ROUTLEDGE

THIRD EDITION

MORE!

TEACHING FRACTIONS AND RATIOS FOR UNDERSTANDING

In-Depth Discussion and Reasoning Activities

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{\sqrt{4}}{2}$ $\frac{12.2}{14.4}$ $\frac{1}{2}$ $\frac{1}{4}$

SUSAN J. LAMON

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To Bill

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Preface

This resource book accompanies *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Teaching Strategies for Teachers*. It was originally intended as a scaffold for adults who are reasoning their way through the fraction world for the first time. Reasoning after many years of rule-based computation presents a challenge to one's understanding, logical thinking, problem solving, and ability to communicate! *MORE* still serves that purpose. However, those who have used the previous editions will note several changes in organization and in content.

In this edition, I have included more activities and supplemental problems in each chapter, including more real-world applications. Additional student work has been added for your analysis, and templates for key manipulatives are provided. Finally, following each chapter is a collection of praxis problems geared to the content of the chapter.

MORE is not an answer key; good reasoning should always produce correct answers, but the *process* is the goal. Everyone knows the conventional symbolic representations and algorithms for getting the correct answers. The purpose here is to demonstrate and to help you engage in powerful ways of thinking so that you can, in turn, enhance the mathematical education of your students.

Solutions are offered with this caveat: no solution should be taken as *the* way to think about a situation. *MORE* offers some suggestions, but no effort is made to exhaust all of the possibilities.

CHAPTER 1

Proportional Reasoning: An Overview

GETTING STARTED

1. If 6 men can build a house in 3 days, then to shorten the time, you will need more men on the job. As the number of men goes up, the number of days goes down. Six men build $\frac{1}{3}$ of the house in one day, so 12 men could build $\frac{2}{3}$ of the house in one day, and 18 men could build $\frac{3}{3}$ of one whole house in a day (assuming that all of the men do an honest day's work). This means that it would take 3 times as many men to complete the job in $\frac{1}{3}$ the time.

2. If 6 chocolates cost \$0.93, then 12 would cost \$1.86 and 24 would cost \$3.72. If 6 cost \$0.93, then 2 cost \$0.31. The cost of 22 candies is \$0.31 less than \$3.72 or \$3.41.

3. John has 3 times as many marbles as Mark, so you can think of the whole set of marbles as 4 equal groups, with 3 of the groups in front of John and 1 group in front of Mark. If there are 32 marbles, each group contains 8 marbles. John has 24 marbles and Mark has 8 marbles.

4. If Mac does twice as much as his brother, he will do $\frac{2}{3}$ of the lawn, while his brother does $\frac{1}{3}$. If it takes Mac 45 minutes to do $\frac{3}{3}$ of the job, then it takes 15 minutes to do $\frac{1}{3}$ and 30 minutes to do $\frac{2}{3}$. Meanwhile, during that 30 minutes, his little brother does the other $\frac{1}{3}$ of the lawn.

5. The more people you have working, the faster the job will get done (assuming, of course, that the boys do not goof off on the job). If 6 boys were given 20 minutes to clean up, then 1 boy should be given 6 times as much time or 120 minutes. Then 9 would each require $\frac{1}{9}$ of the time needed by 1 person: $\frac{120}{9}$ or $\frac{40}{3}$ or $13\frac{1}{3}$ minutes. Another way. If 6 boys can do the job in 20 minutes, then 3 would take

40 minutes. If there are 9 people working, then each set of 3 people does $\frac{1}{3}$ of the job in 40 minutes, so the total time needed is $\frac{40}{3}$ minutes.

6. No answer. Knowing the weight of one player is not helpful in determining the weight of 11 players. People's weights are not related to each other.

7. Every time they put away \$7, Sandra pays \$2 and her mom pays \$5. To get \$210, they will need to make their respective contributions 30 times. In all, Sandra will contribute \$60 and her mom will contribute \$150.

8. If you decrease the number of people doing a job, it will take longer to finish the job. If you have $\frac{1}{3}$ the number of people working, they can get only $\frac{1}{3}$ as much done in the 96 minutes. Each $\frac{1}{3}$ of the job will take them 96 minutes, so they can do $\frac{2}{3}$ of work in 192 minutes and $\frac{3}{3}$ of the job (the whole job) in 288 minutes (4 hours and 48 minutes).

9. Reason down then reason up. The bike can run for 5 minutes on \$0.65 worth of fuel, and for 1 minute on \$0.13 worth of fuel. It can run for 6 minutes on \$0.78 worth of fuel and for 7 minutes on \$0.91 worth of fuel.

10. 15 to 1 = 150 to 10. Therefore, decreasing the number of faculty by 8 will give the required ratio.

11. Your shadow ($8'$) is 1.6 times as tall as you are ($5'$), so the shadow cast by the telephone pole must be 1.6 times as tall as the pole. If the shadow is $48'$ and that is 1.6 times the real height of the telephone pole, the pole must be $30'$ tall. Also, the telephone pole's shadow is 6 times as long as yours, so it must be 6 times as tall as you are.

12. In a square, two adjacent sides have the same length. That means that the ratio of the measure of one side to the measure of the other side would be 1. The rectangle that is most square is the one whose ratio of width to length is closest to 1. For the $35'' \times 39''$ rectangle, the ratio is $\frac{35}{39}$ or about 0.90; for the $22'' \times 25''$ rectangle, the ratio is $\frac{22}{25}$ or 0.88. This means that the $35'' \times 39''$ rectangle is most square.

13. Gear A has 1.5 times the number of teeth on gear B. So every time A turns once, B turns 1.5 times. If A makes 5.5 revolutions, B makes 5.5(1.5) or 8.25 revolutions.

14. The density or crowdedness of a town with cars is given by comparing the number of cars to the number of square miles in the town. For town A, the crowdedness is $\frac{12555}{15} = 837 \frac{\text{cars}}{\text{sq mi}}$. For town B, it is $\frac{2502}{3} = 834 \frac{\text{cars}}{\text{sq mi}}$. For town C, it is $\frac{14212}{17} = 836 \frac{\text{cars}}{\text{sq mi}}$. The town least crowded with cars is town B. B must be Birmingham.

15. Several different approaches are given:

    a. In pitcher A, 4 out of 7 total cubes are cranberry; in B, 3 out of 5 total cubes are cranberry. Because $\frac{3}{5} > \frac{4}{7}$, B has a stronger cranberry taste.
    b. In pitcher A, the ratio of cranberry to apple is 4 to 3; in B, the ratio of cranberry to apple is 3 to 2. Because $\frac{3}{2} > \frac{4}{3}$, B has a stronger cranberry taste.
    c. Using common denominators, pitcher A has 4 of the 7 total cubes or $\frac{4}{7} = \frac{20}{35}$ cranberry; B has $\frac{3}{5}$ of its total cubes or $\frac{3}{5} = \frac{21}{35}$ cranberry. So B has a stronger cranberry taste.
    d. Using common denominators to compare ratios within each pitcher, $\frac{3}{2} = \frac{9}{6} > \frac{4}{3} = \frac{8}{6}$, so B has a stronger cranberry taste.

16. In a true enlargement, all dimensions grow by the same factor. Suppose the original picture measured 5 cm × 6.5 cm. If the picture were enlarged and the width increased from 5 to 9, it grew to 1.8 times its original width; the new length should also be 1.8 times the original length, or 11.7 cm. This means that the 9 cm × 10 cm picture is not its enlargement. Check each pair of pictures to find the cases where the length and width were both multiplied by the same factor in going from the smaller to the larger picture. You will find that picture B is an enlargement of picture C. Both dimensions of C are multiplied by 1.25.

17. If you assume that the rats are dumb enough to stand by and wait their turn, that there is some orderly way of assigning the rats to each of the cats, and agree to disregard the foolishness of fractional cats and rats, here are some solutions:

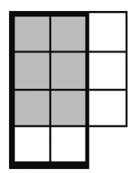
    i. If 6 cats together kill 1 rat in 1 minute, then it would take 12 cats to kill 2 rats in 1 minute. So if the 12 have 50 times as long to do it, they can kill 50 times as many rats (100 rats).

ii. If 3 cats kill 1 rat in 2 minutes, then 12 cats (4 times as many cats) could kill 4 rats (4 times as many rats) in 2 minutes. The same 12 cats could kill 25 times as many rats if they have 25 times as long to do it. So the could kill 100 rats in 50 minutes.

iii. 6 cats/1 rat/1 minute
6 cats/300 rats/300 minutes
2 cats/100 rats/300 minutes
12 cats/100 rats/50 minutes

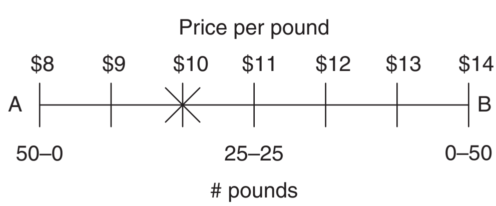
18. When weights are placed farther from the fulcrum, they will exert a greater effect or downward pull. Weights closer to the fulcrum have a lesser effect. A heavier weight closer to the center may be counteracted by a smaller weight placed farther out. How much "tipping" you get depends on the weight you have put on each side and how far along the arm each weight is placed. In A, there are 3 weights on the left side, each 3 units from the fulcrum or $3(3) = 9$ units of pull. On the right side, there are 2 weights, each 4 units from the fulcrum or $2(4) = 8$ units of pull. The balance will tip to the left. In B, you have $1(4) + 2(3) + 1(2) = 12$ units of pull on the left, and $1(4) + 2(2) + 1(1) = 9$ units of pull on the right. Again, the beam will tip to the left. Also notice that in B, the far weights balance, while the closer weights are farther to the left.

19. A picture may be the best way to represent this situation. Shade $\frac{2}{3}$ of a rectangle to represent the married men and $\frac{3}{4}$ of another rectangle to represent the married women. Because the corresponding numbers of men and women are equal, position the rectangles so that the shaded parts overlap.



Then you can clearly see that the total number of women is the same as $\frac{8}{9}$ of the total number of men. The ratio of men to women is $9:8$.

20. Let A be the coffee sold for \$8 per pound, and B, the coffee sold for \$14 per pound. If we buy A alone, we pay \$8 per pound. If equal amounts of each type of coffee are used in the mixture (25 lb. of A and 25 lb. of B), then we will pay \$11 per pound. There must be more of A because the mixture is selling for \$10 per pound.



Because the cost of the mixture is $\frac{2}{3}$ of the way between \$8 and \$11, the amount of A must be $\frac{1}{3}$ of the way between 25 and 50 pounds. There are $33\frac{1}{3}$ pounds of coffee A, and the rest of the mixture $\left(16\frac{2}{3}\text{ pounds}\right)$ must be coffee B.

CHAPTER ACTIVITIES

1. Using Mr. Short's measurements, we can see that 2 buttons are as tall as 3 paperclips. If Mr. Tall measures 6 buttons, that would be the same as 9 paperclips.

2. If 2 buttons are as tall as 3 paperclips, then five times as many buttons are as tall as 5 times as many paper clips. This means that 10 buttons have the same height as 15 paperclips. The length of Mr. Tall's car is 10 buttons. If 2 buttons are as tall as 3 paperclips, then 1 button is as tall as $1\frac{1}{2}$ paperclips. $7\frac{1}{2}$ paperclips are as wide as $2 + 2 + 1 = 5$ buttons.

3. In A, notice that 2 out of every 3 eggs are brown. Color B and C accordingly. B should have 8 brown eggs; C should have 12 brown eggs.

4. a. No. 12 ounces should cost \$0.98 and 15 ounces should be \$0.245 more than 12 ounces.
   b. No. 10 pounds should cost twice as much as 5 pounds.
   c. Yes
   d. No. 50 sheets should cost \$3.38.
   e. Yes
   f. No. Buying 1 box is costs less than the price per box when buying a bundle of 3 boxes.

5. Because of his visual representation of the problem, Josh can tell that 2 buttons have the same height as 3 paperclips. $2:3 = 6:9$.

6. Time is not proportional to speed because time decreases as speed increases.

7. If we assume that you can maintain a constant speed for the duration of the trip, then 20 mph × 3 hours = 60 miles, 40 mph × 1.5 hours = 60 miles, and 50 mph × 1.2 hours = 60 miles. Time is inversely proportional to speed and the constant is $k = 60$, the distance.

8. a. $I = \frac{k}{d^2}$ b. $F = ka$ c. $F = \frac{k}{d^2}$ d. $R = kl$
   e. $m = kr^2$ f. $W = \frac{k}{d^2}$

9. Answers will vary.

10. Mo is 1.5 times as along as Louie, so he should get 45 sardines. Pete is half as long as Louie, so he should get 15. You need 90 sardines to feed all three of them.

11.

a. ↑↑ and ↓↓	b. ↑↑ and ↓↓	c. ↑↑ and ↓↓	d. NR
e. ↑↑ and ↓↓	f. ↑↑ and ↓↓	g. ↑↑ and ↓↓	h. NR
i. ↑↓ and ↓↑	j. ↑↓ and ↓↑	k. ↑↓ and ↓↑	l. NR

12. a, b, c, f, g

13. i, j

14. Variables: number of men working (m), number of days to complete the job (d), number of man days k

    $md = k$

    The number of men working times the number of days they work = the number of man days needed to complete the job.

    The number of men working is inversely proportional to the number of days needed to complete the work.

15. Variables: number of diamonds (d), number of sticks (s)

    constant = k = number of sticks to make one diamond = 4

    $d = 4s$

    The number of diamonds is proportional to the number of sticks.

16. Using the example give, $k = 0.05$. Therefore the speed of the car is given by this equation: $120 = 0.05\ \text{speed}^2$ and the car's speed is just under 49 mph.

17. Charlie is translating at a rate of 5 pages per hour. Working together, they can translate 9 pages in an hour. The constant of proportionality is 350 pages. $R \times T = 350$, so 9 pages per hour = 350. $350 \div 9 = 38.88$ hours.

18. a. Distance is proportional to time and k = your speed.
    b. Circumference is proportional to diameter and $k = \pi$.
    c. Your cost is proportional to gallons purchased and k = price per gallon.
    d. Pay is proportional to hours worked and k = hourly salary.
    e. Number purchased is proportional to total cost and the unit price is k.
    f. Number of centimeters is proportional to number of meters and $k = 0.01$
    g. Map distance is proportional to real distance and k = scale of the map.