Converter-Interfaced Energy Storage Systems

Context, Modelling and Dynamic Analysis

Federico Milano and Álvaro Ortega Manjavacas

Converter-Interfaced Energy Storage Systems

Gain an in-depth understanding of state-of-the-art converter-interfaced energy storage systems with this unique book, covering dynamic behaviour, modelling, stability analysis and control.

- Presents an in-depth treatment of the conceptual, technical and economic frameworks underpinning energy storage in modern power systems
- Includes a comprehensive review of technologies for cutting-edge converterinterfaced energy storage systems
- Addresses the impact of energy storage on the dynamic interaction of microgrids with transmission and distribution systems
- Provides a variety of reference models, and a generalized model for energy storage systems to enable benchmarking of control strategies and stability analysis

Accompanied by a wealth of numerical examples and supporting data online, this is the ideal text for graduate students, researchers and industry professionals working in power system dynamics, renewable energy integration and smart grid development.

Federico Milano is Professor of Power Systems Control and Protection, and Head of Electrical Engineering, at University College Dublin. He is a Fellow of the IEEE and the IET.

Álvaro Ortega Manjavacas is a Senior Power Systems Researcher in the School of Electrical and Electronic Engineering at University College Dublin.

"This is a timely and impressive book on an emerging and important topic. The comprehensive and in-depth overview of energy storage technologies, modelling, and dynamic simulation will make the book a valuable reference for practicing engineers and researchers working with the planning and operation of the future electric power system. The extensive list of references will be of great help for deepened studies."

Göran Andersson, ETH Zürich

"Energy storage systems (ESS) are considered by many as the Holy Grail of the upcoming decarbonised future. From rooftop PV microsystems to giant pumped storage units, virtually all ESS are expected to be interfaced through power converters, for the sake of added flexibility and efficiency. This volume, co-authored by one of the most recognized experts in modelling, analysis and control of power systems dynamic phenomena, constitutes a self-contained and unique blend of general concepts, motivating factors and technical details, satisfying in this way the interests of a wide audience and filling an important gap in the technical literature."

Antonio Gomez-Exposito, University of Seville

"Excellent and timely material written by experienced authors! You must read this book."

Jean Mahseredjian, Polytechnique Montréal

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To my parents, Guido and Silvana.

F.M.

To my parents, Manuel Ángel and Mari Paz, and brother, José Miguel.

Á.O.M.

Energy is a very subtle concept. It is very, very difficult to get right.^{\dagger}

Richard P. Feynman

 $[\]dagger$ Reproduced from [86], with the permission of the American Association of Physics Teachers.

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Preface

Background and Motivations

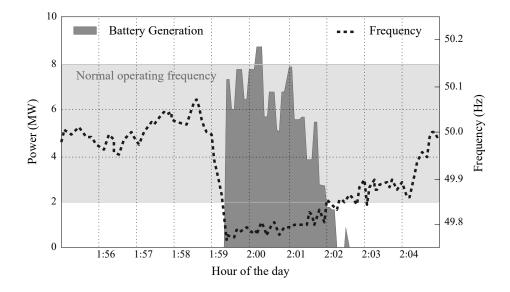
This book is the result of the work of the authors on modelling, simulation, control and stability analysis of converter-interfaced energy storage systems carried out in the period from 2010 to 2018. The first author (FM) started offering final projects on modelling of converter-interfaced energy storage technologies when he was working at the University of Castilla-La Mancha, Spain, and the second author (ÁOM) was brave enough to first undertake one of these final projects and then pursue his PhD on the same subject at University College Dublin, Ireland, which the first author joined in 2013.

Back in 2010, the idea of studying large-size converter-interfaced energy storage devices was considered, to use a euphemism, an oddity. Several colleagues were – and some still are – highly sceptical on the high cost of storage devices and on their scalability. The most common concern that we have learned to expect for every article we have submitted and presentation we have given in these years, has been related to the *inviability* of these devices for power system applications due to economic constraints. Also, the idea that a battery could be utilised to implement a continuous control, and not exclusively discrete on/off operations has often been considered a peculiar idea, again due to economic considerations.

In February 2017, the first author witnessed a heated discussion at a small workshop hold in Champéry, Switzerland, between a supporter and an opponent of battery energy storage devices for power system applications. The only argument of the opponent was economic viability. After the workshop, the storage-device enthusiast was driven home by his Tesla Model S.

Despite mixed feelings, the blooming of energy storage technologies is undeniable. The cover and most articles of the issue of September/October 2017 of the IEEE magazine *Power & Energy* are dedicated to 'Opening the door to energy storage – Challenges for future systems'. Seminars and special sessions on energy storage applications at international conferences on power systems are omnipresent. A one-day tutorial with title 'Energy Storage: An introduction to technologies, applications and best practices' has been organised every year from 2015 to 2018 at the IEEE PES General Meetings. Also relevant were the one-day tutorials of the IEEE PES T&D 2018, Denver, Colorado, and of the Power System Computation Conference (PSCC) 2018, Dublin, Ireland.

In the real world, that is, in the world outside academia and symposia, the years from 2010 to 2017 have been an extraordinary period of intense brainstorming and exper-



The Hornsdale Power Reserve 100 MW battery responding to a drop in system frequency on 14 December 2017. (Courtesy of Australian Energy Market Operator).

imentation on energy storage solutions. Existing technologies, such as batteries, have been and are continuously being dramatically improved in terms of duration and reliability. One of the most well-known outcomes of this research is certainly the exponential growth of hybrid and plug-in electric vehicles. New, often very imaginative, prototypes that exploit a new chemical reaction or a new surprising solution come out on a monthly, if not weekly, basis.

Back in 2010, the cost of a lithium-ion battery was about \$1000/kWh. This cost dropped to about \$270/kWh in 2016 according to a survey carried out by Bloomberg New Energy Finance, i.e. a 73% drop in six years. Predictions are between \$190 and \$130 in 2020 and between \$75 and \$50 in 2030. This dramatic decrease is clearly due to the humongous interest arising from the business of electric vehicles, not power system applications. Yet, a 100 MW, 129 MWh lithium-ion battery has been built in less than 100 days and installed in Hornsdale, South Australia, by Tesla Inc., as a personal bet of the company co-founder and CEO, Elon Musk. This was – and still is at the time of completing this book – the world's largest grid-scale battery and charged for the first time at 8:36 am, on 1 December 2017, and reached 31 MW in 2 minutes.

While the habitual sceptics were wondering whether such a large battery was actually a solid business model, the Hornsdale battery has been utilised to provide frequency control to the Australian system, see figure, and its surprisingly fast time response has helped balance several major energy outages that have occurred since it was installed. We expect that the Hornsdale battery will help save much more money than that required for its construction. Other projects for similar or even larger grid-scale batteries are currently in progress or under evaluation.

Organisation

The matter of the book is organised in nine chapters divided into three parts, as follows.

Part I – Context

Chapter 1 introduces the basic concepts of energy storage through a variety of examples that span several time scales, from the electromagnetic transients of transmission lines to the daily load levelling through pumped hydroelectric power plants. The technical background that motivates a monograph on converter-interfaced energy storage devices is also given in this chapter.

Chapter 2 defines the technical and economic parameters, challenges and issues that characterise energy storage devices. Particular emphasis is given to the definition of relevant quantities as well as grid applications and the levelised cost of electricity, which allow fairly comparing different storage technologies.

Chapter 3 provides high-level descriptions of the most important current technologies for energy storage applications. Emerging technologies that have reached the prototype stage are also considered. The second half of the chapter is dedicated to the description of real-world examples.

Part II – Modelling

Chapter 4 presents the structure and main elements that compose modern electrical energy systems. These include conventional devices, renewable and/or distributed energy resources and the smart grid concept. A dynamic model of microgrids that is adequate for the transient stability analysis of interconnected AC networks is also provided in this chapter.

Chapter 5 introduces the model and the basic controllers of the AC/DC voltage-sourced converter, namely the main device on which all energy storage devices considered in this book are based. The model described in this chapter is based on a dq-axis frame, average, fundamental frequency formulation and includes detailed dynamics of the DC side, primary controllers, and current and PI control limiters.

Chapter 6 presents the detailed models of each storage technology considered in the case studies discussed in Part III. These include battery, compressed air, flywheel and superconducting magnetic energy storage. Models of a few other emerging technologies are also presented along with simplified energy storage models. A description of the procedure to define a generalised yet accurate dynamic model of any converter-interfaced energy storage technology completes the chapter.

Part III – Dynamic Analysis

Chapter 7 provides a comprehensive comparison, through a variety of examples, of the detailed, simplified and generalised energy storage system models of the technologies described in Chapter 6. The features of each model as well as the advantages provided by the generalised model are discussed.

Chapter 8 presents a variety of control strategies for energy storage devices. The primary frequency control and the performance of the ubiquitous PID controller as well as other non-conventional approaches, such as sliding mode and H-infinity, are thoroughly discussed. Then the chapter considers secondary frequency control of storage devices through model predictive control and a decentralised stochastic control of microgrids. **Chapter 9** completes the book with the stability analysis of power systems with inclusion of converter-interfaced energy storage devices. Frequency, small-signal, and voltage stability are considered. Brief outlines of converter-driven instability issues as well

Software Tools

as of microgrid stability are also given in this chapter.

For the reader interested in software technicalities, all simulations included in the book are obtained using the Python-based software tool Dome [188]. The Dome version utilised is based on Fedora Linux 25, Python 3.6.2, CVXOPT 1.1.9, KLU 1.3.8, and MAGMA 2.2.0. The hardware consists of two 20-core 2.2 GHz Intel Xeon CPUs, which are utilised for matrix factorisation and Monte-Carlo time-domain simulations; and one NVidia Tesla K80 GPU, which is utilised for the small-signal stability analysis.

Lessons Learned

Writing a book is a long journey and an engaging learning process. We have learned that blunt economic considerations should not be utilised as an argument to destroy academic research. We wish to thank all our colleagues and anonymous reviewers around the world who, when commenting on our work, could not find any better argument than the high cost of storage devices. Their criticism reinforced our intuition that we were on the right track.

We have been very fortunate to work on converter-interfaced energy storage devices in these years. As for any emerging technology, this has been a period full of brainstorming, unresting ideas and unexpected developments. For this reason, in the book, we consider with the same agnosticism with which we started in 2010 not only batteries but also several less common energy storage technologies. We also propose an approach to model all technologies, including the ones that will not survive the battle with business models and even those that have not been invented yet.

At the time of writing the last paragraphs of this book, there is still no conclusive work on the dynamic analysis of converter-interfaced energy storage systems. There are more questions than answers, which is good. Vladimir Nabokov was wont to say that a good book should end with an open question. We trust that this book moves towards the right direction and provides some useful tools to answer these questions. We hope that the reader will have as much fun reading the book as we had while writing it.

> Federico Milano & Álvaro Ortega Manjavacas Dublin, Chiusa di Pesio, Quintanar de la Orden

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Disclaimer

The opinions, findings, conclusions and recommendations expressed in this work are those of the authors and do not necessarily reflect the views of the European Union or Science Foundation Ireland. The European Commission and Science Foundation Ireland are not responsible for any use that may be made of the information that this work contains.

Notation

Energy storage system models span many technologies and fields, ranging from electromagnetism and electrochemistry to mechanics and thermodynamics. Whenever there is no possible confusion, the simplest and most common notation for physical quantities is used, even if doing so sometimes leads to utilising the same letter for different quantities. In these cases, different styles or fonts are used. The context where such quantities appear also helps avoid confusion. Tables with a list of symbols of relevant variables and parameters and their definition are also included whenever relevant. The remainder of this section only reports the high-level notation adopted throughout the book. Whenever a different notation is used, quantities are defined in the text.

Scalars, Vectors and Matrices

- v, V, V scalar variable, scalar parameter
- *v*, **v** vector, one-dimensional array
- V matrix, two dimensional array

Reference frames

- v(t) time domain quantity (v is used if the context is unambiguous and in schemes for simplicity)
- $v_{abc}(t)$ vector of three-phase time domain quantities
- v(s) frequency domain quantity (Laplace transform)
- \bar{v}_{dq} Park vector in dq-axis reference frame, i.e. $\bar{v}_{dq} = v_d + jv_q$
- \bar{v} phasor or complex quantity, i.e. $\bar{v} = v \angle \theta$
- \bar{v}^* conjugate of \bar{v} , i.e. $\bar{v}^* = v \angle -\theta$
- \underline{v} average quantity

Time derivatives

$\frac{d}{dt}$	time derivative in time domain
<i>S</i>	time derivative in frequency domain (Laplace transform)
$j\omega_o$	time derivative in phasor domain
$\frac{\mathrm{d}}{\mathrm{d}t} + j\omega_o$	time derivative in dq-axis reference frame (Park transform)
<i>v</i>	rate of change with respect to time

Common Quantities

drift term of stochastic processes; transformer tap ratio
cross-sectional area
diffusion term of stochastic processes
susceptance
specific heat capacity
volumetric heat capacity
capacitance
concentration of ions
duty cycle
damping
electromotive force (EMF); standard cell potential
energy
expectation
electric field
electrical frequency
vector of differential equations
mass or molar fraction
vector of algebraic equations
conductance; Gibbs free energy
height
hentalpy; inertia constant
magnetic field
current
imaginary unit
moment of inertia
coefficient in empirical formulas
controller gain
length
inductance
specific latent heat
mass; modulation amplitude of AC/DC converters
machine starting time $(M = 2H)$
molar mass
number of moles
active power
pressure
Park tensor
reactive power
electric charge
heat

Q	volumetric flow
r	radius
R	resistance
R	droop of primary frequency control
Ī	complex power
s	Laplace transform variable
S	entropy
S	sliding surface
t	time (r within integrals)
Т	time constant
и	input signal
и	vector of input signals
U	internal energy
v	voltage
V	volume
w	Wiener process
W	mechanical work
x	position; state variable; control signal
x	vector of state variables
X	reactance
У	measured grid signal
у	vector of algebraic variables
\bar{Y}	admittance
z	valency number
\overline{z}	extended state variable vector
Ī	impedance
α	autocorrelation
β	pitch angle of the blades of wind turbines; gate position of hydro turbines
γ	polytropic coefficient
δ	angular position
ε	permittivity
ζ	stochastic variable
η	efficiency
θ	phase angle of voltage phasors
Θ λ	temperature
	electricity price; tip speed ratio of wind turbines
μ ε	mean value; permeability white noise
ξ Ξ	total resource capacity available for AIMD control
	probability
$\overline{\sigma}$	density
ho	uchony

xix

ϱ	discount rate
σ	standard deviation
5	switch status
au	torque, time delay
υ	speed
ϕ	magnetic flux
arphi	phase shift
ψ	total magnetic flux
ω	angular speed

Note. In per unit, the angular speed ω has the same value of the frequency f. For this reason, whenever the context is unambiguous, ω is also loosely referred to as *frequency*.

Common Superscripts and Subscripts

а	first phase of a 3-phase system
ac	AC quantity
b	base quantity
b	second phase of a 3-phase system
С	converter
с	third phase of a 3-phase system
d	direct axis of the dqo transform
D	demand quantity
dc	DC quantity
dq	dq-frame (Park) vector
е	electrical
G	generator quantity
L	transmission line quantity
т	mechanical
max	maximum value
min	minimum value
n	nominal or rated quantity
n	neutral point
0	reference, initial or base-case condition
0	zero axis quantity of the dqo transform
q	quadrature-axis quantity of the dqo transform
r	rotor quantity
ref	reference or set-point quantity
S	stator quantity
t	turbine quantity
Т	transformer quantity
tot	total quantity
	total quality
W	wind-related quantity

Constants

$F = 96.487 \left[\mathrm{kC} \mathrm{mol}^{-1} \right]$	Faraday constant
$k_B = 5.670 \cdot 10^{-8} \left[W m^{-2} K^{-4} \right]$	Boltzmann constant
R = 8.314 [J mol ⁻¹ K ⁻¹]	Universal gas constant
$\varepsilon_o = 8.854187817 \cdot 10^{-12} \text{ [Fm}^{-1]}$	Vacuum permittivity
$\mu_o = 4\pi \cdot 10^7 \left[\mathrm{N} \mathrm{A}^{-2} \right]$	Vacuum permeability
$\pi = 3.14159265359$ [rad]	

Numbers

The order of vector and matrices is indicated with *n* and a suffix to indicate the variable to which such a number refers. For example, n_x indicates the order of the vector of state variables $\mathbf{x}(t)$.

Units

The units of absolute quantities follow the International System of Units (SI). Unless explicitly indicated, however, the equations that describe AC circuits are in per unit values, as usual in power system analysis. The bases are the three-phase apparent power, s_n , the phase-to-phase voltage v_n and the frequency f_n . All other bases are derived from these three quantities. For example, the bases of the impedance and the line current are, respectively:

$$Z_n = \frac{v_n^2}{s_n} , \qquad i_n = \frac{s_n}{\sqrt{3} v_n} .$$

The main device discussed in the book is the AC/DC converter (see Chapter 5). Hence equations of DC circuits appear very often. These are expressed in absolute values. When DC and AC quantities appear in the same expression, the units of each quantity are indicated explicitly.

Acronyms and Abbreviations

AA-CAES	Advanced Adiabatic Compressed Air Energy Storage
ABB	ASEA Brown Boveri
AC	Alternating Current
AFC	Alkaline Fuel Cell
AGC	Automatic Generation Control
AHI	Aqueous Hybrid Ion
AIMD	Additive Increase Multiplicative Decrease
ALAB	Advanced Lead-Acid Battery
ARES	Advanced Rail Energy Storage
ATES	Aquifer Thermal Energy Storage
AVR	Automatic Voltage Regulator
BDF	Backward Differentiation Formulas
BEM	Backward Euler Method
BES	Battery Energy Storage
BTES	Borehole Thermal Energy Storage
C-CAES	Cavern-based Compressed Air Energy Storage
CAES	Compressed Air Energy Storage
CCGT	Combined-Cycle Gas Turbine
ССТ	
001	Critical Clearing Time
CDF	Critical Clearing Time Cumulative Distribution Function
	-
CDF	Cumulative Distribution Function
CDF CESI	Cumulative Distribution Function Centro Elettrotecnico Sperimentale Italiano
CDF CESI CHP	Cumulative Distribution Function Centro Elettrotecnico Sperimentale Italiano Combined Heat Power
CDF CESI CHP CI-ESS	Cumulative Distribution Function Centro Elettrotecnico Sperimentale Italiano Combined Heat Power Converter-Interfaced Energy Storage System
CDF CESI CHP CI-ESS CIG	Cumulative Distribution Function Centro Elettrotecnico Sperimentale Italiano Combined Heat Power Converter-Interfaced Energy Storage System Converter-Interfaced Generation
CDF CESI CHP CI-ESS CIG Col	Cumulative Distribution Function Centro Elettrotecnico Sperimentale Italiano Combined Heat Power Converter-Interfaced Energy Storage System Converter-Interfaced Generation Centre of Inertia
CDF CESI CHP CI-ESS CIG CoI CPV	Cumulative Distribution Function Centro Elettrotecnico Sperimentale Italiano Combined Heat Power Converter-Interfaced Energy Storage System Converter-Interfaced Generation Centre of Inertia Concentrated Photovoltaic
CDF CESI CHP CI-ESS CIG CoI CPV CSC	Cumulative Distribution Function Centro Elettrotecnico Sperimentale Italiano Combined Heat Power Converter-Interfaced Energy Storage System Converter-Interfaced Generation Centre of Inertia Concentrated Photovoltaic Current-Sourced Converter

СТ	Clearing Time
CTES	Cavern Thermal Energy Storage
DAE	Differential Algebraic Equation
DC	Direct Current
DER	Distributed Energy Resource
DFIG	Doubly-Fed Induction Generator
DFIM	Doubly-Fed Induction Machine
DoE	US Department of Energy
EAC	Equal Area Criterion
EC	Electrochemical Capacitor
ECES	Electrochemical Capacitor Energy Storage
EDF	Electricité de France
EDLC	Electric Double-Layer Capacitor
EIA	Energy Information Administration
EMF	Electromotive Force
EMF	
EMIS	Energy Management System Electromagnetic Transient
ESS	-
ESS	Energy Storage System Electric Vehicle
Ev	Electric venicle
FACTS	Flexible AC Transmission System
FDF	Frequency Divider Formula
FeCrFB	Iron-Chromium Flow Battery
FERC	Federal Energy Regulatory Commission
FES	Flywheel Energy Storage
FLC	Fuzzy Logic Control
G-CAES	General Compressed Air Energy Storage
GEM	Generalised Energy Storage System Model
GPES	Gravel Potential Energy Storage
GTES	Gravel Thermal Energy Storage
HESS	Hybrid Energy Storage System
HEV	Hybrid Electric Vehicle
HIC	H-Infinity Control
	-
HT-UTES	High Temperature Underground Thermal Energy Storage
HI-UTES HVAC	High Temperature Underground Thermal Energy Storage Heating, Ventilation and Air Conditioning
	High Temperature Underground Thermal Energy Storage Heating, Ventilation and Air Conditioning High-Voltage Direct Current

I-CAES	Isothermal Compressed Air Energy Storage
ICT	Information and Communications Technology
IEA	International Energy Agency
IEEE	Institute of Electrical and Electronics Engineers
IES	Inductive Energy Storage
IFAC	International Federation on Automatic Control
IGBT	Integrated Gate Bipolar Transistor
IGCT	Integrated Gate Commutated Thyristor
ISO	Independent System Operator
ІТМ	Implicit Trapezoidal Method
KFSM	Kalman Filter-based Synchronisation Method
LAES	Liquid Air Energy Storage
LCoE	Levelised Cost of Electricity
LCoS	Levelised Cost of Storage
LEC	Levelised Energy Cost
LQC	Linear-Quadratic Control
MCFC	Molten Carbonate Fuel Cell
MG	Microgrid
MOST	Molecular Solar Thermal
MPC	Model Predictive Control
MPPT	Maximum Power Point Tracking
MRI	Magnetic Resonance Image
MSTES	Molten-Salt Thermal Energy Storage
NASA	National Aeronautics and Space Administration
NEA	Nuclear Energy Agency
ODE	Ordinary Differential Equation
OECD	Organisation for Economic Co-operation and Development
OMIB	One-Machine Infinite-Bus
PAFC	Phosphoric Acid Fuel Cell
РСМ	Phase Change Material
PDF	Probability Density Function
PEMFC	Polymer Exchange Membrane Fuel Cell
PES	Power & Energy Society
PI	Proportional Integral
PIC	Proportional Integral Control

PID	Proportional Integral Derivative			
PJM	Pennsylvania-New Jersey-Maryland Interconnection			
PFC	Primary Frequency Control			
PHES	Pumped Hydroelectric Energy Storage			
PHTES	Pumped Heat Thermal Energy Storage			
PLL	Phase-Locked Loop			
PMSM	Permanent-Magnet Synchronous Machine			
PSCC	Power System Computation Conference			
PSDP	Power System Dynamic Performance			
PSS	Power System Stabiliser			
PV	Photovoltaic			
PWM	Pulse-Width Modulation			
RDFT	Recursive Discrete Fourier Transform			
RES	Renewable Energy Source			
RFB	Redox Flow Battery			
RMS	Root Mean Square			
RoCoF	Rate of Change of Frequency			
SCADA	Supervisory Control And Data Acquisition			
SDAE	Stochastic Differential Algebraic Equation			
SDE	Stochastic Differential Equation			
SFC	Secondary Frequency Control			
SH	Smart House			
SI-DAE	Semi-Implicit Differential Algebraic Equation			
SIL	Storage Input Limiter			
SLH	Specific Latent Heat			
SMC	Sliding Mode Control			
SMES	Superconducting Magnetic Energy Storage			
SMPES	Solid-Masses Potential Energy Storage			
SNG	Synthetic Natural Gas			
SoC	State of Charge			
SoE	State of Energy			
SOFC	Solid Oxide Fuel Cell			
SoH	State of Health			
SR-PHES	Surface-Reservoir Pumped Hydroelectric Energy Storage			
SRAM	Static Random-Access Memory			
SS-PHES	Sub-Surface Pumped Hydroelectric Energy Storage			
STATCOM	Static Synchronous Compensator			
STF	Solar Thermal Fuel			

STSA	Stochastic Transient Stability Analysis				
T-CAES TCL TES	Tank-based Compressed Air Energy Storage Thermostatically Controlled Load				
TG TSA	Thermal Energy Storage Turbine Governor Transient Stability Analysis				
ULTC UTES	Under-Load Tap Changer Underground Thermal Energy Storage				
VCO VRFB VRLAB VS-PHES VSC	Voltage Controlled Oscillator Vanadium Redox Flow Battery Valve Regulated Lead-Acid Battery Variable-Speed Pumped Hydroelectric Energy Storage Voltage-Sourced Converter				
WECC WSCC	Western Electricity Coordinating Council (former WSCC) Western Systems Coordinating Council				
ZnBrFB	Zinc-Bromine Flow Battery				

Part I

Context

1.1 Introduction

Storage is a fundamental part of any system. Goods are stored in warehouses. Internal combustion engines utilise fuel to store energy. Computers need hard disks. Communication systems and internet networks require data concentrators. Biological cells utilise adenosine triphosphate. Hydrogen is the fuel of stars. Ultimately, all matter stores energy according to the well-known mass–energy equivalence, as deduced from the special relativity postulates and the symmetries of space and time.

This book focuses exclusively on electric system applications and only on those particular kinds of energy storage technologies that are able to *cycle* the energy. The chemical energy stored in fossil fuel is not part of what we consider hereinafter as energy storage devices. We look for storage technologies which are also fully *reversible*, except of course for inevitable losses. A byproduct of the energy storage technologies and devices considered in this book is to reduce the *grey energy* due to fossil fuels.

Electric systems naturally include several kinds of energy storage, spanning several time scales. This book focuses on the specific mechanism to store and exchange energy through power electronic converters in the time scales from a few hundred milliseconds to tens of minutes. Electromechanical and control dynamics fall within these time scales. The focus is not on how energy is stored *per se* but, rather, how stored energy can be efficiently and conveniently exchanged during power system transients.

The remainder of this chapter discusses the crucial role of energy storage for power systems in the relevant time scales. In particular, the chapter provides a high level overview of the *natural* energy storage in power systems. Section 1.2 shows that energy storage is a fundamental, intrinsic part of such systems. There are, in fact, several time scales and means where energy is stored. Without such storage means, the systems cannot work. Seven quantitative examples are provided. Section 1.3 discusses the impact of the integration of renewable energy resources and elaborates on the effects of reducing the inertia due to non-synchronous generation. The concept of flexibility is also discussed in this section. Section 1.4 states the need for new devices to store energy and why such devices are more important today than ever in the history of power systems. Section 1.5 discusses the features and roles of conventional and converter-interfaced energy storage devices. Section 1.6 proposes a variety of symbols for generic converter-interfaced energy storage devices for single-line electric diagrams.

1.2 Power Balance in Electric Energy Systems

One of the first concepts taught in a module on electrical energy systems is that power consumption and losses have to be balanced at every instant by generation. This statement allows introducing the need for frequency control, power reserve and, on a longer time scale, unit commitment and seasonal storage.

After providing this example, the lecturer often also proposes to the students some counter-examples of other systems that do not have such a strict power balance requirement. In the internet, if the download requests of a user cannot be satisfied, they simply wait until the communication band is less congested. Similarly, in the transport sector, traffic jams happen and people simply have to wait until roads again become accessible. The list of counter-examples completes with saying that, in power systems, a load cannot 'wait' for the congestion to be resolved and this makes the control of a power system quite challenging.

While fundamentally correct, the statement above lacks some important remarks and clarifications which are crucial for this book and constitute its motivation. The first remark is that balancing the variations of the load cannot be achieved 'instantaneously', even assuming an ideal control. The control can only 'follow' load variations and there is no way to perfect tracking load variations. To complicate the situation, in recent years part of the generation has also become stochastic – because of the penetration of wind and solar power – and the balance of the consumption/losses/generation has become even more challenging.

The question is thus: how is the power balanced during the inevitable time that elapses from the load variation and the action of the fastest control? Loads have actually to 'wait' for their demand to be satisfied. Only such a wait is way shorter than what may happen in congested internet and road networks. The physical principle, however, is the same. As soon as the load consumption increases, the extra energy required is harvested from the energy stored in the system itself. In the same vein, if the load consumption decreases, the surplus of energy temporarily available is stored in the system.

It has to be expected that this energy storage process back and forth within the system cannot take too long. In engineering, the concepts of *long* and *short* – as well as those of *big* and *small* – are relative. The time scale of the phenomenon considered here is in the range of a few hundred milliseconds up to a few seconds. This is the equivalent in power systems of the several minutes that the download of a large film from the internet can take and the hours that a traffic jam can last. If the frequency control does not intervene within a few seconds, then the electric system can face serious stability issues and even collapse. The blackout in Europe in 2006 is a good example of what happens if the frequency control fails [28].

So far, we have focused only on the instantaneous power balance. Power system dynamics and operation, however, span several time scales, from milliseconds to years. Depending on the time scale, different devices and/or controllers take over the role of handling the energy storage. Storage is crucial for every time scale. Without storage, the whole system would crash as a house of cards.

The centrality of the role of energy storage is the common thread of this book. Let us illustrate this concept with some examples, as follows:

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1 ms	· alactromogna	tio transiant	ot a long	tronomicción lina
1 1115	. CICCHOHIA2HC	не паняет	OI = a = OII P	transmission line.

10 ms : charge and discharge of the capacitor on the DC side of an AC/DC converter.

100 ms : transient response of the DC exciter of a large synchronous generator.

1 s : electromechanical oscillations of synchronous machines.

10 s : primary frequency control of synchronous machines.

100 s : secondary frequency control of synchronous machines.

1 h : load levelling through pumped hydroelectric power plants.

These examples span relevant time scales of power system dynamics, control and operation.

1.2.1 Example 1: Storage in Transmission Lines

Three-phase AC transmission systems are huge RLC circuits where resistances, inductances and capacitances are due to, respectively, transmission losses; conductor inductive effects and transformer flux leakages and magnetisation; and overhead line and cable parasite capacitive charging.

This example discusses the ability of transmission lines to store energy and release it to the load during a voltage dip. Let us consider the simplified lumped π -model of the transmission line shown in Figure 1.1 (see Section 4.3.1 for further details on the model of the transmission system).

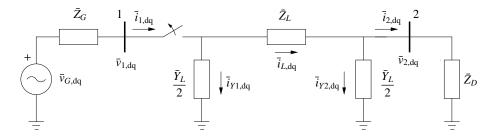


Figure 1.1 Simplified transmission system including a voltage source that feeds a load through an overhead transmission line.

This first example is rather unconventional as the primary purpose of three-phase AC circuits in general and of transmission lines in particular is to transmit energy from the generators to the loads, not to store such energy. As a matter of fact, transmission lines are not the best energy storage systems as we show below. It is considered important, however, to provide a quantitative example and show the effective time scale of the discharge of a line. We trust that associating 'numbers' to the concepts is useful to avoid gross misunderstandings and have a better idea of the actual behaviour of the electrical system and its components.

Length, ℓ	R_L	X_L	$G_L/2$	$B_L/2$
[km]	[Ω]	[Ω]	[µS]	[µS]
200	3.64573	73.74022	0.1260473	449.89134
400	6.80630	142.59986	1.0503017	915.21820
600	9.04484	202.02055	3.8012269	1,413.60348
800	10.01682	248.96718	9.9799844	1,967.73599

Table 1.1 Equivalent π -circuit parameters per phase for long transmission lines at 60 Hz [11].

The parameters of the line are indicated in Table 1.1 and represent the equivalent π -model parameters for a standard long transmission line [11]. The load connected to bus 2 is assumed to be purely resistive, i.e. $\bar{Z}_D = R_D$ and consumes the rated current at the nominal voltage. Finally, the source is a constant 550 kV, 60 Hz voltage generator and is assumed to be the phase reference:

$$\bar{v}_{G,dq} = \frac{\sqrt{2}}{\sqrt{3}} 550 + j0 \text{ kV},$$
 (1.1)

where the dq-axis magnitude is assumed to be the peak value of the Root Mean Square (RMS) phase-to-neutral voltage and the equivalent Thévenin equivalent impedance per phase of the source is set as:

$$\bar{Z}_G = 10 \,\Omega/\text{phase} \,. \tag{1.2}$$

The equations of the circuit in dq-frame coordinates rotating at ω_o are:

$$\begin{aligned} 0 &= \bar{v}_{G,dq} - R_G \,\bar{i}_{1,dq}(t) - \bar{v}_{1,dq}(t) \\ 0 &= \bar{i}_{1,dq}(t) - \bar{i}_{Y1,dq}(t) - \bar{i}_{L,dq}(t) \\ 0 &= 0.5 \, B_L \left(\frac{d}{dt} + j\omega_o \right) \,\bar{v}_{1,dq}(t) + 0.5 \, G_L \,\bar{v}_{1,dq}(t) - \bar{i}_{Y1,dq}(t) \\ 0 &= \bar{v}_{1,dq}(t) - \bar{v}_{2,dq}(t) - R_L \,\bar{i}_{L,dq}(t) - X_L \left(\frac{d}{dt} + j\omega_o \right) \,\bar{i}_{L,dq}(t) \\ 0 &= \bar{i}_{L,dq}(t) - \bar{i}_{Y2,dq}(t) - \bar{v}_{2,dq}(t) \\ 0 &= 0.5 \, B_L \left(\frac{d}{dt} + j\omega_o \right) \,\bar{v}_{2,dq}(t) + 0.5 \, G_L \,\bar{v}_{2,dq}(t) - \bar{i}_{Y2,dq}(t) \\ 0 &= \bar{v}_{2,dq}(t) - R_D \,\bar{i}_{2,dq}(t) , \end{aligned}$$

where

$$\bar{Z}_L = R_L + jX_L
\bar{Y}_L = G_L + jB_L ,$$
(1.4)

are the series impedance and the shunt admittance, respectively, of the transmission line (see also Section 4.3.1.1), and $\omega_o = 2\pi f_o = 2\pi 60 = 377$ rad/s is the fundamental angular speed of the system.

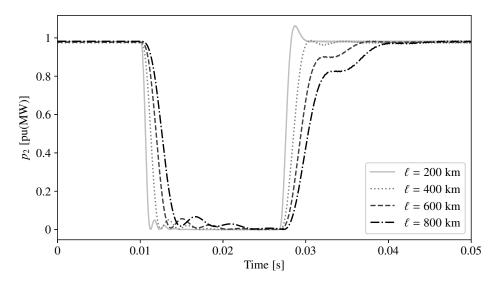


Figure 1.2 Effect of the line length on the power consumed by the load connected to bus 2 following the disconnection of the source at t = 0.01 s during 1 cycle. ℓ is the length of the transmission line. The power base is 500 MVA.

Figure 1.2 shows the dynamic behaviour of the power consumed by the load connected to bus 2 after the disconnection of the source at t = 0.01 s during 1 cycle, namely 16.7 ms at 60 Hz. As expected, the transmission line, even if very long, is not able to sustain the power of the load with the energy stored in its equivalent impedance and capacitance. The discharge of the line stored energy lasts, in the best scenario, a few milliseconds. Despite its fast discharge rate, this energy prevents the power instantaneously dropping to zero during the transient and is thus useful to reduce the effects of micro imbalances in the load and 'helps', along with the energy stored in machine fluxes, cope with the very first instants after a disturbance.

1.2.2 Example 2: Capacitive Storage

The RC circuit shown in Figure 1.3 includes an electrostatic storage. When the switch is closed, the equation that models the circuit is:

$$C \frac{d}{dt}v(t) = \frac{1}{R_1} \left(v_o - v(t) \right) - \frac{1}{R_2} v(t) .$$
(1.5)

When the switch is open, one has:

$$C \frac{d}{dt}v(t) = -\frac{1}{R_2}v(t)$$
. (1.6)

As long as the switch is closed, the capacitor stores energy in the form of electric charge on its plates, whereas it releases energy when the switch is open. This concept is wellknown to any first year engineering student, of course. However, it is important to emphasise the role of the capacitor and, hence, of the energy storage in the circuit. If the circuit did not include the capacitor, during the dip the power delivered to the resistance R_2 would be null during the voltage dip. The capacitor can release its stored energy and, hence, the current flowing in the resistance R_2 is non-null when the voltage source is disconnected, provided that the capacitance is *big enough* and the dip duration is *short enough*.

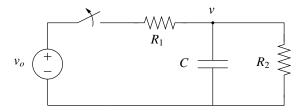


Figure 1.3 Simple RC circuit with a constant voltage source v_o and a switch that interrupts the supply.

The RC circuit shown in Figure 1.3 can model the capacitor on the DC side of an AC/DC converter. Such a capacitor is required to reduce the voltage ripple due to the converter switching and provide fault-ride-through capability during at least half a cycle of the AC fundamental frequency. For a converter with rated power $p_n = 1$ MW and voltage $v_n = 800$ V, the capacitor is of the order of tens of millifarads.

In power converters, the DC capacitor is designed for either (i) reducing the ripple due to the converter switching; or (ii) enabling a fault-ride-through capability when operating as a current source converter (CSC). Generally criterion (ii) provides larger values for the capacitance.

For the fault-ride-through design, typical hypotheses are that the converter is operating as CSC and that it should at least withstand a voltage dip of 100% for a duration of half a cycle, e.g. $t_{1/2} = 10$ ms. It is assumed that, before the voltage dip, the DC bus voltage is the rated one and that the converter is delivering its rated power. During the dip, the voltage on the DC bus is entirely sustained by the DC capacitor until a minimum voltage value, v_{dc}^{min} , below which the converter is disconnected.

The assumptions above lead to the following constraint:

$$p_n t_{1/2} = \frac{1}{2} C \left(v_n^2 - v_{\rm dc}^{\rm min^2} \right), \qquad (1.7)$$

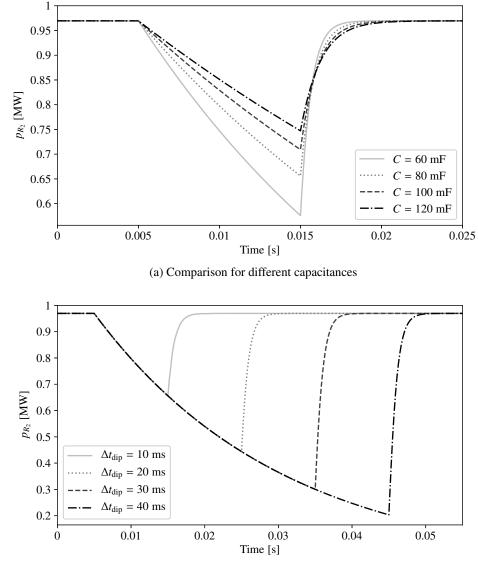
and, substituting the values above for p_n , v_n and $t_{1/2}$, and assuming that $v_{dc}^{min} = 600$ V, (1.7) gives:

$$10^6 \cdot 0.01 = \frac{1}{2}C(800^2 - 600^2),$$

which leads to $C = 7.14 \cdot 10^{-2}$ F.

Back to the circuit of Figure 1.3, assume that the resistance R_2 models a load that consumes 1 MW at the rated voltage, i.e. $R_2 = v_n^2/p_n = 0.64 \Omega$; and $v_o = v_n$ and $R_1 = 1 \Omega$ is the Thévenin equivalent voltage and resistance, respectively, of the DC voltage source that feeds the load. Finally, assume that the time during which the switch is open, namely Δt_{dip} , is a multiple of half a cycle of a 50 Hz system.

Figure 1.4a shows the power dissipated by R_2 during the transient for different values of the capacitance C and same duration $\Delta t_{dip} = 10$ ms of the voltage dip. Figure 1.4b shows the power dissipated by R_2 during the transient for a fixed value of the capacitance C = 80 mF and different lengths Δt_{dip} of the voltage dip.



(b) Comparison for different voltage dip durations

Figure 1.4 Power dissipated in the resistance R_2 as a function of: (a) the value of the capacitance *C* following a voltage dip of duration $\Delta t_{dip} = 10$ ms starting at t = 5 ms; and (b) the duration of the voltage dip Δt_{dip} for a capacitor C = 80 mF. The voltage dip is caused by the opening of the switch occurring at t = 5 ms.

The key aspects of these otherwise quite expected results are the *size* of the capacitance and the *duration* of the dip. It is clear that the power in the resistance R_2 will not drop to zero only if the dip lasts for a sufficiently short time and/or the capacitance is sufficiently big. Another fundamental parameter that decides the response of the circuit is the *initial charge* of the capacitor. The simulation results shown in Figure 1.4 have been obtained assuming that the capacitance is fully charged at t = 0, and hence the energy initially stored in the capacitance is

$$E_{\mathrm{C},o} = \frac{1}{2} C \frac{R_2^2}{(R_1 + R_2)^2} v_o^2 \approx \frac{1}{2} C v_n^2 , \qquad (1.8)$$

where $\frac{R_2}{R_1+R_2}v_o \approx v_n$ is the voltage on the capacitor in steady state with the switch closed, as can be deduced from (1.5). For example, assuming C = 80 mF, $E_{C,o} = 24.8$ kJ, i.e. about 2.5% of the energy consumed by the load in 1 s. It is also relevant to note that C and R_2 are directly and inversely proportional to the rated power of the converter, respectively, so the time constant and, hence, the response time of the circuit does not actually vary.

We have chosen the parameters of the circuit to show a realistic transient behaviour of the capacitor connected to the DC side of an AC/DC converter on purpose. The goal is to remove any doubt about the fact that the energy storage capability of these devices alone is limited both in terms of duration (about a cycle) and in terms of energy (a small percentage of the rated converter capacity). The need for dedicated energy storage devices to be connected to the DC side of the converters is thus apparent.

1.2.3 Example 3: Inductive Storage

Let us consider a dual example with respect to the RC circuit above. The RL circuit shown in Figure 1.5 includes magnetic storage in the form of a conventional iron-core winding. When the switch is open, the inductance stores magnetic energy in its core according to the equation:

$$L\frac{d}{dt}i(t) = R_1(i_o - i(t)) - R_2 i(t) .$$
(1.9)

When the switch is closed, the circuit is described by the following equation:

$$L\frac{d}{dt}i(t) = -R_2 i(t) , \qquad (1.10)$$

and the inductance prevents the current that flows in the resistance R_2 from dropping instantaneously by releasing the stored magnetic energy.

Let us assume that the circuit represents the main exciter of a round rotor synchronous generator. The machine nominal values are $s_n = 588$ MVA, $p_n = 500$ MW, $v_n = 21$ kV, and $f_n = 50$ Hz. The exciter nominal current and voltage are $i_n = 6,300$ A and $v_n = 600$ V, respectively. The inductance of an exciter of this size is typically around a few hundred mH.

No load is generally connected to the exciter but, for the sake of example, let assume that R_2 represents a resistive load that consumes the rated active power of the generator,

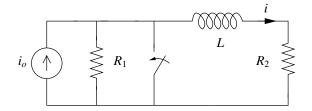


Figure 1.5 Simple RL circuit with a constant current i_o and a switch that interrupts the supply.

i.e. $R_2 = p_n/i_n^2 = 12.6 \ \Omega$,¹ and $R_1 = 0.05 \ \Omega$ and $i_o = v_n/R_1$ are the equivalent Norton resistance and current, respectively, of the source that feeds the exciter. The source i_o is chosen so that the current initially circulating in the inductance *L* is approximately i_n .

Figure 1.6a shows the transient behaviour of the power in the resistance R_2 for different values of the inductance L; whereas Figure 1.6b shows the transient behaviour of the power in the resistance R_2 for different values of the duration of the current sag Δt_{sag} . The two plots are dual to those shown in Figure 1.4. The initial energy in the inductance is:

$$E_{\mathrm{L},o} = \frac{1}{2} L \frac{R_1^2}{(R_1 + R_2)^2} \, i_o^2 \approx \frac{1}{2} L \, i_n^2 \,, \tag{1.11}$$

where $\frac{R_1}{R_1+R_2}i_o \approx i_n$ is the current flowing in the resistance R_2 in steady state when the switch is open, as can be deduced from (1.9). In nominal conditions, $E_{L,o} = 11.9$ MJ for L = 600 mH, which is about 2.2% of the energy generated by the synchronous machine in 1 s.

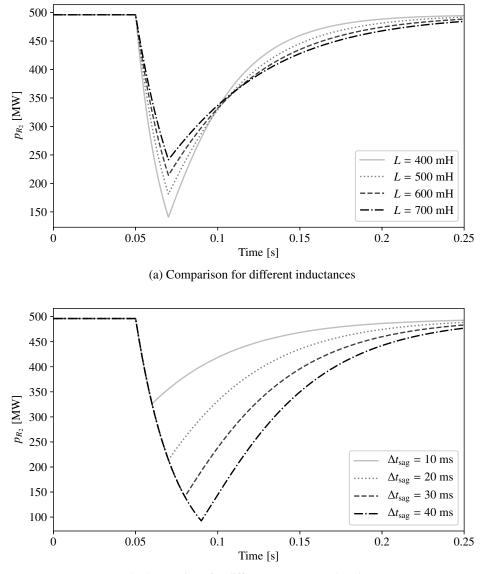
The response time of the inductor is about an order of magnitude slower than that of the capacitor discussed in the previous example. Even so, the power delivered by the inductance lowers between 50 and 70% in 20 ms, i.e. one cycle at 50 Hz, depending on the size of L and thus its capacity is clearly too small to be usable as energy storage. Fortunately, synchronous machines have other storage resources, such as inertia, spinning reserve and the prime mover, which are discussed in the Examples 4 to 6 below.

1.2.4 Example 4: Synchronous Machine Inertia

Starting with this example, we discuss a series of simulations based on the Western Systems Coordinating Council (WSCC) 3-machine 9-bus system, which is a well-known and widely utilised IEEE benchmark system and whose scheme is shown in Figure 1.7. The complete set of data can be found in Appendix D.

This first example considers exclusively the effect of the inertia of synchronous machines. No primary controllers are included and synchronous machines are modelled with their classical electromechanical model. Loads are modelled as constant impedances. The base-case operating point is the same as that described in [12].

¹ The resistance of the coil is about $v_n/i_n = 0.095 \Omega$ and is thus negligible in this example.



(b) Comparison for different current sag durations

Figure 1.6 Power dissipated in the resistance R_2 as a function of: (a) the value of the inductance L following a current sag of duration $\Delta t_{sag} = 0.02$ s starting at t = 0.05 s; and (b) the duration of the current sag Δt_{sag} for a capacitor L = 600 mH. The current sag is caused by the closure of the switch at t = 0.05 s.

According to these assumptions, the only state variables are the rotor angles, $\delta_{r,i}$, and rotor angular speeds, $\omega_{r,i}$, of the synchronous machines. In per unit, the classical equations of the *i*th machine can be written in the following compact form: