

volume 238

A Dekker Series of Lecture Notes in Pure and Applied Mathematics

Stochastic Processes and Functional Analysis

A Volume of Recent Advances in Honor of M. M. Rao

edited by

Alan C. Krinik

Randall J. Swift

Stochastic Processes and Functional Analysis

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Preface

An AMS Special Session in honor of M.M. Rao was held at the 2002 joint meetings of the American Mathematical Society and the Mathematical Association of America. That Special Session was on Stochastic Processes and Functional Analysis and was organized by Professors Alan C. Krinik and Randall J. Swift, both of California State Polytechnic University, Pomona.

Professor M.M. Rao has had a long and distinguished research career. His research spans the areas of probability, statistics, stochastic processes, Banach space theory, measure theory and differential equations - both deterministic and stochastic. The prolific published research of M.M. Rao impacts each of these broad areas of mathematics.

The purpose of the Special Session was to highlight the key role played by abstract analysis in simplifying and solving fundamental problems in stochastic theory. This notion is fundamental to the mathematics research of M.M. Rao, who uses functional analytic methods to bring questions in these diverse areas to light.

The Sessions were a great success, bringing together a diverse group of research mathematicians whose work has been influenced by M.M.'s work and who, in turn, have influenced his work. Not only did this diverse group of speakers benefit from the common unifying thread of the session, but also there were often lively discussions and questions from the session audience.

This volume contains most of the talks given at the Sessions as well as several that were contributed later. This collection of papers reflects the depth and enormous breadth of M.M. Rao's work. A major highlight of the Sessions was M.M.'s talk entitled "Stochastic analysis and function spaces", which was a remarkable unifying survey of recent work in the area. This volume features that talk as an article, which includes a broad bibliography of the important works in the area.

The volume begins with a biography of M.M. Rao, a complete bibliography of his published writings, a list of his Ph.D. students and, notably, a collection of essays about M.M. written by some of his Ph.D students. Many of M.M. students have remained devoted to him, decades after completing their degrees. Their loyal devotion arises from M.M.'s complete dedication

to them. He consistently put their concerns and welfare as his first priority. Their essays are a remarkable tribute.

This volume complements the Festschrift volume Stochastic Processes and Functional Analysis, which was published by Marcel Dekker, Inc. in 1997. That volume was in celebration of M.M.'s 65th birthday. As M.M. continues to work on, develop and expand mathematics, we look to future collections of articles that honor him and his love of mathematics.

R. J. Swift

A. C. Krinik

Biography of M. M. Rao

M.M. Rao was born Malempati Madhusudana Rao in the village of Nimmagadda in the state of Andhra Pradesh in India on June 6, 1929. He came to the United States after completing his studies at the College of Andhra University and the Presidency College of Madras University. He obtained his Ph.D in 1959 at the University of Minnesota under the supervision of Monroe Donsker (as well as Bernard R. Gelbaum, Leonid Hurwicz, and I. Richard Savage).

His first academic appointment was at Carnegie Institute of Technology (now called Carnegie Mellon University) in 1959. In 1972, he joined the faculty at the University of California, Riverside where he remains today. He has held visiting positions at the Institute for Advanced Study (Princeton), the Indian Statistical Institute, University of Vienna, University of Strassbourg, and the Mathematical Sciences Research Institute (Berkeley).

In 1966 he married Durgamba Kolluru in India. They have twin daughters Leela and Uma and one granddaughter.

M.M.'s research interests were initially in probability and mathematical statistics, but his intense mathematical interest and natural curiosity found him pursuing a wide range of mathematical analysis including stochastic processes, functional analysis, ergodic theory and related asymptotics, differential equations and difference equations. His breadth of interest is mirrored by his students, many of whom are recognized as experts in diverse fields such as measure theory, operator theory, partial differential equations and stochastic processes.

M.M. has always strived for complete understanding and generality in mathematics and rarely accepts less from others. This view of mathematics has played a central role in his teaching. M.M. Rao is truly a gifted lecturer and he has inspired many generations of students. He is a demanding Ph.D. advisor that expects the most from his students. The guidance and mentoring he provides them has led to many of his students becoming successful mathematicians. It is no wonder that he has had his share of the best available graduate students.

M.M. is a prolific writer. His first published writings were not on math-

ematics, but rather Indian poetry. He wrote poetry in his late teenage years and had a collection of his poems published when he was 21. His mathematical research publications are many and span five decades. He is active and vital as ever. He has just completed a second edition of his well-received measure theory text and is currently working upon revised and expanded second editions of his probability and conditional measures texts.



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Jerome A. Goldstein (1967), "Stochastic differential equations and nonlinear semi-groups."

Neil E. Gretsky (1967), "Representation theorems on Banach function spaces."

William T. Kraynek (1968), "Interpolation of sub-linear operators on generalized Orlicz and Hardy spaces."

Robert L. Rosenberg (1968), "Compactness in Orlicz spaces based on sets of probability measures."

George Y. H. Chi (1969), "Nonlinear prediction and multiplicity of generalized random processes."

At University of California, Riverside:

Vera Darlean Briggs (1973), "Densities for infinitely divisible processes."

Stephen V. Noltie (1975), "Integral representations of chains and vector measures."

Theodore R. Hillmann (1977), "Besicovitch - Orlicz spaces of almost periodic functions."

Michael D. Brennan (1978), "Planar semi-martingales and stochastic integrals."

James P. Kelsh (1978), "Linear analysis of harmonizable time series."

Alan C. Krinik (1978), "Stroock - Varadhan theory of diffusion in a Hilbert space and likelihood ratios."

Derek K. Chang (1983), "Bimeasures, harmonizable process and filtering."

Marc H. Mehlman (1990), "Moving average representation and prediction for multidimensional strongly harmonizable process."

Randall J. Swift (1992), "Structural and sample path analysis of harmonizable random fields."

Michael L. Green (1995), "Multi-parameter semi-martingale integrals and boundedness principles."

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For M.M. Rao

Professor M.M. Rao has had a long and distinguished research career. His research spans the areas of probability, statistics, stochastic processes, Banach space theory, measure theory and differential equations - both deterministic and stochastic. The prolific published research of M.M. Rao impacts each of these broad areas of mathematics

During M.M.'s career, he has had eighteen Ph.D. students. Many of his students have gone on to very successful careers in mathematics and are recognized experts in their field of study. Six of his former students have written tribute essays about M.M. and each are affectionately dedicated to him. These essays were written by J.A. Goldstein, M.L. Green, N.E. Gretskey, A.C. Krinik, R.J. Swift and J.J. Uhl.

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An Appreciation of my teacher, M.M. Rao

I want to record my thoughts about M. M. Rao as a teacher. He was a really great teacher and his teaching continues to have a major impact on my career.

As a first year graduate student at Carnegie Tech, in 1963-64, I took Rao's year long course on Functional Analysis. There were a lot of good students around Tech at that time; included in Rao's class were second year students Neil Gretskey and Jerry Uhl. Rao's ambitious style was to cover one major result in each lecture, or three per week. And all major theorems had descriptive names, some standard ("Dominated Convergence Theorem") and some not ("Law of the Unconscious Statistician"). The use of those names made the results easier to remember; I think Rao got this idea from Michel Loeve's book (from which I learned probability theory). Our text was by Angus E. Taylor, but we didn't use it much. Rao taught mostly out of Dunford & Schwartz (Vol. 1) and Hille & Phillips. His organization of the topics was excellent. An unusually large amount of material was covered per class. So much so that details were often omitted (or, sometimes in our minds, incorrectly given). With great regularity Gretskey, Uhl and I would stay after class and work out the complete details of the arguments we had just seen. Sometimes we realized that Rao really had given all the details; after all we were merely beginners and not yet well versed in mathematics. We always found that all of his results had correct versions, occasionally slightly different from what one of us thought when the discussion began. But by the end of the year, I learned so much that, for the first time, I considered myself a mathematician. Gretskey, Uhl and I were somehow teaching assistants to Rao, helping to teach one another. At the time I didn't give Rao credit for orchestrating this, but I think he did, at least to a substantial extent. He conveyed his love of mathematical depth and understanding and his passion for intense mathematical discussions.

I took many more grad courses from Rao prior to graduating in May 1967. They were all great courses, but none matched that extraordinary course in Functional Analysis. That course had a permanent influence on me, and for the rest of my life I will feel close and grateful to M.M. and to Neil and Jerry

as well.

Having gotten my BS at Tech in 1963 and anticipating my MS in 1964, I decided to apply elsewhere (in the fall of 1963) to do my Ph.D. away from my birthplace, Pittsburgh. But my wife had a teaching job in Pittsburgh and her applications elsewhere were unsuccessful. So, despite fellowship offers from more prestigious institutions, I was happy to stay at Tech because I knew (from the Functional Analysis course) that a great thesis advisor was available. Gretskey and Uhl were already doing research reading under Rao, and in the spring of 1964 I told Rao I wished to work with him in Functional Analysis (as Neil and Jerry were doing). He said he would be glad to be my advisor, but he had a problem in probability theory for me. I protested, saying I didn't know any probability theory. He pointed out that I had taken a year long junior level probability course from Morrie DeGroot, an excellent teacher and probabilist. (Of course, he was right, but I was mystified, being so in love with Functional Analysis). But, as my main focus was to work under M.M., I said OK. The first paper he gave me to read was by Dynkin, and it defined a Markov process as a 21 tuple (or something like that). Numbers larger than my combined fingers and toes made me nervous; and I was unable to read Dynkin's paper, Rao suggested I try Loeve's book and work a lot of the problems. This was a great suggestion, and Rao helped me a lot when I got stuck. And, happily, 6 or 8 months later I was able to read and understand Dynkin's paper (which was indeed a toughie).

M.M. ran great seminars and, among other things forced his students to present papers they read and their own work. His ferocious but kind questioning taught us never to give a seminar less than fully prepared. And he taught us to work together and learn from one another. This is a very important point which was evident, but I didn't realize it at the time. Rao's teaching and advising styles were shamelessly adopted by me in my capacity as a teacher and advisor. I have had over 20 Ph.D. students ("children") and at least 8 "grandchildren", most of whom never met Rao and probably are largely unaware of the major hidden role he played in their education.

I love to reread occasionally the article Rao wrote in the Raofest volume, celebrating his 65th birthday. Rao did something special and unusual; he gave his best research problems to his students. I have tried to follow his lead, and I believe our profession would be better off if more thesis advisors did the same thing.

Rao was uncompromising in his high standards, but he was gentle and helpful. Not all of his Ph.D. students had the native brilliance of Gretskey or Uhl, but all of them (that I know) wrote excellent theses. Rao got his students to live up to their potential. I think that is the highest praise one can give to a teacher.

Rao was also an excellent researcher. As a departmental citizen he was pretty feisty. He objected to (mathematical) political issues taking prece-

dence over issues of quality and scholarship. Doing the “right thing” is not always the way to maximize one’s popularity. But M.M. never hesitated to stand up and fight for his principles.

I suppose I should tell one anecdote. The enormous length of Rao’s first two names reminds me of Dynkin’s definition of a Markov process (which took me 6 or 8 months to understand). So once I asked Dick Moore what Rao’s nickname was. Dick’s response: “He doesn’t have one. People call him M.M. But he should have one.”- Dick thought about it and hit on the nickname Mmmmmmmmm. But it never stuck.

I feel much affection for M.M. I always respected and admired him, and there were moments during my grad student days, when the term “affection” did not characterize my feelings toward him. But I was young, brash and impatient; some things I could figure out very quickly and some not. Was I lucky to have M.M. as my principal teacher and mentor? Absolutely yes. Could I have done better either at Tech or elsewhere? I don’t think so. Rao shaped my passion for mathematics, my desires to understand things fully, my standards, and my teaching and advising techniques. I owe him so much, more than I can usually imagine. Thank you M.M., for being such an inspiration and such a friend.

J.A. Goldstein



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1001 words about Rao

My first contact with M.M. Rao was in the fall quarter of 1989 in an Advanced Calculus class. Before this course, like a typical undergraduate, I inquired of other students about him. Most of my information came from the graduate students at UCR, since they were the ones who had taken courses from him. The graduate students generally regarded him as a hard, but fair teacher. This positive tone, however, was laced with an undertone, not unlike the sort one would receive about a blind date, who in all other respects was perfect, except for some peculiar habit. It required only one lecture to discover the peculiarity of M.M. Rao. He is so absorbed into mathematics that where the man ended and the math began was blurred until the separation of the two is unimaginable.

His lectures are wonderful. The students of Rao have coined the phrase “Rao Math” for the rather distinctive style he has when presenting mathematics. He carefully prepares all his lectures, often writing them out in their entirety before the beginning of the quarter. An appropriate motto he has given is “If we do this for the general case, the rest will follow as corollaries.” One need only read one of his books to see the verity of this motto. A good example is his text for Real Analysis, *Measure Theory and Integration*. He often immediately began lecture upon entering the room and always went over the allotted time leaving the next class waiting at the door. On more than one occasion, he was writing as he walked out the door! These peculiarities are symptoms of his strength, a single-minded dedication to his profession coupled with a deep interest and curiosity in the subject. In M.M. Rao, I met someone that hit the 35th level of MathTM, a true math guru. To be fair, not everyone prospered under Rao. The lack of concrete examples was the typical student complaint about M.M. Rao’s instruction. I guess M.M. Rao had been getting some grief about not being concrete enough, for during one class he declared, “I am an applied mathematician! I apply this theorem to prove that one!” This is a typical Raoism.

The beauty of mathematics as presented by him seduced me. I know that I am not the only one to experience this and like others I started taking more courses from Rao after my first introduction to him in advanced calculus. I

began learning more analysis and in particular probability as a consequence. A tremendous benefit to my education was the open door I always found at his office and the many conversations I had with Rao about mathematics and his research have enriched my life. As a student, I never felt belittled or talked down to by Rao even when discussing his research. During my seeking of an advisor for my Ph.D. thesis, some of the other professors cautioned me about Rao, concerning his ability to win the best students. A reputation well deserved. I mulled over several individuals, all very capable mathematicians, but the accessibility of M.M. Rao won me over, even though my first interests were in algebra and topology.

His work ethic was intimidating. Sleeping four or fewer hours per night working on mathematics most of the day, seven days a week he labored with "a devil on his back" to complete his projects. He only took a half-day off on Sunday. He once said, "Mathematics is a harsh mistress. Either you love her, or she will leave you."

My thesis topic was to extend stochastic integration to multi-parameter manifold valued semi-martingales using the generalized Bochner boundedness principle. The students of Rao have termed his theses topics as "topics in the clouds". A few of the completed theses are "Orlicz spaces of additive set functions and set martingales", "Integral representations of chains and vector measures" and "Multi-parameter semi-martingales integrals and boundedness principles." The last being my own, coming up short on the manifold part. The theses that Rao has guided tend to be on the long side, my own was 138 pages, not the longest.

This propensity to generalize has worked well for M.M. Rao. Take for instance what Rao has done with ideas from S. Bochner. In 1956, Bochner wrote "Stationarity, boundedness, almost periodicity of random valued functions" in the Proceedings of the Third Berkeley Symposium. In this paper Bochner defined V-bounded processes and noted that these processes were an extension Loeve's harmonizable processes. Rao's idea was to define two classes of processes, the V-bounded being called weakly harmonizable processes which includes the processes of Loeve, now called strongly harmonizable. This definition opened up a whole new area of research in harmonizability being still actively pursued. Another idea Rao gleaned from this paper is to define stochastic integration via a boundedness principle. His generalized Bochner boundedness principle provides a unified approach to stochastic integration including all known stochastic integrals under one umbrella. This principle would still be unknown if M.M. Rao had not pursued mathematics in his own distinctive manner. For the Young functions from Orlicz space theory were necessary for the result. Rao met with Bochner three times. Bochner must have been impressed, since he communicated three of Rao's papers to the National Academy of Sciences. Rao still has not been entered as a member of the Academy.

Rao still lies dear in my heart as he does in the hearts of many others who have come across his path. M.M. Rao asked me to not compare him as an equal to Bochner, his modesty is showing, but in my eyes, he is a great mathematician and a great man. He still shows his faith in me and has many expectations for my work, encouraging me to continue my labors. My wish is that he finds a satisfaction in his life and work that brings him peace. I look forward to the years that come to see what new worlds he will open in mathematics.

M.L. Green



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A Guide to Life, Mathematical and Otherwise

When I went to graduate school in the early '60s I started in the Systems & Communications Sciences interdisciplinary program at Carnegie Tech. I knew that I wanted to study and work in Numerical Analysis and Computing. In my second year I decided to take a Functional Analysis course because I had some half-formed idea that this would be a valuable tool for Numerical Analysis. I did not in any way anticipate the ensuing life-changing event of meeting M.M. Rao. The course became an almost-religious epiphany for me: this was truly the way, the truth, and the light! M.M.'s lectures were magnificent; the material was spell-binding; the problem sets were *really* challenging. Several of my fellow students felt the same way = especially Jerry Uhl and Jerry Goldstein. We took more courses and seminars with M.M. and we chose to write our theses under his direction. The three of us spent a lot of time challenging each other and guiding each other under M.M.'s firm but insightful hand. In the last vestiges of the medieval guild system, we apprenticed ourselves to a guild master – a true master.

There was certainly a deep love for Mathematics and a lifelong friendship and bond that we developed together under M.M.'s direction, but there was much more to M.M.'s influence. M.M. had a deep concern for, and loyalty to, his students. No matter how busy he was, he always had time and energy for us in all aspects of our development. When my wife left me in the final year of my thesis work, M.M. was there to counsel and comfort me. Unbeknownst to me at the time, he had also spoken with my wife to see if there were any possible solution. When I mistakenly thought that one of my thesis results was contained in an earlier paper, he brought me out of my depression and led me to see the positive differences in the work. When I succumbed to procrastination and other earthly temptations, he was there to inspire me with his example. He was never accusatory or condemning, just exemplary and inspiring. When a new department chair took a personal dislike to me, M.M. was there to defend his student. Of course, this was the same M.M. who liked to put an occasional (unannounced) unsolved problem on his take-home exams in the advanced topics courses.

When I received a job offer from the Mathematics Department at UCR,

he told me that it was a great opportunity because Howard Tucker was there and he repeatedly advised me that “... you will really like Professor Tucker”. This stuck in my mind so deeply that when I finally met Howard and he told me to call him Howard, my natural response was “Yes sir, Professor Tucker”.

A few years later, my new department wanted to recruit a senior person in Functional Analysis and Probability. M.M. was not in the market for a new job, but I knew that he was not happy with his department chair. Our department managed to interview him twice and convinced him to come. So we wound up in the same department for thirty years. Once again he led me to learn life's great lessons. At first I needed to assert my independence from him. That must have been painful for him, but he never showed it. Then I needed to again succumb to procrastination and earthly temptations. Once again, he was over the space of many years non-accusatory and supportive. Coming back into the fold, I started to drift into areas of applications of Functional Analysis and Measure Theory. He renamed our continuing Functional Analysis Seminar the Functional Analysis and Related Topics Seminar.

It has been a very large feature of my life as well as a remarkable pleasure and privilege to be his student, his colleague, and his friend.

N.E. Gretsky

Rao and the early Riverside years

M.M. Rao first came to the University of California, Riverside Mathematics Department from Carnegie-Melon in 1973. There was much excitement and anticipation of his coming by both new colleagues and graduate students in the UCR Math Department. Neil Gretskey (a former Rao graduate student from Carnegie-Melon) was already on the faculty at UCR. Neil and others had alerted UCR graduate students of M.M. Rao's prominence in probability and functional analysis. M.M. Rao was a welcome addition of a talented research mathematician who was receptive to graduate students. This enhanced an already formidable mathematics department that had F. Burton Jones in topology, Richard Block in algebra and Victor Shapiro in differential equations among other notable faculty members.

As a new graduate student at UCR (coming from UCLA) in 1973, I knew very little of the anticipation surrounding M.M. Rao's first academic year at UCR. However, I became quickly familiar with Rao's teaching style as I took his inaugural graduate sequence in real analysis at UCR: Math 209, 210, 211 starting in September 1973. The course was taught at a high level of abstraction. The first quarter was measure theory developed on general sigma rings using an outer measure approach restricted down to measurable sets via the Caratheodory construction. The second quarter contained the major results of general integration theory. The third quarter included an introduction to Choquet's capacity theory. There was no specific textbook for the course but rather a list of several recommended texts. The course was carefully and clearly presented by M.M. Rao, a man in his early forties (originally from India) with a lively personality, who wore a suit to class. I tried to take careful notes and absorb the material since I knew a comprehensive qualifying exam on real analysis based on this course was waiting for me at the end of the academic year. However, the material was not easy for me. I passed the qualifying exam but considered myself fortunate. As for this introduction to Rao, I found him an animated professor completely engaged in the subject of real analysis. He developed the theory from a modern abstract viewpoint but was concerned about the history of the subject and was careful to credit various mathematicians as we proceeded (Lebesgue,

Caratheodory, Vitali, Saks, Fubini, Egoroff, Choquet, etc.).

Several very talented UCR graduate students, including Stephen Noltie and Michael Brennan, were seemingly planning to work with Rao even before he arrived at UCR. By the time, I asked Professor Rao to be my advisor in 1975-76, I was his sixth Ph.D. student at UCR. I was grateful he agreed to take me as his student. From the beginning, Rao had the reputation of being more demanding than most other professors at UCR. Rao would oversee your progress but he would not help you in the writing of your thesis. Rao expected his students to be prepared in many different areas of mathematics. As a consequence, Rao students routinely took additional coursework past the qualifiers. For example, I took graduate sequences in functional analysis, advanced statistical inference and probability theory after my qualifying exams. The idea was to be prepared to solve our dissertation problem from a variety of different possible perspectives.

Another important aspect of being a Rao graduate student in the 1970's was an ongoing quarterly seminar on functional analysis or stochastic processes. This seminar (which still meets) consisted of Rao, his students and any other interested parties. Everyone attending talked sooner or later. When the discussion became very specialized, the seminar often reduced down to Rao and his graduate students. For me, I recall having to present material that originated from a seminal 1969 article written by D.W. Stroock and S.R.S. Varadhan on solutions of diffusion processes in d -dimensions using the martingale problem approach. I vividly recall preparing this challenging material for what seemed like an endless number of consecutive weeks. It was stressful but very helpful in forcing me to understand this paper which I eventually generalized into my dissertation. Understanding came slowly (and in phases). I learned how to present material when there were holes or unresolved problem areas and how to talk around topics until I was able to make complete arguments. The whole experience also brought the Rao students together in a common misery and made me appreciative of the mathematical abilities of my fellow grad student, Michael Brennan, who kindly helped me understand the more incomprehensible parts of this paper. This seminar experience was a common learning experience for Rao students in the 1970's. It is an activity that I still do today on a modified basis with my own graduate students.

At UCR, M.M. Rao was primarily known among graduate students as a consummate researcher in mathematics—a man whose research interests connected functional analysis and integration theory with probability theory and stochastic processes. He was also recognized as an engaging professor who attracted some of the stronger graduate students to work with him and take a wide range of graduate classes. From a work ethic point of view role, no one worked harder than Rao. In the 70's, Rao occupied the (eastward) end office of a string of about twelve windowed offices on the third floor of Sproul

Hall that faced south overlooking University drive. Any passer-by, looking up at these offices in the evening would customarily see only two or three lights on after dark. Sometimes if the hour was late only one light burned. M.M. Rao's window was almost always lit. He was up there doing research, reviewing articles and in the 70's writing his first book. His colleagues and graduate students knew he was there. They also knew that he would be back in his office at least one day over the weekend as well. Rao displayed a commitment to his profession that was hard to match. From a graduate student's point of view, no one could complain that Rao was inaccessible or difficult to find.

M.M. Rao of the 1970's was a confident, forceful and demanding advisor. As an outstanding mathematician, Rao had expectations or intuition of how the solution of mathematical problems should turn out. Whenever graduate student progress did not fit his long range view, he expected to be consulted or convinced as to why these mathematical objectives were not possible. He also expected graduate students to make a dedicated effort and work hard. Finally, Rao expected his stronger graduate students to make significant contributions by doing future mathematical research. After all, Rao himself lived according to these standards. These expectations sometimes caused tensions between Rao and his students. For example, graduate students, myself included, would at times "disappear" for weeks or even months. When this happened there could be many possible reasonable explanations (and some unreasonable explanations as well)-including outside life factors effecting the unreal graduate school existence. Sometimes, a graduate student just rather "lay low" while trying to achieve progress rather than share their "failed attempts" at solving a problem. I can remember Rao asking "Where is ___?-I haven't seen him in weeks!" These incidents had both good and bad consequences. Rao students developed an independence and self reliance in doing mathematical research and also provided more opportunities for Rao students to bond together. Rao stories, like war stories, were swapped over lunch or over a few beers. Sometimes even an old Rao story from the legendary days of Gretsky, Uhl and Goldstein would be recycled when pertinent. In the end, Rao's forceful personality and expectations played differently among his graduate students (some of whom also had strong personalities and different goals).

M.M. Rao views his graduate students as one big family. Certainly, there are many of his former students who have flourished in mathematical careers engaged in many of the same aspects of the profession that have occupied Rao. There are also former, highly capable, graduate students who presently have little interest in mathematical research and have chosen exciting alternative career paths. M.M. Rao is interested and always enjoys hearing (and talking at length about) how each of his graduate students is doing. However, make no mistake about it, Rao is a true believer. M.M. Rao's career

in mathematics is distinguished by his talent, passion and energy in doing mathematics. There has never been any doubt in his mind that (if one has the ability) being a research mathematician is the best way to go. I think that even today, Rao would not understand how a graduate student in mathematics with outstanding potential in research could choose to do anything else. It is also very difficult to imagine M.M. not being engaged in mathematical activities. Rao is a lifer. Currently, at age 74, he is going strong. Rao is busy writing books with plans for additional books in the future.

M.M. Rao did a wonderful job of protecting and promoting his graduate students. He was influential and resourceful in securing teaching assistantships and research assistantships to support his graduate students throughout graduate school. During the 1970's, Rao was preparing his first book, *Stochastic Processes and Integration*. I, along with several other grad students, worked as a research assistant, proof-reading this monograph. Professor Rao was very receptive to student reaction to his writing. At first, I was hesitant to mention where I had difficulty in understanding his text but it became very clear that he was sincerely interested in both my mathematical and stylistic comments. Rao, in discussion, would often tell me of the history of various portions of the text and what different mathematicians had contributed. These were good times for me. I was seeing mathematics from an insider's perspective. Sometimes, Rao would go off describing some current mathematicians. For example, he knew I was studying a paper of the Russian mathematician Daletskii and Rao would tell me of his personal meeting with Daletskii on a visit to Russia and how nice a man he was even presenting M.M. a bottle of Vodka (or Cognac) as a gift. Rao still had the gift somewhere in his office. These exchanges were memorable.

The academic environment and spirit for faculty and graduate students in the UCR Mathematics Department of the 1970's was very good. The Department was a friendly place and a good place to study mathematics. Al Stralka chaired the Department. I recall colloquia given by Erdos, Halmos, Bing, Stein and Uhl. I recall the excitement of the four color problem being solved at that time. There was an entertaining talk on properties of Fibonacci numbers as well. The colloquia were preceded by a reception that usually included cookies—a sure way to attract graduate students. For at least two years, the math graduate students participated in intramural basketball games. Our team names of “Zorn's Dilemma” and “The Hardy-Haar Measure” accurately reflected our team's abilities. We had measure 1 of going the whole season without a victory. Except, there was one anomalous game where we actually nipped the lowly and equally winless Physics team on a last second miracle shot—which demonstrated once and for all that events of measure 0 can indeed happen! We had fun with basketball but actually looked forward to the pizza and beer get-togethers after the game more than the game itself.

During the mid 1970's, Rao students were united by the knowledge that we were committed to a challenging route working under M.M. Rao and hopeful of his influence to secure us academic employment at a notoriously difficult time period for new Ph.D.'s to find jobs as professors in mathematics. We were also united by having taken an unusually large number of courses from Rao. The following pet phrases (and situations) were often repeated (or experienced) many times in class and today serve to help us recapture, with affection, his unique personality and style:

"We make the following definition with complete 'malice of forethought'."

"Did you think it was going to be easy? No! That is why his name is on the theorem."

"Be careful whenever you see that word 'consider' for what follows is a new idea..."

"From there he went on to develop (pronounced 'devil-up') the whole theory..."

"You ask me if I can change the order of integration? I DID IT!"

"That's the one, that's the condition you need..."

"You work and work and work and that is what comes out..."

"Now we have proved the Dynkin-Doob Lemma which is also used by statisticians who have no idea why it is true, so we call this result the Theorem of the 'Unconscious Statistician'..."

"If you wish to avoid making any mistakes, do nothing at all and that, of course, would be the biggest mistake of all..."

"What a loss...that is the death of his career as research mathematician", (Rao's reaction upon hearing a local mathematics professor had become dean.)

"You can take this book and throw it in the ditch..."

Many times Rao would smile and laugh as he repeated these sayings in different settings. Occasionally, M.M. would re-tell a joke or funny story and break out laughing aloud before reaching his own punch-line.

And finally the signature (literally and figuratively) of most Rao chalk

talks was the amazing amount of mathematical material he was able to cram into the lower right hand corner of the board as class time expired. His writing became a space filling curve as he adjusted by writing smaller and smaller-working several minutes past when the class was scheduled to stop-leaving students dazed and hopelessly trying to decipher his final scribbling.

Rao could push and posture. During my last year in graduate school Rao had monitored all my dissertation work. I had passed my oral exams and was in the process of writing up my final results. We were 3 months away from being done. He looked at my folder of dissertation results and then back at me and announced, "It's not enough". I felt my heart sink and had nothing to say. I went home wondering what more I could do. There was no more but he was still seeing if he could squeeze some new results out of me. I did not like the pressure but I understood his intent. It soon became clear to him (if it wasn't already) that there was nothing else to do on my problem. It never came up again and I finished my Ph.D. as originally scheduled three months later.

In 1978, Rao still had three of his six graduate students (Brennan, Kelsh and Krinik) anxious to get out. Rao was leaving on a sabbatical (I believe to France) starting Fall 1978 and the realization dawned on us that Rao's sabbatical was our best chance of finally finishing up. Otherwise, we would have to postpone our graduation until Rao's return to UCR a year later and, of course, no one wanted to wait. In a furious finish, we all made it. I was the last of the three to finish and remember happily driving M.M. to the airport.

After graduation, my relationship and appreciation of M.M. Rao grew and matured. As a graduate student, I was always appreciative of his financial support for all his students and his academic support for me in particular. After graduation, Rao became a major player in my career. He was always in my corner, helping me. From key letters of recommendation to help me secure positions at JPL, University of Nevada, Reno and Cal Poly Pomona, to advising me where to try to publish my results, to being supportive when my efforts were not always successful, to providing me with opportunities to resume research activities and to finally just being there as a good friend. His encouragement and assistance in developing my professional activities has been and remains a constant.

In 1985, I invited M.M. to give a colloquium at Cal Poly Pomona. M. M. did his usual super job and in the audience sat a talented graduating senior who would not forget the talk nor the speaker. That senior was Randy Swift who eventually went on to earn his Ph.D. under M.M. Rao and who today is a good friend and valuable colleague at Cal Poly Pomona. Randy is also the real editor of this volume which we both affectionately dedicate to M.M., our mutual mentor. In tribute to M.M. Rao's stellar career, Randy compiled this volume of research articles.

The eighteen Rao students share a special bond and understanding of what it means to earn your doctorate degree in mathematics under the direction of M.M. This collection of essays and articles in honor of M.M. illustrates this bond crosses four decades and bridges his Carnegie-Mellon University students of the 60's with the University of California, Riverside students of the 70's , 80's and 90's. It's been a pleasure to have the opportunity to celebrate M.M. Rao's many contributions and to be "one of Rao's students".

Alan Krinik



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On M.M. Rao

I first met Professor M.M. Rao in 1985, when I was an undergraduate attending California State Polytechnic University, Pomona. Alan Krinik had invited M.M. to give a colloquium talk in the department.

At the time, I was a senior math major and one of the department's promising students. I had attended departmental colloquium talks, but never had I been exposed to a mathematician of the caliber of M.M. Rao.

His talk began in a very elementary fashion, but the breadth and depth of the mathematics it spanned greatly impressed me. I was struck by the passion for mathematics that he displayed. I had not been in the presence of someone totally devoted to his discipline.

After I completed my Masters degree, I worked for a while in the aerospace industry. I found myself desiring to pursue a PhD. My interest in probability theory and my strong recollection of M.M. led me to apply to the University of California, Riverside.

As fate would have it, and in my good fortune, I took M.M. for a graduate course in Probability, his lectures were absolutely beautiful. Spanning the subject with depth and presented with crystal clarity. Of course, he used his text *Probability Theory with Applications*, perhaps the finest graduate text written on the subject.

This course, and indeed, this text, set the tone for what working with M.M. would be like. M.M. believes that homework should challenge his students. During his courses, he assigns a problem or two per week. These problems are not routine homework problems, rather they are problems from the research literature. They are not mere exercises. Indeed, his students spend vast amounts of time working on them. To this end, he is preparing his students for research. Many of these problems aid his students in their future works.

M. M. greatly respects effort. If he sees that a student is working, he will guide the student gently down the appropriate path. He has an incredible memory for details, often if a student was stuck on an idea, he would say, go see this page of a particular paper or text. On that page, you would find what you needed to get going again on the problem. From these interactions,

M.M. seems to gauge a student's ability. I became a student of his after I had completed the course in probability theory, a seminar on random fields and a course in stochastic processes. I never asked him to be my advisor; rather, it seemed to be a natural evolution.

The first problem that he asked me to study involved the sample path behavior of harmonizable processes. I spent a large part of that first summer developing my facility with these processes. By the end of the summer, I obtained my first minor result; it was on the analyticity of the sample paths. However, the goal was to consider the almost periodic behavior of these sample paths and I was stuck.

I toiled in vain for the next few months on this problem. One day, late in November, I went to talk with M.M. about the problem, he listened intently and then said, "let it rest there, for now, I would like you to look at this calculation I have been working upon with harmonizable isotropic random fields."

He had obtained a representation for the covariance that involved some rather complicated special functions. He asked me to see what I could do with it, in particular, could it be made to look similar to the representation obtained by Bochner for the stationary isotropic case.

I told him that I would try, and he said "There is no try, there is only do, and I *know* that you can."

By Sunday afternoon of that weekend, I had simplified the expression and had showed that it reduced to Bochner's representation in a very natural way.

That Monday, I gave him the result. He, in a very delighted manner, then said to me, "See if you can push it. Look at Yadrenko's book and use this representation to extend his results."

This began a glorious 3-month stretch of research production. I obtained several major results for harmonizable isotropic random fields.

Riding the tide of this success, he said to me "And what of the almost periodicity?"

With the confidence I had obtained, I went back to the problem. Within a month or so, I had obtained the results I had long sought. This experience gave me great confidence in my ability to do research. It also gave me a very broad research program to pursue. The confidence that M.M. showed in my abilities as a mathematician remains with me today. It has allowed me to flourish.

Many years later, M.M. told me that the first string of results I had obtained after I had obtained the representation was likely enough for the PhD. However, he saw that I was on track and he was going to have me do as much as I could.

This story is very typical of the relationships he has with his students. He works them very hard, always encouraging, and yet unyielding in his

determination that they do their very best work.

In this setting, many of his students have become mathematicians completely devoted to the discipline. Whether this devotion is shown through excellence in teaching, or excellence in research, or both, for each of us, it is likely attributable to the role M.M. played, and continues to play, in our lives.

It is no wonder then, that for some of us, M.M. holds a place in our hearts, and we remain devoted to him as former students and now colleagues.

R.J. Swift



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