effective learning & teaching in

MATHEMATICS & its applications

Edited by peter kahn & joseph kyle

the institute for learning and teaching in higher education

THE TIMES HIGHER EDUCATION SUPPLEMENT

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KOGAN PAGE

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John Mason has taught mathematics for over 40 years and has worked at the Open University for over 30 years. He remains focused on supporting those who wish to think mathematically, and those who wish to support others who wish to think mathematically. *Thinking Mathematically* (Addison Wesley, 1982) and *Learning and Doing Mathematics* (Macmillan, 1988) are just two of the many books, pamphlets and papers which he has written for students and teachers. His concerns and interests have led him to examine the role of mental imagery in mathematical thinking, the role of metaphor and metonymy in how choices are made when thinking mathematically, and the design of tasks intended to promote mathematical thinking.

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David Pountney is a Principal Lecturer and Subject Leader for Mathematics and Statistics at Liverpool John Moores University. He teaches mathematics, including the use of computer algebra systems, to specialist mathematics undergraduates and non-specialists such as engineering and science students, and he has published textbooks and numerous articles related to the teaching of mathematics and mathematics applications. In particular, he and his co-authors were co-chairs of the 4th International *DERIVE*-TI89/92 (Computer Algebra in Mathematics Education) at Liverpool John Moores University in July 2000.

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Honor Williams is a Reader in Mathematics Education and Dean of the Faculty of Sciences at University College Chichester. The main focus of her research, development and publications concern the role of assessment in enhancing learning and teaching of mathematics at school and tertiary level. She has contributed to the IAEA (International Association of Educational Assessment) as well as the International Commission on Mathematical Instruction (ICMI) studies, and has directed projects on Teaching, Learning and Assessing Mathematics at sixth form and university level.

Acknowledgements

The editors are grateful to Sally Brown and Jonathan Simpson for their roles in bringing this book to publication. We would also like to thank all of the contributors for sharing their experience of, and insight into, learning and teaching mathematics and its applications. Finally, we are grateful to John Blake for his foreword.

Forewords

Mathematics dates from antiquity, built upon the minds of the world's intellectual giants: Euclid, Pythagoras, Archimedes, Newton, Euler, Gauss, Einstein and many, many others. It is perhaps the most developed science, deploying a huge corpus of theory and technique. Music, art and mathematics transcend all cultures but perhaps mathematics most of all is universal, for it is a truly common language, regrettably appreciated by and accessible to far too few of us.

Among the sciences, mathematical sciences have undergone extraordinary growth over the last decade, primarily stimulated by advances in computing facilities, both software and hardware. The advancement of science, indeed society more generally, has always depended upon mathematics and will do so even more in the future.

Communicating this corpus of knowledge, understanding the theoretical and logical base and deploying this knowledge to the benefit of mankind is one of the biggest challenges facing education at the beginning of this new century (or millennium!). It is therefore pleasing to see a text on *Effective Teaching and Learning in Mathematics and its Applications,* edited by Peter Kahn and Joseph Kyle, with chapters by leading practitioners in the discipline of mathematics, statistics and operational research. Chapters in the book cover the key areas of assessment (diagnostic, formative, summative), learning developments, course design and reflection, application of mathematics in modelling phenomena, the analysis of data and preparation for future employment. There is also recognition of the need to support the non-specialists who need mathematics or statistics for the full understanding and appreciation of their own discipline.

I strongly encourage you to read the chapters of this book, implement and further develop the ideas and concepts relevant to your interests, but to also draw your colleagues' attention to the book as well, for there is much to understand about the learning, teaching and assessment of mathematics, statistics and operational research.

Professor John R Blake
Director, LTSN Mathematics, Statistics and Operational Research Network
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http://www.mathstore.ac.uk

I am delighted to see in print this latest volume in the Effective Learning and Teaching series. The editors are to be congratulated on the immense amount of hard work they have invested in putting together this important contribution to the literature on learning and teaching in mathematics. Teaching maths is never easy and I am sure that readers will find much useful information and advice in this book as well as useful contact information and references to further reading. I am particularly happy to see the contributions that have been made by our colleagues in the LTSN subject centre, since it is imperative that the ILT and subject centres work together to provide a 'joined up' approach to supporting staff working in teaching and the support of learning in higher education.

The ILT is now at an important stage in its development with more than 10,000 members as we go to press, 112 programmes in higher education teaching and learning accredited by the ILT and a full programme of publications, activities and events on offer. Now with members in the majority on our governing council, active on all our committees and working parties and getting involved in the running our members' forums and other events, members are increasingly taking charge of shaping and directing the policy and direction of the organisation.

This book has a part to play in the ILT's mission to enhance the status of teaching, improve the experience of learning and support innovation in higher education. I commend it to you, whether you are an ILT member or not and welcome the contribution it makes to the higher education context.

Sally Brown
Director of Membership Services
Institute for Learning and Teaching

Preface

Setting the scene

Recent years have seen a greater focus on learning and teaching in mathematics and its applications in higher education. What should the typical undergraduate programme contain and how should it be taught? How best do we serve the needs of those who require mathematics as part of their study of another discipline? There will, no doubt, be many valid answers to these questions. And it is one of the strengths of this text that it attempts to cover a fair slice of the spectrum of these views.

As we write, there is in the UK what we might refer to as the 'official' answer, embodied in the December 2001 draft of the Quality Assurance Agency for Higher Education (QAA, 2000) Benchmarking statement, which covers mathematics, statistics and operational research (QAA, 2001). In this we see that a graduate who has reached the modal level should be able to:

- Demonstrate a reasonable understanding of the main body of knowledge for the programme of study.
- Demonstrate a good level of skill in calculation and manipulation of the material within this body of knowledge.
- Apply a range of concepts and principles in loosely defined contexts, showing effective judgement in the selection and application of tools and techniques.
- Develop and evaluate logical arguments.
- Demonstrate skill in abstracting the essentials of problems, formulating them mathematically and obtaining solutions by appropriate methods.
- Present arguments and conclusions effectively and accurately.
- Demonstrate appropriate transferable skills and the ability to work with relatively little guidance or support.

The authors of this statement go to some lengths to qualify and set the context for this list. In particular it is stressed that 'students should meet this standard in an overall sense, not necessarily in respect of each and every of the statements listed' (QAA, 2001). There is no attempt to set a 'national curriculum' but a clear generic description of the type of skills and qualities we should look to be fostering in our programmes.

Not surprisingly, many of the themes in the bullet points above surface again and again in the chapters that follow. At the same time we have been keenly aware that the learning experiences of students who have to engage with mathematics and its applications cover a wide spectrum. This will range from foundation level material, preparing students for entry to other numerate disciplines, to advanced level specialist mathematical study at or near the contemporary frontiers of the subject. It is our intention therefore that the tone of this text is both contemporary and inclusive.

Identifying the audience

The main purpose of this book is to serve as a guide for everyone who contributes to the learning experiences of students who have to engage with mathematics and its applications. Although it is anticipated that college and university-based lecturers, whether newly appointed or experienced, will constitute the core readership, many others including engineers and scientists, educational developers, teaching assistants and those in staff development units should also find something of interest and value in what follows. Departmental heads and those involved in mentoring other members of staff will also find the book helpful.

Moreover, although the editors are based in the UK, the themes developed are not unique to any one country but, rather like mathematics itself, are applicable across international boundaries.

Explaining the aim

Throughout the book, the underlying aim of the authors is to assist those engaged in facilitating learning and teaching in mathematics and its applications at higher education level. The contributors are firmly grounded in the discipline and have followed quite specific briefs relevant to mathematics and its applications. In the use of the term mathematics, we have allowed our authors a reasonable amount of latitude, believing that the context renders the necessary clarity. Typically, the term 'mathematics' is used by the contributors to refer both to mathematics itself and to applications of mathematics. Thus, this text does not focus on generic aspects of learning and teaching in higher education for which there is now a substantial and rapidly expanding literature.

Nor is the book intended as a direct classroom resource with immediately 'consumable' exercises and activities. There are abundant materials of this nature available, as can be seen by visiting www.mathstore.ac.uk, the Web site of the Learning and Teaching Support Network (LTSN) Mathematics, Statistics and Operational Research (OR.) Network. (It is worth noting in passing that the Mathematics, Statistics and OR Network supports learning and teaching within these disciplines in the UK, as part of the wider network of subject centres called the LTSN.) Instead, the book seeks to explain the challenges of mathematics and its applications in education and give examples of good practice from experienced facilitators in the field. Hence, it provides a guide for reflective practice and offers routes to explore wherein readers may develop their own pedagogic principles.

In planning any text of this nature, a number of sub-headings arise almost naturally. These are:

- Teaching and the support of learning.
- Design and planning of learning activities.
- Assessment and giving feedback to students.
- Effective environments and student learning support systems.
- Reflective practice and personal development.

In this book we have endeavoured to see that this fifth point is not seen as isolated or independent from the other points above, but rather features as a crucial component in them all. It is no accident that these are descriptions of the five key areas of professional activity as identified by the UK's Institute for Learning and Teaching in Higher Education (ILT) and we are happy to see the aims of this text aligned closely with the work of the ILT.

Clearly, professional commitment to teaching and the support of learning needs to be espoused by all engaged in higher education. However, it may well be argued that such commitment is particularly apposite for contemporary mathematics and its applications. There are clear signs that the wider world is becoming aware of the issues, many of them international, that currently surround the discipline. Whether it is as a result of changing policy affecting school mathematics or whether it is the impact of rapidly developing technology, we face many new challenges in our discipline. Furthermore, the range and diversity of those engaged in learning our subject is considerable and is destined to become wider still in the near future.

An important consideration for the editors and the contributors has been to maintain a clear focus on those issues that are almost unique to mathematics and its applications. On one hand it is the science of strict logical deduction and reasoning, which are very severe taskmasters both for the learner and the teacher. On the other hand the breadth of the applicability of mathematics is immense. Mathematics is fundamental not only to much of science and technology but also to almost all situations that require an analytical model-building approach, whatever the discipline. In recent decades there has been an explosive growth of the use of mathematics in areas outside the traditional base of science, technology and engineering. Even in considering the development of Web-based resources, mathematics with its specialist symbols and fonts presents challenges that do not disturb many other disciplines or, it would seem, those developing commercial software.

A further factor in the design of the content has been to offer an insight into a range of approaches for teachers and learners. We know that, in the UK at least, 'most teaching comprises formal lectures' and that more innovative methods are only used 'occasionally' (QAA, 2000). Similarly we learn that most assessment strategies rely on formal examinations and, more damagingly, that in some cases too narrow a range of assessment methods had led to deficiencies in the measurement of learning. There is clearly some value therefore in exploring, as many of the subsequent chapters do, tried and tested alternatives to the traditional approaches.

Outlining the structure

The book is divided into two main parts. Part A focuses on issues that are common to learning and teaching across mathematics and its applications as a whole and at any level. It takes in such topics as the use of technology, assessment and course design. Part B, beginning at Chapter 10, focuses on specific areas of mathematics and its applications. It also covers specific contexts, such as learning and teaching that involves non-specialist students of mathematics.

Chapter 1 is designed to provide a context for later contributions. The focus is on the transition from largely school mathematics to further study of the subject at a higher education level. There is a close analysis of what skills might be expected from various backgrounds and advice on how to use and interpret the results of initial assessment or diagnosis of students' mathematical abilities. Thus Chapter 1 sets the scene and tells us exactly where we are when we start to work with our students.

In Chapter 2, we explore the contrast between the ways in which mathematics is discovered and the ways in which it is presented, especially to students. It looks therefore at the nature of the process that leads students to develop an understanding of formal mathematical structures. The initial focus is on the role of pattern recognition and conjecture, taking particular account of proof, and leads on to a wider consideration of the role that special cases play in the genetic process.

This chapter is appropriately followed by the topic of Chapter 3, which takes as one of its starting points the idea that for the student the genesis of a new mathematical concept or result is very often an active process. This, the authors claim, is a powerful argument in favour of active learning, by which they mean learning where the students' minds are actively engaged in creating knowledge through interacting with their teachers, peers and the subject matter.

Chapter 4 begins with a concise resume of the standard technical terms that have now become part of the assessment landscape. There then follows a survey of various attempts to use the computer to assist in assessing mathematics. There is a discussion on matching assessment to learning outcomes and a substantial section outlining good practice in the design of assessment elements. The chapter closes with a look at what developments are likely to emerge in the near future.

As an acknowledgement of the ever-increasing power and ease of access of computer technology one of the themes in Chapter 4, that of computer-assisted assessment, is examined in greater detail in Chapter 5. Technology continues to have a significant impact on the learning, teaching and assessment of mathematics throughout higher education establishments worldwide. In this chapter the authors look particularly at the impact of computer algebra systems, that is to say software systems that can perform symbolic as well as numerical manipulations and which include graphical display capabilities. Examples include *Mathematica, Maple, Macsyma* and DERIVE. Some of these systems are available not only on PCs but also on hand-held ‘super-calculators’ such as the TI-92 plus and the TI-89. The chapter concentrates on the use of DERIVE to illustrate the issues raised, but these issues are generic in nature and the translation to other systems is straightforward.

The increasingly important area of transferable skills is the topic for Chapter 6. The authors here take the point of view that transferable skills are best taught and practised in the context of a student’s main study: mathematical sciences. In other words, the development of these skills should be embedded in the mathematics curriculum. After describing the range of skills that come under this heading, the authors describe how one might go about embedding transferable skills in the curriculum.

Such an approach is also advocated within Chapter 7, which addresses a common challenge, that of designing courses that open up mathematics to students in ways that both attract them and serve their needs. The author argues that while it is essential that courses are built around mathematical considerations, course designers also need to take account of wider issues. Taking learning outcomes as the fundamental element of design, the author shows how the remaining elements of course structure fall more naturally into place. In order to illustrate these principles the author includes a number of case studies, two of which seek to draw on the approach of problem-based learning.

Chapter 8 explores the notion of the Total Learning Environment, a concept in which all participants, curriculum, assessments, evaluations and perceptions are considered to be part of the one learning entity. In many ways this draws together themes from earlier chapters. In this chapter, the authors describe the various components of the Total Learning Environment through the use of a model, which although it might vary from institution to institution, highlights the important issue of improving student learning at all levels. The approach is illustrated through a case study of how one mathematics department used this model to improve the first-year mathematics learning environment.

As indicated above, there is an increasing emphasis being given to the concept of the reflective practitioner and Chapter 9 concentrates on the role of reflection in learning from experience as a lecturer or tutor. After considering some reasons for engaging in active reflection in the first section, some suggestions are made concerning effective ways to use memories of incidents as the basis for methodical reflection with a view to improving the learning and teaching experience. It is argued that an important corollary of such reflection is that it is possible to help students develop more efficient and effective learning processes, and one of the ways of doing this is by engaging them in a similar process of reflection.

For many students of other disciplines in higher education ‘mathematics’ means ‘numeracy’. Chapter 10 examines the related issues that have always posed something of a problem for the university sector, especially for students studying subjects other than mathematics. The chapter begins with a case study examining how numeracy support has developed at one university. From this detailed consideration of one approach to developing students’ numeracy one can distil more general issues inherent in offering numeracy support. The chapter concludes by giving a wider picture of the current situation in the UK, using case studies from a selection of other universities.

An allied but quite distinct issue is that of providing support for the mathematical requirements of the non-specialist student. Chapter 11 explores both the needs of non-specialist students of mathematics and the ways in which these needs might be met more effectively than is presently the case. In the context of this chapter, 'non-specialists' are students who come to university to read subjects other than mathematics, either as single- or joint-honours students, but for whom mathematics is nevertheless a compulsory part of their university experience. These students may be contrasted with the wider body of students considered in the previous chapter, where the focus was on all students in higher education and their subsequent employment-related needs. After an initial consideration of who should teach non-specialists, the chapter goes on to review a number of recent reports and the concerns they raise about the state of mathematics education of non-specialists in universities. The author then looks at some practical steps that might be taken and concludes by suggesting future measures that will go some way to ensure that the problems that have been highlighted are tackled in a serious way.

In October 1970, the *Mathematical Gazette* carried an article describing 'What makes a mathematician?' in which we were told that 'in the application of mathematics, one needs an appreciation of model-making'. Now more commonly referred to as mathematical modelling, this area has become increasingly important over the intervening years and is examined in Chapter 12. The term 'mathematical modelling' is closely connected to the concept of a mathematical model but reflects more the higher-level interactive skills of formulating, analysing, evaluating and reporting of physical situations through the use of mathematical models. This chapter discusses the main features of this complex, interactive, multifaceted activity that is based on applying mathematics. Some structural features of the process are delineated after which there are concrete suggestions on implementing these ideas at various levels in the curriculum.

Statistics is often regarded as being difficult to understand, especially by non-specialist students of the subject. If for no other reason it merits a full and separate chapter. In Chapter 13, the author outlines the issues pertinent to this relatively youthful branch of knowledge. The case is persuasively argued that for many students, especially non-specialists, a data-driven approach to learning statistics has best chance of success. The approach is illustrated with a description of the innovative Web-based random data selector. The chapter then describes a new approach to teaching statistics that has been developed in the motor industry and could be deployed with good effect in engineering departments.

Mathematics graduates are rightly seen as possessing considerable skill in abstract reasoning, logical deduction and problem solving, and for this reason they find employment in a great variety of careers and professions. However, these skills are rarely well developed when they enter higher education. In particular there is very little appreciation of the central role of proof in mathematics and hardly any real facility in the various styles of proof and logical reasoning. In Chapter 14 we examine these issues and look at some suggestions for developing these much valued skills.

The conclusion to the book in Chapter 15 seeks to draw out a number of characteristics of effective learning and teaching in mathematics and its applications. The chapter takes each of the identified characteristics in turn, drawing out their role in effective learning and teaching and looking to see how future practice might take greater account of them. While it is relatively straightforward to draw out various characteristics of effective learning and teaching, it is more challenging to ensure that one's own practice is in fact effective. The chapter thus ends by considering the professional development of lecturers and tutors with regard to their teaching, both on an individual basis and more widely.

Finally, the book also gives pointers to a variety of resources that are available to support learning and teaching in mathematics and its applications. In particular, the Internet allows access to a vast range of resources, and one person's selection is provided in the Appendix. Otherwise, references to a number of texts on learning and teaching in mathematics are given in the section on further reading. Several of these place an emphasis on specific aspects, such as giving lectures.

And now…?

Read the book! Engage with it and reflect upon it. We would encourage you frequently to consider how you might apply or adapt the ideas proposed in this text to your own practice. Even where you disagree with the text, we would hope that you would take steps to enhance your understanding of the contentious issue and possibly to adapt your own teaching as a result. Our aim in presenting this text is to help all who are looking to improve the learning experiences of students as they study one of the most inspiring and most challenging subjects in higher education.

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Part A
Issues in learning and teaching

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1
The transition to higher education

John Appleby and William Cox

Introduction

The transition from one educational stage to another is usually a difficult and uncertain process. Examples include the first day at school, the primary/secondary interface, GCSE to A level (in the UK), school to university and even embarking upon postgraduate research. This article considers the transition to university and how we might better influence students' prior expectations and preparation for this new phase of their life. In addition, we look at the implications for the current curriculum of the common problems of the transition as well as discussing initial assessment of students as they enter higher education (HE).

In mathematics there has been a great deal of recent publicity about the issues around the transition to HE, especially in the UK. Notable among these are the report from the London Mathematical Society (1992) and the more recent publication by the Engineering Council (2000). Other commentators include Cox (1994), Lawson (1997) and Gardiner (1997). Mustoe (1992), Sutherland and Pozzi (1995) and others look at the problem as it affects engineering programmes. Developments that impact on these issues include: changes in school/pre-university curricula, widening access and participation, the wide range of degrees on offer in mathematical subjects, IT in schools and social factors. In the UK, problems arising in the transition are mentioned in many Quality Assurance Agency (QAA) Subject Review reports in Mathematics, Statistics and Operational Research (MSOR), and also in some engineering Subject Review reports.

Various reports, including those listed above, point to the changes in schools as the source of problems in the transition, and make recommendations as to how things could be put right there. Indeed, in response to wider concerns about literacy and numeracy, recent government initiatives within the UK have, perhaps, partially restored some of the skills that providers of numerate degrees need; and these might feed into HE in the next decade. As well as changes at primary level, there is much talk of extending these to the 11–13 age group. But notwithstanding such changes, it is doubtful that we will ever return to a situation where school qualifications are designed solely as a foundation for HE, ensuring that any chosen university entrant will be capable in all the skills required by any university programme with a mathematics component.

However, difficulties in transition are caused not just by changes in school curricula, but also by changes in the ability range, in student attitudes and expectations, student financial burdens, in the resourcing available in HE, and in the curricula we expect them to undertake. Some of these are not at all under our control, and the best we can do is to be aware of them and how they affect our students. Others, including curriculum design and, to an extent, pastoral support, may need attention if those who choose our courses are to have the best chance of success. We might also usefully consider what our students know about our courses when they choose them, and how they might prepare themselves a little before they come: in attitude as well as in knowledge. For example, Loughborough University sends new students a revision booklet before they come (see Croft, 2001; and also the discussion in Chapter 11). Since 50 per cent of MSOR providers within England have been criticized for poor progression rates in Subject Reviews, it is clear that there is much to be done.

We can make an analogy with the position of English as a Second Language provision in adult education, where students apparently equally well qualified have widely variable skills and needs. An important component of such provision is rigorous initial assessment to determine students' current skills profiles before designing a realistic programme that will meet their needs. We are not used to this in HE because there are usually a large number of students and they are traditionally expected to be capable of catching up for themselves if need be, and we are used to the idea that they have already covered everything that we need. It is therefore likely that a system of initial assessment will form an important part of our degree programmes, especially in the transition phase (see the section on initial assessment below).

Understanding the transition to higher education

A recent Teaching and Learning Undergraduate Mathematics (TaLUM) symposium categorized the issues impacting on the HE transition in mathematics as follows:

- The nature of mathematics: does 'mathematics' mean the same thing to schoolteachers, mathematics lecturers, engineering lecturers, etc? Mathematics taught in the context of mathematics honours courses and in 'service' courses for students whose study focuses on other relevant disciplines may be seen as having quite different purposes, by lecturers as well as students.
- Social component. HE is more student centred, students are less biddable, there are financial, emotional and other conflicting pressures.
- Diversity. There is wider participation, wider access, more variable backgrounds and more variable requirements.
- University requirements. Expansion in HE has led to a wide range of not always clearly articulated aims and objectives, so that entry qualifications no longer specify adequately what students should know and be able to do; some attempts have been made to address this through the work of the QAA.
- School provision. This is no longer geared to feed a coherent spectrum of HE institutions, and university staff are sometimes ignorant of it. There have been changes in school curricula in other subjects such as physics, which used to provide useful support and motivation for students learning mathematics.
- University provision. There are changes in content and style of university teaching compared with school. This is partly due to the fact that many university teachers are

not trained (based on an assumption of self-motivated, able students).

The TaLUM symposium further noted that the following were also needed:

- Representation. There is a need to expand fora for school and university teachers to air views and concerns, debate issues, etc; the Learning and Teaching Support Network (LTSN) subject centres may now contribute to this within the UK.
- Liaison. There is a need for closer liaison between schools and universities.

The preceding list focuses on changes in the transition to HE, attempting to list some of the main categories of reasons for the increased difficulties. It is also useful to consider the students and their perspective, the university including its staff and its courses, and the change that is required for the one to adapt to the other. In this section we explore the contrast between attitudes and characteristics typical of lecturers and those of students, together with our intentions and expectations, usually implicit, of the course and of the students. We examine the curriculum in more detail in the following section.

University lecturers were not typical students even at the time they were students, and, in many cases, that was 10, 20, or even 40 years ago. We liked our subject, worked adequately if not hard, and also perhaps had a style of learning that was atypical. Understanding what we were doing, finding out for ourselves, checking our results, as well as having a natural bent for mathematics, were likely to be our characteristics. This attitude continues (except that there is far more emphasis on research compared to teaching these days), and we probably most like teaching those who are interested and make an effort. Most of us probably had family members who had been to university: not true for many students for some years after any major expansion.

In addition, we may well have enjoyed reading as a pastime, perhaps did not have a telephone or television in our lodgings, and certainly did not have a mobile phone or access to the Internet. We probably did not have term-time work, and also spent less on clothes, entertainment and drink, etc. School teaching also followed a different style as well as different syllabuses, with more routine practice though perhaps with fewer topics and possibly no calculator. On the other hand, we were less used to visiting the library, interviewing people, project work and group work and more used to textbook problems little connected with everyday problems. Furthermore, we also did not have to address the issues associated with IT and transferable skills, both of which are discussed in other chapters.

Our current student group, as well as having a great variety of qualifications and ability, also have variety in social background, age, work experience and ambitions or intentions. Lastly, some students, especially mature ones, may have had some gap since they last formally studied mathematics.

Other factors not often mentioned are the changes in other areas of the school curriculum. In the UK the most obvious are that double mathematics A level, formerly common for applicants to mathematics, physics or engineering courses, is now rare, and the changes in physics. Far fewer sixth-formers now study physics, so much so that many engineering courses now accept students without this qualification, but also the nature of Physics A level has changed. Partly because GCSE Mathematics contains no calculus, A level Physics has become less quantitative, and topics like simple harmonic motion and