Strength of Materials

Fundamentals and Applications

T. D. Gunneswara Rao
Mudimby Andal

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Strength of Materials

While designing engineering structures, whether they are supporting girders, shock absorbers, or wings of an aircraft, an understanding of structural behavior and the influence of stresses is necessary. Written with a distinct approach of explaining concepts through solved problems this text discusses all fundamental concepts of the strength of materials including stress, strain, elastic constants, shear force, bending moment, and bending stress.

The study of flexural shear stress, conjugate beam method, method of sections and joints, statically determinate trusses, and thin cylinders is presented in detail with the help of solved numerical exercises. The text also discusses advanced concepts of strength of materials such as shear center, rotating discs, unsymmetrical bending, and deflection of trusses. Designed as a foundation text for undergraduates pursuing courses in civil engineering, mechanical engineering, and metallurgical engineering, this book also has use value for candidates appearing for competitive examinations.

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To
Late T. Venkateswara Sarma and Late T. Anasuyamma
(Parents of T. D. Gunneswara Rao)
and
Late M. S. Krishnamacharyulu and Late M. Varalakshmi
(Parents of Mudimby Andal)

Contents

PREFACE

This book *Strength of Materials: Fundamentals and Applications* is brought to the readers with an aim to provide sufficient information and point out ways this information can be applied to practical problems in the domain of mechanics, as is done by design engineers in real life situations. It is structured in the form of a text book for undergraduates pursuing civil engineering, mechanical engineering and metallurgical engineering. Our experience in teaching 'strength of materials' over the past 30 years finds its place in this book. Many questions frequently raised by our students, and are also common problems faced by a lot of other students, while attempting to understand the subjects 'strength of materials' or 'mechanics of solids' are addressed in this book in the form of worked out examples and in the detailed treatment of the theoretical aspects.

Each chapter is provided with objectives and these objectives are mapped to the worked-out example problems and the exercise problems. This mapping strategy will also help the teaching faculty in deciding the course objectives and in evaluating the course objectives.

At the end of every chapter, previous GATE examination and UPSC competitive examination objective-type questions are provided with solutions. This book is useful for students preparing for competitive examinations.

The first ten chapters are devoted to understanding the effects of basic structural actions, which would be sufficient for an elementary treatment of the 'strength of materials' course. The remaining seven chapters focus on advanced topics wherein combined structural actions are considered.

Care is exercised so that mistakes or typographical errors are minimized. All the same we request the readers to comment and provide suggestions for improving the next edition of this book.

ACKNOWLEDGEMENTS

We thank God Almighty for giving us the moral support and good health required for bringing this book into existence. We would like to thank each and every person who helped us, directly or indirectly, in the successful completion of this project.

We thank the reviewers for their patience in going through each chapter of the book and providing valuable suggestions for improving the technical aspects as well as readability of the book. We sincerely thank Gauravjeet Singh Reen, Commissioning Editor at Cambridge University Press, who spent a good amount of time making suggestions for editorial improvements and for better appearance of this book.

We are in debt to our research scholars Mallikarjuna Rao, T. Chaitanya, M. Venu, K. V. Ramana and Vinay Kumar who helped us at various stages during the writing of this book.

We cannot find words to describe the blessings and wishes of our family members, without whose support we would not have completed this book.

Last but not least, thanks are due to T. A. Kamakshi, a student of final year B. Tech, Civil Engineering, who acted as a critical reader for each and every worked-out example and unsolved problem, suggesting modifications wherever required.

CHAPTER 1

STRESS–STRAIN

UNIT OBJECTIVE

This chapter provides information about the theory and derivation of formulae for stresses, strains, and deformations. The presentation attempts to help the student achieve the following:

Objective 1: Determine the normal stress and shear stresses.

Objective 2: Determine normal strain and shear strains.

Objective 3: Determine deformations of different structural elements under axial loads.

Objective 4: Calculate the variation in the dimensions caused due to loads.

1.1 INTRODUCTION

The study of strength of materials includes the understanding of internal stresses and deformations of members subjected to external loading. It also includes the study of failure criterion applicable for the solids subjected to loads. The major actions on the bodies subjected to external loading can be considered as axial force, which include axial compression and axial tension, shear force, bending moment, and torsion. Often members are subjected to the mentioned actions either individually or in combined state such as combined bending, torsion, and axial thrust. The internal reactions due to the external forces cannot be visualized, whereas the deformations can be observed, thus can be measured. Hence generally, the failure criterion of a body subjected to external loads depends not only on the internal actions but also on the deformations. To quantify the internal actions due to the above-said forces, the action of the forces and the corresponding deformations are to be studied in detail in the subsequent chapters. The different individual actions on members were presented in Figure 1.1(a)–(e). However, in this chapter concept of internal reactions and their effects will be discussed. In the subsequent chapters, the effects of mentioned individual actions and combined actions will be discussed in detail.

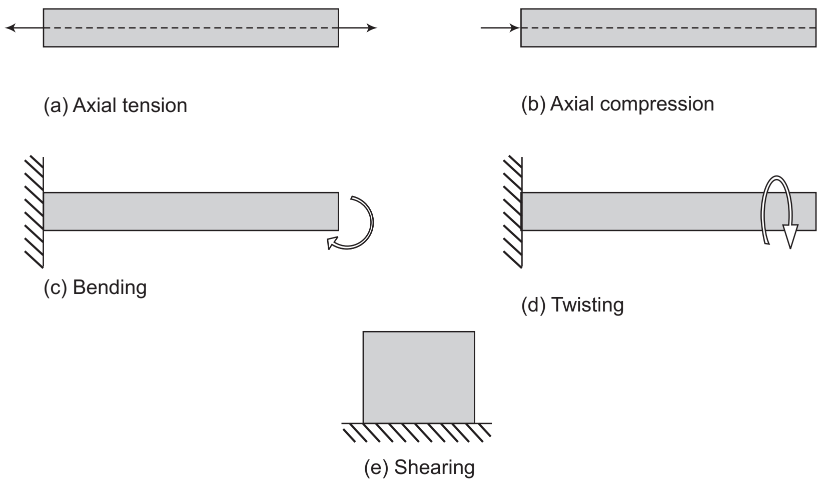


FIGURE 1.1

1.2 NORMAL STRESS AND SHEAR STRESS

Consider a body subjected to several forces and surface tractions as shown in Figure 1.2. Take a section 1-1, to observe the effect of all forces on the section considered. Let '*P*' be the net resultant of the forces. The resistance developed by the body to this resultant at any point within the domain of the body is referred as stress.

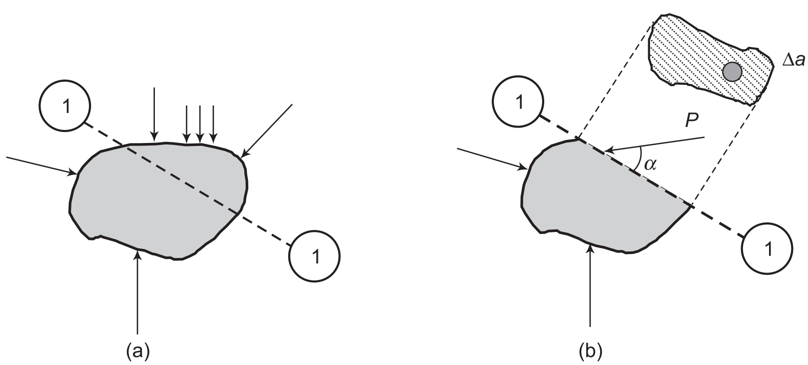


FIGURE 1.2 (a) A body acted upon by external forces; (b) resultant force '*P*' acting on section 1-1.

The resultant of forces acting on one side of the section may be resolved into two components. One component is along the plane $P \cos \alpha$, whereas the other component is perpendicular to the plane $P \sin \alpha$.

Consider an elemental area Δa in the plane 1-1, the internal resistance offered by this elemental area for the normal force $P \sin \alpha$ may be written as $Lt_{\Delta a \to 0} \dfrac{\Delta(P \sin \alpha)}{\Delta a}$. This quantity reduces a particular value called normal stress, as the direction of the component is normal to the plane. The letter σ generally denotes this normal stress.

$$Lt_{\Delta a \to 0} \frac{\Delta(P \sin\alpha)}{\Delta a} = \sigma.$$

Similarly there exists internal resistance in tune of the tangential force $P\cos\alpha$. The resistance to tangential force offered by the elemental area can be written as $Lt_{\Delta a \to 0} \frac{\Delta(P \cos\alpha)}{\Delta a}$. This quantity also reduces to a particular value called shear stress or tangential stress. The letter τ generally denotes this shear stress.

$$Lt_{\Delta a \to 0} \frac{\Delta(P \cos\alpha)}{\Delta a} = \tau.$$

In simple terms, the stress may be defined as the internal resistance offered by a body per unit area.

PROBLEM 1.1 **Objective 1**

Referring to Figure 1.3, determine the normal stress and shear stress induced along the sections 1-1 and 2-2 inclined 30° to the longitudinal axis of the bar of square section 40 mm × 40 mm and length 0.5 m. The axial force acting on the bar is 120 kN.

SOLUTION

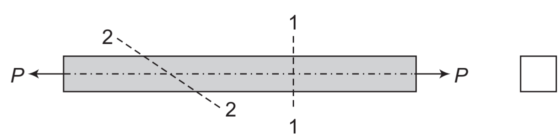


FIGURE 1.3 Bar subjected to axial tension P.

Along section 1-1:

The resultant force normal to the section is = P = 120 kN.

Normal stress at this section $\sigma = \frac{P}{A} = \frac{120 \times 1000}{40 \times 40} = 75$ MPa (tensile stress).

Shear force along the section = 0.

Hence the shear stress at this section is zero.

Along section 2-2:

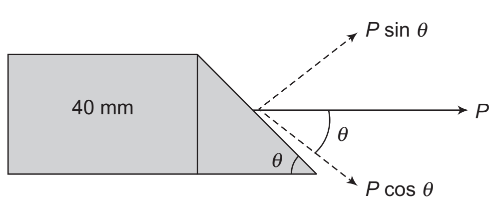


FIGURE 1.4 Internal forces at section 2-2.

The resultant force normal to the section is = 120 sin θ = 120 × sin 30 = 60 kN.

Cross-sectional area of the normal to section 2-2 is 40 × 40/sin 30 = 3200 mm^2.

Normal stress at this section $\sigma = \frac{P}{A} = \frac{60 \times 1000}{3200} = 18.75 \text{ MPa}$ (tensile stress).

Shear force along the section = P = 120 cos θ = 120 × cos 30 = 103.92 kN.

Hence the shear stress at this section $\tau = \frac{103.92 \times 1000}{3200} = 32.48$ MPa.

For a general force system on a body, any plane will carry three stress components due to external loading. Of these three stress components, one is normal stress and the other two are shear stresses. To get the state of stress at a point, we shall represent the stresses over a cube, when this cube reduces to a point, the resulting stresses would be the stresses at point. The possible stresses over such a cube were shown in Figure 1.5.

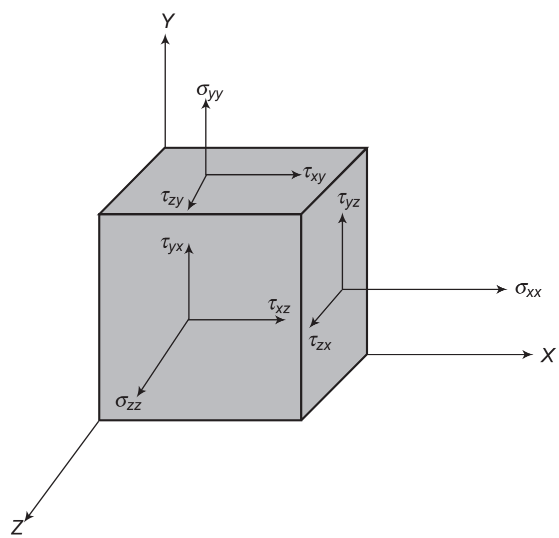


FIGURE 1.5 Possible stresses on a body at a point

The nine stress components are generally represented in a tensor form.

$$[(\sigma)] = \begin{bmatrix} \sigma_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{bmatrix}$$

$\sigma_{\mathbf{xx}}$ = normal stress component acting along x axis on a plane whose normal is along x axis.

$\tau_{\mathbf{yx}}$ = shear stress component acting along y axis on a plane whose normal is along x axis.

Consider a body subjected to pure shear stress τ at top in the horizontal direction of the body shown in Figure 1.6. Let 'a' be the length, 'h' be the height and width of the body perpendicular to the plane of the paper be unit.

Net force $\vec{F}$ acting at the top of the block = $\tau \times a \times 1$.

The resisting force that develops at the base = F (←).

Now the tangential force at top and bottom are same and hence produce a clockwise couple equal to $\tau \times a \times 1 \times h$.

To maintain equilibrium, an anticlockwise couple of same intensity should develop.

The shear stress components (τ_*) developed on orthogonal planes gives anticlockwise couple of $\tau_* \times h \times 1 \times a$.

Equating the clockwise couple to the anticlockwise couple

$$\tau \times a \times 1 \times h = \tau_* \times h \times 1 \times a$$

$$\Rightarrow \qquad \tau = \tau_*$$

in which τ_* is referred as complimentary shear stress.

Thus, complimentary shear stress is always equal to the applied shear stress and acts on a plane orthogonal to the plane in which the applied shear stress acts.

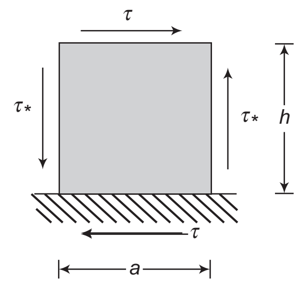


FIGURE 1.6 Shear stress and complimentary shear stress.

1.3 NORMAL STRAIN AND SHEAR STRAIN

Normal stress and shear stress are the internal resistances thus cannot be visualized. The deformations are only the measurable quantities. Thus, normal strain is a measurable deformation parameter corresponding to normal stress and shear strain corresponds to shear stress. Consider the deformations of a block in a plane $ABCD$ shown in Figure 1.7. After the load application, let the deformed shape be $A'B'C'D'$.

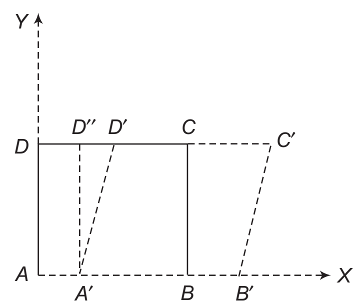


FIGURE 1.7 Deformed shape of a block in a plane.

Change in the length of the part $AB = A'B' - AB$.

Normal strain is defined as the ratio of change in length of a segment to the original length of the segment.

Hence, normal strain along AB is given by $\dfrac{\text{Change in } AB}{\text{Original length of } AB} = \dfrac{A'B' - AB}{AB}$.

The normal strain is denoted by the letter ε. As this quantity is a ratio, strain does not have any units.

Shear strain is defined as the change of angle between two planes due to loading. In Figure 1.7, consider the plane AB and AD. Before the loading, the angle between AB and AD is 90°. After the loading, the included angle between AB and AD reduced by $\angle D''AD'$. This angular change ($\angle D''AD'$) is referred as shear strain, generally denoted by the letter γ. Shear strain also does not have any units like normal strain.

To represent the strain at a point, we shall represent the strains over a cube, as it was done in the case of stresses, when this cube reduces to a point, the resulting strains would be the stain at point.

Thus, the strain tensor at a point can be presented as

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{xx} & \gamma_{yx} & \gamma_{yz} \\ \gamma_{xy} & \varepsilon_{yy} & \gamma_{yz} \\ \gamma_{xz} & \gamma_{yz} & \varepsilon_{zz} \end{bmatrix}$$

in which ε_{xx} is the normal strain along x axis. γ_{yx} is the shear strain or change in the included angle between the planes, which are along y and x axes.

PROBLEM 1.2

Objective 2

Referring to Figure 1.8, determine the normal strain and shear strain along the diagonal AC in a strained body. The strain along X axis is 0.2×10^{-3}. AB = 40 mm and AD = 30 mm. Face AD is fixed.

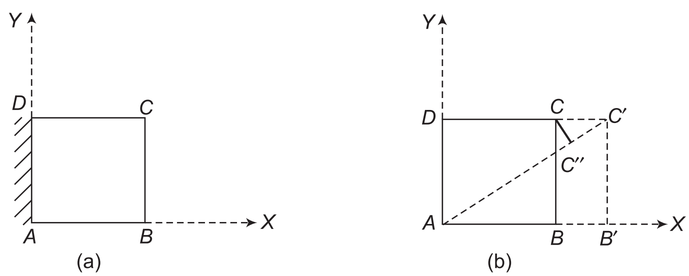


FIGURE 1.8 (a) Block *ABCD* before deformations; (b) deformed configuration of block *ABCD*.

SOLUTION

The deformed configuration of the block *ABCD* is shown as *AB′C′D* to an exaggerated view. Join *AC′*. Draw a perpendicular *C′C″* on to *AC′* from *C*. *AC* is approximately equal to *AC″*.

When the deformations are very small, the following approximation holds good.

$$\angle CAB \approx \angle C'AB = \angle CC'C' = \tan^{-1}\left(\frac{BC}{AB}\right) = 36.87°$$

Normal strain along AC = $\dfrac{\text{Increase in the length } AC}{AC} = \dfrac{C''C'}{AC} = \dfrac{C''C'}{AC''}$

$C''C' = CC' \cos 36.87°$.

$CC' = BB' = \varepsilon_x \times AB = 40 \times 0.2 \times 10^{-3} = 0.008$ mm.

$C''C' = 0.008 \times \cos 36.87° = 0.0064$ mm.

Normal strain along $AC = \dfrac{0.0064}{50} = 0.000128$.

Shear strain between the planes AC and AB is $\gamma = \angle CAC''$.

Shear strain along $AC = \gamma = \dfrac{CC''}{AC} = \dfrac{CC'' \sin 36.87°}{AC} = \dfrac{0.008 \times 0.75}{50} = 0.00012$.

1.4 RELATIONSHIP BETWEEN STRESS AND STRAIN

The stress (may be normal stress or shear stress) is related with the corresponding strain (normal strain or shear strain) in terms of elastic constants called modulus of elasticity and rigidity modulus.

The modulus of elasticity is the ratio of normal stress to normal strain, whereas the rigidity modulus is the ratio of shear stress to shear strain. The units for modulus of elasticity or shear modulus are gigapascal (GPa).

$$\text{Modulus of elasticity } (E) = \frac{\text{Normal stress } (\sigma)}{\text{Normal strain } (\varepsilon)}$$

$$\text{Rigidity modulus } (G) = \frac{\text{Shear stress } (\tau)}{\text{Shear strain } (\gamma)}.$$

These elastic constants are constant for individual materials and largely depend on the crystalline structure, orientation, and bond energies. Table 1.1 gives the details of modulus of elasticity and rigidity modulus of different materials within the elastic limit.

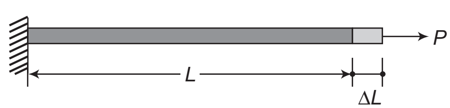
Table 1.1 Modulus of elasticity of different materials

Sl. No.	Material	Modulus of Elasticity (GPa)
1	Steel	200
2	Copper	100–80
3	Aluminum	60–80
4	Concrete	25–35
5	Timber	10–15

1.5 ADDITIONAL PROBLEMS ON DIRECT OR AXIAL STRESSES

PROBLEM 1.3 **Objective ***

Derive an expression for the extension of a prismatic bar subjected to axial tension.



SOLUTION

Normal stress in the bar $\sigma = \dfrac{P}{A}$.

Let ΔL be the extension of the bar.

Then, normal strain in the bar $= \varepsilon = \dfrac{\Delta L}{L}$.

Let E be the modulus of elasticity of the bar.

Then, $E = \dfrac{\text{Normal stress}}{\text{Normal strain}} = \dfrac{P/A}{\Delta L/L}$

$$\Rightarrow \qquad \Delta L = \frac{PL}{AE}$$

Stiffness of the bar is defined as the load required for unit extension. Hence, axial stiffness of the bar generally denoted as 'k' is given by

$$k = \frac{P}{\Delta} = \frac{AE}{L}$$

In the above expression of stiffness, the term 'AE' is referred as axial rigidity. It depends on the cross-section as well as material of the member.

A possible doubt to the reader: If the extension of the bar is ΔL due to P, then for additional load say P' in the expression to determine additional extension $\dfrac{P'L}{\text{AE}}$, should L be used or $L + \Delta L$?

This possible doubt makes the reader to understand many important assumptions to be followed in solid mechanics.

1. **Order of loading should not have any effect on the deformations or internal stresses.** This means that whether P is applied first then P', or P' first then P or P and P' be applied simultaneously should not have any effect on the deformation. This is true for the materials, which follow linear force–displacement relationship. This law of superposition does not hold well in case materials which exhibit nonlinear force–displacement relationship. This can be observed from the figures shown below.

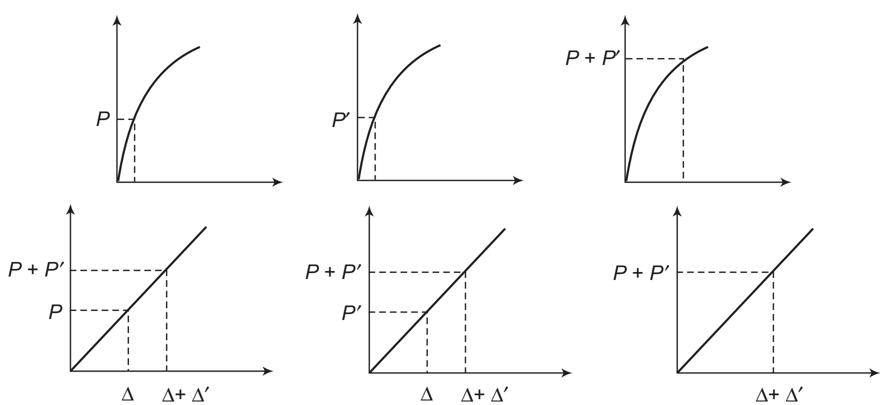


FIGURE 1.9 A case, where law of superposition is valid.

2. **Higher order deformations are neglected.** This means that deformations due to deformation are very small and can be neglected. That is in the axial extension, if $L + \Delta L$ is used in place of L, then $\Delta L + \Delta\Delta L = \dfrac{PL}{AE} + \dfrac{P(\Delta L)}{AE}$. If this is accepted, the next question that may arise is should we use $L + \Delta L + \Delta(\Delta L)$ in place

of L? If we continue like this there will be no end for it. For most of the materials within the working range of loads, $\Delta(\Delta L)$ is very small compared to ΔL, and hence $\Delta(\Delta L)$ can be ignored. This $\Delta(\Delta L)$ is referred as deformation due to deformation or second-order deformation. Thus, in strength of materials the effect of deformations due to deformations or second-order deformations is neglected.

PROBLEM 1.4

Objective 3

Estimate the deformation of points B, C, and D of the compound bar $ABCD$ subjected to loading as shown in Figure 1.10. Take modulus of elasticity of the material as 200 GPa. $P = 10$ kN, section at 1-1 is 50 mm × 50 mm solid, section at 2-2 is hollow section of external dimensions 50 mm × 50 mm and inner dimensions 25 mm × 25 mm, and section at 3-3 is circular section of 40 mm diameter. $AB = 1$ m, $BC = 1.2$ m, and $CD = 1.1$ m.

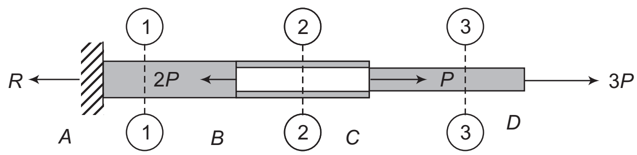


FIGURE 1.10

SOLUTION

Structural systems or components of structural systems must be in equilibrium. This is an essential condition.

Consider the equilibrium of the system.

Thus, sum of the forces in X direction must be equal to zero.

$$\Rightarrow \quad \Sigma F_x = 0$$

$$\Rightarrow \quad 3P + P - 2P - R = 0$$

$$\Rightarrow \quad R = 2P.$$

Consider the free body diagram of the three parts of the $ABCD$ as shown in Figure 1.11.

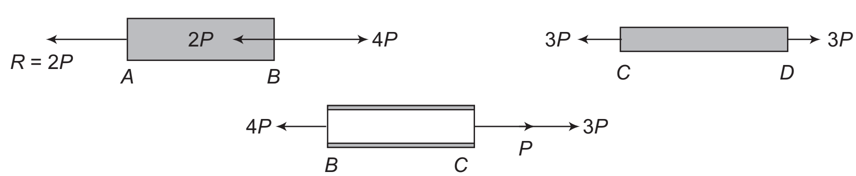


FIGURE 1.11

It is always convenient to draw the free body diagrams from free end of the member. In the portion CD, axial tensile load of $3P$ is acting at the free end. To keep the member CD in equilibrium, there should an axial force of $3P$ acting at C in the opposite direction, that is, toward left. Thus, the portion CD of the member is acted upon by a tensile force of $3P$.

Tensile stress in the region CD of the member = $\sigma_{CD} = \dfrac{3P}{A_{CD}} = \dfrac{3 \times 10 \times 1000}{\dfrac{\pi}{4} \times 40 \times 40} = 23.87$ MPa

Extension in the portion $CD = \Delta_{CD} = \dfrac{\text{Stress}}{\text{Modulus of elasticity}} \times \text{Length } (CD)$

$$\Delta_{CD} = \frac{23.87}{200 \times 10^3} \times 1100 = 0.131 \text{ mm}$$

In the portion CD, at C a tensile load of $3P$ was applied to maintain equilibrium. Hence, a tensile load of $3P$ at C in the portion BC should be applied. Then, apply the load P acting at C in the portion BC. Thus, apply a tensile load of $4P$ at C. Apply a tensile load of $4P$ at B to maintain equilibrium.

Therefore, the portion BC of the member is subjected to a tensile load of $4P$.

Hence, tensile stress in the portion $BC = \sigma_{BC} = \dfrac{4P}{A_{BC}} = \dfrac{4 \times 10 \times 1000}{[50 \times - 50 - 25 \times 25]} = 21.33 \text{ MPa}$.

Extension in the portion $BC = \Delta_{BC} = \dfrac{\text{Stress}}{\text{Modulus of elasticity}} \times \text{Length } (BC)$

$$\Delta_{BC} = \frac{21.33}{200 \times 10^3} \times 1200 = 0.128 \text{ mm}$$

Similarly applying equilibrium for the portion AB, the tensile load acting = $2P$.

Hence, tensile stress in the portion $AB = \sigma_{AB} = \dfrac{2P}{A_{AB}} = \dfrac{2 \times 10 \times 1000}{[50 \times 50]} = 8.00 \text{ MPa}$

Extension in the portion $AB = \Delta_{AB} = \dfrac{\text{Stress}}{\text{Modulus of elasticity}} \times \text{Length } (AB)$

$$\Delta_{AB} = \frac{8.00}{200 \times 10^3} \times 1000 = 0.04 \text{ mm}$$

Finally, the displacement at A = 0.0

Displacement of B = 0.04 mm

Displacement of C = 0.04 + 0.128 = 0.168 mm

Displacement of D = 0.04 + 0.128 + 0.131 = 0.299 mm

Hence, the total extension of the member is 0.299 mm

Maximum tensile stress is 23.87 MPa in the portion CD of the member.

PROBLEM 1.5

Objective 3

Derive an expression for the extension of a conical bar of length L fixed at the base and hanging due to its own weight. Specific weight of the material of the bar is γ and modulus of elasticity is E.

SOLUTION

Let d_0 be the base diameter of the conical bar.

Consider a fiber located at a distant x from the bottom of the bar of thickness Δx.

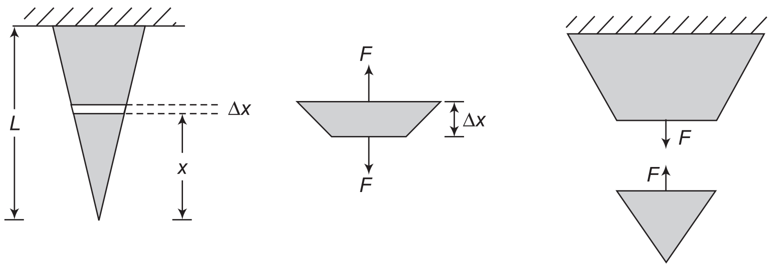


FIGURE 1.12

The free body diagram of the elemental strip of length Δx is shown in Figure 1.12. The force acting on the elemental strip is nothing but the weight of the portion clinging to the section under consideration.

Diameter of the conical bar at the section under consideration $d_x = \dfrac{d_0}{L}x$.

Therefore, weight of the portion of the conical bar up to the section = $F = \dfrac{1}{12}\pi d_x^2 x\gamma$

$$\Rightarrow \qquad F = \frac{1}{12}\pi\gamma\frac{d_0^2}{L^2}x^3.$$

Extension of this elemental strip of length Δx is $\Delta(\Delta L) = \dfrac{F\Delta x}{A_x E}$

$$\Rightarrow \qquad \Delta(\Delta L) = \frac{1}{12E}\pi\gamma\frac{d_0^2}{L^2}x^3 \times \frac{1}{\frac{\pi}{4}\frac{d_0^2}{L^2}x^2} \times \Delta x$$

$$\Rightarrow \qquad \Delta L = \int_0^L \frac{1}{3E}\,\gamma\, x\; dx = \frac{\gamma L^2}{6E}$$

Thus, the extension of conical bar due to its own weight is $\dfrac{\gamma L^2}{6E}$.

Similarly, it can be shown that the extension of a prismatic member (same cross-section throughout the length) hanging on its own weight is $\dfrac{\gamma L^2}{2E}$.

PROBLEM 1.6

Objective 3

Derive an expression for the extension of a tapering bar of length L subjected to an axial pull of intensity P. Modulus of elasticity of the material of the bar is E. The diameter at one end is d_1 and at the other end it is d_2.

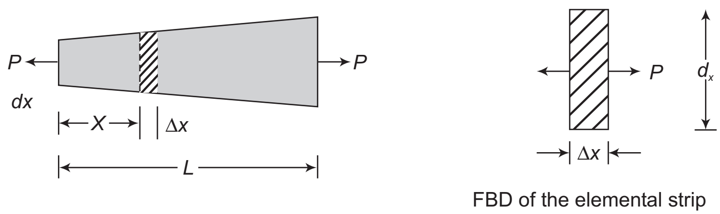


FIGURE 1.13

Consider an elemental strip of length Δx, located at a distant X from the one end of the member. Let d_x be the diameter of the bar located at a distant x from one end of the bar, where the diameter is d_1.

Then, $$d_x = d_1 + \frac{d_2 - d_1}{L} x$$

Consider an elemental strip of length Δx at the section under consideration.

The free body diagram of the elemental strip of length Δx is as shown in Figure 1.13.

Extension of this elemental strip of length Δx is $\Delta(\Delta L) = \dfrac{P\Delta x}{A_x E}$

$$\Rightarrow \quad \Delta(\Delta L) = \frac{P}{E} \times \frac{1}{\frac{\pi}{4} d_x^2} \times \Delta x$$

$$\Rightarrow \quad \Delta L = \int_0^L \frac{P}{E} \times \frac{1}{\frac{\pi}{4}\left[d_1 + \frac{d_2 - d_1}{L} x\right]^2} \times dx$$

$$= \frac{P}{E} \times \left[- \frac{\left(d_1 + \frac{d_2 - d_1}{L} x\right)^{-1}}{\frac{\pi}{4}\left(\frac{d_2 - d_1}{L}\right)} \right]_0^L$$

$$= \frac{PL}{E} \left[\frac{\frac{1}{d_1} - \frac{1}{d_2}}{\frac{\pi}{4}(d_2 - d_1)} \right]$$

$$= \frac{PL}{E\left(\frac{\pi}{4} d_1 d_2\right)}$$

In the above expression if $d_1 = d_2 = d$ then, the expression for extension reduces to the expression for the extension of a solid circular uniform bar $\dfrac{PL}{AE}$.

PROBLEM 1.7

Objective 4

A prismatic solid circular bar of length L is subjected to axial load. This solid bar is bored for a length of $0.5L$, such that the inner diameter of the bored portion is 0.6 times the diameter of the solid portion. Estimate the percentage increase in the extension of the bored bar under same load, when compared with that of the solid prismatic bar.

SOLUTION

Let P be the axial load acting on the bar. Let d be the diameter of the bar.

The extension of the prismatic bar = $\dfrac{4PL}{\pi d^2 E}$.

If half portion of the bar is bored then,

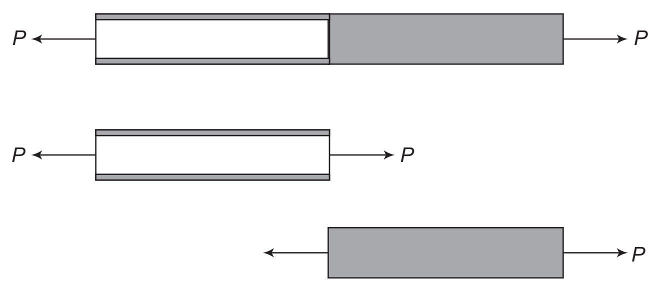


FIGURE 1.14

Free body diagram of the two parts of the bored bar is shown in Figure 1.14.

The extension of the bored half portion of the bar = $\Delta_1 = \dfrac{P(0.5L)}{\frac{\pi}{4}[d^2 - (0.6d)^2 E]}$

$$\Rightarrow \qquad \Delta_1 = \frac{2PL}{0.64\pi d^2 E}.$$

Extension of the remaining half portion = $\Delta_2 = \dfrac{P(0.5L)}{\frac{\pi}{4} d^2 E} = \dfrac{2PL}{\pi d^2 E}$.

The total extension of the bored bar is $\Delta_* = \Delta_1 + \Delta_2 = (1.281)\dfrac{4PL}{\pi d^2 E}$.

Percentage increase in the extension = $\dfrac{\Delta_* - \Delta}{\Delta} \times 100 = 28.1\%$.

PROBLEM 1.8

Objective 4

A 5-kg mass rotates in a horizontal circle with constant angular speed at the end of 1.5 m steel wire such that the steel wire makes 30° with the vertical. Determine the speed, stress in the steel wire due to the rotation, and also evaluate the extension of the steel wire taking E = 200 GPa. Diameter of the steel wire is 0.5 mm.

SOLUTION

Let P be the tension in the steel wire and ω be the angular speed of the steel ball. As the ball is rotating about a fixed vertical axis, the ball is subjected to normal force $\left(m\dfrac{v^2}{r}\right)$ and tangential force $\left(m\dfrac{dv}{dt}\right)$, in which v is the speed of the ball, in the present case this v is constant. Hence, the tangential force acting on the ball vanishes.

Velocity of the ball = $v = r\omega$

If we draw the free body diagram of the ball, the forces acting on that will be

(a) Self-weight of the ball = mg = 5 × 9.81 = 49.5 ≈ 50 N.
(b) Normal force (acting normal to the path) = $mr\omega^2$.
(c) Tension in the steel wire = P inclined 30° to the vertical.

These forces were shown in the free body diagram.

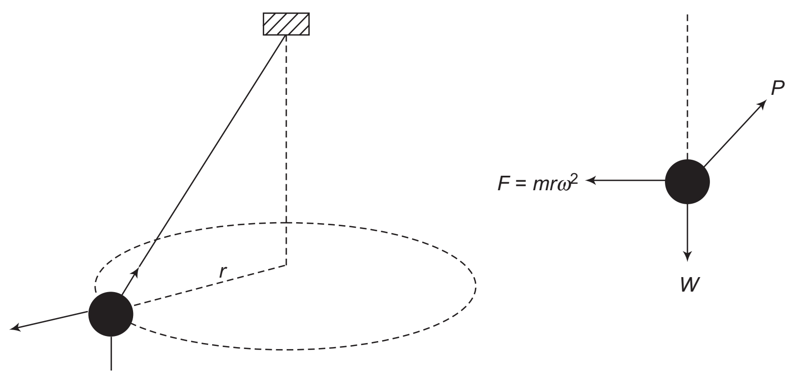


FIGURE 1.15

Resolving the forces vertically,

$$W = P\cos\theta$$

$$\Rightarrow \quad P\cos 30 = 50$$

$$\Rightarrow \quad P = \frac{50}{\cos 30} = 57.735 \text{ N.}$$

$$\Rightarrow \quad \text{Normal stress in the wire } \sigma = \frac{57.735}{\frac{\pi}{4}(0.5)^2} = 294 \text{ MPa.}$$

$$\text{Extension of the steel wire} = \Delta = \frac{\sigma L}{E} = \frac{294 \times 1500}{200 \times 1000} = 2.205 \text{ mm.}$$

To determine the angular speed of the steel ball, resolve the forces horizontally.

$$\Rightarrow \quad P\sin\theta = F = mr\omega^2$$

$$\Rightarrow \qquad \omega^2 = \frac{57.735 \sin 30}{5 \times 1.5 \sin 30}$$

$$\Rightarrow \qquad \omega = 2.775 \text{ radians/s}$$

Angular speed of the ball about the vertical axis is $\dfrac{60\omega}{2\pi} = 26.5$ RPM.

PROBLEM 1.9

Objective 1

A rigid bar of length 0.5 m and negligible weight hangs by means of two wires of length 1.2 m each, as shown in Figure 1.16. If a gravity load of 100 kN is applied at the left middle third point of the rigid bar, determine the stresses in the two wires and the inclination of the rigid bar with the horizontal. The cross-sectional area of aluminum bar is 1000 mm^2 and that of steel bar is 500 mm^2. Modulus of elasticity of steel bar is 202 GPa while that of aluminum is 65 GPa.

SOLUTION

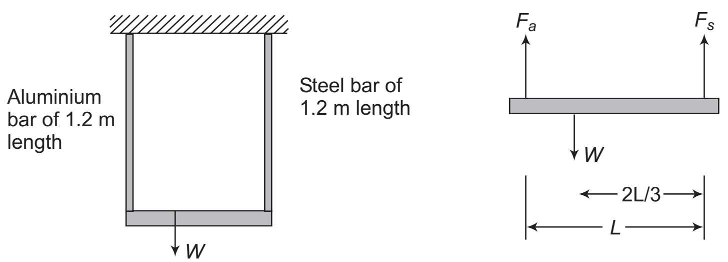


FIGURE 1.16

Sketch the free body diagram of the rigid bar.
Let F_a be the force the aluminum bar and F_s be the force in the steel bar.
Use equilibrium equations to determine the forces F_a and F_s.
That is, sum of the horizontal forces is equal to zero. (Equilibrium equation (1))

$$\Rightarrow \qquad F_a + F_s = W = 100 \text{ kN}.$$

Sum of the moments about any arbitrary point is zero. (Equilibrium equation (2))
Taking moments about a point through which F_a is passing,

$$\Rightarrow \qquad F_s \times L - 100 \times \frac{2L}{3} = 0$$

$$\Rightarrow \qquad F_s = 66.67 \text{ kN}$$

$$\Rightarrow \qquad F_a = 33.33 \text{ kN}.$$

As the bar is rigid, the bar itself will not undergo any deformations, but aluminum and steel bars undergo extensions.

Extension in the aluminum bar $\Delta_a = \dfrac{F_a \times L_a}{A_a E_a} = \dfrac{66.67 \times 1000 \times 1200}{1000 \times 65 \times 1000} = 1.231$ mm

Extension in the steel $\Delta_s = \dfrac{F_s \times L_s}{A_s E_s} = \dfrac{33.33 \times 1000 \times 1200}{500 \times 202 \times 1000} = 0.396$ mm .

From the extensions of steel bar and aluminum bar, it is clear that the left end of rigid bar moves down by 1.231 mm and the right end of the same moves down by 0.369 mm. Thus, rigid bar rotates in the anticlockwise direction.

Let θ be the rotation of the rigid bar.

Then, $\theta = \dfrac{\Delta_a - \Delta_s}{L} = \dfrac{1.231 - 0.396}{500} = 0.00167$ radians.

Inclination of the rigid bar in anticlockwise direction is 0.00167 radians.

PROBLEM 1.10

Objective 3

Estimate the vertical and horizontal deflection at the point C of the two-member truss shown in Figure 1.17. Take E = 200 GPa. Cross-sectional area of each member is 2000 mm^2. AC = 1.5 m; BC = 0.9 m.

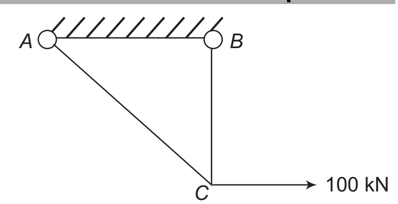


FIGURE 1.17

SOLUTION

The forces in the members of the truss are to be found out using equilibrium equations. The member AC is subjected to tension, whereas BC is subjected to compression.

$$\angle BCA = \theta = \cos^{-1}\left(\frac{0.9}{1.5}\right) = 53.13$$

$\Rightarrow$ $\cos\theta = 0.6$

$\Rightarrow$ $\sin\theta = 0.8$

Resolving the forces horizontally, $F_{AC} \sin\theta = 100$ kN

$\Rightarrow$ $F_{AC} = 125$ kN (tensile).

Resolving the forces vertically, $F_{AC}\cos\theta - F_{BC} = 0$

$\Rightarrow$ $F_{BC} = 75$ kN (compressive).

Normal tensile stress in the bar AC is $\dfrac{125 \times 1000}{2000} = 62.5$ MPa .

Normal compressive stress in the bar BC is $\dfrac{75 \times 1000}{2000} = 37.5$ MPa.

We are required to evaluate the vertical and horizontal deflection of point C. The force in the bar AC moves the point C in the direction of AC, CC_1 as shown in Figure 1.18, whereas the compressive force in the bar BC tries to move the point C in the upward direction along CB, CC_2 as shown in Figure 1.18. Because of this, the final position of the point C will be the meeting point of arcs drawn to the points C_1 and C_2, taking centers as A and B, respectively. As the displacements are negligible, rather than drawing arcs, it is convenient to draw perpendiculars. Thus, draw perpendiculars at C_1 to

AC_1 (CC_1) and C_2 to BC_2 (CC_2). These perpendiculars meet at point C_3. Thus, net movement of the point C will be from C to C_3, as shown in Figure 1.18. The horizontal and vertical projection of CC_3 will be the horizontal and vertical displacement of the point C.

Free extension of $AC = CC_1 = \dfrac{62.5 \times 1500}{200 \times 1000} = 0.469$ mm.

Free extension of $BC = CC_2 = \dfrac{37.5 \times 900}{200 \times 1000} = 0.169$ mm.

Angle $C_2CC_1 = 180° - \theta = 126.87°$.

C_1C_3 is perpendicular to CC_1; C_2C_3 is perpendicular to CC_2; and CC_3 is the net displacement of the point C.

It is clear that $CC_3 = CC_1/\cos\theta_1 = CC_2/\cos\theta_2$. (1)

Moreover,

$$\theta_1 + \theta_2 = 126.87° \tag{2}$$

From equation (1)

$$\frac{\cos\theta_1}{\cos\theta_2} = \frac{1.563}{0.169} = 2.775$$

From equation (2)

$$\frac{\cos\theta_1}{\cos(126.87 - \theta_1)} = 2.775$$

$$\Rightarrow \qquad \theta_1 = 50.2°.$$

Thus, $CC_3 = \dfrac{0.169}{\cos(50.2)} = 0.264$ mm .

Hence, the horizontal displacement of C is $CC_3 \sin\theta_1 = 0.264 \sin\theta_1 = 0.768$ mm. Toward right of C, vertical displacement of the point C is $CC_3 \cos\theta_1 = 0.264 \times \cos(50.2) = 0.169$ mm (upward).

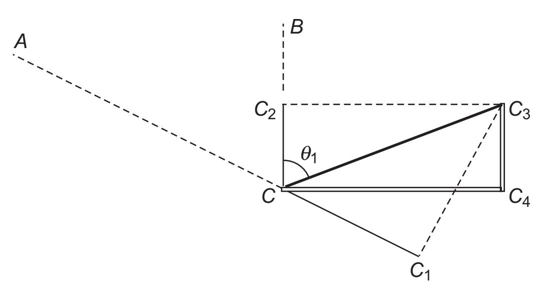


FIGURE 1.18

PROBLEM 1.11 **Objective 3**

A steel bar *AB* of length 3.5 m and diameter 25 mm are connected by four inextensible cables of length 2.5 m each, forming a rhombus with *AB* as diagonal, as shown in Figure 1.19. A 100 kN forces act at the points *C* and *D*. Determine the decrease in the length of the strut *AB* and increase in the length between the points *C* and *D*. Take modulus of elasticity of steel as 201 GPa.

SOLUTION

The cables *CA*, *CB*, *DA*, and *DB* are inextensible means that they do not undergo any deformation but the points *C* and *D* move due to the deformation of the strut *AB*. A strut is a member subjected to axial compression. The axial deformation of the strut is to be found, to determine the displacement between the points *C* and *DE*. Using equilibrium equations, determine the force in the strut *AB*.

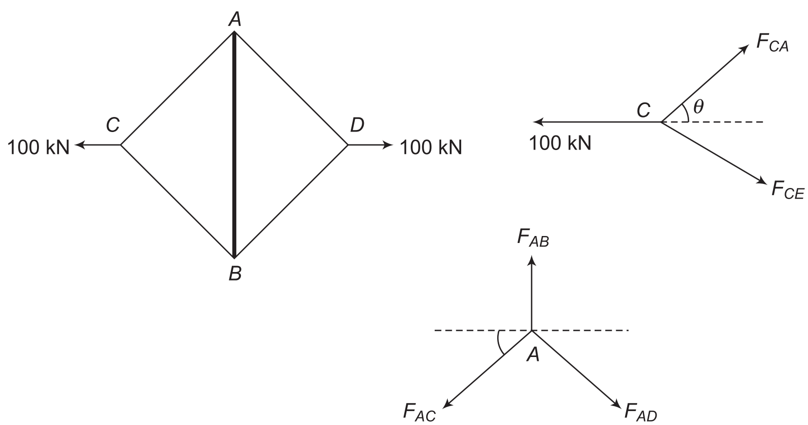


FIGURE 1.19

Let F_{AC} be the force in the cable *AC* and F_{CB} be the force in the cable *CB*. Let θ be the inclination of AC with the horizontal.

$$\sin\theta = \frac{(AB/2)}{AC}$$

$$\Rightarrow \qquad \theta = \sin^{-1}\left(\frac{1.75}{2.5}\right) = 44.43°.$$

Consider the joint equilibrium at *C*, that is, sum of the vertical forces is equal to zero.

$$\Rightarrow \qquad F_{CA}\sin\theta - F_{CB}\sin\theta = 0$$

$$\Rightarrow \qquad F_{CA} = F_{CB}.$$

Sum of the horizontal forces is equal to zero.

$$\Rightarrow \qquad F_{CA}\cos\theta - F_{CB}\cos\theta = 100 \text{ kN}.$$

$$\Rightarrow \qquad F_{CA} = F_{CB} = \frac{100}{2\cos\theta} = 70.01 \text{ kN}.$$

To determine the force in the strut *AB*, consider the equilibrium of the joint *A*.

Sum of the horizontal forces is equal to zero.

$$\Rightarrow \quad F_{CA}\cos\theta - F_{AD}\cos\theta = 0.$$

$$\Rightarrow \quad F_{CA} = F_{AD}\ 70.01\text{ kN.}$$

Sum of the vertical forces is equal to zero.

$$\Rightarrow \quad F_{CA}\sin\theta + F_{AD}\sin\theta = F_{AB}$$

$$\Rightarrow \quad F_{AB} = 2F_{CA}\sin\theta = 98.02\text{ kN.}$$

Axial deformation (compression) of the strut AB is $\dfrac{98.02\times1000\times3500}{\frac{\pi}{4}(25)^2\times201\times1000} = 3.48$ mm.

To determine the displacement between the points C and D, consider the triangle ACD.

$$L_{\mathrm{CD}} = 2\times\left(\frac{AB}{2}\right)\times\tan\theta.$$

Differentiating on both sides

$$\Delta(L_{CD}) = 2\times\tan\theta\times\Delta\left(\frac{AB}{2}\right)$$

$$\Rightarrow \quad \Delta(L_{CD}) = 2\times0.98\times\frac{3.48}{2} = 3.41\text{ mm.}$$

Displacement between the points C and D is 3.41 mm.

PROBLEM 1.12

Objective 4

Two bars of length 1.5 m each are connected to a rigid bar of length L, as shown in Figure 1.20. The axial stiffness of the bar AB is 2.5 N/mm. Determine the axial stiffness as well as axial rigidity of the bar CD, if the rigid bar has to be horizontal, when a load $W = 600$ N is applied at $L/3$ distant from the bar AB.

SOLUTION

Let F_{AB} and F_{CD} be the forces in the members AB and CD. The condition is that the rigid bar should be horizontal. Thus, the extension of the bar AB and CD should be same.

$$\therefore \quad \left(\frac{F_{AB}L}{AE}\right)_{AB} = \left(\frac{F_{CD}L}{AE}\right)_{CD}$$

$\dfrac{AE}{L}$ of member AB is the stiffness of AB and is given by 2.5 N/mm. Axial stiffness member CD is to be determined.

$$\therefore \quad \frac{F_{AB}}{\left(\dfrac{AE}{L}\right)_{AB}} = \frac{F_{CD}}{\left(\dfrac{AE}{L}\right)_{CD}}.$$

F_{AB} and F_{CD} are to be determined from equilibrium equations. Consider the equilibrium of the rigid bar.

That is, sum of the horizontal forces is equal to zero.

$$\Rightarrow \qquad F_{AB} + F_{CD} = W = 600 \text{ N}.$$

Sum of the moments about any arbitrary point is zero.

Taking moments about a point through which F_a is passing

$$\Rightarrow \qquad F_{CD} \times L - 600 \times \frac{L}{3} = 0$$

$$\Rightarrow \qquad F_{CD} = 200 \text{ N}$$

$$\Rightarrow \qquad F_{AB} = 400 \text{ N}$$

$$\therefore \qquad \frac{400}{2.5} = \frac{200}{\left(\dfrac{AE}{L}\right)_{CD}}$$

$$\Rightarrow \qquad \left(\frac{AE}{L}\right)_{CD} = k_{CD} = 1.25 \text{ N/mm}.$$

Axial stiffness of the member CD required is 1.25 N/mm.

Axial rigidity AE of the member $CD = L_{CD} \times k_{CD} = 1.25 \times 1500 = 1875 \text{ N} \cdot \text{mm}^2$.

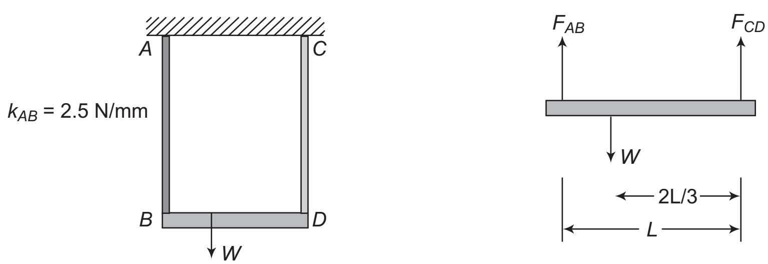


FIGURE 1.20

Statically Indeterminate Structures

Statically indeterminate structures are those which cannot be analyzed with the help of equilibrium equations alone. Most of the structures fall under the category of indeterminate structures. In this section, we consider the analysis of few statically indeterminate structures having axially loaded members. For example, consider a member clamped at both ends subjected to axial forces as shown in Figure 1.21.

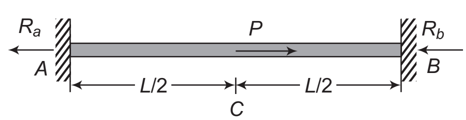


FIGURE 1.21

Let the reactions at A and B, R_a and R_b, respectively, use the condition of equilibrium that sum of the horizontal force is equal to zero.

$$\Rightarrow \qquad R_a + R_b = P. \tag{1.1}$$

The two unknown quantities R_a and R_b cannot be found in equation (1.1), that is, R_a and R_b cannot be determined using equilibrium equation. Thus, this problem falls under the category of statically indeterminate problem. To solve this problem, condition pertaining to deformations shall be used. The conditions pertaining to deformations are called 'compatibility conditions'. In the present example, the compatibility condition is that the total extension of the bar AB between the fixed supports is zero. That is, extension in the portion AC must be equal to the compressive deformation in the portion CB. From this compatibility condition, one more equation can be formed, thus R_a and R_b (two unknowns from two equations) can be determined.

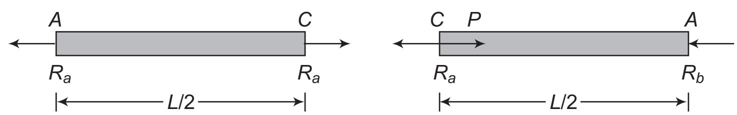


FIGURE 1.22

$$R_b = P - R_a. \tag{1.1}$$

From compatibility condition,

Extension in the bar AC = Compressive deformation in the bar CB

$$\frac{R_a(L/2)}{[AE]_{AC}} = \frac{R_b(L/2)}{[AE]_{CB}} \tag{1.2}$$

From equations (1.1) and (1.2), R_a and R_b can be evaluated.

PROBLEM 1.13

Objective 1

A bar having cross-sectional arcs of 1500 mm^2 is fixed between two rigid walls. Two loads P_1 and P_2 are applied at points C and DE shown in Figure 1.23. Determine the stresses induced in the portions AC, CD, and DB. Take E = 200 GPa; P_1 = 150 kN; and P_2 = 100 kN.

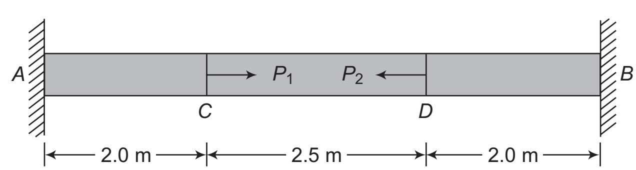


FIGURE 1.23

SOLUTION

Let R_a and R_b be the reactions at A and B, respectively.

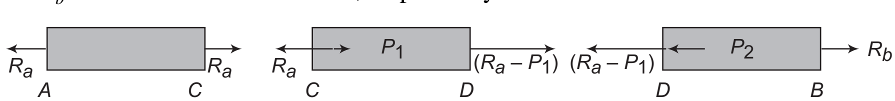


FIGURE 1.24 *FBD* of members *AC*, *CD*, and *DB*.

From the equilibrium condition, the sum of horizontal force is equal to zero.

$$R_a - P_1 + P_2 - R_b = 0 \tag{1.3}$$

$\Rightarrow$ $$R_a - R_b = P_1 - P_2 = 150 - 100.$$

$\therefore$ $$R_a - R_b = 50 \tag{1.4}$$

As per the compatibility condition, total extension between *A* and *B* shall be zero.

$\Rightarrow$ $$\delta_{AC} + \delta_{CD} + \delta_{DB} = 0 \tag{1.5}$$

$$\delta_{AC} = \frac{R_a \times L_{AC}}{(AE)_{AC}} = \frac{R_a(2000)}{1500 \times 2 \times 10^5}$$

$$\delta_{CD} = \frac{(R_a - P_1)L_{CD}}{(AE)_{CD}} = \frac{(R_a - 150) \times 2500}{1500 \times 2 \times 10^5}$$

$$\delta_{DB} = \frac{R_b \times L_{DB}}{(AE)_{DB}} = \frac{R_b \times 2000}{1500 \times 2 \times 10^5}.$$

Substituting the above in equation (1.5)

$\therefore$ $$\frac{R_a(2000)}{3 \times 10^8} + \frac{(R_a - 150) \times 2500}{3 \times 10^8} + \frac{R_b \times 2000}{3 \times 10^8} = 0 \tag{1.6}$$

Using R_b value from equation (1.4) into equation (1.6)

$$2R_a + 2.5(R_a - 150) + 2 \times (R_a - 50) = 0$$

$\Rightarrow$ $$R_a = 475/6.5 = 73.08 \text{ kN.}$$

$$R_b = 23.08 \text{ kN}$$

$\therefore$ Stress in the portion $AC = \sigma_{AC} = \dfrac{73.08 \times 10^3}{1500} = 48.72$ MPa

Stress in the portion $CD = \sigma_{CD} = \dfrac{R_a - P_1}{A} = \dfrac{-76.92 \times 10^3}{1500} = -51.28$ MPa (compressive)

Stress in the portion $DB = \sigma_{DB} = \dfrac{R_b}{A}$

$$\sigma_{DB} = \frac{23.08 \times 10^3}{1500} = 15.39 \text{ MPa (tensile)}$$

$$\sigma_{AC} = 48.72 \text{ MPa (tensile)}$$

$$\sigma_{CD} = 51.28 \text{ MPa (compressive)}$$

$$\sigma_{DB} = 15.39 \text{ MPa (tensile).}$$

Axial thrust diagram: The axial force varies in different portions of the bar *AB*. Diagrammatic representation of this variation is called thrust diagram.

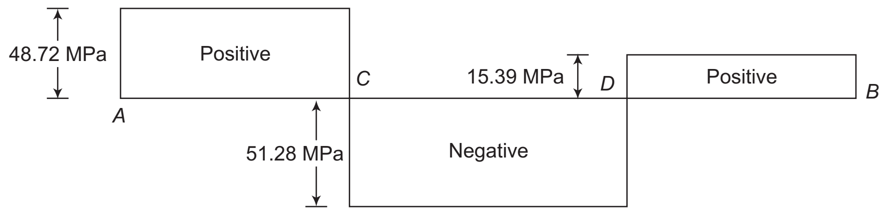


FIGURE 1.25

Axial tension is +ve (positive) and axial compression is –ve (negative).

PROBLEM 1.14

Objective 4

A bar *AB* of length 3 m is fixed between the rigid supports. If an axial load of 200 kN at 2 m from support A, determine the axial force in the portion *AC* and *CB*. Also determine the variation in the axial forces, if support *B* yields by 0.5 mm. Take cross-sectional area of the bar *AB* as 1000 mm^2 and E = 200 GPa.

SOLUTION

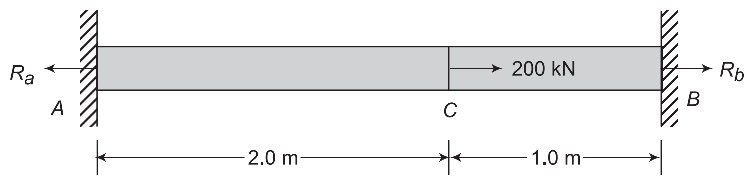


FIGURE 1.26

Case (i): supports do not yield:

$$\sum F_x = 0 \text{ (equilibrium equation)}$$

$$\Rightarrow \quad R_a - R_b = 200 \text{ kN} \tag{1.7}$$

Total extension between *A* and *B* is zero.

$$\Rightarrow \quad \delta_{AC} + \delta_{CB} = 0$$

$$\Rightarrow \quad \frac{R_a \times L_{AC}}{(AE)_{AC}} + \frac{R_b \times L_{CB}}{(AE)_{CB}} = 0$$

$$\Rightarrow \quad \frac{R_a \times 2000}{1000 \times 2 \times 10^5} + \frac{R_b \times 1000}{1000 \times 2 \times 10^5} = 0$$

$$\Rightarrow \quad 2R_a + R_b = 0. \tag{1.8}$$

Solving equations (1.7) and (1.8)

$$R_a = 66.67 \text{ kN}$$

$$R_b = -133.33 \text{ kN}.$$

In this problem, the axial rigidity of the members *AC* and *CB* is not affecting the forces in the portions *AC* and *CB*.

Case (ii): If the support '*B*' yields by 0.5 mm/yielding of support means that, the support B can relax (deform) by 0.5 mm; then the force in the portion *AC* is 200 kN and *CD* becomes zero.

Otherwise, the deformation over and above 0.5 mm induces different types of forces. Therefore, release the support *B*. Then

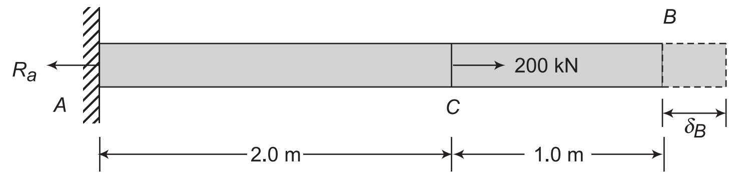


FIGURE 1.27

Free extension $\delta_B = \delta_{AC} + \delta_{CB}$

$$\delta_{AC} = \frac{200 \times 10^3 \times 2000}{1000 \times 2 \times 10^5} = 2 \text{ mm}$$

$$\delta_{CB} = 0$$

As the free extension is more than 0.5 mm, the fixidity at '*B*' should develop a reaction R_b to compensate the deformation of $(\delta_B - 0.5) = 1.5$ mm.

Axial compression in the member due to

$$R_b = \frac{R_b \times L}{AE} = 1.5 \text{ mm}$$

$$\frac{R_b \times 3000}{1000 \times 2 \times 10^5} = 1.5$$

$$R_b = 100 \times 10^3 = 100 \text{ kN.}$$

Force in the portion *AC* = 100 kN (T)

Force in the portion *CB* = 100 kN (C)

	Force in the Portion *AC*	Force in the Portion *CB*
Case (i)	66.67 kN (T)	133.33 kN (C)
Case (ii)	100 kN (T)	100 kN (C)

PROBLEM 1.15 **Objective 4**

Three bars 1, 2, and 3 of length 2, 1.5, and 2.5 m respectively, are connected to a rigid plate, which carries a 200 kN point load as shown in Figure 1.28. The cross-sectional area of each bar is 1000 mm^2. Bars 1 and 3 are of mild steel, while the bar 2 is aluminum. Before placing 200 kN load, the rigid plate is horizontal. Determine the final configuration of the plate due to 200 kN load. Take E_S = 200 GPa and E_a = 60 GPa.

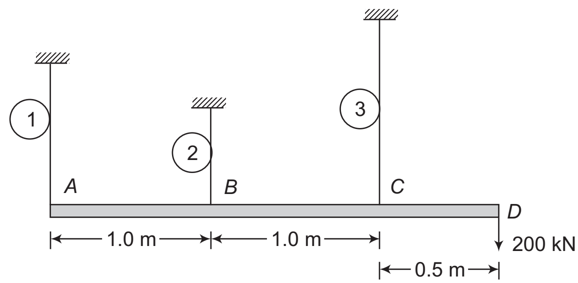


FIGURE 1.28

SOLUTION

Let F_1, F_2, and F_3 be the forces in the members 1, 2, and 3, respectively. Considering the free body diagram of the bar *ABCD*,

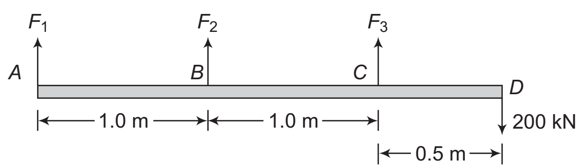


FIGURE 1.29

Sum of the forces in vertical direction is zero.

$$= F_1 + F_2 + F_3 = 200 \text{ kN}.$$

Sum of the moments about point '*A*' is zero.

$$= 2F_3 + F_2 = 200 \times 2.5 = 500.$$

From the two equilibrium equations, the three unknown quantities F_1, F_2, and F_3 cannot be found. Hence, compatibility condition is to be used.

The rigid bar *ABCD* cannot deform, but owing to the extensions of bars 1, 2, and 3 the rigid bar rotates. Let δ_1, δ_2, and δ_3 be the extensions of the bars 1, 2, and 3.

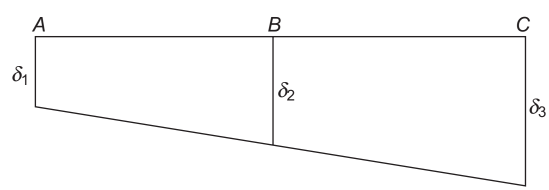


FIGURE 1.30

As the rigid bar just rotates, the deformations δ_1, δ_2, and δ_3 adjust in such way that,

$$\frac{\delta_2 - \delta_1}{1} = \frac{\delta_3 - \delta_1}{2} = \frac{\delta_3 - \delta_2}{1}$$

$$\delta_2 - \delta_1 = \delta_3 - \delta_2$$

$$= \delta_1 + \delta_3 = 2\delta_2 \qquad \text{Compatibility condition} \qquad (1.9)$$

$$= \frac{F_1 \times 2000}{1000 \times 2 \times 10^5} + \frac{F_3 \times 2500}{1000 \times 2 \times 10^5} = \frac{2F_2 \times 1500}{1000 \times 0.6 \times 10^5}$$

$$= 2F_1 + 2.5F_3 = 10F_2 \quad \text{Compatibility condition} \qquad (1.10)$$

Solving equations (1), (2), and (4)

$F_1 = -70.428$ kN (compressive);

$F_2 = 42.857$ kN (tensile);

$F_3 = 228.571$ kN (tensile).

Rotation of the rigid bar *ABCD* is given by

$$\theta = \frac{\delta_2 - \delta_1}{1000} \ ::\ \delta_2 = \frac{F_2 L_2}{(\text{AE})_2} \ ::\ \delta_1 = \frac{F_1 L_1}{(\text{AE})_1}$$

$\delta_1 = -0.714$ mm

$\delta_2 = 1.071$ mm

$\therefore \qquad \theta = 0.00179$ radians.

PROBLEM 1.16

Objective 1

A rigid bar is supplied by a pin at 'A' and two bars (1) and (2) at points B and C as shown in Figure 1.31. The bar (1) is copper having 100 mm^2 cross-sectional area. Bar (2) is of steel with cross-sectional area of 120 mm^2. Determine the stresses in bars 1 and 2 due to a 10 kN load at 'D'. E_S = 200 GPa and E_C = 80 GPa.

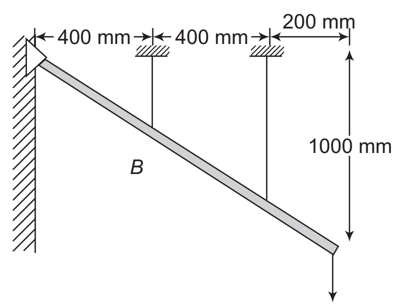


Figure 1.31

SOLUTION

Let F_1 and F_2 be the forces in the bars 1 and 2, respectively. Using static equilibrium equations, that is, by taking moments about A,

$$F_1\,(0.4) + F_2\,(0.8) = 10 \times 1.0$$

$$= F_1 + 2F_2 = 25. \ (1)$$

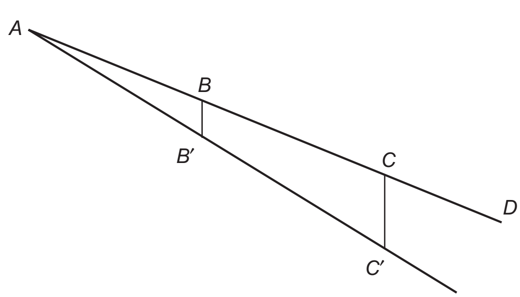


Figure 1.32

Bar $ABCD$ is rigid bar. Hence, it takes configurations $AB'C'D$

$$BB' = \delta_B = \frac{F_1 L_1}{A_1 E_1}$$

$$CC' = \delta_C = \frac{F_2 L_2}{A_2 E_2}$$

$$L_1 = 0.4 \text{ m} \qquad \left(\because\ L_1 = \frac{1000}{400 + 400 + 200} \times 400 \text{ mm}\right)$$

$$L_2 = 0.8 \text{ m}$$

'θ' rotation of bar is equal to

$$\frac{\delta_B}{(400/\cos\alpha)} = \frac{\delta_c}{(800/\cos\alpha)}$$

$$\delta_B = \frac{\delta_C}{2}$$

$$\frac{F_1 \times 400}{100 \times 0.8 \times 10^5} = \frac{F_2 \times 800}{120 \times 2 \times 10^5} \times \frac{1}{2} \tag{1.11}$$

$$F_1 = \frac{1}{3} F_2 \tag{1.12}$$

From equations (1.11) and (1.12)

$$F_1 = 3.571 \text{ kN}; F_2 = 10.714 \text{ kN}$$

Stress in the copper bar (1) = 35.71 N/mm^2 (tensile)

Stress in the steel bar (2) = 89.28 N/mm^2 (tensile).

PROBLEM 1.17

Objective 1

A rigid plate form rests on two aluminum rods having 1200 mm^2 cross-sectional area and length 250 mm. A third bar made of steel 249 mm long and cross-sectional area 2400 mm^2 is in between the two aluminum bars as shown in Figure 1.33. A load of 750 kN is applied on the platform. Determine the stresses induced in aluminum and steel bars. Take E_S = 210 GPa and E_{Al} = 70 GPa.

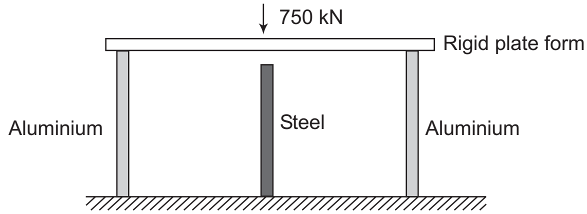


FIGURE 1.33

SOLUTION

In the given system, aluminum bars alone take the load till the gap of 1 mm is closed. Once the gap is closed, then aluminum and steel bars resist the load. Thus, the solution of the problem can be split into two cases. Final stresses will be the algebraic sum of stresses from two cases.

Case (i): Load required by the aluminum bars to deform by 1 mm.

Case (ii): After the gap is closed, sharing the remaining load between aluminum and steel.

Case (i): Let σ_{1A} be the stress in aluminum, which closes the gap between aluminum and steel. In this stage, steel bar is not at all stressed $\sigma_{1S} = 0$.

Then $$\frac{\sigma_{1A} \times L_A}{(AE)_A} = 1 \text{ mm}$$

$$\Rightarrow \quad \frac{\sigma_{1A} \times 250}{70 \times 10^3} = 1 \text{ mm}$$

$$\Rightarrow \quad \sigma_{1A} = 280 \text{ N/mm}^2$$

$\Rightarrow$Load required to close the gap = $\sigma_{1A} \times 2 \times A_{A1}$

$$P_1 = 672 \text{ kN.}$$

$\therefore$ In case (i), stress in aluminum = 280 MPa

$$= 0$$

Case (ii): Let P_2 be the load remaining after closing the gap of 1 mm.

$$P_2 = 750 - 672 = 78 \text{ kN.}$$

Let σ_{2A} and σ_{2S} be the stress developed in aluminum and steel, respectively, in the second stage.

$$\therefore \quad \sigma_{2A} \cdot A_{Sl} + \sigma_{2S} \cdot A_S = P_2 \quad \text{(equilibrium condition)}$$

$$\Rightarrow \quad \sigma_{2A}(2400) + \sigma_{2s}(2400) = 78{,}000 \tag{1.13}$$

$$\Rightarrow \quad \sigma_{2A} + \sigma_{2S} = 32.5.$$

Once the gap is closed, the shortcoming of aluminum bar and steel bar should be same for compatibility.

$$\Rightarrow \quad \frac{\sigma_{2A} \times 250}{E_{Al}} = \frac{\sigma_{2S} \times 250}{E_S}$$

$$\therefore \quad \sigma_{2A} = \sigma_{2s}\left[\frac{70 \times 10^3}{210 \times 10^3}\right]$$

$$\sigma_{2A} = \frac{\sigma_{2S}}{3} \tag{1.14}$$

Using equations (1.14) and (1.13)

$$\sigma_{2S} = 24.375 \text{ MPa}$$

$$\sigma_{2A} = 8.125 \text{ MPa}$$

$\therefore$ Final stress in aluminum = $\sigma_A = \sigma_{1A} + \sigma_{2A} = 288.13$ MPa

Final stress in steel = $\sigma_S = \sigma_{1S} + \sigma_{2S} = 24.375$ MPa.

PROBLEM 1.18

Objective 1

A concrete column of cross-section 230 mm × 230 mm carries four numbers of 12 mm diameter steel bars. If an axial load of 200 kN acts on the composite column, determine the stress induced in concrete and steel. E_S/E_C = modular ratio = 10.

SOLUTION

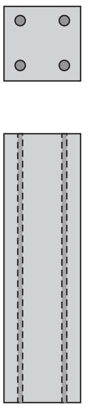


FIGURE 1.34

Let σ_S and σ_C be the stress developed in concrete. For static equilibrium condition

$$\sigma_S \times A_S + \sigma_C \times A_C = P$$

in which A_S is cross-sectional area of steel; A_C is cross-sectional area of concrete.

$$A_S = 4 \times \frac{\pi}{4}(12^2) = 452 \text{ mm}^2$$

$$A_C = 52{,}447.6 \text{ mm}^2$$

$$\therefore \quad 452\,\sigma_S + 52{,}447.6\,\sigma_C = 200{,}000 \tag{1.15}$$

Compatibility condition: Perfect bond exists between concrete and steel.

$$\Rightarrow \quad \delta_C = \delta_S$$

$$\Rightarrow \quad \frac{\sigma_C \times L}{E_C} = \frac{\sigma_S \times L}{E_S}$$

$$\Rightarrow \quad \sigma_S = \frac{E_S}{E_C} \times \sigma_C.$$

Here $\frac{E_S}{E_C}$ is called as modular ratio.

$$\therefore \quad \sigma_S = 10\sigma_C \tag{1.16}$$

From equations (1.15) and (1.16)

$$\sigma_S = 35.11 \text{ MPa}; \ \sigma_C = 3.511 \text{ N/mm}^2$$

Load shared by concrete = 184.131 kN

Load shared by steel = 15.869 kN

% load shared by steel is 7.9% only.

Thus in reinforced concrete columns, capacity of the member can be better increased by increasing concrete rather than steel.

PROBLEM 1.19

Objective 1

A brass tubes of cross-sectional area 200 mm^2 and length 1 m is clamped between two rigid plates as shown in Figure 1.35. A steel bar of cross-sectional area 150 mm^2 passes centrally through the rigid plates and tightened by ruts. Pitch of the nut is 2 mm. If the nut is rotated by ½ revolutions, estimate the stresses produced in steel and brass. Take E_S = 210 GPa and E_B = 100 GPa.

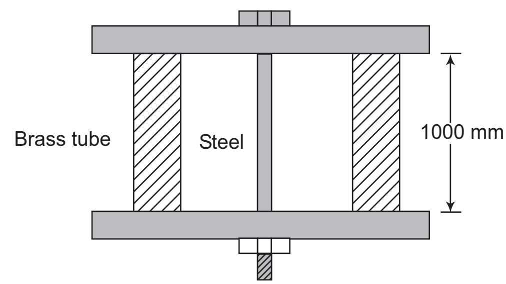


FIGURE 1.35

SOLUTION

When the nut is tightened, the steel both extend and brass bars between the rigid plates compress. Let P_s be the tensile force induced in the steel bolt and P_b be the compressive force induced in the brass tube.

For equilibrium,

$$P_s = P_b = P. \tag{1.17}$$

For compatibility, movement of the nut is equal to the sum of extension in the steel bolt and compression in the brass tube. The above compatibility condition can be explained as below. The steel bar can slide between the rigid plates freely.

Forget tightening of nut for the time being. Now, apply P_s force in steel bolt, the steel bolt extends by δ_s. Then, apply P_b compressive force in the brass tube. Brass tube compress by δ_b when the nut is not tightened, but the force P_b and P_s were applied by external means, then the nut will be $\sigma_s + \delta_b$ distant from the rigid plate. Now, rotate the nut to cover the distance $\Delta = \delta_s + \delta_b$ freely.

This is same as tightening the nut, such that P_b and P_s forces are induced in the brass tube and bolt, simultaneously displacing the nut by δ.

$$\therefore \qquad \Delta = \delta_s + \delta_b \tag{1.18}$$

Δ = distance traveled by the nut in ½ revolution

$$= \text{pitch} \times \frac{1}{2}$$

$$= 2 \times \frac{1}{2} = 1 \text{ mm}$$

$$\delta_S = \frac{P_s \times L_s}{A_S E_S} = \frac{P_s \times 1000}{150 \times 2 \times 10^5}$$

$$\delta_b = \frac{P_b \times L_b}{A_b E_b} = \frac{P_b \times 1000}{200 \times 1 \times 10^5}.$$

Substituting the above in equations (2) and (1)

$$P\left[\frac{1000}{200 \times 1 \times 10^5} + \frac{1000}{150 \times 2 \times 10^5}\right] = 1.0 \text{ mm}$$

$$P = 12{,}000 \text{ N}.$$

Stress in the steel bolt (tensile stress) $\sigma_s = \dfrac{120{,}00}{150} = 80$ MPa (tensile)

Stress in the brass tube = $\sigma_s = \dfrac{12{,}000}{200} = 60$ MPa (compressive stress).

PROBLEM 1.20

Objective 1

A rigid bar AB, hinged at C, is connected by two bars (1) and (2) at A and B, respectively, as shown in Figure 1.36. Bar (1) is 2 mm short in length. Forcibly bar (1) is connected to complete the form. Estimate the stresses induced in the bars for the following data:

$A_1 = 200$ mm^2; $A_2 = 400$ mm^2; $E_1 = 100$ GPa; $E_2 = 10$ GPa.

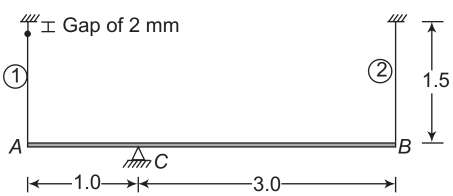


FIGURE 1.36

SOLUTION

To close the gap of 2 mm, say force F_1 is to be applied for the first bar. Because of F_1, let F_2 be the force developed in bar (2). Apply condition of equilibrium,

$$\sum M_C = 0$$

$$= F_1 \times 1 = 3F_2. \tag{1.19}$$

Let δ_1 be the extension in the bar (1) due to F_1. Let δ_2 be the extension in the bar (2) due to F_2. If B moves down by δ_1, point A moves up by $\dfrac{\delta_1}{3}$.

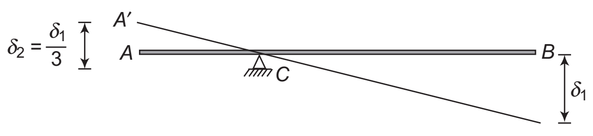


FIGURE 1.37

For closing the gap, $\delta_1 + \dfrac{\delta_2}{3} = \Delta$. (1.20)

This is the compatibility condition.

$$\delta_1 = \frac{F_1 L_1}{A_1 E_1}\ ;\ \delta_2 = \frac{F_2 L_2}{A_2 E_2}$$

$$\therefore \quad \frac{F_1 \times 1500}{200 \times 1 \times 10^5} + \frac{1}{3} \times \frac{F_2 \times 1500}{400 \times 0.7 \times 10^5} = 2 \text{ mm.} \tag{1.21}$$

Using the value of F_1 in the above equation

$$F_1 = 8.235 \text{ kN}$$

$$F_2 = 24.706 \text{ kN}$$

$\therefore$ Stress in the bar (1) $\sigma_1 = 123.53$ MPa (tensile stress)

Stress in the bar (2) $\sigma_2 = 20.588$ MPa (tensile stress).

PROBLEM 1.21

Objective 1

Three horizontal bars of same cross-sectional area connected between two rigid plates of length L between them. Because of a fabrication error, the central bar is $0.0005L$ short. Find the stress in each bar after the system has been mechanically closed. $E = 75$ GPa.

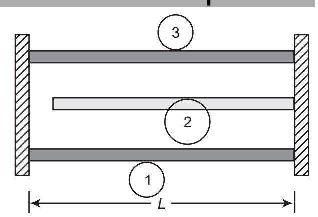


FIGURE 1.38

SOLUTION

$P_1 = P_3$ due to symmetry of the structure

$$\Sigma H = 0 = P_2 = 2P_1 \tag{1.22}$$

When the middle bar is pulled to close the gap, compressive stress is induced in bars (1) and (3).

Hence, as per the compatibility condition

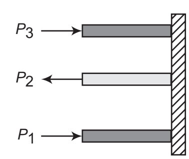


FIGURE 1.39

$$\delta_2 + \delta_1 = \delta_2 + \delta_3 = \Delta.$$

$$\frac{P_2 L}{AE} + \frac{P_1 L}{AE} = \Delta \tag{1.23}$$

$$\frac{P_2 L}{AE} + \frac{P_2 L}{2AE} = 0.0005\ L$$

Stress in the bar 2 is $\sigma_2 = \dfrac{P_2}{A}$

$$\left(\frac{\sigma_2}{E} + \frac{\sigma_2}{2E}\right) L = 0.0005 \text{ L}$$

$$= \frac{75 \times 10 \quad \times 0.0005}{1.5} = 25 \text{ MPa}$$

Stress in the middle bar $\sigma_2 = 25$ MPa (T)
Stress in the bar (1) = $\sigma_1 = \sigma_2/2 = 12.5$ MPa (C)
Stress in the bar (3) = $\sigma_3 = \sigma_1 = 12.5$ MPa (C).

PROBLEM 1.22

Objective 3

A rigid bar *AB* is pinned at A and supported by a steel rod at *D* as shown in Figure 1.40. A linear spring of stiffness 20 kN/mm is located at *C* and concentrated load of 30 kN at *D*. Determine the vertical displacement of the point *B* A_s = 100 mm^2, E_S = 200 GPa; L_S = 0.5 m.

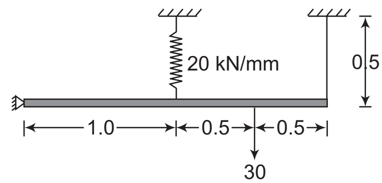


FIGURE 1.40

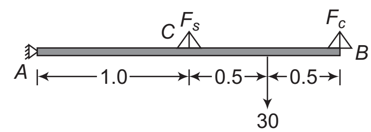


FIGURE 1.41

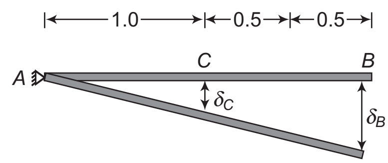


FIGURE 1.42

SOLUTION

A linear spring means force–displacement response of the spring is linear. Let F_C be the force in the spring and F_S be the force in the string.

Taking moments about *A*,

$$2 \times F_S + 1 \times F_C = 30 \times 1.5$$

$$\Rightarrow \qquad 2F_S + F_C = 45$$

Compatibility condition,

$$\delta_C/AC = \delta_B/AB$$

$$\delta_C/1 = \delta_B/2$$

$$\Rightarrow \qquad 2\delta_C = \delta_B$$

δ_C = extension of the spring = F_C/K

$$\frac{2F_C}{20} = \frac{F_S \times 500 \times 1000}{100 \times 2 \times 10^5} \quad (F_S \text{ and } F_C \text{ are in kN})$$

$$F_C = \frac{F_S}{4}$$

$$\Rightarrow \qquad F_S = 20 \text{ kN};\ F_C = 5 \text{ kN}.$$

Displacement at $B = \delta_B = 2 \times \delta_C = 2 \times F_C/K = 2 \times 5/20 = 0.5$ mm.

1.5.1 Thermal Stress

Temperature variations cause change in the dimensions of the body depending on the material properties. If the deformations with change in temperature are restrained, the stress induced in the thermal property of the body is called 'coefficient of thermal expansion'. Coefficient of thermal expansion is the increase in length per unit length of the body due to unit rise in temperature. Generally, temperature is expressed in Kelvin (K) or degree Celsius (°C). Coefficient of thermal expansion is denoted by 'α'.

Increase in length with change in temperature = $\Delta = L\,\alpha\,T$

in which L = length of the body

α = coefficient of thermal expansion

T = rise in temperature.

Thermal strain = $\Delta L/L = \alpha \cdot T$

Thermal strain is positive, if T is rise in temperature and is negative, if T decreases in temperature.

PROBLEM 1.23

Objective 1

A stepped steel bar shown in Figure 1.43 is fixed between two rigid walls at room temperature of 27 °C. If the temperature is raised to 50 °C determine the maximum stress produced in the bar. Take E_S = 210 GPa, $\alpha_s = 12 \times 10^{-6}$/°C.

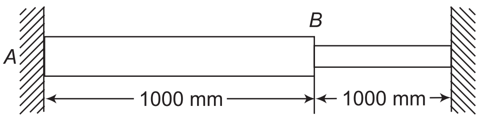


FIGURE 1.43

SOLUTION

Release a support and allow free expansion due to rise in temperature. Then, apply compressive force at the released end, such that the net extension becomes zero.

Released structure

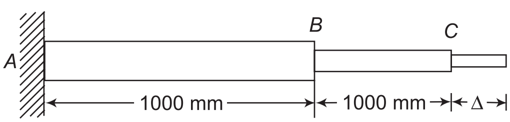


FIGURE 1.44

$$\Delta = L\,\alpha\,T \text{ (It do not depend on cross-sectional area)}$$

$$\Delta = 2000 \times 12 \times 10^{-6} \times (50 - 27)$$

$$= 0.552 \text{ mm.}$$

Apply force 'P' at 'C' such that, the bars compress by 0.552 mm so that net extension at C due to P and rise in temperature is zero.

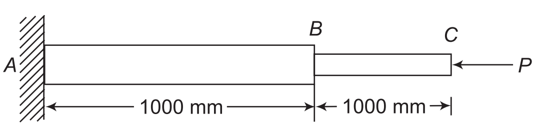


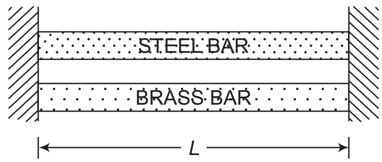
FIGURE 1.45

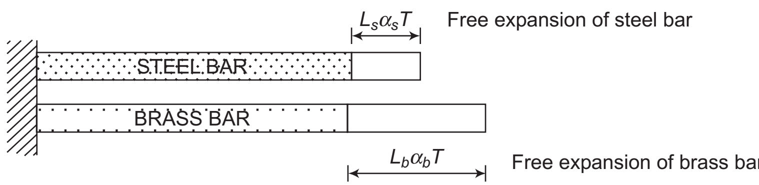
$$\Rightarrow \quad (P \times 1000)/(75 \times 2.10 \times 10^5) + (P \times 1000)/(150 \times 2.1 \times 10^5) = 0.552$$

$$P = 5796 \text{ N}; \ \sigma_{\max} = 77.28 \text{ MPa (C).}$$

1.5.2 Thermal Stresses in Composite Members

Composite members are more common in structural members. Change in temperature induces stress in the composite members as the thermal properties of the individual materials vary. Consider two bars of different materials say steel and brass; brazed together and subjected to rise in temperature by ΔT. To find the stresses produced due to the temperature variation, release the structure and apply force in the bars to maintain compatibility condition; in this case steel and brass bars undergo same extension or strain.





In the absence of rigid plate at the end,

Free expansion of steel = $\alpha_s(L)\,\Delta T$

Free expansion of brass = $\alpha_b(L)\,\Delta T$

in which α_s = coefficient of thermal expansion and α_b = coefficient of thermal expansion of brass.

But as per compatibility condition, both steel and brass bars have to undergo same extension. Thus, steel bar extends little more than the free expansion, whereas brass undergoes extension less than free expansion.

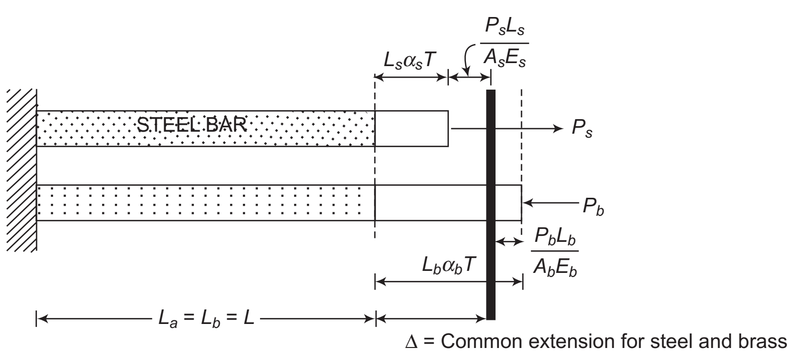


FIGURE 1.46

P_s is the tensile force in the steel bar, responsible for deformation $\Delta - \alpha_s(L)\Delta T$).

P_b is the compressive force in the brass bar, responsible for deformation $(L)\ \alpha_b\Delta T - \Delta$.

Extension due to $P_s = \dfrac{P_S \times L}{A_s E_s}$.

Compression due to $P_b = \dfrac{P_b \times L}{A_b E_b}$.

As per equilibrium condition,

$$P_b = P_s = P.$$

Using the compatibility,

$$\Delta - L(\alpha_s)\,\Delta T = \frac{P_S L}{A_s E_s} \tag{1.24}$$

$$L(\alpha_b)\,\Delta T - \Delta = \frac{P_B L}{A_b E_b} \tag{1.25}$$

Equating 'Δ' from equations (1.24) and (1.25)

$$\frac{PL}{A_s E_s} + L(\alpha_s)\,\Delta T = L(\alpha_b)\,\Delta T - \frac{PL}{A_b E_b}$$

$$P\left[\frac{1}{A_s E_s} + \frac{1}{A_b E_b}\right] = \Delta T\,[\alpha_b - \alpha_s]$$

$$P = \frac{\Delta T(\alpha_b - \alpha_s) \times A_s E_s A_b E_b}{(A_b E_b + A_s E_s)}$$

Stress in steel (tension) = $\sigma_s = \dfrac{\Delta T(\alpha_b - \alpha_s) \times E_s A_b E_b}{(A_b E_b + A_s E_s)}$

Stress in brass (compressive) = $\sigma_b = \dfrac{\Delta T(\alpha_b - \alpha_s) \times E_b A_s E_s}{(A_b E_b + A_s E_s)}$

PROBLEM 1.24

Objective 1

A rigid bar *ABC* is pinned at *C* and attached to two vertical bars (1) and (2) as shown in Figure 1.47. Estimate the stresses in the bars, if the temperature of the bar (1) is decreased by 40°C. Data: L_1 = 0.9 m; A_1 = 300 mm^2; E_1 = 200 GPa; $\alpha_1 = 11.7 \times 10^{-6}$/°C; L_2 = 1.2 m; A_2 = 1200 mm^2; E_2 = 70 GPa; $\alpha_2 = 23 \times 10^{-6}$/°C;

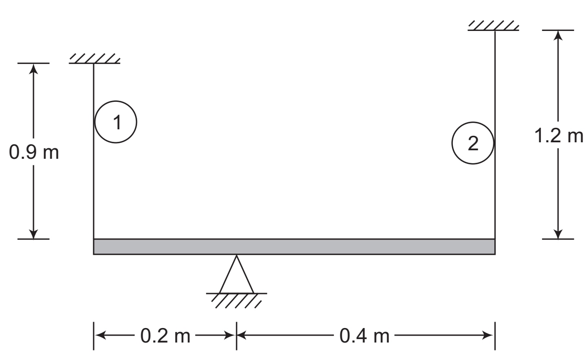


FIGURE 1.47

SOLUTION

Let P_1 and P_2 be the force in the members due to change in temperature.

From the condition of equilibrium

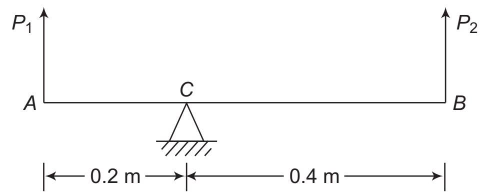


FIGURE 1.48

$$\Sigma M_C = 0$$

$$\Rightarrow \quad P_1\,(0.2) = P_2\,(0.4) \tag{1.26}$$

$$\Rightarrow \quad P_1 = 2P_2.$$

Let the displacement at A be δ_A and at B be δ_B.

$$\frac{\delta_A}{0.2} = \frac{\delta_B}{0.4} \Rightarrow \delta_A = \frac{\delta_B}{2} \tag{1.27}$$

δ_A is displacement, which is the decrease in length due to thermal variation and increase in length due to P_1.

$$\therefore \quad \delta_A = L_1\alpha_1 T - \frac{P_1 L_1}{A_1 E_1}$$

As the bar (2) is not subjected to any thermal movement

$$\therefore \quad \delta_B = \frac{P_2 L_2}{A_2 E_2}$$

$$\delta_B = 2\delta_A \text{ (compatibility condition)}$$

$$\therefore \quad 2\left\{900 \times 11.7 \times 10^{-6} \times 40 - \frac{P_1 \times 900}{300 \times 2 \times 10^5}\right\} = \frac{P_2 \times 1200}{1200 \times 0.7 \times 10^5}$$

$$\Rightarrow \quad 0.8424 - 0.3 \times 10^{-4} P_1 = 1.429 \times 10^{-5} P_2 \tag{1.28}$$

Using equation (1.26) in equation (1.28)

$$0.8424 - 0.6 \times 10^{-4} P_1 = 1.429 \times 10^{-5} P_2$$

$$\Rightarrow \quad P_2 = 11{,}339 \text{ N}$$

$$\Rightarrow \quad P_1 = 22678 \text{ kN.}$$

Stress in bar (1) $\sigma_1 = 75.59$ MPa.

Stress in bar (2) $\sigma_2 = 9.45$ MPa.

PROBLEM 1.25

Objective 1

A rigid bar AD pinned at 'A' and attached to the bars BC and ED is shown in Figure 1.49. Temperature of the bar CB is decreased by 25 °C and that of the bar ED is increased by 25 °C. Find the stress induced in the bars CB and ED. Bar ED is steel and bar CB is brass.

$$E_S = 200 \text{ GPa};\ \alpha_s = 12 \times 10^{-6}/°\text{C};\ A_s = 250 \text{ mm}^2;\ E_b = 90 \text{ GPa};\ \alpha_b = 20 \times 10^{-6}/°\text{C};\ A_b = 500 \text{ mm}^2$$

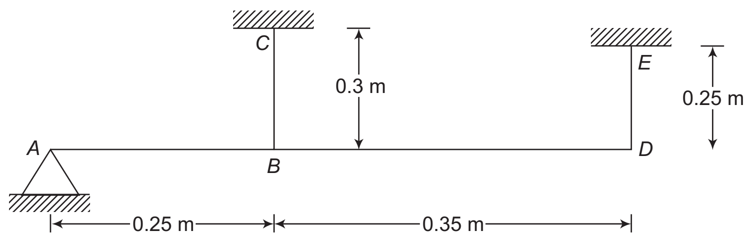


FIGURE 1.49

SOLUTION

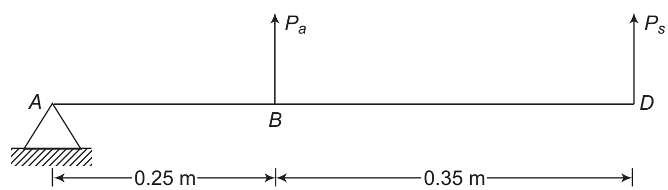


FIGURE 1.50

Let P_s and P_b be the forces developed in the steel and brass bars, respectively. As per equilibrium condition,

Algebraic sum of moments about $A = 0$

$$\Sigma M_A = 0$$

$$\Rightarrow \qquad P_s \times 0.6 = P_b \times 0.25$$

$$\Rightarrow \qquad P_b = 2.4 P_s \tag{1.29}$$

For compatibility condition,

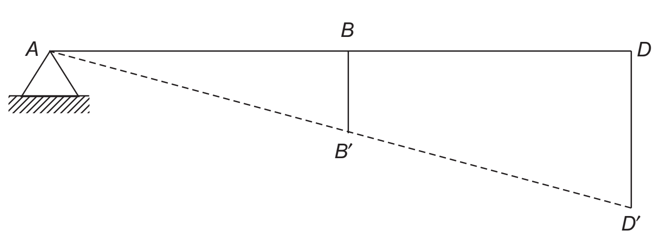


FIGURE 1.51

$BB' = \delta_B$ = displacement of the point $B\downarrow$

= [Extension of the bar BC due to P_b] – [Contraction of BC due to drop in temperature]

$$= \frac{P_b L_b}{A_b E_b} - L_b \alpha_b T$$

$$= \frac{P_b \times 300}{500 \times 0.9 \times 10^5} - 300 \times 20 \times 10^{-6} \times 25$$

$= 0.667 \times 10^{-5} P_b$

$DD' = \delta_D$ = displacement of the point $D\downarrow$

= [Extension of the bar ED due to P_s] – [Extension of ED due to rise in temperature]

$$= \frac{P_s L_s}{A_s E_s} + L_s \alpha_s T$$

$$= \frac{P_s \times 250}{250 \times 2 \times 10^5} + 250 \times 12 \times 10^{-6} \times 25$$

$= 0.5 \times 10^{-5} P_s + 0.075.$

For compatibility

$$\frac{BB'}{AB} = \frac{DD'}{AD}$$

$\Rightarrow$ $DD' = BB'$

$\therefore$ $\delta_D = 2.4\delta_B$

$\therefore$ $0.5 \times 10^{-5} P_S + 0.075 = 2.4[0.667 \times 10^{-5} P_b - 0.15]$

$\Rightarrow$ $1.60 \times 10^{-5} P_b - 0.5 \times P_S = 0.435$

$\Rightarrow$ $1.60\ P_b - 0.5 \times P_S = 43{,}500.$

From equation (1), $P_b = 2.4P_s$

$\therefore$ $P_s = 13{,}024$ N

$\therefore$ $P_b = 31{,}257.5$ N

Stress in steel bar = $\sigma_s = 52.10$ MPa (tensile stress)

Stress in steel bar = $\sigma_b = 62.52$ MPa (tensile stress).

PROBLEM 1.26

Objective 1

A rigid bar of negligible weight is supported by two bars (1) and (2) and a change at A as shown in Figure 1.52. Determine the temperature change required to cause a stress of 55 MPa in bar (1) for the following data.

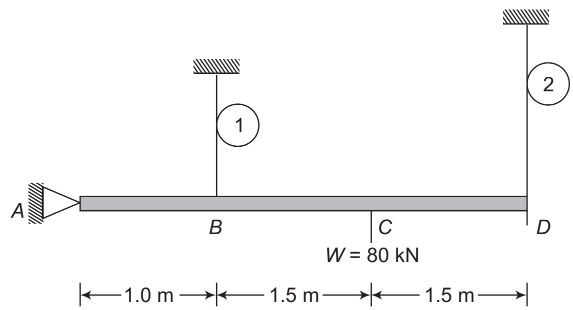


FIGURE 1.52

$L_1 = 1.5$ m; $A_1 = 320$ mm^2; $E_1 = 200$ GPa; $\alpha_1 = 11.7 \times 10^{-6}$/°C
$L_2 = 3.0$ m; $A_2 = 1300$ mm^2; $E_2 = 83$ GPa; $\alpha_2 = 18.9 \times 10^{-6}$/°C

SOLUTION

Let σ_1 and σ_2 be the tensile stress induced in bars (1) and (2), respectively.

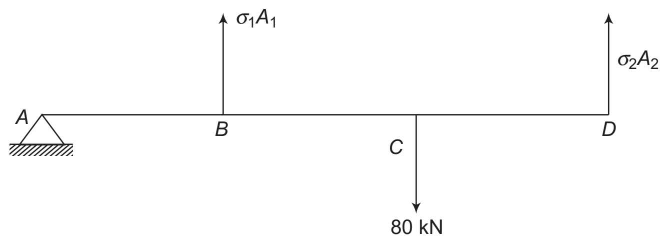


FIGURE 1.53

Algebraic sum of moments about $A = 0$

$$\Sigma M_A = 0$$

$$\Rightarrow \quad \sigma_1 A_1 \times 1 + \sigma_2 A_2 \times 4 = 80 \times 1000 \times 2.5$$

$$\Rightarrow \quad 55 \times 320 \times 1 + 4 \times \sigma_2 \times 1300 = 2 \times 10^5$$

$$\Rightarrow \quad \sigma_2 = 35.08 \text{ MPa.}$$

Let 'T' be the rise in temperature that develops stress in bar (1) by 55 MPa.

From compatibility consideration,

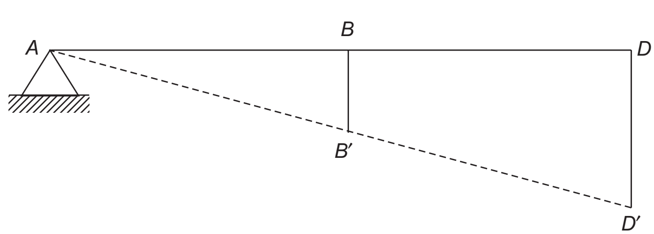


FIGURE 1.54

$$\frac{\delta_B}{1} = \frac{\delta_D}{4}$$

$$\delta_D = 4\delta_B \tag{1.30}$$

δ_B = Displacement of point B, due to rise in temperature and σ_1

$$\delta_B = \frac{\sigma_1 L_1}{E_1} + L_1\alpha_1 T = 0.4124 + 0.0176T$$

δ_D = Displacement of point D due to σ_2 and rise in temperature T

$$= \frac{\sigma_2 L_2}{E_2} + L_2\alpha_2 T$$

$$= \frac{35.08 \times 3000}{0.83 \times 10^5} + 3000 \times 18.9 \times 10^{-6} \times T$$

$$= 1.2678 + 0.0567T.$$

Using δ_D and δ_B values in equation (1.30)

$$1.2678 + 0.0567T = 4 \times [0.4125 + 0.0176T]$$

$$\Rightarrow \qquad T = -27.9\ °\text{C}.$$

A drop in temperature about 27.9 °C is required to create tensile stress of 55 MPa in the bar (1).

PROBLEM 1.27

Objective 1

In an assembly of brass tube and steel bolt shown in Figure 1.55, the pitch of the bolt thread is 1 mm. The cross-sectional area of the tube is 1000 mm^2 and that of steel bolt is 500 mm^2. If the nut is turned by 1.5 revolutions and the temperature of the system is raised by 100 °C. Find the stresses in the tube and the bolt. Take E_s = 210 GPa; E_b = 85 GPa; $\alpha_s = 12 \times 10^{-6}$/°C; and $\alpha_b = 20 \times 10^{-6}$/°C.

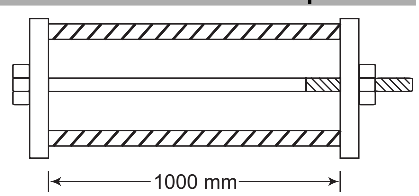


FIGURE 1.55

SOLUTION

This problem can be solved by dividing the problem into two cases.

Case (i): Stresses due problem into only nut tightening. Because of this, steel bolt will be tensioned and brass tube will be compressed.

Case (ii): Stresses tube due to only temperature rise. Because of this, steel bolt will be tensioned and brass tube will be compressed; final stress in the materials will be the algebraic sum of stresses of case (i) and case (ii).

Case (i): Stresses due to nut tightening

Let P_{s1} and P_{b1} be the tensile force in the bolt and compressive force in the tube due to nut tightening by 1.5 revolutions.

For equilibrium,

$$P_{s1} = P_{b1} = P_1 \text{ (say)}.$$

For compatibility

Distance traveled by the nut (Δ) = Extension in the steel bolt + compression in the tube

Distance traveled by the nut in 1.5 evolutions = Δ = 1.5 × pitch

= 1.5 × 1

= 1.5 mm.

$$\therefore \quad \frac{P_{s1}L_{s1}}{A_s E_s} + \frac{P_{b1}L}{A_b E_b} = \Delta$$

$$\Rightarrow \quad \frac{P_{s1} \times 1000}{500 \times 210 \times 10^3} + \frac{P_{b1} \times 1000}{1000 \times 85000} = 1.5$$

As $P_{s1} = P_{b1} = P_1$

$$P_1 = 70{,}461 \text{ N.}$$

$$\therefore \quad \sigma_{s1} = 140.92 \text{ MPa (tensile)}$$

$$\therefore \quad \sigma_{b1} = 70.46 \text{ MPa (compressive).}$$

Case (ii): Stresses due to only thermal variation

Let P_{s2} and P_{b2} be the tensile force in the steel bolt and compressive force in the brass tube, respectively.

For equilibrium, $P_{s2} = P_{b2} = P_2$ say. (1.31)

For compatibility, extension of steel bolt and compression of brass tube should be same.

$$\frac{P_{s2}L}{A_s E_s} + L\alpha_s T = L\alpha_b T - \frac{P_{b2}L}{A_b E_b}$$

$$\therefore \quad \frac{P_{s2}}{500 \times 2.1 \times 10^5} + 12 \times 10^{-6} \times 100 = 20 \times 10^{-6} \times 100 - \frac{P_{b2}}{1000 \times 0.85 \times 10^5}$$

$$P_{s2} = P_{b2} = P_2$$

$$P_2 = 37{,}579 \text{ N}$$

Stress in bolt $\sigma_{s2} = \dfrac{37{,}579}{500} = 75.16$ MPa (tensile)

Stress in tube $\sigma_{b2} = \dfrac{37{,}579}{1000} = 37.58$ MPa (compressive)

Final stress in the steel bolt $\sigma_s = \sigma_{s1} + \sigma_{s2}$

= 140.92 + 75.16

= 216.08 MPa (T)

Final stress in brass tube $\sigma_b = \sigma_{b1} + \sigma_{b2}$

= 70.46 + 37.58

= 108.04 MPa (C)

1.5.3 Strain Energy due to Axial Loading

Work done by a force is the product of the magnitude of the force and the displacement or deformation of the body in the direction of the force. Thus, forces acting on deformable bodies 'work' as bodies deform, though the bodies are at rest configuration. This work done by the external force must be conserved.

The internal stresses and the corresponding strains due to external force produce internal work (within the body). From low conservation of energy, external work must be equal to internal work. This internal work is defined as strain energy.

$$\text{We} = U + I$$

$$\text{We} = \text{External work}$$

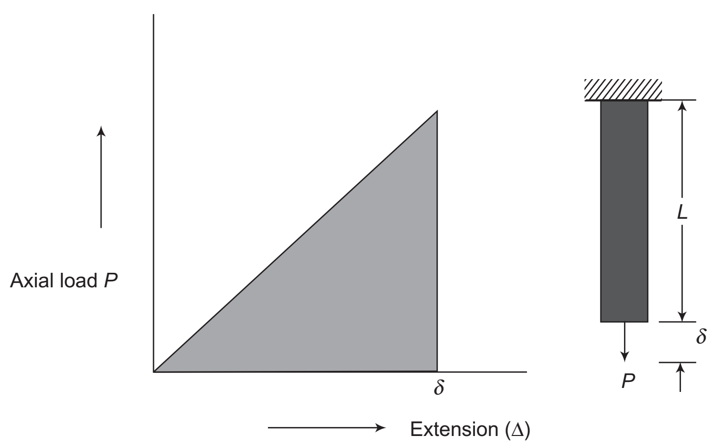
$$U = \text{Strain energy}$$

$$I = \text{Internal heat energy}$$

In strength of materials, we assume that the system is adiabatic. Thus, no heat is given to the system or taken out of the system. Thus, internal heat energy becomes zero ($I = 0$).

$$\therefore \quad \text{We} = U$$

In general, the loads applied are gradual and corresponding deformations are also gradual.
Thus, the external work done (We)



FIGURES 1.56 AND 1.57

$$\therefore \text{ Strain energy } U = \frac{1}{2}P\delta$$

$$U = \frac{1}{2}P\frac{PL}{AE}$$

Rearranging the above expression,

$$U = \frac{1}{2}P\frac{PL}{AE} \times \frac{A}{A}\text{; Taking } P/A \text{ as } \sigma,$$

$$U = \frac{1}{2}\sigma\frac{\sigma L}{E}A = \frac{1}{2}\sigma\frac{\sigma}{E} \times \text{ (Volume of the bar)}$$

or strain energy density = U/volume = $\frac{1}{2}\frac{\sigma^2}{E}$

in which 'σ' is stress and E is modulus of elasticity.

Resilience

Strain energy stored in the body, when the stress at its proportionality limit is referred as 'resilience modulus'. This represents the ability of the body to absorb energy. (This value is high for springs.)

$$U_R = \frac{1}{2}\frac{\sigma^2{}_y}{E}$$

Toughness Modulus

Toughness modulus is the strain energy stored in the body up to complete rupture. This can be obtained from the area bounded by the $P - \Delta$ (curve) up to rupture/failure.

Strain energy concept is of great help, especially when we deal with suddenly applied loads or moving loads or impact loads, etc.

1.5.4 Stress and Deformation of Bars under Impact Loading

Consider a bar of length 'L' cross-sectional area 'A' and modulus of elasticity 'E', having a movable washer 'W' as collar in Figure 1.58. A rigid collar present at bottom receives the washer. Let 'h' be the height through which the washer falls. This is a best example to give an idea about impact loading and suddenly applied loading. If the height of fall 'h' is zero, then the loading becomes suddenly applied load, otherwise the loading is impact loading. Let 'δ' be the extension of the bar due to impact loading.

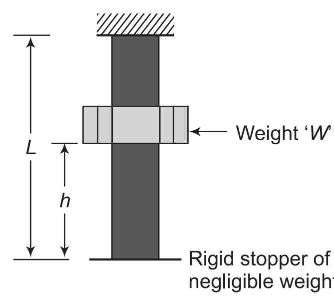


FIGURE 1.58

External work done by washer = $W(h + \delta)$ (1.32)

Let W_* be the equivalent gradually applied, which will give the same effect of impact loading. Then, the strain energy stored in the bar is U

$$U = \frac{1}{2}W_*\delta$$

Strain energy stored in the body due to impact loading of $W_* = U$

As W_* is a gradually applied and gives the same effect of impact loading, 'δ' should be replaced by $\frac{W_* L}{\text{AE}}$.

$$U = \frac{1}{2}W_*\frac{W_* L}{\text{AE}} \tag{1.33}$$

Equating the external work done to strain energy absorbed, that is, equating equations (1.32) and (1.33) and replacing δ by $\frac{W_* L}{AE}$,

$$\frac{1}{2} W_* \frac{W_* L}{AE} = W\left[h + \frac{W_* L}{AE}\right]$$

$$\Rightarrow \qquad W_*^2\left(\frac{L}{AE}\right) - 2W_*\left(\frac{WL}{AE}\right) - 2Wh = 0 \tag{1.34}$$

Solving the above quadratic equation and taking the higher value of W_*

$$W_* = W\left\{1 + \sqrt{1 + \frac{2h}{\left(\frac{WL}{AE}\right)}}\right\}. \tag{1.35}$$

$\frac{WL}{AE}$ is extension for the bar considering the load 'W' as a gradually applied load.

Thus, $\frac{WL}{AE}$ is referred as static deformation

$$W_* = W\left\{1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right\}. \tag{1.36}$$

δ_{st} = Static deformation due to 'W'.

The instantaneous maximum extension is given by

$$\delta = \frac{W_* L}{AE}$$

Instantaneous maximum stress $\sigma = \frac{W_*}{A}$

For suddenly applied load, $h = 0$ and hence $W_* = 2W$

$$\sigma = \frac{W_*}{A} = \frac{2W}{A}.$$

In the design of energy absorbing structures, such as shock absorbers and springs, it is required that δ_{st} should be high. That means, body must be more flexible. Then, the force attracted by the system W_* is going to be less. Otherwise, if the structure is more rigid, that is, δ_{st} is very less, then W_* force attracted by the system will be very high. Even in the case of earthquake-resistant design of structures, this principle holds good. In case of heavy winds, small plants will not break owing to their flexibility, while very big trees break down/get uprooted because of their high rigidity.

The ratio $\frac{W_*}{W}$ is referred as impact factor.

PROBLEM 1.28

Objective 1

A short steel piece of length 200 mm and cross-sectional area 500 mm^2 receives a falling weight of mass 5 kg on its top; height of the fall is 50 mm. If $E = 210$ GPa, estimate the instantaneous maximum stress induced in the steel piece. Estimate instantaneous maximum stress, if cross-sectional area of the piece is reduced by 20% in the top half portion of the piece.

SOLUTION

From the first part of the problem, data are

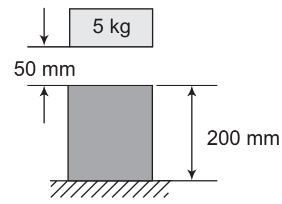


FIGURE 1.59

$$W = 5 \times 9.81 = 49.05 \text{ N}$$

$$L = 200 \text{ mm}$$

$$A = 500 \text{ mm}^2$$

$$E = 210 \text{ GPa}$$

$$h = \text{Height of the fall} = 50 \text{ mm}$$

$$\delta_{\text{St}} = \text{Static compression due to } W$$

$$= \frac{WL}{AE}$$

$$= \frac{49.05 \times 200}{500 \times 2.1 \times 10^5} = 0.934 \times 10^{-4}.$$

Equivalent gradually applied load be W_*

$$W_* = W\left\{1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right\}$$

$$W_* = 49.05\left\{1 + \sqrt{1 + \frac{2 \times 50}{0.934 \times 10^{-4}}}\right\} = 50{,}802.5 \text{ N}$$

Instantaneous maximum stress developed

$$\sigma = \frac{50802.5}{500} = 101.61 \text{ MPa} \quad \text{(C)}$$

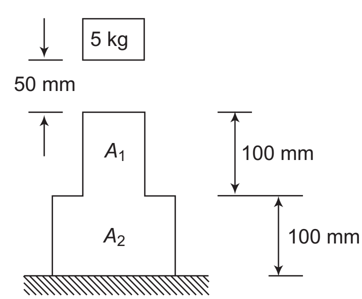


FIGURE 1.60

In second part of the problem, cross-sectional area of the top half of the steel piece is reduced by 20%.

$\therefore$ C/S area of top portion = 400 mm^2.

Let W_{*2} be the statically equivalent load in second case.

δ_{St2} = static compression in stepped column

$$\delta_{\text{st2}} = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2}$$

$$= \frac{49.05 \times 100}{400 \times 2.1 \times 10^5} + \frac{49.05 \times 100}{500 \times 2.1 \times 10^5}$$

$$= 1.05 \times 10^{-4} \text{ mm}$$

$$W_{*2} = 49.05 \times \left\{1 + \sqrt{1 + \frac{2h}{\delta_{st2}}}\right\}$$

$$W_{*2} = 47{,}916.97 \text{ N}$$

Instantaneous maximum stress developed = $\sigma = \dfrac{47{,}916.97}{400} = 119.73$ MPa .

Reduction in cross-sectional area is not much beneficial in reducing the impact effect, but reduction in 'E' value will give lot of advantage. This can be verified by decreasing the value of E in this problem.

PROBLEM 1.29 **Objective 1**

A steel piece of height 200 mm receives an impact load of 5 kg mass falling through 50 mm height. Estimate the instantaneous stress produced in the steel piece. To reduce the impact effect, a short rubber pad of same cross-sectional area and thickness of 20 mm is used. Estimate reduction in impact factor with the provision of rubber pad. Take E_s = 200 GPa; E_r = 1.5 GPa; and A = 250 mm^2.

SOLUTION

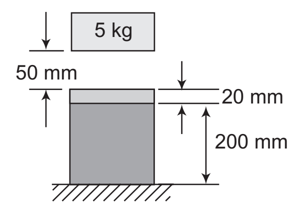


FIGURE 1.61

$$\text{Impact factor} = \frac{\text{Statically equivalent load}}{\text{Falling weight}}$$

Let W_* be the statically equivalent load.

$$W = 9.81 \times 5 = 49.05 \text{ N}$$

$$W_* = W\{1 + \sqrt{\frac{2h}{\delta_{st}} + 1}\}$$

$$\delta_{St} = \frac{5 \times 9.81 \times 200}{250 \times 2 \times 10^5} = 1.962 \times 10^{-4} \text{ mm}$$

$$W_* = 49.05\{1 + \sqrt{\frac{2 \times 50}{1.962 \times 10^{-4}} + 1}\}$$

$$= 49.05 \times 714.92 = 35{,}067 \text{ N}$$

$$\therefore \text{Impact factor} = \frac{W_*}{W} = 714.92.$$

Effect of providing rubber pad

Let W_{*2} be the statically applied equivalent load.

$$\delta_{St} = \frac{49.05 \times 200}{250 \times 2 \times 10} + \frac{49.05 \times 20}{250 \times 1.5 \times 1000}$$

$$= 0.00281 \text{ mm}$$

$$W_{*2} = 49.05\left\{\sqrt{1 + \frac{2 \times 50}{0.00281}} + 1\right\}$$

$$= 49.05 \times 189.65 = 9302.25 \text{ N}.$$

Impact factor $\dfrac{W_{*2}}{W} = 189.65.$

From this example, it is clear that provision rubber pad reduces the impact effect.

Impact factor in the absence of rubber pad = 714.92.

Impact factor with the provision of rubber pad is 189.65.

PROBLEM 1.30

Objective 1

A rigid bar ABC, shown in Figure 1.62, is subjected to an impact load factor of 100 N, falling through a height of 0.2 m. A steel bar of cross-sectional area 1200 mm^2 and 1.5 m long is attached to the rigid bar at B. Determine the instantaneous maximum stress induced in the steel bar. Take E_s = 100 GPa.

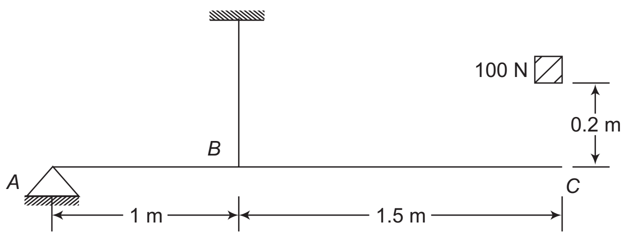


FIGURE 1.62

SOLUTION

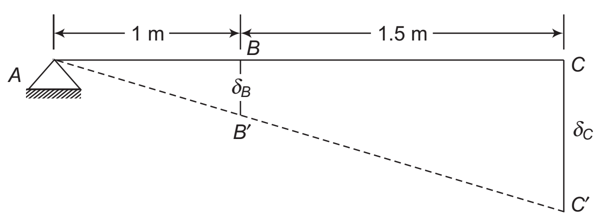


FIGURE 1.63

Let δ_c and δ_b be the deformation at C and B, respectively.

From compatibility,

$$\frac{\delta_C}{2.5} = \frac{\delta_B}{1}$$

Or $$\delta_c = 2.5\ \delta_B$$

External work done by falling weight

$$= 100\ (h + \delta_C)$$

$$= 100\ (200 + \delta_C)$$

Energy absorbed by the steel bar

Let W_* be the statically applied equivalent tensile load in the steel bar.

$$\text{Energy absorbed} = \frac{1}{2} W_* \delta_B$$

$$= \frac{1}{2} W_* \frac{W_* L}{AE}.$$

Equating the energy absorbed to the external work done,

$$100\,(200 + \delta_C) = \frac{1}{2}\frac{W_*^2 L}{AE}$$

Also $\delta_C = 2.5\ \delta_B$ (from compatibility condition)

$$= 2.5 \times \frac{W_* L}{AE}$$

∴ $$100\left\{200 + 2.5\frac{W_* L}{AE}\right\} = \frac{1}{2}\frac{W_*^2 L}{AE}$$

⇒ $$100\left[200 + \frac{2.5 W_* \times 1500}{1200 \times 2 \times 10^5}\right] = \frac{1}{2} \times \frac{W_*^2 \times 1500}{1200 \times 2 \times 10^5}$$

⇒ $$0.3125 \times 10^{-5} W_*^2 - 1.5625 \times 10^{-3} W_* - 20000 = 0$$

⇒ $$W_* = 80{,}250.4\ \text{N}.$$

∴ Instantaneous maximum stress produced in steel = 66.88 MPa (tensile).

PROBLEM 1.31 **Objective 1**

An unknown weight falls 4 cm on to a collar rigidly attached to the lower end of a vertical bar 4 m long and 8 cm^2 in section. If the maximum instantaneous extension is found to be 0.42 cm, find the corresponding stress and the value of the unknown weight. $E = 200$ kN/mm^2.

SOLUTION

Height of fall $h = 40$ mm

Length of the bar $L = 4$ m $= 4000$ mm

C/S area of the bar $A = 8$ cm^2 $= 800$ mm^2

Max. instantaneous extension $\delta = 0.42$ cm $= 4.2$ mm

Modulus of elasticity $E = 200$ GPa $= 2 \times 10^5$ MPa.

Let W_* be the equivalent gradually applied load that gives the same effect of impact load.

Then $$\delta = \frac{W_* L}{AE}$$

⇒ $$4.2 = \frac{W_* \times 4000}{800 \times 200 \times 10^3}$$

⇒ $$W_* = 168{,}000\ \text{N}\,.$$

Let W be the falling weight.

Work done, that is, energy associated with falling weight $= W(h + \delta)$

$$= W[40 + 4.2]$$

Energy absorbed by the wire $U = \frac{1}{2} W_* \delta = \frac{1}{2} \times 168{,}000 \times 4.2$

Energy work done to the energy absorbed $W[44.2] = \frac{1}{2} \times 168{,}000 \times 4.2$

$\Rightarrow$ $$W = 7981.9 \text{ N}.$$

PROBLEM 1.32

Objective 4

An aluminum bar 60 mm diameter when subjected to an axial tensile load 100 kN elongates 0.20 mm in a gauge length 300 mm and the diameter is decreased by 0.012 mm. Calculate the modulus of elasticity and the Poisson's ratio of the material.

SOLUTION

Diameter of the bar $= 60$ mm

Tensile load $(P) = 100 \text{ kN} = 100 \times 10^3$ N

Extension or elongation $\delta = 0.2$

Gauge length $L = 300$ mm

Decrease in diameter $\delta d = 0.012$ mm

Linear strain $$\in = \frac{\delta}{L} = \frac{0.2}{300} = 0.00067$$

Linear stress $$\sigma = \frac{P}{A} = \frac{100 \times 10^3}{\frac{\Pi}{4} 60^2} = 35.368 \text{ N/mm}^2$$

Modulus of elasticity $$\frac{\sigma}{\in} = \frac{35.368}{0.00067} = 0.5305 \times 10^5 \text{ MPa}$$

Poisson's ratio $$\mu = \frac{\text{Linear strain}}{\text{Lateral strain}}$$

Lateral strain $$= \frac{\text{Change in diameter}}{\text{Diameter}} = \frac{0.012}{60} = 0.0002$$

Poisson's ratio $$\mu = \frac{0.0002}{0.00067} = 0.3$$

$\mu = 0.3$ and $E = 0.53 \times 10^5$ MPa $= 53$ GPa.

PROBLEM 1.33

Objective 4

A compound bar 1 m long is 40 mm diameter for 300 mm length, 30 mm diameter for the next 350 mm length. Determine the diameter of the remaining length, so that its elongation under an axial load of 100 kN does not exceed 1 mm. Take $E = 2 \times 10^5$ N/mm^2.

SOLUTION

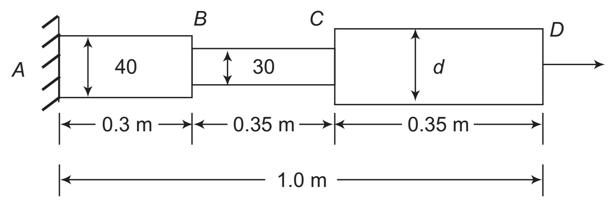


FIGURE 1.64

Length of proportion $CD = 1000 - 300 - 350 = 350$ mm

Total extension: $\delta_{AB} + \delta_{BC} + \delta_{CD} < 1$ mm

Given that, diameter of bar in portion $AB = d_{AB} = 40$ mm

$$E = 200 \text{ GPa and } P = 100 \text{ kN}$$

$$d_{BC} = 30 \text{ mm}$$

$$d_{CD} = d \text{ (to be found)}$$

Length of proportion AB $\quad L_{AB} = 300$ mm

$$L_{BC} = 350 \text{ mm}$$

$$L_{CD} = 350 \text{ mm}$$

$$\delta_{AB} = \frac{PL_{AB}}{A_{AB}E} = \frac{100 \times 10^3 \times 300}{\frac{\Pi}{4}(40)^2 \times 2 \times 10^5} = 0.119 \text{ mm}$$

$$\delta_{BC} = \frac{PL_{BC}}{A_{BC}E} = \frac{100 \times 10^3 \times 350}{\frac{\Pi}{4}(30)^2 \times 2 \times 10^5} = 0.248 \text{ mm}$$

$$\delta_{CD} = \frac{PL_{CD}}{A_{CD}E} = \frac{100 \times 10^3 \times 350}{\frac{\Pi}{4}(d)^2 \times 2 \times 10^5} = \frac{222.82}{d^2}$$

Given that, $\delta_{AB} + \delta_{BC} + \delta_{CD} \leq 1$ mm

$$\Rightarrow \quad 0.119 + 0.248 + \frac{222.82}{d^2} \leq 1$$

$$\Rightarrow \quad \frac{222.82}{d^2} \leq 1$$

$$\Rightarrow \quad d \leq 18.76 \text{ mm.}$$