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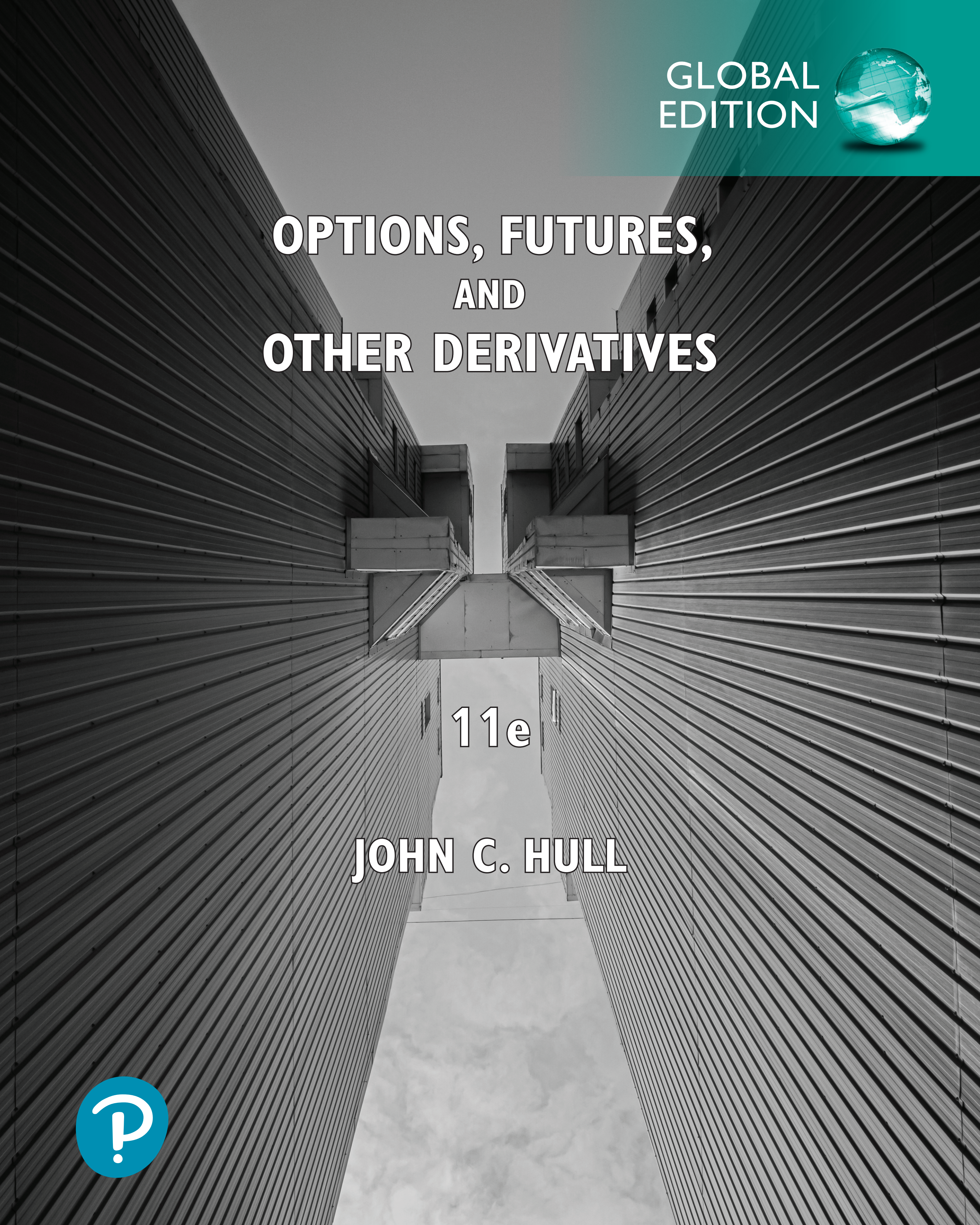
OPTIONS, FUTURES, AND OTHER DERIVATIVES



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JOHN C. HULL

OPTIONS, FUTURES, AND OTHER DERIVATIVES

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To Michelle

CONTENTS IN BRIEF

CONTENTS

BUSINESS SNAPSHOTS

TECHNICAL NOTES

Available on the Author's Website
www-2.rotman.utoronto.ca/~hull/technicalnotes

1. Convexity Adjustments to Eurodollar Futures
2. Properties of the Lognormal Distribution
3. Warrant Valuation When Value of Equity plus Warrants Is Lognormal
4. Exact Procedure for Valuing American Calls on Stocks Paying a Single Dividend
5. Calculation of the Cumulative Probability in a Bivariate Normal Distribution
6. Differential Equation for Price of a Derivative on a Stock Paying a Known Dividend Yield
7. Differential Equation for Price of a Derivative on a Futures Price
8. Analytic Approximation for Valuing American Options
9. Generalized Tree-Building Procedure
10. The Cornish–Fisher Expansion to Estimate VaR
11. Manipulation of Credit Transition Matrices
12. Calculation of Cumulative Noncentral Chi-Square Distribution
13. Efficient Procedure for Valuing American-Style Lookback Options
14. The Hull–White Two-Factor Model
15. Valuing Options on Coupon-Bearing Bonds in a One-Factor Interest Rate Model
16. Construction of an Interest Rate Tree with Nonconstant Time Steps and Nonconstant Parameters
17. The Process for the Short Rate in an HJM Term Structure Model
18. Valuation of a Compounding Swap
19. Valuation of an Equity Swap
20. Changing the Market Price of Risk for Variables That Are Not the Prices of Traded Securities
21. Hermite Polynomials and Their Use for Integration
22. Valuation of a Variance Swap
23. The Black, Derman, Toy Model
24. Proof that Forward and Futures Prices are Equal When Interest Rates Are Constant
25. A Cash-Flow Mapping Procedure
26. A Binomial Measure of Credit Correlation
27. Calculation of Moments for Valuing Asian Options
28. Calculation of Moments for Valuing Basket Options
29. Proof of Extensions to Itô's Lemma
30. The Return of a Security Dependent on Multiple Sources of Uncertainty
31. Properties of Ho–Lee and Hull–White Interest Rate Models

PREFACE

Derivatives markets have seen many changes over the last 30 years. Successive editions of *Options, Futures, and Other Derivatives* have managed to keep up to date. The book has an applied approach. It is a very popular college text, but it can also be found on trading-room desks throughout the world. (Indeed, I receive emails from derivatives practitioners about the book almost every day.) The blending of material useful for practitioners with material appropriate for university courses is what makes the book unique.

NEW TO THIS EDITION

- A major change in financial markets will be the phase-out of LIBOR. This has led to important changes throughout the 11th edition. The overnight reference rates that will replace LIBOR, and the way they are used to determine zero curves, are discussed carefully.
- Within-chapter examples and end-of-chapter problems that were previously based on LIBOR have been largely replaced by examples based on the new reference rates or by generic examples.
- The likely impact of the new reference rates on valuation models is discussed.
- The new reference rates are considered to be risk-free whereas LIBOR incorporates a time-varying credit spread. The book discusses the desire on the part of banks to augment the new reference rates with a measure of the level of credit spreads in the market.
- The chapter on Wiener processes now covers fractional Brownian motion. This is becoming increasingly used in modeling volatility.
- Rough volatility models which have in the last few years been found to fit volatility surfaces well are added to the models considered in Chapter 27.
- Machine learning is becoming increasingly used in pricing and hedging derivatives. The reader is introduced to these applications at various points in the book.
- Changes in the regulatory environment, including Basel IV, are covered.
- The end-of-chapter problems have been updated. To make the book as easy to use as possible, solutions to all end-of-chapter problems are now on www.pearsonglobaleditions.com and www-2.rotman.utoronto.ca/~hull.

- Instructor support material has been revised. In particular, there are now many more suggestions on assignment questions that can be used in conjunction with chapters.
- The DerivaGem software is less LIBOR-focused and is available for download from www-2.rotman.utoronto.ca/~hull/software.
- Tables, charts, market data, and examples have been updated throughout the book.

SOLVING TEACHING AND LEARNING CHALLENGES

Most instructors find that courses in derivatives are fun to teach. There is not a big gap between theory and practice. Most students know a little about the subject and are motivated to learn more. Usually there is some current news that can be discussed in class, e.g., the level of the VIX index or events that affect particular option prices.

Math Knowledge

Math is the key challenge for many students taking a course in derivatives. I have kept this in mind in the way material is presented throughout the book. Instructors are often faced with a trade-off between mathematical rigor and the simplicity with which an idea is explained. My preference is always to look for the simplest way of explaining an idea in the first instance. Sometimes using words rather than equations is effective. I avoid using notation that has lots of subscripts, superscripts, and function arguments as far as possible because this can be off-putting to a reader who is new to the material. Nonessential mathematical material has been either eliminated or included in technical notes on my website.

The reality is that many students only understand an equation when they have seen numbers substituted into it. For that reason, many numerical examples have been included in the text. The software DerivaGem (discussed below) allows students to get a feel for equations by trying different inputs.

I am often asked about the math prerequisites for *Options, Futures and Other Derivatives*. Students will be able to cope with a course based on this book if they are comfortable with algebra and understand probabilities and probability distributions. A knowledge of calculus concepts is useful for parts of the book. But no knowledge of stochastic calculus is assumed. The basic knowledge of stochastic processes that is needed for a more advanced understanding of derivatives is explained carefully in Chapter 14.

End of Chapter Problems

As in earlier editions, there are many other end-of-chapter problems to help students apply the ideas presented in the chapters. These have been updated. The distinction between "practice questions" and "further questions" has been eliminated. Answers to all end-of-chapter problems are on my website and available through www.pearsonglobaleditions.com.

Designing a Course

There are many ways in which the material in the book can be used. Instructors teaching an introductory course in derivatives tend to spend most time on the first 20 chapters, and often choose to omit Chapter 14 and Section 15.6. Instructors teaching a more advanced course find that many different combinations of chapters in the second half of the book can be used. I find that Chapter 37 is a fun chapter that works well at the end of either an introductory or an advanced course.

Software

The DerivaGem software is an important part of the book. Students get comfortable with the models presented in the book when they use DerivaGem to value transactions under different assumptions. The use of the software is explained at the end of the book.

I recommend giving students assignments that involve using the basic DG400a.xls software. There are many types of assignments that can be developed. For example, students can be asked to compare American or European option prices given by a binomial model with those from the Black–Scholes–Merton model. They can be asked to report what happens as the number of time steps is increased in a binomial model and can use the software to display trees. (DerivaGem can display trees with up to 10 time steps and can calculate prices and Greek letters using up to 500 time steps.)

Many charts can be produced using the software and students can include those charts in reports produced for the instructor. The calculation of zero curves and swap valuation is made easy with DerivaGem. I like to use DerivaGem in class when I illustrate some key concepts.

Students taking a more advanced course in derivatives can be asked to compare prices given by different models using the *Alternative Models* worksheet in DG400a.xls. Alternatives to Black–Scholes that are covered include CEV, Merton mixed jump–diffusion, variance gamma, Heston, and SABR. Students can also be asked to carry out assignments concerned with the use of different models for pricing bond options. The CDS and CDO worksheets can be used in conjunction with each other for an assignment if CDOs are covered.

DerivaGem can be used in conjunction with current market data that can be downloaded from Yahoo Finance or other providers. For example, students can be asked to compare implied volatilities for options on different stocks that have been in the news. They can also be asked to calculate volatility term structures and volatility smiles for stock indices. Assignments such as these can be important because they make the underlying concepts more “real” and lead to interesting classroom discussions.

The *DG400 Applications* software enables students to carry out assignments where they are asked investigate issues such as how the performance of delta hedging is improved as the interval between rebalancing is decreased or how managing gamma can improve the performance of delta hedging. Assuming students have a basic knowledge of Excel, they should have no difficulty using this software and changing instructions as necessary.

The *DG400 Functions* software is a little more challenging. It contains the functions used by DG400a.xls. Students can use these functions to develop their own Excel worksheets in order to investigate particular issues and answer assignment questions.

Many instructors find DerivaGem to be a really useful resource. DerivaGem can be downloaded from www-2.rotman.utoronto.ca/~hull/software.

Slides

Several hundred PowerPoint slides accompany this book. They can be a useful starting point for instructors. Those who adopt the text are welcome to adapt the slides to meet their needs. These slides are available on www.pearsonglobaleditions.com and www-2.rotman.utoronto.ca/~hull.

Technical Notes

There are over 30 technical notes available. They are referred to in the text and can be downloaded from www-2.rotman.utoronto.ca/~hull/TechnicalNotes.

By not including the Technical Notes in the book, I am able to streamline the presentation of material so that it is more reader-friendly.

EMPLOYABILITY

A natural question for students is: "Will a course in derivatives improve my chances of a getting a job in finance?" The answer is an overwhelming yes. Probably the first thing many students think about when considering options or other derivatives is an exchange such as the CBOE. In fact, as Chapter 1 makes clear, the over-the-counter (OTC) market is much larger than the exchange-traded market and likely to be much more important to students in their first job (or subsequent jobs). *Options, Futures, and Other Derivatives* has a much bigger focus on the OTC market than most other derivatives texts.

Derivatives have steadily increased in importance. Potential employers can be classified as "buy side" and "sell side". The buy side includes nonfinancial corporations, insurance companies, fund managers, and some other financial institutions. The sell side consists of large financial institutions who act as market makers. Many students who take courses in derivatives may not become derivatives traders or derivatives analysts. However, derivatives now permeate all aspects of finance. If you work in investment banking, there is likely to be a derivatives component to some of the deals you are involved in; if you work in fund management, you will probably find derivatives to be convenient tools for some purposes; if you work for a nonfinancial corporation, you may be involved in using derivative contracts for hedging and negotiating with a sell-side institution; and so on. Whatever your role in finance, it is important that you be able to talk about derivatives knowledgeably, use the right words, and understand the motivations of a counterparty to a transaction. A course based on *Options, Futures, and Other Derivatives* will help you do this.

What about those of you who want to specialize in derivatives? I have literally lost count of the many successful derivative executives who have told me "Thank you for your book. I read it before the interview, and it got me my first job in derivatives." (My joking response has typically been: "Great, but you realize that means you owe me 20% of your first year's salary.") The people I am talking about typically had engineering, physics, or other quantitative backgrounds at the time of the interview but had never taken a course in finance! So, while the book is important for those planning a career in finance, it is absolutely essential reading for all those aspiring to a career in derivatives. As mentioned earlier, it is found on trading-room desks throughout the world.

This book will help you develop your quant skills so that you become more marketable in finance. But other skills are of course important. Good communication skills are necessary. Many instructors ask students to present the results of projects in class. Students should take full advantage of these opportunities to practice and improve. If presentations are recorded, they should review the recording carefully.

At my business school, we used to run optional mock interviews and other self-development activities for students. Interestingly, the students that took advantage of them tended to be the ones that already had fairly good skills. The students that really needed help did not participate. (We have since made the activities mandatory.) I would urge all students to take advantage of all opportunities to improve their soft skills. Do not dismiss them as unimportant.

What are other important skills? The book discusses the regulatory environment for derivatives which changed a lot following the 2008 financial crisis. Make sure you understand the issues and are familiar with the latest developments. You should also use a derivatives course to help develop your critical thinking skills. Ask questions in class and do not be afraid to express an opinion about an issue.

A potential employer will want to be convinced that you can work well with others. While at university you will be involved in many group projects and should take this opportunity to develop good collaboration skills. You may find some members of your group difficult to work with, but this is also likely to be true in your first full-time job. Go to an interview prepared to talk about your experiences working with other students.

In addition to quant skills and knowledge of derivatives, I have mentioned that communication skills, the ability to work collaboratively, and critical thinking are soft skills that you should try and develop to make sure you get that first job. Another I might add is social responsibility. It is not an accident that most successful corporate executives are actively involved in community activities. Be prepared to talk about sustainable finance, which is an aspect of social responsibility and becoming an increasingly important area within finance.

ACKNOWLEDGMENTS

Many people have played a part in the development of successive editions of this book. Indeed, the list of people who have provided me with feedback on the book is now so long that it is not possible to mention everyone. I have benefited from the advice of many academics who have taught from the book and from the comments of many derivatives practitioners. I would like to thank the students in my courses at the University of Toronto who have made many suggestions on how the material can be improved. Eddie Mizzi from The Geometric Press did an excellent job editing the final manuscript and handling page composition. Emilio Barone from Luiss Guido Carli University in Rome provided many detailed comments. Andrés Olivé provided valuable research assistance.

Alan White, a colleague at the University of Toronto, deserves a special acknowledgment. Alan and I have been carrying out joint research and consulting in the areas of derivatives and risk management for over 30 years. During that time, we have spent many hours discussing key issues. Many of the new ideas in this book, and many of the new ways used to explain old ideas, are as much Alan's as mine. Alan has done most of the development work on the DerivaGem software.

Special thanks are due to the many people at Pearson I have worked with for over 30 years. Those who have worked with me on the 11th edition include Neeraj Bhalla,

Sugandh Juneja, and Emily Biberger. I would like to thank them for their enthusiasm, advice, and encouragement.

I welcome comments on the book from readers. My e-mail address is:

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John Hull

About the Author

John Hull is the Maple Financial Professor of Derivatives and Risk Management at the Joseph L. Rotman School of Management, University of Toronto. He was in 2016 awarded the title of University Professor (an honor granted to only 2% of faculty at the University of Toronto). He is an internationally recognized authority on derivatives and risk management and has many publications in this area. His work has an applied focus. He has acted as consultant to many financial institutions throughout the world and has won many teaching awards, including the University of Toronto's prestigious Northrop Frye award. His research and teaching activities include risk management, regulation, and machine learning, as well as derivatives. He is co-director of Rotman's Master of Finance and Master of Financial Risk Management programs.

1 CHAPTER Introduction

In the last 40 years, derivatives have become increasingly important in finance. Futures and options are actively traded on many exchanges throughout the world. Many different types of forward contracts, swaps, options, and other derivatives are entered into by financial institutions, fund managers, and corporate treasurers in the over-the-counter market. Derivatives are added to bond issues, used in executive compensation plans, embedded in capital investment opportunities, used to transfer risks in mortgages from the original lenders to investors, and so on. We have now reached the stage where those who work in finance, and many who work outside finance, need to understand how derivatives work, how they are used, and how they are priced.

Whether you love derivatives or hate them, you cannot ignore them! The derivatives market is huge—much bigger than the stock market when measured in terms of underlying assets. The value of the assets underlying outstanding derivatives transactions is several times the world gross domestic product. As we shall see in this chapter, derivatives can be used for hedging or speculation or arbitrage. They can transfer a wide range of risks in the economy from one entity to another.

A *derivative* involves two parties agreeing to a future tranasaction. Its value depends on (or derives from) the values of other underlying variables. Very often the variables underlying derivatives are the prices of traded assets. A stock option, for example, is a derivative whose value is dependent on the price of a stock. However, derivatives can be dependent on almost any variable, from the price of hogs to the amount of snow falling at a certain ski resort.

Since the first edition of this book was published in 1988 there have been many developments in derivatives markets. For example:

- Many new instruments such as credit derivatives, electricity derivatives, weather derivatives, and insurance derivatives have been developed.
- Many new types of interest rate, foreign exchange, and equity derivatives now trade.
- There have been many new ideas in risk management and risk measurement.
- Real option methods for capital investment appraisal have been developed.
- The financial crisis of 2008 occurred, with derivatives (perhaps unfairly) getting much of the blame.

- Many regulations affecting the over-the-counter derivatives market have been introduced.
- The "risk-free" discount rate used to value derivatives has changed and the decision has been taken to phase out LIBOR.
- Derivatives dealers now adjust the way they price derivatives to allow for credit risks, funding costs, and capital requirements.
- Collateral and credit issues are now given much more attention and have led to changes in the way derivatives are traded.
- Machine learning is now becoming widely used for managing derivatives portfolios.

The book has evolved to keep up to date with these developments. For example: the 2008 financial crisis is discussed in Chapter 8; changes in the interest rates used for derivatives pricing are discussed in Chapter 4; valuation adjustments are covered in Chapter 9; real options are explained in Chapter 36; credit derivatives are covered in Chapter 25; energy, weather, and insurance derivatives are covered in Chapter 35. Machine learning applications are discussed at various points in the book.

In this opening chapter, we take a first look at derivatives markets and how they are changing. We contrast exchange-traded and over-the-counter derivatives markets and review recent regulatory changes affecting the markets. We describe forward, futures, and options markets and provide examples of how they are used by hedgers, speculators, and arbitrageurs. Later in the book we will elaborate on many of the points made in this chapter.

1.1 EXCHANGE-TRADED MARKETS

A derivatives exchange is a market where individuals and companies trade standardized contracts that have been defined by the exchange. Derivatives exchanges have existed for a long time. The Chicago Board of Trade (CBOT) was established in 1848 to bring farmers and merchants together. Initially its main task was to standardize the quantities and qualities of the grains that were traded. Within a few years, the first futures-type contract was developed. It was known as a *to-arrive contract*. Speculators soon became interested in the contract and found trading the contract to be an attractive alternative to trading the grain itself. A rival futures exchange, the Chicago Mercantile Exchange (CME), was established in 1919. Now futures exchanges exist all over the world. (See table at the end of the book.) The CME and CBOT have merged to form the CME Group (www.cmegroup.com), which also includes the New York Mercantile Exchange (NYMEX), and the Kansas City Board of Trade (KCBT).

The Chicago Board Options Exchange (CBOE, www.cboe.com) started trading call option contracts on 16 stocks in 1973. Options had traded prior to 1973, but the CBOE succeeded in creating an orderly market with well-defined contracts. Put option contracts started trading on the exchange in 1977. The CBOE now trades options on thousands of stocks and many different stock indices. Like futures, options have proved to be very popular contracts. Many other exchanges throughout the world now trade options. (See table at the end of the book.) The underlying assets include foreign currencies and futures contracts as well as stocks and stock indices.

Once two traders have agreed to trade a product offered by an exchange, it is handled by the exchange clearing house. This stands between the two traders and manages the risks. Suppose, for example, that trader A enters into a futures contract to buy 100 ounces of gold from trader B in six months for $1,750 per ounce. The result of this trade will be that A has a contract to buy 100 ounces of gold from the clearing house at $1,750 per ounce in six months and B has a contract to sell 100 ounces of gold to the clearing house for $1,750 per ounce in six months. The advantage of this arrangement is that traders do not have to worry about the creditworthiness of the people they are trading with. The clearing house takes care of credit risk by requiring each of the two traders to deposit funds (known as margin) with the clearing house to ensure that they will live up to their obligations. Margin requirements and the operation of clearing houses are discussed in more detail in Chapter 2.

Electronic Markets

Traditionally derivatives exchanges have used what is known as the *open outcry system*. This involves traders physically meeting on the floor of the exchange, shouting, and using a complicated set of hand signals to indicate the trades they would like to carry out. Exchanges have largely replaced the open outcry system by *electronic trading*. This involves traders entering their desired trades at a keyboard and a computer being used to match buyers and sellers. The open outcry system has its advocates, but, as time passes, it is becoming less and less used.

Electronic trading has led to a growth in high-frequency trading. This involves the use of algorithms to initiate trades, often without human intervention, and has become an important feature of derivatives markets.

1.2 OVER-THE-COUNTER MARKETS

Not all derivatives trading is on exchanges. Many trades take place in the *over-the-counter* (OTC) market. Banks, other large financial institutions, fund managers, and corporations are the main participants in OTC derivatives markets. Once an OTC trade has been agreed, the two parties can either present it to a central counterparty (CCP) or clear the trade bilaterally. A CCP is like an exchange clearing house. It stands between the two parties to the derivatives transaction so that one party does not have to bear the risk that the other party will default. When trades are cleared bilaterally, the two parties have usually signed an agreement covering all their transactions with each other. The issues covered in the agreement include the circumstances under which outstanding transactions can be terminated, how settlement amounts are calculated in the event of a termination, and how the collateral (if any) that must be posted by each side is calculated. CCPs and bilateral clearing are discussed in more detail in Chapter 2.

Large banks often act as market makers for the more commonly traded instruments. This means that they are always prepared to quote a bid price (at which they are prepared to take one side of a derivatives transaction) and an ask price (at which they are prepared to take the other side).

Business Snapshot 1.1 The Lehman Bankruptcy

On September 15, 2008, Lehman Brothers filed for bankruptcy. This was the largest bankruptcy in U.S. history and its ramifications were felt throughout derivatives markets. Almost until the end, it seemed as though there was a good chance that Lehman would survive. A number of companies (e.g., the Korean Development Bank, Barclays Bank in the United Kingdom, and Bank of America) expressed interest in buying it, but none of these was able to close a deal. Many people thought that Lehman was "too big to fail" and that the U.S. government would have to bail it out if no purchaser could be found. This proved not to be the case.

How did this happen? It was a combination of high leverage, risky investments, and liquidity problems. Commercial banks that take deposits are subject to regulations on the amount of capital they must keep. Lehman was an investment bank and not subject to these regulations. By 2007, its leverage ratio had increased to 31:1, which means that a 3–4% decline in the value of its assets would wipe out its capital. Dick Fuld, Lehman's Chairman and Chief Executive Officer, encouraged an aggressive deal-making, risk-taking culture. He is reported to have told his executives: "Every day is a battle. You have to kill the enemy." The Chief Risk Officer at Lehman was competent, but did not have much influence and was even removed from the executive committee in 2007. The risks taken by Lehman included large positions in the instruments created from subprime mortgages, which will be described in Chapter 8. Lehman funded much of its operations with short-term debt. When there was a loss of confidence in the company, lenders refused to renew this funding, forcing it into bankruptcy.

Lehman was very active in the over-the-counter derivatives markets. It had over a million transactions outstanding with about 8,000 different counterparties. Lehman's counterparties were often required to post collateral and this collateral had in many cases been used by Lehman for various purposes. Litigation aimed at determining who owes what to whom continued for many years after the bankruptcy filing.

Prior to the financial crisis, which started in 2007 and is discussed in some detail in Chapter 8, OTC derivatives markets were largely unregulated. Following the financial crisis and the failure of Lehman Brothers (see Business Snapshot 1.1), we have seen the development of many new regulations affecting the operation of OTC markets. The main objectives of the regulations are to improve the transparency of OTC markets and reduce systemic risk (see Business Snapshot 1.2). The over-the-counter market in some respects is being forced to become more like the exchange-traded market. Three important changes are:

1. Standardized OTC derivatives between two financial institutions in the United States must, whenever possible, be traded on what are referred to a *swap execution facilities* (SEFs). These are platforms similar to exchanges where market participants can post bid and ask quotes and where market participants can trade by accepting the quotes of other market participants.
2. There is a requirement in most parts of the world that a CCP be used for most standardized derivatives transactions between financial institutions.
3. All trades must be reported to a central repository.

Business Snapshot 1.2 Systemic Risk

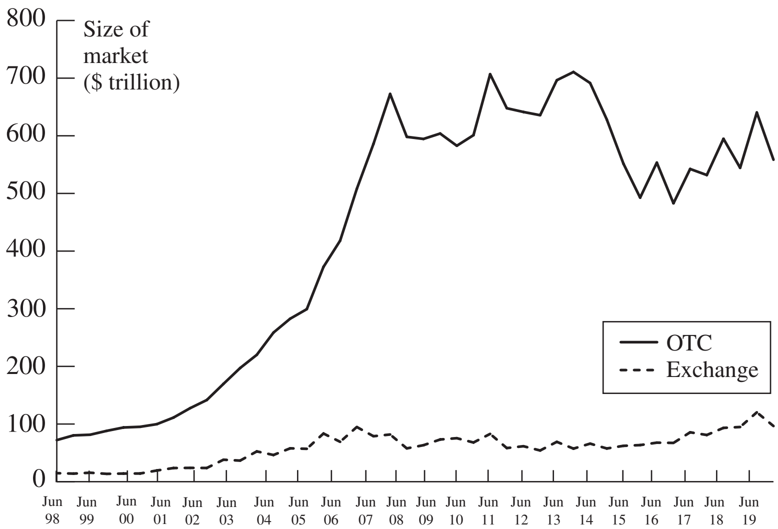
Systemic risk is the risk that a default by one financial institution will create a "ripple effect" that leads to defaults by other financial institutions and threatens the stability of the financial system. There are huge numbers of over-the-counter transactions between banks. If Bank A fails, Bank B may take a huge loss on the transactions it has with Bank A. This in turn could lead to Bank B failing. Bank C that has many outstanding transactions with both Bank A and Bank B might then take a large loss and experience severe financial difficulties; and so on.

The financial system has survived defaults such as Drexel in 1990 and Lehman Brothers in 2008, but regulators continue to be concerned. During the market turmoil of 2007 and 2008, many large financial institutions were bailed out, rather than being allowed to fail, because governments were concerned about systemic risk.

Market Size

Both the over-the-counter and the exchange-traded market for derivatives are huge. The number of derivatives transactions per year in OTC markets is smaller than in exchange-traded markets, but the average size of the transactions is much greater. Although the statistics that are collected for the two markets are not exactly comparable, it is clear that the volume of business in the over-the-counter market is much larger than in the exchange-traded market. The Bank for International Settlements (www.bis.org) started collecting statistics on the markets in 1998. Figure 1.1 compares (a) the estimated total principal amounts underlying transactions that were outstanding in the over-the-counter markets between June 1998 and December 2019 and (b) the estimated total value of the assets underlying exchange-traded contracts during the same period. Using these measures, the size of the over-the-counter market in December 2019 was $558.5 trillion

Figure 1.1 Size of over-the-counter and exchange-traded derivatives markets.



and the size of the exchange-traded market was \$96.5 trillion.[1] Figure 1.1 shows that the OTC market grew rapidly up to 2007, but has seen very little net growth since then. One reason for the lack of growth is the popularity of *compression*. This is a procedure where two or more counterparties restructure transactions with each other with the result that the underlying principal is reduced.

In interpreting Figure 1.1, we should bear in mind that the principal underlying an over-the-counter transaction is not the same as its value. An example of an over-the-counter transaction is an agreement to buy 100 million U.S. dollars with British pounds at a predetermined exchange rate in 1 year. The total principal amount underlying this transaction is \$100 million. However, the value of the transaction might be only \$1 million. The Bank for International Settlements estimates the gross market value of all over-the-counter transactions outstanding in December 2019 to be about \$11.6 trillion.[2]

1.3 FORWARD CONTRACTS

A relatively simple derivative is a *forward contract*. It is an agreement to buy or sell an asset at a certain future time for a certain price. It can be contrasted with a *spot contract*, which is an agreement to buy or sell an asset almost immediately. A forward contract is traded in the over-the-counter market—usually between two financial institutions or between a financial institution and one of its clients.

One of the parties to a forward contract assumes a *long position* and agrees to buy the underlying asset on a certain specified future date for a certain specified price. The other party assumes a *short position* and agrees to sell the asset on the same date for the same price.

Forward contracts on foreign exchange are very popular. Most large banks employ both spot and forward foreign-exchange traders. As we shall see in Chapter 5, there is a relationship between forward prices, spot prices, and interest rates in the two currencies. Table 1.1 provides quotes for the exchange rate between the British pound (GBP) and the U.S. dollar (USD) that might be made by a large international bank on May 21, 2020. The quote is for the number of USD per GBP. The first row indicates that the

Table 1.1 Spot and forward quotes for the exchange rate between USD and GBP on May 21, 2020 (GBP = British pound; USD = U.S. dollar; quote is number of USD per GBP).

	Bid	*Ask*
Spot	1.2217	1.2220
1-month forward	1.2218	1.2222
3-month forward	1.2220	1.2225
6-month forward	1.2224	1.2230

[1] When a CCP stands between two sides in an OTC transaction, two transactions are considered to have been created for the purposes of the BIS statistics.

[2] A contract that is worth \$1 million to one side and −\$1 million to the other side would be counted as having a gross market value of \$1 million.

bank is prepared to buy GBP (also known as sterling) in the spot market (i.e., for virtually immediate delivery) at the rate of \$1.2217 per GBP and sell sterling in the spot market at \$1.2220 per GBP. The second, third, and fourth rows indicate that the bank is prepared to buy sterling in 1, 3, and 6 months at \$1.2218, \$1.2220, and \$1.2224 per GBP, respectively, and to sell sterling in 1, 3, and 6 months at \$1.2222, \$1.2225, and \$1.2230 per GBP, respectively.

Forward contracts can be used to hedge foreign currency risk. Suppose that, on May 21, 2020, the treasurer of a U.S. corporation knows that the corporation will pay £1 million in 6 months (i.e., on November 21, 2020) and wants to hedge against exchange rate moves. Using the quotes in Table 1.1, the treasurer can agree to buy £1 million 6 months forward at an exchange rate of 1.2230. The corporation then has a long forward contract on GBP. It has agreed that on November 21, 2020, it will buy £1 million from the bank for \$1.2230 million. The bank has a short forward contract on GBP. It has agreed that on November 21, 2020, it will sell £1 million for \$1.2230 million. Both sides have made a binding commitment.

Payoffs from Forward Contracts

Consider the position of the corporation in the trade we have just described. What are the possible outcomes? The forward contract obligates the corporation to buy £1 million for \$1,223,000. If the spot exchange rate rose to, say, 1.3000, at the end of the 6 months, the forward contract would be worth \$77,000 (= \$1,300,000 − \$1,223,000) to the corporation. It would enable £1 million to be purchased at an exchange rate of 1.2230 rather than 1.3000. Similarly, if the spot exchange rate fell to 1.2000 at the end of the 6 months, the forward contract would have a negative value to the corporation of \$23,000 because it would lead to the corporation paying \$23,000 more than the market price for the sterling.

In general, the payoff from a long position in a forward contract on one unit of an asset is

$$S_T - K$$

where K is the delivery price and S_T is the spot price of the asset at maturity of the contract. This is because the holder of the contract is obligated to buy an asset worth S_T for K. Similarly, the payoff from a short position in a forward contract on one unit of an asset is

$$K - S_T$$

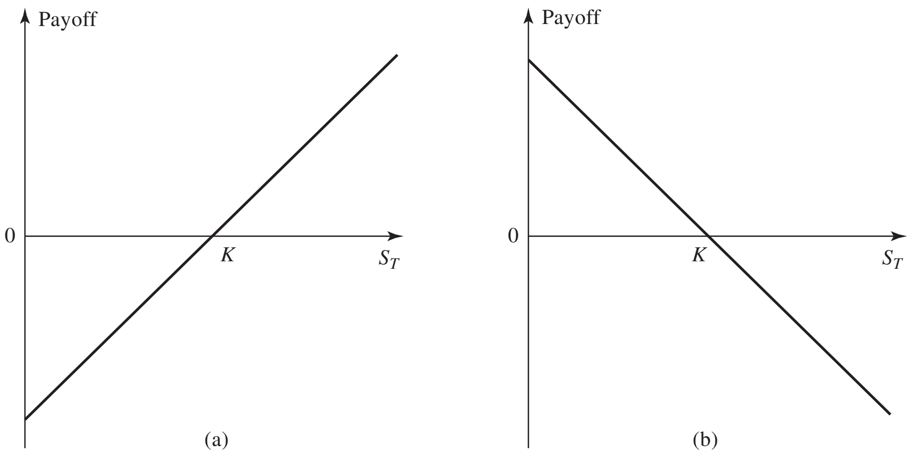
These payoffs can be positive or negative. They are illustrated in Figure 1.2. Because it costs nothing to enter into a forward contract, the payoff from the contract is also the trader's total gain or loss from the contract.

In the example just considered, $K = 1.2230$ and the corporation has a long contract. When $S_T = 1.3000$, the payoff is \$0.077 per £1; when $S_T = 1.2000$, it is −\$0.023 per £1.

Forward Prices and Spot Prices

We shall be discussing in some detail the relationship between spot and forward prices in Chapter 5. For a quick preview of why the two are related, consider a stock that pays no dividend and is worth \$60. You can borrow or lend money for 1 year at 5%. What should the 1-year forward price of the stock be?

Figure 1.2 Payoffs from forward contracts: (a) long position, (b) short position. Delivery price $= K$; price of asset at contract maturity $= S_T$.



The answer is \$60 grossed up at 5% for 1 year, or \$63. If the forward price is more than this, say \$67, you could borrow \$60, buy one share of the stock, and sell it forward for \$67. After paying off the loan, you would net a profit of \$4 in 1 year. If the forward price is less than \$63, say \$58, an investor owning the stock as part of a portfolio would sell the stock for \$60 and enter into a forward contract to buy it back for \$58 in 1 year. The proceeds of investment would be invested at 5% to earn \$3. The investor would end up \$5 better off than if the stock were kept in the portfolio for the year.

1.4 FUTURES CONTRACTS

Like a forward contract, a futures contract is an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price. Unlike forward contracts, futures contracts are normally traded on an exchange. To make trading possible, the exchange specifies certain standardized features of the contract. As the two parties to the contract do not necessarily know each other, the exchange clearing house stands between them as mentioned earlier.

Two large exchanges on which futures contracts are traded are the Chicago Board of Trade (CBOT) and the Chicago Mercantile Exchange (CME), which have now merged to form the CME Group. On these and other exchanges throughout the world, a very wide range of commodities and financial assets form the underlying assets in the various contracts. The commodities include pork bellies, live cattle, sugar, wool, lumber, copper, aluminum, gold, and tin. The financial assets include stock indices, currencies, and Treasury bonds. Futures prices are regularly reported in the financial press. Suppose that, on September 1, the December futures price of gold is quoted as \$1,750. This is the price, exclusive of commissions, at which traders can agree to buy or sell gold for December delivery. It is determined in the same way as other prices (i.e., by the laws of supply and demand). If more traders want to go long than to go short, the price goes up; if the reverse is true, then the price goes down.

Further details on issues such as margin requirements, daily settlement procedures, delivery procedures, bid–ask spreads, and the role of the exchange clearing house are given in Chapter 2.

1.5 OPTIONS

Options are traded both on exchanges and in the over-the-counter market. There are two types of option. A *call option* gives the holder the right to buy the underlying asset by a certain date for a certain price. A *put option* gives the holder the right to sell the underlying asset by a certain date for a certain price. The price in the contract is known as the *exercise price* or *strike price*; the date in the contract is known as the *expiration date* or *maturity*. *American options* can be exercised at any time up to the expiration date. *European options* can be exercised only on the expiration date itself.[3] Most of the options that are traded on exchanges are American. In the exchange-traded equity option market, one contract is usually an agreement to buy or sell 100 shares. European options are generally easier to analyze than American options, and some of the properties of an American option are frequently deduced from those of its European counterpart.

It should be emphasized that an option gives the holder the right to do something. The holder does not have to exercise this right. This is what distinguishes options from forwards and futures, where the holder is obligated to buy or sell the underlying asset. Whereas it costs nothing to enter into a forward or futures contract, except for margin requirements which will be discussed in Chapter 2, there is a cost to acquiring an option.

The largest exchange in the world for trading stock options is the Chicago Board Options Exchange (CBOE; www.cboe.com). Table 1.2 gives the bid and ask quotes for some of the call options trading on Apple (ticker symbol: AAPL), on May 21, 2020. Table 1.3 does the same for put options trading on Apple on that date. The quotes are taken from the CBOE website. The Apple stock price at the time of the quotes was bid 316.23, ask 316.50. The bid–ask spread for an option (as a percent of the price) is usually

Table 1.2 Prices of call options on Apple, May 21, 2020; stock price: bid \$316.23, ask \$316.50 (*Source*: CBOE).

Strike price	*June 2020*		*September 2020*		*December 2020*	
(\$)	*Bid*	*Ask*	*Bid*	*Ask*	*Bid*	*Ask*
290	29.80	30.85	39.35	40.40	46.20	47.60
300	21.55	22.40	32.50	33.90	40.00	41.15
310	14.35	15.30	26.35	27.25	34.25	35.65
320	8.65	9.00	20.45	21.70	28.65	29.75
330	4.20	5.00	15.85	16.25	23.90	24.75
340	1.90	2.12	11.35	12.00	19.50	20.30

[3] Note that the terms *American* and *European* do not refer to the location of the option or the exchange. Some options trading on North American exchanges are European.

Table 1.3 Prices of put options on Apple, May 21, 2020; stock price: bid \$316.23, ask \$316.50 (*Source*: CBOE).

Strike price	*June 2020*		*September 2020*		*December 2020*	
(\$)	*Bid*	*Ask*	*Bid*	*Ask*	*Bid*	*Ask*
290	3.00	3.30	12.70	13.65	20.05	21.30
300	4.80	5.20	15.85	16.85	23.60	24.90
310	7.15	7.85	19.75	20.50	28.00	28.95
320	11.25	12.05	24.05	24.80	32.45	33.35
330	17.10	17.85	28.75	29.85	37.45	38.40
340	24.40	25.45	34.45	35.65	42.95	44.05

much greater than that for the underlying stock and depends on the volume of trading. The option strike prices in Tables 1.2 and 1.3 are \$290, \$300, \$310, \$320, \$330, and \$340. The maturities are June 2020, September 2020, and December 2020. The precise expiration day is the third Friday of the expiration month. The June options expire on June 19, 2020, the September options on September 18, 2020, and the December options on December 18, 2020.

The tables illustrate a number of properties of options. The price of a call option decreases as the strike price increases, while the price of a put option increases as the strike price increases. Both types of option tend to become more valuable as their time to maturity increases. These properties of options will be discussed further in Chapter 11.

Suppose a trader instructs a broker to buy one December call option contract on Apple with a strike price of \$340. The broker will relay these instructions to a trader at the CBOE and the deal will be done. The (ask) price indicated in Table 1.2 is \$20.30. This is the price for an option to buy one share. In the United States, an option contract is a contract to buy or sell 100 shares. Therefore, the trader must arrange for \$2,030 to be remitted to the exchange through the broker. The exchange will then arrange for this amount to be passed on to the party on the other side of the transaction.

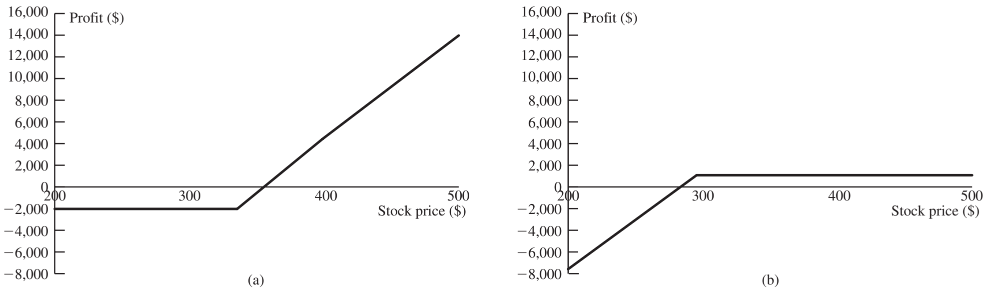
In our example, the trader has obtained at a cost of \$2,030 the right to buy 100 Apple shares for \$340 each. If the price of Apple does not rise above \$340 by December 18, 2020, the option is not exercised and the trader loses \$2,030.[4] But if Apple does well and the option is exercised when the bid price for the stock is \$400, the trader is able to buy 100 shares at \$340 and immediately sell them for \$400 for a profit of \$6,000, or \$3,970 when the initial cost of the option contract is taken into account.[5]

An alternative trade would be to sell one September put option contract with a strike price of \$290 at the bid price of \$12.70. The trader receives $100 \times 12.70 = \$1{,}270$. If the Apple stock price stays above \$290, the option is not exercised and the trader makes a \$1,270 profit. However, if stock price falls and the option is exercised when the stock price is \$250, there is a loss. The trader must buy 100 shares at \$290 when they are worth only \$250. This leads to a loss of \$4,000, or \$2,730 when the initial amount received for the option contract is taken into account.

[4] The calculations here ignore any commissions paid by the trader.

[5] The calculations here ignore the effect of discounting. The \$6,000 should be discounted from the time of exercise to the purchase date when calculating the profit.

Figure 1.3 Net profit from (a) purchasing a contract consisting of 100 Apple December call options with a strike price of $340 and (b) selling a contract consisting of 100 Apple September put options with a strike price of $290.



The stock options trading on the CBOE are American. If we assume for simplicity that they are European, so that they can be exercised only at maturity, the trader's profit as a function of the final stock price for the two trades we have considered is shown in Figure 1.3.

Further details about the operation of options markets and how prices such as those in Tables 1.2 and 1.3 are determined by traders are given in later chapters. At this stage we note that there are four types of participants in options markets:

1. Buyers of calls
2. Sellers of calls
3. Buyers of puts
4. Sellers of puts.

Buyers are referred to as having *long positions*; sellers are referred to as having *short positions*. Selling an option is also known as *writing the option*.

1.6 TYPES OF TRADERS

Derivatives markets have been outstandingly successful. The main reason is that they have attracted many different types of traders and have a great deal of liquidity. When a trader wants to take one side of a contract, there is usually no problem in finding someone who is prepared to take the other side.

Three broad categories of traders can be identified: hedgers, speculators, and arbitrageurs. Hedgers use derivatives to reduce the risk that they face from potential future movements in a market variable. Speculators use them to bet on the future direction of a market variable. Arbitrageurs take offsetting positions in two or more instruments to lock in a profit. As described in Business Snapshot 1.3, hedge funds have become big users of derivatives for all three purposes.

In the next few sections, we will consider the activities of each type of trader in more detail.

Business Snapshot 1.3 Hedge Funds

Hedge funds have become major users of derivatives for hedging, speculation, and arbitrage. They are similar to mutual funds in that they invest funds on behalf of clients. However, they accept funds only from professional fund managers or financially sophisticated individuals and do not publicly offer their securities. Mutual funds are subject to regulations requiring that the shares be redeemable at any time, that investment policies be disclosed, that the use of leverage be limited, and so on. Hedge funds are relatively free of these regulations. This gives them a great deal of freedom to develop sophisticated, unconventional, and proprietary investment strategies. The fees charged by hedge fund managers are dependent on the fund's performance and are relatively high—typically 1 to 2% of the amount invested plus 20% of the profits. Hedge funds have grown in popularity, with about $2 trillion being invested in them throughout the world. "Funds of funds" have been set up to invest in a portfolio of hedge funds.

The investment strategy followed by a hedge fund manager often involves using derivatives to set up a speculative or arbitrage position. Once the strategy has been defined, the hedge fund manager must:

1. Evaluate the risks to which the fund is exposed
2. Decide which risks are acceptable and which will be hedged
3. Devise strategies (usually involving derivatives) to hedge the unacceptable risks.

Here are some examples of the labels used for hedge funds together with the trading strategies followed:

Long/Short Equities: Purchase securities considered to be undervalued and short those considered to be overvalued in such a way that the exposure to the overall direction of the market is small.

Convertible Arbitrage: Take a long position in a thought-to-be-undervalued convertible bond combined with an actively managed short position in the underlying equity.

Distressed Securities: Buy securities issued by companies in, or close to, bankruptcy.

Emerging Markets: Invest in debt and equity of companies in developing or emerging countries and in the debt of the countries themselves.

Global Macro: Carry out trades that reflect anticipated global macroeconomic trends.

Merger Arbitrage: Trade after a possible merger or acquisition is announced so that a profit is made if the announced deal takes place.

1.7 HEDGERS

In this section we illustrate how hedgers can reduce their risks with forward contracts and options.

Hedging Using Forward Contracts

Suppose that it is May 21, 2020, and ImportCo, a company based in the United States, knows that it will have to pay £10 million on August 21, 2020, for goods it has purchased from a British supplier. The GBP/USD exchange rate quotes made by a financial institution are shown in Table 1.1. ImportCo could hedge its foreign exchange risk by buying pounds (GBP) from the financial institution in the 3-month forward market at 1.2225. This would have the effect of fixing the price to be paid to the British exporter at $12,225,000.

Consider next another U.S. company, which we will refer to as ExportCo, that is exporting goods to the United Kingdom and, on May 21, 2020, knows that it will receive £30 million 3 months later. ExportCo can hedge its foreign exchange risk by selling £30 million in the 3-month forward market at an exchange rate of 1.2220. This would have the effect of locking in the U.S. dollars to be realized for the sterling at $36,660,000.

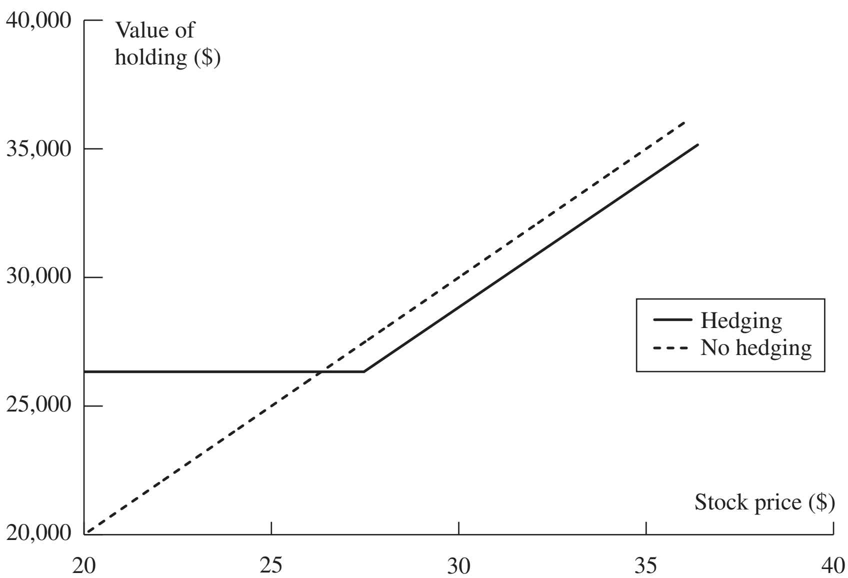
Note that a company might do better if it chooses not to hedge than if it chooses to hedge. Alternatively, it might do worse. Consider ImportCo. If the exchange rate is 1.2000 on August 21 and the company has not hedged, the £10 million that it has to pay will cost $12,000,000, which is less than $12,225,000. On the other hand, if the exchange rate is 1.3000, the £10 million will cost $13,000,000—and the company will wish that it had hedged! The position of ExportCo if it does not hedge is the reverse. If the exchange rate in August proves to be less than 1.2220, the company will wish that it had hedged; if the rate is greater than 1.2220, it will be pleased that it has not done so.

This example illustrates a key aspect of hedging. The purpose of hedging is to reduce risk. There is no guarantee that the outcome with hedging will be better than the outcome without hedging.

Hedging Using Options

Options can also be used for hedging. Consider an investor who in May of a particular year owns 1,000 shares of a particular company. The share price is $28 per share. The investor is concerned about a possible share price decline in the next 2 months and wants protection. The investor could buy ten July put option contracts on the

Figure 1.4 Value of the stock holding in 2 months with and without hedging.



company's stock with a strike price of \$27.50. Each contract is on 100 shares, so this would give the investor the right to sell a total of 1,000 shares for a price of \$27.50. If the quoted option price is \$1, then each option contract would cost $100 \times \$1 = \$100$ and the total cost of the hedging strategy would be $10 \times \$100 = \$1{,}000$.

The strategy costs \$1,000 but guarantees that the shares can be sold for at least \$27.50 per share during the life of the option. If the market price of the stock falls below \$27.50, the options will be exercised, so that \$27,500 is realized for the entire holding. When the cost of the options is taken into account, the amount realized is \$26,500. If the market price stays above \$27.50, the options are not exercised and expire worthless. However, in this case the value of the holding is always above \$27,500 (or above \$26,500 when the cost of the options is taken into account). Figure 1.4 shows the net value of the portfolio (after taking the cost of the options into account) as a function of the stock price in 2 months. The dotted line shows the value of the portfolio assuming no hedging.

A Comparison

There is a fundamental difference between the use of forward contracts and options for hedging. Forward contracts are designed to neutralize risk by fixing the price that the hedger will pay or receive for the underlying asset. Option contracts, by contrast, provide insurance. They offer a way for investors to protect themselves against adverse price movements in the future while still allowing them to benefit from favorable price movements. Unlike forwards, options involve the payment of an up-front fee.

1.8 SPECULATORS

We now move on to consider how futures and options markets can be used by speculators. Whereas hedgers want to avoid exposure to adverse movements in the price of an asset, speculators wish to take a position in the market. Either they are betting that the price of the asset will go up or they are betting that it will go down.

Speculation Using Futures

Consider a U.S. speculator who in May thinks that the British pound will strengthen relative to the U.S. dollar over the next 2 months and is prepared to back that hunch to the tune of £250,000. One thing the speculator can do is purchase £250,000 in the spot market in the hope that the sterling can be sold later at a higher price. (The sterling once purchased would be kept in an interest-bearing account.) Another possibility is to take a long position in four CME July futures contracts on sterling. (Each futures contract is for the purchase of £62,500 in July.) Table 1.4 summarizes the two alternatives on the assumption that the current exchange rate is 1.2220 dollars per pound and the July futures price is 1.2223 dollars per pound. If the exchange rate turns out to be 1.3000 dollars per pound in July, the futures contract alternative enables the speculator to realize a profit of $(1.3000 - 1.2223) \times 250{,}000 = \$19{,}425$. The spot market alternative leads to 250,000 units of an asset being purchased for \$1.2220 in May and sold for \$1.3000 in July, so that a profit of $(1.3000 - 1.2220) \times 250{,}000 = \$19{,}500$ is made. If the exchange rate falls to 1.2000 dollars per pound, the futures contract gives rise to a $(1.2223 - 1.2000) \times 250{,}000 = \$5{,}575$ loss, whereas the spot market alternative gives

Table 1.4 Speculation using spot and futures contracts. One futures contract is on £62,500. Initial margin on four futures contracts = \$20,000.

	Possible trades	
	Buy £250,000 *Spot price = 1.2220*	*Buy 4 futures contracts* *Futures price = 1.2223*
Investment	\$305,500	\$20,000
Profit if July spot = 1.3000	\$19,500	\$19,425
Profit if July spot = 1.2000	−\$5,500	−\$5,575

rise to a loss of $(1.2220 - 1.2000) \times 250{,}000 = \$5{,}500$. The futures market alternative appears to give rise to slightly worse outcomes for both scenarios. But this is because the calculations do not reflect the interest that is earned or paid.

What then is the difference between the two alternatives? The first alternative of buying sterling requires an up-front investment of $250{,}000 \times 1.2220 = \$305{,}500$. In contrast, the second alternative requires only a small amount of cash to be deposited by the speculator in what is termed a "margin account". (The operation of margin accounts is explained in Chapter 2.) In Table 1.4, the initial margin requirement is assumed to be \$5,000 per contract, or \$20,000 in total. The futures market allows the speculator to obtain leverage. With a relatively small initial outlay, a large speculative position can be taken.

Speculation Using Options

Options can also be used for speculation. Suppose that it is October and a speculator considers that a stock is likely to increase in value over the next 2 months. The stock price is currently \$20, and a 2-month call option with a \$22.50 strike price is currently selling for \$1. Table 1.5 illustrates two possible alternatives, assuming that the speculator is willing to invest \$2,000. One alternative is to purchase 100 shares; the other involves the purchase of 2,000 call options (i.e., 20 call option contracts). Suppose that the speculator's hunch is correct and the price of the stock rises to \$27 by December. The first alternative of buying the stock yields a profit of

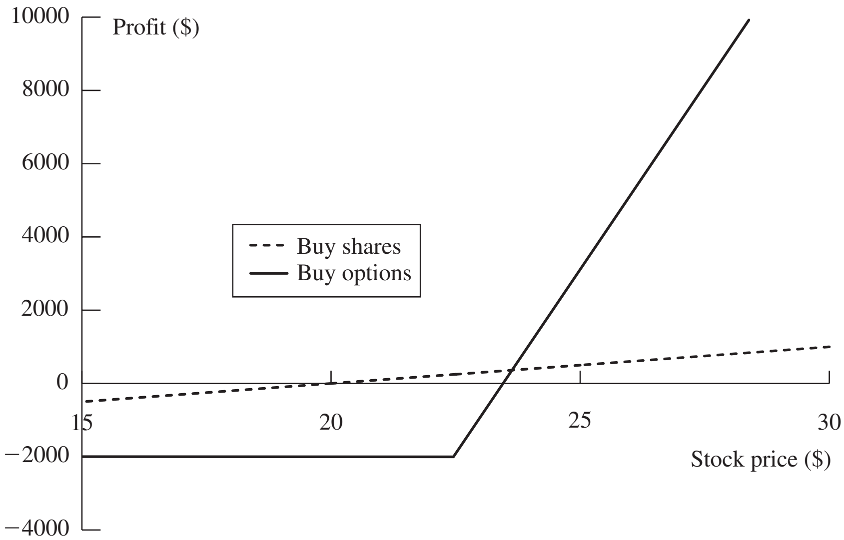
$$100 \times (\$27 - \$20) = \$700$$

However, the second alternative is far more profitable. A call option on the stock with a strike price of \$22.50 gives a payoff of \$4.50, because it enables something worth \$27 to

Table 1.5 Comparison of profits from two alternative strategies for using \$2,000 to speculate on a stock worth \$20 in October.

	December stock price	
Speculator's strategy	*\$15*	*\$27*
Buy 100 shares	−\$500	\$700
Buy 2,000 call options	−\$2,000	\$7,000

Figure 1.5 Profit or loss from two alternative strategies for speculating on a stock currently worth $20.



be bought for $22.50. The total payoff from the 2,000 options that are purchased under the second alternative is

$$2{,}000 \times \$4.50 = \$9{,}000$$

Subtracting the original cost of the options yields a net profit of

$$\$9{,}000 - \$2{,}000 = \$7{,}000$$

The options strategy is, therefore, 10 times more profitable than directly buying the stock.

Options also give rise to a greater potential loss. Suppose the stock price falls to $15 by December. The first alternative of buying stock yields a loss of

$$100 \times (\$20 - \$15) = \$500$$

Because the call options expire without being exercised, the options strategy would lead to a loss of $2,000—the original amount paid for the options. Figure 1.5 shows the profit or loss from the two strategies as a function of the stock price in 2 months.

Options like futures provide a form of leverage. For a given investment, the use of options magnifies the financial consequences. Good outcomes become very good, while bad outcomes result in the whole initial investment being lost.

A Comparison

Futures and options are similar instruments for speculators in that they both provide a way in which a type of leverage can be obtained. However, there is an important difference between the two. When a speculator uses futures, the potential loss as well as the potential gain is very large. When options are purchased, no matter how bad things get, the speculator's loss is limited to the amount paid for the options.

1.9 ARBITRAGEURS

Arbitrageurs are a third important group of participants in futures, forward, and options markets. Arbitrage involves locking in a riskless profit by simultaneously entering into transactions in two or more markets. In later chapters we will see how arbitrage is sometimes possible when the futures price of an asset gets out of line with its spot price. We will also examine how arbitrage can be used in options markets. This section illustrates the concept of arbitrage with a very simple example.

Let us consider a stock that is traded on both the New York Stock Exchange (www.nyse.com) and the London Stock Exchange (www.londonstockexchange.com). Suppose that the stock price is \$120 in New York and £100 in London at a time when the exchange rate is \$1.2300 per pound. An arbitrageur could simultaneously buy 100 shares of the stock in New York and sell them in London to obtain a risk-free profit of

$$100 \times [(\$1.23 \times 100) - \$120]$$

or \$300 in the absence of transactions costs. Transactions costs would probably eliminate the profit for a small trader. However, a large investment bank faces very low transactions costs in both the stock market and the foreign exchange market. It would find the arbitrage opportunity very attractive and would try to take as much advantage of it as possible.

Arbitrage opportunities such as the one just described cannot last for long. As arbitrageurs buy the stock in New York, the forces of supply and demand will cause the dollar price to rise. Similarly, as they sell the stock in London, the sterling price will be driven down. Very quickly the two prices will become equivalent at the current exchange rate. Indeed, the existence of profit-hungry arbitrageurs makes it unlikely that a major disparity between the sterling price and the dollar price could ever exist in the first place. Generalizing from this example, we can say that the very existence of arbitrageurs means that in practice only very small arbitrage opportunities are observed in the prices that are quoted in most financial markets. In this book most of the arguments concerning futures prices, forward prices, and the values of option contracts will be based on the assumption that no arbitrage opportunities exist.

1.10 DANGERS

Derivatives are very versatile instruments. As we have seen, they can be used for hedging, for speculation, and for arbitrage. It is this very versatility that can cause problems. Sometimes traders who have a mandate to hedge risks or follow an arbitrage strategy become (consciously or unconsciously) speculators. The results can be disastrous. One example of this is provided by the activities of Jérôme Kerviel at Société Général (see Business Snapshot 1.4).

To avoid the sort of problems Société Général encountered, it is very important for both financial and nonfinancial corporations to set up controls to ensure that derivatives are being used for their intended purpose. Risk limits should be set and the activities of traders should be monitored daily to ensure that these risk limits are adhered to.

Unfortunately, even when traders follow the risk limits that have been specified, big mistakes can happen. Some of the activities of traders in the derivatives market during

Business Snapshot 1.4 SocGen's Big Loss in 2008

Derivatives are very versatile instruments. They can be used for hedging, speculation, and arbitrage. One of the risks faced by a company that trades derivatives is that an employee who has a mandate to hedge or to look for arbitrage opportunities may become a speculator.

Jérôme Kerviel joined Société Général (SocGen) in 2000 to work in the compliance area. In 2005, he was promoted and became a junior trader in the bank's Delta One products team. He traded equity indices such as the German DAX index, the French CAC 40, and the Euro Stoxx 50. His job was to look for arbitrage opportunities. These might arise if a futures contract on an equity index was trading for a different price on two different exchanges. They might also arise if equity index futures prices were not consistent with the prices of the shares constituting the index. (This type of arbitrage is discussed in Chapter 5.)

Kerviel used his knowledge of the bank's procedures to speculate while giving the appearance of arbitraging. He took big positions in equity indices and created fictitious trades to make it appear that he was hedged. In reality, he had large bets on the direction in which the indices would move. The size of his unhedged position grew over time to tens of billions of euros.

In January 2008, his unauthorized trading was uncovered by SocGen. Over a three-day period, the bank unwound his position for a loss of 4.9 billion euros. This was at the time the biggest loss created by fraudulent activity in the history of finance. (Later in the year, a much bigger loss from Bernard Madoff's Ponzi scheme came to light.)

Rogue trader losses were not unknown at banks prior to 2008. For example, in the 1990s, Nick Leeson, who worked at Barings Bank, had a mandate similar to that of Jérôme Kerviel. His job was to arbitrage between Nikkei 225 futures quotes in Singapore and Osaka. Instead he found a way to make big bets on the direction of the Nikkei 225 using futures and options, losing \$1 billion and destroying the 200-year old bank in the process. In 2002, it was found that John Rusnak at Allied Irish Bank had lost \$700 million from unauthorized foreign exchange trading. The lessons from these losses are that it is important to define unambiguous risk limits for traders and then to monitor what they do very carefully to make sure that the limits are adhered to.

the period leading up to the start of the financial crisis in July 2007 proved to be much riskier than they were thought to be by the financial institutions they worked for. As will be discussed in Chapter 8, house prices in the United States had been rising fast. Most people thought that the increases would continue—or, at worst, that house prices would simply level off. Very few were prepared for the steep decline that actually happened. Furthermore, very few were prepared for the high correlation between mortgage default rates in different parts of the country. Some risk managers did express reservations about the exposures of the companies for which they worked to the U.S. real estate market. But, when times are good (or appear to be good), there is an unfortunate tendency to ignore risk managers and this is what happened at many financial institutions during the 2006–2007 period. The key lesson from the financial crisis is that financial institutions should always be dispassionately asking "What can go wrong?", and they should follow that up with the question "If it does go wrong, how much will we lose?"

SUMMARY

One of the exciting developments in finance over the last 40 years has been the growth of derivatives markets. In many situations, both hedgers and speculators find it more attractive to trade a derivative on an asset than to trade the asset itself. Some derivatives are traded on exchanges; others are traded by financial institutions, fund managers, and corporations in the over-the-counter market, or added to new issues of debt and equity securities. Much of this book is concerned with the valuation of derivatives. The aim is to present a unifying framework within which all derivatives—not just options or futures—can be valued.

In this chapter we have taken a first look at forward, futures, and option contracts. A forward or futures contract involves an obligation to buy or sell an asset at a certain time in the future for a certain price. There are two types of options: calls and puts. A call option gives the holder the right to buy an asset by a certain date for a certain price. A put option gives the holder the right to sell an asset by a certain date for a certain price. Forwards, futures, and options trade on a wide range of different underlying assets.

The success of derivatives can be attributed to their versatility. They can be used by: hedgers, speculators, and arbitrageurs. Hedgers are in the position where they face risk associated with the price of an asset. They use derivatives to reduce or eliminate this risk. Speculators wish to bet on future movements in the price of an asset. They use derivatives to get extra leverage. Arbitrageurs are in business to take advantage of a discrepancy between prices in two different markets. If, for example, they see the futures price of an asset getting out of line with the cash price, they will take offsetting positions in the two markets to lock in a profit.

FURTHER READING

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Practice Questions

1.1. Explain carefully the difference between selling a call option and buying a put option.

1.2. An investor enters into a short forward contract to sell 100,000 British pounds for U.S. dollars at an exchange rate of 1.3000 USD per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is (a) 1.2900 and (b) 1.3200?

1.3. A trader enters into a short cotton futures contract when the futures price is 50 cents per pound. The contract is for the delivery of 50,000 pounds. How much does the trader gain or lose if the cotton price at the end of the contract is (a) 48.20 cents per pound and (b) 51.30 cents per pound?

1.4. Suppose that you write a put contract with a strike price of $40 and an expiration date in 3 months. The current stock price is $41 and the contract is on 100 shares. What have you committed yourself to? How much could you gain or lose?

1.5. You would like to speculate on a rise in the price of a certain stock. The current stock price is $29 and a 3-month call with a strike price of $30 costs $2.90. You have $5,800 to invest. Identify two alternative investment strategies, one in the stock and the other in an option on the stock. What are the potential gains and losses from each?

1.6. Suppose that you own 5,000 shares worth $25 each. How can put options be used to provide you with insurance against a decline in the value of your holding over the next 4 months?

1.7. When first issued, a stock provides funds for a company. Is the same true of a stock option? Discuss.

1.8. Explain why a futures contract can be used for either speculation or hedging.

1.9. Suppose that a March call option to buy a share for $50 costs $2.50 and is held until March. Under what circumstances will the holder of the option make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity of the option.

1.10. Suppose that a June put option to sell a share for $60 costs $4 and is held until June. Under what circumstances will the seller of the option (i.e., the party with the short position) make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity of the option.

1.11. It is May and a trader writes a September call option with a strike price of $20. The stock price is $18 and the option price is $2. Describe the trader's cash flows if the option is held until September and the stock price is $25 at that time.

1.12. A trader writes a December put option with a strike price of \$30. The price of the option is \$4. Under what circumstances does the trader make a gain?

1.13. A company knows that it is due to receive a certain amount of a foreign currency in 4 months. What type of option contract is appropriate for hedging?

1.14. A U.S. company expects to have to pay 1 million Canadian dollars in 6 months. Explain how the exchange rate risk can be hedged using (a) a forward contract and (b) an option.

1.15. A trader enters into a short forward contract on 100 million yen. The forward exchange rate is \$0.0090 per yen. How much does the trader gain or lose if the exchange rate at the end of the contract is (a) \$0.0084 per yen and (b) \$0.0101 per yen?

1.16. The CME Group offers a futures contract on long-term Treasury bonds. Characterize the traders likely to use this contract.

1.17. "Options and futures are zero-sum games." What do you think is meant by this?

1.18. Describe the profit from the following portfolio: a long forward contract on an asset and a long European put option on the asset with the same maturity as the forward contract and a strike price that is equal to the forward price of the asset at the time the portfolio is set up.

1.19. In the 1980s, Bankers Trust developed *index currency option notes* (ICONs). These were bonds in which the amount received by the holder at maturity varied with a foreign exchange rate. One example was its trade with the Long Term Credit Bank of Japan. The ICON specified that if the yen/USD exchange rate, S_T, is greater than 169 yen per dollar at maturity (in 1995), the holder of the bond receives \$1,000. If it is less than 169 yen per dollar, the amount received by the holder of the bond is

$$1{,}000 - \max\left[0,\ 1{,}000\left(\frac{169}{S_T} - 1\right)\right]$$

When the exchange rate is below 84.5, nothing is received by the holder at maturity. Show that this ICON is a combination of a regular bond and two options.

1.20. On July 1, 2021, a company enters into a forward contract to buy 10 million Japanese yen on January 1, 2022. On September 1, 2021, it enters into a forward contract to sell 10 million Japanese yen on January 1, 2022. Describe the payoff from this strategy.

1.21. Suppose that USD/sterling spot and forward exchange rates are as follows:

Spot	1.2580
90-day forward	1.2556
180-day forward	1.2518

What opportunities are open to an arbitrageur in the following situations?

(a) A 180-day European call option to buy £1 for \$1.22 costs 2 cents.
(b) A 90-day European put option to sell £1 for \$1.29 costs 2 cents.

1.22. A trader buys a call option with a strike price of \$30 for \$3. Does the trader ever exercise the option and lose money on the trade? Explain your answer.

1.23. A trader sells a put option with a strike price of \$40 for \$5. What is the trader's maximum gain and maximum loss? How does your answer change if it is a call option?

1.24. "Buying a put option on a stock when the stock is owned is a form of insurance." Explain this statement.

1.25. On May 21, 2020, as indicated in Table 1.2, the spot ask price of Apple stock is $316.50 and the ask price of a call option with a strike price of $320 and a maturity date of September is $21.70. A trader is considering two alternatives: buy 100 shares of the stock and buy 100 September call options. For each alternative, what is (a) the upfront cost, (b) the total gain if the stock price in September is $400, and (c) the total loss if the stock price in September is $300. Assume that the option is not exercised before September and positions are unwound at option maturity.

1.26. What is arbitrage? Explain the arbitrage opportunity when the price of a dually listed mining company stock is $50 (USD) on the New York Stock Exchange and $60 (CAD) on the Toronto Stock Exchange. Assume that the exchange rate is such that 1 U.S. dollar equals 1.21 Canadian dollars. Explain what is likely to happen to prices as traders take advantage of this opportunity.

1.27. Trader A enters into a forward contract to buy an asset for $1,000 in one year. Trader B buys a call option to buy the asset for $1,000 in one year. The cost of the option is $100. What is the difference between the positions of the traders? Show the profit as a function of the price of the asset in one year for the two traders.

1.28. In March, a U.S. investor instructs a broker to sell one July put option contract on a stock. The stock price is $42 and the strike price is $40. The option price is $3. Explain what the investor has agreed to. Under what circumstances will the trade prove to be profitable? What are the risks?

1.29. A U.S. company knows it will have to pay 3 million euros in three months. The current exchange rate is 1.1500 dollars per euro. Discuss how forward and options contracts can be used by the company to hedge its exposure.

1.30. A stock price is $29. A trader buys one call option contract on the stock with a strike price of $30 and sells a call option contract on the stock with a strike price of $32.50. The market prices of the options are $2.75 and $1.50, respectively. The options have the same maturity date. Describe the trader's position.

1.31. The price of gold is currently $1,200 per ounce. The forward price for delivery in 1 year is $1,300 per ounce. An arbitrageur can borrow money at 3% per annum. What should the arbitrageur do? Assume that the cost of storing gold is zero and that gold provides no income.

1.32. On May 21, 2020, an investor owns 100 Apple shares. As indicated in Table 1.3, the share price is about $316 and a December put option with a strike price of $290 costs $21.30. The investor is comparing two alternatives to limit downside risk. The first involves buying one December put option contract with a strike price of $290. The second involves instructing a broker to sell the 100 shares as soon as Apple's price reaches $290. Discuss the advantages and disadvantages of the two strategies.

1.33. A bond issued by Standard Oil some time ago worked as follows. The holder received no interest. At the bond's maturity the company promised to pay $1,000 plus an additional amount based on the price of oil at that time. The additional amount was equal to the product of 170 and the excess (if any) of the price of a barrel of oil at maturity over $25. The maximum additional amount paid was $2,550 (which corresponds to a price of $40 per barrel). Show that the bond is a combination of a regular bond, a long position in call options on oil with a strike price of $25, and a short position in call options on oil with a strike price of $40.

1.34. Suppose that in the situation of Table 1.1 a corporate treasurer said: "I will have £1 million to sell in 6 months. If the exchange rate is less than 1.19, I want you to give me 1.19. If it is greater than 1.25, I will accept 1.25. If the exchange rate is between 1.19 and 1.25, I will sell the sterling for the exchange rate." How could you use options to satisfy the treasurer?

CHAPTER 2

Futures Markets and Central Counterparties

In Chapter 1 we explained that both futures and forward contracts are agreements to buy or sell an asset at a future time for a certain price. A futures contract is traded on an exchange, and the contract terms are standardized by that exchange. A forward contract is traded in the over-the-counter market and can be customized to meet the needs of users.

This chapter covers the details of how futures markets work. We examine issues such as the specification of contracts, the operation of margin accounts, the organization of exchanges, the regulation of markets, the way in which quotes are made, and the treatment of futures transactions for accounting and tax purposes. We explain how some of the ideas pioneered by futures exchanges have been adopted by over-the-counter markets.

2.1 BACKGROUND

Examples of large futures exchanges are the CME Group (www.cmegroup.com), the Intercontinental Exchange (www.theice.com), Eurex (www.eurexchange.com), B3, Brazil (www.b3.com.br), the National Stock Exchange of India (www.nse-india.com), the China Financial Futures Exchange (www.cffex.com.cn), and the Tokyo Financial Exchange (www.tfx.co.jp). (See the table at the end of this book for a more complete list.)

We examine how a futures contract comes into existence by considering the corn futures contract traded by the CME Group. On June 5, a trader in New York might call a broker with instructions to buy 5,000 bushels of corn for delivery in September of the same year. The broker would immediately issue instructions to a trader to buy (i.e., take a long position in) one September corn contract. (Each corn contract is for the delivery of exactly 5,000 bushels.) At about the same time, another trader in Kansas might instruct a broker to sell 5,000 bushels of corn for September delivery. This broker would then issue instructions to sell (i.e., take a short position in) one corn contract. A price would be determined and the deal would be done. Under the traditional open outcry system, floor traders representing each party would physically meet to determine the price. With electronic trading, a computer matches the traders.

Business Snapshot 2.1 The Unanticipated Delivery of a Futures Contract

This story (which may well be apocryphal) was told to the author of this book a long time ago by a senior executive of a financial institution. It concerns a new employee of the financial institution who had not previously worked in the financial sector. One of the clients of the financial institution regularly entered into a long futures contract on live cattle for hedging purposes and issued instructions to close out the position on the last day of trading. (Live cattle futures contracts are traded by the CME Group and each contract is on 40,000 pounds of cattle.) The new employee was given responsibility for handling the account.

When the time came to close out a contract the employee noted that the client was long one contract and instructed a trader at the exchange to buy (not sell) one contract. The result of this mistake was that the financial institution ended up with a long position in two live cattle futures contracts. By the time the mistake was spotted trading in the contract had ceased.

The financial institution (not the client) was responsible for the mistake. As a result, it started to look into the details of the delivery arrangements for live cattle futures contracts—something it had never done before. Under the terms of the contract, cattle could be delivered by the party with the short position to a number of different locations in the United States during the delivery month. Because it was long, the financial institution could do nothing but wait for a party with a short position to issue a *notice of intention to deliver* to the exchange and for the exchange to assign that notice to the financial institution.

It eventually received a notice from the exchange and found that it would receive live cattle at a location 2,000 miles away the following Tuesday. The new employee was sent to the location to handle things. It turned out that the location had a cattle auction every Tuesday. The party with the short position that was making delivery bought cattle at the auction and then immediately delivered them. Unfortunately the cattle could not be resold until the next cattle auction the following Tuesday. The employee was therefore faced with the problem of making arrangements for the cattle to be housed and fed for a week. This was a great start to a first job in the financial sector!

The trader in New York who agreed to buy has a *long futures position* in one contract; the trader in Kansas who agreed to sell has a *short futures position* in one contract. The price agreed to is the current *futures price* for September corn, say 600 cents per bushel. This price, like any other price, is determined by the laws of supply and demand. If, at a particular time, more traders wish to sell rather than buy September corn, the price will go down. New buyers then enter the market so that a balance between buyers and sellers is maintained. If more traders wish to buy rather than sell September corn, the price goes up. New sellers then enter the market and a balance between buyers and sellers is maintained.

Closing Out Positions

The vast majority of futures contracts do not lead to delivery. The reason is that most traders choose to close out their positions prior to the delivery period specified in the contract. Closing out a position means entering into the opposite trade to the original one. For example, the New York trader who bought a September corn futures contract on June 5 can close out the position by selling (i.e., shorting) one September corn futures contract on, say, July 20. The Kansas trader who sold (i.e., shorted) a September contract on June 5 can close out the position by buying one September contract on, say, August 25. In each case, the trader's total gain or loss is determined by the change in the futures price between June 5 and the day when the contract is closed out.

Delivery is so unusual that traders sometimes forget how the delivery process works (see Business Snapshot 2.1). Nevertheless, we will review delivery procedures later in this chapter. This is because it is the possibility of final delivery that ties the futures price to the spot price.[1]

2.2 SPECIFICATION OF A FUTURES CONTRACT

When developing a new contract, the exchange must specify in some detail the exact nature of the agreement between the two parties. In particular, it must specify the asset, the contract size (exactly how much of the asset will be delivered under one contract), where delivery can be made, and when delivery can be made.

Sometimes alternatives are specified for the grade of the asset that will be delivered or for the delivery locations. As a general rule, it is the party with the short position (the party that has agreed to sell the asset) that chooses what will happen when alternatives are specified by the exchange.[2] When the party with the short position is ready to deliver, it files a *notice of intention to deliver* with the exchange. This notice indicates any selections it has made with respect to the grade of asset that will be delivered and the delivery location.

The Asset

When the asset is a commodity, there may be quite a variation in the quality of what is available in the marketplace. When the asset is specified, it is therefore important that the exchange stipulate the grade or grades of the commodity that are acceptable. The Intercontinental Exchange (ICE) has specified the asset in its orange juice futures contract as frozen concentrates that are U.S. Grade A with Brix value of not less than 62.5 degrees.

For some commodities a range of grades can be delivered, but the price received depends on the grade chosen. For example, in the CME Group's corn futures contract, the standard grade is "No. 2 Yellow," but substitutions are allowed with the price being adjusted in a way established by the exchange. No. 1 Yellow is deliverable for 1.5 cents per bushel more than No. 2 Yellow. No. 3 Yellow is deliverable for 2 to 4 cents per bushel less than No. 2 Yellow depending on indicators of quality.

The financial assets in futures contracts are generally well defined and unambiguous. For example, there is no need to specify the grade of a Japanese yen. However, there are

[1] As mentioned in Chapter 1, the spot price is the price for almost immediate delivery.

[2] There are rare exceptions. As pointed out by J. E. Newsome, G. H. F. Wang, M. E. Boyd, and M. J. Fuller in "Contract Modifications and the Basic Behavior of Live Cattle Futures," *Journal of Futures Markets*, 24, 6 (2004), 557–90, the CME gave the buyer some delivery options in live cattle futures starting in 1995.

some interesting features of the Treasury bond and Treasury note futures contracts traded on the Chicago Board of Trade. For example, the underlying asset in the Treasury bond contract is any U.S. Treasury bond that has a maturity between 15 and 25 years; in the 10-year Treasury note futures contract, the underlying asset is any Treasury note with a maturity of between 6.5 and 10 years. The exchange has a formula for adjusting the price received according to the coupon and maturity date of the bond delivered. This is discussed in Chapter 6.

The Contract Size

The contract size specifies the amount of the asset that has to be delivered under one contract. This is an important decision for the exchange. If the contract size is too large, many traders who wish to hedge relatively small exposures or who wish to take relatively small speculative positions will be unable to use the exchange. On the other hand, if the contract size is too small, trading may be expensive as there is a cost associated with each contract traded.

The correct size for a contract clearly depends on the likely user. Whereas the value of what is delivered under a futures contract on an agricultural product might be $10,000 to $20,000, it is much higher for some financial futures. For example, under the Treasury bond futures contract traded by the CME Group, instruments with a face value of $100,000 are delivered.

In some cases exchanges have introduced “mini” contracts to attract smaller traders. For example, the CME Group’s Mini Nasdaq 100 contract is on 20 times the Nasdaq 100 index, whereas the regular contract is on 100 times the index. (We will cover futures on indices more fully in Chapter 3.)

Delivery Arrangements

The place where delivery will be made must be specified by the exchange. This is particularly important for commodities that involve significant transportation costs. In the case of the ICE frozen concentrate orange juice contract, delivery is to exchange-licensed warehouses in Florida, New Jersey, or Delaware.

When alternative delivery locations are specified, the price received by the party with the short position is sometimes adjusted according to the location chosen by that party. The price tends to be higher for delivery locations that are relatively far from the main sources of the commodity.

Delivery Months

A futures contract is referred to by its delivery month. The exchange must specify the precise period during the month when delivery can be made. For many futures contracts, the delivery period is the whole month.

The delivery months vary from contract to contract and are chosen by the exchange to meet the needs of market participants. For example, corn futures traded by the CME Group have delivery months of March, May, July, September, and December. At any given time, contracts trade for the closest delivery month and a number of subsequent delivery months. The exchange specifies when trading in a particular month’s contract will begin. The exchange also specifies the last day on which trading can take place for a given contract. Trading generally ceases a few days before the last day on which delivery can be made.

Price Quotes

The exchange defines how prices will be quoted. For example, crude oil futures prices are quoted in dollars and cents. Treasury bond and Treasury note futures prices are quoted in dollars and thirty-seconds of a dollar.

Price Limits and Position Limits

For most contracts, daily price movement limits are specified by the exchange. If in a day the price moves down from the previous day's close by an amount equal to the daily price limit, the contract is said to be *limit down*. If it moves up by the limit, it is said to be *limit up*. A *limit move* is a move in either direction equal to the daily price limit. Normally, trading ceases for the day once the contract is limit up or limit down. However, in some instances the exchange has the authority to step in and change the limits.

The purpose of daily price limits is to prevent large price movements from occurring because of speculative excesses. However, limits can become an artificial barrier to trading when the price of the underlying commodity is advancing or declining rapidly. Whether price limits are, on balance, good for futures markets is controversial.

Position limits are the maximum number of contracts that a speculator may hold. The purpose of these limits is to prevent speculators from exercising undue influence on the market.

2.3 CONVERGENCE OF FUTURES PRICE TO SPOT PRICE

As the delivery period for a futures contract is approached, the futures price converges to the spot price of the underlying asset. When the delivery period is reached, the futures price equals—or is very close to—the spot price.

To see why this is so, we first suppose that the futures price is above the spot price during the delivery period. Traders then have a clear arbitrage opportunity:

1. Sell (i.e., short) a futures contract
2. Buy the asset
3. Make delivery.

These steps are certain to lead to a profit equal to the amount by which the futures price exceeds the spot price. As traders exploit this arbitrage opportunity, the futures price will fall. Suppose next that the futures price is below the spot price during the delivery period. Companies interested in acquiring the asset will find it attractive to enter into a long futures contract and then wait for delivery to be made. As they do so, the futures price will tend to rise.

The result is that the futures price is very close to the spot price during the delivery period. Figure 2.1 illustrates the convergence of the futures price to the spot price. In Figure 2.1a the futures price is above the spot price prior to the delivery period. In

Figure 2.1 Relationship between futures price and spot price as the delivery period is approached: (a) Futures price above spot price; (b) futures price below spot price.

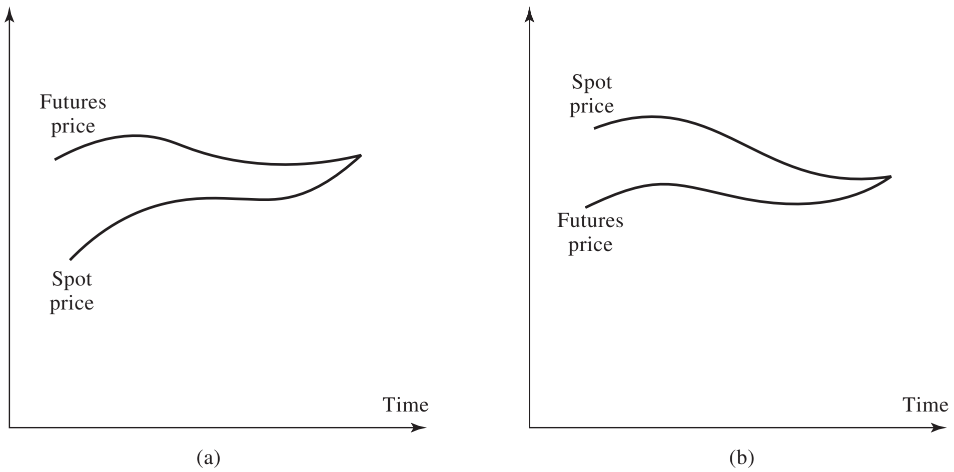

Figure 2.1b the futures price is below the spot price prior to the delivery period. The circumstances under which these two patterns are observed are discussed in Chapter 5.

2.4 THE OPERATION OF MARGIN ACCOUNTS

If two traders get in touch with each other directly and agree to trade an asset in the future for a certain price, there are obvious risks. One of the traders may regret the deal and try to back out. Alternatively, the trader simply may not have the financial resources to honor the agreement. One of the key roles of the exchange is to organize trading so that contract defaults are avoided. This is where margin accounts come in.

Daily Settlement

To illustrate how margin accounts work, we consider a trader who buys (i.e., takes a long position in) two December gold futures contracts. We suppose that the current futures price is $1,750 per ounce. Because the contract size is 100 ounces, the trader has contracted to buy a total of 200 ounces at this price. The trader has to keep funds in what is known as a margin account. The amount that must be deposited at the time the contract is entered into is known as the *initial margin*. We suppose this is $6,000 per contract, or $12,000 in total. At the end of each trading day, the margin account is adjusted to reflect the trader's gain or loss. This practice is referred to as *daily settlement* or *marking to market*.

Suppose, for example, that by the end of the first day the futures price has dropped by $9 from $1,750 to $1,741. The trader has a loss of $1,800 (= 200 × $9), because the 200 ounces of December gold, which the trader contracted to buy at $1,750, can now be sold for only $1,741. The balance in the margin account would therefore be reduced by $1,800 to $10,200. Similarly, if the price of December gold rose to $1,759 by the end of the first day, the balance in the margin account would be increased by $1,800 to $13,800. A trade is first settled at the close of the day on which it takes place. It is then settled at the close of trading on each subsequent day.

Daily settlement leads to funds flowing each day between traders with long positions and traders with short positions. If the futures price increases from one day to the next, funds flow from traders with short positions to traders with long positions. If the futures price decreases from one day to the next, funds flow in the opposite direction, from traders with short positions to traders with long positions. This daily flow of funds between traders to reflect gains and losses is known as *variation margin*.

Most individuals have to contact their brokers to trade. They are subject to what is termed a *maintenance margin*. This is somewhat lower than the initial margin. If the balance in the margin account falls below the maintenance margin, the trader receives a margin call and is expected to top up the margin account to the initial margin level within a short period of time. If the trader does not provide this variation margin, the broker closes out the position. In the case of the trader considered earlier, closing out the position would involve neutralizing the existing contract by selling 200 ounces of gold for delivery in December.

If the trader's contract increases in value, the balance in the margin account increases. The trader is entitled to withdraw any balance in the margin account that is in excess of the initial margin.

Table 2.1 Operation of margin account for a long position in two gold futures contracts. The initial margin is $6,000 per contract, or $12,000 in total; the maintenance margin is $4,500 per contract, or $9,000 in total. The contract is entered into on Day 1 at $1,750 and closed out on Day 16 at $1,726.90.

Day	*Trade price ($)*	*Settlement price ($)*	*Daily gain ($)*	*Cumulative gain ($)*	*Margin account balance ($)*	*Margin call ($)*
1	1,750.00				12,000	
1		1,741.00	−1,800	−1,800	10,200	
2		1,738.30	−540	−2,340	9,660	
3		1,744.60	1,260	−1,080	10,920	
4		1,741.30	−660	−1,740	10,260	
5		1,740.10	−240	−1,980	10,020	
6		1,736.20	−780	−2,760	9,240	
7		1,729.90	−1,260	−4,020	7,980	4,020
8		1,730.80	180	−3,840	12,180	
9		1,725.40	−1,080	−4,920	11,100	
10		1,728.10	540	−4,380	11,640	
11		1,711.00	−3,420	−7,800	8,220	3,780
12		1,711.00	0	−7,800	12,000	
13		1,714.30	660	−7,140	12,660	
14		1,716.10	360	−6,780	13,020	
15		1,723.00	1,380	−5,400	14,400	
16	1,726.90		780	−4,620	15,180	

Table 2.1 illustrates the operation of the margin account for one possible sequence of futures prices in the case of the trader considered earlier who buys two gold futures contracts. The maintenance margin is assumed to be $4,500 per contract, or $9,000 in total. On Day 7, the balance in the margin account falls $1,020 below the maintenance margin level. This drop triggers a margin call from the broker for an additional $4,020 to bring the account balance up to the initial margin level of $12,000. It is assumed that the trader provides this margin by the close of trading on Day 8. On Day 11, the balance in the margin account again falls below the maintenance margin level, and a margin call for $3,780 is sent out. The trader provides this margin by the close of trading on Day 12. On Day 16, the trader decides to close out the position by selling two contracts. The futures price on that day is $1,726.90, and the trader has a cumulative loss of $4,620. Note that the trader has excess margin on Days 8, 13, 14, and 15. It is assumed that the excess is not withdrawn.

Most brokers pay traders interest on the balance in a margin account. The balance in the account does not, therefore, represent a true cost, provided that the interest rate is competitive with what could be earned elsewhere. To satisfy the initial margin requirements, but not subsequent margin calls, a trader can usually deposit securities with the broker. Treasury bills are usually accepted in lieu of cash at about 90% of their market value. Shares are also sometimes accepted in lieu of cash, but at about 50% of their market value.

Whereas a forward contract is settled at the end of its life, a futures contract is, as we have seen, settled daily. At the end of each day, the trader's gain (loss) is added to (subtracted from) the margin account, bringing the value of the contract back to zero. A futures contract is in effect closed out and rewritten at a new price each day.

Minimum levels for the initial and maintenance margin are set by the exchange clearing house. Individual brokers may require greater margins from their clients than the minimum levels specified by the exchange clearing house. Minimum margin levels are determined by the variability of the price of the underlying asset and are revised when necessary. The higher the variability, the higher the margin levels. The maintenance margin is usually about 75% of the initial margin.

Note that margin requirements are the same on short futures positions as they are on long futures positions. It is just as easy to take a short futures position as it is to take a long one. The spot market does not have this symmetry. Taking a long position in the spot market involves buying the asset for immediate delivery and presents no problems. Taking a short position involves selling an asset that you do not own. This is a more complex transaction that may or may not be possible in a particular market. It is discussed further in Chapter 5.

The Clearing House and Its Members

A *clearing house* acts as an intermediary in futures transactions. It guarantees the performance of the parties to each transaction. The clearing house has a number of members. Brokers who are not members themselves must channel their business through a member and post margin with the member. The main task of the clearing house is to keep track of all the transactions that take place during a day, so that it can calculate the net position of each of its members.

The clearing house member is required to provide to the clearing house initial margin (sometimes referred to as clearing margin) reflecting the total number of contracts that are being cleared. The maintenance margin is set equal to the initial margin. At the end of each day, the transactions being handled by the clearing house member are settled through the clearing house. If in total the transactions have lost money, the member is required to provide variation margin to the exchange clearing house equal to the loss; if there has been a gain on the transactions, the member receives variation margin from the clearing house equal to the gain. Intraday variation margin payments may also be required by a clearing house from its members in times of significant price volatility; if margin requirements are not met, a member is closed out.

In determining margin requirements, the number of contracts outstanding is usually calculated on a net basis rather than a gross basis. This means that short positions the clearing house member is handling for clients are netted against long positions. Suppose, for example, that the clearing house member has two clients: one with a long position in 20 contracts, the other with a short position in 15 contracts. The initial margin would be calculated on the basis of five contracts. The calculation of the margin requirement is usually designed to ensure that the clearing house is about 99% certain that the margin will be sufficient to cover any losses in the event that the member defaults and has to be closed out. Clearing house members are required to contribute to a guaranty fund. This may be used by the clearing house in the event that a member defaults and the member's margin proves insufficient to cover losses.

Credit Risk

The whole purpose of the margining system is to ensure that funds are available to pay traders when they make a profit. Overall the system has been very successful. Traders entering into contracts at major exchanges have always had their contracts honored. Futures markets were tested on October 19, 1987, when the S&P 500 index declined by over 20% and traders with long positions in S&P 500 futures found they had negative margin balances with their brokers. Traders who did not meet margin calls were closed out but still owed their brokers money. Some did not pay and as a result some brokers went bankrupt because, without their clients' money, they were unable to meet margin calls on contracts they entered into on behalf of their clients. However, the clearing houses had sufficient funds to ensure that everyone who had a short futures position on the S&P 500 got paid.

2.5 OTC MARKETS

Over-the-counter (OTC) markets, introduced in Chapter 1, are markets where companies agree to derivatives transactions without involving an exchange. Credit risk has traditionally been a feature of OTC derivatives markets. Consider two companies, A and B, that have entered into a number of derivatives transactions. If A defaults when the net value of the outstanding transactions to B is positive, a loss is likely to be taken by B. Similarly, if B defaults when the net value of outstanding transactions to A is positive, a loss is likely to be taken by company A. In an attempt to reduce credit risk, the OTC market has borrowed some ideas from exchange-traded markets. We now discuss this.

Central Counterparties

We briefly mentioned CCPs in Section 1.2. These are clearing houses for standard OTC transactions that perform much the same role as exchange clearing houses. Members of the CCP, similarly to members of an exchange clearing house, have to provide both initial margin and daily variation margin. Like members of an exchange clearing house, they are also required to contribute to a guaranty fund.

Once an OTC derivative transaction has been agreed between two parties A and B, it can be presented to a CCP. Assuming the CCP accepts the transaction, it becomes the counterparty to both A and B. (This is similar to the way the clearing house of a futures exchange becomes the counterparty to the two sides of a futures trade.) For example, if the transaction is a forward contract where A has agreed to buy an asset from B in one year for a certain price, the clearing house agrees to

1. Buy the asset from B in one year for the agreed price, and
2. Sell the asset to A in one year for the agreed price.

It takes on the credit risk of both A and B.

All members of the CCP are required to provide initial margin to the CCP. Transactions are valued daily and there are daily variation margin payments to or from the member. If an OTC market participant is not itself a member of a CCP, it can arrange to clear its trades through a CCP member. It will then have to provide margin to the CCP member. Its relationship with the CCP member is similar to the relationship between a broker and a futures exchange clearing house member.

Following the financial crisis that started in 2007, regulators have become more concerned about systemic risk (see Business Snapshot 1.2). One result of this, mentioned in Section 1.2, has been legislation requiring that most standard OTC transactions between financial institutions be handled by CCPs.

Bilateral Clearing

Those OTC transactions that are not cleared through CCPs are cleared bilaterally. In the bilaterally cleared OTC market, two companies A and B usually enter into a master agreement covering all their trades.[3] This agreement usually includes an annex, referred to as the credit support annex or CSA, requiring A or B, or both, to provide collateral. The collateral is similar to the margin required by exchange clearing houses or CCPs from their members.

Collateral agreements in CSAs usually require transactions to be valued each day. A simple two-way agreement between companies A and B might work as follows. If, from one day to the next, the transactions between A and B increase in value to A by X (and therefore decrease in value to B by X), B is required to provide collateral worth X to A. If the reverse happens and the transactions increase in value to B by X (and decrease in value to A by X), A is required to provide collateral worth X to B. (To use the terminology of exchange-traded markets, X is the variation margin provided.) Collateral agreements and the way counterparty credit risk is assessed for bilaterally cleared transactions is discussed further in Chapter 24.

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[3] The most common such agreement is an International Swaps and Derivatives Association (ISDA) Master Agreement.

Business Snapshot 2.2 Long-Term Capital Management's Big Loss

Long-Term Capital Management (LTCM), a hedge fund formed in the mid-1990s, always collateralized its bilaterally cleared transactions. The hedge fund's investment strategy was known as convergence arbitrage. A very simple example of what it might do is the following. It would find two bonds, X and Y, issued by the same company that promised the same payoffs, with X being less liquid (i.e., less actively traded) than Y. The market places a value on liquidity. As a result the price of X would be less than the price of Y. LTCM would buy X, short Y, and wait, expecting the prices of the two bonds to converge at some future time.

When interest rates increased, the company expected both bonds to move down in price by about the same amount, so that the collateral it paid on bond X would be about the same as the collateral it received on bond Y. Similarly, when interest rates decreased, LTCM expected both bonds to move up in price by about the same amount, so that the collateral it received on bond X would be about the same as the collateral it paid on bond Y. It therefore expected that there would be no significant outflow of funds as a result of its collateralization agreements.

In August 1998, Russia defaulted on its debt and this led to what is termed a "flight to quality" in capital markets. One result was that investors valued liquid instruments more highly than usual and the spreads between the prices of the liquid and illiquid instruments in LTCM's portfolio increased dramatically. The prices of the bonds LTCM had bought went down and the prices of those it had shorted increased. It was required to post collateral on both. The company experienced difficulties because it was highly leveraged. Positions had to be closed out and LTCM lost about $4 billion. If the company had been less highly leveraged, it would probably have been able to survive the flight to quality and could have waited for the prices of the liquid and illiquid bonds to move back closer to each other.

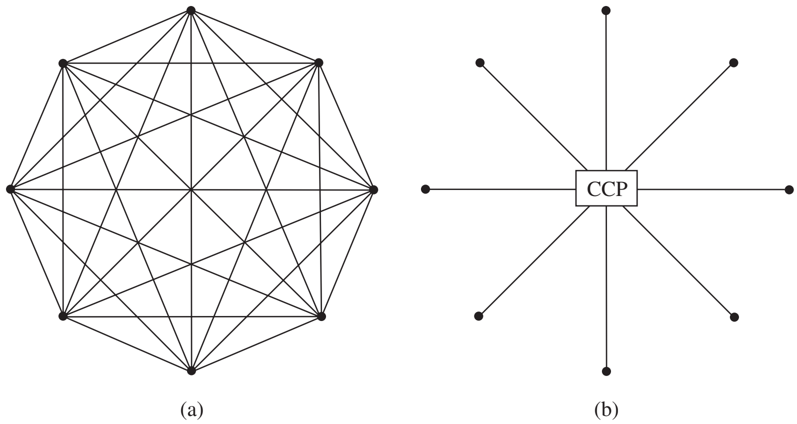
It has historically been relatively rare for a CSA to require initial margin. This is changing. Starting in 2016, regulations were introduced to require both initial margin and variation margin for bilaterally cleared transactions between financial institutions.[4] The initial margin is posted with a third party.

Collateral significantly reduces credit risk in the bilaterally cleared OTC market (and so the use of CCPs for standard transactions between financial institutions and regulations requiring initial margin for other transactions between financial institutions should reduce risks for the financial system). Collateral agreements were used by hedge fund Long-Term Capital Management (LTCM) for its bilaterally cleared derivatives in the 1990s. The agreements allowed LTCM to be highly levered. They did provide credit protection, but as described in Business Snapshot 2.2, the high leverage left the hedge fund exposed to other risks.

Figure 2.2 illustrates the way bilateral and central clearing work. (It makes the simplifying assumption that there are only eight market participants and one CCP). Under bilateral clearing there are many different agreements between market partici-

[4] For both this regulation and the regulation requiring standard transactions between financial institutions to be cleared through CCPs, "financial institutions" include banks, insurance companies, pension funds, and hedge funds. Transactions with most nonfinancial corporations and some foreign exchange transactions are exempt from the regulations.

Figure 2.2 (a) The traditional way in which OTC markets have operated: a series of bilateral agreements between market participants; (b) how OTC markets would operate with a single central counterparty (CCP) acting as a clearing house.



pants, as indicated in Figure 2.2a. If all OTC contracts were cleared through a single CCP, we would move to the situation shown in Figure 2.2b. In practice, because not all OTC transactions are routed through CCPs and there is more than one CCP, the market has elements of both Figure 2.2a and Figure 2.2b.[5]

Futures Trades vs. OTC Trades

Regardless of how transactions are cleared, initial margin when provided in the form of cash usually earns interest. The daily variation margin provided by a clearing house member for futures contracts does not earn interest. This is because the variation margin constitutes the daily settlement. Transactions in the OTC market, whether cleared through CCPs or cleared bilaterally, are usually not settled daily. For this reason, the daily variation margin that is provided by the member of a CCP or, as a result of a CSA, earns interest when it is in the form of cash.

Securities can be often be used to satisfy margin/collateral requirements.[6] The market value of the securities is reduced by a certain amount to determine their value for margin purposes. This reduction is known as a *haircut*.

2.6 MARKET QUOTES

Futures quotes are available from exchanges and several online sources. Table 2.2 is constructed from quotes provided by the CME Group for a number of different commodities on May 21, 2020. Quotes for index, currency, and interest rate futures are given in Chapters 3, 5, and 6, respectively.

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[5] The impact of CCPs on credit risk depends on the number of CCPs and proportions of all trades that are cleared through them. See D. Duffie and H. Zhu, "Does a Central Clearing Counterparty Reduce Counterparty Risk?," *Review of Asset Pricing Studies*, 1 (2011): 74–95.

[6] As already mentioned, the variation margin for futures contracts must be provided in the form of cash.

The asset underlying the futures contract, the contract size, and the way the price is quoted are shown at the top of each section of Table 2.2. The first asset is gold. The contract size is 100 ounces and the price is quoted as dollars per ounce. The maturity month of the contract is indicated in the first column of the table.

Prices

The first three numbers in each row of Table 2.2 show the opening price, the highest price in trading so far during the day, and the lowest price in trading so far during the day. The opening price is representative of the prices at which contracts were trading immediately after the start of trading on a day. For the June 2020 gold contract, the opening price on May 21, 2020 was $1,751.7 per ounce. The highest price during the day was also $1,751.7 per ounce and the lowest price during the day was $1,713.3 per ounce.

Settlement Price

The *settlement price* is the price used for calculating daily gains and losses and margin requirements. It is usually calculated as the price at which the contract traded immediately before the end of a day's trading session. The fourth number in Table 2.2 shows the settlement price the previous day. The fifth number shows the most recent trading price, and the sixth number shows the price change from the previous day's settlement price. In the case of the June 2020 gold contract, the previous day's settlement price was $1,752.1. The most recent trade was at $1,725.5, $26.6 lower than the previous day's settlement price. If $1,725.5 proved to be the settlement price on May 21, 2020, the margin account of a trader with a long position in one contract would lose 2,660 on May 21, 2020 and the margin account of a trader with a short position in one contract would gain this amount on that date.

Trading Volume and Open Interest

The final column of Table 2.2 shows the *trading volume*. The trading volume is the number of contracts traded in a day. It can be contrasted with the *open interest*, which is the number of contracts outstanding, that is, the number of long positions or, equivalently, the number of short positions.

If there is a large amount of trading by day traders (i.e., traders who enter into a position and close it out on the same day), the volume of trading in a day can be greater than either the beginning-of-day or end-of-day open interest.

Patterns of Futures

Futures prices can show a number of different patterns. They can increase with the maturity of the futures contract, decrease with the maturity of the futures contract, or show a mixed pattern where the futures price sometimes increases and sometimes decreases with maturity. A futures market where prices increase with maturity is known as a *normal market*. A futures market where futures prices decrease with maturity is known as an *inverted market*. Table 2.2 indicates that gold, crude oil, corn, and wheat exhibited a normal market on May 21, 2020 for the range of maturities considered. Soyabeans and live cattle exhibit a mixture of the two patterns: normal for some maturity ranges and inverted for others.

Table 2.2 Futures quotes for a selection of CME Group contracts on commodities on May 21, 2020.

	Open	*High*	*Low*	*Prior settlement*	*Last trade*	*Change*	*Volume*
Gold 100 oz, $ per oz							
June 2020	1751.7	1751.7	1715.3	1752.1	1725.5	−26.6	223,200
Aug. 2020	1765.3	1765.3	1731.2	1765.6	1740.7	−24.9	54,503
Oct. 2020	1768.0	1768.8	1739.0	1774.0	1747.4	−26.6	2,559
Dec. 2020	1778.8	1779.8	1743.8	1781.7	1752.7	−29.0	5,280
Dec. 2021	1779.0	1779.0	1755.1	1790.7	1757.2	−33.5	345
Crude Oil 1000 barrels, $ per barrel							
July 2020	33.53	34.66	33.26	33.49	33.96	+0.47	356,081
Aug. 2020	33.93	35.05	33.78	33.94	34.40	+0.46	118,534
Dec. 2020	35.18	36.08	35.06	35.23	35.76	+0.53	78,825
Dec. 2021	37.87	38.49	37.78	37.91	38.15	+0.24	22,542
Dec. 2022	40.30	40.74	39.92	40.27	40.24	−0.03	3,732
Corn 5000 bushels, cents per bushel							
July 2020	317.75	320.25	316.25	319.50	318.00	−1.50	104,099
Sept. 2020	323.50	325.00	321.25	324.25	323.00	−1.25	25,967
Dec. 2020	333.25	334.50	331.00	334.00	333.00	−1.00	32,855
Mar. 2021	346.00	347.00	344.00	346.50	345.75	−0.75	4,449
May 2021	353.75	354.50	351.50	354.00	353.50	−0.50	1,077
Dec. 2021	365.25	365.75	363.50	365.75	365.75	0.00	2,775
Soybeans 5000 bushels, cents per bushel							
July 2020	835.00	848.50	833.75	846.75	835.25	−11.50	89,375
Sept. 2020	849.25	851.50	839.00	849.75	840.50	−9.25	5,502
Nov. 2020	851.50	856.00	844.00	854.00	846.25	−7.75	42,274
Jan. 2021	856.75	859.00	847.75	857.00	849.75	−7.25	9,173
Mar. 2021	850.25	852.75	843.00	850.25	844.75	−5.50	13,531
May 2021	846.50	851.00	842.50	848.25	844.25	−4.00	3,736
July 2021	855.50	858.25	850.50	855.75	852.25	−3.50	1,953
Wheat 5000 bushels, cents per bushel							
July 2020	520.00	524.00	512.00	513.75	515.75	+2.00	72,667
Sept. 2020	520.75	525.00	514.50	515.25	518.50	+3.25	26,565
Dec. 2020	528.00	532.25	523.00	522.75	526.50	+3.75	18,522
Mar. 2021	531.75	538.75	530.25	530.00	534.50	+4.50	6,020
May 2021	535.50	540.25	532.75	532.75	537.00	+4.25	1,333
Live Cattle 40,000 lbs, cents per lb							
June 2020	98.775	99.200	97.975	98.400	98.650	+0.250	6,567
Oct. 2020	99.800	99.975	98.775	99.625	99.800	+0.175	6,875
Dec. 2020	102.750	102.950	102.050	102.725	102.800	+0.075	5,511
June 2021	101.750	102.750	101.625	101.975	102.675	+0.700	290

2.7 DELIVERY

As mentioned earlier in this chapter, very few of the futures contracts that are entered into lead to delivery of the underlying asset. Most are closed out early. Nevertheless, it is the possibility of eventual delivery that determines the futures price. An understanding of delivery procedures is therefore important.

The period during which delivery can be made is defined by the exchange and varies from contract to contract. The decision on when to deliver is made by the party with the short position, whom we shall refer to as trader A. When trader A decides to deliver, trader A's broker issues a notice of intention to deliver to the exchange clearing house. This notice states how many contracts will be delivered and, in the case of commodities, also specifies where delivery will be made and what grade will be delivered. The exchange then chooses a party with a long position to accept delivery.

Suppose that the party on the other side of trader A's futures contract when it was entered into was trader B. It is important to realize that there is no reason to expect that it will be trader B who takes delivery. Trader B may well have closed out his or her position by trading with trader C, trader C may have closed out his or her position by trading with trader D, and so on. The usual rule chosen by the exchange is to pass the notice of intention to deliver on to the party with the oldest outstanding long position. Parties with long positions must accept delivery notices. However, if the notices are transferable, traders with long positions usually have a short period of time to find another party with a long position that is prepared to take delivery in place of them.

In the case of a commodity, taking delivery usually means accepting a warehouse receipt in return for immediate payment. The party taking delivery is then responsible for all warehousing costs. In the case of livestock futures, there may be costs associated with feeding and looking after the animals (see Business Snapshot 2.1). In the case of financial futures, delivery is usually made by wire transfer. For all contracts, the price paid is usually the most recent settlement price. If specified by the exchange, this price is adjusted for grade, location of delivery, and so on.

There are three critical days for a contract. These are the first notice day, the last notice day, and the last trading day. The *first notice day* is the first day on which a notice of intention to make delivery can be submitted to the exchange. The *last notice day* is the last such day. The *last trading day* is generally a few days before the last notice day. To avoid the risk of having to take delivery, a trader with a long position should close out his or her contracts prior to the first notice day.

Cash Settlement

Some financial futures, such as those on stock indices discussed in Chapter 3, are settled in cash because it is inconvenient or impossible to deliver the underlying asset. In the case of the futures contract on the S&P 500, for example, delivering the underlying asset would involve delivering a portfolio of 500 stocks. When a contract is settled in cash, all outstanding contracts are declared closed on a predetermined day. The final settlement price is set equal to the spot price of the underlying asset at either the open or close of trading on that day. For example, in the S&P 500 futures contract traded by the CME Group, the predetermined day is the third Friday of the delivery month and final settlement is at the opening price on that day.

2.8 TYPES OF TRADERS AND TYPES OF ORDERS

There are two main types of traders executing trades: *futures commission merchants* (FCMs) and *locals*. FCMs are following the instructions of their clients and charge a commission for doing so; locals are trading on their own account.

Individuals taking positions, whether locals or the clients of FCMs, can be categorized as hedgers, speculators, or arbitrageurs, as discussed in Chapter 1. Speculators can be classified as scalpers, day traders, or position traders. *Scalpers* are watching for very short-term trends and attempt to profit from small changes in the contract price. They usually hold their positions for only a few minutes. *Day traders* hold their positions for less than one trading day. They are unwilling to take the risk that adverse news will occur overnight. *Position traders* hold their positions for much longer periods of time. They hope to make significant profits from major movements in the markets.

Orders

The simplest type of order placed with a broker is a *market order*. It is a request that a trade be carried out immediately at the best price available in the market. However, there are many other types of orders. We will consider those that are more commonly used.

A *limit order* specifies a particular price. The order can be executed only at this price or at one more favorable to the trader. Thus, if the limit price is $30 for a trader wanting to buy, the order will be executed only at a price of $30 or less. There is, of course, no guarantee that the order will be executed at all, because the limit price may never be reached.

A *stop order* or *stop-loss order* also specifies a particular price. The order is executed at the best available price once a bid or ask is made at that particular price or a less-favorable price. Suppose a stop order to sell at $30 is issued when the market price is $35. It becomes an order to sell when and if the price falls to $30. In effect, a stop order becomes a market order as soon as the specified price has been hit. The purpose of a stop order is usually to close out a position if unfavorable price movements take place. It limits the loss that can be incurred.

A *stop–limit order* is a combination of a stop order and a limit order. The order becomes a limit order as soon as a bid or ask is made at a price equal to or less favorable than the stop price. Two prices must be specified in a stop–limit order: the stop price and the limit price. Suppose that at the time the market price is $35, a stop–limit order to buy is issued with a stop price of $40 and a limit price of $41. As soon as there is a bid or ask at $40, the stop–limit becomes a limit order at $41. If the stop price and the limit price are the same, the order is sometimes called a *stop-and-limit order*.

A *market-if-touched* (MIT) *order* is executed at the best available price after a trade occurs at a specified price or at a price more favorable than the specified price. In effect, an MIT becomes a market order once the specified price has been hit. An MIT is also known as a *board order*. Consider a trader who has a long position in a futures contract and is issuing instructions that would lead to closing out the contract. A stop order is designed to place a limit on the loss that can occur in the event of unfavorable price movements. By contrast, a market-if-touched order is designed to ensure that profits are taken if sufficiently favorable price movements occur.

A *discretionary order* or *market-not-held order* is traded as a market order except that execution may be delayed at the broker's discretion in an attempt to get a better price.

Some orders specify time conditions. Unless otherwise stated, an order is a day order and expires at the end of the trading day. A *time-of-day order* specifies a particular period of time during the day when the order can be executed. An *open order* or a *good-till-canceled order* is in effect until executed or until the end of trading in the particular contract. A *fill-or-kill order*, as its name implies, must be executed immediately on receipt or not at all.

2.9 REGULATION

Futures markets in the United States are currently regulated federally by the Commodity Futures Trading Commission (CFTC; www.cftc.gov), which was established in 1974.

The CFTC looks after the public interest. It is responsible for ensuring that prices are communicated to the public and that futures traders report their outstanding positions if they are above certain levels. The CFTC also licenses all individuals who offer their services to the public in futures trading. The backgrounds of these individuals are investigated, and there are minimum capital requirements. The CFTC deals with complaints brought by the public and ensures that disciplinary action is taken against individuals when appropriate. It has the authority to force exchanges to take disciplinary action against members who are in violation of exchange rules.

With the formation of the National Futures Association (NFA; www.nfa.futures.org) in 1982, some of responsibilities of the CFTC were shifted to the futures industry itself. The NFA is an organization of individuals who participate in the futures industry. Its objective is to prevent fraud and to ensure that the market operates in the best interests of the general public. It is authorized to monitor trading and take disciplinary action when appropriate. The agency has set up an efficient system for arbitrating disputes between individuals and its members.

The Dodd–Frank act, signed into law by President Obama in 2010, expanded the role of the CFTC. For example, it is now responsible for rules requiring that standard over-the-counter derivatives between financial institutions be traded on swap execution facilities and cleared through central counterparties (see Section 1.2).

Trading Irregularities

Most of the time futures markets operate efficiently and in the public interest. However, from time to time, trading irregularities do come to light. One type of trading irregularity occurs when a trader group tries to "corner the market."[7] The trader group takes a huge long futures position and also tries to exercise some control over the supply of the underlying commodity. As the maturity of the futures contracts is approached, the trader group does not close out its position, so that the number of outstanding futures contracts may exceed the amount of the commodity available for delivery. The holders of short positions realize that they will find it difficult to deliver and become desperate to close out their positions. The result is a large rise in both futures and spot prices. Regulators usually deal with this type of abuse of the market by

[7] Possibly the best known example of this was the attempt by the Hunt brothers to corner the silver market in 1979–80. Between the middle of 1979 and the beginning of 1980, their activities led to a price rise from $6 per ounce to $50 per ounce.

increasing margin requirements or imposing stricter position limits or prohibiting trades that increase a speculator's open position or requiring market participants to close out their positions.

2.10 ACCOUNTING AND TAX

The full details of the accounting and tax treatment of futures contracts are beyond the scope of this book. A trader who wants detailed information on this should obtain professional advice. This section provides some general background information.

Accounting

Accounting standards require changes in the market value of a futures contract to be recognized when they occur unless the contract qualifies as a hedge. If the contract does qualify as a hedge, gains or losses are generally recognized for accounting purposes in the same period in which the gains or losses from the item being hedged are recognized. The latter treatment is referred to as *hedge accounting*.

Consider a company with a December year end. In September 2020 it buys a March 2021 corn futures contract and closes out the position at the end of February 2021. Suppose that the futures prices are 350 cents per bushel when the contract is entered into, 370 cents per bushel at the end of 2020, and 380 cents per bushel when the contract is closed out. The contract is for the delivery of 5,000 bushels. If the contract does not qualify as a hedge, the gains for accounting purposes are

$$5{,}000 \times (3.70 - 3.50) = \$1{,}000$$

in 2020 and

$$5{,}000 \times (3.80 - 3.70) = \$500$$

in 2021. If the company is hedging the purchase of 5,000 bushels of corn in February 2021 so that the contract qualifies for hedge accounting, the entire gain of $1,500 is realized in 2021 for accounting purposes.

The treatment of hedging gains and losses is sensible. If the company is hedging the purchase of 5,000 bushels of corn in February 2021, the effect of the futures contract is to ensure that the price paid (inclusive of the futures gain or loss) is close to 350 cents per bushel. The accounting treatment reflects that this price is paid in 2021.

The Financial Accounting Standards Board has issued FAS 133 and ASC 815 explaining when companies can and cannot use hedge accounting. The International Accounting Standards Board has similarly issued IAS 39 and IFRS 9.

Tax

Under the U.S. tax rules, two key issues are the nature of a taxable gain or loss and the timing of the recognition of the gain or loss. Gains or losses are either classified as capital gains or losses or alternatively as part of ordinary income.

For a corporate taxpayer, capital gains are taxed at the same rate as ordinary income, and the ability to deduct losses is restricted. Capital losses are deductible only to the extent of capital gains. A corporation may carry back a capital loss for three years and carry it forward for up to five years. For a noncorporate taxpayer, short-term capital gains are taxed at the same rate as ordinary income, but long-term capital gains are subject to a maximum capital gains tax rate of 20%. (Long-term capital gains are gains from the sale of a capital asset held for longer than one year; short-term capital gains are the gains from the sale of a capital asset held one year or less.) Taxpayers earning income above certain thresholds pay an additional 3.8% on all investment income. For a noncorporate taxpayer, capital losses are deductible to the extent of capital gains plus ordinary income up to $3,000 and can be carried forward indefinitely.

Generally, positions in futures contracts are treated as if they are closed out on the last day of the tax year. For the noncorporate taxpayer, this gives rise to capital gains and losses that are treated as if they were 60% long term and 40% short term without regard to the holding period. This is referred to as the "60/40" rule. A noncorporate taxpayer may elect to carry back for three years any net losses from the 60/40 rule to offset any gains recognized under the rule in the previous three years.

Hedging transactions are exempt from this rule. The definition of a hedge transaction for tax purposes is different from that for accounting purposes. The tax regulations define a hedging transaction as a transaction entered into in the normal course of business primarily for one of the following reasons:

1. To reduce the risk of price changes or currency fluctuations with respect to property that is held or to be held by the taxpayer for the purposes of producing ordinary income
2. To reduce the risk of price or interest rate changes or currency fluctuations with respect to borrowings made by the taxpayer.

A hedging transaction must be clearly identified in a timely manner in the company's records as a hedge. Gains or losses from hedging transactions are treated as ordinary income. The timing of the recognition of gains or losses from hedging transactions generally matches the timing of the recognition of income or expense associated with the transaction being hedged.

2.11 FORWARD vs. FUTURES CONTRACTS

The main differences between forward and futures contracts are summarized in Table 2.3. Both contracts are agreements to buy or sell an asset for a certain price at a certain future time. A forward contract is traded in the over-the-counter market and there is no standard contract size or standard delivery arrangements. A single delivery date is usually specified and the contract is usually held to the end of its life and then settled. A futures contract is a standardized contract traded on an exchange. A range of delivery dates is usually specified. It is settled daily and usually closed out prior to maturity.

Profits from Forward and Futures Contracts

Suppose that the sterling exchange rate for a 90-day forward contract is 1.2000 and that this rate is also the futures price for a contract that will be delivered in exactly 90 days. What is the difference between the gains and losses under the two contracts?

Under the forward contract, the whole gain or loss is realized at the end of the life of the contract. Under the futures contract, the gain or loss is realized day by day because of the daily settlement procedures. Suppose that trader A is long £1 million in

Table 2.3 Comparison of forward and futures contracts.

Forward	*Futures*
Private contract between two parties	Traded on an exchange
Not standardized	Standardized contract
Usually one specified delivery date	Range of delivery dates
Settled at end of contract	Settled daily
Delivery or final cash settlement usually takes place	Contract is usually closed out prior to maturity
Some credit risk	Virtually no credit risk

a 90-day forward contract and trader B is long £1 million in 90-day futures contracts. (Because each futures contract is for the purchase or sale of £62,500, trader B must purchase a total of 16 contracts.) Assume that the spot exchange rate in 90 days proves to be 1.4000 dollars per pound. Trader A makes a gain of \$200,000 on the 90th day. Trader B makes the same gain—but spread out over the 90-day period. On some days trader B may realize a loss, whereas on other days he or she makes a gain. However, in total, when losses are netted against gains, there is a gain of \$200,000 over the 90-day period.

Foreign Exchange Quotes

Both forward and futures contracts trade actively on foreign currencies. However, there is sometimes a difference in the way exchange rates are quoted in the two markets. For example, futures prices where one currency is the U.S. dollar are always quoted as the number of U.S. dollars per unit of the foreign currency or as the number of U.S. cents per unit of the foreign currency. Forward prices are always quoted in the same way as spot prices. This means that, for the British pound, the euro, the Australian dollar, and the New Zealand dollar, the forward quotes show the number of U.S. dollars per unit of the foreign currency and are directly comparable with futures quotes. For other major currencies, forward quotes show the number of units of the foreign currency per U.S. dollar (USD). Consider the Canadian dollar (CAD). A futures price quote of 0.7500 USD per CAD corresponds to a forward price quote of 1.3333 CAD per USD ($1.3333 = 1/0.7500$).

SUMMARY

A very high proportion of the futures contracts that are traded do not lead to the delivery of the underlying asset. Traders usually enter into offsetting contracts to close out their positions before the delivery period is reached. However, it is the possibility of final delivery that drives the determination of the futures price. For each futures contract, there is a range of days during which delivery can be made and a well-defined delivery procedure. Some contracts, such as those on stock indices, are settled in cash rather than by delivery of the underlying asset.

The specification of contracts is an important activity for a futures exchange. The two sides to any contract must know what can be delivered, where delivery can take place, and when delivery can take place. They also need to know details on the trading hours, how prices will be quoted, maximum daily price movements, and so on. New contracts must be approved by the Commodity Futures Trading Commission before trading starts.

Margin accounts are an important aspect of futures markets. A trader keeps a margin account with his or her broker. The account is adjusted daily to reflect gains or losses, and from time to time the broker may require the account to be topped up if adverse price movements have taken place. The broker either must be a clearing house member or must maintain a margin account with a clearing house member. Each clearing house member maintains a margin account with the exchange clearing house. The balance in the account is adjusted daily to reflect gains and losses on the business for which the clearing house member is responsible.

In over-the-counter derivatives markets, transactions are cleared either bilaterally or centrally. When bilateral clearing is used, collateral frequently has to be posted by one or both parties to reduce credit risk. When central clearing is used, a central counterparty (CCP) stands between the two sides. It requires each side to provide margin and performs much the same function as an exchange clearing house.

Forward contracts differ from futures contracts in a number of ways. Forward contracts are private arrangements between two parties, whereas futures contracts are traded on exchanges. There is generally a single delivery date in a forward contract, whereas futures contracts frequently involve a range of such dates. Because they are not traded on exchanges, forward contracts do not need to be standardized. A forward contract is not usually settled until the end of its life, and most contracts do in fact lead to delivery of the underlying asset or a cash settlement at this time.

In the next few chapters we shall examine in more detail the ways in which forward and futures contracts can be used for hedging. We shall also look at how forward and futures prices are determined.

FURTHER READING

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Practice Questions

2.1. Suppose that you enter into a short futures contract to sell July silver for \$17.20 per ounce. The size of the contract is 5,000 ounces. The initial margin is \$4,000, and the maintenance margin is \$3,000. What change in the futures price will lead to a margin call? What happens if you do not meet the margin call?

2.2. Suppose that in September 2021 a company takes a long position in a contract on May 2022 crude oil futures. It closes out its position in March 2022. The futures price (per barrel) is \$48.30 when it enters into the contract, \$50.50 when it closes out its position, and \$49.10 at the end of December 2021. One contract is for the delivery of 1,000 barrels. What is the company's total profit? When is it realized? How is it taxed if it is (a) a hedger and (b) a speculator? Assume that the company has a December 31 year end.

2.3. What does a stop order to sell at \$2 mean? When might it be used? What does a limit order to sell at \$2 mean? When might it be used?

2.4. What differences exist in the way prices are quoted in the foreign exchange futures market, the foreign exchange spot market, and the foreign exchange forward market?

2.5. The party with a short position in a futures contract sometimes has options as to the precise asset that will be delivered, where delivery will take place, when delivery will take place, and so on. Do these options increase or decrease the futures price? Explain your reasoning.

2.6. Explain how margin accounts protect futures traders against the possibility of default.

2.7. A trader buys two July futures contracts on frozen orange juice concentrate. Each contract is for the delivery of 15,000 pounds. The current futures price is 160 cents per pound, the initial margin is \$6,000 per contract, and the maintenance margin is \$4,500 per contract. What price change would lead to a margin call? Under what circumstances could \$2,000 be withdrawn from the margin account?

2.8. Show that, if the futures price of a commodity is greater than the spot price during the delivery period, then there is an arbitrage opportunity. Does an arbitrage opportunity exist if the futures price is less than the spot price? Explain your answer.

2.9. Explain the difference between a market-if-touched order and a stop order.

2.10. Explain what a stop–limit order to sell at 20.30 with a limit of 20.10 means.

2.11. At the end of one day a clearing house member is long 100 contracts, and the settlement price is \$50,000 per contract. The original margin is \$2,000 per contract. On the following day the member becomes responsible for clearing an additional 20 long contracts, entered into at a price of \$51,000 per contract. The settlement price at the end of this day is \$50,200. How much does the member have to add to its margin account with the exchange clearing house?

2.12. Explain why collateral requirements increased in the OTC market as a result of regulations introduced since the 2008 financial crisis.

2.13. The forward price of the Swiss franc for delivery in 45 days is quoted as 1.1000. The futures price for a contract that will be delivered in 45 days is 0.9000. Explain these two quotes. Which is more favorable for a trader wanting to sell Swiss francs?

2.14. Suppose you call your broker and issue instructions to sell one July hogs contract. Describe what happens.

2.15. "Speculation in futures markets is pure gambling. It is not in the public interest to allow speculators to trade on a futures exchange." Discuss this viewpoint.

2.16. What do you think would happen if an exchange started trading a contract in which the quality of the underlying asset was incompletely specified?

2.17. "When a futures contract is traded on the floor of the exchange, it may be the case that the open interest increases by one, stays the same, or decreases by one." Explain this statement.

2.18. Suppose that, on October 24, 2022, a company sells one April 2023 live cattle futures contract. It closes out its position on January 21, 2023. The futures price (per pound) is 121.20 cents when it enters into the contract, 118.30 cents when it closes out its position, and 118.80 cents at the end of December 2022. One contract is for the delivery of 40,000 pounds of cattle. What is the total profit? How is it taxed if the company is (a) a hedger and (b) a speculator? Assume that the company has a December 31 year end.

2.19. A cattle farmer expects to have 120,000 pounds of live cattle to sell in 3 months. The live cattle futures contract traded by the CME Group is for the delivery of 40,000 pounds of cattle. How can the farmer use the contract for hedging? From the farmer's viewpoint, what are the pros and cons of hedging?

2.20. It is July 2021. A mining company has just discovered a small deposit of gold. It will take 6 months to construct the mine. The gold will then be extracted on a more or less continuous basis for 1 year. Futures contracts on gold are available with delivery months every 2 months from August 2021 to December 2022. Each contract is for the delivery of 100 ounces. Discuss how the mining company might use futures markets for hedging.

2.21. Explain how CCPs work. What are the advantages to the financial system of requiring CCPs to be used for all standardized derivatives transactions between financial institutions?

2.22. Trader A enters into futures contracts to buy 1 million euros for 1.1 million dollars in three months. Trader B enters in a forward contract to do the same thing. The exchange rate (dollars per euro) declines sharply during the first two months and then increases for the third month to close at 1.1300. Ignoring daily settlement, what is the total profit of each trader? When the impact of daily settlement is taken into account, which trader has done better?

2.23. Explain what is meant by open interest. Why does the open interest usually decline during the month preceding the delivery month? On a particular day, there were 2,000 trades in a particular futures contract. This means that there were 2,000 buyers (going long) and 2,000 sellers (going short). Of the 2,000 buyers, 1,400 were closing out positions and 600 were entering into new positions. Of the 2,000 sellers, 1,200 were closing out positions and 800 were entering into new positions. What is the impact of the day's trading on open interest?

2.24. A company enters into a short futures contract to sell 5,000 bushels of wheat for 750 cents per bushel. The initial margin is \$3,000 and the maintenance margin is \$2,000. What price change would lead to a margin call? Under what circumstances could \$1,500 be withdrawn from the margin account?

2.25. Suppose that there are no storage costs for crude oil and the interest rate for borrowing or lending is 4% per annum. How could you make money if the June and December futures contracts for a particular year trade at \$50 and \$56, respectively?

2.26. What position is equivalent to a long forward contract to buy an asset at K on a certain date and a put option to sell it for K on that date.

2.27. A bank's derivatives transactions with a counterparty are worth +\$10 million to the bank and are cleared bilaterally. The counterparty has posted \$10 million of cash collateral. What credit exposure does the bank have?

CHAPTER 3 Hedging Strategies Using Futures

Many of the participants in futures markets are hedgers. Their aim is to use futures markets to reduce a particular risk that they face. This risk might relate to fluctuations in the price of oil, a foreign exchange rate, the level of the stock market, or some other variable. A *perfect hedge* is one that completely eliminates the risk. Perfect hedges are rare. For the most part, therefore, a study of hedging using futures contracts is a study of the ways in which hedges can be constructed so that they perform as close to perfectly as possible.

In this chapter we consider a number of general issues associated with the way hedges are set up. When is a short futures position appropriate? When is a long futures position appropriate? Which futures contract should be used? What is the optimal size of the futures position for reducing risk? At this stage, we restrict our attention to what might be termed *hedge-and-forget* strategies. We assume that no attempt is made to adjust the hedge once it has been put in place. The hedger simply takes a futures position at the beginning of the life of the hedge and closes out the position at the end of the life of the hedge. In Chapter 19 we will examine dynamic hedging strategies in which the hedge is monitored closely and frequent adjustments are made.

The chapter initially treats futures contracts as forward contracts (that is, it ignores daily settlement). Later it explains adjustments that are necessary to take account of the difference between futures and forwards.

3.1 BASIC PRINCIPLES

When an individual or company chooses to use futures markets to hedge a risk, the objective is often to take a position that neutralizes the risk as far as possible. Consider a company that knows it will gain $10,000 for each 1 cent increase in the price of a commodity over the next 3 months and lose $10,000 for each 1 cent decrease in the price during the same period. To hedge, the company's treasurer should take a short futures position that is designed to offset this risk. The futures position should lead to a loss of $10,000 for each 1 cent increase in the price of the commodity over the 3 months and a gain of $10,000 for each 1 cent decrease in the price during this period. If the price of the commodity goes down, the gain on the futures position offsets the loss on the rest of the company's business. If the price of the commodity goes up, the loss on the futures position is offset by the gain on the rest of the company's business.

Short Hedges

A *short hedge* is a hedge, such as the one just described, that involves a short position in futures contracts. A short hedge is appropriate when the hedger already owns an asset and expects to sell it at some time in the future. For example, a short hedge could be used by a farmer who owns some hogs and knows that they will be ready for sale at the local market in two months. A short hedge can also be used when an asset is not owned right now but will be owned and ready for sale at some time in the future. Consider, for example, a U.S. exporter who knows that he or she will receive euros in 3 months. The exporter will realize a gain if the euro increases in value relative to the U.S. dollar and will sustain a loss if the euro decreases in value relative to the U.S. dollar. A short futures position leads to a loss if the euro increases in value and a gain if it decreases in value. It has the effect of offsetting the exporter's risk.

To provide a more detailed illustration of the operation of a short hedge in a specific situation, we assume that it is May 15 today and that an oil producer has just negotiated a contract to sell 1 million barrels of crude oil. It has been agreed that the price that will apply in the contract is the market price on August 15. The oil producer is therefore in the position where it will gain $10,000 for each 1 cent increase in the price of oil over the next 3 months and lose $10,000 for each 1 cent decrease in the price during this period. Suppose that on May 15 the spot price is $50 per barrel and the crude oil futures price for August delivery is $49 per barrel. Because each futures contract is for the delivery of 1,000 barrels, the company can hedge its exposure by shorting (i.e., selling) 1,000 futures contracts. If the oil producer closes out its position on August 15, the effect of the strategy should be to lock in a price close to $49 per barrel.

To illustrate what might happen, suppose that the spot price on August 15 proves to be $45 per barrel. The company realizes $45 million for the oil under its sales contract. Because August is the delivery month for the futures contract, the futures price on August 15 should be very close to the spot price of $45 on that date. The company therefore gains approximately

$$\$49 - \$45 = \$4$$

per barrel, or $4 million in total from the short futures position. The total amount realized from both the futures position and the sales contract is therefore approximately $49 per barrel, or $49 million in total.

For an alternative outcome, suppose that the price of oil on August 15 proves to be $55 per barrel. The company realizes $55 per barrel for the oil and loses approximately

$$\$55 - \$49 = \$6$$

per barrel on the short futures position. Again, the total amount realized is approximately $49 million. It is easy to see that in all cases the company ends up with approximately $49 million.

Long Hedges

Hedges that involve taking a long position in a futures contract are known as *long hedges*. A long hedge is appropriate when a company knows it will have to purchase a certain asset in the future and wants to lock in a price now.

Suppose that it is now January 15. A copper fabricator knows it will require 100,000 pounds of copper on May 15 to meet a certain contract. The spot price of copper is 340 cents per pound, and the futures price for May delivery is 320 cents per pound. The fabricator can hedge its position by taking a long position in four futures contracts offered by the CME Group and closing its position on May 15. Each contract is for the delivery of 25,000 pounds of copper. The strategy has the effect of locking in the price of the required copper at close to 320 cents per pound.

Suppose that the spot price of copper on May 15 proves to be 325 cents per pound. Because May is the delivery month for the futures contract, this should be very close to the futures price. The fabricator therefore gains approximately

$$100{,}000 \times (\$3.25 - \$3.20) = \$5{,}000$$

on the futures contracts. It pays $100{,}000 \times \$3.25 = \$325{,}000$ for the copper, making the net cost approximately $\$325{,}000 - \$5{,}000 = \$320{,}000$. For an alternative outcome, suppose that the spot price is 305 cents per pound on May 15. The fabricator then loses approximately

$$100{,}000 \times (\$3.20 - \$3.05) = \$15{,}000$$

on the futures contract and pays $100{,}000 \times \$3.05 = \$305{,}000$ for the copper. Again, the net cost is approximately \$320,000, or 320 cents per pound.

Note that, in this case, it is clearly better for the company to use futures contracts than to buy the copper on January 15 in the spot market. If it does the latter, it will pay 340 cents per pound instead of 320 cents per pound and will incur both interest costs and storage costs. For a company using copper on a regular basis, this disadvantage would be offset by the convenience of having the copper on hand.[1] However, for a company that knows it will not require the copper until May 15, the futures contract alternative is likely to be preferred.

The examples we have looked at assume that the futures position is closed out in the delivery month. The hedge has the same basic effect if delivery is allowed to happen. However, making or taking delivery can be costly and inconvenient. For this reason, delivery is not usually made even when the hedger keeps the futures contract until the delivery month. As will be discussed later, hedgers with long positions usually avoid any possibility of having to take delivery by closing out their positions before the delivery period.

We have also assumed in the two examples that there is no daily settlement. In practice, daily settlement does have a small effect on the performance of a hedge. As explained in Chapter 2, it means that the payoff from the futures contract is realized day by day throughout the life of the hedge rather than all at the end.

3.2 ARGUMENTS FOR AND AGAINST HEDGING

The arguments in favor of hedging are so obvious that they hardly need to be stated. Most nonfinancial companies are in the business of manufacturing, or retailing or wholesaling, or providing a service. They have no particular skills or expertise in predicting variables such as interest rates, exchange rates, and commodity prices.

[1] See Section 5.11 for a discussion of convenience yields.

(Indeed, even experts are often wrong when they make predictions about these variables.) It makes sense for them to hedge the risks associated with these variables as they become aware of them. The companies can then focus on their main activities. By hedging, they avoid unpleasant surprises such as sharp rises in the price of a commodity that is being purchased.

In practice, many risks are left unhedged. In the rest of this section we will explore some of the reasons for this.

Hedging and Shareholders

One argument sometimes put forward is that the shareholders can, if they wish, do the hedging themselves. They do not need the company to do it for them. This argument is, however, open to question. It assumes that shareholders have as much information as the company's management about the risks faced by a company. In most instances, this is not the case. The argument also ignores commissions and other transactions costs. These are less expensive per dollar of hedging for large transactions than for small transactions. Hedging is therefore likely to be less expensive when carried out by the company than when it is carried out by individual shareholders.

One thing that shareholders can do far more easily than a corporation is diversify risk. A shareholder with a well-diversified portfolio may be immune to many of the risks faced by a corporation. For example, in addition to holding shares in a company that uses copper, a well-diversified shareholder may hold shares in a copper producer, so that there is very little overall exposure to the price of copper. If companies are acting in the best interests of well-diversified shareholders, it can be argued that hedging is unnecessary in many situations. However, the extent to which managers are in practice influenced by this type of argument is open to question.

Hedging and Competitors

If hedging is not the norm in a certain industry, it may not make sense for one particular company to choose to be different from all others. Competitive pressures within the industry may be such that the prices of the goods and services produced by the industry fluctuate to reflect raw material costs, interest rates, exchange rates, and so on. A company that does not hedge can expect its profit margins to be roughly constant. However, a company that does hedge can expect its profit margins to fluctuate!

To illustrate this point, consider two manufacturers of gold jewelry, SafeandSure Company and TakeaChance Company. We assume that most companies in the industry do not hedge against movements in the price of gold and that TakeaChance Company is no exception. However, SafeandSure Company has decided to be different from its competitors and to use futures contracts to hedge its purchase of gold over the next 18 months. If the price of gold goes up, economic pressures will tend to lead to a corresponding increase in the wholesale price of jewelry, so that TakeaChance Company's gross profit margin is unaffected. By contrast, SafeandSure Company's profit margin will increase after the effects of the hedge have been taken into account. If the price of gold goes down, economic pressures will tend to lead to a corresponding decrease in the wholesale price of jewelry. Again, TakeaChance Company's profit margin is unaffected. However, SafeandSure Company's profit margin goes down. In extreme conditions,

Table 3.1 Danger in hedging when competitors do not hedge.

Change in gold price	*Effect on price of gold jewelry*	*Effect on profits of TakeaChance Co.*	*Effect on profits of SafeandSure Co.*
Increase	Increase	None	Increase
Decrease	Decrease	None	Decrease

SafeandSure Company's profit margin could become negative as a result of the "hedging" carried out! The situation is summarized in Table 3.1.

This example emphasizes the importance of looking at the big picture when hedging. All the implications of price changes on a company's profitability should be taken into account in the design of a hedging strategy to protect against the price changes.

Hedging Can Lead to a Worse Outcome

It is important to realize that a hedge using futures contracts can result in a decrease or an increase in a company's profits relative to the position it would be in with no hedging. In the example involving the oil producer considered earlier, if the price of oil goes down, the company loses money on its sale of 1 million barrels of oil, and the futures position leads to an offsetting gain. The treasurer can be congratulated for having had the foresight to put the hedge in place. Clearly, the company is better off than it would be with no hedging. Other executives in the organization, it is hoped, will appreciate the contribution made by the treasurer. If the price of oil goes up, the company gains from its sale of the oil, and the futures position leads to an offsetting loss. The company is in a worse position than it would be with no hedging. Although the hedging decision was perfectly logical, the treasurer may in practice have a difficult time justifying it. Suppose that the price of oil at the end of the hedge is $59, so that the company loses $10 per barrel on the futures contract. We can imagine a conversation such as the following between the treasurer and the president:

President: This is terrible. We've lost $10 million in the futures market in the space of three months. How could it happen? I want a full explanation.

Treasurer: The purpose of the futures contracts was to hedge our exposure to the price of oil, not to make a profit. Don't forget we made $10 million from the favorable effect of the oil price increases on our business.

President: What's that got to do with it? That's like saying that we do not need to worry when our sales are down in California because they are up in New York.

Treasurer: If the price of oil had gone down . . .

President: I don't care what would have happened if the price of oil had gone down. The fact is that it went up. I really do not know what you were doing playing the futures markets like this. Our shareholders will expect us to have done particularly well this quarter. I'm going to have to explain to them that your actions reduced profits by $10 million. I'm afraid this is going to mean no bonus for you this year.

Business Snapshot 3.1 Hedging by Gold Mining Companies

It is natural for a gold mining company to consider hedging against changes in the price of gold. Typically it takes several years to extract all the gold from a mine. Once a gold mining company decides to go ahead with production at a particular mine, it has a big exposure to the price of gold. Indeed a mine that looks profitable at the outset could become unprofitable if the price of gold plunges.

Gold mining companies are careful to explain their hedging strategies to potential shareholders. Some gold mining companies do not hedge. They tend to attract shareholders who buy gold stocks because they want to benefit when the price of gold increases and are prepared to accept the risk of a loss from a decrease in the price of gold. Other companies choose to hedge. They estimate the number of ounces of gold they will produce each month for the next few years and enter into short futures or forward contracts to lock in the price for all or part of this.

Suppose you are Goldman Sachs and are approached by a gold mining company that wants to sell you a large amount of gold in 1 year at a fixed price. How do you set the price and then hedge your risk? The answer is that you can hedge by borrowing the gold from a central bank, selling it immediately in the spot market, and investing the proceeds at the risk-free rate. At the end of the year, you buy the gold from the gold mining company and use it to repay the central bank. The fixed forward price you set for the gold reflects the risk-free rate you can earn and the lease rate you pay the central bank for borrowing the gold.

Treasurer: That's unfair. I was only . . .

President: Unfair! You are lucky not to be fired. You lost $10 million.

Treasurer: It all depends on how you look at it . . .

It is easy to see why many treasurers are reluctant to hedge! Hedging reduces risk for the company. However, it may increase risk for the treasurer if others do not fully understand what is being done. The only real solution to this problem involves ensuring that all senior executives within the organization fully understand the nature of hedging before a hedging program is put in place. Ideally, hedging strategies are set by a company's board of directors and are clearly communicated to both the company's management and the shareholders. (See Business Snapshot 3.1 for a discussion of hedging by gold mining companies.)

3.3 BASIS RISK

The hedges in the examples considered so far have been almost too good to be true. The hedger was able to identify the precise date in the future when an asset would be bought or sold. The hedger was then able to use futures contracts to remove almost all the risk arising from the price of the asset on that date. In practice, hedging is often not quite as straightforward as this. Some of the reasons are as follows:

1. The asset whose price is to be hedged may not be exactly the same as the asset underlying the futures contract.

2. There may be uncertainty as to the exact date when the asset will be bought or sold.
3. The hedge may require the futures contract to be closed out before its delivery month.

These problems give rise to what is termed *basis risk*. This concept will now be explained.

The Basis

The *basis* in a hedging situation is as follows:[2]

$$\text{Basis} = \text{Spot price of asset to be hedged} - \text{Futures price of contract used}$$

If the asset to be hedged and the asset underlying the futures contract are the same, the basis should be zero at the expiration of the futures contract. Prior to expiration, the basis may be positive or negative.

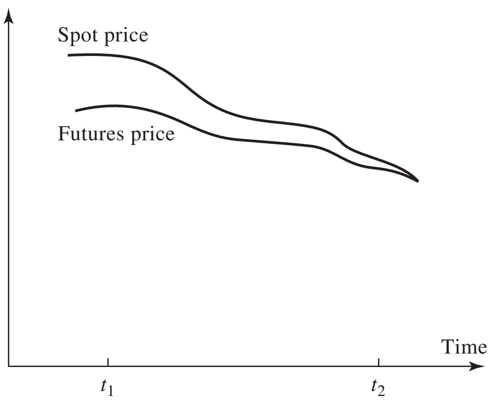
As time passes, the spot price and the futures price for a particular month do not necessarily change by the same amount. As a result, the basis changes. An increase in the basis is referred to as a *strengthening of the basis*; a decrease in the basis is referred to as a *weakening of the basis*. Figure 3.1 illustrates how a basis might change over time in a situation where the basis is positive prior to expiration of the futures contract.

To examine the nature of basis risk, we will use the following notation:

S_1: Spot price at time t_1
S_2: Spot price at time t_2
F_1: Futures price at time t_1
F_2: Futures price at time t_2
b_1: Basis at time t_1
b_2: Basis at time t_2.

We will assume that a hedge is put in place at time t_1 and closed out at time t_2. As an example, we will consider the case where the spot and futures prices at the time the

Figure 3.1 Variation of basis over time.



[2] This is the usual definition. However, the alternative definition Basis = Futures price − Spot price is sometimes used, particularly when the futures contract is on a financial asset.

hedge is initiated are \$2.50 and \$2.20, respectively, and that at the time the hedge is closed out they are \$2.00 and \$1.90, respectively. This means that $S_1 = 2.50$, $F_1 = 2.20$, $S_2 = 2.00$, and $F_2 = 1.90$.

From the definition of the basis, we have

$$b_1 = S_1 - F_1 \quad \text{and} \quad b_2 = S_2 - F_2$$

so that, in our example, $b_1 = 0.30$ and $b_2 = 0.10$.

Consider first the situation of a hedger who knows that the asset will be sold at time t_2 and takes a short futures position at time t_1. The price realized for the asset is S_2 and the profit on the futures position is $F_1 - F_2$. The effective price that is obtained for the asset with hedging is therefore

$$S_2 + F_1 - F_2 = F_1 + b_2$$

In our example, this is \$2.30. The value of F_1 is known at time t_1. If b_2 were also known at this time, a perfect hedge would result. The hedging risk is the uncertainty associated with b_2 and is known as *basis risk*. Consider next a situation where a company knows it will buy the asset at time t_2 and initiates a long hedge at time t_1. The price paid for the asset is S_2 and the loss on the hedge is $F_1 - F_2$. The effective price that is paid with hedging is therefore

$$S_2 + F_1 - F_2 = F_1 + b_2$$

This is the same expression as before and is \$2.30 in the example. The value of F_1 is known at time t_1, and the term b_2 represents basis risk.

Note that basis changes can lead to an improvement or a worsening of a hedger's position. Consider a company that uses a short hedge because it plans to sell the underlying asset. If the basis strengthens (i.e., increases) unexpectedly, the company's position improves because it will get a higher price for the asset after futures gains or losses are considered; if the basis weakens (i.e., decreases) unexpectedly, the company's position worsens. For a company using a long hedge because it plans to buy the asset, the reverse holds. If the basis strengthens unexpectedly, the company's position worsens because it will pay a higher price for the asset after futures gains or losses are considered; if the basis weakens unexpectedly, the company's position improves.

The asset that gives rise to the hedger's exposure is sometimes different from the asset underlying the futures contract that is used for hedging. This is known as *cross hedging* and is discussed in the next section. It leads to an increase in basis risk. Define S_2^* as the price of the asset underlying the futures contract at time t_2. As before, S_2 is the price of the asset being hedged at time t_2. By hedging, a company ensures that the price that will be paid (or received) for the asset is

$$S_2 + F_1 - F_2$$

This can be written as

$$F_1 + (S_2^* - F_2) + (S_2 - S_2^*)$$

The terms $S_2^* - F_2$ and $S_2 - S_2^*$ represent the two components of the basis. The $S_2^* - F_2$ term is the basis that would exist if the asset being hedged were the same as the asset underlying the futures contract. The $S_2 - S_2^*$ term is the basis arising from the difference between the two assets.

Choice of Contract

One key factor affecting basis risk is the choice of the futures contract to be used for hedging. This choice has two components:

1. The choice of the asset underlying the futures contract
2. The choice of the delivery month.

If the asset being hedged exactly matches an asset underlying a futures contract, the first choice is generally fairly easy. In other circumstances, it is necessary to carry out a careful analysis to determine which of the available futures contracts has futures prices that are most closely correlated with the price of the asset being hedged.

The choice of the delivery month is likely to be influenced by several factors. In the examples given earlier in this chapter, we assumed that, when the expiration of the hedge corresponds to a delivery month, the contract with that delivery month is chosen. In fact, a contract with a later delivery month is usually chosen in these circumstances. The reason is that futures prices are in some instances quite erratic during the delivery month. Moreover, a long hedger runs the risk of having to take delivery of the physical asset if the contract is held during the delivery month. Taking delivery can be expensive and inconvenient. (Long hedgers normally prefer to close out the futures contract and buy the asset from their usual suppliers.)

In general, basis risk increases as the time difference between the hedge expiration and the delivery month increases. A good rule of thumb is therefore to choose a delivery month that is as close as possible to, but later than, the expiration of the hedge. Suppose delivery months are March, June, September, and December for a futures contract on a particular asset. For hedge expirations in December, January, and February, the March contract will be chosen; for hedge expirations in March, April, and May, the June contract will be chosen; and so on. This rule of thumb assumes that there is sufficient liquidity in all contracts to meet the hedger's requirements. In practice, liquidity tends to be greatest in short-maturity futures contracts. Therefore, in some situations, the hedger may be inclined to use short-maturity contracts and roll them forward. This strategy is discussed later in the chapter.

Example 3.1

It is March 1. A U.S. company expects to receive 50 million Japanese yen at the end of July. Yen futures contracts on the CME Group have delivery months of March, June, September, and December. One contract is for the delivery of 12.5 million yen. The company therefore shorts four September yen futures contracts on March 1. When the yen are received at the end of July, the company closes out its position. We suppose that the futures price on March 1 in cents per yen is 1.0800 and that the spot and futures prices when the contract is closed out are 1.0200 and 1.0250, respectively.

The gain on the futures contract is $1.0800 - 1.0250 = 0.0550$ cents per yen. The basis is $1.0200 - 1.0250 = -0.0050$ cents per yen when the contract is closed out. The effective price obtained in cents per yen is the final spot price plus the gain on the futures:

$$1.0200 + 0.0550 = 1.0750$$

This can also be written as the initial futures price plus the final basis:

$$1.0800 + (-0.0050) = 1.0750$$

The total amount received by the company for the 50 million yen is 50×0.01075 million dollars, or \$537,500.

Example 3.2

It is June 8 and a company knows that it will need to purchase 20,000 barrels of crude oil at some time in October or November. Oil futures contracts are currently traded for delivery every month by the CME Group and the contract size is 1,000 barrels. The company therefore decides to use the December contract for hedging and takes a long position in 20 December contracts. The futures price on June 8 is \$48.00 per barrel. The company finds that it is ready to purchase the crude oil on November 10. It therefore closes out its futures contract on that date. The spot price and futures price on November 10 are \$50.00 per barrel and \$49.10 per barrel.

The gain on the futures contract is $49.10 - 48.00 = \$1.10$ per barrel. The basis when the contract is closed out is $50.00 - 49.10 = \$0.90$ per barrel. The effective price paid (in dollars per barrel) is the final spot price less the gain on the futures, or

$$50.00 - 1.10 = 48.90$$

This can also be calculated as the initial futures price plus the final basis,

$$48.00 + 0.90 = 48.90$$

The total price paid is $48.90 \times 20{,}000 = \$978{,}000$.

3.4 CROSS HEDGING

In Examples 3.1 and 3.2, the asset underlying the futures contract was the same as the asset whose price is being hedged. *Cross hedging* occurs when the two assets are different. Consider, for example, an airline that is concerned about the future price of jet fuel. Because jet fuel futures are not actively traded, it might choose to use heating oil futures contracts to hedge its exposure.

The *hedge ratio* is the ratio of the size of the position taken in futures contracts to the size of the exposure. When the asset underlying the futures contract is the same as the asset being hedged, it is natural to use a hedge ratio of 1.0. This is the hedge ratio we have used in the examples considered so far. For instance, in Example 3.2, the hedger's exposure was on 20,000 barrels of oil, and futures contracts were entered into for the delivery of exactly this amount of oil.

When cross hedging is used, setting the hedge ratio equal to 1.0 is not always optimal. The hedger should choose a value for the hedge ratio that minimizes the variance of the value of the hedged position. We now consider how the hedger can do this.

Calculating the Minimum Variance Hedge Ratio

We first present an analysis assuming no daily settlement of futures contracts. The minimum variance hedge ratio depends on the relationship between changes in the spot price and changes in the futures price. Define:

ΔS: Change in spot price, S, during a period of time equal to the life of the hedge

ΔF: Change in futures price, F, during a period of time equal to the life of the hedge.

If we assume that the relationship between ΔS and ΔF is approximately linear (see Figure 3.2), we can write:

$$\Delta S = a + b\Delta F + \epsilon$$

where a and b are constants and ϵ is an error term. Suppose that the hedge ratio is h (i.e., a percentage h of the exposure to S is hedged with futures). Then the change in the value of the position per unit of exposure to S is

$$\Delta S - h\Delta F = a + (b - h)\Delta F + \epsilon$$

The standard deviation of this is minimized by setting $h = b$ (so that the second term on the right-hand side disappears).

Denote the minimum variance hedge ratio by h^*. We have shown that $h^* = b$. It follows from the formula for the slope in linear regression that

$$h^* = \rho\frac{\sigma_S}{\sigma_F} \tag{3.1}$$

Figure 3.2 Regression of change in spot price against change in futures price.

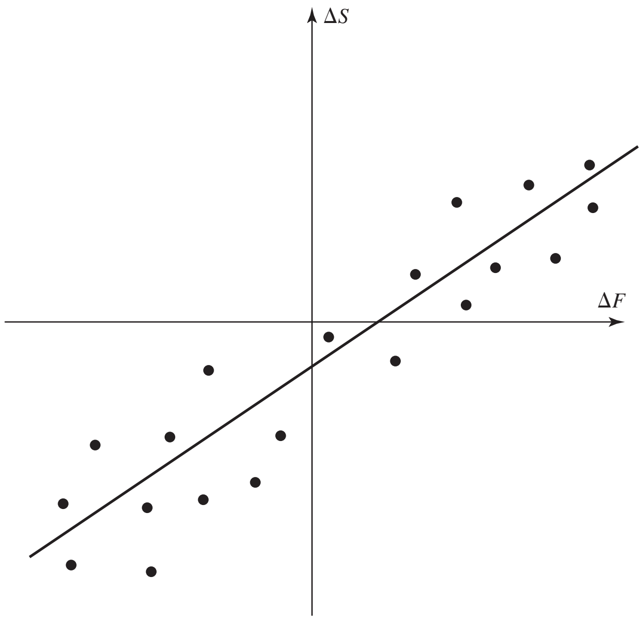
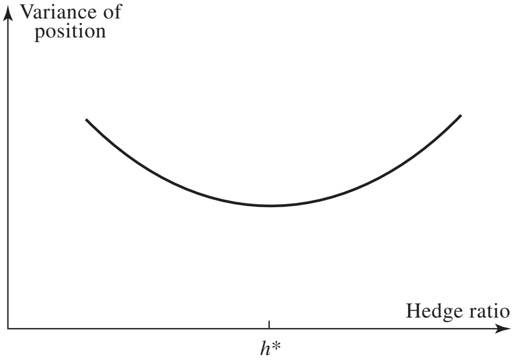


Figure 3.3 Dependence of variance of hedger's position on hedge ratio.



where σ_S is the standard deviation of ΔS, σ_F is the standard deviation of ΔF, and ρ is the coefficient of correlation between the two.

Equation (3.1) shows that the optimal hedge ratio is the product of the coefficient of correlation between ΔS and ΔF and the ratio of the standard deviation of ΔS to the standard deviation of ΔF. Figure 3.3 shows how the variance of the value of the hedger's position depends on the hedge ratio chosen.

If $\rho = 1$ and $\sigma_F = \sigma_S$, the hedge ratio, h^*, is 1.0. This result is to be expected, because in this case the futures price mirrors the spot price perfectly. If $\rho = 1$ and $\sigma_F = 2\sigma_S$, the hedge ratio h^* is 0.5. This result is also as expected, because in this case the futures price always changes by twice as much as the spot price. The *hedge effectiveness* can be defined as the proportion of the variance that is eliminated by hedging. This is the R^2 from the regression of ΔS against ΔF and equals ρ^2.

The parameters ρ, σ_F, and σ_S in equation (3.1) are usually estimated from historical data on ΔS and ΔF. (The implicit assumption is that the future will in some sense be like the past.) A number of equal nonoverlapping time intervals are chosen, and the values of ΔS and ΔF for each of the intervals are observed. Ideally, the length of each time interval is the same as the length of the time interval for which the hedge is in effect. In practice, this sometimes severely limits the number of observations that are available, and a shorter time interval is used.

Optimal Number of Contracts

To calculate the number of contracts that should be used in hedging, define:

Q_A: Size of position being hedged (units)

Q_F: Size of one futures contract (units)

N^*: Optimal number of futures contracts for hedging.

The futures contracts should be on $h^* Q_A$ units of the asset. The number of futures contracts required is therefore given by

$$N^* = \frac{h^* Q_A}{Q_F} \tag{3.2}$$

Example 3.3 shows how the results in this section can be used by an airline hedging the purchase of jet fuel.[3]

Example 3.3

An airline expects to purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil futures for hedging. We suppose that Table 3.2 gives, for 15 successive months, data on the change, ΔS, in the jet fuel price per gallon and the corresponding change, ΔF, in the futures price for the contract on heating oil that would be used for hedging price changes during the month. In this case, the usual formulas for calculating standard deviations and correlations give $\sigma_F = 0.0313$, $\sigma_S = 0.0263$, and $\rho = 0.928$.

From equation (3.1), the minimum variance hedge ratio, h^*, is therefore

$$0.928 \times \frac{0.0263}{0.0313} = 0.78$$

Each heating oil contract traded by the CME Group is on 42,000 gallons of heating oil. From equation (3.2), the optimal number of contracts is

$$\frac{0.78 \times 2{,}000{,}000}{42{,}000}$$

which is 37 when rounded to the nearest whole number.

Table 3.2 Data to calculate minimum variance hedge ratio when heating oil futures contract is used to hedge purchase of jet fuel.

Month *i*	*Change in heating oil futures price per gallon* $(= \Delta F)$	*Change in jet fuel price per gallon* $(= \Delta S)$
1	0.021	0.029
2	0.035	0.020
3	−0.046	−0.044
4	0.001	0.008
5	0.044	0.026
6	−0.029	−0.019
7	−0.026	−0.010
8	−0.029	−0.007
9	0.048	0.043
10	−0.006	0.011
11	−0.036	−0.036
12	−0.011	−0.018
13	0.019	0.009
14	−0.027	−0.032
15	0.029	0.023

[3] Derivatives with payoffs dependent on the price of jet fuel do exist, but heating oil futures are often used to hedge an exposure to jet fuel prices because they are traded more actively.

Impact of Daily Settlement

The analysis we have presented so far is appropriate when forward contracts are used for hedging. The daily settlement of futures contract means that, when futures contracts are used, there are a series of one-day hedges, not a single hedge. Define:

$\hat{\sigma}_S$: Standard deviation of percentage one-day changes in the spot price
$\hat{\sigma}_F$: Standard deviation of percentage one day changes in the futures price
$\hat{\rho}$: Correlation between percentage one-day changes in the spot and futures.

The standard deviation of one-day changes in spot and futures are $\hat{\sigma}_S S$ and $\hat{\sigma}_F F$. Also, $\hat{\rho}$ is the correlation between one-day changes. It follows from equation (3.1) that the optimal one-day hedge is

$$h^* = \hat{\rho}\frac{\hat{\sigma}_S S}{\hat{\sigma}_F F}$$

so that from equation (3.2)

$$N^* = \hat{\rho}\frac{\hat{\sigma}_S S Q_A}{\hat{\sigma}_F F Q_F}$$

The hedge ratio in equation (3.1) is based on regressing actual changes in spot prices against actual changes in futures prices. An alternative hedge ratio, $\hat{h}$, can be derived in the same way by regressing daily percentage changes in spot against daily percentage changes in futures:

$$\hat{h} = \hat{\rho}\frac{\hat{\sigma}_S}{\hat{\sigma}_F}$$

Then

$$N^* = \frac{\hat{h} V_A}{V_F} \tag{3.3}$$

where $V_A = SQ_A$ is the value of the position being hedged and $V_F = FQ_F$ is the futures price times the size of one contract.

Consider another situation where 2 million gallons of jet fuel is being hedged with heating oil futures. Suppose that the spot and futures price are 1.10 and 1.30, respectively. In this case, $V_A = 2,000,000 \times 1.10 = 2,200,000$ while $V_F = 42{,}000 \times 1.30 = 54{,}600$. If $\hat{\rho} = 0.8$, the optimal number of contracts for a one-day hedge is

$$\frac{0.8 \times 2{,}200{,}000}{54{,}600} = 32.23$$

or 32 when rounded to the nearest whole number. The optimal hedge is liable to change from day to day as the relative values of spot and futures prices change. But the changes are usually small and often ignored.

The analysis can be refined by taking account of the the interest that is earned or paid over the remaining life of the hedge. Suppose that the interest rate is 5% per annum and the hedge has a remaining life of one year. It is then appropriate to divide N^* by 1.05. This is referred to as *tailing the hedge*.

3.5 STOCK INDEX FUTURES

We now move on to consider stock index futures and how they are used to hedge or manage exposures to equity prices.

A *stock index* tracks changes in the value of a hypothetical portfolio of stocks. The weight of a stock in the portfolio at a particular time equals the proportion of the hypothetical portfolio invested in the stock at that time. The percentage increase in the stock index over a small interval of time is set equal to the percentage increase in the value of the hypothetical portfolio. Dividends are usually not included in the calculation so that the index tracks the capital gain/loss from investing in the portfolio.[4]

If the hypothetical portfolio of stocks remains fixed, the weights assigned to individual stocks in the portfolio do not remain fixed. When the price of one particular stock in the portfolio rises more sharply than others, more weight is automatically given to that stock. Sometimes indices are constructed from a hypothetical portfolio consisting of one of each of a number of stocks. The weights assigned to the stocks are then proportional to their market prices, with adjustments being made when there are stock splits. Other indices are constructed so that weights are proportional to market capitalization (stock price $\times$ number of shares outstanding). The underlying portfolio is then automatically adjusted to reflect stock splits, stock dividends, and new equity issues.

Stock Indices

Table 3.3 shows futures prices for contracts on three different stock indices on May 21, 2021.

The *Dow Jones Industrial Average* is based on a portfolio consisting of 30 blue-chip stocks in the United States. The weights given to the stocks are proportional to their prices. The CME Group trades two futures contracts on the index. One is on $10 times

Table 3.3 Futures quotes for a selection of CME Group contracts on stock indices on May 21, 2020.

	Open	*High*	*Low*	*Prior settlement*	*Last trade*	*Change*	*Volume*
Mini Dow Jones Industrial Average, $5 times index							
June 2020	24,545	24,660	24,300	24,519	24,421	−98	210,202
Sept. 2020	24,426	24,542	24,212	24,411	24,311	−100	140
Mini S&P 500, $50 times index							
June 2020	2,972.25	2,973.50	2,933.00	2,968.50	2,944.00	−24.50	1,542,213
Sept. 2020	2,963.00	2,964.75	2,924.75	2,959.75	2,935.50	−24.25	3,048
Dec. 2020	2,939.50	2,955.50	2,928.25	2,954.50	2,930.00	−24.50	183
Mini NASDAQ-100, $20 times index							
Mar. 2019	9,494.75	9,510.75	9,355.00	9,485.50	9,372.00	−113.50	420,841
June 2019	9,470.75	9,495.00	9,349.50	9,469.75	9,360.75	−109.00	279

[4] An exception to this is a *total return index*. This is calculated by assuming that dividends on the hypothetical portfolio are reinvested in the portfolio.

the index. The other (the Mini DJ Industrial Average) is on \$5 times the index. The Mini contract trades most actively.

The *Standard & Poor's 500* (S&P 500) *Index* is based on a portfolio of 500 different stocks: 400 industrials, 40 utilities, 20 transportation companies, and 40 financial institutions. The weights of the stocks in the portfolio at any given time are proportional to their market capitalizations. The stocks are those of large publicly held companies that trade on NYSE Euronext or Nasdaq OMX. The CME Group trades two futures contracts on the S&P 500. One is on \$250 times the index; the other (the Mini S&P 500 contract) is on \$50 times the index. The Mini contract trades most actively.

The *Nasdaq-100* is based on a portfolio of 100 stocks traded on the Nasdaq exchange with weights proportional to market capitalizations. The CME Group trades two futures contracts. One is on \$100 times the index; the other (the Mini Nasdaq-100 contract) is on \$20 times the index. The Mini contract trades most actively.

Some futures contracts on indices outside the United States are also traded actively. An example is the contract on the CSI 300 index, a market-capitalization-weighted index of 300 Chinese stocks, which trades on the China Financial Futures Exchange (CFFEX, www.cffex.com.cn).

As mentioned in Chapter 2, futures contracts on stock indices are settled in cash, not by delivery of the underlying asset. All contracts are marked to market to either the opening price or the closing price of the index on the last trading day, and the positions are then deemed to be closed. For example, contracts on the S&P 500 are closed out at the opening price of the S&P 500 index on the third Friday of the delivery month.

Hedging an Equity Portfolio

Stock index futures can be used to hedge a well-diversified equity portfolio. Define:

V_A: Current value of the portfolio

V_F: Current value of one futures contract (the futures price times the contract size).

If the portfolio mirrors the index, the optimal hedge ratio can be assumed to be 1.0 and equation (3.3) shows that the number of futures contracts that should be shorted is

$$N^* = \frac{V_A}{V_F} \tag{3.4}$$

Suppose, for example, that a portfolio worth \$5,050,000 mirrors a well-diversified index. The index futures price is 1,010 and each futures contract is on \$250 times the index. In this case $V_A = 5{,}050{,}000$ and $V_F = 1{,}010 \times 250 = 252{,}500$, so that 20 contracts should be shorted to hedge the portfolio.

When the portfolio does not mirror the index, we can use the capital asset pricing model (see the appendix to this chapter). The parameter beta (β) from the capital asset pricing model is the slope of the best-fit line obtained when excess return on the portfolio over the risk-free rate is regressed against the excess return of the index over the risk-free rate. When $\beta = 1.0$, the return on the portfolio tends to mirror the return on the index; when $\beta = 2.0$, the excess return on the portfolio tends to be twice as great as the excess return on the index; when $\beta = 0.5$, it tends to be half as great; and so on.

A portfolio with a β of 2.0 is twice as sensitive to movements in the index as a portfolio with a beta 1.0. It is therefore necessary to use twice as many contracts to hedge the portfolio. Similarly, a portfolio with a beta of 0.5 is half as sensitive to market movements as a portfolio with a beta of 1.0 and we should use half as many contracts to hedge it. In general,

$$N^* = \beta \frac{V_A}{V_F} \tag{3.5}$$

This formula assumes that the maturity of the futures contract is close to the maturity of the hedge.

Comparing equation (3.5) with equation (3.3), we see that they imply $\hat{h} = \beta$. This is not surprising. The hedge ratio $\hat{h}$ is the slope of the best-fit line when percentage one-day changes in the portfolio are regressed against percentage one-day changes in the futures price of the index. Beta (β) is the slope of the best-fit line when the return from the portfolio is regressed against the return for the index.

We illustrate that this formula gives good results by extending our earlier example. Suppose that a futures contract with 4 months to maturity is used to hedge the value of a portfolio over the next 3 months in the following situation:

$$\begin{aligned}
&\text{Index level} = 1{,}000\\
&\text{Index futures price} = 1{,}010\\
&\text{Value of portfolio} = \$5{,}050{,}000\\
&\text{Risk-free interest rate} = 4\% \text{ per annum}\\
&\text{Dividend yield on index} = 1\% \text{ per annum}\\
&\text{Beta of portfolio} = 1.5
\end{aligned}$$

One futures contract is for delivery of \$250 times the index. As before, $V_F = 250 \times 1{,}010 = 252{,}500$. From equation (3.5), the number of futures contracts that should be shorted to hedge the portfolio is

$$1.5 \times \frac{5{,}050{,}000}{252{,}500} = 30$$

Suppose the index turns out to be 900 in 3 months and the futures price is 902. The gain from the short futures position is then

$$30 \times (1010 - 902) \times 250 = \$810{,}000$$

The loss on the index is 10%. The index pays a dividend of 1% per annum, or 0.25% per 3 months. When dividends are taken into account, an investor in the index would therefore earn −9.75% over the 3-month period. Because the portfolio has a β of 1.5, the capital asset pricing model gives

$$\begin{aligned}
&\text{Expected return on portfolio} - \text{Risk-free interest rate}\\
&\qquad = 1.5 \times (\text{Return on index} - \text{Risk-free interest rate})
\end{aligned}$$

The risk-free interest rate is approximately 1% per 3 months. It follows that the expected return (%) on the portfolio during the 3 months when the 3-month return on the index is −9.75% is

$$1.0 + [1.5 \times (-9.75 - 1.0)] = -15.125$$

The expected value of the portfolio (inclusive of dividends) at the end of the 3 months is

Table 3.4 Performance of stock index hedge.

Value of index in three months:	900	950	1,000	1,050	1,100
Futures price of index today:	1,010	1,010	1,010	1,010	1,010
Futures price of index in three months:	902	952	1,003	1,053	1,103
Gain on futures position ($):	810,000	435,000	52,500	−322,500	−697,500
Return on market:	−9.750%	−4.750%	0.250%	5.250%	10.250%
Expected return on portfolio:	−15.125%	−7.625%	−0.125%	7.375%	14.875%
Expected portfolio value in three months including dividends ($):	4,286,187	4,664,937	5,043,687	5,422,437	5,801,187
Total value of position in three months ($):	5,096,187	5,099,937	5,096,187	5,099,937	5,103,687

therefore

$$\$5{,}050{,}000 \times (1 - 0.15125) = \$4{,}286{,}187$$

It follows that the expected value of the hedger's position, including the gain on the hedge, is

$$\$4{,}286{,}187 + \$810{,}000 = \$5{,}096{,}187$$

Table 3.4 summarizes these calculations together with similar calculations for other values of the index at maturity. It can be seen that the total expected value of the hedger's position in 3 months is almost independent of the value of the index. This is what one would expect if the hedge is a good one.

The only thing we have not covered so far is the relationship between futures prices and spot prices. We will see in Chapter 5 that the 1,010 assumed for the futures price today is roughly what we would expect given the interest rate and dividend we are assuming. The same is true of the futures prices in 3 months shown in Table 3.4.[5]

Reasons for Hedging an Equity Portfolio

Table 3.4 shows that the hedging procedure results in a value for the hedger's position at the end of the 3-month period being about 1% higher than at the beginning of the 3-month period. There is no surprise here. The risk-free rate is 4% per annum, or 1% per 3 months. The hedge results in the investor's position growing at the risk-free rate.

It is natural to ask why the hedger should go to the trouble of using futures contracts. To earn the risk-free interest rate, the hedger can simply sell the portfolio and invest the proceeds in a risk-free security.

One answer to this question is that hedging can be justified if the hedger feels that the stocks in the portfolio have been chosen well. In these circumstances, the hedger might be very uncertain about the performance of the market as a whole, but

[5] The calculations in Table 3.4 assume that the dividend yield on the index is predictable, the risk-free interest rate remains constant, and the return on the index over the 3-month period is perfectly correlated with the return on the portfolio. In practice, these assumptions do not hold perfectly, and the hedge works rather less well than is indicated by Table 3.4.

confident that the stocks in the portfolio will outperform the market (after appropriate adjustments have been made for the beta of the portfolio). A hedge using index futures removes the risk arising from market moves and leaves the hedger exposed only to the performance of the portfolio relative to the market. This will be discussed further shortly. Another reason for hedging may be that the hedger is planning to hold a portfolio for a long period of time and requires short-term protection in an uncertain market situation. The alternative strategy of selling the portfolio and buying it back later might involve unacceptably high transaction costs.

Changing the Beta of a Portfolio

In the example in Table 3.4, the beta of the hedger's portfolio is reduced to zero so that the hedger's expected return is almost independent of the performance of the index. Sometimes futures contracts are used to change the beta of a portfolio to some value other than zero. Continuing with our earlier example:

$$\begin{aligned}&\text{Index level} = 1{,}000\\&\text{Index futures price} = 1{,}010\\&\text{Value of portfolio} = \$5{,}050{,}000\\&\text{Beta of portfolio} = 1.5\end{aligned}$$

As before, $V_F = 250 \times 1{,}010 = 252{,}500$ and a complete hedge requires

$$1.5 \times \frac{5{,}050{,}000}{252{,}500} = 30$$

contracts to be shorted. To reduce the beta of the portfolio from 1.5 to 0.75, the number of contracts shorted should be 15 rather than 30; to increase the beta of the portfolio to 2.0, a long position in 10 contracts should be taken; and so on. In general, to change the beta of the portfolio from β to β^*, where $\beta > \beta^*$, a short position in

$$(\beta - \beta^*)\frac{V_A}{V_F}$$

contracts is required. When $\beta < \beta^*$, a long position in

$$(\beta^* - \beta)\frac{V_A}{V_F}$$

contracts is required.

Locking in the Benefits of Stock Picking

Suppose you consider yourself to be good at picking stocks that will outperform the market. You own a single stock or a small portfolio of stocks. You do not know how well the market will perform over the next few months, but you are confident that your portfolio will do better than the market. What should you do?

You should short $\beta V_A/V_F$ index futures contracts, where β is the beta of your portfolio, V_A is the total value of the portfolio, and V_F is the current value of one index futures contract. If your portfolio performs better than a well-diversified portfolio with the same beta, you will then make money.