)'), grid
```

$G(s)G_c(s)$

Compute triangular wave input.

Linear simulation.

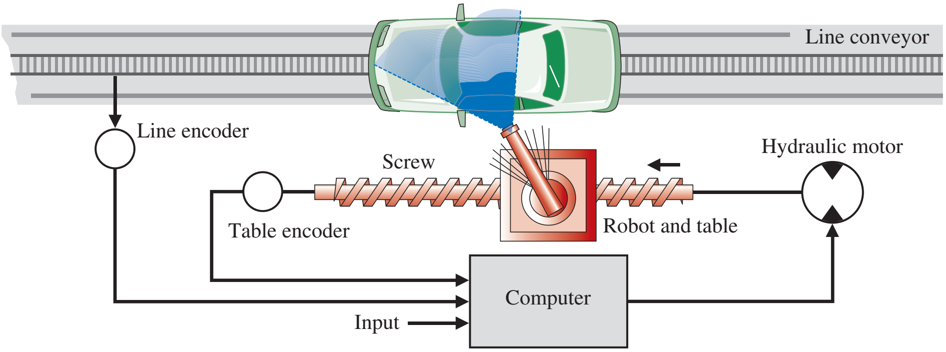
(b)

Chapter 2 Mathematical Models of Systems. Mathematical models of physical systems in input–output or transfer function form are developed in Chapter 2. A wide range of systems are considered.

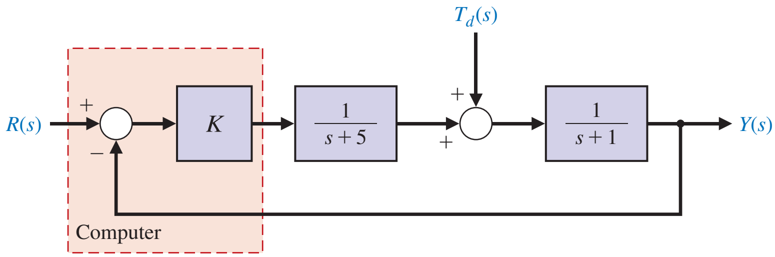
Chapter 3 State Variable Models. Mathematical models of systems in state variable form are developed in Chapter 3. The transient response of control systems and the performance of these systems are examined.

Chapter 4 Feedback Control System Characteristics. The characteristics of feedback control systems are described in Chapter 4. The advantages of feedback are discussed, and the concept of the system error signal is introduced.

Chapter 5 The Performance of Feedback Control Systems. In Chapter 5, the performance of control systems is examined. The performance of a control system is correlated with the s-plane location of the poles and zeros of the transfer function of the system.



(a)



(b)

Chapter 6 The Stability of Linear Feedback Systems. The stability of feedback systems is investigated in Chapter 6. The relationship of system stability to the characteristic equation of the system transfer function is studied. The Routh–Hurwitz stability criterion is introduced.

Chapter 7 The Root Locus Method. Chapter 7 deals with the motion of the roots of the characteristic equation in the *s*-plane as one or two parameters are varied. The locus of roots in the *s*-plane is determined by a graphical method. We also introduce the popular PID controller and the Ziegler-Nichols PID tuning method.

Chapter 8 Frequency Response Methods. In Chapter 8, a steady-state sinusoid input signal is utilized to examine the steady-state response of the system as the frequency of the sinusoid is varied. The development of the frequency response plot, called the Bode plot, is considered.

Chapter 9 Stability in the Frequency Domain. System stability utilizing frequency response methods is investigated in Chapter 9. Relative stability and the Nyquist criterion are discussed. Stability is considered using Nyquist plots, Bode plots, and Nichols charts.

Chapter 10 The Design of Feedback Control Systems. Several approaches to designing and compensating a control system are described and developed in Chapter 10. Various candidates for service as compensators are presented and it is shown how they help to achieve improved performance. The focus is on lead and lag compensators.

Chapter 11 The Design of State Variable Feedback Systems. The main topic of Chapter 11 is the design of control systems using state variable models. Full-state feedback design and observer design methods based on pole placement are discussed. Tests for controllability and observability are presented, and the concept of an internal model design is discussed.

Chapter 12 Robust Control Systems. Chapter 12 deals with the design of highly accurate control systems in the presence of significant uncertainty. Five methods for robust design are discussed, including root locus, frequency response, ITAE methods for robust PID controllers, internal models, and pseudo-quantitative feedback.

Chapter 13 Digital Control Systems. Methods for describing and analyzing the performance of computer control systems are described in Chapter 13. The stability and performance of sampled-data systems are discussed.

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CONTRIBUTORS

Benjamin Chong, University of Leeds
Murat Doğruel, Marmara University
Quang Ha, University of Technology Sydney
Ashish Rajeshwar Kulkarni, Delhi Technological University
Savita Nema, Maulana Azad National Institute of Technology Bhopal
Mark Ovinis, Universiti Teknologi PETRONAS
Bidyadhar Subudhi, National Institute of Technology Rourkela

REVIEWERS

Quang Ha, University of Technology Sydney
Shen Hin Lim, University of Waikato
Mark Ovinis, Universiti Teknologi PETRONAS
Fuwen Yang, Griffith University

OPEN LINES OF COMMUNICATION

The authors would like to establish a line of communication with the users of *Modern Control Systems*. We encourage all readers to send comments and suggestions for this and future editions. By doing this, we can keep you informed of any general-interest news regarding the textbook and pass along comments of other users.

Keep in touch!

Robert H. Bishop robertbishop@usf.edu

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About the Authors

Richard C. Dorf was Emeriti Faculty of Electrical and Computer Engineering at the University of California, Davis. Known as an instructor who was highly concerned with the discipline of electrical engineering and its application to social and economic needs, Professor Dorf wrote and edited several successful engineering textbooks and handbooks, including the best selling *Engineering Handbook*, second edition and the third edition of the *Electrical Engineering Handbook*. Professor Dorf was also co-author of *Technology Ventures*, a leading textbook on technology entrepreneurship. Professor Dorf was a Fellow of the IEEE and a Fellow of the ASEE. Dr. Dorf held a patent for the PIDA controller.

Robert H. Bishop is the Dean of Engineering at the University of South Florida, President and CEO of the Institute of Applied Engineering, and a Professor in the Department of Electrical Engineering. Prior to coming to The University of South Florida, he was the Dean of Engineering at Marquette University and before that a Department Chair and Professor of Aerospace Engineering and Engineering Mechanics at The University of Texas at Austin where he held the Joe J. King Professorship and was a Distinguished Teaching Professor. Professor Bishop started his engineering career as a member of the technical staff at the Charles Stark Draper Laboratory. He authors the well-known textbook for teaching graphical programming entitled *Learning with LabVIEW* and is also the editor-in-chief of the *Mechatronics Handbook*. Professor Bishop remains an active teacher and researcher and has authored/co-authored over one hundred and forty-five journal and conference papers. He is a Fellow of the AIAA, a Fellow of the American Astronautical Society (AAS), a Fellow of the American Association for the Advancement of Science (AAAS) and active in ASEE and in the Institute of Electrical and Electronics Engineers (IEEE).

CHAPTER 1

Introduction to Control Systems

PREVIEW

A control system consists of interconnected components to achieve a desired purpose. In this chapter, we discuss open- and closed-loop feedback control systems. We examine examples of control systems through the course of history. Early systems incorporated many of the basic ideas of feedback that are employed in modern control systems. A design process is presented that encompasses the establishment of goals and variables to be controlled, definition of specifications, system definition, modeling, and analysis. The iterative nature of design allows us to handle the design gap effectively while accomplishing necessary trade-offs in complexity, performance, and cost. Finally, we introduce the Sequential Design Example: Disk Drive Read System. This example will be considered sequentially in each chapter of this book. It represents a practical control system design problem while simultaneously serving as a useful learning tool.

DESIRED OUTCOMES

Upon completion of Chapter 1, students should be able to:

- ❑ Give illustrative examples of control systems and describe their relationship to key contemporary issues.
- ❑ Recount a brief history of control systems and their role in society.
- ❑ Predict the future of controls in the context of their evolutionary pathways.
- ❑ Recognize the elements of control system design and possess an appreciation of appreciate controls in the context of engineering design.

1.1 INTRODUCTION

Engineers create products that help people. Our quality of life is sustained and enhanced through engineering. To accomplish this, engineers strive to understand, model, and control the materials and forces of nature for the benefit of humankind. A key area of engineering that reaches across many technical areas is the multidisciplinary field of control system engineering. Control engineers are concerned with understanding and controlling segments of their environment, often called **systems**, which are interconnections of elements and devices for a desired purpose. The system might be something as clear-cut as an automobile cruise control system, or as extensive and complex as a direct brain-to-computer system to control a manipulator. Control engineering deals with the design (and implementation) of control systems using linear, time-invariant mathematical models representing actual physical nonlinear, time-varying systems with parameter uncertainties in the presence of external disturbances. As computer systems—especially embedded processors—have become less expensive, require less power and space, while growing more computationally powerful, at the same time that sensors and actuators have simultaneously experienced the same evolution to more capability in smaller packages, the application of control systems has grown in number and complexity. A **sensor** is a device that provides a measurement of a desired external signal. For example, resistance temperature detectors (RTDs) are sensors used to measure temperature. An **actuator** is a device employed by the control system to alter or adjust the environment. An electric motor drive used to rotate a robotic manipulator is an example of a device transforming electric energy to mechanical torque.

The face of control engineering is rapidly changing. The age of the Internet of Things (IoT) presents many intriguing challenges in control system applications in the environment (think about more efficient energy use in homes and businesses), manufacturing (think 3D printing), consumer products, energy, medical devices and healthcare, transportation (think about automated cars!), among many others [14]. A challenge for control engineers today is to be able to create simple, yet reliable and accurate mathematical models of many of our modern, complex, interrelated, and interconnected systems. Fortunately, many modern design tools are available, as well as open source software modules and Internet-based user groups (to share ideas and answer questions), to assist the modeler. The implementation of the control systems themselves is also becoming more automated, again assisted by many resources readily available on the Internet coupled with access to relatively inexpensive computers, sensors, and actuators. **Control system engineering** focuses on the modeling of a wide assortment of physical systems and using those models to design controllers that will cause the closed-loop systems to possess desired performance characteristics, such as stability, relative stability, steady-state tracking with prescribed maximum errors, transient tracking (percent overshoot, settling time, rise time, and time to peak), rejection of external disturbances, and robustness to modeling uncertainties. The extremely important step of the overall design and implementation process is designing the control systems, such as PID controllers, lead and lag controllers, state variable feedback controllers, and other popular controller structures. That is what this textbook is all about!

FIGURE 1.1 Process to be controlled.

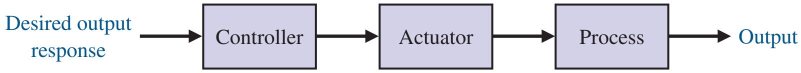


FIGURE 1.2 Open-loop control system (without feedback).

Control system engineering is based on the foundations of feedback theory and linear system analysis, and it integrates the concepts of network theory and communication theory. It is founded on a strong mathematical foundation, yet is very practical and impacts our lives every day in almost all we do. Indeed, control engineering is not limited to any engineering discipline but is equally applicable to aerospace, agricultural, biomedical, chemical, civil, computer, industrial, electrical, environmental, mechanical, nuclear engineering, and even computer science. Many aspects of control engineering can also be found in studies in systems engineering.

A **control system** is an interconnection of components forming a system configuration that will provide a desired system response. The basis for analysis of a system is the foundation provided by linear system theory, which assumes a cause–effect relationship for the components of a system. A component, or **process**, to be controlled can be represented graphically, as shown in Figure 1.1. The input–output relationship represents the cause-and-effect relationship of the process, which in turn represents a processing of the input signal to provide a desired output signal. An **open-loop control system** uses a controller and an actuator to obtain the desired response, as shown in Figure 1.2. An open-loop system is a system without feedback.

An open-loop control system utilizes an actuating device to control the process directly without using feedback.

In contrast to an open-loop control system, a closed-loop control system utilizes an additional measure of the actual output to compare the actual output with the desired output response. The measure of the output is called the **feedback signal**. A simple **closed-loop feedback control system** is shown in Figure 1.3. A feedback control system is a control system that tends to maintain a prescribed relationship of one system variable to another by comparing functions of these variables and using the difference as a means of control. With an accurate sensor, the measured output is a good approximation of the actual output of the system.

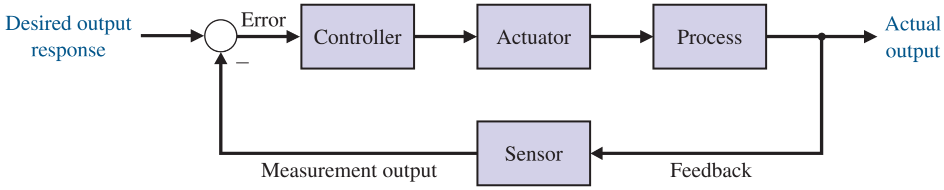


FIGURE 1.3 Closed-loop feedback control system (with feedback).

A feedback control system often uses a function of a prescribed relationship between the output and reference input to control the process. Often the difference between the output of the process under control and the reference input is amplified and used to control the process so that the difference is continually reduced. In general, the difference between the desired output and the actual output is equal to the error, which is then adjusted by the controller. The output of the controller causes the actuator to modulate the process in order to reduce the error. For example, if a ship is heading incorrectly to the right, the rudder is actuated to direct the ship to the left. The system shown in Figure 1.3 is a **negative feedback** control system, because the output is subtracted from the input and the difference is used as the input signal to the controller. The feedback concept is the foundation for control system analysis and design.

A closed-loop control system uses a measurement of the output and feedback of this signal to compare it with the desired output (reference or command).

A closed-loop control has many advantages over open-loop control, including the ability to reject external **disturbances** and improve **measurement noise** attenuation. We incorporate disturbances and measurement noise in the block diagram as external inputs, as illustrated in Figure 1.4. External disturbances and measurement noise are inevitable in real-world applications and must be addressed in practical control system designs.

The feedback systems in Figures 1.3 and 1.4 are single-loop feedback systems. Many feedback control systems contain more than one feedback loop. A common **multiloop feedback control system** is illustrated in Figure 1.5 with an inner loop and an outer loop. In this scenario, the inner loop has a controller and a sensor and the outer loop has a controller and sensor. Other varieties of multiloop feedback systems are considered throughout the book as they represent more practical situations found in real-world applications. However, we use the single-loop feedback system for learning about the benefits of feedback control systems since the outcomes readily scale to multiloop systems.

Due to the increasing complexity of systems under active control and the interest in achieving optimum performance, the importance of control system engineering continues to grow. Furthermore, as the systems become more complex, the interrelationship of many controlled variables must be considered in the control scheme. A block diagram depicting a **multivariable control system** is shown in Figure 1.6.

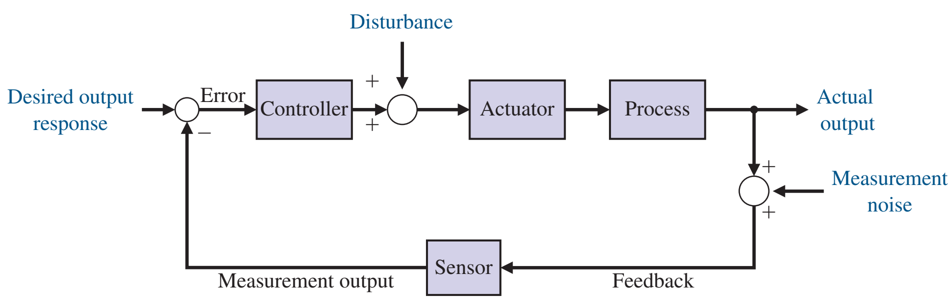


FIGURE 1.4 Closed-loop feedback system with external disturbances and measurement noise.

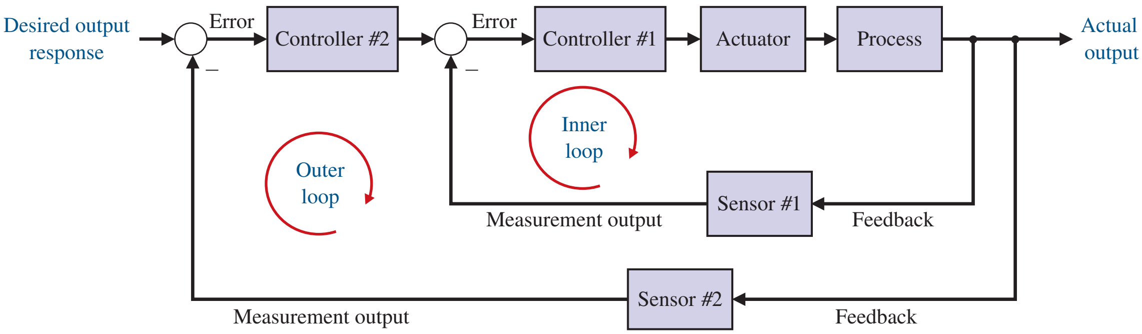


FIGURE 1.5 Multiloop feedback system with an inner loop and an outer loop.

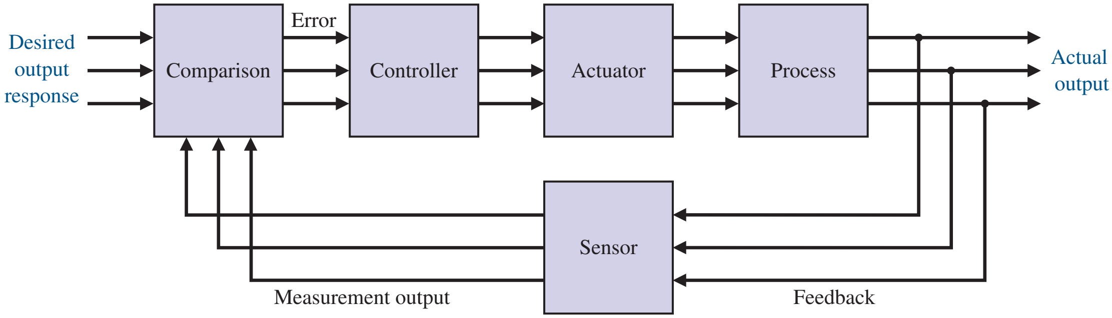


FIGURE 1.6 Multivariable control system.

A common example of an open-loop control system is a microwave oven set to operate for a fixed time. An example of a closed-loop control system is a person steering an automobile (assuming his or her eyes are open) by looking at the auto's location on the road and making the appropriate adjustments.

The introduction of feedback enables us to control a desired output and can improve accuracy, but it requires attention to the issues of stability and performance.

1.2 BRIEF HISTORY OF AUTOMATIC CONTROL

The use of feedback to control a system has a fascinating history. The first applications of feedback control appeared in the development of float regulator mechanisms in Greece in the period 300 to 1 B.C. [1, 2, 3]. The water clock of Ktesibios used a float regulator. An oil lamp devised by Philon in approximately 250 B.C. used a float regulator in an oil lamp for maintaining a constant level of fuel oil. Heron of Alexandria, who lived in the first century A.D., published a book entitled *Pneumatica*, which outlined several forms of water-level mechanisms using float regulators [1].

The first feedback system to be invented in modern Europe was the temperature regulator of Cornelis Drebbel (1572–1633) of Holland [1]. Dennis Papin (1647–1712) invented the first pressure regulator for steam boilers in 1681. Papin's pressure regulator was a form of safety regulator similar to a pressure-cooker valve.

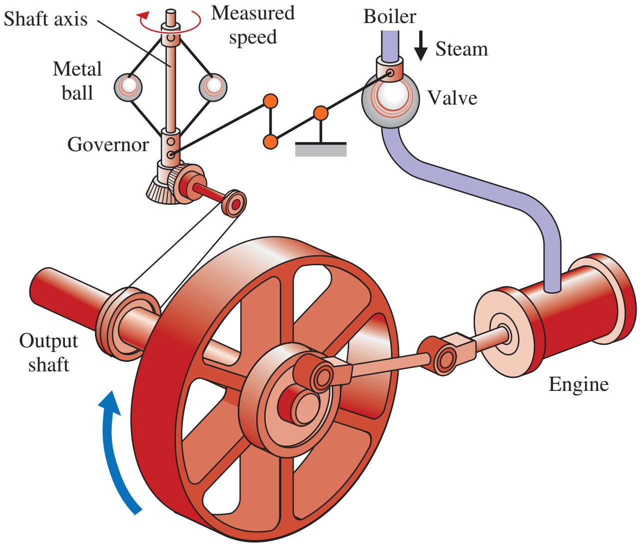


FIGURE 1.7
Watt's flyball governor.

The first automatic feedback controller used in an industrial process is generally agreed to be James Watt's **flyball governor**, developed in 1769 for controlling the speed of a steam engine [1, 2]. The all-mechanical device, illustrated in Figure 1.7, measured the speed of the output shaft and utilized the movement of the flyball to control the steam valve and therefore the amount of steam entering the engine. As depicted in Figure 1.7, the governor shaft axis is connected via mechanical linkages and beveled gears to the output shaft of the steam engine. As the steam engine output shaft speed increases, the ball weights rise and move away from the shaft axis and through mechanical linkages the steam valve closes and the engine slows down.

The first historical feedback system is the water-level float regulator said to have been invented by I. Polzunov in 1765 [4]. The level regulator system is illustrated in Figure 1.8. The float detects the water level and controls the valve that covers the water inlet in the boiler.

The next century was characterized by the development of automatic control systems through intuition and invention. Efforts to increase the accuracy of the control system led to slower attenuation of the transient oscillations and even to unstable systems. It then became imperative to develop a theory of automatic control. In 1868, J. C. Maxwell formulated a mathematical theory related to control theory using a differential equation model of a governor [5]. Maxwell's study was concerned with the effect various system parameters had on the system performance. During the same period, I. A. Vyshnegradskii formulated a mathematical theory of regulators [6].

Prior to World War II, control theory and practice developed differently in the United States and western Europe than in Russia and eastern Europe. The main

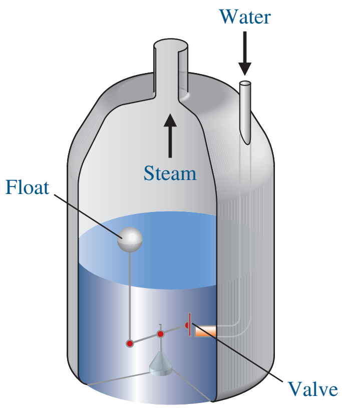


FIGURE 1.8
Water-level float regulator.

impetus for the use of feedback in the United States was the development of the telephone system and electronic feedback amplifiers by Bode, Nyquist, and Black at Bell Telephone Laboratories [7–10, 12].

Harold S. Black graduated from Worcester Polytechnic Institute in 1921 and joined Bell Laboratories of American Telegraph and Telephone (AT&T). At that time, the major task confronting Bell Laboratories was the improvement of the telephone system and the design of improved signal amplifiers. Black was assigned the task of linearizing, stabilizing, and improving the amplifiers that were used in tandem to carry conversations over distances of several thousand miles. After years of working on oscillator circuits, Black had the idea of negative feedback amplifiers as a way to avoid self-oscillations. His idea would enhance the stability of circuit stability over a wide range of frequency bands [8].

The frequency domain was used primarily to describe the operation of the feedback amplifiers in terms of bandwidth and other frequency variables. In contrast, the eminent mathematicians and applied mechanicians in the former Soviet Union inspired and dominated the field of control theory. The Russian theory tended to utilize a time-domain formulation using differential equations.

The control of an industrial process (manufacturing, production, and so on) by automatic rather than manual means is often called **automation**. Automation is prevalent in the chemical, electric power, paper, automobile, and steel industries, among others. The concept of automation is central to our industrial society. Automatic machines are used to increase the production of a plant. Industries are concerned with the productivity per worker of their plants. **Productivity** is defined as the ratio of physical output to physical input [26]. In this case, we are referring to labor productivity, which is real output per hour of work.

A large impetus to the theory and practice of automatic control occurred during World War II when it became necessary to design and construct automatic airplane piloting, gun-positioning systems, radar antenna control systems, and other military systems based on the feedback control approach.

The complexity and expected performance of these military systems necessitated an extension of the available control techniques and fostered interest in control systems and the development of new insights and methods. Prior to 1940, for most cases, the design of control systems was an art involving a trial-and-error approach. During the 1940s, mathematical and analytical methods increased in number and utility, and control engineering became an engineering discipline in its own right [10–12].

Another example of the discovery of an engineering solution to a control system problem was the creation of a gun director by David B. Parkinson of Bell Telephone Laboratories. In the spring of 1940, Parkinson was intent on improving the automatic level recorder, an instrument that used strip-chart paper to plot the record of a voltage. A critical component was a small potentiometer used to control the pen of the recorder through an actuator. If a potentiometer could be used to control the pen on a level recorder, might it be capable of controlling other machines such as an antiaircraft gun? [13].

After considerable effort, an engineering model was delivered for testing to the U.S. Army on December 1, 1941. Production models were available by early 1943, and eventually 3000 gun controllers were delivered. Input to the controller was provided by radar, and the gun was aimed by taking the data of the airplane's present position and calculating the target's future position.

Frequency-domain techniques continued to dominate the field of control following World War II with the increased use of the Laplace transform and the complex frequency plane. During the 1950s, the emphasis in control engineering theory was on the development and use of the s-plane methods and, particularly, the root locus approach. Furthermore, during the 1980s, the use of digital computers for control components became routine. The technology of these new control elements to perform accurate and rapid calculations was formerly unavailable to control engineers. These computers are now employed especially for process control systems in which many variables are measured and controlled simultaneously by the computer.

With the advent of Sputnik and the space age, another new impetus was imparted to control engineering. It became necessary to design complex, highly accurate control systems for missiles and space probes. Furthermore, the necessity to minimize the weight of satellites and to control them very accurately has spawned the important field of optimal control. Due to these requirements, the time-domain methods developed by Liapunov, Minorsky, and others have been met with great interest. Theories of optimal control developed by L. S. Pontryagin in the former Soviet Union and R. Bellman in the United States, as well as studies of robust systems, have contributed to the interest in time-domain methods. Control engineering must consider both the time-domain and the frequency-domain approaches simultaneously in the analysis and design of control systems.

A notable advance with worldwide impact is the U.S. space-based radionavigation system known as the Global Positioning System or GPS [82–85]. In the distant past, various strategies and sensors were developed to keep explorers on the oceans from getting lost, including following coastlines, using compasses to point north, and sextants to measure the angles of stars, the moon, and the sun above the horizon. The early explorers were able to estimate latitude accurately, but not longitude. It was not until the 1700s with the development of the chronometer that, when used with the sextant, the longitude could be estimated. Radio-based navigation systems began to appear in the early twentieth century and were used in World War II. With the advent of Sputnik and the space age, it became known that radio signals from satellites could be used to navigate on the ground by observing the Doppler shift of the received radio signals. Research and development culminated in the 1990s with 24 navigation satellites (known as the GPS) that solved the fundamental problem that explorers faced for centuries by providing a dependable mechanism to pinpoint the current location. Freely available on a continuous worldwide basis, GPS provides very reliable location and time information anytime, day or night, anywhere in the world. Using GPS as a sensor to provide position (and velocity) information is a mainstay of active control systems for transportation systems in the air, on the ground, and on the oceans. The GPS assists relief and emergency workers to save lives, and helps us with our everyday activities including the control of power grids, banking, farming, surveying, and many other tasks.

Global navigation satellite services (such as GPS, GLONASS, and Galileo) providing position, navigation, and timing data coupled with evolving wireless mobile technology, highly capable mobile computing systems and devices, global geographic information systems, and semantic web are supporting the evolving field of **ubiquitous positioning** [100-103]. These systems can provide information on the location of people, vehicles, and other objects as a function of time across the globe. As personal **ubiquitous computing** [104] contiues to push active control technology to the edge where the action is taking place, we will be faced with many opportunities to design and field autonomous systems based on the firm ground of system theoretic concepts covered in this introductory text on modern control systems.

The evolution of the **Internet of Things (IoT)** is having a transformational impact on the field of control engineering. The idea of the IoT, first proposed by Kevin Ashton in 1999, is the network of physical objects embedded with electronics, software, sensors, and connectivity—all elements of control engineering [14]. Each of the "things" on the network has an embedded computer with connectivity to the Internet. The ability to control connected devices is of great interest to control engineers, but there remains much work to be done, especially in establishing standards [24]. The International Data Corporation estimates that there will be 41.6 billion IoT devices generating 79.4 zettabytes (ZB) of data by the year 2025 [106]. One ZB is equal to one trillion GB! Figure 1.9 presents a technology roadmap that illustrates that in the near future control engineering is likely to play a role in creating active control applications for connected devices (adopted from [27]).

A selected history of control system development is summarized in Table 1.1.

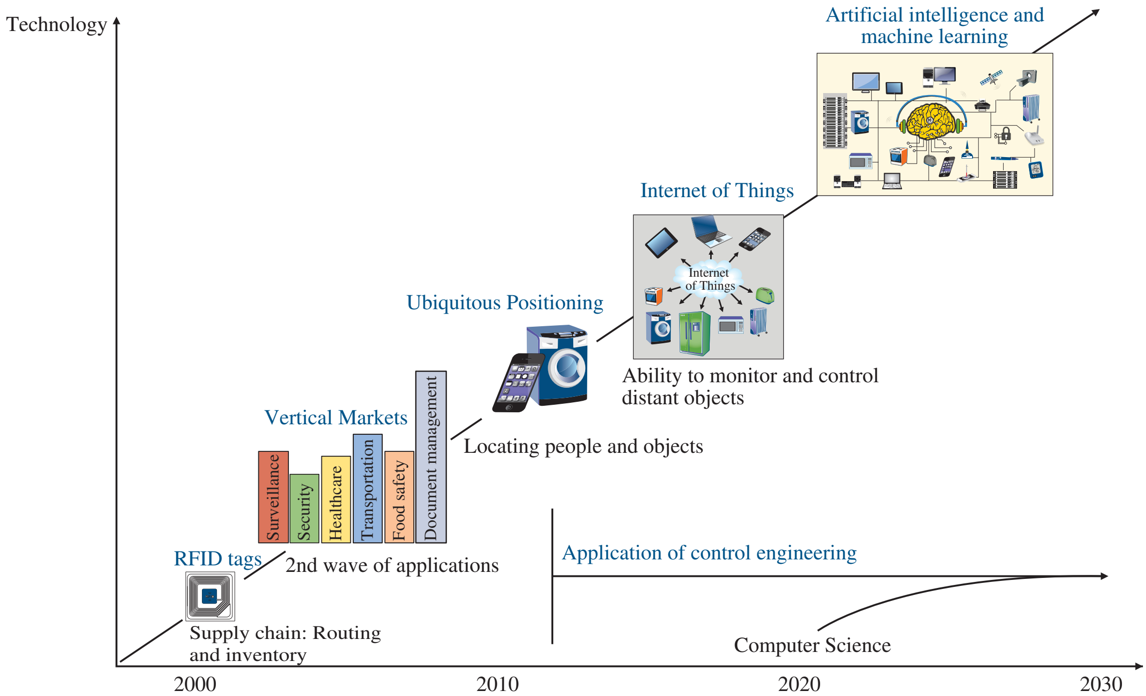


FIGURE 1.9 Technology roadmap to the Internet of Things enhanced with artificial intelligence with applications to control engineering (Source: SRI Business Intelligence).

Table 1.1 Selected Historical Developments of Control Systems

1769	James Watt's steam engine and governor developed.
1868	J. C. Maxwell formulates a mathematical model for a governor control of a steam engine.
1913	Henry Ford's mechanized assembly machine introduced for automobile production.
1927	H. S. Black conceives of the negative feedback amplifier and H. W. Bode analyzes feedback amplifiers.
1932	H. Nyquist develops a method for analyzing the stability of systems.
1941	Creation of first antiaircraft gun with active control.
1952	Numerical control (NC) developed at Massachusetts Institute of Technology for control of machine-tool axes.
1954	George Devol develops "programmed article transfer," considered to be the first industrial robot design.
1957	Sputnik launches the space age leading, in time, to miniaturization of computers and advances in automatic control theory.
1960	First Unimate robot introduced, based on Devol's designs. Unimate installed in 1961 for tending die-casting machines.
1970	State-variable models and optimal control developed.

Table 1.1 *(continued)*

1980	Robust control system design widely studied.
1983	Introduction of the personal computer (and control design software soon thereafter) brought the tools of design to the engineer's desktop.
1990	The government ARPANET (the first network to use the Internet Protocol) was decommissioned and private connections to the Internet by commercial companies rapidly spread.
1994	Feedback control widely used in automobiles. Reliable, robust systems demanded in manufacturing.
1995	The Global Positioning System (GPS) was operational providing positioning, navigation, and timing services worldwide.
1997	First ever autonomous rover vehicle, known as Sojourner, explores the Martian surface.
2007	The Orbital Express mission performed the first autonomous space rendezvous and docking.
2011	The NASA Robonaut R2 became the first US-built robot on the International Space Station designed to assist with crew extravehicular activities (EVAs).
2013	For the first time, a vehicle—known as BRAiVE and designed at the University of Parma, Italy—moved autonomously on a mixed traffic route open to public traffic without a passenger in the driver seat.
2014	Internet of Things (IoT) enabled by convergence of key systems including embedded systems, wireless sensor networks, control systems, and automation.
2016	Space X successfully lands the first rocket on an autonomous spaceport drone ship controllrd by an autonomus robot.
2019	Alphabet's Wing begins making first commercial drone deliveries in the US.

1.3 EXAMPLES OF CONTROL SYSTEMS

Control engineering is concerned with the analysis and design of goal-oriented systems. Therefore the mechanization of goal-oriented policies has grown into a hierarchy of goal-oriented control systems. Modern control theory is concerned with systems that have self-organizing, adaptive, robust, learning, and optimum qualities.

EXAMPLE 1.1 **Automated vehicles**

Driving an automobile is a pleasant task when the auto responds rapidly to the driver's commands. The era of autonomous or self-driving vehicles is almost upon us [15, 19, 20]. The autonomous vehicle must be able to sense the changing environment, perform trajectory planning, prescribe the control inputs that include steering and turning, accelerating and braking, and many other functions typically handled by the driver, and actually implement the control strategy. Steering is one of the critical functions of autonomous vehicles. A simple block diagram of an automobile steering control system is shown in Figure 1.10(a). The desired course is compared with a measurement of the actual course in order to generate a measure of the error, as shown in Figure 1.10(b). This measurement is obtained by visual and tactile (body movement) feedback, as provided by the feel of the steering wheel by the hand (sensor). This feedback system is a familiar version of the steering control system in an ocean liner or the flight controls in a large airplane. A typical direction-of-travel response is shown in Figure 1.10(c). ■

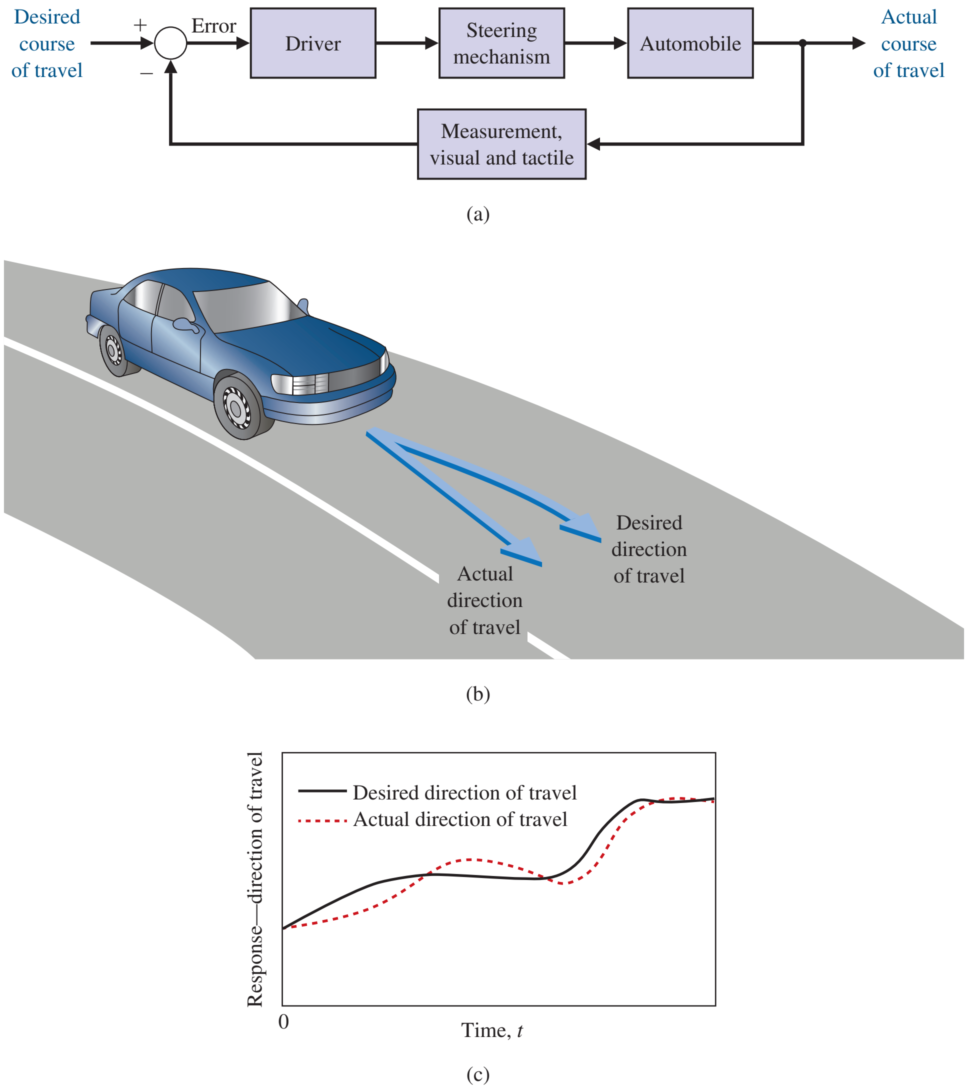


FIGURE 1.10 (a) Automobile steering control system. (b) The driver uses the difference between the actual and the desired direction of travel to generate a controlled adjustment of the steering wheel. (c) Typical direction-of-travel response.

EXAMPLE 1.2 Human-in-the-loop control

A basic, manually controlled closed-loop system for regulating the level of fluid in a tank is shown in Figure 1.11. The input is a reference level of fluid that the operator is instructed to maintain. (This reference is memorized by the operator.) The power amplifier is the operator, and the sensor is visual. The operator compares the actual level with the desired level and opens or closes the valve (actuator), adjusting the fluid flow out, to maintain the desired level. ■

EXAMPLE 1.3 Humanoid robots

The use of computers integrated with machines that perform tasks like a human worker has been foreseen by several authors. In his famous 1923 play, entitled

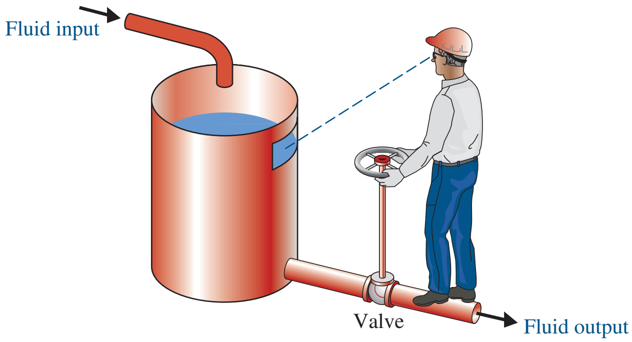


FIGURE 1.11 A manual control system for regulating the level of fluid in a tank by adjusting the output valve. The operator views the level of fluid through a port in the side of the tank.

R.U.R. [48], Karel Capek called artificial workers *robots*, deriving the word from the Czech noun *robota*, meaning "work."

A **robot** is a computer-controlled machine and involves technology closely associated with automation. Industrial robotics can be defined as a particular field of automation in which the automated machine (that is, the robot) is designed to substitute for human labor [18, 33]. Thus robots possess certain humanlike characteristics. Today, the most common humanlike characteristic is a mechanical manipulator that is patterned somewhat after the human arm and wrist. Some devices even have anthropomorphic mechanisms, including what we might recognize as mechanical arms, wrists, and hands [28]. An example of an anthropomorphic robot is shown in Figure 1.12. We recognize that the automatic machine is well suited to some tasks, as noted in Table 1.2, and that other tasks are best carried out by humans [106]. ■

EXAMPLE 1.4 Electric power industry

There has been considerable discussion recently concerning the gap between practice and theory in control engineering. However, it is natural that theory precedes the applications in many fields of control engineering. Nonetheless, it is interesting to note that in the electric power industry, the largest industry in the United States, the gap is relatively insignificant. The electric power industry is primarily interested in energy conversion, control, and distribution. It is critical that computer control be increasingly applied to the power industry in order to improve the efficient use of energy resources. Also, the control of power **plants** for minimum waste emission has become increasingly important. The modern, large-capacity plants, which exceed several hundred megawatts, require automatic control systems that account for the interrelationship of the process variables and optimum power production. It is common to have 90 or more manipulated variables under coordinated control. A simplified model showing several of the important control variables of a large boiler–generator system is shown in Figure 1.13. This is an example of the importance of measuring many variables, such as pressure and oxygen, to provide information to the computer for control calculations. ■

The electric power industry has used the modern aspects of control engineering for significant and interesting applications. It appears that in the process industry, the factor that maintains the applications gap is the lack of instrumentation to measure all the important process variables, including the quality and composition of

FIGURE 1.12 The Honda ASIMO humanoid robot. ASIMO walks, climbs stairs, and turns corners. (David Coll Blanco/ Alamy Stock Photo)

Table 1.2 Task Difficulty: Human Versus Automatic Machine

Tasks Difficult for a Machine	Tasks Difficult for a Human
Displaying real emotions	Operating in toxic environments
Acting based on ethical principles	Highly repetitive activities
Precise coordination with other robots	Deep underwater surveys
Anticipating human actions and responses	Outer planet space exploration
Acquiring new skills on its own	Working diligently with no breaks for long periods

the product. As these instruments become available, the applications of modern control theory to industrial systems should increase measurably.

EXAMPLE 1.5 **Biomedical engineering**

There have been many applications of control system theory to biomedical experimentation, diagnosis, prosthetics, and biological control systems [22, 23, 48]. The control systems under consideration range from the cellular level to the central nervous system and include temperature regulation and neurological, respiratory, and cardiovascular control. Most physiological control systems are closed-loop systems. However, we find not one controller but rather control loop within control loop, forming a hierarchy of systems. The modeling of the structure of biological processes confronts the analyst with a high-order model and a complex structure. Prosthetic devices aid millions of people worldwide. Recent advances in feedback control

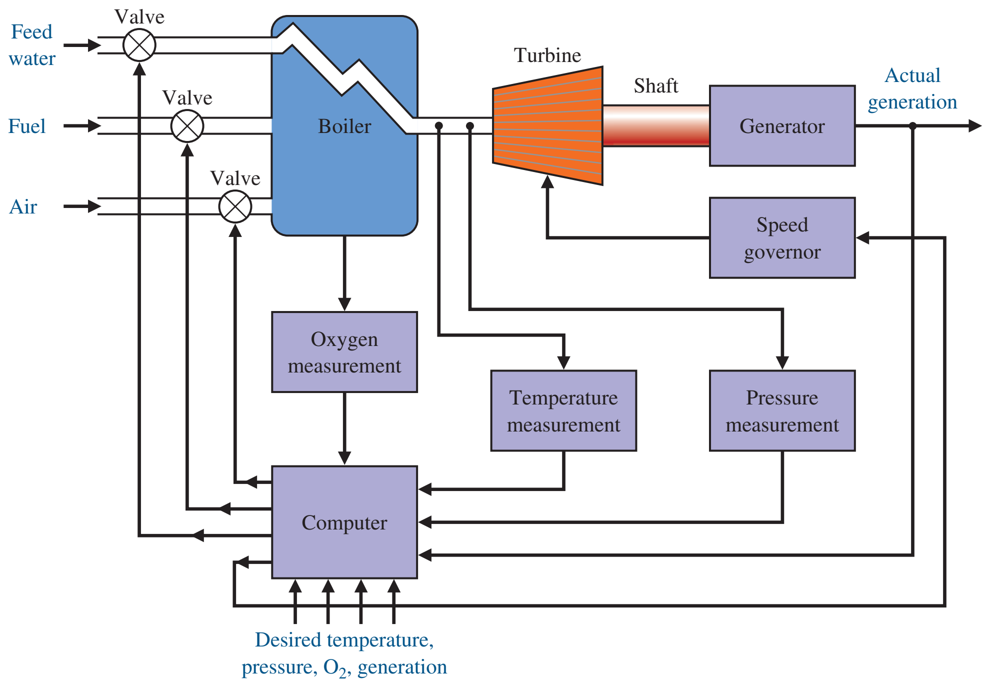


FIGURE 1.13 Coordinated control system for a boiler–generator.

technology will profoundly transform the lives of amputees and people living with paralysis. Much progress has been made in the restoration of sensation of touch and pain and in connecting prosthetic limb sensors with haptic feedback directly back to the brain. Figure 1.14 depicts a prosthetic had and arm with the same dexterity as a human arm. Especially fascinating are advances in brain-controlled feedback of prosthetic limbs enabling the power of the human brain to guide the movement [39]. Another fascinating advance in the development of prosthetic limbs is to make possible the sense of touch and pain [22]. Much progress has been made in the restoration of sensation of touch and pain and in connecting sensors to the prosthetic limbs with haptic feedback directly back to the brain. ■

EXAMPLE 1.6 **Social, economic, and political systems**

It is interesting and valuable to attempt to model the feedback processes prevalent in the social, economic, and political spheres. This approach is undeveloped at present but appears to have a bright future. Society is composed of many feedback systems and regulatory bodies, which are controllers exerting the forces on society necessary to maintain a desired output. A simple lumped model of the national income feedback control system is shown in Figure 1.15. This type of model helps the analyst to understand the effects of government control and the dynamic effects of government spending. Of course, many other loops not shown also exist, since, theoretically, government spending cannot exceed the tax collected without generating a deficit, which is itself a control loop containing the Internal Revenue Service and the Congress. In a socialist country, the loop due to consumers is deemphasized and government control is emphasized. In that case, the measurement

FIGURE 1.14
Recent advances in electronic prosthetics have resulted in the development of a prosthetic hand and arm that has the same dexterity as a human arm. (Kuznetsov Dmitriy/Shutterstock).

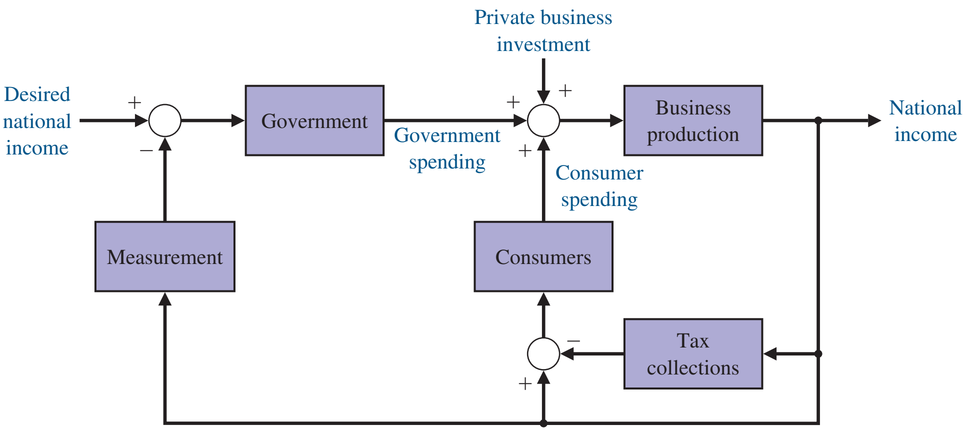


FIGURE 1.15
A feedback control system model of the national income.

block must be accurate and must respond rapidly; both are very difficult characteristics to realize from a bureaucratic system. This type of political or social feedback model, while usually nonrigorous, does impart information and understanding. ■

EXAMPLE 1.7 **Unmanned aerial vehicles**

The ongoing area of research and development of unmanned aerial vehicles (UAVs) is full of potential for the application of control systems. These aircrafts are also known as drones. An example of a drone is shown in Figure 1.16. Drones are unmanned but are usually controlled by ground operators. Typically they do not operate autonomously, and their inability to provide the level of safety required in a complex airspace keeps them from flying freely in the commercial airspace although package delivery via drones has begun. One significant challenge is to develop control systems that will avoid in-air collisions. Ultimately, the goal is to employ the drone autonomously in

FIGURE 1.16
A commercial drone (GuruXOX/ Shuttterstock).

such applications as aerial photography to assist in disaster mitigation, survey work to assist in construction projects, crop monitoring, and continuous weather monitoring. An intriguing emerging area of applied research is the integration of artificial intelligence (AI) and drones [74]. Smart unmanned aircraft will require significant deployment of advanced control systems throughout the airframe. ■

EXAMPLE 1.8 **Industrial control systems**

Other familiar control systems have the same basic elements as the system shown in Figure 1.3. A refrigerator has a temperature setting or desired temperature, a thermostat to measure the actual temperature and the error, and a compressor motor for power amplification. Other examples in the home are the oven, furnace, and water heater. In industry, there are many examples, including speed controls; process temperature and pressure controls; and position, thickness, composition, and quality controls [17, 18].

Feedback control systems are used extensively in industrial applications. Thousands of industrial and laboratory robots are currently in use. Manipulators can pick up objects weighing hundreds of pounds and position them with an accuracy of one-tenth of an inch or better [28]. Automatic handling equipment for home, school, and industry is particularly useful for hazardous, repetitious, dull, or simple tasks. Machines that automatically load and unload, cut, weld, or cast are used by industry to obtain accuracy, safety, economy, and productivity [28, 41].

Another important industry, the metallurgical industry, has had considerable success in automatically controlling its processes. In fact, in many cases, the control theory is being fully implemented. For example, a hot-strip steel mill is controlled for temperature, strip width, thickness, and quality.

There has been considerable interest recently in applying the feedback control concepts to automatic warehousing and inventory control. Furthermore, automatic control of agricultural systems (farms) is receiving increased interest. Automatically controlled silos and tractors have been developed and tested. Automatic control of wind turbine generators, solar heating and cooling, and automobile engine performance are important modern examples [20, 21]. ■

1.4 ENGINEERING DESIGN

Engineering design is the central task of the engineer. It is a complex process in which both creativity and analysis play major roles.

> **Design is the process of conceiving or inventing the forms, parts, and details of a system to achieve a specified purpose.**

Design activity can be thought of as planning for the emergence of a particular product or system. Design is an innovative act whereby the engineer creatively uses knowledge and materials to specify the shape, function, and material content of a system. The design steps are (1) to determine a need arising from the values of various groups, covering the spectrum from public policy makers to the consumer; (2) to specify in detail what the solution to that need must be and to embody these values; (3) to develop and evaluate various alternative solutions to meet these specifications; and (4) to decide which one is to be designed in detail and fabricated.

An important factor in realistic design is the limitation of time. Design takes place under imposed schedules, and we eventually settle for a design that may be less than ideal but considered "good enough." In many cases, time is the *only* competitive advantage.

A major challenge for the designer is writing the specifications for the technical product. **Specifications** are statements that explicitly state what the device or product is to be and do. The design of technical systems aims to provide appropriate design specifications and rests on four characteristics: complexity, trade-offs, design gaps, and risk.

Complexity of design results from the wide range of tools, issues, and knowledge to be used in the process. The large number of factors to be considered illustrates the complexity of the design specification activity, not only in assigning these factors their relative importance in a particular design, but also in giving them substance either in numerical or written form, or both.

The concept of **trade-off** involves the need to resolve conflicting design goals, all of which are desirable. The design process requires an efficient compromise between desirable but conflicting criteria.

In making a technical device, we generally find that the final product does not appear as originally visualized. For example, our image of the problem we are solving does not appear in written description and ultimately in the specifications. Such **design gaps** are intrinsic in the progression from an abstract idea to its realization.

This inability to be absolutely sure about predictions of the performance of a technological object leads to major uncertainties about the actual effects of the designed devices and products. These uncertainties are embodied in the idea of unintended consequences or **risk**. The result is that designing a system is a risk-taking activity.

Complexity, trade-off, gaps, and risk are inherent in designing new systems and devices. Although they can be minimized by considering all the effects of a given design, they are always present in the design process.

Within engineering design, there is a fundamental difference between the two major types of thinking that must take place: engineering **analysis** and **synthesis**. Attention is focused on models of the physical systems that are analyzed to provide insight and that indicate directions for improvement. On the other hand, synthesis is the process by which these new physical configurations are created.

Design is a process that may proceed in many directions before the desired one is found. It is a deliberate process by which a designer creates something new in response to a recognized need while recognizing realistic constraints. The design process is inherently iterative—we must start somewhere! Successful engineers learn to simplify complex systems appropriately for design and analysis purposes. A gap between the complex physical system and the design model is inevitable. Design gaps are intrinsic in the progression from the initial concept to the final product. We know intuitively that it is easier to improve an initial concept incrementally than to try to create a final design at the start. In other words, engineering design is not a linear process. It is an iterative, nonlinear, creative process.

The main approach to the most effective engineering design is parameter analysis and optimization. Parameter analysis is based on (1) identification of the key parameters, (2) generation of the system configuration, and (3) evaluation of how well the configuration meets the needs. These three steps form an iterative loop. Once the key parameters are identified and the configuration synthesized, the designer can **optimize** the parameters. Typically, the designer strives to identify a limited set of parameters to be adjusted.

1.5 CONTROL SYSTEM DESIGN

The design of control systems is a specific example of engineering design. The goal of control engineering design is to obtain the configuration, specifications, and identification of the key parameters of a proposed system to meet an actual need.

The control system design process is illustrated in Figure 1.17. The design process consists of seven main building blocks, which we arrange into three groups:

1. Establishment of goals and variables to be controlled, and definition of specifications (metrics) against which to measure performance.
2. System definition and modeling.
3. Control system design and integrated system simulation and analysis.

In each chapter of this book, we will highlight the connection between the design process illustrated in Figure 1.17 and the main topics of that chapter. The objective is to demonstrate different aspects of the design process through illustrative examples. We have established the following connections between the chapters in this book and the design process block diagram:

1. Establishment of goals, control variables, and specifications: Chapters 1, 3, 4, and 13.
2. System definition and modeling: Chapters 2–4, and 11–13.
3. Control system design, simulation, and analysis: Chapters 4–13.

The first step in the design process consists of establishing the system goals. For example, we may state that our goal is to control the velocity of a motor accurately. The second step is to identify the variables that we desire to control (for example, the velocity of the motor). The third step is to write the specifications in terms of the accuracy we must attain. This required accuracy of control will then lead

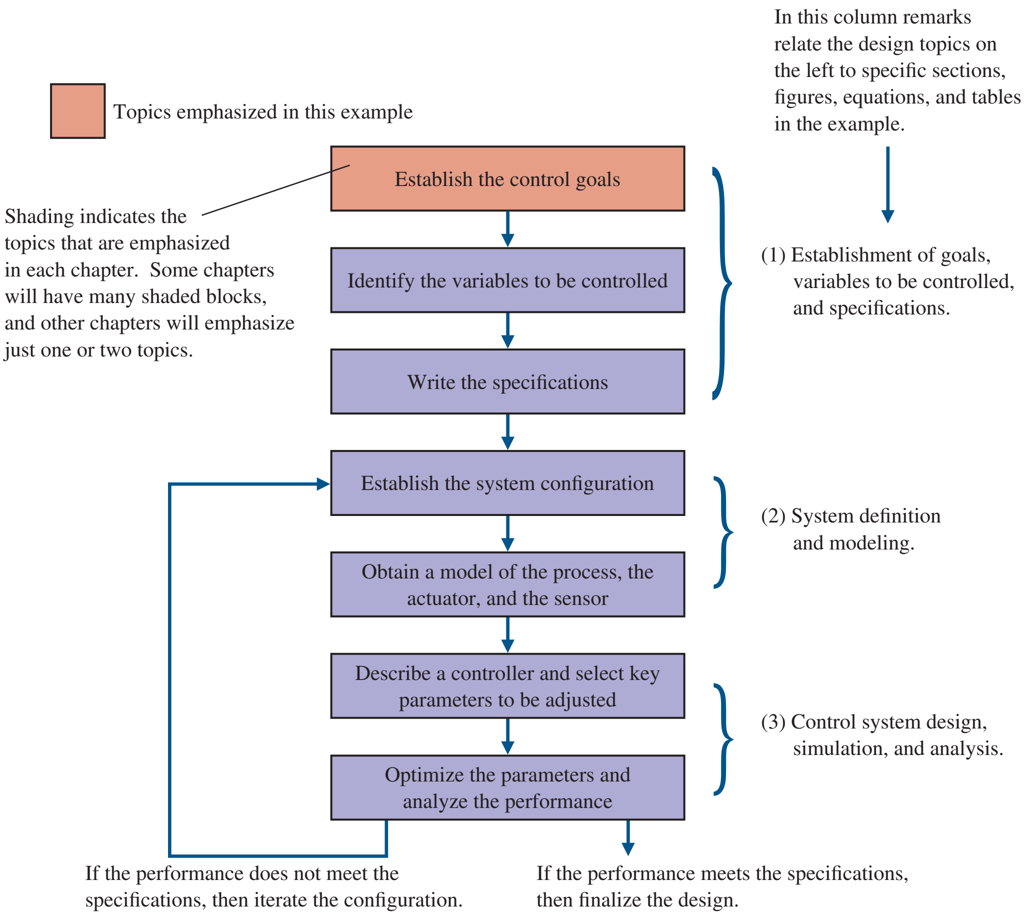


FIGURE 1.17 The control system design process.

to the identification of a sensor to measure the controlled variable. The performance specifications will describe how the closed-loop system should perform and will include (1) good regulation against disturbances, (2) desirable responses to commands, (3) realistic actuator signals, (4) low sensitivities, and (5) robustness.

As designers, we proceed to the first attempt to configure a system that will result in the desired control performance. This system configuration will normally consist of a sensor, the process under control, an actuator, and a controller, as shown in Figure 1.3. The next step consists of identifying a candidate for the actuator. This will, of course, depend on the process, but the actuation chosen must be capable of effectively adjusting the performance of the process. For example, if we wish to control the speed of a rotating flywheel, we will select a motor as the actuator. The sensor, in this case, must be capable of accurately measuring the speed. We then obtain a model for each of these elements.

Students studying controls are often given the models, frequently represented in transfer function or state variable form, with the understanding that they represent the underlying physical systems, but without further explanation. An obvious question is, where did the transfer function or state variable model come from? Within the context of a course in control systems, there is a need to address key questions surrounding modeling. To that end, in the early chapters, we will provide insight into key modeling concerns and answer fundamental questions: How is the transfer function obtained? What basic assumptions are implied in the model development? How general are the transfer functions? However, mathematical modeling of physical systems is a subject in and of itself. We cannot hope to cover the mathematical modeling in its entirety, but interested students are encouraged to seek outside references (see, for example, [76–80]).

The next step is the selection of a controller, which often consists of a summing amplifier that will compare the desired response and the actual response and then forward this error-measurement signal to an amplifier.

The final step in the design process is the adjustment of the parameters of the system to achieve the desired performance. If we can achieve the desired performance by adjusting the parameters, we will finalize the design and proceed to document the results. If not, we will need to establish an improved system configuration and perhaps select an enhanced actuator and sensor. Then we will repeat the design steps until we are able to meet the specifications, or until we decide the specifications are too demanding and should be relaxed.

The design process has been dramatically affected by the advent of powerful and inexpensive computers, and effective control design and analysis software. For example, the Boeing 777 was the world's first 100% digitally designed civilian aircraft.The benefits of this design approach to Boeing was a 50% saving in development costs, a 93% reduction in design change and rework rate, and a 50–80% reduction in problems compared with traditional manufacturing [56]. The follow-on project, known as the Boeing 787 Dreamliner, was developed without physical prototypes. In many applications, the availability of digital design tools, including the certification of the control system in realistic computer simulations, represents a significant cost reduction in terms of money and time.

Another notable innovation in design is the generative design process coupled with artificial intelligence [57]. Generative design is an iterative design process that typically utilizes a computer program to generate a (potentially large) number of designs based on a given set of constraints provided by the designer. The designer then fine-tunes the feasible solution provided by the computer program by adjusting the constraint space to reduce the number of viable solutions. For example, the generative design is revolutionizing aircraft design [58]. The application of the highly computer-intensive generative design process in feedback control theory remains an open question. However, the generative design process concept can also be applied in a more traditional (less computationally intensive) environment to enhance the design process in Figure 1.17. For example, once a single design has been found that meets the specifications, the process can be repeated by selecting different system configurations and controller structures. After a number of controllers are designed that meet the specifications, the designer can then begin to narrow the design by adjusting the constraints. There are facets of the generative design process that will be illuminated in this book as we discuss the control system design process.

In summary, the controller design problem is as follows: Given a model of the system to be controlled (including its sensors and actuators) and a set of design goals, find a suitable controller, or determine that none exists. As with most of engineering design, the design of a feedback control system is an iterative and nonlinear process. A successful designer must consider the underlying physics of the plant under control, the control design strategy, the controller design architecture (that is, what type of controller will be employed), and effective controller tuning strategies. In addition, once the design is completed, the controller is often implemented in hardware, and hence issues of interfacing with hardware can appear. When taken together, these different phases of control system design make the task of designing and implementing a control system quite challenging [73].

1.6 MECHATRONIC SYSTEMS

A natural stage in the evolutionary process of modern engineering design is encompassed in the area known as **mechatronics** [64]. The term mechatronics was coined in Japan in the 1970s [65–67]. Mechatronics is the synergistic integration of mechanical, electrical, and computer systems and has evolved over the past 30 years, leading to a new breed of intelligent products. Feedback control is an integral aspect of modern mechatronic systems. One can understand the extent that mechatronics reaches into various disciplines by considering the components that make up mechatronics [68–71]. The key elements of mechatronics are (1) physical systems modeling, (2) sensors and actuators, (3) signals and systems, (4) computers and logic systems, and (5) software and data acquisition. Feedback control encompasses aspects of all five key elements of mechatronics, but is associated primarily with the element of signals and systems, as illustrated in Figure 1.18.

Advances in computer hardware and software technology coupled with the desire to increase the performance-to-cost ratio has revolutionized engineering design. New products are being developed at the intersection of traditional disciplines of engineering, computer science, and the natural sciences. Advancements in traditional disciplines are fueling the growth of mechatronics systems by providing

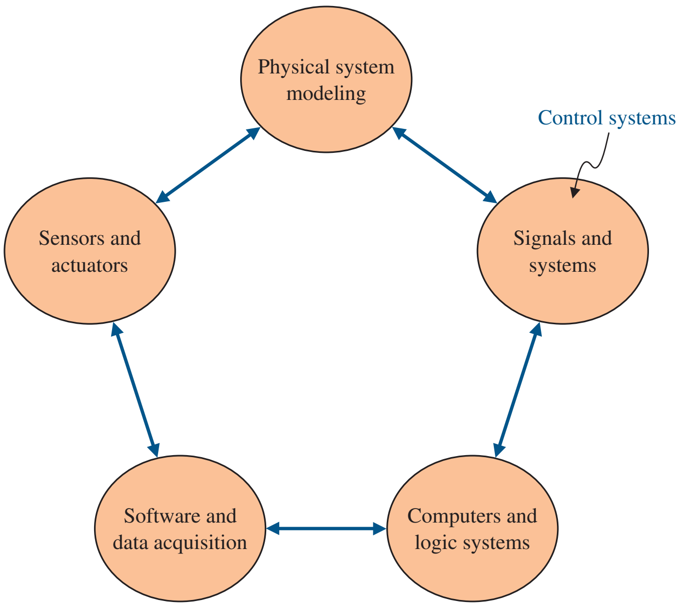


FIGURE 1.18
The key elements of mechatronics [64].

"enabling technologies." A critical enabling technology was the microprocessor which has had a profound effect on the design of consumer products. We should expect continued advancements in cost-effective microprocessors and microcontrollers, novel sensors and actuators enabled by advancements in applications of microelectromechanical systems (MEMS), advanced control methodologies and real-time programming methods, networking and wireless technologies, and mature computer-aided engineering (CAE) technologies for advanced system modeling, virtual prototyping, and testing. The continued rapid development in these areas will only accelerate the pace of smart (that is, actively controlled) products.

An exciting area of mechatronic system development in which control systems will play a significant role is the area of alternative energy production and consumption. Hybrid fuel automobiles and efficient wind power generation are two examples of systems that can benefit from mechatronic design methods. In fact, the mechatronic design philosophy can be effectively illustrated by the example of the evolution of the modern automobile [64]. Before the 1960s, the radio was the only significant electronic device in an automobile. Today, many automobiles have many microcontrollers, and a multitude of sensors, and thousands of lines of software code. A modern automobile can no longer be classified as a strictly mechanical machine—it has been transformed into a comprehensive mechatronic system.

EXAMPLE 1.9 **Hybrid fuel vehicles**

A **hybrid fuel automobile**, depicted in Figure 1.19, utilizes a conventional internal combustion engine in combination with a battery (or other energy storage device such as a fuel cell or flywheel) and an electric motor to provide a propulsion system capable of doubling the fuel economy over conventional automobiles. Although these hybrid vehicles will never be zero-emission vehicles (since they have internal combustion engines), they can reduce the level of harmful emissions by one-third to one-half, and with future improvements, these emissions may reduce even further. As stated earlier, the modern automobile requires many advanced control systems to operate. The control systems must regulate the performance of

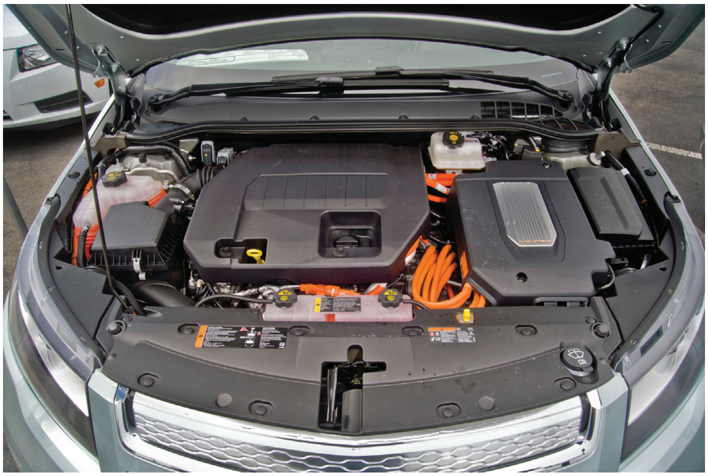


FIGURE 1.19 The hybrid fuel automobile can be viewed as a mechatronic system. (Marmaduke St. John/Alamy Stock Photo.)

the engine, including fuel–air mixtures, valve timing, transmissions, wheel traction control, antilock brakes, and electronically controlled suspensions, among many other functions. On the hybrid fuel vehicle, there are additional control functions that must be satisfied. Especially necessary is the control of power between the internal combustion engine and the electric motor, determining power storage needs and implementing the battery charging, and preparing the vehicle for low-emission start-ups. The overall effectiveness of the hybrid fuel vehicle depends on the combination of power units that are selected (e.g., battery versus fuel cell for power storage). Ultimately, however, the control strategy that integrates the various electrical and mechanical components into a viable transportation system strongly influences the acceptability of the hybrid fuel vehicle concept in the marketplace. ■

The second example of a mechatronic system is the advanced wind power generation system.

EXAMPLE 1.10 **Wind power**

Many nations in the world today are faced with unstable energy supplies. Additionally, the negative effects of fossil fuel utilization on the quality of our air are well documented. Many nations have an imbalance in the supply and demand of energy, consuming more than they produce. To address this imbalance, many engineers are considering developing advanced systems to access other sources of energy, such as wind energy. In fact, wind energy is one of the fastest-growing forms of energy generation in the United States and in other locations around the world. A wind farm is illustrated in Figure 1.20.

By the end of 2019, the installed global wind energy capacity was over 650.8 GW. In the United States, there was enough energy derived from wind to power over 27.5 million homes, according to the American Wind Energy Association. For the past 40 years, researchers have concentrated on developing technologies that work well in high wind areas (defined to be areas with a wind speed of at least 6.7 m/s at a height of 10 m). Most of the easily accessible high wind sites in the United States are now utilized, and improved technology must be developed to make lower wind areas more cost effective. New developments are required in materials and

FIGURE 1.20 Efficient wind power generation. (Photo courtesy of NASA)

aerodynamics so that longer turbine rotors can operate efficiently in the lower winds, and in a related problem, the towers that support the turbine must be made taller without increasing the overall costs. In addition, advanced controls will be required to achieve the level of efficiency required in the wind generation drive train. Newer wind turbines can operate in wind speeds less than 1 mph. ■

EXAMPLE 1.11 Wearable computers

Many contemporary control systems are **embedded control** systems [81]. Embedded control systems employ on-board special-purpose digital computers as integral components of the feedback loop. Many new wearable products include embedded computers. This includes wristwatches, eyeglasses, sports wristbands, e-textiles, and computer garments. Figure 1.21 illustrates the popular computer eyeglasses. For example, the glasses devices might enable physicians to access and manage data and display the data when they need it during a patient examination. One might imagine future applications where the device would monitor and track the doctor's eye movements and use that information in a feedback loop to very precisely control a medical instrument during a procedure. The utilization of wearable computers in feedback control applications is in its infancy and the possibilities are enormous. ■

Advances in sensors, actuators, and communication devices are leading to a new class of embedded control systems that are networked using wireless technology, thereby enabling spatially-distributed control. Embedded control system designers must be able to understand and work with various network protocols, diverse operating systems and programming languages. While the theory of systems and controls serves as the foundation for the modern control system design, the design process is rapidly expanding into a multi-disciplinary enterprise encompassing multiple engineering areas, as well as information technology and computer science.

Advances in alternate energy products, such as the hybrid automobile and the generation of efficient wind power generators, provide vivid examples of mechatronics development. There are numerous other examples of intelligent systems poised to enter our everyday life, including autonomous rovers, smart home appliances (e.g., dishwashers, vacuum cleaners, and microwave ovens), wireless network-enabled devices, "human-friendly machines" [72] that perform robot-assisted surgery, and implantable sensors and actuators.

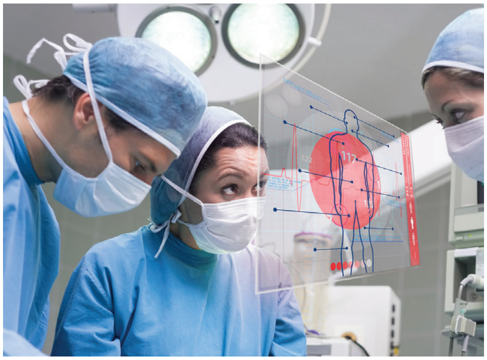


FIGURE 1.21 Wearable computers can assist a physician provide better healthcare delivery. (Wavebreak Media Ltd/123RF.)

1.7 GREEN ENGINEERING

Global issues such as climate change, clean water, sustainability, waste management, emissions reduction, and minimizing raw material and energy use have caused many engineers to re-think existing approaches to engineering design in critical areas. One outcome of the evolving design strategy is to consider an approach that has come to be known as "green engineering." The goal of green engineering is to design products that will minimize pollution, reduce the risk to human health, and improve the environment. The basic principles of green engineering are [86]:

1. Engineer processes and products holistically, use systems analysis, and integrate environmental impact assessment tools.
2. Conserve and improve natural ecosystems while protecting human health and well-being.
3. Use life-cycle thinking in all engineering activities.
4. Ensure that all material and energy inputs and outputs are as inherently safe and benign as possible.
5. Minimize depletion of natural resources.
6. Strive to prevent waste.
7. Develop and apply engineering solutions, while being cognizant of local geography, aspirations, and cultures.
8. Create engineering solutions beyond current or dominant technologies; improve, innovate, and invent technologies to achieve sustainability.
9. Actively engage communities and stakeholders in development of engineering solutions.

Putting the principles of green engineering into practice leads us to a deeper understanding of the power of feedback control systems as an enabling technology. For example, in Section 1.9, we present a discussion on smart grids. Smart grids aim to deliver electrical power more reliably and efficiently in an environmentally friendly fashion. This in turn will potentially enable the large-scale use of renewable energy sources, such as wind and solar, that are naturally intermittent. Sensing and feedback are key technology areas that enable the smart grids [87]. Green engineering applications can be classified into one of five categories [88]:

1. Environmental Monitoring
2. Energy Storage Systems
3. Power Quality Monitoring
4. Solar Energy
5. Wind Energy

As the field of green engineering matures, it is almost certain that more applications will evolve, especially as we apply the eighth principle (listed above) of green engineering to create engineering solutions beyond current or dominant technologies and improve, innovate, and invent technologies. In the subsequent chapters, we present examples from each of these areas.

There is a global effort underway to reduce greenhouse gases from all sources. To accomplish this, it is necessary to improve both the quality and quantity of our environmental monitoring systems. An example is using wireless measurements on a cabled robotic controlled mobile sensing platform moving along the forest understory to measure key environmental parameters in a rain forest.

Energy storage systems are critical technologies for green engineering. There are many types of energy storage systems. The energy storage system we are most familiar with is the battery. Batteries are used to power most of the electronic devices in use today; some batteries are rechargeable and some are single-use throwaways. To adhere to green engineering principles, we would favor energy storage systems that are renewable. A very important energy storage device for green engineering systems is the fuel cell.

The problems associated with power quality monitoring are varied and can include leading and lagging power, voltage variations, and waveform harmonics. Many of the green engineering systems and components require careful monitoring of current and voltages. An interesting example would be the modeling of current transformers that are used in various capacities for measuring and monitoring within the power grid network of interconnected systems used to deliver electricity.

Efficiently converting solar energy into electricity is an engineering challenge. Two technologies for generation of electricity using sunshine are solar photovoltaic and solar thermal. With photovoltaic systems the sunlight is converted directly to electricity, and with solar thermal the sun heats water to create steam that is used to power steam engines. Designing and deploying solar photovoltaic systems for solar power generation is one approach employing green engineering principles to utilize the sun's energy to power our homes, offices, and businesses.

Power derived from wind is an important source of renewable energy around the world. Wind energy conversion to electric power is achieved by wind energy turbines connected to electric generators. The intermittency characteristic of wind energy makes the smart grid development essential to bring the energy to the power grid when it is available and to provide energy from other sources when the wind dies down or is disrupted. The irregular character of wind direction and power also results in the need for reliable, steady electric energy by using control systems on the wind turbines themselves. The goal of these control devices is to reduce the effects of wind intermittency and the effect of wind direction change.

The role of control systems in green engineering will continue to expand as the global issues facing us require ever increasing levels of automation and precision.

1.8 THE FUTURE EVOLUTION OF CONTROL SYSTEMS

The continuing goal of control systems is to provide extensive flexibility and a high level of autonomy. Two system concepts are approaching this goal by different evolutionary pathways, as illustrated in Figure 1.22. Today's industrial robot is perceived as quite autonomous—once it is programmed, further intervention is not normally required. Because of sensory limitations, these robotic systems have limited flexibility in adapting to work environment changes; improving perception is the motivation of computer vision research. The control system is very adaptable,

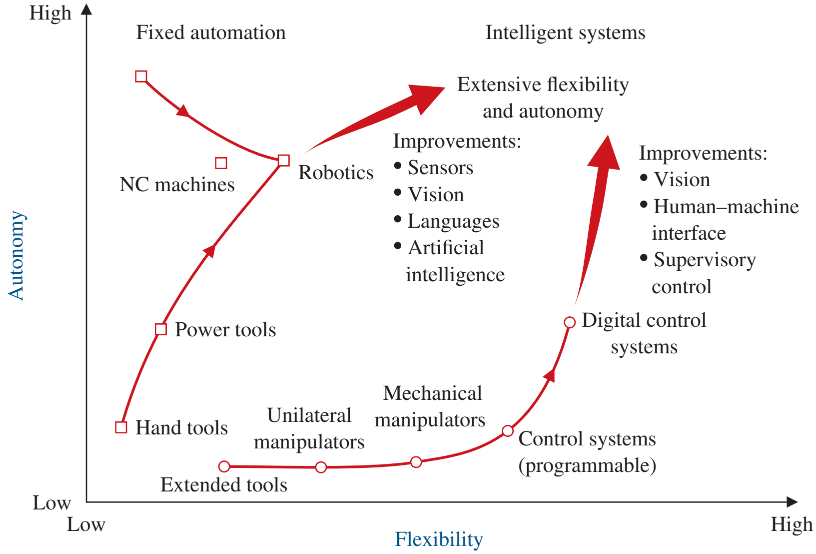


FIGURE 1.22 Evolution of control systems and autonomy.

but it relies on human supervision. Advanced robotic systems are striving for task adaptability through enhanced sensory feedback. Research areas concentrating on artificial intelligence, sensor integration, computer vision, and off-line CAD/CAM programming will make systems more universal and economical. Control systems are moving toward autonomous operation as an enhancement to human control. Research in supervisory control, human–machine interface methods, and computer database management are intended to reduce operator burden and improve operator efficiency. Many research activities are common to robotics and control systems and are aimed at reducing implementation cost and expanding the realm of application. These include improved communication methods and advanced programming languages.

The easing of human labor by technology, a process that began in prehistory, is entering a new stage. The acceleration in the pace of technological innovation inaugurated by the Industrial Revolution has until recently resulted mainly in the displacement of human muscle power from the tasks of production. The current revolution in computer technology is causing an equally momentous social change, the expansion of information gathering and information processing as computers extend the reach of the human brain [16].

Control systems are used to achieve (1) increased productivity and (2) improved performance of a device or system. Automation is used to improve productivity and obtain high-quality products. Automation is the automatic operation or control of a process, device, or system. We use automatic control of machines and processes to produce a product reliably and with high precision [28]. With the demand for flexible, custom production, a need for flexible automation and robotics is growing [17, 25].

The theory, practice, and application of automatic control is a large, exciting, and extremely useful engineering discipline. One can readily understand the motivation for a study of modern control systems.

1.9 DESIGN EXAMPLES

In this section we present illustrative design examples. This is a pattern that we will follow in all subsequent chapters. Each chapter will contain a number of interesting examples in a special section entitled Design Examples meant to highlight the main topics of the chapter. At least one example among those presented in the Design Example section will be a more detailed problem and solution that demonstrates one or more of the steps in the design process shown in Figure 1.17. In the first example, we discuss the development of the smart grid as a concept to deliver electrical power more reliably and efficiently as part of a strategy to provide a more environmentally friendly energy delivery system. The smart grid will enable the large-scale use of renewable energy sources that depend on the natural phenomenon to generate power and which are intermittent, such as wind and solar. Providing clean energy is an engineering challenge that must necessarily include active feedback control systems, sensors, and actuators. In the second example presented here, a rotating disk speed control illustrates the concept of open-loop and closed-loop feedback control. The third example is an insulin delivery control system in which we determine the design goals, the variables to control, and a preliminary closed-loop system configuration.

EXAMPLE 1.12 **Smart grid control systems**

A smart grid is as much a concept as it is a physical system. In essence, the concept is to deliver power more reliably and efficiently while remaining environmentally friendly, economical, and safe [89, 90]. A smart grid can be viewed as a system comprised of hardware and software that routes power more reliably and efficiently to homes, businesses, schools, and other users of power. One view of the smart grid is illustrated schematically in Figure 1.23. Smart grids can be national or local in scope. One can even consider home smart grids (or microgrids). In fact, smart grids encompass a wide and rich field of investigation. As we will find, control systems play a key role in smart grids at all levels.

One interesting aspect of the smart grid is real-time demand side management requiring a two-way flow of information between the user and the power generation system [91]. For example, smart meters are used to measure electricity use in the home and office. These sensors transmit data to utilities and allow the utility to transmit control signals back to a home or building. These smart meters can control and turn on or off home and office appliances and devices. Smart home-energy devices enable the homeowners to control their usage and respond to price changes at peak-use times.

The five key technologies required to implement a successful modern smart grid include (i) integrated communications, (ii) sensing and measurements, (iii) advanced components, (iv) advanced control methods, and (v) improved interfaces and decision support [87]. Two of the five key technologies fall under the general category of control systems, namely (ii) sensing and measurements and (iii) advanced control methods. It is evident that control systems will play a key role in realizing the modern smart grid. The potential impact of the smart grid on delivery of power is very high. Currently, the total U.S. grid includes 9,200 units generating over 1 million MW of capacity over 300,000 miles of transmission lines. A smart grid will use sensors,

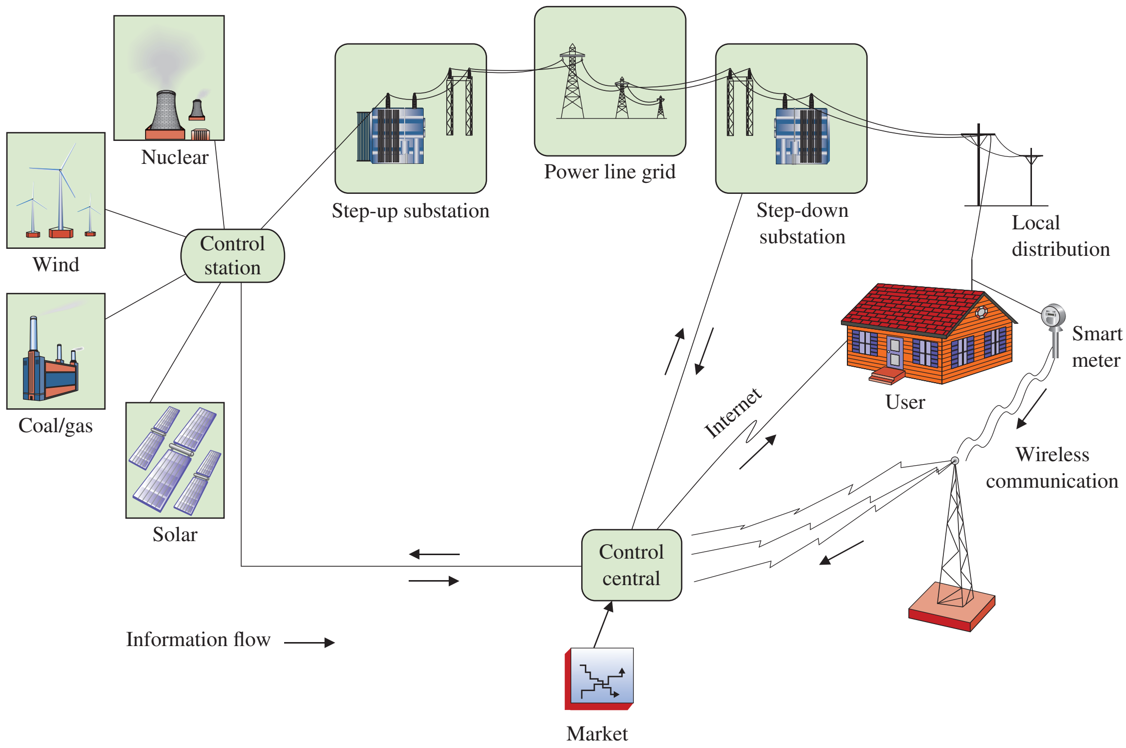


FIGURE 1.23 Smart grids are distribution networks that measure and control usage.

controllers, the Internet, and communication systems to improve the reliability and efficiency of the grid. It is estimated that deployment of smart grids could reduce emissions of CO_2 by 12 percent by 2030 [91].

One of the elements of the smart grid are the distribution networks that measure and control usage. In a smart grid, the power generation depends on the market situation (supply/demand and cost) and the power source available (wind, coal, nuclear, geothermal, biomass, etc.). In fact, smart grid customers with solar panels or wind turbines can sell their excess energy to the grid and get paid as microgenerators [92]. In the subsequent chapters, we discuss various control problems associated with pointing solar panels to the sun and with prescribing the pitch of the wind turbine blades to manage the rotor speed thereby controlling the power output.

Transmission of power is called power flow and the improved control of power will increase its security and efficiency. Transmission lines have inductive, capacitive, and resistive effects that result in dynamic impacts or disturbances. The smart grid must anticipate and respond to system disturbances rapidly. This is referred to as self-healing. In other words, a smart grid should be capable of managing significant disturbances occurring on very short time scales. To accomplish this, the self-healing process is constructed around the idea of a feedback control system where self-assessments are used to detect and analyze disturbances so that corrective action can be applied to restore the grid. This requires sensing and measurements to provide information to the control systems. One of the benefits of using smart grids is that renewable energy sources that depend on intermittent natural phenomena (such as wind and sunshine) can potentially be utilized more efficiently by allowing for load shedding when the wind dies out or clouds block the sunshine.

Feedback control systems will play an increasingly important role in the development of smart grids as we move to the target date. It may be interesting to recall the various topics discussed in this section in the context of control systems as each chapter in this textbook unfolds new methods of control system design and analysis. ■

EXAMPLE 1.13 **Rotating disk speed control**

Many modern devices employ a rotating disk held at a constant speed. For example, spinning disk conformal microscopes enable line-cell imaging in biomedical applications. Our goal is to design a system for rotating disk speed control that will ensure that the actual speed of rotation is within a specified percentage of the desired speed [40, 43]. We will consider a system without feedback and a system with feedback.

To obtain disk rotation, we will select a DC motor as the actuator because it provides a speed proportional to the applied motor voltage. For the input voltage to the motor, we will select an amplifier that can provide the required power.

The open-loop system (without feedback) is shown in Figure 1.24(a). This system uses a battery source to provide a voltage that is proportional to the desired speed. This voltage is amplified and applied to the motor. The block diagram of the open-loop system identifying the controller, actuator, and process is shown in Figure 1.24(b).

To obtain a feedback system, we need to select a sensor. One useful sensor is a tachometer that provides an output voltage proportional to the speed of its shaft. Thus the closed-loop feedback system takes the form shown in Figure 1.25(a). The block diagram model of the feedback system is shown in Figure 1.25(b). The error voltage is generated by the difference between the input voltage and the tachometer voltage.

We expect the feedback system of Figure 1.25 to be superior to the open-loop system of Figure 1.24 because the feedback system will respond to errors and act to

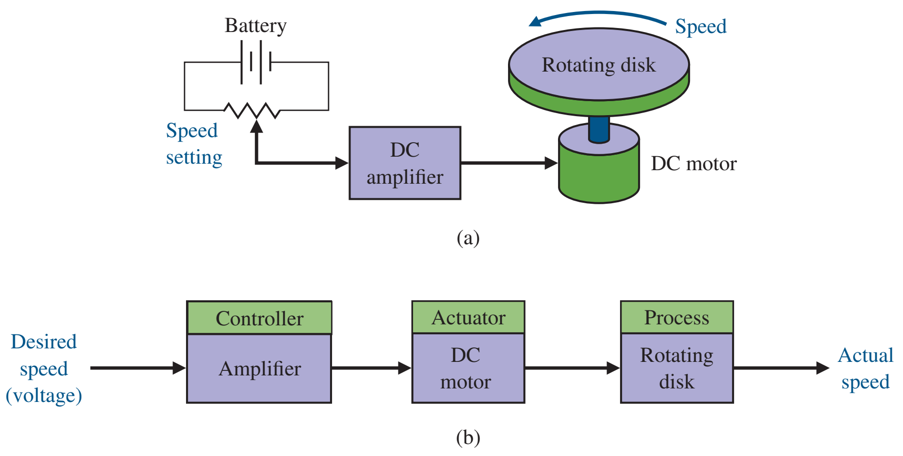


FIGURE 1.24
(a) Open-loop (without feedback) control of the speed of a rotating disk. (b) Block diagram model.

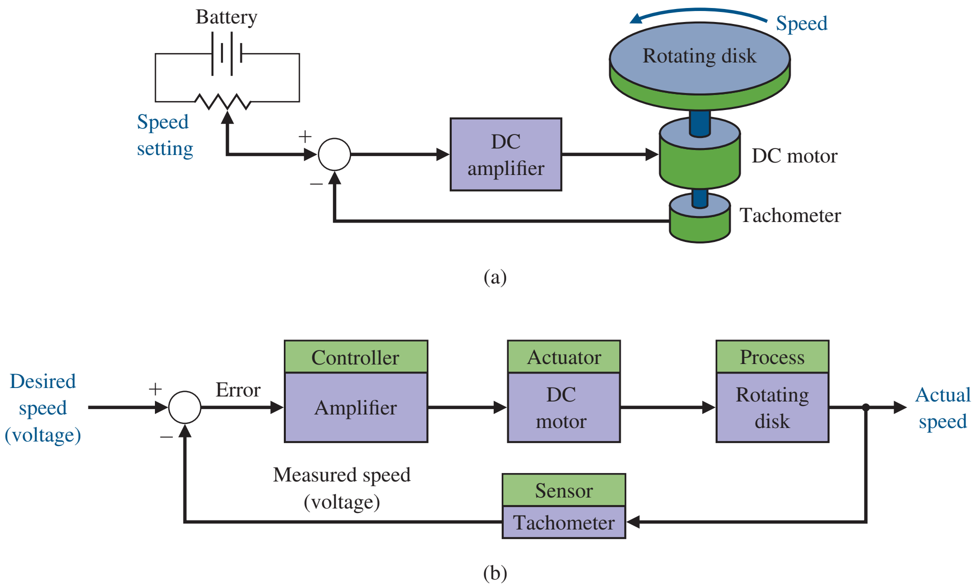


FIGURE 1.25
(a) Closed-loop control of the speed of a rotating disk. (b) Block diagram model.

reduce them. With precision components, we could expect to reduce the error of the feedback system to one-hundredth of the error of the open-loop system. ■

EXAMPLE 1.14 Insulin delivery control system

Control systems have been utilized in the biomedical field to create implanted automatic drug-delivery systems to patients [29–31]. Automatic systems can be used to regulate blood pressure, blood sugar level, and heart rate. A common application of control engineering is in the field of drug delivery in which mathematical models of the dose–effect relationship of the drugs are used. A drug-delivery system implanted in the body uses a closed-loop system since miniaturized glucose sensors are now available. The best solutions rely on individually programmable, pocket-sized insulin pumps that can deliver insulin.

The blood glucose and insulin concentrations for a healthy person are shown in Figure 1.26. The system must provide the insulin from a reservoir implanted within the diabetic person. Therefore, the control goal is:

Control Goal

Design a system to regulate the blood sugar concentration of a diabetic by controlled dispensing of insulin.

Referring to Figure 1.26, the next step in the design process is to define the variable to be controlled. Associated with the control goal we can define the variable to be controlled to be:

Variable to Be Controlled

Blood glucose concentration

In subsequent chapters, we will have the tools to quantitatively describe the control design specifications using a variety of steady-state performance

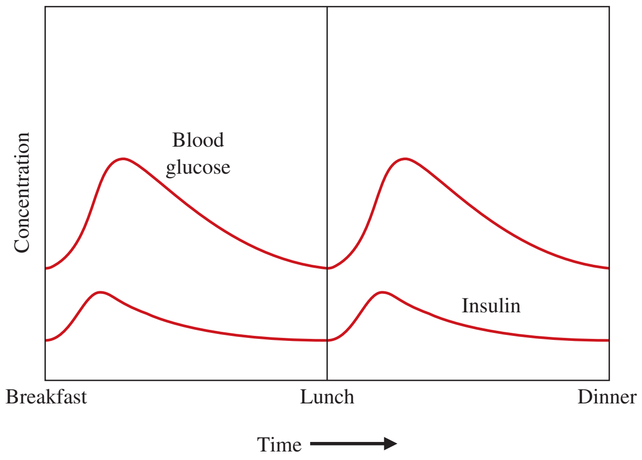


FIGURE 1.26 The blood glucose and insulin levels for a healthy person.

specifications and transient response specifications, both in the time-domain and in the frequency domain. At this point, the control design specifications will be qualitative and imprecise. In that regard, for the problem at hand, we can state the design specification as:

Control Design Specifications

Provide a blood glucose level for the diabetic that closely approximates (tracks) the glucose level of a healthy person.

Given the design goals, variables to be controlled, and control design specifications, we can now propose a preliminary system configuration. A closed-loop system uses a fully implantable glucose sensor and miniature motor pump to regulate the insulin delivery rate as shown in Figure 1.27. The feedback control system uses a sensor to measure the actual glucose level and compare that level with the desired level, thus turning the motor pump on when it is required. ■

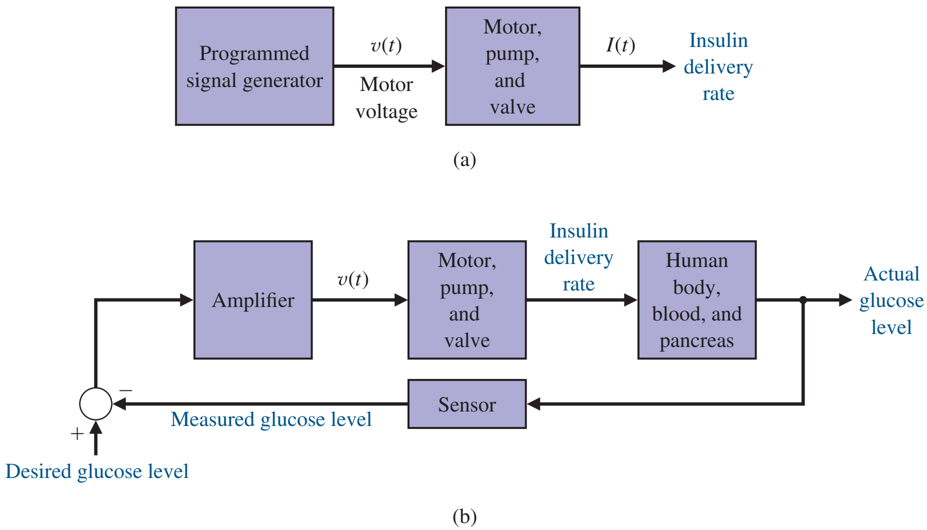
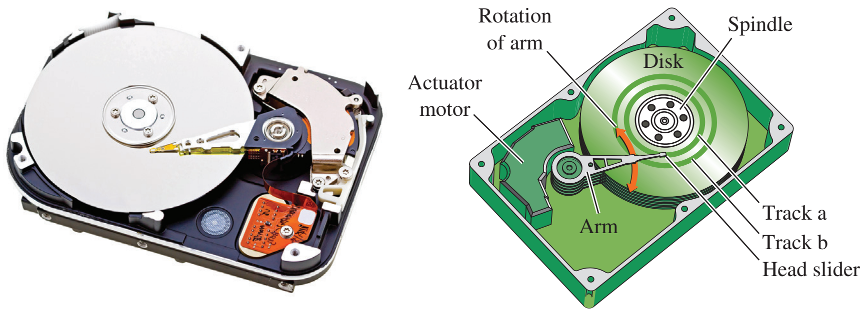


FIGURE 1.27 (a) Open-loop (without feedback) control and (b) closed-loop control of blood glucose.

1.10 SEQUENTIAL DESIGN EXAMPLE: DISK DRIVE READ SYSTEM

We will use the design process of Figure 1.17 in each chapter to identify the steps that we are accomplishing. For example, in Chapter 1 we (1) identify the control goal, (2) identify the variables to control, (3) write the initial specifications for the variables, and (4) establish the preliminary system configuration.

Information can be readily and efficiently stored on magnetic disks. Hard disk drives (HDD) are used in notebook computers and larger computers of all sizes and are essentially all standardized as defined by ANSI standards. Even with the advent of advanced storage technologies, such as cloud storage, flash memory, and solid-state drives (SSDs), the HDD remains an important storage media. The role of the HDD is changing from fast and primary storage to slow storage with enormous capacity [50]. The installation of SSD units are surpassing HDD units for the first time. The SSD units are known to have much better performance than HDD, however, the difference in cost per gigabyte ratio is about 6:1, and that is expected to remain that way until 2030. Among the many reasons to keep our interest in HDD units is that it is anticipated that about 90% of the required capacity for cloud computing applications will be realized with HHDs moving into the foreseeable future [51, 62]. In the past, disk drive designers have concentrated on increasing data density and data access times. Designers are now considering employing disk drives to perform tasks historically delegated to central processing units (CPUs), thereby leading to improvements in the computing environment [63]. Three areas of "intelligence" under investigation include off-line error recovery, disk drive failure warnings, and storing data across multiple disk drives. Consider the basic diagram of a disk drive shown in Figure 1.28. The goal of the disk drive reader device is to position the reader head to read the data stored on a track on the disk. The variable to accurately control is the position of the reader head (mounted on a slider device). The disk rotates at a speed between 1800 and 10,000 rpm, and the head "flies" above the disk at a distance of less than 100 nm. The initial specification for the position accuracy is 1 μm. Furthermore, we plan to be able to move the head from track a to track b within 50 ms, if possible. Thus, we establish an initial system configuration



(a) (b)

FIGURE 1.28 (a) A disk drive (Ragnarock/Shutterstock.) (b) Diagram of a disk drive.

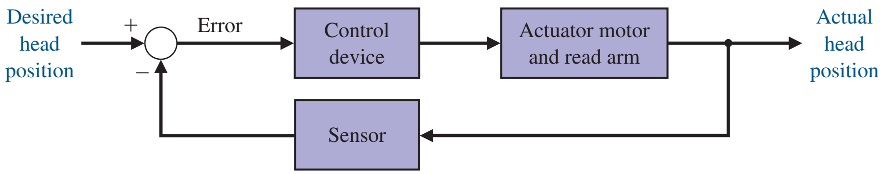


FIGURE 1.29 Closed-loop control system for disk drive.

as shown in Figure 1.29. This proposed closed-loop system uses a motor to actuate (move) the arm to the desired location on the disk. We will consider the design of the disk drive further in Chapter 2.

1.11 SUMMARY

In this chapter, we discussed open- and closed-loop feedback control systems. Examples of control systems through the course of history were presented to motivate and connect the subject to the past. In terms of contemporary issues, key areas of application were discussed, including humanoid robots, unmanned aerial vehicles, wind energy, hybrid automobiles, and embedded control. The central role of controls in mechatronics was discussed. Mechatronics is the synergistic integration of mechanical, electrical, and computer systems. Finally, the design process was presented in a structured form and included the following steps: the establishment of goals and variables to be controlled, definition of specifications, system definition, modeling, and analysis. The iterative nature of design allows us to handle the design gap effectively while accomplishing necessary trade-offs in complexity, performance, and cost.

SKILLS CHECK

In this section, we provide three sets of problems to test your knowledge: True or False, Multiple Choice, and Word Match. To obtain direct feedback, check your answers with the answer key provided at the conclusion of the end-of-chapter problems.

In the following **True or False** and **Multiple Choice** problems, circle the correct answer.

1. The flyball governor is generally agreed to be the first automatic feedback controller used in an industrial process. *True or False*
2. A closed-loop control system uses a measurement of the output and feedback of the signal to compare it with the desired input. *True or False*
3. Engineering synthesis and engineering analysis are the same. *True or False*
4. The block diagram in Figure 1.30 is an example of a closed-loop feedback system. *True or False*

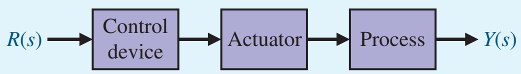


FIGURE 1.30 System with control device, actuator, and process.

5. A multivariable system is a system with more than one input and/or more than one output. *True or False*

6. Early applications of feedback control include which of the following?
   a. Water clock of Ktesibios
   b. Watt's flyball governor
   c. Drebbel's temperature regulator
   d. All of the above

7. Important modern applications of control systems include which of the following?
   a. Safe automobiles
   b. Autonomous robots
   c. Automated manufacturing
   d. All of the above

8. Complete the following sentence:
   Control of an industrial process by automatic rather than manual means is often called ________.
   a. negative feedback
   b. automation
   c. a design gap
   d. a specification

9. Complete the following sentence:
   ________ *are intrinsic in the progression from an initial concept to the final product.*
   a. Closed-loop feedback systems
   b. Flyball governors
   c. Design gaps
   d. Open-loop control systems

10. Complete the following sentence:
    Control engineers are concerned with understanding and controlling segments of their environments, often called ________.
    a. systems
    b. design synthesis
    c. trade-offs
    d. risk

11. Early pioneers in the development of systems and control theory include:
    a. H. Nyquist
    b. H. W. Bode
    c. H. S. Black
    d. All of the above

12. Complete the following sentence:
    An open-loop control system utilizes an actuating device to control a process ________.
    a. without using feedback
    b. using feedback
    c. in engineering design
    d. in engineering synthesis

13. A system with more than one input variable or more than one output variable is known by what name?

a. Closed-loop feedback system

b. Open-loop feedback system

c. Multivariable control system

d. Robust control system

14. Control engineering is applicable to which fields of engineering?

a. Mechanical and aerospace

b. Electrical and biomedical

c. Chemical and environmental

d. All of the above

15. Closed-loop control systems should have which of the following properties:

a. Good regulation against disturbances

b. Desirable responses to commands

c. Low sensitivity to changes in the plant parameters

d. All of the above

In the following **Word Match** problems, match the term with the definition by writing the correct letter in the space provided.

a. Optimization	The output signal is fed back so that it subtracts from the input signal.	________
b. Risk	A system that uses a measurement of the output and compares it with the desired output.	________
c. Complexity of design	A set of prescribed performance criteria.	________
d. System	A measure of the output of the system used for feedback to control the system.	________
e. Design	A system with more than one input variable or more than one output variable.	________
f. Closed-loop feedback control system	The result of making a judgment about how much compromise must be made between conflicting criteria.	________
g. Flyball governor	An interconnection of elements and devices for a desired purpose.	________
h. Specifications	A reprogrammable, multifunctional manipulator used for a variety of tasks.	________
i. Synthesis	A gap between the complex physical system and the design model intrinsic to the progression from the initial concept to the final product.	________
j. Open-loop control system	The intricate pattern of interwoven parts and knowledge required.	________
k. Feedback signal	The ratio of physical output to physical input of an industrial process.	________
l. Robot	The process of designing a technical system.	________
m. Multivariable control system	A system that utilizes a device to control the process without using feedback.	________

n. Design gap	Uncertainties embodied in the unintended consequences of a design.	______
o. Positive feedback	The process of conceiving or inventing the forms, parts, and details of a system to achieve a specified purpose.	______
p. Negative feedback	The device, plant, or system under control.	______
q. Trade-off	The output signal is fed back so that it adds to the input signal.	______
r. Productivity	An interconnection of components forming a system configuration that will provide a desired response.	______
s. Engineering design	The control of a process by automatic means.	______
t. Process	The adjustment of the parameters to achieve the most favorable or advantageous design.	______
u. Control system	The process by which new physical configurations are created.	______
v. Automation	A mechanical device for controlling the speed of a steam engine.	______

EXERCISES

Exercises are straightforward applications of the concepts of the chapter.

The following systems can be described by a block diagram showing the cause–effect relationship and the feedback (if present). Identify the function of each block and the desired input variable, output variable, and measured variable. Use Figure 1.3 as a model where appropriate.

E1.1 Describe typical sensors that can measure each of the following [93]:
- **a.** Linear position
- **b.** Velocity (or speed)
- **c.** Nongravitational acceleration
- **d.** Rotational position (or angle)
- **e.** Rotational velocity
- **f.** Temperature
- **g.** Pressure
- **h.** Liquid (or gas) flow rate
- **i.** Torque
- **j.** Force
- **k.** Earth's magnetic field
- **p.** Heart rate

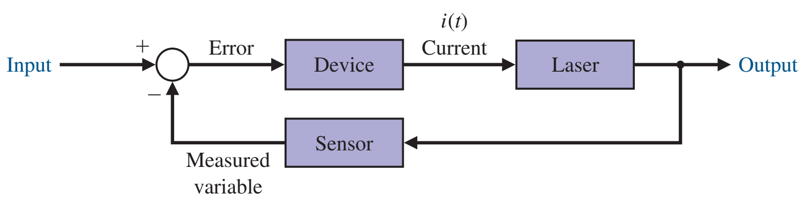


FIGURE E1.3 Partial block diagram of an optical source.

E1.2 Describe typical actuators that can convert the following [93]:
- **a.** Mechanical energy to fluidic energy
- **b.** Mechanical energy to electrical energy
- **c.** Electrical energy to mechanical energy
- **d.** Kinetic energy to electrical energy
- **e.** Electrical energy to heat

E1.3 A CD player laser beam focusing system has an array of photodiodes that is used to determine if the laser beam is in focus. The laser beam focus is controlled by an input current to a lens focusing motor. A microprocessor controls the input current to the motor by comparing the output from the array of photodiodes. Complete the block diagram representing this closed-loop control system shown in Figure E1.3, identifying the output, input, and measured variables, and the control device.

E1.4 A surgeon uses a control system, that is a robot surgical system, to perform surgery remotely. Sketch a block diagram to illustrate this feedback system.

E1.5 Fly-fishing is a sport that challenges the person to cast a small feathery fly using a light rod and line. The goal is to place the fly accurately and lightly on the distant surface of the stream [59]. Describe the fly-casting process and a model of this process.

E1.6 An autofocus camera will adjust the distance of the lens from the film by using a beam of infrared or ultrasound to determine the distance to the subject [42]. Sketch a block diagram of this control system, and briefly explain its operation.

E1.7 Because a sailboat cannot sail directly into the wind, and traveling straight downwind is usually slow, the shortest sailing distance is rarely a straight line. Thus sailboats tack upwind—the familiar zigzag course—and jibe downwind. A tactician's decision of when to tack and where to go can determine the outcome of a race.

Describe the process of tacking a sailboat as the wind shifts direction. Sketch a block diagram depicting this process.

E1.8 An autonomous self-driving vehicle can sense its environment and navigate without human input. Describe a simplified feedback control system for a guidance system that ensures the vehicle navigates its surroundings safely.

E1.9 Describe the block diagram of the control system of a skateboard with a human rider.

E1.10 Describe the process of human biofeedback used to regulate factors such as pain or body temperature. Biofeedback is a technique whereby a human can, with some success, consciously regulate pulse, reaction to pain, and body temperature.

E1.11 Future advanced commercial aircraft will be E-enabled. This will allow the aircraft to take advantage of continuing improvements in computer power and network growth. Aircraft can continuously communicate their location, speed, and critical health parameters to ground controllers, and gather and transmit local meteorological data. Sketch a block diagram showing how the meteorological data from multiple aircraft can be transmitted to the ground, combined using ground-based powerful networked computers to create an accurate weather situational awareness, and then transmitted back to the aircraft for optimal routing.

E1.12 Unmanned aerial vehicles (UAVs) are being developed to operate in the air autonomously for long periods of time. By autonomous, we mean that there is no interaction with human ground controllers. Sketch a block diagram of an autonomous UAV that is tasked for crop monitoring using aerial photography. The UAV must photograph and transmit the entire land area by flying a pre-specified trajectory as accurately as possible.

E1.13 Consider the inverted pendulum shown in Figure E1.13. Sketch the block diagram of a feedback control system. Identify the process, sensor, actuator, and controller. The objective is keep the pendulum in the upright position, that is to keep $\theta = 0$, in the presence of disturbances.

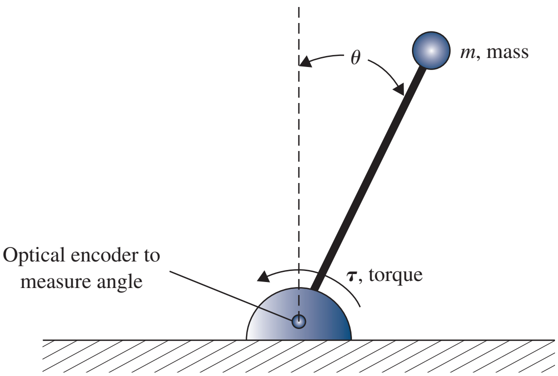


FIGURE E1.13 Inverted pendulum control.

E1.14 Sketch a block diagram of a person playing a video game. Suppose that the input device is a joystick and the game is being played on a desktop computer.

E1.15 For people with diabetes, keeping track of and maintaining blood glucose at safe levels is very important. Continuous blood glucose monitors and readers are available that enable a measurement of blood glucose with a painless scan rather than a fingerprick, as illustrated in Figure E1.15. Sketch a block diagram with a continuous blood glucose monitor and a reader and their possible control actions they might implement as they manage a high blood glucose reading.

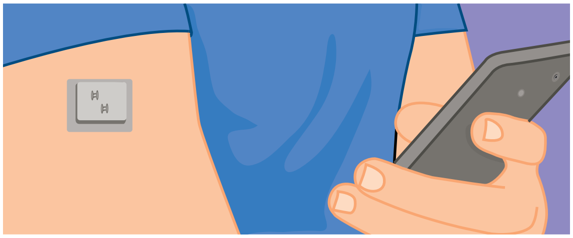


FIGURE E1.15 A continuous blood glucose monitoring system

PROBLEMS

Problems require extending the concepts of this chapter to new situations.

The following systems may be described by a block diagram showing the cause–effect relationship and the feedback (if present). Each block should describe its function. Use Figure 1.3 as a model where appropriate.

P1.1 Automobiles have variable windshield wiper speed settings for different rain intensity. Sketch a block diagram of a wiper system where the driver sets the wiper speed. Identify the function of each element of the variable speed control of the wiper system.

P1.2 Control systems can use a human operator as part of a closed-loop control system. Sketch the block diagram of the valve control system shown in Figure P1.2.

P1.3 In a chemical process control system, it is valuable to control the chemical composition of the product. To do so, a measurement of the composition can be obtained by using an infrared stream analyzer, as shown in Figure P1.3. The valve on the additive stream may be controlled. Complete the control feedback loop, and sketch a block diagram describing the operation of the control loop.

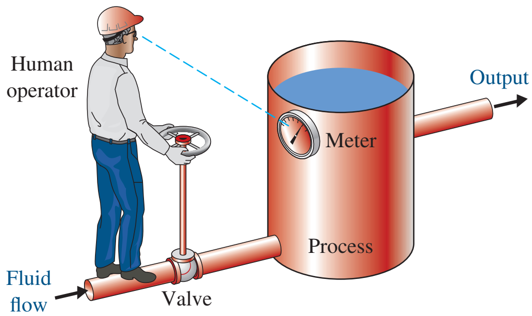


FIGURE P1.2 Fluid-flow control.

P1.4 The accurate control of a nuclear reactor is important for power system generators. Assuming the number of neutrons present is proportional to the power level, an ionization chamber is used to measure the power level. The current i_o is proportional to the power level. The position of the graphite control rods moderates the power level. Complete the control system of the nuclear reactor shown in Figure P1.4 and sketch the block diagram describing the operation of the feedback control loop.

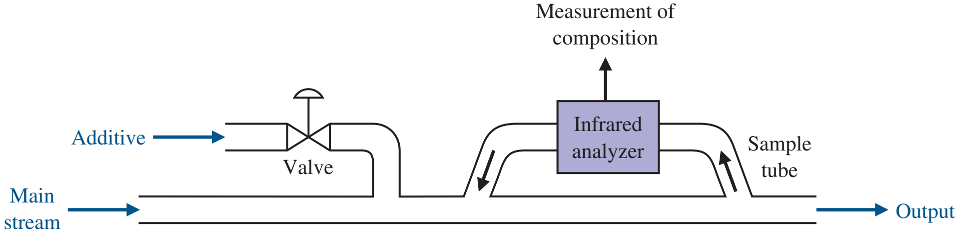


FIGURE P1.3 Chemical composition control.

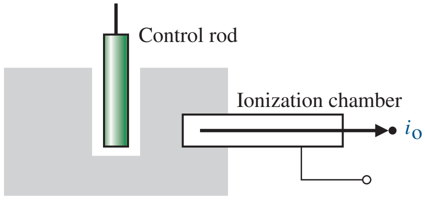


FIGURE P1.4 Nuclear reactor control.

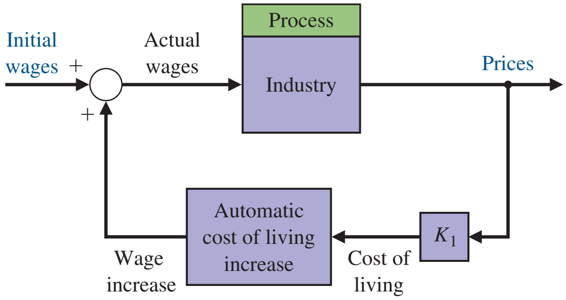


FIGURE P1.6 Positive feedback.

P1.5 A light-seeking control system, used to track the sun, is shown in Figure P1.5. The output shaft, driven by the motor through a worm reduction gear, has a bracket attached on which are mounted two photocells. Complete the closed-loop system so that the system follows the light source.

P1.6 Feedback systems do not always involve negative feedback. Economic inflation, which is evidenced by continually rising prices, is a **positive feedback** system. A positive feedback control system, as shown in Figure P1.6, adds the feedback signal to the input signal, and the resulting signal is used as the input to the process. A simple model of the price–wage inflationary spiral is shown in Figure P1.6. Add additional feedback loops, such as legislative control or control of the tax rate, to stabilize the system. It is assumed that an increase in workers' salaries, after some time delay, results in an increase in prices. Under what conditions could prices be stabilized by falsifying or delaying the availability of cost-of-living data? How would a national wage and price economic guideline program affect the feedback system?

P1.7 The story is told about the sergeant who stopped at the jewelry store every morning at nine o'clock and compared and reset his watch with the chronometer in the window. Finally, one day the sergeant went into the store and complimented the owner on the accuracy of the chronometer.

"Is it set according to time signals from Arlington?" asked the sergeant.

"No," said the owner, "I set it by the five o'clock cannon fired from the fort each afternoon. Tell me, Sergeant, why do you stop every day and check your watch?"

The sergeant replied, "I'm the gunner at the fort!"

Is the feedback prevalent in this case positive or negative? The jeweler's chronometer loses two minutes each 24-hour period and the sergeant's watch loses three minutes during each eight hours. What is the net time error of the cannon at the fort after 12 days?

P1.8 In a public address system, when the microphone is placed too close to the loudspeaker, a positive feedback system is inadvertently created. The audio input from the microphone is amplified, which comes out through the loudspeaker. This audio output is received by the microphone again, which gets amplified further, and comes out through the loudspeaker again. This positive loop gain is known as audio feedback or the Larsen effect, and causes the system to overload, resulting in a high-pitched sound. Construct the corresponding feedback model, and identify each block of the model.

P1.9 Models of physiological control systems are valuable aids to the medical profession. A model of the heart-rate control system is shown in Figure P1.9 [23, 48]. This model includes the processing of the nerve signals by the brain. The heart-rate control system is, in fact, a multivariable system, and the variables x, y, w, v, z, and u are vector

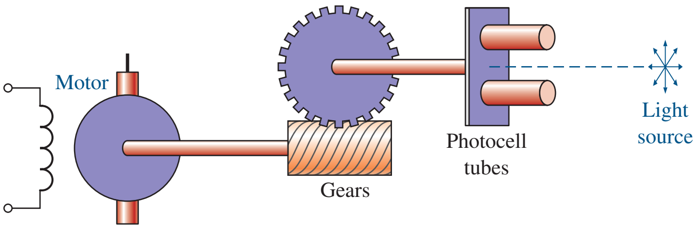


FIGURE P1.5 A photocell is mounted in each tube. The light reaching each cell is the same in both only when the light source is exactly in the middle as shown.

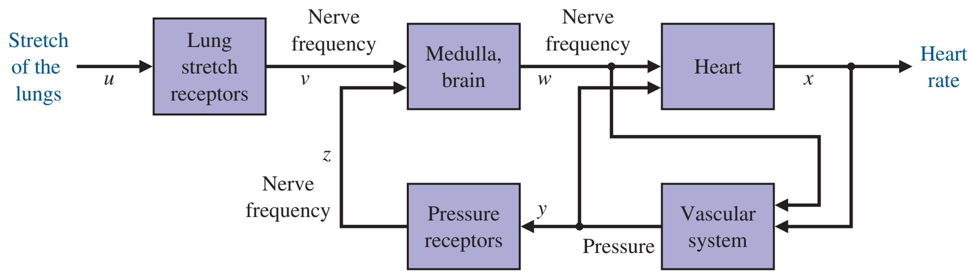


FIGURE P1.9 Heart-rate control.

variables. In other words, the variable x represents many heart variables $x_1, x_2, \ldots, x_n$. Examine the model of the heart-rate control system and add or delete blocks, if necessary. Determine a control system model of one of the following physiological control systems:

1. Respiratory control system
2. Adrenaline control system
3. Human arm control system
4. Eye control system
5. Pancreas and the blood-sugar-level control system
6. Circulatory system

P1.10 The role of air traffic control systems is increasing as airplane traffic increases at busy airports. Engineers are developing air traffic control systems and collision avoidance systems using the Global Positioning System (GPS) navigation satellites [34, 55]. GPS allows each aircraft to know its position in the airspace landing corridor very precisely. Sketch a block diagram depicting how an air traffic controller might use GPS for aircraft collision avoidance.

P1.11 Automatic control of water level using a float level was used in the Middle East for a water clock [1, 11]. The water clock (Figure P1.11) was used from sometime before Christ until the 17th century. Discuss the operation of the water clock, and establish how the float provides a feedback control that maintains the accuracy of the clock. Sketch a block diagram of the feedback system.

P1.12 An automatic turning gear for windmills was invented by Meikle in about 1750 [1, 11]. The fantail gear shown in Figure P1.12 automatically turns the windmill into the wind. The fantail windmill at right angle to the mainsail is used to turn the turret. The gear ratio is of the order of 3000 to 1. Discuss the operation of the windmill, and establish the feedback operation that maintains the main sails into the wind.

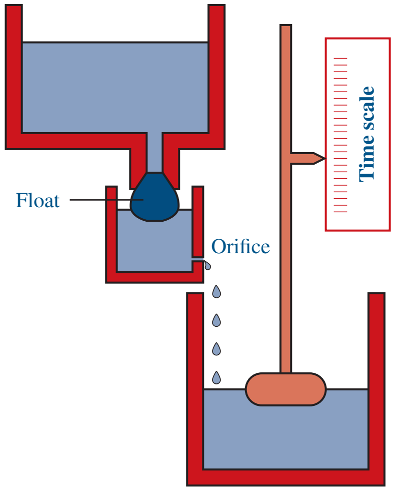


FIGURE P1.11 Water clock. (From Newton, Gould, and Kaiser, *Analytical Design of Linear Feedback Controls*. Wiley, New York, 1957, with permission.)

P1.13 A common example of a two-input control system is an automobile power transmission system, with a gear shifter and an accelerator pedal. The objective is to obtain (1) a desired speed and (2) a desired torque. Sketch a block diagram of the closed-loop control system.

P1.14 Adam Smith (1723–1790) discussed the issue of free competition between the participants of an economy in his book *Wealth of Nations*. It may be said that Smith employed social feedback mechanisms to explain his theories [41]. Smith suggests that (1) the available workers as a whole compare the various possible employments and enter that one offering the greatest rewards, and (2) in any employment the

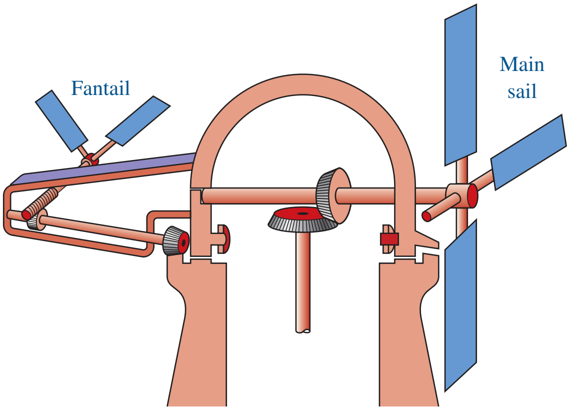


FIGURE P1.12 Automatic turning gear for windmills. (From Newton, Gould, and Kaiser, *Analytical Design of Linear Feedback Controls*. Wiley, New York, 1957, with permission.)

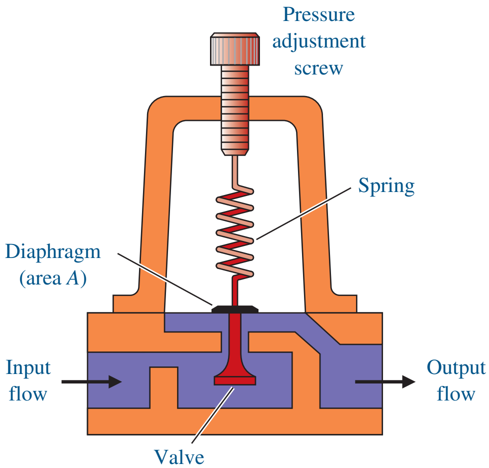


FIGURE P1.18 Pressure regulator.

rewards diminish as the number of competing workers rises. Let r = total of rewards averaged over all trades, c = total of rewards in a particular trade, and q = influx of workers into the specific trade. Sketch a feedback system to represent this system.

P1.15 Small computers are used as part of a start-stop system in automobiles to control emissions and obtain improved gas mileage. A computer-controlled start-stop system that automatically stops and restarts an engine to reduce the time the engine idles could improve gas mileage and reduce unwanted polluting emissions significantly. Sketch a block diagram for such a system for an automobile.

P1.16 All humans have experienced a fever associated with an illness. A fever is related to the changing of the control input in the body's thermostat. This thermostat, within the brain, normally regulates temperature near 98°F in spite of external temperatures ranging from 0°F to 100°F or more. For a fever, the input, or desired, temperature is increased. Even to many scientists, it often comes as a surprise to learn that fever does not indicate something wrong with body temperature control but rather well-contrived regulation at an elevated level of desired input. Sketch a block diagram of the temperature control system and explain how aspirin will lower a fever.

P1.17 Baseball players use feedback to judge a fly ball and to hit a pitch [35]. Describe a method used by a batter to judge the location of a pitch so that he can have the bat in the proper position to hit the ball.

P1.18 A cutaway view of a commonly used pressure regulator is shown in Figure P1.18. The desired pressure is set by turning a calibrated screw. This compresses the spring and sets up a force that opposes the upward motion of the diaphragm. The bottom side of the diaphragm is exposed to the water pressure that is to be controlled. Thus the motion of the diaphragm is an indication of the pressure difference between the desired and the actual pressures. It acts like a comparator. The valve is connected to the diaphragm and moves according to the pressure difference until it reaches a position in which the difference is zero. Sketch a block diagram showing the control system with the output pressure as the regulated variable.

P1.19 Ichiro Masaki of General Motors has patented a system that automatically adjusts a car's speed to keep a safe distance from vehicles in front. Using a video camera, the system detects and stores a reference image of the car in front. It then compares this image with a stream of incoming live images as the two cars move down the highway and calculates the distance. Masaki suggests that the system could control steering as well as speed, allowing drivers to lock on to the car ahead and get a "computerized tow." Sketch a block diagram for the control system.

P1.20 A high-performance race car with an adjustable wing (airfoil) is shown in Figure P1.20. Develop a block diagram describing the ability of the airfoil to keep a constant road adhesion between the car's tires and the race track surface. Why is it important to maintain good road adhesion?

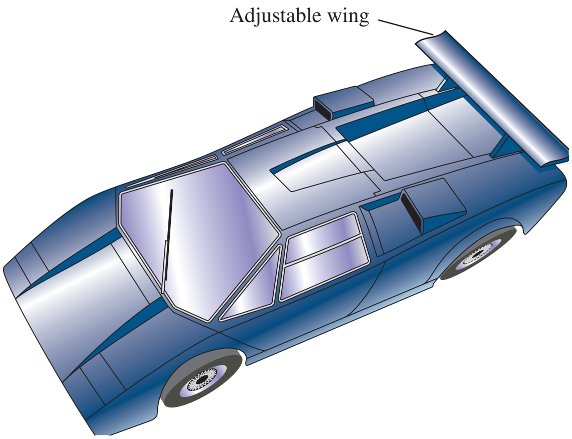


FIGURE P1.20 A high-performance race car with an adjustable wing.

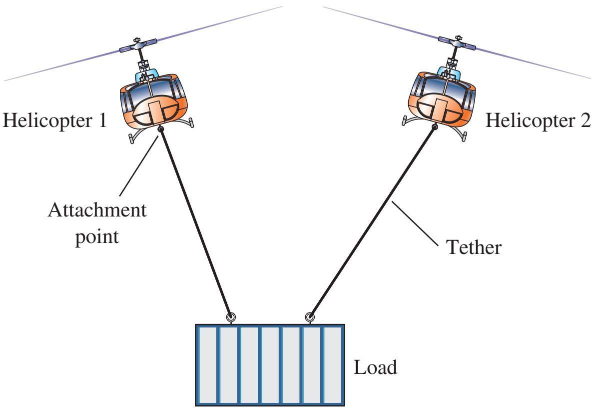


FIGURE P1.21 Two helicopters used to lift and move a large load.

P1.21 The potential of employing two or more helicopters for transporting payloads that are too heavy for a single helicopter is a well-addressed issue in the civil and military rotorcraft design arenas [37]. Overall requirements can be satisfied more efficiently with a smaller aircraft by using multilift for infrequent peak demands. Hence the principal motivation for using multilift can be attributed to the promise of obtaining increased productivity without having to manufacture larger and more expensive helicopters. A specific case of a multilift arrangement where two helicopters jointly transport payloads has been named **twin lift**. Figure P1.21 shows a typical "two-point pendant" twin lift configuration in the lateral/vertical plane.

Develop the block diagram describing the pilots' action, the position of each helicopter, and the position of the load.

P1.22 Engineers want to design a control system that will allow a building or other structure to react to the force of an earthquake much as a human would. The structure would yield to the force, but only so much, before developing strength to push back [47]. Develop a block diagram of a control system to reduce the effect of an earthquake force.

P1.23 Engineers at the Science University of Tokyo are developing a robot with a humanlike face [52]. The robot can display facial expressions, so that it can work cooperatively with human workers. Sketch a block diagram for a facial expression control system of your own design.

P1.24 An innovation for an intermittent automobile windshield wiper is the concept of adjusting its wiping cycle according to the intensity of the rain [54]. Sketch a block diagram of the wiper control system.

P1.25 In the past 50 years, over 20,000 metric tons of hardware have been placed in Earth's orbit. During the same time span, over 15,000 metric tons of hardware returned to Earth. The objects remaining in Earth's orbit range in size from large operational spacecraft to tiny flecks of paint. There are over 500,000 objects in Earth's orbit 1 cm or larger in size. About 20,000 of the space objects are currently tracked from groundstations on the Earth. Space traffic control [61] is becoming an important issue, especially for commercial satellite companies that plan to "fly" their satellites through orbit altitudes where other satellites are operating, and through areas where high concentrations of space debris may exist. Sketch a block diagram of a space traffic control system that commercial companies might use to keep their satellites safe from collisions while operating in space.

P1.26 NASA is developing a compact rover designed to transmit data from the surface of an asteroid back to Earth, as illustrated in Figure P1.26. The rover will use a camera to take panoramic shots of the asteroid surface. The rover can position itself so that the camera can be pointed straight down

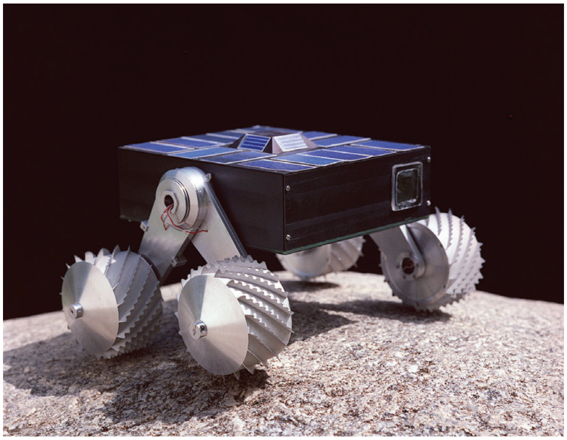


FIGURE P1.26 Microrover designed to explore an asteroid. (Photo courtesy of NASA.)

at the surface or straight up at the sky. Sketch a block diagram illustrating how the microrover can be positioned to point the camera in the desired direction. Assume that the pointing commands are relayed from the Earth to the microrover and that the position of the camera is measured and relayed back to Earth.

P1.27 A direct methanol fuel cell is an electrochemical device that converts a methanol water solution to electricity [75]. Like rechargeable batteries, fuel cells directly convert chemicals to energy; they are very often compared to batteries, specifically rechargeable batteries. However, one significant difference between rechargeable batteries and direct methanol fuel cells is that, by adding more methanol water solution, the fuel cells recharge instantly. Sketch a block diagram of the direct methanol fuel cell recharging system that uses feedback to continuously monitor and recharge the fuel cell.

ADVANCED PROBLEMS

Advanced problems represent problems of increasing complexity.

AP1.1 The development of robotic microsurgery devices will have major implications on delicate eye and brain surgical procedures. One such device is shown in Figure AP1.1. Haptic (force and tactile) feedback can greatly help a surgeon by mimicking the physical interaction that takes place between the microsurgery robotic manipulator and human tissue. Sketch a block diagram for a haptic and tactile subsystem with a microsurgical device in the loop being operated by a surgeon. Assume that the force of the end-effector on the microsurgical device can be measured and is available for feedback.

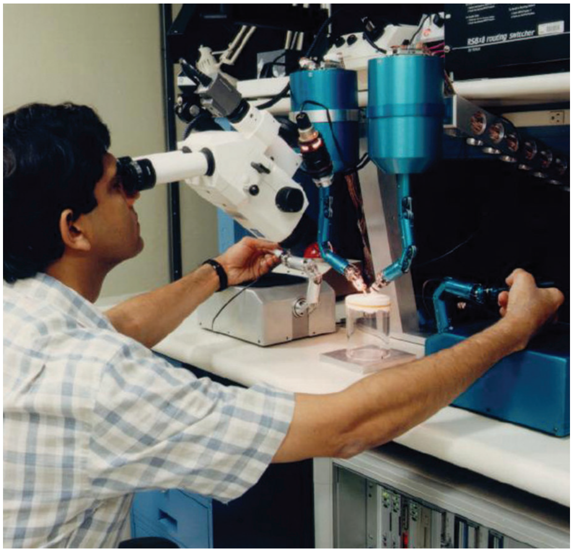


FIGURE AP1.1 Microsurgery robotic manipulator. (Photo courtesy of NASA.)

AP1.2 Advanced wind energy systems are being installed in many locations throughout the world as a way for nations to deal with rising fuel prices and energy shortages, and to reduce the negative effects of fossil fuel utilization on the quality of the air. The modern windmill can be viewed as a mechatronic system. Think about how an advanced wind energy system would be designed as a mechatronic system. List the various components of the wind energy system and associate each component with one of the five elements of a mechatronic system: physical system modeling, signals and systems, computers and logic systems, software and data acquisition, and sensors and actuators.

AP1.3 Many modern luxury automobiles have an advanced driver-assistance systems (ADAS) option. The collision avoidance feature of an ADAS system uses radars to detect nearby obstacles to notify drivers of potential collisions. Figure AP1.3 illustrates the collision avoidance feature of an ADAS system. Sketch a block diagram of this ADAS feedback control system. In your own words, describe the control problem and the challenges facing the designers of the control system.

AP1.4 Adaptive optics has applications to a wide variety of key control problems, including imaging of the human retina and large-scale, ground-based astronomical

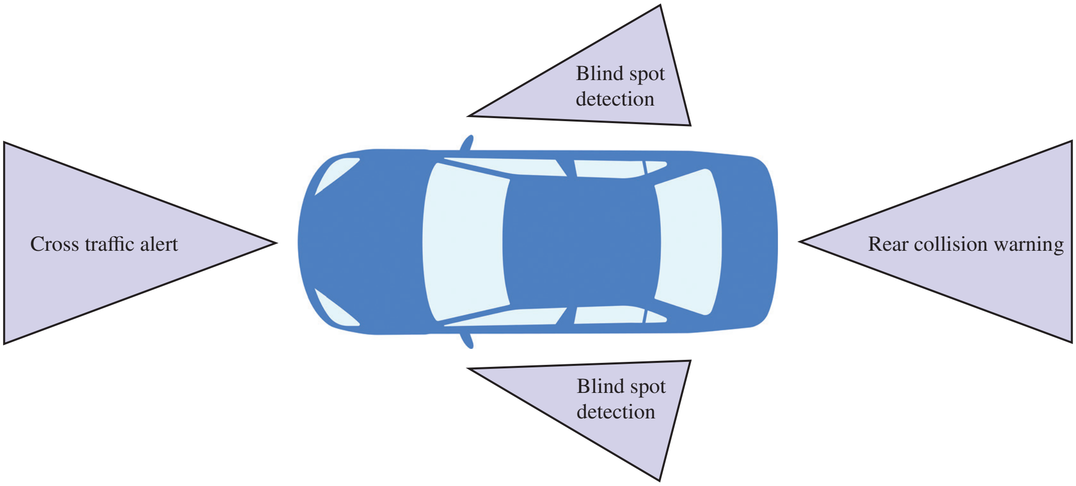


FIGURE AP1.3 A collision avoidance feature of an ADAS system.

observations [98]. In both cases, the approach is to use a wavefront sensor to measure distortions in the incoming light and to actively control and compensate to the errors induced by the distortions. Consider the case of an extremely large ground-based optical telescope, possibly an optical telescope up to 100 meters in diameter. The telescope components include deformable mirrors actuated by micro-electro-mechanical (MEMS) devices and sensors to measure the distortion of the incoming light as it passes through the turbulent and uncertain atmosphere of Earth.

There is at least one major technological barrier to constructing a 100-m optical telescope. The numerical computations associated with the control and compensation of the extremely large optical telescope can be on the order of 10^{10} calculations each 1.5 ms. If we assume that the computational capability is available, then one can consider the design of a feedback control system that uses the available computational power. We can consider many control issues associated with the large-scale optical telescope. Some of the controls problems that might be considered include controlling the pointing of the main dish, controlling the individual deformable mirrors, and attenuating the deformation of the dish due to changes in outside temperature.

Describe a closed-loop feedback control system to control one of the deformable mirrors to compensate for the distortions in the incoming light. Figure AP1.4 shows a diagram of the telescope with a single deformable mirror. Suppose that the mirror has an associated MEMS actuator that can be used to

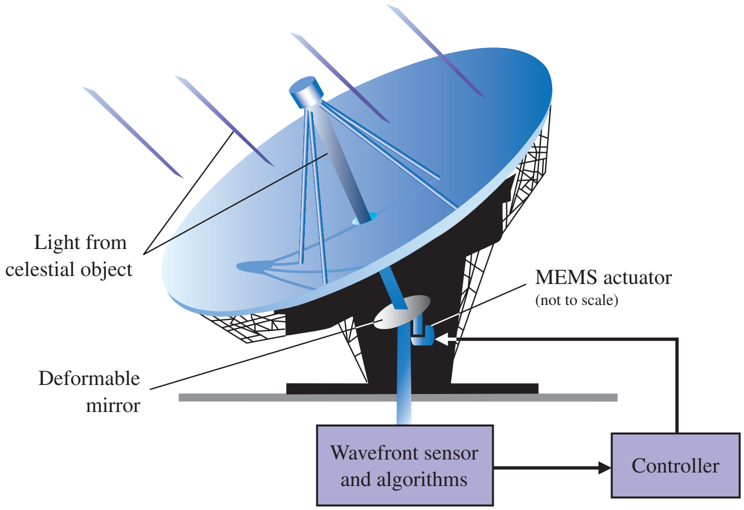


FIGURE AP1.4 Extremely large optical telescope with deformable mirrors for atmosphere compensation.

vary the orientation. Also, assume that the wavefront sensor and associated algorithms provide the desired configuration of the deformable mirror to the feedback control system.

AP1.5 The Burj Dubai is the tallest building in the world [94]. The building, shown in Figure AP1.5, stands at over 800 m with more than 160 stories. There are 57 elevators servicing this tallest free-standing structure in the world. Traveling at up to 10 m/s, the elevators have the world's longest travel distance from lowest to highest stop. Describe a closed-loop feedback control system that guides an elevator of a high-rise building to a desired floor while maintaining a reasonable transit time [95]. Remember that high accelerations will make the passengers uncomfortable.

FIGURE AP1.5 The world's tallest building in Dubai. (Photo courtesy of Obstando Images/Alamy.)

AP1.6 The robotic vacuum cleaner depicted in Figure AP1.6 is an example of a mechatronic system that aids humans in maintaining their homes. A dirt detection control system would enable the robotic vacuum cleaner to vacuum the same area more than once if the dirt level is unsatisfactory, since a single pass may not be enough to adequately remove a high level of dirt. If the robotic vacuum cleaner detects more dirt than usual, it should vacuum the same area until the sensors detect lesser dirt in that area. Describe a closed-loop feedback control system to

FIGURE AP1.6 A robotic vacuum cleaner communicates with the base station as it maneuvers around the room. (Photo courtesy of Hugh Threlfall/Alamy.)

detect an acceptable level of dirt, so that the robotic vacuum cleaner will vacuum the same area again.

AP1.7 Space X has developed a very important system to allow for recovery of the first stage of their Falcon rocket at sea, as depicted in Figure AP1.7. The landing ship is an autonomous drone ship. Sketch a block diagram describing a control system that would control the pitch and roll of the landing ship on the sea.

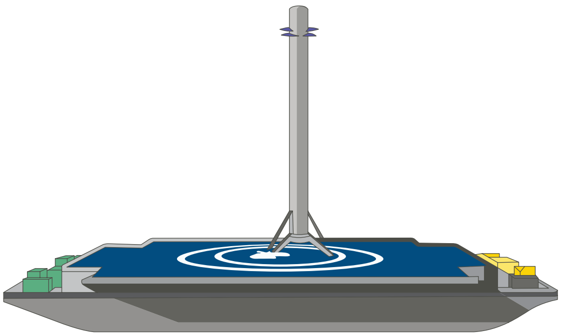


FIGURE AP1.7 Space X return landing on sea- based drone ship.

DESIGN PROBLEMS

Design problems emphasize the design task. Continuous design problems (CDP) build upon a design problem from chapter to chapter.

CDP1.1 Increasingly stringent requirements of modern, high-precision machinery are placing increasing demands on slide systems [53]. The typical goal is to accurately control the desired path of the table shown in Figure CDP1.1. Sketch a block diagram model of a feedback system to achieve the desired goal. The table can move in the x direction as shown.

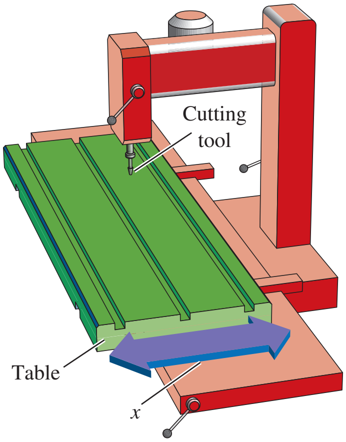


FIGURE CDP1.1 Machine tool with table.

FIGURE DP1.3 Using a smart phone to remotely monitor and control a washing machine. (Photo courtesy of Mikkel William/E+/Getty Images.)

DP1.1 Background noise affects the audio output quality of a headphone. Noise-cancelling headphones use active noise control to reduce this unwanted ambient noise. Sketch a block diagram of an "active noise control" feedback system that will reduce the effect of unwanted noise. Indicate the device within each block.

DP1.2 Aircraft are fitted with autopilot control that, at the press of a button, automatically controls the flight path of an aircraft, without manual control by a pilot. In this way, the pilot can focus on monitoring the flight path, weather, and onboard systems. Design a feedback control in block diagram form for an autopilot system.

DP1.3 Describe a feedback control system in which a user utilizes a smart phone to remotely monitor and control a washing machine as illustrated in Figure DP1.3. The control system should be able to start and stop the wash cycle, control the amount of detergent and the water temperature, and provide notifications on the status of the cycle.

DP1.4 As part of the automation of a dairy farm, the automation of cow milking is under study [36]. Design a milking machine that can milk cows four or five times a day at the cow's demand. Sketch a block diagram and indicate the devices in each block.

DP1.5 A large, braced robot arm for welding large structures is shown in Figure DP1.5. Sketch the block diagram of a closed-loop feedback control system for accurately controlling the location of the weld tip.

DP1.6 Vehicle traction control, which includes antiskid braking and antispin acceleration, can enhance vehicle performance and handling. The objective of this control is to maximize tire traction by preventing locked brakes as well as tire spinning during acceleration. Wheel slip, the difference between the vehicle speed and the wheel speed, is chosen as the controlled variable because of its strong influence on the tractive force between the tire and the road [19]. The adhesion coefficient between the wheel and the road reaches a maximum at a low slip. Develop a block diagram model of one wheel of a traction control system.

DP1.7 The Hubble space telescope was repaired and modified in space on several occasions [44, 46, 49]. One

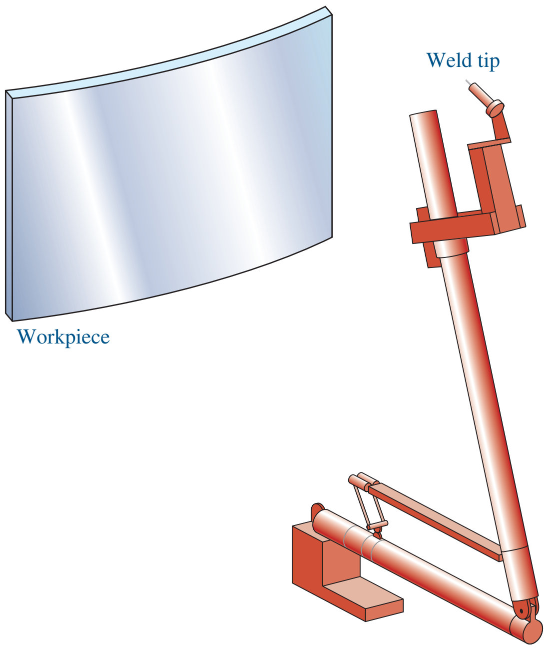


FIGURE DP1.5 Robot welder.

challenging problem with controlling the Hubble is damping the jitter that vibrates the spacecraft each time it passes into or out of the Earth's shadow. The worst vibration has a period of about 20 seconds, or a frequency of 0.05 hertz. Design a feedback system that will reduce the vibrations of the Hubble space telescope.

DP1.8 A challenging application of control design is the use of nanorobots in medicine. Nanorobots will require onboard computing capability, and very tiny sensors and actuators. Fortunately, advances in biomolecular computing, bio-sensors, and actuators are promising to enable medical nanorobots to emerge within the next decade [99]. Many interesting medical applications will benefit from nanorobotics. For example, one use might be to use the robotic devices to precisely deliver anti-HIV drugs or to combat cancer by targeted delivering of chemotherapy as illustrated in Figure DP1.8.

At the present time, we cannot construct practical nanorobots, but we can consider the control design process that would enable the eventual development and installation of these tiny devices in the medical field. Consider the problem of designing a nanorobot to deliver a cancer drug to a specific location within the human body. The target site might be the location of a tumor, for example. Suggest one or more control goals that might guide the design process. Recommend the variables that should be controlled and provide a list of reasonable specifications for those variables.

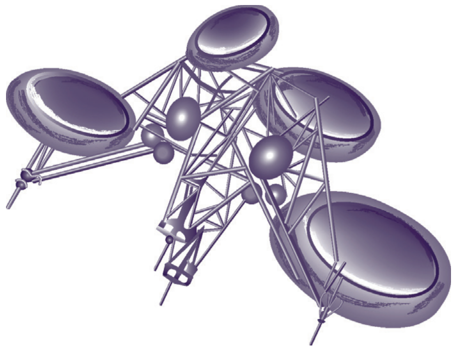


FIGURE DP1.8 An artist illustration of a nanorobot interacting with human blood cells.

DP1.9 Consider the human transportation vehicle (HTV) depicted in Figure DP1.9. The self-balancing HTV is actively controlled to allow safe and easy transportation of a single person [97]. Describe a closed-loop feedback control system to assist the rider of the HTV in balancing and maneuvering the vehicle.

FIGURE DP1.9 Personal transportation vehicle. (Photo courtesy of Sergiy Kuzmin/Shutterstock.)

DP1.10 In addition to maintaining automobile speed, many vehicles can also maintain a prescribed distance to an automobile in front, as illustrated in Figure DP1.10. Design a feedback control sysytem that can maintain cruise speed at a prescribed distance to the vehicle in front. What happens if the leading vehicle slows down below the desired cruise speed?

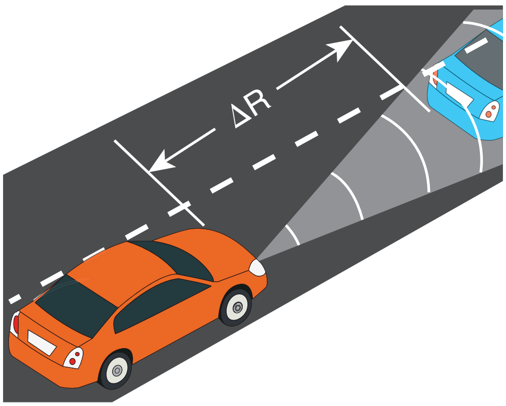


FIGURE DP1.10 Maintaining cruise speed at a prescribed distance.

ANSWERS TO SKILLS CHECK

True or False: (1) True; (2) True; (3) False; (4) False; (5) True

Multiple Choice: (6) d; (7) d; (8) b; (9) c; (10) a; (11) d; (12) a; (13) c; (14) d; (15) d

Word Match (in order, top to bottom): p, f, h, k, m, q, d, l, n, c, r, s, j, b, e, t, o, u, v, a, i, g

TERMS AND CONCEPTS

Actuator A device employed by the control system to alter or adjust the environment.

Analysis The process of examining a system in order to gain a better understanding, provide insight, and find directions for improvement.

Automation The control of a process by automatic means.

Closed-loop feedback control system A system that uses a measurement of the output and compares it with the desired output to control the process.

Complexity of design The intricate pattern of interwoven parts and knowledge required.

Control system An interconnection of components forming a system configuration that will provide a desired response.

Control system engineering An engineering discipline that focuses on the modeling of a wide assortment of physical systems and using those models to design controllers that will cause the closed-loop systems to possess desired performance characteristics.

Design The process of conceiving or inventing the forms, parts, and details of a system to achieve a specified purpose.

Design gap A gap between the complex physical system and the design model intrinsic to the progression from the initial concept to the final product.

Disturbance An unwanted input signal that affects the output signal.

Embedded control Feedback control system that employs on-board special-purpose digital computers as integral components of the feedback loop.

Engineering design The process of designing a technical system.

Feedback signal A measure of the output of the system used for feedback to control the system.

Flyball governor A mechanical device for controlling the speed of a steam engine.

Hybrid fuel automobile An automobile that uses a conventional internal combustion engine in combination with an energy storage device to provide a propulsion system.

Internet of Things (IoT) Network of physical objects embedded with electronics, software, sensors, and connectivity.

Measurement noise An unwanted input signal that affects the measured output signal.

Mechatronics The synergistic integration of mechanical, electrical, and computer systems.

Multiloop feedback control system A feedback control system with more than one feedback control loop.

Multivariable control system A system with more than one input variable or more than one output variable.

Negative feedback An output signal fed back so that it subtracts from the input signal.

Open-loop control system A system that uses a device to control the process without using feedback. Thus the output has no effect upon the signal to the process.

Optimization The adjustment of the parameters to achieve the most favorable or advantageous design.

Plant *See* Process.

Positive feedback An output signal fed back so that it adds to the input signal.

Process The device, plant, or system under control.

Productivity The ratio of physical output to physical input of an industrial process.

Risk Uncertainties embodied in the unintended consequences of a design.

Robot Programmable computers integrated with a manipulator. A reprogrammable, multifunctional manipulator used for a variety of tasks.

Sensor A device that provides a measurement of a desired external signal.

Specifications Statements that explicitly state what the device or product is to be and to do. A set of prescribed performance criteria.

Synthesis The process by which new physical configurations are created. The combining of separate elements or devices to form a coherent whole.

System An interconnection of elements and devices for a desired purpose.

Trade-off The result of making a judgment about how to compromise between conflicting criteria.

Ubiquitous computing A concept in which computing is made available everywhere at any time and can occur on any device.

Ubiquitous positioning A concept in which positioning systems identify the location and position of people, vehicles and objects in time at any location indoors and outdoors.

CHAPTER

2

Mathematical Models of Systems

PREVIEW

Mathematical models of physical systems are key elements in the design and analysis of control systems. The dynamic behavior is generally described by ordinary differential equations. We will consider a wide range of systems. Since most physical systems are nonlinear, we will discuss linearization approximations which allow us to use Laplace transform methods. We will then proceed to obtain the input–output relationship in the form of transfer functions. The transfer functions can be organized into block diagrams or signal-flow graphs to graphically depict the interconnections. Block diagrams and signal-flow graphs are very convenient and natural tools for designing and analyzing complicated control systems. We conclude the chapter by developing transfer function models for the various components of the Sequential Design Example: Disk Drive Read System.

DESIRED OUTCOMES

Upon completion of Chapter 2, students should be able to:

- Recognize that differential equations can describe the dynamic behavior of physical systems.
- Utilize linearization approximations through Taylor series.
- Understand the application of Laplace transforms and their role in obtaining transfer functions.
- Interpret block diagrams and signal-flow graphs and explain their role in analyzing control systems.
- Describe the important role of modeling in the control system design process.

2.1 INTRODUCTION

To understand and control complex systems, one must obtain quantitative **mathematical models** of these systems. It is necessary therefore to analyze the relationships between the system variables and to obtain a mathematical model. Because the systems under consideration are dynamic in nature, the descriptive equations are usually **differential equations**. Furthermore, if these equations can be **linearized**, then the **Laplace transform** can be used to simplify the method of solution. In practice, the complexity of systems and our ignorance of all the relevant factors necessitate the introduction of **assumptions** concerning the system operation. Therefore we will often find it useful to consider the physical system, express any necessary assumptions, and linearize the system. Then, by using the physical laws describing the linear equivalent system, we can obtain a set of time-invariant, ordinary linear differential equations. Finally, using mathematical tools, such as the Laplace transform, we obtain a solution describing the operation of the system. In summary, the approach to dynamic system modeling can be listed as follows:

1. Define the system and its components.
2. Formulate the mathematical model and fundamental necessary assumptions based on basic principles.
3. Obtain the differential equations representing the mathematical model.
4. Solve the equations for the desired output variables.
5. Examine the solutions and the assumptions.
6. If necessary, reanalyze or redesign the system.

2.2 DIFFERENTIAL EQUATIONS OF PHYSICAL SYSTEMS

The differential equations describing the dynamic performance of a physical system are obtained by utilizing the physical laws of the process [1–4]. Consider the torsional spring–mass system in Figure 2.1 with applied torque $T_a(t)$. Assume the torsional spring element is massless. Suppose we want to measure the torque $T_s(t)$ transmitted to the mass m. Since the spring is massless, the sum of the torques acting on the spring itself must be zero, or

$$T_a(t) - T_s(t) = 0,$$

which implies that $T_s(t) = T_a(t)$. We see immediately that the external torque $T_a(t)$ applied at the end of the spring is transmitted *through* the torsional spring. Because of this, we refer to the torque as a **through-variable**. In a similar manner, the angular rate difference associated with the torsional spring element is

$$\omega(t) = \omega_s(t) - \omega_a(t).$$

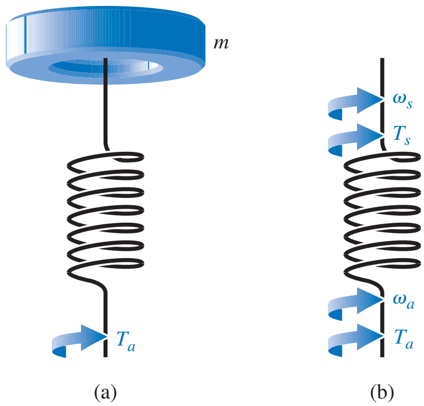


FIGURE 2.1 (a) Torsional spring–mass system. (b) Spring element.

Thus, the angular rate difference is measured across the torsional spring element and is referred to as an **across-variable**. These same types of arguments can be made for most common physical variables (such as force, current, volume, flow rate, etc.). A more complete discussion on through- and across-variables can be found in [26, 27]. A summary of the through- and across-variables of dynamic systems is given in Table 2.1 [5]. Information concerning the International System (SI) of units associated with the various variables discussed in this section can be found online, as well in many handy references, such as the MCS website.† For example, variables that measure temperature are degrees Kelvin in SI units, and variables that measure length are meters. A summary of the describing equations for lumped, linear, dynamic elements is given in Table 2.2 [5]. The equations in

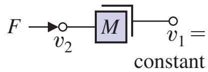
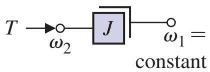
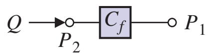
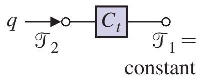
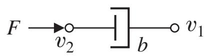
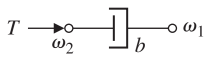
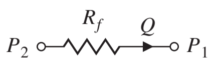
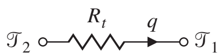
Table 2.1 Summary of Through- and Across-Variables for Physical Systems

System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, $\mathcal{T}_{21}$	

†The companion website is available at www.pearsonglobaleditions.com.

Table 2.2 are idealized descriptions and only approximate the actual conditions (for example, when a linear, lumped approximation is used for a distributed element).

Table 2.2 Summary of Governing Differential Equations for Ideal Elements

Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\dfrac{di}{dt}$	$E = \dfrac{1}{2}Li^2$	
	Translational spring	$v_{21} = \dfrac{1}{k}\dfrac{dF}{dt}$	$E = \dfrac{1}{2}\dfrac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \dfrac{1}{k}\dfrac{dT}{dt}$	$E = \dfrac{1}{2}\dfrac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\dfrac{dQ}{dt}$	$E = \dfrac{1}{2}IQ^2$	
Capacitive storage	Electrical capacitance	$i = C\dfrac{dv_{21}}{dt}$	$E = \dfrac{1}{2}Cv_{21}^2$	
	Translational mass	$F = M\dfrac{dv_2}{dt}$	$E = \dfrac{1}{2}Mv_2^2$	
	Rotational mass	$T = J\dfrac{d\omega_2}{dt}$	$E = \dfrac{1}{2}J\omega_2^2$	
	Fluid capacitance	$Q = C_f\dfrac{dP_{21}}{dt}$	$E = \dfrac{1}{2}C_fP_{21}^2$	
	Thermal capacitance	$q = C_t\dfrac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	
Energy dissipators	Electrical resistance	$i = \dfrac{1}{R}v_{21}$	$\mathcal{P} = \dfrac{1}{R}v_{21}^2$	
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}^2$	
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}^2$	
	Fluid resistance	$Q = \dfrac{1}{R_f}P_{21}$	$\mathcal{P} = \dfrac{1}{R_f}P_{21}^2$	
	Thermal resistance	$q = \dfrac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \dfrac{1}{R_t}\mathcal{T}_{21}$	

Nomenclature

- *Through-variable:* F = force, T = torque, i = current, Q = fluid volumetric flow rate, q = heat flow rate.
- *Across-variable:* v = translational velocity, ω = angular velocity, v = voltage, P = pressure, $\mathcal{T}$ = temperature.
- *Inductive storage:* L = inductance, $1/k$ = reciprocal translational or rotational stiffness, I = fluid inertance.
- *Capacitive storage:* C = capacitance, M = mass, J = moment of inertia, C_f = fluid capacitance, C_t = thermal capacitance.
- *Energy dissipators:* R = resistance, b = viscous friction, R_f = fluid resistance, R_t = thermal resistance.

The symbol v is used for both voltage in electrical circuits and velocity in translational mechanical systems and is distinguished within the context of each differential equation. For mechanical systems, one uses Newton's laws; for electrical systems, Kirchhoff's voltage laws. For example, the simple spring-mass-damper mechanical system shown in Figure 2.2(a) is described by Newton's second law of motion. The free-body diagram of the mass M is shown in Figure 2.2(b). In this spring-mass-damper example, we model the wall friction as a **viscous damper**, that is, the friction force is linearly proportional to the velocity of the mass. In reality the friction force may behave in a more complicated fashion. For example, the wall friction may behave as a **Coulomb damper**. Coulomb friction, also known as dry friction, is a nonlinear function of the mass velocity and possesses a discontinuity around zero velocity. For a well-lubricated, sliding surface, the viscous friction is appropriate and will be used here and in subsequent spring-mass-damper examples. Summing the forces acting on M and utilizing Newton's second law yields

$$M\frac{d^2y(t)}{dt^2} + b\frac{dy(t)}{dt} + ky(t) = r(t), \tag{2.1}$$

where k is the spring constant of the ideal spring and b is the friction constant. Equation (2.1) is a second-order linear constant-coefficient (time-invariant) differential equation.

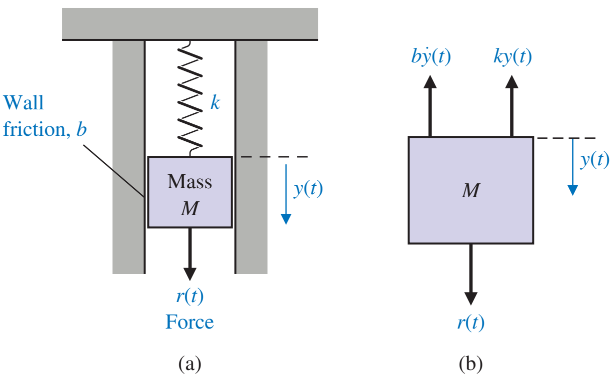


FIGURE 2.2 (a) Spring-mass-damper system. (b) Free-body diagram.

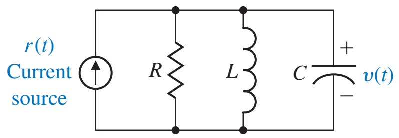


FIGURE 2.3 RLC circuit.

Alternatively, one may describe the electrical *RLC* circuit of Figure 2.3 by utilizing Kirchhoff's current law. Then we obtain the following integrodifferential equation:

$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} + \frac{1}{L}\int_0^t v(t)\,dt = r(t). \tag{2.2}$$

The solution of the differential equation describing the process may be obtained by classical methods such as the use of integrating factors and the method of undetermined coefficients [1]. For example, when the mass is initially displaced a distance $y(0) = y_0$ and released, the dynamic response of the system can be represented by an equation of the form

$$y(t) = K_1 e^{-\alpha_1 t}\sin(\beta_1 t + \theta_1). \tag{2.3}$$

A similar solution is obtained for the voltage of the *RLC* circuit when the circuit is subjected to a constant current $r(t) = I$. Then the voltage is

$$v(t) = K_2 e^{-\alpha_2 t}\cos(\beta_2 t + \theta_2). \tag{2.4}$$

A voltage curve typical of an *RLC* circuit is shown in Figure 2.4.

To reveal further the close similarity between the differential equations for the mechanical and electrical systems, we shall rewrite Equation (2.1) in terms of velocity:

$$v(t) = \frac{dy(t)}{dt}.$$

Then we have

$$M\frac{dv(t)}{dt} + bv(t) + k\int_0^t v(t)\,dt = r(t). \tag{2.5}$$

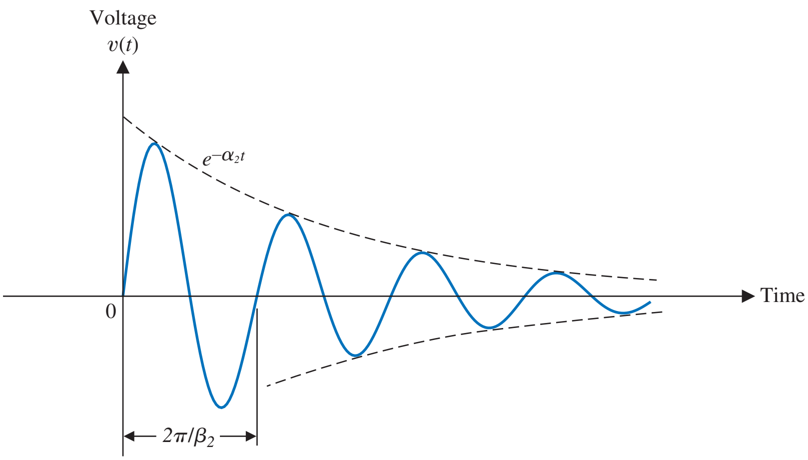


FIGURE 2.4 Typical voltage response for an *RLC* circuit.

One immediately notes the equivalence of Equations (2.5) and (2.2) where velocity $v(t)$ and voltage $v(t)$ are equivalent variables, usually called **analogous variables**, and the systems are analogous systems. Therefore the solution for velocity is similar to Equation (2.4), and the response for an underdamped system is shown in Figure 2.4. The concept of analogous systems is a very useful and powerful technique for system modeling. The voltage–velocity analogy, often called the force–current analogy, is a natural one because it relates the analogous through- and across-variables of the electrical and mechanical systems. Another analogy that relates the velocity and current variables is often used and is called the force–voltage analogy [21, 23].

Analogous systems with similar solutions exist for electrical, mechanical, thermal, and fluid systems. The existence of analogous systems and solutions provides the analyst with the ability to extend the solution of one system to all analogous systems with the same describing differential equations. Therefore what one learns about the analysis and design of electrical systems is immediately extended to an understanding of fluid, thermal, and mechanical systems.

2.3 LINEAR APPROXIMATIONS OF PHYSICAL SYSTEMS

A great majority of physical systems are linear within some range of the variables. In general, systems ultimately become nonlinear as the variables are increased without limit. For example, the spring-mass-damper system of Figure 2.2 is linear and described by Equation (2.1) as long as the mass is subjected to small deflections $y(t)$. However, if $y(t)$ were continually increased, eventually the spring would be overextended and break. Therefore the question of linearity and the range of applicability must be considered for each system.

A system is defined as linear in terms of the system excitation and response. In the case of the electrical network, the excitation is the input current $r(t)$ and the response is the voltage $v(t)$. In general, a **necessary condition** for a linear system can be determined in terms of an excitation $x(t)$ and a response $y(t)$. When the system at rest is subjected to an excitation $x_1(t)$, it provides a response $y_1(t)$. Furthermore, when the system is subjected to an excitation $x_2(t)$, it provides a corresponding response $y_2(t)$. For a linear system, it is necessary that the excitation $x_1(t) + x_2(t)$ result in a response $y_1(t) + y_2(t)$. This is the **principle of superposition**.

Furthermore, the magnitude scale factor must be preserved in a **linear system**. Again, consider a system with an input $x(t)$ that results in an output $y(t)$. Then the response of a linear system to a constant multiple β of an input x must be equal to the response to the input multiplied by the same constant so that the output is equal to $\beta y(t)$. This is the property of **homogeneity**.

A linear system satisfies the properties of superposition and homogeneity.

A system characterized by the relation $y(t) = x^2(t)$ is not linear, because the superposition property is not satisfied. A system represented by the relation $y(t) = mx(t) + b$ is not linear, because it does not satisfy the homogeneity property. However, this second system may be considered linear about an operating point x_0, y_0 for small changes Δx and Δy. When $x(t) = x_0 + \Delta x(t)$ and $y(t) = y_0 + \Delta y(t)$, we have

$$y(t) = mx(t) + b$$

or

$$y_0 + \Delta y(t) = mx_0 + m\Delta x(t) + b.$$

Therefore, $\Delta y(t) = m\Delta x(t)$, which satisfies the necessary conditions.

The linearity of many mechanical and electrical elements can be assumed over a reasonably large range of the variables [7]. This is not usually the case for thermal and fluid elements, which are more frequently nonlinear in character. Fortunately, however, one can often linearize nonlinear elements assuming small-signal conditions. This is the normal approach used to obtain a linear equivalent circuit for electronic circuits and transistors. Consider a general element with an excitation (through-) variable $x(t)$ and a response (across-) variable $y(t)$. Several examples of dynamic system variables are given in Table 2.1. The relationship of the two variables is written as

$$y(t) = g\big(x(t)\big), \tag{2.6}$$

where $g(x(t))$ indicates $y(t)$ is a function of $x(t)$. The normal operating point is designated by x_0. Because the curve (function) is continuous over the range of interest, a **Taylor series** expansion about the operating point may be utilized [7]. Then we have

$$y(t) = g\big(x(t)\big) = g(x_0) + \left.\frac{dg}{dx}\right|_{x(t)=x_0} \frac{\big(x(t) - x_0\big)}{1!} + \left.\frac{d^2g}{dx^2}\right|_{x(t)=x_0} \frac{\big(x(t) - x_0\big)^2}{2!} + \cdots. \tag{2.7}$$

The slope at the operating point,

$$m = \left.\frac{dg}{dx}\right|_{x(t)=x_0},$$

is a good approximation to the curve over a small range of $x(t) - x_0$, the deviation from the operating point. Then, as a reasonable approximation, Equation (2.7) becomes

$$y(t) = g(x_0) + \left.\frac{dg}{dx}\right|_{x(t)=x_0} \big(x(t) - x_0\big) = y_0 + m\big(x(t) - x_0\big). \tag{2.8}$$

Finally, Equation (2.8) can be rewritten as the linear equation

$$y(t) - y_0 = m\big(x(t) - x_0\big)$$

or

$$\Delta y(t) = m\Delta x(t). \tag{2.9}$$

Consider the case of a mass, M, sitting on a nonlinear spring, as shown in Figure 2.5(a). The normal operating point is the equilibrium position that occurs when the spring force balances the gravitational force Mg, where g is the gravitational constant. Thus, we obtain $f_0 = Mg$, as shown. For the nonlinear spring with $f(t) = y^2(t)$, the equilibrium position is $y_0 = (Mg)^{1/2}$. The linear model for small deviation is

$$\Delta f(t) = m\Delta y(t),$$

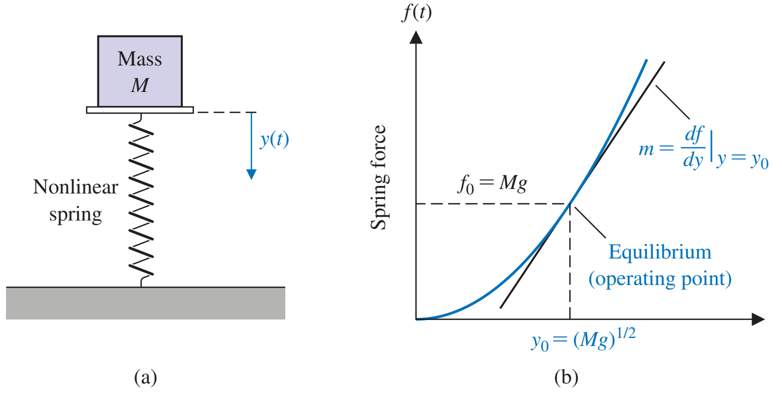


FIGURE 2.5 (a) A mass sitting on a nonlinear spring. (b) The spring force versus $y(t)$.

where

$$m = \left.\frac{df}{dy}\right|_{y(t)=y_0},$$

as shown in Figure 2.5(b). Thus, $m = 2y_0$. A **linear approximation** is as accurate as the assumption of small signals is applicable to the specific problem.

If the dependent variable $y(t)$ depends upon several excitation variables, $x_1(t), x_2(t), \ldots, x_n(t)$, then the functional relationship is written as

$$y(t) = g\big(x_1(t), x_2(t), \ldots, x_n(t)\big). \tag{2.10}$$

The Taylor series expansion about the operating point $x_{1_0}, x_{2_0}, \ldots, x_{n_0}$ is useful for a linear approximation to the nonlinear function. When the higher-order terms are neglected, the linear approximation is written as

$$y(t) = g\big(x_{1_0}, x_{2_0}, \ldots, x_{n_0}\big) + \left.\frac{\partial g}{\partial x_1}\right|_{x(t)=x_0}\big(x_1(t) - x_{1_0}\big) + \left.\frac{\partial g}{\partial x_2}\right|_{x(t)=x_0}\big(x_2(t) - x_{2_0}\big)$$
$$+ \cdots + \left.\frac{\partial g}{\partial x_n}\right|_{x(t)=x_0}\big(x_n(t) - x_{n_0}\big), \tag{2.11}$$

where x_0 is the operating point. Example 2.1 will clearly illustrate the utility of this method.

EXAMPLE 2.1 **Pendulum oscillator model**

Consider the pendulum oscillator shown in Figure 2.6(a). The torque on the mass is

$$T(t) = MgL \sin\theta(t), \tag{2.12}$$

where g is the gravity constant. The equilibrium condition for the mass is $\theta_0 = 0°$. The nonlinear relation between $T(t)$ and $\theta(t)$ is shown graphically in Figure 2.6(b). The first derivative evaluated at equilibrium provides the linear approximation, which is

$$T(t) - T_0 \cong MgL \left.\frac{\partial \sin\theta}{\partial\theta}\right|_{\theta(t)=\theta_0}\big(\theta(t) - \theta_0\big),$$

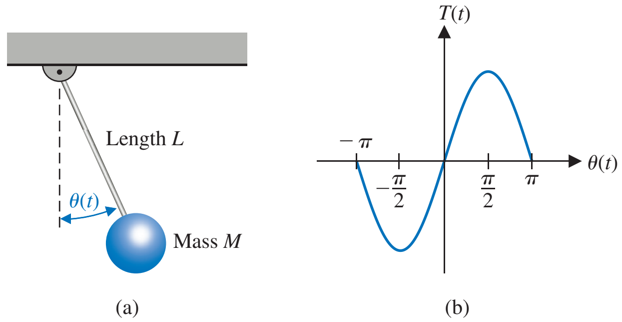


FIGURE 2.6 Pendulum oscillator.

where $T_0 = 0$. Then, we have

$$T(t) = MgL\theta(t). \tag{2.13}$$

This approximation is reasonably accurate for $-\pi/4 \leq \theta \leq \pi/4$. For example, the response of the linear model for the swing through $\pm 30°$ is within 5% of the actual nonlinear pendulum response. ■

2.4 THE LAPLACE TRANSFORM

The ability to obtain linear time-invariant approximations of physical systems allows the analyst to consider the use of the **Laplace transformation**. The Laplace transform method substitutes relatively easily solved algebraic equations for the more difficult differential equations [1, 3]. The time-response solution is obtained by the following operations:

1. Obtain the linearized differential equations.
2. Obtain the Laplace transformation of the differential equations.
3. Solve the resulting algebraic equation for the transform of the variable of interest.

The Laplace transform exists for linear differential equations for which the transformation integral converges. Therefore, for $f(t)$ to be transformable, it is sufficient that

$$\int_{0^-}^{\infty} |f(t)| e^{-\sigma_1 t}\, dt < \infty,$$

for some real, positive σ_1 [1]. The 0^- indicates that the integral should include any discontinuity, such as a delta function at $t = 0$. If the magnitude of $f(t)$ is $|f(t)| < Me^{\alpha t}$ for all positive t, the integral will converge for $\sigma_1 > \alpha$. The region of convergence is therefore given by $\infty > \sigma_1 > \alpha$, and σ_1 is known as the abscissa of absolute convergence. Signals that are physically realizable always have a Laplace transform. The Laplace transformation for a function of time, $f(t)$, is

$$\boxed{F(s) = \int_{0^-}^{\infty} f(t) e^{-st}\, dt = \mathcal{L}\{f(t)\}.} \tag{2.14}$$

The **inverse Laplace transform** is written as

$$\boxed{f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s) e^{+st}\, ds.} \tag{2.15}$$

The transformation integrals have been employed to derive tables of Laplace transforms that are used for the great majority of problems. A table of important Laplace transform pairs is given in Table 2.3. A more complete list of Laplace transform pairs can be found in many references, including at the MCS website.

Table 2.3 Important Laplace Transform Pairs

$f(t)$	$F(s)$
Step function, $u(t)$	$\dfrac{1}{s}$
e^{-at}	$\dfrac{1}{s+a}$
$\sin \omega t$	$\dfrac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\dfrac{s}{s^2+\omega^2}$
t^n	$\dfrac{n!}{s^{n+1}}$
$f^{(k)}(t) = \dfrac{d^k f(t)}{dt^k}$	$s^k F(s) - s^{k-1} f(0^-) - s^{k-2} f'(0^-) - \ldots - f^{(k-1)}(0^-)$
$\displaystyle\int_{-\infty}^{t} f(t)\,dt$	$\dfrac{F(s)}{s} + \dfrac{1}{s}\displaystyle\int_{-\infty}^{0} f(t)\,dt$
Impulse function $\delta(t)$	1
$e^{-at} \sin \omega t$	$\dfrac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\dfrac{s+a}{(s+a)^2+\omega^2}$
$\dfrac{1}{\omega}\left[(\alpha-a)^2+\omega^2\right]^{1/2} e^{-at} \sin(\omega t+\phi)$, $\phi = \tan^{-1}\dfrac{\omega}{\alpha-a}$	$\dfrac{s+\alpha}{(s+a)^2+\omega^2}$
$\dfrac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n\sqrt{1-\zeta^2}\,t$, $\zeta < 1$	$\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
$\dfrac{1}{a^2+\omega^2} + \dfrac{1}{\omega\sqrt{a^2+\omega^2}} e^{-at} \sin(\omega t-\phi)$, $\phi = \tan^{-1}\dfrac{\omega}{-a}$	$\dfrac{1}{s\left[(s+a)^2+\omega^2\right]}$
$1 - \dfrac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\phi\right)$, $\phi = \cos^{-1}\zeta$, $\zeta < 1$	$\dfrac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$
$\dfrac{\alpha}{a^2+\omega^2} + \dfrac{1}{\omega}\left[\dfrac{(\alpha-a)^2+\omega^2}{a^2+\omega^2}\right]^{1/2} e^{-at} \sin(\omega t+\phi)$. $\phi = \tan^{-1}\dfrac{\omega}{\alpha-a} - \tan^{-1}\dfrac{\omega}{-a}$	$\dfrac{s+\alpha}{s\left[(s+a)^2+\omega^2\right]}$

Alternatively, the Laplace variable s can be considered to be the differential operator so that

$$s \equiv \frac{d}{dt}. \tag{2.16}$$

Then we also have the integral operator

$$\frac{1}{s} \equiv \int_{0^-}^{t} dt. \tag{2.17}$$

The inverse Laplace transformation is usually obtained by using the Heaviside partial fraction expansion. This approach is particularly useful for systems analysis and design because the effect of each characteristic root or eigenvalue can be clearly observed.

To illustrate the usefulness of the Laplace transformation and the steps involved in the system analysis, reconsider the spring-mass-damper system described by Equation (2.1), which is

$$M\frac{d^2y(t)}{dt^2} + b\frac{dy(t)}{dt} + ky(t) = r(t). \tag{2.18}$$

We wish to obtain the response, $y(t)$, as a function of time. The Laplace transform of Equation (2.18) is

$$M\left(s^2Y(s) - sy\left(0^-\right) - \frac{dy}{dt}\left(0^-\right)\right) + b\left(sY(s) - y\left(0^-\right)\right) + kY(s) = R(s). \tag{2.19}$$

When

$$r(t) = 0, \quad \text{and} \quad y\left(0^-\right) = y_0, \quad \text{and} \quad \left.\frac{dy}{dt}\right|_{t=0^-} = 0,$$

we have

$$Ms^2Y(s) - Msy_0 + bsY(s) - by_0 + kY(s) = 0. \tag{2.20}$$

Solving for $Y(s)$, we obtain

$$Y(s) = \frac{(Ms + b)y_0}{Ms^2 + bs + k} = \frac{p(s)}{q(s)}. \tag{2.21}$$

The denominator polynomial $q(s)$, when set equal to zero, is called the **characteristic equation** because the roots of this equation determine the character of the time response. The roots of this characteristic equation are also called the **poles** of the system. The roots of the numerator polynomial $p(s)$ are called the **zeros** of the system; for example, $s = -b/M$ is a zero of Equation (2.21). Poles and zeros are critical frequencies. At the poles, the function $Y(s)$ becomes infinite, whereas at the zeros, the function becomes zero. The complex frequency ***s*-plane** plot of the poles and zeros graphically portrays the character of the natural transient response of the system.

For a specific case, consider the system when $k/M = 2$ and $b/M = 3$. Then Equation (2.21) becomes

$$Y(s) = \frac{(s + 3)y_0}{(s + 1)(s + 2)}. \tag{2.22}$$

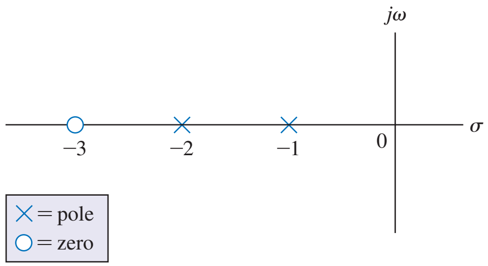


FIGURE 2.7 An s-plane pole and zero plot.

The poles and zeros of $Y(s)$ are shown on the s-plane in Figure 2.7.

Expanding Equation (2.22) in a partial fraction expansion, we obtain

$$Y(s) = \frac{k_1}{s+1} + \frac{k_2}{s+2}, \tag{2.23}$$

where k_1 and k_2 are the coefficients of the expansion. The coefficients k_i are called **residues** and are evaluated by multiplying through by the denominator factor of Equation (2.22) corresponding to k_i and setting s equal to the root. Evaluating k_1 when $y_0 = 1$, we have

$$\begin{aligned} k_1 &= \left.\frac{(s-s_1)p(s)}{q(s)}\right|_{s=s_1} \\ &= \left.\frac{(s+1)(s+3)}{(s+1)(s+2)}\right|_{s_1=-1} = 2 \end{aligned} \tag{2.24}$$

and $k_2 = -1$. Alternatively, the residues of $Y(s)$ at the respective poles may be evaluated graphically on the s-plane plot, since Equation (2.24) may be written as

$$\begin{aligned} k_1 &= \left.\frac{s+3}{s+2}\right|_{s=s_1=-1} \\ &= \left.\frac{s_1+3}{s_1+2}\right|_{s_1=-1} = 2. \end{aligned} \tag{2.25}$$

The graphical representation of Equation (2.25) is shown in Figure 2.8. The graphical method of evaluating the residues is particularly valuable when the order of the characteristic equation is high and several poles are complex conjugate pairs.

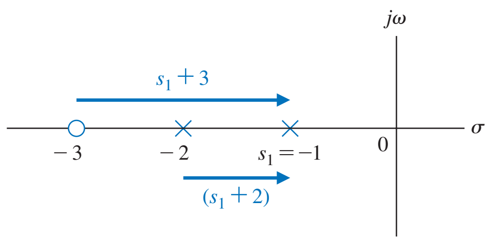


FIGURE 2.8 Graphical evaluation of the residues.

The inverse Laplace transform of Equation (2.22) is then

$$y(t) = \mathcal{L}^{-1}\left\{\frac{2}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{-1}{s+2}\right\}. \tag{2.26}$$

Using Table 2.3, we find that

$$y(t) = 2e^{-t} - 1e^{-2t}. \tag{2.27}$$

Finally, it is usually desired to determine the **steady-state** or **final value** of the response of $y(t)$. For example, the final or steady-state rest position of the spring-mass-damper system may be calculated. The **final value theorem** states that

$$\boxed{\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s),} \tag{2.28}$$

where a simple pole of $Y(s)$ at the origin is permitted, but poles on the imaginary axis and in the right half-plane and repeated poles at the origin are excluded. Therefore, for the specific case of the spring-mass-damper, we find that

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 0. \tag{2.29}$$

Hence the final position for the mass is the normal equilibrium position $y = 0$.

Reconsider the spring-mass-damper system. The equation for $Y(s)$ may be written as

$$Y(s) = \frac{(s + b/M)y_0}{s^2 + (b/M)s + k/M} = \frac{(s + 2\zeta\omega_n)y_0}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{2.30}$$

where ζ is the dimensionless **damping ratio**, and ω_n is the **natural frequency** of the system. The roots of the characteristic equation are

$$s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}, \tag{2.31}$$

where, in this case, $\omega_n = \sqrt{k/M}$ and $\zeta = b/\left(2\sqrt{kM}\right)$. When $\zeta > 1$, the roots are real and the system is **overdamped**; when $\zeta < 1$, the roots are complex and the system is **underdamped**. When $\zeta = 1$, the roots are repeated and real, and the condition is called **critical damping**.

When $\zeta < 1$, the response is underdamped, and

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}. \tag{2.32}$$

The s-plane plot of the poles and zeros of $Y(s)$ is shown in Figure 2.9, where $\theta = \cos^{-1}\zeta$. As ζ varies with ω_n constant, the complex conjugate roots follow a

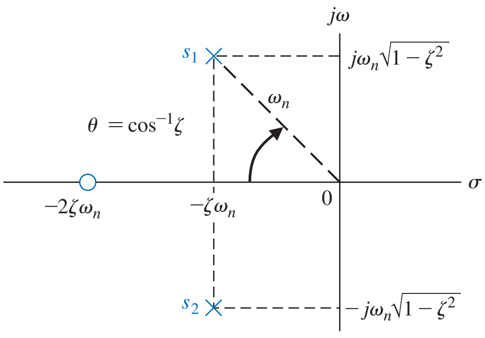


FIGURE 2.9 An s-plane plot of the poles and zeros of $Y(s)$.

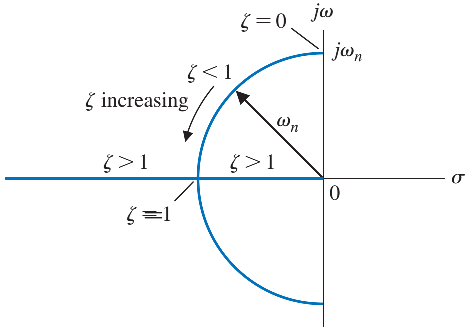


FIGURE 2.10 The locus of roots as ζ varies with ω_n constant.

circular locus, as shown in Figure 2.10. The transient response is increasingly oscillatory as the roots approach the imaginary axis when ζ approaches zero.

The inverse Laplace transform can be evaluated using the graphical residue evaluation. The partial fraction expansion of Equation (2.30) is

$$Y(s) = \frac{k_1}{s - s_1} + \frac{k_2}{s - s_2}. \tag{2.33}$$

Since s_2 is the complex conjugate of s_1, the residue k_2 is the complex conjugate of k_1 so that we obtain

$$Y(s) = \frac{k_1}{s - s_1} + \frac{\hat{k}_1}{s - \hat{s}_1}$$

where the hat indicates the conjugate relation. The residue k_1 is evaluated from Figure 2.11 as

$$k_1 = \frac{y_0(s_1 + 2\zeta\omega_n)}{s_1 - \hat{s}_1} = \frac{y_0 M_1 e^{j\theta}}{M_2 e^{j\pi/2}}, \tag{2.34}$$

where M_1 is the magnitude of $s_1 + 2\zeta\omega_n$, and M_2 is the magnitude of $s_1 - \hat{s}_1$. A review of complex numbers can be found in many online references, as well as on the MCS website. In this case, we obtain

$$k_1 = \frac{y_0(\omega_n e^{j\theta})}{2\omega_n\sqrt{1-\zeta^2}\,e^{j\pi/2}} = \frac{y_0}{2\sqrt{1-\zeta^2}\,e^{j(\pi/2-\theta)}}, \tag{2.35}$$

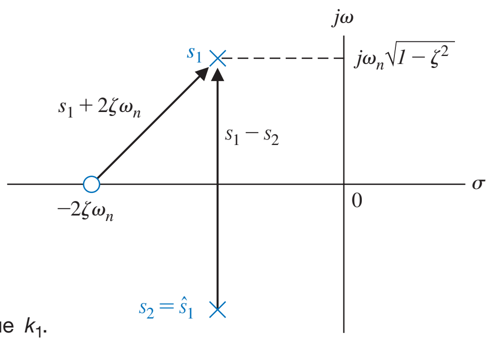


FIGURE 2.11 Evaluation of the residue k_1.

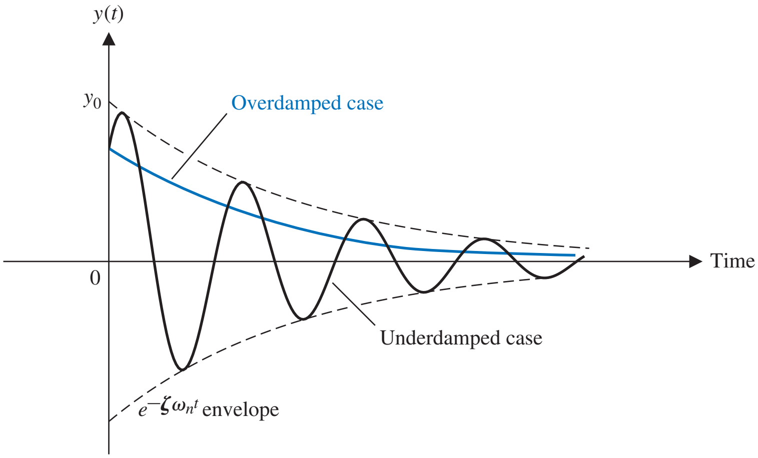


FIGURE 2.12 Response of the spring-mass-damper system.

where $\theta = \cos^{-1}\zeta$. Therefore,

$$k_2 = \frac{y_0}{2\sqrt{1-\zeta^2}}\, e^{j(\pi/2-\theta)}. \tag{2.36}$$

Finally, letting $\beta = \sqrt{1-\zeta^2}$, we find that

$$\begin{aligned} y(t) &= k_1 e^{s_1 t} + k_2 e^{s_2 t} \\ &= \frac{y_0}{2\sqrt{1-\zeta^2}}\left(e^{j(\theta-\pi/2)}e^{-\zeta\omega_n t}e^{j\omega_n\beta t} + e^{j(\pi/2-\theta)}e^{-\zeta\omega_n t}e^{-j\omega_n\beta t}\right) \\ &= \frac{y_0}{\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\theta\right). \end{aligned} \tag{2.37}$$

The solution, Equation (2.37), can also be obtained using item 11 of Table 2.3. The transient responses of the overdamped ($\zeta > 1$) and underdamped ($\zeta < 1$) cases are shown in Figure 2.12. The transient response that occurs when $\zeta < 1$ exhibits an oscillation in which the amplitude decreases with time, and it is called a **damped oscillation**.

The relationship between the s-plane location of the poles and zeros and the form of the transient response can be interpreted from the s-plane pole–zero plots. For example, as seen in Equation (2.37), adjusting the value of $\zeta\omega_n$ varies the $e^{-\zeta\omega_n t}$ envelope, hence the response $y(t)$ shown in Figure 2.12. The larger the value of $\zeta\omega_n$, the faster the damping of the response, $y(t)$. In Figure 2.9, we see that the location of the complex pole s_1 is given by $s_1 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$. So, making $\zeta\omega_n$ larger moves the pole further to the left in the s-plane. Thus, the connection between the location of the pole in the s-plane and the step response is apparent—moving the pole s_1 farther in the left half-plane leads to a faster damping of the transient step response. Of course, most control systems will have more than one complex pair of poles, so the transient response will be the result of the contributions of all the poles. In fact, the magnitude of the response of each pole, represented by the residue, can be visualized by examining the graphical residues on the s-plane. We will discuss the connection between the pole and zero locations and the transient and steady-state response more in subsequent chapters. We will find that the Laplace transformation and the s-plane approach are very useful techniques for system analysis and design where emphasis is placed on the transient and steady-state performance. In fact, because the study of control systems is concerned primarily with the transient and steady-state performance of dynamic systems, we have real cause to appreciate the value of the Laplace transform techniques.

2.5 THE TRANSFER FUNCTION OF LINEAR SYSTEMS

The **transfer function** of a linear system is defined as the ratio of the Laplace transform of the output variable to the Laplace transform of the input variable, with all initial conditions assumed to be zero. The transfer function of a system (or element) represents the relationship describing the dynamics of the system under consideration.

A transfer function may be defined only for a linear, stationary (constant parameter) system. A nonstationary system, often called a time-varying system, has one or more time-varying parameters, and the Laplace transformation may not be utilized. Furthermore, a transfer function is an input–output description of the behavior of a system. Thus, the transfer function description does not include any information concerning the internal structure of the system and its behavior.

The transfer function of the spring-mass-damper system is obtained from the original Equation (2.19), rewritten with zero initial conditions as follows:

$$Ms^2Y(s) + bsY(s) + kY(s) = R(s). \tag{2.38}$$

Then the transfer function is the ratio of the output to the input, or

$$G(s) = \frac{Y(s)}{R(s)} = \frac{1}{Ms^2 + bs + k}. \tag{2.39}$$

The transfer function of the RC network shown in Figure 2.13 is obtained by writing the Kirchhoff voltage equation, yielding

$$V_1(s) = \left(R + \frac{1}{Cs}\right)I(s), \tag{2.40}$$

expressed in terms of transform variables. We shall frequently refer to variables and their transforms interchangeably. The transform variable will be distinguishable by the use of an uppercase letter or the argument (s).

The output voltage is

$$V_2(s) = I(s)\left(\frac{1}{Cs}\right). \tag{2.41}$$

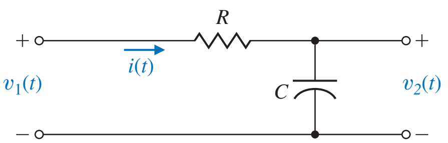


FIGURE 2.13
An RC network.

Therefore, solving Equation (2.40) for $I(s)$ and substituting in Equation (2.41), we have

$$V_2(s) = \frac{(1/Cs)V_1(s)}{R + 1/Cs}.$$

Then the transfer function is obtained as the ratio $V_2(s)/V_1(s)$,

$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{RCs + 1} = \frac{1}{\tau s + 1} = \frac{1/\tau}{s + 1/\tau}, \tag{2.42}$$

where $\tau = RC$, the **time constant** of the network. The single pole of $G(s)$ is $s = -1/\tau$. Equation (2.42) could be immediately obtained if one observes that the circuit is a voltage divider, where

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}, \tag{2.43}$$

and $Z_1(s) = R$, $Z_2 = 1/Cs$.

A multiloop electrical circuit or an analogous multiple-mass mechanical system results in a set of simultaneous equations in the Laplace variable. It is usually more convenient to solve the simultaneous equations by using matrices and determinants [1, 3, 15]. An introduction to matrices and determinants can be found in many references online, as well as on the MCS website.

Let us consider the long-term behavior of a system and determine the response to certain inputs that remain after the transients fade away. Consider the dynamic system represented by the differential equation

$$\begin{aligned}\frac{d^n y(t)}{dt^n} + q_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \ldots + q_0 y(t) \\ = p_{n-1}\frac{d^{n-1}r(t)}{dt^{n-1}} + p_{n-2}\frac{d^{n-2}r(t)}{dt^{n-2}} + \cdots + p_0 r(t),\end{aligned} \tag{2.44}$$

where $y(t)$ is the response, and $r(t)$ is the input or forcing function. If the initial conditions are all zero, then the transfer function is the coefficient of $R(s)$ in

$$Y(s) = G(s)R(s) = \frac{p(s)}{q(s)}R(s) = \frac{p_{n-1}s^{n-1} + p_{n-2}s^{n-2} + \cdots + p_0}{s^n + q_{n-1}s^{n-1} + \cdots + q_0}R(s). \tag{2.45}$$

The output response consists of a natural response (determined by the initial conditions) plus a forced response determined by the input. We now have

$$Y(s) = \frac{m(s)}{q(s)} + \frac{p(s)}{q(s)}R(s),$$

where $q(s) = 0$ is the characteristic equation. If the input has the rational form

$$R(s) = \frac{n(s)}{d(s)},$$

then

$$Y(s) = \frac{m(s)}{q(s)} + \frac{p(s)}{q(s)}\frac{n(s)}{d(s)} = Y_1(s) + Y_2(s) + Y_3(s), \tag{2.46}$$

where $Y_1(s)$ is the partial fraction expansion of the natural response, $Y_2(s)$ is the partial fraction expansion of the terms involving factors of $q(s)$, and $Y_3(s)$ is the partial fraction expansion of terms involving factors of $d(s)$.

Taking the inverse Laplace transform yields

$$y(t) = y_1(t) + y_2(t) + y_3(t).$$

The transient response consists of $y_1(t) + y_2(t)$, and the steady-state response is $y_3(t)$.

EXAMPLE 2.2 **Solution of a differential equation**

Consider a system represented by the differential equation

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2r(t),$$

where the initial conditions are $y(0) = 1, \dfrac{dy}{dt}(0) = 0$, and $r(t) = 1,\ t \geq 0$.

The Laplace transform yields

$$\left[s^2Y(s) - sy(0)\right] + 4\left[sY(s) - y(0)\right] + 3Y(s) = 2R(s).$$

Since $R(s) = 1/s$ and $y(0) = 1$, we obtain

$$Y(s) = \frac{s+4}{s^2+4s+3} + \frac{2}{s\left(s^2+4s+3\right)},$$

where $q(s) = s^2 + 4s + 3 = (s+1)(s+3) = 0$ is the characteristic equation, and $d(s) = s$. Then the partial fraction expansion yields

$$Y(s) = \left[\frac{3/2}{s+1} + \frac{-1/2}{s+3}\right] + \left[\frac{-1}{s+1} + \frac{1/3}{s+3}\right] + \frac{2/3}{s} = Y_1(s) + Y_2(s) + Y_3(s).$$

Hence, the response is

$$y(t) = \left[\frac{3}{2}e^{-t} - \frac{1}{2}e^{-3t}\right] + \left[-1e^{-t} + \frac{1}{3}e^{-3t}\right] + \frac{2}{3},$$

and the steady-state response is

$$\lim_{t\to\infty} y(t) = \frac{2}{3}.$$

EXAMPLE 2.3 **Transfer function of an op-amp circuit**

The operational amplifier (op-amp) belongs to an important class of analog integrated circuits commonly used as building blocks in the implementation of control systems and in many other important applications. Op-amps are active elements (that is, they have external power sources) with a high gain when operating in their linear regions. A model of an ideal op-amp is shown in Figure 2.14.

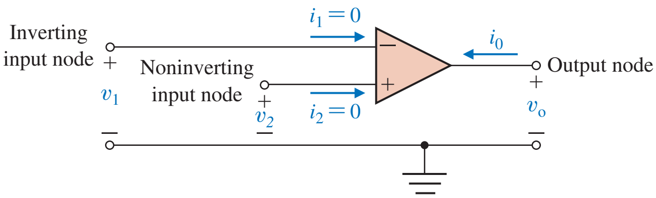


FIGURE 2.14 The ideal op-amp.

The operating conditions for the ideal op-amp are (1) $i_1 = 0$ and $i_2 = 0$, thus implying that the input impedance is infinite, and (2) $v_2 - v_1 = 0$ (or $v_1 = v_2$). The input–output relationship for an ideal op-amp is

$$v_0 = K(v_2 - v_1) = -K(v_1 - v_2),$$

where the gain K approaches infinity. In our analysis, we will assume that the linear op-amps are operating with high gain and under idealized conditions.

Consider the inverting amplifier shown in Figure 2.15. Under ideal conditions, we have $i_1 = 0$, so that writing the node equation at v_1 yields

$$\frac{v_1 - v_{in}}{R_1} + \frac{v_1 - v_0}{R_2} = 0.$$

Since $v_2 = v_1$ (under ideal conditions) and $v_2 = 0$ (see Figure 2.15 and compare it with Figure 2.14), it follows that $v_1 = 0$. Therefore,

$$-\frac{v_{in}}{R_1} - \frac{v_0}{R_2} = 0,$$

and rearranging terms, we obtain

$$\frac{v_0}{v_{in}} = -\frac{R_2}{R_1}.$$

We see that when $R_2 = R_1$, the ideal op-amp circuit inverts the sign of the input, that is, $v_0 = -v_{in}$ when $R_2 = R_1$. ■

EXAMPLE 2.4 Transfer function of a system

Consider the mechanical system shown in Figure 2.16 and its electrical circuit analog shown in Figure 2.17. The electrical circuit analog is a force–current analog as outlined in Table 2.1. The velocities $v_1(t)$ and $v_2(t)$ of the mechanical system are

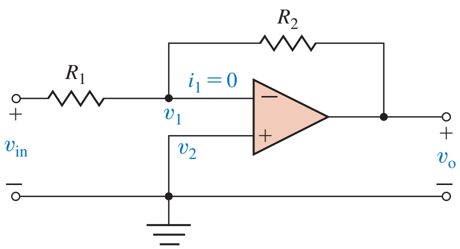


FIGURE 2.15 An Inverting amplifier operating with ideal conditions.

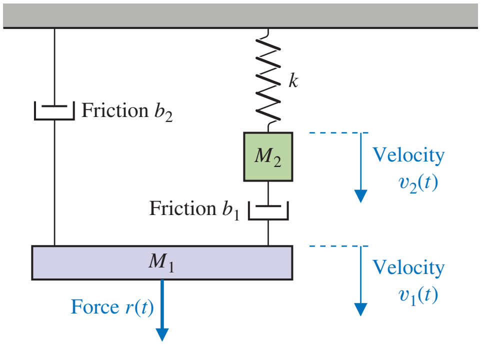


FIGURE 2.16 Two-mass mechanical system.

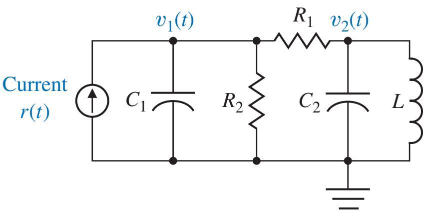


FIGURE 2.17 Two-node electric circuit analog $C_1 = M_1$, $C_2 = M_2$, $L = 1/k$, $R_1 = 1/b_1$, $R_2 = 1/b_2$.

directly analogous to the node voltages $v_1(t)$ and $v_2(t)$ of the electrical circuit. The simultaneous equations, assuming that the initial conditions are zero, are

$$M_1 s V_1(s) + (b_1 + b_2)V_1(s) - b_1 V_2(s) = R(s), \tag{2.47}$$

and

$$M_2 s V_2(s) + b_1 V_2((s) - V_1(s)) + k\frac{V_2(s)}{s} = 0. \tag{2.48}$$

These equations are obtained using the force equations for the mechanical system of Figure 2.16. Rearranging Equations (2.47) and (2.48), we obtain

$$(M_1 s + (b_1 + b_2))V_1(s) + (-b_1)V_2(s) = R(s),$$

$$(-b_1)V_1(s) + \left(M_2 s + b_1 + \frac{k}{s}\right)V_2(s) = 0,$$

or, in matrix form,

$$\begin{bmatrix} M_1 s + b_1 + b_2 & -b_1 \\ -b_1 & M_2 s + b_1 + \dfrac{k}{s} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} R(s) \\ 0 \end{bmatrix}. \tag{2.49}$$

Assuming that the velocity of M_1 is the output variable, we solve for $V_1(s)$ by matrix inversion or Cramer's rule to obtain [1, 3]

$$V_1(s) = \frac{(M_2 s + b_1 + k/s)R(s)}{(M_1 s + b_1 + b_2)(M_2 s + b_1 + k/s) - b_1^2}. \tag{2.50}$$

Then the transfer function of the mechanical (or electrical) system is

$$G(s) = \frac{V_1(s)}{R(s)} = \frac{(M_2 s + b_1 + k/s)}{(M_1 s + b_1 + b_2)(M_2 s + b_1 + k/s) - b_1^2}$$

$$= \frac{(M_2 s^2 + b_1 s + k)}{(M_1 s + b_1 + b_2)(M_2 s^2 + b_1 s + k) - b_1^2 s}. \tag{2.51}$$

If the transfer function in terms of the position $x_1(t)$ is desired, then we have

$$\frac{X_1(s)}{R(s)} = \frac{V_1(s)}{sR(s)} = \frac{G(s)}{s}. \tag{2.52}$$ ■

As an example, let us obtain the transfer function of an important electrical control component, the **DC motor** [8]. A DC motor is used to move loads and is called an **actuator**.

An actuator is a device that provides the motive power to the process.

EXAMPLE 2.5 Transfer function of the DC motor

The DC motor is a power actuator device that delivers energy to a load, as shown in Figure 2.18(a); a sketch of a DC motor is shown in Figure 2.18(b). The DC motor converts direct current (DC) electrical energy into rotational mechanical energy. A major fraction of the torque generated in the rotor (armature) of the motor is available to drive an external load. Because of features such as high torque, speed controllability over a wide range, portability, well-behaved speed–torque characteristics, and adaptability to various types of control methods, DC motors are widely used in numerous control applications, including robotic manipulators, tape transport mechanisms, disk drives, machine tools, and servovalve actuators.

The transfer function of the DC motor will be developed for a linear approximation to an actual motor, and second-order effects, such as hysteresis and the voltage drop across the brushes, will be neglected. The input voltage may be applied to the field or armature terminals. The air-gap flux $\phi(t)$ of the motor is proportional to the field current, provided the field is unsaturated, so that

$$\phi(t) = K_f i_f(t). \tag{2.53}$$

The torque developed by the motor is assumed to be related linearly to $\phi(t)$ and the armature current as follows:

$$T_m(t) = K_1 \phi(t) i_a(t) = K_1 K_f i_f(t) i_a(t). \tag{2.54}$$

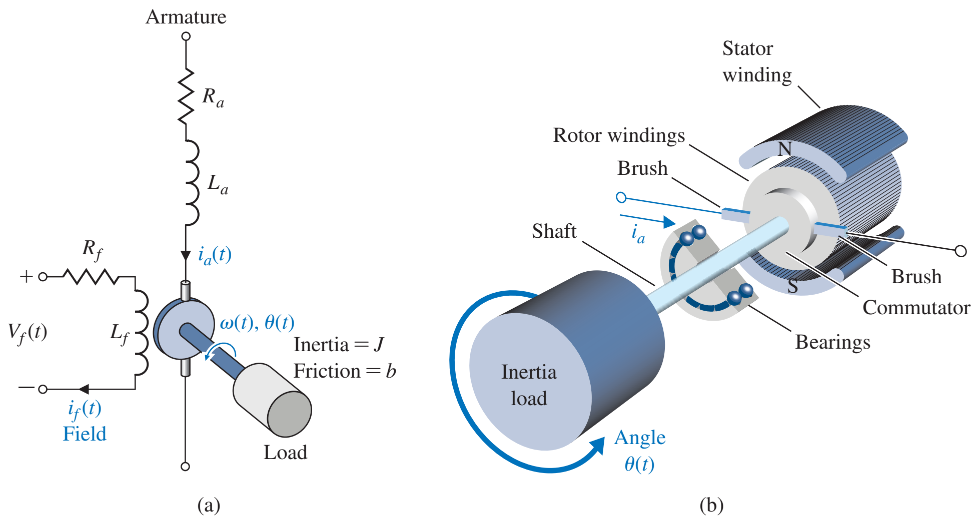


FIGURE 2.18 A DC motor (a) electrical diagram and (b) sketch.

It is clear from Equation (2.54) that, to have a linear system, one current must be maintained constant while the other current becomes the input current. First, we shall consider the field current controlled motor, which provides a substantial power amplification. Then we have, in Laplace transform notation,

$$T_m(s) = \left(K_1 K_f I_a\right) I_f(s) = K_m I_f(s), \tag{2.55}$$

where $i_a = I_a$ is a constant armature current, and K_m is defined as the motor constant. The field current is related to the field voltage as

$$V_f(s) = \left(R_f + L_f s\right) I_f(s). \tag{2.56}$$

The motor torque $T_m(s)$ is equal to the torque delivered to the load. This relation may be expressed as

$$T_m(s) = T_L(s) + T_d(s), \tag{2.57}$$

where $T_L(s)$ is the load torque and $T_d(s)$ is the disturbance torque, which is often negligible. However, the disturbance torque often must be considered in systems subjected to external forces such as antenna wind-gust forces. The load torque for rotating inertia, as shown in Figure 2.18, is written as

$$T_L(s) = Js^2\theta(s) + bs\theta(s). \tag{2.58}$$

Rearranging Equations (2.55)–(2.57), we have

$$T_L(s) = T_m(s) - T_d(s), \tag{2.59}$$

$$T_m(s) = K_m I_f(s), \tag{2.60}$$

$$I_f(s) = \frac{V_f(s)}{R_f + L_f s}. \tag{2.61}$$

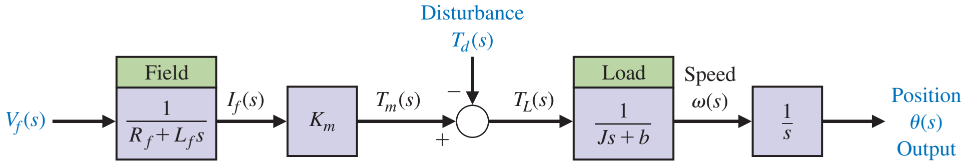


FIGURE 2.19 Block diagram model of field-controlled DC motor.

Therefore, the transfer function of the motor–load combination, with $T_d(s) = 0$, is

$$\frac{\theta(s)}{V_f(s)} = \frac{K_m}{s(Js+b)(L_f s + R_f)} = \frac{K_m/(JL_f)}{s(s+b/J)(s+R_f/L_f)}. \tag{2.62}$$

The block diagram model of the field-controlled DC motor is shown in Figure 2.19. Alternatively, the transfer function may be written in terms of the time constants of the motor as

$$\frac{\theta(s)}{V_f(s)} = G(s) = \frac{K_m/(bR_f)}{s(\tau_f s + 1)(\tau_L s + 1)}, \tag{2.63}$$

where $\tau_f = L_f/R_f$ and $\tau_L = J/b$. Typically, one finds that $\tau_L > \tau_f$ and often the field time constant may be neglected.

The armature-controlled DC motor uses the armature current i_a as the control variable. The stator field can be established by a field coil and current or a permanent magnet. When a constant field current is established in a field coil, the motor torque is

$$T_m(s) = (K_1 K_f I_f) I_a(s) = K_m I_a(s). \tag{2.64}$$

When a permanent magnet is used, we have

$$T_m(s) = K_m I_a(s),$$

where K_m is a function of the permeability of the magnetic material.

The armature current is related to the input voltage applied to the armature by

$$V_a(s) = (R_a + L_a s) I_a(s) + V_b(s), \tag{2.65}$$

where $V_b(s)$ is the back electromotive-force voltage proportional to the motor speed. Therefore, we have

$$V_b(s) = K_b \omega(s), \tag{2.66}$$

where $\omega(s) = s\theta(s)$ is the transform of the angular speed and the armature current is

$$I_a(s) = \frac{V_a(s) - K_b \omega(s)}{R_a + L_a s}. \tag{2.67}$$

Equations (2.58) and (2.59) represent the load torque, so that

$$T_L(s) = Js^2\theta(s) + bs\theta(s) = T_m(s) - T_d(s). \tag{2.68}$$