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Introduction to Logical Theory

P. F. Strawson



Introduction to Logical Theory

First published in 1952, Professor Strawson's highly influential *Introduction to Logical Theory* provides a detailed examination of the relationship between the behaviour of words in common language and the behaviour of symbols in a logical system. He seeks to explain both the exact nature of the discipline known as Formal Logic, and also to reveal something of the intricate logical structure of ordinary unformalised discourse.

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Introduction to LOGICAL THEORY

by

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PREFACE

THERE are in existence many text-books and technical treatises on formal logic, and I have not sought in this book to add to their number. Many such books, excellent as they often are in their expositions of the technical and systematic aspects of logic, deal comparatively sketchily, and often rather misleadingly, with the relations between the formal systems they expound and the logical features of ordinary discourse. As a result of this omission, the true character of formal logic itself is apt to be left obscure. So this book has two complementary aims: one is to bring out some points of contrast and of contact between the behaviour of words in ordinary speech and the behaviour of symbols in a logical system; the other is to make clear, at an introductory level, the nature of formal logic itself. I have included enough of the elementary material of formal logic to provide a basis for the philosophical discussion of its nature, and to serve, if desired, as an introduction to more advanced technical treatises. Since the book is designed to be used as a general introduction to logic, I have added a concluding chapter on induction and probability.

I wish to acknowledge my great indebtedness to the many Oxford colleagues from whose discussions of the topics of this book I have profited; and among these, in particular, to Mr. H. P. Grice, from whom I have never ceased to learn about logic since he was my tutor in the subject; and to Professor Gilbert Ryle, Mr. G. A. Paul and Miss Ruby Meager, all of whom read the book either in manuscript or in proof and saved me from many inelegancies and mistakes.

P. F. S.

Oxford,
May, 1952.

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CHAPTER I

LOGICAL APPRAISAL

1. WHEN a man says or writes something, there are many different ways in which his performance may be judged. Among other things, we may question his truthfulness or criticize his style, we may assess the morality of what he says, or we may appraise its logic; though not all these types of assessment are appropriate to all kinds of utterance. The words 'logical' and 'illogical' are themselves among the words of logical appraisal. If you call a discourse logical, you are in some degree commending it. If you call it illogical, you are, so far, condemning it. Words and phrases which go with 'logical' are 'consistent', 'cogent', 'valid', 'it follows'; words and phrases which go with 'illogical' are 'inconsistent', 'self-contradictory', 'invalid', 'a *non sequitur*'. Part of our problem is to see what sort of appraisal these words are used for, to what kind of standards we appeal in using them. It is easy to see that these are not moral or aesthetic standards; that logical criticism is not, say, a kind of literary criticism. A slightly more difficult distinction is that between the criticism we offer when we declare a man's remarks to be untrue and the criticism we offer when we declare them to be inconsistent. In the first case we criticize his remarks on the ground that they fail to square with the facts; in the second case we criticize them on the ground that they fail to square with one another. The charge of untruth refers beyond the words and sentences the man uses to that in the world about which he talks. We deny his assertion, and, in doing so, make a counter-assertion of our own about the subject of his discourse. We contradict him. But the charge of inconsistency does not in this way refer to anything outside the statements that the man makes. We simply consider the way his statements hang together. Just from considering the sentences themselves, as they are used, we can, perhaps, see that not all the statements he makes can be true together. It is not that we contradict him, and in doing so, make a counter-assertion about the subject of his remarks;

we assert that he has contradicted himself, and, in doing this, we make no appeal to the facts and express no opinion about them. It is this kind of internal criticism that is appraisal of the logic of a piece of discourse.

I. INCONSISTENCY

2. Words of logical appraisal have connected meanings. To be clear about the meaning of one such word is to be clear about the meanings of the others. For example, in a proof or argument, one statement (the conclusion) is said to follow logically from, or to be logically implied by, others (the premises), if the argument is valid; and an argument is valid only if it would be inconsistent (or self-contradictory) to assert the premises while denying the conclusion; or, in other words, only if the truth of the premises is inconsistent with the falsity of the conclusion. A deductive argument is a sort of threat, which takes the form: if you accept these premises as true, then you must accept this conclusion as true as well, on pain of self-contradiction. From among the various concepts of logical appraisal, I shall select this notion of inconsistency or self-contradiction for detailed discussion. Other choices could have been made, but there are reasons, which will emerge as we go on, for making this choice.

3. What is inconsistency? It is better to approach this question indirectly, by asking a series of others. One might ask first: Why bother to avoid inconsistency? What is wrong with contradicting yourself? There is nothing morally wrong about it. It may not even be entirely pointless. Suppose a man sets out to walk to a certain place; but, when he gets half-way there, turns round and comes back again. This may not be pointless. He may, after all, have wanted only exercise. But, from the point of view of a change of position, it is as if he had never set out. And so a man who contradicts himself may have succeeded in exercising his vocal chords. But from the point of view of imparting information, of communicating facts (or falsehoods) it is as if he had never opened his mouth. He utters words, but does not say anything. Or he might be compared with a man who makes as if to give something away and then takes it back again. He arouses expectations which he does not

fulfil; and this may have been his purpose. Similarly, it may have been the purpose of a man who contradicts himself just to create puzzlement. The point is that the *standard* purpose of speech, the intention to communicate something, is frustrated by self-contradiction. Contradicting oneself is like writing something down and then erasing it, or putting a line through it. A contradiction cancels itself and leaves nothing. Consequently one cannot explain what a contradiction is just by indicating, as one might be tempted to do, a certain form of words. One might be tempted to say that a contradiction was anything of the form 'X is the case and X is not the case'. But this will not do. If someone asks you whether you were pleased by something, you may reply: 'Well, I was and I wasn't', and you will communicate perfectly well. Or there might be a convention that when one said anything of this form, the second part of the sentence was to be neglected. Then the minimum requirement for such a contradiction would be to say, first, 'X is the case and X is not the case' and, after that, 'X is not the case and X is the case'. Nevertheless, the temptation to explain a contradiction as anything of this form is, we shall see, not without point.

4. The next two questions to ask are more difficult. They are: (a) when we use these words of logical appraisal, what is it exactly that we are appraising? and (b) how does logical appraisal become possible? That is, we shall ask: what is it exactly that we declare to be inconsistent? and: what makes inconsistency possible? I have spoken of *statements* as being inconsistent with each other; and there is a temptation to think that in this context we mean by a statement the same thing as a sentence of a certain kind; or, perhaps, the meaning of such a sentence. But suppose I write on the blackboard the following two pairs of sentences: (i) 'I am under six foot tall' and 'I am over six foot tall'; (ii) 'The conductor is a bachelor' and 'The conductor is married'. In writing the sentences on the blackboard, I have, of course, not contradicted myself; for I may have written them there with a purely illustrative intention, in giving an English lesson. Someone might say: Nevertheless, the sentences in each pair *are* inconsistent with each other. But what would this mean? Would it mean that if they were

ever uttered with the intention of making a statement, an inconsistency would result? But suppose the first two sentences were uttered by different people, or by the same person at an interval of years; and that the second two sentences were uttered in different omnibuses, or in the same omnibus, but on different days. Then there would be no inconsistency. Earlier, I paraphrased 'seeing that two statements are inconsistent' as 'seeing that they cannot both be true together'. And it is clear that that of which we can say that it is true or false is also that of which we can say that it is consistent or inconsistent with another of its kind. What these examples show is that we cannot identify that which is true or false (the statement) with the sentence used in making it; for the same sentence may be used to make quite different statements, some of them true and some of them false. And this does not arise from any ambiguity in the sentence. The sentence may have a single meaning which is precisely what, as in these cases, allows it to be used to make quite different statements. So it will not do to identify the statement either with the sentence or with the meaning of the sentence. A particular statement is identified, not only by reference to the words used, but also by reference to the circumstances in which they are used, and, sometimes, to the identity of the person using them. No one would be tempted to say that the sentence 'I am over six foot tall' was inconsistent with the sentence 'You are under six foot tall'. But plainly they can be used, in certain circumstances, to make statements which are inconsistent with each other; i.e., in the case where the second sentence is addressed to the man by whom the first sentence is uttered.

It is easy to see why one is tempted to think of the sentence 'I am over six foot tall' as being inconsistent with the sentence 'I am under six foot tall'. One thinks of both sentences as being uttered, in the same breath, by the same person. In this case we should ordinarily regard that person as having contradicted himself, i.e., we should regard him as having said something and then unsaid it; and so as having said nothing. The important assumption is that the two expressions 'over six foot tall' and 'under six foot tall' are applied to the same person at the same time. Let us give the name 'incompatible predicates' to any pair of expressions the application of which

to the same person or thing at the same time results in an inconsistency. Thus we can say that one of the ways in which it is possible to say something inconsistent is by applying incompatible predicates to the same person or thing at the same time.

5. But must a language have incompatible predicates in it? And what makes predicates incompatible? I want to answer the first question by saying, not that a language must have incompatible predicates in it; only that it is very natural that it should. And I want to answer the second question by saying that it is we, the makers of language, who make predicates incompatible. One of the main purposes for which we use language is to report events and to describe things and persons. Such reports and descriptions are like answers to questions of the form: what was it like? what is it (he, she) like? We describe something, say what it is like, by applying to it words that we are also prepared to apply to other things. But not to all other things. A word that we are prepared to apply to everything without exception (such as certain words in current use in popular, and especially military, speech) would be useless for the purposes of description. For when we say what a thing is like, we not only compare it with other things, we also distinguish it from other things. (These are not two activities, but two aspects of the same activity.) Somewhere, then, a boundary must be drawn, limiting the applicability of a word used in describing things; and it is we who decide where the boundaries are to be drawn.

This metaphor of drawing boundaries is in some ways misleading. I do not mean by it that we often make conscious decisions of this kind (though we sometimes do); nor that our boundary-drawing is a quite arbitrary matter; nor that the boundaries are fixed and definite; nor that the decisions we make when we make them, are purely verbal decisions. The boundaries are more like areas of indeterminate ownership than frontier-lines. We show ourselves to be near such a boundary, and we show also its indeterminacy, when, in reply to such a question as 'Was it red?', we give such an answer as 'Well, I suppose you could call it red'. We show ourselves on the point of making a boundary-decision when, with all the facts before us, we hesitate over the application of a certain word. Does

such and such an act constitute an act of aggression or not? This case shows, too, how our decision is not a purely verbal matter; for important consequences may follow from our deciding that it is, or is not, an act of aggression. What makes our decisions, for a word already in use, non-arbitrary, is this: that our normal purpose will be defeated if the comparison implicit in the use of the word is too unnatural, if the similarity is too tenuous.

We may say: two predicates are incompatible when they lie on different sides of a boundary we have drawn: 'under six foot tall' and 'over six foot tall'; 'red' and 'orange'; 'aggressive' and 'pacific'. But this needs some explanation. Suppose you draw a closed figure on a piece of paper and then someone indicates a point on the ceiling and says: 'Does this point lie inside or outside the boundaries of the figure?' Of course, one might answer by imagining the boundaries of the figure extended in another dimension, up to the ceiling. But you might refuse to answer the question, by saying that you were drawing the boundary line only in the plane of the paper. Whatever lay outside the line in the plane of the paper was excluded from the figure. Things lying in a different plane were not excluded from it, but neither were they included in it. The figure has a certain plane of exclusiveness. And so with a word: it has a certain range of incompatibilities. 'Under six foot tall' is incompatible with 'over six foot tall'; but neither is incompatible with 'aggressive'. The last expression has a different incompatibility-range from the other two. There may sometimes be objections of a logical kind to applying expressions with different incompatibility-ranges to the same thing; but these will not be the objection that inconsistency will result from doing so.

When we apply a predicate to something, we implicitly exclude from application to that thing the predicates which lie outside the boundaries of the predicate we apply, but in the same incompatibility-range. By this I mean that if we go on to apply to the thing, in the same breath, one of the predicates which lie outside those boundaries, we shall be taken to have contradicted ourselves and said nothing. (This might be taken as a definition of 'incompatible predicates'.) But there is a qualification to be made here. Just as we might reply to the

query 'Were you pleased?' with the words 'Well, I was and I wasn't' without inconsistency, so we might apply to the same thing, in the same breath, two predicates, which would ordinarily be regarded as incompatible, without contradicting ourselves. If we do this, we invite the question 'What do you mean?'; and if we can explain what we mean, or show the point of saying what we say, then we have not contradicted ourselves. But if there is no way of doing this, we are inconsistent. Thus we might say, in answer to a question, 'He is both over six foot tall and under six foot tall', and then explain that he has a disease which makes him stoop, but that if he were cured and were able to stand upright, he would top the six-foot mark. This shows again that one cannot fully explain what self-contradiction is, just by reference to groupings of words.

6. So long as we bear this qualification in mind, we can safely speak of incompatible predicates and can safely say that, when we apply a predicate to something by way of describing it, we implicitly exclude from application to it any predicates incompatible with that which we apply. (We should be said to have contradicted anyone who had just applied any of those predicates to the thing.) When we notice that this function of exclusion is implicit in all descriptive uses of language, we should not find it surprising that language contains devices for rendering the function explicit; devices of which, in English, the word 'not' is the most prominent. There are many very different kinds of occasion on which our primary concern is with the explicit exclusion of a predicate; e.g., when we wish to contradict a previous assertion; or to correct a possible false impression; or to express the contrast between what had been expected, feared, suggested, or hoped, and the reality; sometimes, when we are answering a direct question; sometimes, when we grope towards the right description by eliminating the wrong ones. What is common to such cases is that they create a need or a motive for emphasizing a difference rather than a resemblance. It is instructive to compare the use of 'not' with the use of those words which begin with negative prefixes; like 'intolerable', 'unpretentious', 'impolite', 'non-aggressive'. These words bear their incompatibilities on their faces as surely as any

phrase containing 'not'; but one would hardly say of them that they have the same function of explicitly rejecting a suggested description. They do not point more emphatically to differences than to likenesses; they rather serve to underline the fact that the two are complementary. One might ask why some words have such manifest incompatibles (viz., words which are the same except for a negative prefix), while others do not; why we do not speak of things as 'unblue', for example. One might be inclined to suggest that it is because 'not being blue' is relatively so indeterminate; i.e., that, where there is a wide range of incompatible predicates, like colour-words, it is unnatural to have a single word expressly excluding one of them. But I do not think this is a complete answer. There is a wide range of races and nationalities, but we have words (e.g., 'foreign', 'non-English', 'non-European') to indicate 'not being of a particular nationality (or range of races)'. I think the answer is, rather, that if we had a constant and persistent interest in things not being blue, as opposed to such a temporary interest as may arise from, e.g., wishing to correct a false impression, then we *should* have a word for this. Then we might say that in calling a thing 'unblue' we should be as much emphasizing its likeness to other unblue things as its difference from blue things. (It was characteristic of those formal logicians who framed unnatural-looking negative terms, like 'non-blue', not to concern themselves with questions and differences of this kind.)

This discussion of the function of 'not' helps us to see part of the point of the saying, incorrect though it is, that a contradiction is simply something of the form 'X is the case and X is not the case'. The standard and primary use of 'not' is specifically to contradict or correct; to cancel a suggestion of one's own or another's. And there is no restriction on the sphere in which it may exercise this function. Not all predicates have corresponding negatively prefixed terms, and not all statements are of the kind in which we simply apply a descriptive predicate to some person or thing. But any statement, whether or not it is of this simple kind, can be contradicted by the use of 'not'. So we are strongly inclined to regard a statement involving something of the form 'X is the case and X is not the case' as a self-contradiction; though always the indeterminacy

of the verbal boundaries we draw, the different points of view which may tempt us both to apply and to withhold an expression, allow of the possibility of a consistent meaning being given to something of this form.

7. It is, then, our own activity of making language through using it, our own determination of the limits of the application of words, that makes inconsistency possible; and it is no accident that, when we want to form for ourselves a general pattern or type of inconsistency, we employ the two words 'and' and 'not', together with a repetition of some one phrase or expression. Since all concepts of logical appraisal may be explained in terms of inconsistency, it is not surprising that these two words should play an important role in logic.

But we can create the possibility of inconsistency in statement, and hence of validity in argument, in a way more deliberate and self-conscious than those I have so far discussed. We can deliberately fix the boundaries of some words in relation to those of other words. This is what we do when we *define*¹ words or phrases. To introduce or to accept a definition¹ is to announce or to agree that conjoining the defined (or defining) expression with the defining (or defined) expression by the words 'and' and 'not' in their standard use (or in any equivalent way), and referring this conjunction to one and the same situation, is to count as an inconsistency. Accepting a definition is agreeing to be bound by a rule of language of this kind.

8. Let us now return to the questions we asked earlier: namely, what is it to which we apply words of logical appraisal? and: what makes logical appraisal possible? We saw that the answer to the first question was not 'sentences or groups of words', but 'statements or groups of statements'. It is statements and not sentences that are inconsistent with one another, follow from one another, etc. We see that the answer to the second question is: the boundaries of application that we draw between one expression and another, the rules we come to

¹ The words 'define' and 'definition' have many connected, though distinguishable, uses, of some of which what I say here is not true. I use the words here in a 'strict' sense.

observe for using expressions of all kinds.¹ And the answer to the second question shows the full point of the temptation to answer the first by talking of sentences (groups of words) as being inconsistent. Behind inconsistencies between statements, stand rules for the use of expressions. If one understands this relationship, a lot of things which have puzzled people become clear. One sees how a linguistic rule for expressions in a particular language can lead to a general statement of logical appraisal which transcends individual languages altogether. Suppose someone says: 'A statement to the effect that a certain person is someone's son-in-law is inconsistent with the statement that he has never been married.' Let us call the statement he makes in saying this a general statement of logical appraisal or, for short, a logical statement. Now suppose someone says: 'In English the words "son-in-law of" mean the same as the words "married to the daughter of".' Let us call the statement he makes in saying this a linguistic statement. Now what is the relation between the logical statement and the linguistic statement? Well, suppose we translate into French the sentence used to make the logical statement. We shall obtain a sentence with no English words in it. If we also translate into French the sentence used to make the linguistic statement (i.e., the sentence beginning 'In English'), we shall obtain a French sentence, beginning 'En anglais', in which the expressions '"son-in-law of"' and '"married to the daughter of"' reappear unchanged. It seems that, whereas we are inclined to say that the French and English versions of the logical statement mean the same thing, or are used to make the same statement, we are not inclined to say that the English sentence used to make the linguistic statement means the same as the French sentence: 'En français les mots "gendre de" veulent dire la même chose que les mots "marié avec la fille de"'. For we are inclined to say that anyone uttering *this* sentence would be talking about a rule of French, whereas anyone uttering the English sentence used to make the linguistic statement would be talking about

¹ But we must notice that, as far as ordinary speech is concerned, and apart from the introduction of words by *definition*, this talk of 'rules' may mislead us. We do not *generally* (in ordinary speech) draw up rules and make our practice conform to them; it is rather that we extract the rules from our practice, from noticing when we *correct* one another, when we are inclined to say that something is *inconsistent*, and so on.

a rule of English. So these sentences are used to make quite different statements about quite different things, namely French words and English words. And if one says this, as one is strongly inclined to do, and also says that the French and English versions of the logical statement mean the same (are versions of the same statement), then, of course, it would seem to follow that the logical statement is not about what the linguistic statements are about, that the truth of the logical statement is independent of the truth of the linguistic statements; and from here it is an easy step to thinking of logical facts as independent of linguistic facts, and to adopting an attitude of reverence to logical facts. But to take this step is to forget that the fact that the English and French versions of the logical statement mean the same is itself in part the linguistic fact that 'son-in-law of' and 'married' in English mean the same as 'gendre de' and 'marié' in French. We might express this by saying that there is, after all, an alternative translation of the English sentence used for making the linguistic statement; namely the French sentence quoted above, beginning 'En français . . .' We might say that these were really different versions of the same rule; that in laying down inconsistency-rules in one language, we were implicitly laying down inconsistency-rules for the corresponding expressions in all languages; and that thus a linguistic statement *of the kind quoted* transcends the language of the words which it mentions. Only it is less natural to say this than to say that a logical statement transcends the language in which it is framed.

We see that there are difficulties in identifying logical statements with linguistic statements; in saying that sentences used to make logical statements mean the same as corresponding sentences used to make linguistic statements; and seeing this is apt to give us the illusion of an independent realm of logical facts, of which linguistic rules are merely the adventitious verbal clothing. We feel that, while it is a mere matter of linguistic history, which could easily have been different, that the expression 'son-in-law' means what it does mean, the statement we make when we say 'The statement that a man is a son-in-law is inconsistent with the statement that he has never been married' is one that could not be false, even though it is an historical accident that we make it in these words. But when

we voice this feeling we are voicing the truism that a word could not both have the sense it in fact has (the sense in which we use it in making statements) and not have that sense.

The important thing is to see that when you draw the boundaries of the applicability of words in one language and then connect the words of that language with those of another by means of translation-rules, there is no need to draw the boundaries again for the second language. They are already drawn. (I am not suggesting that this is the order in which things are done; though it is the order in which things are learned.) This is why (or partly why) logical statements framed in one language are not just about that language.

It is important also to notice that this reason for not regarding statements of logical appraisal as about particular groups of words (e.g., sentences) is different from, though connected with, that which we have discussed earlier. Earlier we pointed out that it is not sentences which we say are inconsistent with one another, follow from one another, etc., but statements; the question of what statement is made, and of whether a statement is made at all, depends upon other things than simply what words are used. But rules about words lie behind all statements of logical appraisal; and it remains to be seen whether we can best do logic in terms of rules directly about representative expressions, or in terms of logical relations between statements.

II. REASONING

9. People often say that logic is the study of the principles of deductive reasoning. But this is too narrow, and includes irrelevant suggestions. Arguing, proving, inferring, concluding, solving a mathematical problem, might all be said to be kinds of reasoning. Their aims and purposes are different. The aim of argument is conviction; one tries to get someone to agree that some statement is true or false. You may get a man to agree that a statement is true by showing him that it follows from other statements which he already accepts. You may get him to agree that a statement is false because from it there follows another which he rejects. Proving is different: a man may argue successfully, and even validly, without proving; for an invalid argument may convince, and the premises of a

valid argument may be false. Moreover, a man may prove something without arguing, without seeking to convince. When you prove a mathematical theorem in an examination, you are not trying to convince the examiner of its truth; your object is to exhibit your mathematical knowledge by writing down a set of statements of which the last is the theorem to be proved and of which each follows from the ones written down already, together with earlier theorems. Inferring, drawing conclusions, is different again. Here you know some facts or truths already, and are concerned to see what further information can be derived from them; to find out their logical consequences. Though inferring, proving, arguing have different purposes, they seem usually ¹ to have also the common purpose of connecting truths with truths. The validity of the steps is, in general, prized for the sake of the truth of the conclusions to which they lead. But neither the common purpose, nor the different purposes, of arguing, proving, inferring, are a logical concern. The logical question, of the validity of the steps, is one that can be raised and answered independently of the question of whether these purposes are achieved. The validity of the steps does not alone guarantee the truth of the conclusion, nor their invalidity, its falsity. For to say that the steps are valid, that the conclusion follows from the premises, is simply to say that it would be inconsistent to assert the premises and deny the conclusion; that the truth of the premises is inconsistent with the falsity of the conclusion. The assessment of the reasoning as valid rules out a certain *combination* of truth and falsity; viz., truth in the premises and falsity in the conclusion. But it leaves open the possibility of other combinations: falsity with falsity and falsity with truth, as well as truth with truth. We are not told, when we are told that the reasoning is valid, that it would be inconsistent to deny both premises and conclusion or to assert the conclusion and deny the premises.

10. We often signalize a claim to be making a valid step in reasoning by the use of certain expressions to link one statement,

¹ Not always. A child solves problems in applied arithmetic. What he aims at is not the *true* answer but the *right* answer. And what he is given marks for is not the answer, but the way he gets it.

or set of statements, and another. These are words and phrases like 'so', 'consequently', 'therefore', 'since', 'for', 'it follows that', &c. And other expressions are sometimes used to signalize steps, which we should rightly hesitate to call steps in reasoning, but which are of no less interest to the logician. I have in mind such expressions as 'that is to say', 'in other words', 'more briefly', 'I mean'. These are expressions which we sometimes (though not always or only) use on occasions on which we should describe ourselves, not as inferring or arguing, but rather as, say, putting into other words something that has already been said, or repeating it with something left out, or summarizing it, or making a *précis*. There is no sharply definite line separating those steps which we should call steps in reasoning, and those steps which we should describe in one of the alternative ways I have listed. Obviously, there are extremes, which we should classify without hesitation. Where the steps are numerous and intricate, we unhesitatingly apply such words as 'inference', or 'argument'; where something that has been said is simply repeated, in whole or in part, we unhesitatingly withhold these words. But there are borderline cases. A man who linked one part of his discourse with another by the phrase, 'in other words', thus disclaiming anything so portentous as an inference, might be met with the rejoinder 'But that doesn't follow', which imputes, and disallows, the claim to have validly inferred. The differences between the steps which are steps in reasoning and the steps we should not so describe are, from some points of view, important. From our present point of view, they are less important than the resemblances. What is common to all the cases I refer to is the claim, signalized by the linking expressions,¹ that it would be inconsistent to assert what precedes those expressions and to deny what follows them. The logician interests himself in cases in which this relationship holds between statements, irrespective of whether or not the transition from one statement to another so related to it is a transition which we should dignify by the name 'step in reasoning'; irrespective even of whether it is something we should acknow-

¹ Of course, the linking expressions I listed are not always used to make just this claim. Cf. Chapter 2, p. 37, and Chapter 9.

ledge as a transition. (Later, we shall see the reason for his catholicity of interest.) This explains why 'study of the principles of valid deductive reasoning' is too narrow a description of logic. A man who repeats himself does not reason. But it is inconsistent to assert and deny the same thing. So a logician will say that a statement has to itself the relationship he is interested in.

III. THE LOGICIAN'S SECOND-ORDER VOCABULARY

11. Most of the statements we make are not themselves about statements but about people or things. Statements which are not themselves about statements we shall call first-order statements; statements about first-order statements we shall call second-order statements; and so on. Since words of logical appraisal are used for talking about statements, the statements we make in using such words must at least be of the second order. We shall say that such words constitute a part of the logician's second-order vocabulary. Later, we shall speak analogously of first-order sentences (i.e., sentences used for making statements not about sentences or statements), and second-order sentences (i.e., sentences used for making statements about first-order sentences or first-order statements).

The phrases 'follows from' and 'logically implies' carry with them a suggestion of those mind-exercising situations in which we are prepared to talk of reasoning being carried on, of inferences being made, &c. The word 'valid', applied to a group of statements linked by some expression (e.g., 'therefore') signaling the claim that one of the statements follows from the others, carries the same suggestion. We want a word, to signify that one statement is so related to another that it would be inconsistent to assert the first and deny the second, which does not carry this suggestion. It is customary to use the word 'entails' for this purpose. But, when it is convenient to do so, I shall license myself also to use ordinary words and phrases of logical appraisal in a manner which disregards the suggestion that reasoning-situations are involved. Such a departure from ordinary usage need not be misleading, if it is self-conscious.