Fast Track to Forcing

MIRNA DŽAMONJA

London Mathematical Society
Student Texts **98**

LONDON
MATHEMATICAL
SOCIETY
EST. 1865

CAMBRIDGE

LONDON MATHEMATICAL SOCIETY STUDENT TEXTS

Managing Editor: Ian J. Leary,
Mathematical Sciences, University of Southampton, UK

58 Computational algebraic geometry, HAL SCHENCK
59 Frobenius algebras and 2D topological quantum field theories, JOACHIM KOCK
60 Linear operators and linear systems, JONATHAN R. PARTINGTON
61 An introduction to noncommutative Noetherian rings (2nd Edition), K. R. GOODEARL & R. B. WARFIELD, JR
62 Topics from one-dimensional dynamics, KAREN M. BRUCKS & HENK BRUIN
63 Singular points of plane curves, C. T. C. WALL
64 A short course on Banach space theory, N. L. CAROTHERS
65 Elements of the representation theory of associative algebras I, IBRAHIM ASSEM, DANIEL SIMSON & ANDRZEJ SKOWROŃSKI
66 An introduction to sieve methods and their applications, ALINA CARMEN COJOCARU & M. RAM MURTY
67 Elliptic functions, J. V. ARMITAGE & W. F. EBERLEIN
68 Hyperbolic geometry from a local viewpoint, LINDA KEEN & NIKOLA LAKIC
69 Lectures on Kähler geometry, ANDREI MOROIANU
70 Dependence logic, JOUKU VÄÄNÄNEN
71 Elements of the representation theory of associative algebras II, DANIEL SIMSON & ANDRZEJ SKOWROŃSKI
72 Elements of the representation theory of associative algebras III, DANIEL SIMSON & ANDRZEJ SKOWROŃSKI
73 Groups, graphs and trees, JOHN MEIER
74 Representation theorems in Hardy spaces, JAVAD MASHREGHI
75 An introduction to the theory of graph spectra, DRAGOŠ CVETKOVIĆ, PETER ROWLINSON & SLOBODAN SIMIĆ
76 Number theory in the spirit of Liouville, KENNETH S. WILLIAMS
77 Lectures on profinite topics in group theory, BENJAMIN KLOPSCH, NIKOLAY NIKOLOV & CHRISTOPHER VOLL
78 Clifford algebras: An introduction, D. J. H. GARLING
79 Introduction to compact Riemann surfaces and dessins d'enfants, ERNESTO GIRONDO & GABINO GONZÁLEZ-DIEZ
80 The Riemann hypothesis for function fields, MACHIEL VAN FRANKENHUIJSEN
81 Number theory, Fourier analysis and geometric discrepancy, GIANCARLO TRAVAGLINI
82 Finite geometry and combinatorial applications, SIMEON BALL
83 The geometry of celestial mechanics, HANSJÖRG GEIGES
84 Random graphs, geometry and asymptotic structure, MICHAEL KRIVELEVICH et al
85 Fourier analysis: Part I - Theory, ADRIAN CONSTANTIN
86 Dispersive partial differential equations, M. BURAK ERDOĞAN & NIKOLAOS TZIRAKIS
87 Riemann surfaces and algebraic curves, R. CAVALIERI & E. MILES
88 Groups, languages and automata, DEREK F. HOLT, SARAH REES & CLAAS E. RÖVER
89 Analysis on Polish spaces and an introduction to optimal transportation, D. J. H. GARLING
90 The homotopy theory of $(\infty,1)$-categories, JULIA E. BERGNER
91 The block theory of finite group algebras I, M. LINCKELMANN
92 The block theory of finite group algebras II, M. LINCKELMANN
93 Semigroups of linear operators, D. APPLEBAUM
94 Introduction to approximate groups, M. C. H. TOINTON
95 Representations of finite groups of Lie type (2nd Edition), F. DIGNE & J. MICHEL
96 Tensor products of C^*-algebras and operator spaces, G. PISIER
97 Topics in cyclic theory, D. G. QUILLEN & G. BLOWER

London Mathematical Society Student Texts 98

Fast Track to Forcing

MIRNA DŽAMONJA
Associate Member
IHPST – Institute for the History and
Philosophy of Science and Technology
CNRS – Université Panthéon-Sorbonne
Paris, France

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre,
New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of
education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108420150
DOI: 10.1017/9781108303866

First published 2021

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Names: Džamonja, Mirna, 1965– author.
Title: Fast track to forcing / Mirna Džamonja.
Description: New York : Cambridge University Press, 2020. | Series: London
Mathematical Society student texts ; 98 | Includes bibliographical
references and index.
Identifiers: LCCN 2020018740 | ISBN 9781108420150 (hardback)
Subjects: LCSH: Forcing (Model theory) | Axiomatic set theory.
Classification: LCC QA9.7 .D93 2020 | DDC 511.3/4–dc23
LC record available at https://lccn.loc.gov/2020018740

ISBN 978-1-108-42015-0 Hardback
ISBN 978-1-108-41314-5 Paperback

Dedicated to Jean–Marc Vanden–Broeck, for his never–ending support in all my mathematical adventures.

Contents

Preface

The idea of this book grew out of the lecture notes for a first-year postgraduate module I taught for the MAGIC network of universities in Britain in 2009–2013. Part One of the book roughly corresponds to what was taught there, while the second part is new material. The MAGIC network, at the time of the writing of this book, consists of 21 UK universities and offers postgraduate courses to students enrolled for a Ph.D. in Mathematics, with the idea of providing a quick access to a subject to students who specialise in subjects away from it. The set theory module had ten one-hour lectures. The challenge, for me, was to start these students from the ground zero in set theory and present the subject up to and including the method of forcing. The module proved very popular not only with the students over the remote network but also with some unexpected listeners, such as Ms Eva Roberts, who was the excellent technology assistant responsible for the technical aspects of our video conferencing system, and who proved to be a very interested and informed member of the set theory audience! They encouraged me to make the material available to broader audience.

This book does not really aim to make a set theorist out of you, but it might happen if you read everything. If you only read the first part, it will not even make you competent enough to pass a serious first-year graduate module in set theory in a mathematical logic department. For this you will have to start by consulting one of the classic references, such as [51], [63] (this one being my personal favourite), or a more recent [48]. You will have to work through a large number of exercises and read and reproduce many proofs. There is no fast track to this. However, this book, which does not have a single exercise and skips many proofs, will help you inform yourself of this exciting area of research, unjustly considered too complex to be explained to an interested uninformed listener. If you are a mathematician, it may be that this knowledge will influence you to see some foundational aspects in your own work. If you are a budding set theorist then this book will allow you to plunge into those more serious references with confidence–they are not a particularly easy read if you have not seen any of this material before! And if you are not a mathematician at all, this book will form part of your general culture, somewhat different than the usual aspects of it - but if one can learn Marcel Proust by heart in order to cite him at parties, why not some more esoteric stuff such as foundations of mathematics!

At some point though, maybe you have read the first part of the book, cited foundations of mathematics at many parties and started being bored by the parties. You want to learn more forcing because you fell in love with it! Part Two is made for you. It will tell you much more about forcing from the time it was invented to now. It will tell you about the successes and the challenges and it will tell you about many open questions. It aims to share with you what only the cognoscenti seem to know: combinatorial set theory did not die with the invention of forcing. It was reborn. I hope that some of my colleagues in set theory will find a few interesting sentences in this part of the book, as some of it is rather new material and some not yet published.

And now for Something Completely Different

Finally, you, the reader who goes to parties and cites Proust, know that there is another reason I wrote this book. Many years ago, when I was a finishing undergraduate student in the city of Sarajevo, in what is now Bosnia and Herzegovina and then was Yugoslavia, I was supposed to produce a 4th year thesis on a topic of my choice. I chose forcing, obtained a book on forcing (in this instance, Kunen's book [63] that I recommended you above) and tried to read it on my own. It was impenetrable to me, none of my teachers knew forcing and could not help me, and I got seriously stuck on the exercises in Chapter II. Although I knew a lot of classical set theory, I did not know any logic and it was blocking me, but I did not understand that this was what was blocking me. In fact, probably the most serious problem was that I could follow the arguments line-by-line, but I did not have any intuition. At any rate, I wrote my 4th year thesis on a different topic (Category Theory) and was fortunate enough to be accepted as a graduate student at the University of Wisconsin-Madison and be supervised by Prof. Kenneth Kunen himself. His wonderful graduate course complemented his book, just as it was intended in his writing, and I finally broke through to become an insider. Since that time I have lived in New York, London and Paris and most importantly if one is a set theorist, in Jerusalem, but I have never forgotten how difficult it is to become an insider of set theory if one lives in a place where there are no others to talk to about the subject.

I like to share. I have taught logic and set theory in many places, including the unforgettable African Institute of Mathematical Sciences in Mbour, Senegal, where I had the privilege to address the students chosen from 17 different African countries, for many of whom this was the first time they had seen my subject. Seeing their smiles at the end of every lecture, knowing that we have shared a previously unknown secret, gives a unique thrill.

I wrote this book as if I were teaching to such an audience. I hope the book will help you get an intuition for set theory and I hope you will enjoy it.

I would like to gratefully acknowledge the support of the School of Mathematics at the University of East Anglia (UEA) in Norwich, UK, where I was a Professor of Mathematics at the time of writing this book, and of the scientific consulting company Logique Consult in Paris, where I am the CEO. All my thanks are due to my former Ph.D students Dr Omar Selim and Dr Francesco Parente, in chronological order, for their help with various mathematical and technical aspects of this book. Friends and students helped me with proofreading, among them my student Dr Cristina Criste, UEA Ph.D. student Mark Kamsma and my colleagues Prof. David Buhagiar from the University of Malta and Prof. Lorenz Halbeisen from ETH in Zurich. Prof. Uri Abraham from the Ben-Gurion University of the Negev in Beer Sheva pointed out oversimplifications in a previous representation of Theorem 7.8.3. The Cambridge University Press team have been wonderful and I am really grateful to Roger Astley for his enthusiasm for the concept; Roger, Clare Dennison and Anna Scriven for various aspects of the editing process and the copy editor Jon Billam for his informed and interesting comments. Thank you all very much!

PART ONE

LET'S BE INDEPENDENT