A PRIMER ON THEORETICAL SOIL MECHANICS

DIMITRIOS KOLYMBAS

A Primer on Theoretical Soil Mechanics

A Primer on Theoretical Soil Mechanics is about adapting continuum mechanics to granular materials. The field of continuum mechanics offers many fruitful concepts and methods, however there is declining interest in the field due to its complex and fragmented nature. This book's purpose is therefore to facilitate the understanding of the theoretical principles of soil mechanics, as well as introducing the new theory of barodesy. This title argues for barodesy as a simple alternative to the plasticity theory used currently and provides a systematic insight into this new constitutive model for granular materials. This book therefore introduces a complex field from a fresh and innovative perspective using simple concepts, succinct equations and explanatory sketches. Intended for advanced undergraduates, graduates and PhD students, this title is also apt for researchers seeking advanced training on fundamental topics.

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Preface

> Socrates taught us to distinguish between what we understand and what we do not understand.

Theoretical soil mechanics is continuum mechanics adapted to soil, a granular material. Continuum mechanics is a complicated, subtle and difficult science. It suffers from fragmentation; its notation is not unique, and the degree of mathematical abstraction varies greatly. Some researchers get lost in it. The many diverse numerical applications add to the confusion; their continuum mechanical foundations are often hidden.

These aspects imply a declining interest in continuum mechanics for soils, which is also enhanced by the emergence of computer-based micromechanical methods. Instead of the continuum, agglomerates of individual particles are considered. This approach has its merits but still does not provide a global understanding for the behaviour of soil. Certainly, the continuum is an abstraction, just like the mass point of classical mechanics. The discontinuous structure of matter (such as sand) can be reconciled with the assumption of the continuum by considering the continuously distributed field quantities as expected values of discontinuous distributions.

The main purpose of this book is to awaken the reader's interest towards the elegant field of continuum mechanics and its applications in soil mechanics. An important part therein is the constitutive law of soils, now in a completely new frame – the theory of barodesy.

Every science starts from its fundamentals and proceeds to ever higher peaks of increasing complexity and refinement. Thus, the author of a textbook that is intended to be clear and concise faces the questions of 'where to start?' and 'where to stop?'. Clearly, the answers can only be subjective in the hope that the reader can retrieve the missing information in both directions. A good book should rather motivate the reader towards a search in literature and in one's own research.

Deeply convinced that 'less is more', I should like to present the reader with a guide to understanding, rather than a repository of equations. I have also attempted to find new and simpler access to several items. In doing so I have followed Walter Noll's suggestions for the role of the professor:

> 'The professor's focus is on understanding, gaining insight into, judging the significance of, and organizing old knowledge. He is disturbed by the pile-up of undigested and ill-understood new results. He is not happy until he has been able to fit these results into a larger context. He is happy if he can find a new conceptual framework with which to unify and simplify the results that have been found by the researcher.'

1 Granular Materials as Soft Solids

1.1 Soil and Geotechnical Engineering

Our solid underground consists of soil and rock; soil being the more important, as our cities are mainly built on it. One usually considers the underground as fixed, and thus, confidently introduces the load of buildings into the soil. However, engineers gradually realised that soil is a rather soft solid that can be easily deformed (Fig. 1.1). Deformation of soil matters as it can lead to settlement and cracks in buildings. Even worse, inclined soil in slopes can move downhill. This motion can be either slow or fast. In the latter case we have catastrophic landslides that can cause thousands of casualties (Fig. 1.2). Landslides can also occur underwater. Gently inclined submarine slopes comprising thousands of cubic kilometres can suddenly start moving giving rise to mega-tsunamis (e.g. the prehistoric Storegga landslide in the North Atlantic). Thus, soil can behave as a fluid, despite its ability to permanently sustain shear stress. Also, horizontally layered soil, i.e. not inclined soil, can be suddenly transformed into a fluid. This is the case when a water-saturated loose sand deposit undergoes a sudden mechanical excitation (e.g. earthquake). The results are peculiar, buildings can sink into the liquefied sand. Sand can also fly (Fig. 1.3). Jet winds can carry thousands of tons of fine sand to heights of up to several kilometres and move it from, say, the Sahara to Europe. Wind is also responsible for the motion of sand dunes.

Exploiting the softness of soil, geotechnical engineers may intervene applying many operations to it. They undertake big excavations to build, e.g. underground garages, or to extract ore or lignite from the underground. They raise earth dams, which can be destroyed by internal erosion if not properly densified. They improve the bearing capacity of foundations by densification of the underground or by the installation of piles. They support cuts or fills in the underground by retaining walls, etc. For all this, one needs to understand the mechanical behaviour of soil and to this end a large variety of experiments have been carried out over the last decades. Being considerably softer than the usual solids (such as steel or concrete), soil samples allow large and complex deformation in the laboratory. They demonstrate a mechanical behaviour that appears extremely complex on first look. Knowledge of this behaviour however, opens the possibility to assess the deformation (i.e. behaviour under loading) and the stability of soil, and thus, preventing catastrophes such as landslides.

Figure 1.1 Footstep on the soil of the moon (NASA). The irreversible deformation of soil manifests its inelastic nature and also its memory. Soil, in particular sand, was the first material with memory exploited by mankind.

1.2 Granulates in Chemical Engineering

Soil is by far not the only granular material of technical relevance. Chemical engineering considers a vast amount of other granular materials such as flour, sugar, coffee beans and ground coffee, soya beans, cement, ore, pellets, etc., which are of high economic importance (Fig. 1.4). Their mechanical behaviour is dictated by their granular nature which is exactly the same as that of soil. Of technical importance is their storage and transportation, the first accomplished in silos. Here, their granular nature poses some difficulties, especially at the outlets of silos, which often get clogged. As for the transportation of granulates, various techniques have been invented, among them dense and dilute phase pneumatic conveying. These bear similarities with the motion of sand in moving dunes and by jet winds.

1.3 Can We Consider Granular Media as Continua?

Should one treat soil as a continuum or rather as a 'discrete' medium composed of individual particles? In the era of digitalisation, there is a tendency to discretise everything. Also, in soil mechanics too, an increasing number of researchers turn to the discrete approach, as the increasing power of computers allow one to consider grains in large numbers. The idea that grains are the truth, and continuum is merely a fiction is gaining traction. This is a deep ongoing philosophical question: what is truth and what is fiction? In physics, scientists are accustomed to accepting a dual approach to tiny corpuscles, considering them both particles and waves. The problem is thus reduced to which method is more appropriate. The advantages of the continuum approach become clear when we recall the saying that there are those unable to see the forest for the trees. In view of the progressing oblivion of

Figure 1.2 Soil can flow: A mure has covered a vehicle. Courtesy Mag. G. Obwaller, Community Wald im Pinzgau.

the continuum mechanics approach, this book aims to help interested scholars gain insight into it. The biggest merit of continuum mechanics is to allow the application of the powerful tool of calculus.

1.4 Differences between Granulates and Other Solids

Contrary to metals and other solids, granulates have a nearly vanishing tensile strength. They only have shear strength, which is mainly of frictional nature, i.e. proportional to normal stress (see Chapter 9). The part of shear stress that is independent of normal stress is called cohesion. A lengthy dispute ensues in soil mechanics as to whether cohesion should be attributed to electromagnetic attraction between the individual grains (so-called true cohesion) or not [92]. There is no conclusive answer as yet, but the author concurs with Schofield [92] that there is (almost) no true cohesion, and that an apparent cohesion is mainly due to interlocking between grains (caution, 'interlocking' here means merely that the grains are toothed and not that they are interwoven). This interlocking causes dilatancy, giving rise to suction in water-saturated soil, such that in the end the strength by cohesion is also frictional.

The other important peculiarity of granulates is the large range of density variation. One and the same soil can be encountered, at the same pressure, in dense and loose state, the latter being a bad underground for foundations. As such, there is no unique relation between pressure and density. In other words, the same density can prevail at different pressures. The pronounced variability of density gives rise

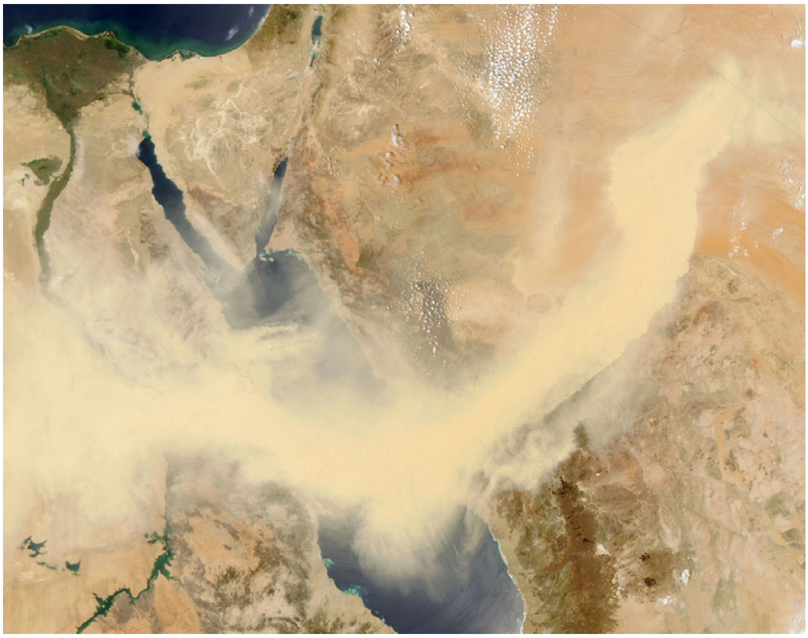

Figure 1.3 Soil can fly: Sand clouds over the Red Sea (NASA).

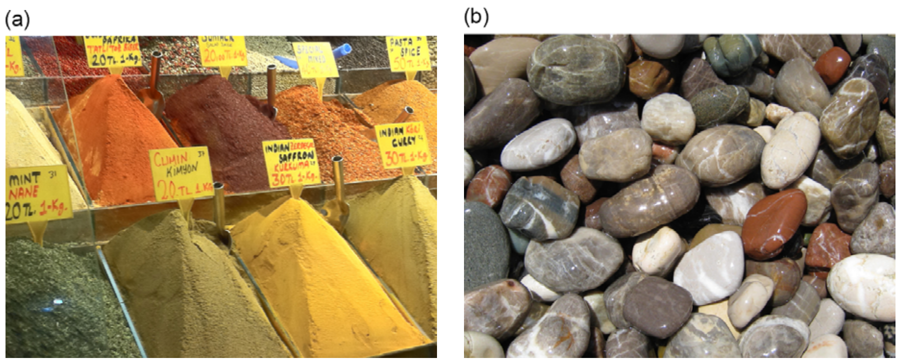

Figure 1.4 There are many different granular materials, such as (a) spices and (b) gravel.

to the phenomena of dilatancy and contractancy with important implications for water-saturated sandy soil: vibrations can easily transform it into a liquid and this liquefaction is a feared side effect of earthquakes.

2 Mechanical Behaviour of Soil

Experimental Results

2.1 The Meaning of Mechanical Behaviour

The mechanical behaviour of a material is the way it responds to deformations. Herein, the response is expressed as stress. We will therefore consider in this book, the strains and stresses that develop during particular loadings. More specifically, we will consider strain and stress paths in the corresponding strain and stress spaces, as well as the correspondence of stress and strain, the so-called stress–strain curves. Taken that stress and strain are tensors with six independent components each, it appears hopeless to get any insight to the underlying processes. Fortunately, there are special cases wherein the symmetries of these will allow us to consider only two principal stress and strain components, and these cases are sufficient to reveal the main aspects of the mechanical behaviour of soil.

The manifold behaviour of soil (and other granular materials) is investigated in the laboratory by several tests. Soil mechanics comprises many laboratory tests, e.g. grain size analysis by sieving. The *mechanical behaviour* of soil is revealed by stress–strain relations and stress paths obtained with deformation of soil samples.

2.2 Element Tests

Stress and strain cannot be measured directly. One may only measure forces acting upon a soil sample and displacements imposed upon the boundaries of the sample. Therefore, one may only infer the stress (as force divided by area) and strain (as elongation divided by length), provided that these quantities are *constant* in the sample. To this end, we need *homogeneous* deformation, i.e. constant deformation throughout the sample. Sources of inhomogeneity can be initial scatter of density, or shear stresses due to friction along rough walls. Such disturbances can be suppressed by improving experimental techniques. More intricate are inhomogeneities that set on spontaneously. They originate from the simple fact that we deform a sample by imposing displacements to its boundaries, but we cannot enforce the distribution of displacements (and stress) *within* the sample. The problem behind is a mathematical one and refers to the loss of uniqueness of the underlying initial boundary value problem (see Chapter 17). The importance of this question is huge and refers not only to the evaluation of laboratory tests but also to the numerical simulation of geotechnical problems.

The ideal case of a test with homogeneous deformation is called an element test. An element test should not be confused with a real test in the laboratory. Rather, it refers to the deformation of a material point and reveals material properties cleared from any system instabilities, such as the onset of inhomogeneous deformation and shear bands. The outcomes of element tests agree to the ones of real tests as long as the deformation of the latter is homogeneous. It is generally believed that, with some care, laboratory tests initially exhibit homogeneous deformation and preserve it until nearly the peak (see Section 2.5). Recent investigations [19, 108] however indicate that the departure from homogeneity sets on much earlier.

2.3 Typical Laboratory Tests

The most widespread tests to explore the mechanical behaviour of a soil are the oedometric and the triaxial tests. Other tests being the shear-box, the simple shear and the true triaxial tests. In the oedometric and triaxial tests, a cylindrical soil specimen is compressed in a vertical direction. Laterally, either the displacement is inhibited (oedometer, kinematical boundary condition, $\varepsilon_2 \equiv \varepsilon_3 = 0$) or the lateral stress is kept constant (triaxial test, static boundary condition, $\sigma_2 \equiv \sigma_3 = \text{const}$). The two different boundary conditions imply a completely different mechanical behaviour; in the oedometric test the stress increases limitlessly, whereas in the triaxial test a limit stress state is reached.

2.4 Oedometric Test

The inhibition of lateral displacement is achieved by a stiff metallic ring (Fig. 2.1). The soil grains move in a vertical direction along this wall, and this evokes shear stresses that disturb the homogeneous distribution of stress. To keep this effect small, the samples are flat, i.e. their diameter is considerably larger than their height. In the course of the test, the vertical load is increased, and the corresponding settlement of the upper plate is monitored. The results are plots of vertical strain ε_1 (usually plotted downwards) versus vertical stress σ_1 or versus $\log \sigma_1$. Instead of ε_1, the compaction of the sample can be expressed as reduction of the void ratio e or of the so-called specific volume $1 + e$. The void ratio e is defined as the ratio V_v/V_s, with V_v and V_s being the volumes of voids and solids, respectively, within a small but representative

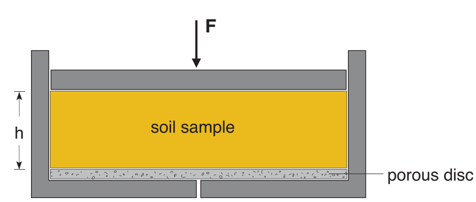

Figure 2.1 Oedometer (schematic). The soil sample is compressed in a vertical direction, whereas lateral expansion is inhibited by a rigid containment. Reproduced from [57], courtesy of Springer Nature.

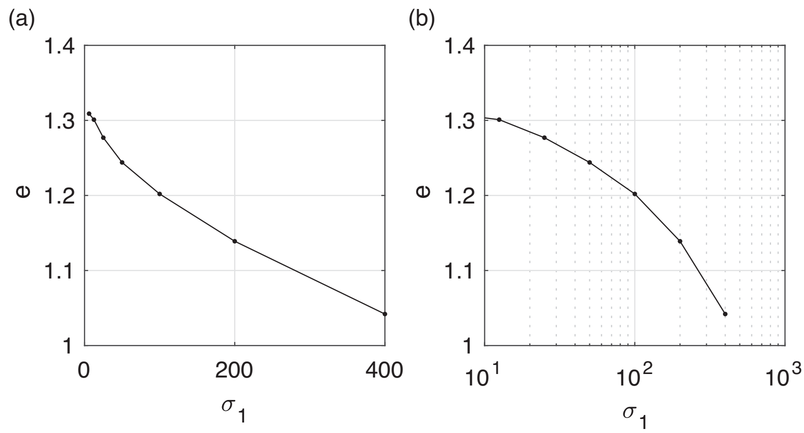

Figure 2.2 Oedometric compression of a clay sample (data from Wichtmann [115]). (a) Linear and (b) semilogarithmic plots. Note the nearly horizontal inclination of the initial part of the curve in the semilogarithmic plot.

volume element. Semilogarithmic plots have to be interpreted with caution, because they change the curvature of the curve: a steep curve becomes nearly horizontal in a semilogarithmic plot and this gives the wrong impression of a high stiffness (Fig. 2.2).

The plots ('compression diagrams') can be approximated equally well either with a logarithmic function

$$e = e_0 - C_c \ln(\sigma_1/\sigma_{10}) \tag{2.1}$$

or with a power function

$$e = (1 + e_0)(\sigma_1/\sigma_{10})^\alpha - 1, \tag{2.2}$$

with e_0 and σ_{10} being the values at the beginning of the test. Sudden increases of the settlement can be attributed to grain crushing or breakage of bonds between grains; these bonds making the difference between *natural* and *reconstituted* soil samples. In natural samples, minute bondings between the grains may increase stiffness (Fig. 2.3).

A supposed bend of the compression curve of undisturbed samples due to the transition from reloading to virgin loading is often attributed to the geological preload of a soil deposit, e.g. due to glaciers in the past. However, this 'geologic bend' is often hard to identify and seems to be rather a misconception emanating from plasticity theory (transition from elastic reloading to plastic virgin loading).

The lateral stress σ_2 can only be monitored if the elongation (assumed as small) of the lateral wall is measured ('soft-oedometer' [45]). At loading, σ_2 increases proportional to the vertical stress, $\sigma_2 = K_0\sigma_1$, but at subsequent unloading it decreases much less than σ_1 (Fig. 16.11).

2.5 Drained Triaxial Test

As in the oedometric test, the sample is compressed in a vertical direction. Laterally it is supported by a hydrostatic pressure that is usually kept constant during the

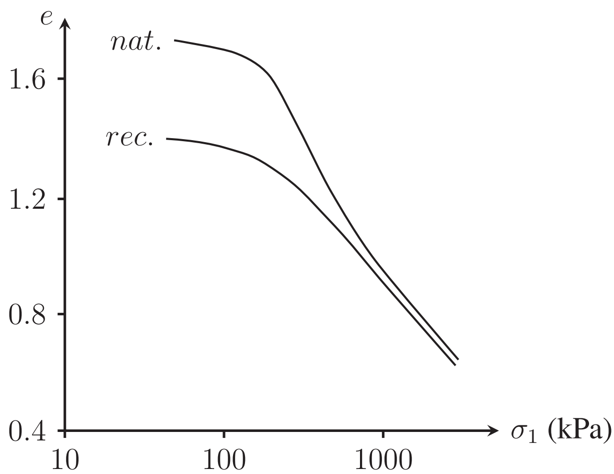

Figure 2.3 Difference between *natural* and *reconstituted* samples, reproduced from [11]. The cementation of an undisturbed (or natural) sample breaks and the compression curve approaches gradually the one of a reconstituted sample.

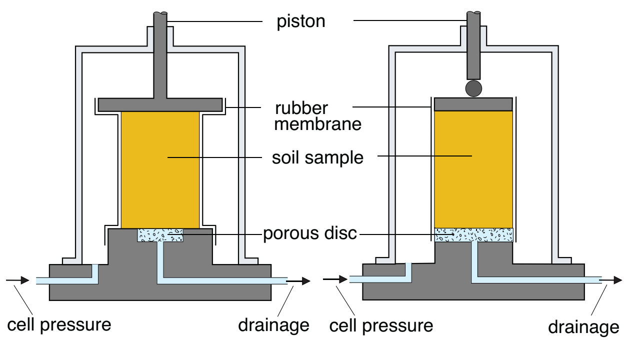

Figure 2.4 Two versions of triaxial tests (schematic). Reproduced from [57], courtesy of Springer Nature.

test: $\sigma_2 = \sigma_3 = \text{const}$ (Fig. 2.4). Contrary to the oedometric test, this is not a kinematical but a statical boundary condition and this implies that the sample can laterally expand. Depending on the amount of this expansion, the volume of the sample is increased or decreased.

The triaxial test reveals rich information on the behaviour of a soil. The resulting plots show

The stress path. The following variables are used:

$$y\text{-axis:} \quad \sigma_1, \quad q := \sigma_1 - \sigma_2, \quad t := \tfrac{\sigma_1-\sigma_2}{2},$$
$$x\text{-axis:} \quad \sigma_2, \quad p := \tfrac{\sigma_1+2\sigma_2}{3}, \quad s := \tfrac{\sigma_1+\sigma_2}{2}.$$

The conventional triaxial test ($\sigma_2 = \text{const}$) is represented as a straight line with inclinations 45° in the s–t-plot, and $\arctan 3 \approx 71.6°$ in the p–q-plot.

The stress–strain curve. The vertical strain is given as $\varepsilon_1 := \Delta u_1/h_0$ or as logarithmic strain: $\epsilon_1 := \ln(h/h_0) = \ln(1+\varepsilon_1)$. h_0 and h are the initial and actual heights of the sample, respectively. Usually, compression is taken as positive. The stress is represented by several variables: σ_1, $\sigma_1 - \sigma_2$, σ_1/σ_2, $\eta := q/p$, $\frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2} = \sin\varphi_m$,