DRUG CALCULATIONS FOR NURSES

A STEP-BY-STEP APPROACH

FIFTH EDITION

Robert Lapham

DRUG CALCULATIONS FOR NURSES

This new edition of *Drug Calculations for Nurses* teaches healthcare professionals how to perform drug calculations with confidence and competence. It provides step-by-step guidance to carry out accurate drug calculations, with units and drug strengths clearly explained.

This bestselling pocket-size book begins with the basic mathematical skills required to perform calculations, including tips on estimating answers. It then covers drug strengths and concentrations, dosage calculations, IV infusion therapies, and pharmacodynamics and pharmacokinetics. Separate chapters focus on children, adults and the elderly, addressing specific challenges encountered in these populations, with new content on pregnancy, and renal and liver function. Helpful worked examples, key points, and objectives are included in every chapter, and this new edition includes more calculations for prescribing and patient-focused scenarios. The companion website, www.drugcalcsnurses.co.uk, provides a comprehensive test bank, with a pre-test and revision test to identify strengths and weaknesses, along with additional practice questions for readers to test themselves on.

Designed for students and practitioners in nursing, midwifery and allied health, this textbook enables readers to improve their numeracy skills for clinical practice and develop their understanding of the broader context for these calculations.

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Fifth Edition

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PREFACE

Drug treatments given to patients in hospital are becoming increasingly more complex. Sometimes, these treatment regimes involve potent and, at times, new and novel drugs. Many of these drugs are toxic or possibly fatal if administered incorrectly or in overdose.

It is therefore very important to be able to carry out drug calculations correctly so as not to put the patient at risk.

In current nursing practice, the need to calculate drug dosages is not uncommon. These calculations have to be performed competently and accurately, not only as to not put the nurse at risk but, more importantly, the patient at risk. This book aims to provide an aid to the basics of mathematics and drug calculations. It is intended to be of use to all nurses of all grades and specialities, and to be a handy reference for use on the ward.

The concept of this book arose from nurses themselves; a frequently asked question was: "Can you help me with drug calculations?" Consequently, a small booklet was written to help nurses with their drug calculations, particularly those studying for their IV certificate. This was very well received, and copies were being produced from original copies, indicating the need for such help and a book like this.

The content of the book was determined by means of a questionnaire, sent to nurses asking them what they would like to see featured in a drug calculations book. As a result, this book was written and, hopefully, covers the topics that nurses would like to see.

Although, this book was primarily written with nurses in mind, others who use drug calculations in their work can also use it. Some topics have been dealt with in greater detail for this reason, e.g. moles and millimoles. So this book can be used by anyone who wishes to improve their skills in drug calculations or to use it as a refresher course.

PART I

This part of the book contains topics that cover the basics of mathematics, particularly multiplication and division.

Following on from this, there is a section on the order of operations (BODMAS or BEDMAS). This describes what to do when you have more than one operation, such as $3 + 3 \times 2$.

These concepts are then applied to various relationships with numbers: how to work with Fractions, Decimals and Percentages.

Chapters in this section cover:

- Basics

1

INTRODUCTION

HOW TO USE THIS BOOK

This book is designed to be used for self-study.

To attain maximum benefit from the book, start at the beginning and work through one chapter at a time as subsequent chapters increase in difficulty. For each chapter attempted, you should understand it a fully and can answer the problems confidently before moving on to the next chapter.

Alternatively, if you wish to quickly skip through any chapter, you can refer to the Key Points found at the beginning of each chapter.

Readers of this book are able to access additional online content at www.drugcalcsnurses.co.uk. The online content consists of a Pre-Test, Revision Test and additional questions. To obtain the maximum benefit from this book, it is a good idea to attempt the pre-test before you start working through the book. The pre-test will enable you assess your current ability in carrying out drug calculations so that you can determine which areas you should concentrate on. Additional questions are available for more practice. After completing the book, repeat the Revision test and compare the two scores to see if you have made any improvement.

BEFORE WE START

Drug calculation questions are a major concern for most healthcare professionals, including nurses and those teaching them. There have been numerous articles highlighting the poor performance of various healthcare professionals.

The vast majority of calculations are likely to be relatively straightforward and it is probably infrequent that you will need to perform any complex calculation. But it is obvious that people are struggling with basic calculations.

It is difficult to explain as to why people find maths difficult, but the best way to overcome this is to try and make maths easy to understand by going back to first principles. The aim is not to demean or offend anyone, but to remind and explain the basics. Maths is just another language that tells us how we measure and estimate and these are the two key words.

It is vital however; that any person performing calculations using any method, formula, or calculator can understand and explain how the final dose is actually arrived at through the calculation.

NUMERACY

Apart from drug calculations, numeracy is used for a range of healthcare activities, such as measuring height and body weight, drug administration and managerial tasks such as budgeting (Hutton 2009) – see table for a list of situations where numeracy is used (Young et al., 2012).

Why do nurses regularly perform badly in numeracy tests? McMullan et al. (2010) provided some suggestions: individuals who are anxious about performing numeracy may be drawn to people-oriented and caring careers such as those in healthcare. They also suggested an over reliance on calculators and no practice in mental arithmetic in secondary schools are contributing factors. 'Maths anxiety' has sometimes been implicated – Williams and Davis (2016) state that students who believe they are not able to master the concepts of, or have had previous failures in, basic mathematics are more likely to suffer with maths anxiety. Bagnasco et al. (2016) also found that undergraduate nursing students had difficult calculating drug dosages because of their lack of knowledge of basic maths principles.

Accurate drug dosage calculations are dependent on accurate mathematical calculations. Thus if anxiety decreases mathematical accuracy, it follows that it may also adversely affect drug dose calculations (Williams and Davis, 2016). However, failure at drug calculations may not always be due to lack of confidence or basic skills A further factor that has been suggested to contribute to lack of ability for some with drug calculations is the conceptual understanding of the problem. Nurses are unable to extract the correct information and do not set up the problem properly and not always necessarily linked to mathematical incompetence (Dopson, 2008).

There are many different methods that can be used to solve drug calculations. It does not really matter which methods are used, but it is important for practitioners to feel comfortable and confident in their chosen method of calculation (Hutton, 2009). Wright (2009) points out that, providing you can clearly explain how you have reached the solution, there are no right ways to solve a drug calculation; just right answers.

Table 1.1 Numeracy skills needed in nursing

Numeracy skill	*Example*
Estimation	All medication dosages IV infusion rate calculations
Addition	Fluid balance Addition of doses of varying strengths (tablets, capsules, suspensions)
Subtraction	Fluid balance
Multiplication	Conversion of units IV infusion rate calculations
Division	Conversion of units IV infusion rate calculations
Fractions	Drip rate calculations Enteral feeding Conversion to SI units
Decimals	Conversion of units IV infusion rate calculations
SI Units	Prescriptions Haematology & biochemistry blood results Patients' weight measurement & conversion
Conversion of units	Paediatric dosages Translation to imperial measures for patients/relatives e.g., birth weight
Understanding percentages	Distinguish between percentage expressions, e.g., 5% Dextrose, grams/100 mL, 98% oxygen saturation, 2% management cuts

(*Continued*)

Table 1.1 (Continued)

Numeracy skill	*Example*
Ratio	Preparation of solutions Medications dosage calculation
Use of formulae	Dosage calculations IV infusion rate calculations Body surface area (BSA) estimation Body mass index (BMI) calculation
Use of tables	Conversion tables (Imperial/SI units) Body mass index (BMI) tables
Use of charts/graphs	Temperature charts Growth charts Prescription charts
Appreciation of statistics	Evidence-based practice
Budgeting	Stock control
Basic bookkeeping	Helping clients with managing their money
Measurement	Fluid balance Vital signs Preparing/drawing-up and dispensing medicines
Negative numbers	Fluid balance Ophthalmics
Recognition of indices	Blood results, e.g., WBC 4.0×10^9/L

NURSING AND MIDWIFERY COUNCIL REQUIREMENTS

The UK Nursing and Midwifery Council (NMC) have always had measures to improve the numeracy skills of nurses and midwives.

For those pre-registration nursing degree programmes which are being delivered under the 2010 *Standards for pre-registration nursing education* (NMC, 2010): Standard R.3.2 states that universities must ensure that selection and admission criteria include evidence of capacity to develop numeracy skills sufficient to meet the competencies required by the programme. That is provide evidence of basic numeracy skills, such as the ability to use numbers accurately in respect of volume, weight and length. These skills must include addition, subtraction, division and multiplication; use of decimals, fractions and percentages; and the use of a calculator. Universities should ensure, wherever possible, that applicants are given feedback on their level of numeracy in relation to their application to support their developmental needs. In addition, Annexe 3 of this document contains what are known as the Essential Skills Clusters. These identify the baseline skills required and include the need to calculate medicines, nutrition, fluids etc. Assessments with regard to these skills must be incorporated into learning outcomes and assessment strategies. Providers will decide on the pass marks and allowed number of resits in order to reach the first and second progression point. After the second progression point and before entry to the register, students must undergo a numerical assessment which must be passed with a 100% pass mark.

For those pre-registration nursing programmes which are being delivered under the 2018 *Standards for pre-registration nursing programmes* (NMC, 2018a), universities must confirm on entry to the programme that students have the capability to develop numeracy skills required to meet programme outcomes. For such programmes, the outcomes are the proficiencies set out in the 2018 publication *Future nurse: Standards of proficiency for registered nurses* (NMC, 2018b). Standard 1.15 states that at the point of registration all registered nurses must be able to 'demonstrate the numeracy, literacy, digital and technological skills required to meet the needs of people in their care to ensure safe and effective nursing practice'.

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2

BASICS

OBJECTIVES

At the end of this chapter, you should be familiar with the following:

- Arithmetic symbols
- Basic maths
  - long multiplication
  - long division
- Rules of arithmetic
- Fractions and decimals
  - reducing or simplifying fractions
  - equivalent fractions
  - adding and subtracting fractions
  - multiplying fractions
  - dividing fractions
  - converting fractions to decimals
  - multiplying decimals
  - dividing decimals
  - rounding of decimal numbers
  - converting decimals to fractions
- Roman numerals
- Powers or exponentials

KEY POINTS

Basic arithmetic rules

- Simple basic rules exist when adding (+), subtracting (-), multiplying (x), and dividing (/ or ÷) numbers – these are known as operations.
- The acronym or word BEDMAS can be used to remember the correct order of operations:

B	Do calculations in brackets first. When you have more than one set of brackets, do the inner brackets first.
E	Next, do any exponentiation (or powers).
D and M	Do the multiplication and division in order from left to right.
A and S	Do the addition and subtraction in order from left to right.

Fractions

- A fraction consists of a numerator and a denominator

  $\frac{2}{5}$ 2 numerator, 5 denominator

- With calculations, it is best to try and simplify or reduce fractions to their lowest terms.
- Equivalent fractions are those with the same value, i.e. $\frac{1}{2}\ \frac{3}{6}\ \frac{4}{8}\ \frac{12}{24}$

  If you reduce them to their simplest form, you will notice that each is exactly a half.
- If you want to convert fractions to equivalent fractions with the *same* denominator, you have to find a common number that is divisible by all the individual denominators.

Operations with fractions

- To add (or subtract) fractions with the *same* denominator, add (or subtract) the numerators and place the result over the common denominator.
- To add (or subtract) fractions with the *different* denominators, first convert them to equivalent fractions with the same denominator,

then add (or subtract) the numerators and place the result over the common denominator as before.

- To multiply fractions, multiply the numerators and the denominators.
- To divide fractions, invert the second fraction and multiply (as above).
- To convert a fraction to a decimal, divide the numerator by the denominator.

Decimals

- When multiplying or dividing decimals, ensure that the decimal point is placed in the correct place.
- Rounding up or down of decimals:

> If the number after the decimal point is 4 or less, then ignore it, i.e. round down
>
> If the number after the decimal point is 5 or more, then add 1 to the whole number, i.e. round up

Roman numerals

- In Roman numerals letters are used to designate numbers.

Powers or exponentials

- Powers or exponentials are a convenient way or writing large or small numbers.

> A positive power or exponent (e.g. 10^5) means *multiply* by the number of times of the power or exponent
>
> A negative power or exponent (e.g. 10^{-5}) means *divide* by the number of times of the power or exponent

Using a calculator

- Ensure that numbers are entered correctly when using a calculator; if necessary, read the manual.

INTRODUCTION

Before dealing with any drug calculations, we will briefly go over a few basic mathematical concepts that may be helpful in some calculations.

This chapter is designed for those who might want to refresh their memories, particularly those that are returning to healthcare after a long absence.

You can simply skip some parts, or all, of this chapter. Alternatively, you can refer back to any part of this chapter as you are working through the rest of the book.

ARITHMETIC SYMBOLS

The following is a table of mathematical symbols generally used in textbooks. The list is not exhaustive, but only covers common symbols you may come across.

Table 2.1 Arithmetic symbols

Symbol	*Meaning*
$+$	plus *or* positive; add in calculations
$-$	minus *or* negative; subtract in calculations
$\pm$	plus *or* minus; positive *or* negative
$\times$	multiply by
/ or $\div$	divide by
$=$	equal to
$\neq$	not equal to
$\equiv$	identically equal to
$\approx$	approximately equal to
$>$	greater than
$<$	less than
$\ngtr$	not greater than
$\nless$	not less than
$\leqslant$	equal to or less than
$\geqslant$	equal to or greater than
%	percent
Σ	sum of

BASIC MATHS

As a refresher, we will look at basic maths. This is quite useful if you don't have a calculator handy as well as understanding how to perform drug calculations from first principles.

First, we will look at long multiplication and division.

Long multiplication

There are various methods for multiplying numbers – many based on ancient methods. A quick Internet search will find many of the methods – Ancient Egyptian; Russian Peasant; Napier's Grids/Rods or Gelosia Method and Using Grids or Boxes. Below is the traditional method for multiplication which many of you will remember from school which relies on splitting numbers into their individual parts (hundreds, tens and units etc.).

Traditional method

To calculate 456 × 78

H	T	U	
4	5	6	First line up the numbers into hundreds (H), tens (T) and units (U)
×	7	8	

When using the traditional method, you multiply the number on the top row by the units and the tens separately, and then add the two together. In this case: 8 units and 7 tens.

First, multiply the numbers in the top row by the units (8), i.e. 8 × 6. Eight times six equals forty-eight. Write the 8 in the units column and carry over the 4 to the tens column:

H	T	U
4	5	6
×	7	8
		8
	4	

Next, multiply by the next number in the top row, i.e. 8 × 5, which equals 40. Also add on the 4 that was carried over from the last step – this makes a total of 44. Write the 4 in the tens column and carry over the 4 to the hundreds column:

H	T	U
4	5	6
×	7	8
	4	8
4	4	

Next, multiply by the next number in the top row, i.e. 8 × 4, which equals 32. Also add on the 4 that was carried over from the last step – this makes a total of 36. Write down 36. You don't need to carry the 3, as there are no more numbers to multiply on this line.

Th	H	T	U
	4	5	6
	×	7	8
3	**6**	4	8
	4	4	

Now we have to multiply by the tens. First, add a zero on the right-hand side of the next row. This is because we want to multiply by 70 (7 tens), which is the same as multiplying by 10 and by 7.

Th	H	T	U
	4	5	6
	×	7	8
3	6	4	8
			0

Multiply as before – this time it is 7 × 6, which equals 42. Place the 2 next to the zero and carry over the 4 to the hundreds column.

Th	H	T	U
	4	5	6
	×	7	8
3	6	4	8
		2	0
	4		

Next, multiply 7 × 5, which equals 35 and add on the 4 carried over to make a total of 39. Write down the 9 and carry over the 3.

Th	H	T	U
	4	5	6
	×	7	8
3	6	4	8
	9	2	0
3	4		

Finally, multiply 7 × 4, which equals 28. Add the 4 to equal 31 and write down 31. You don't need to carry the 3, as there are no more numbers to multiply on this line:

	Th	H	T	U
		4	5	6
		×	7	8
	3	6	4	8
3	**1**	9	2	0

Begin adding up. Now you're done with multiplying; you just need to add together 3,648 and 31,920. Write a plus sign to remind you of this:

		Th	H	T	U
			4	5	6
			×	7	8
		3	6	4	8
+	3	1	9	2	0
	3	5	5	6	8
		1			

As before, carry over numbers (if necessary) when adding together

You should get a final answer of 35,568

For numbers of more than two digits, follow these steps: first multiply the top number by the units, then add a zero and multiply by the tens, then add two zeros and multiply by the hundreds, then add three zeros and multiply by the thousands, and so on. Add up all the numbers at the end.

Decimal numbers can be multiplied using this method. When dealing with decimal numbers, you have to ensure that the decimal point is placed correctly. Otherwise, the steps you follow are the same as before (see Decimals later).

LONG DIVISION

As with multiplication, dividing large numbers can be daunting. But if the process is broken down into several steps, it is made a lot easier.

Before we start, a brief mention of the terms sometimes used might be useful. These are:

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}\left(\text{Answer}\right) \text{ or}$$

$$\begin{array}{rl} & \text{quotient} \quad (\text{Answer}) \\ \text{divisor} & \overline{\left.\right) \text{dividend}} \end{array}$$

The process is as follows:

Example 1

Divide 3,612 by 14

$$1 \;\; 4 \; \overline{\left.\right) 3 \;\; 6 \;\; 1 \;\; 2}$$

Long division works from left to right. First, look at 14 – it is a 2-digit number, it will not go into the first figure (i.e. the one on the left, which is 3). Obviously dividing 14 into 3 goes zero times or will not go. So, we then consider the next number (6) with the 3 to give us 36 – this is greater than 14 (if it wasn't, we would have added successive numbers until a number greater than 14 is found). We then ask the question how many times can 14 go into 36? Twice 14 would equal 28; three times 14 would equal 42. So, the answer is 2.

14 into 36 goes 2
Multiply 2 × 14 =
Subtract the 28 from 36 =

$$\begin{array}{cc|cccc} & & & 2 & & \\ \cline{3-6} 1 & 4 & 3 & 6 & 1 & 2 \\ & & 2 & 8 & & \\ \cline{3-5} & & & 8 & & \end{array}$$

Bring down the next digit (1)

$$\begin{array}{cc|cccc} & & & 2 & & \\ \cline{3-6} 1 & 4 & 3 & 6 & 1 & 2 \\ & & 2 & 8 & \downarrow & \\ \cline{3-5} & & & 8 & 1 & \end{array}$$

Then start the process again:

Divide 14 into 81. Once again, there is no exact number, 5 is the nearest number (6 would be too much). (If you are having trouble a quicker method would be to write down the 14 times table before starting the division).

$14 \times 1 = 14$				2	5	
$14 \times 2 = 28$		14	3	6	1	2
$14 \times 3 = 42$			2	8		
$14 \times 4 = 56$	14 into 81 goes 5			8	1	
$14 \times 5 = 70$	Multiply $14 \times 5 =$			7	0	
$14 \times 6 = 84$	Subtract 70 from 81 =			1	1	
$14 \times 7 = 98$						
$14 \times 8 = 112$						
$14 \times 9 = 126$				2	5	
$14 \times 10 = 140$		14	3	6	1	2
			2	8		
				8	1	
				7	0	
	Bring down the next digit (2)			1	1	2

$14 \times 1 = 14$				2	5	8
$14 \times 2 = 28$		14	3	6	1	2
$14 \times 3 = 42$			2	8		
$14 \times 4 = 56$				8	1	
$14 \times 5 = 70$				7	0	
$14 \times 6 = 84$	14 into 112 goes 8			1	1	2
$14 \times 7 = 98$	Multiply $14 \times 8 =$			1	1	2
$14 \times 8 = 112$	Subtract 112–112 =					0
$14 \times 9 = 126$						
$14 \times 10 = 140$	Answer = 258					

If there was a remainder at the end of the units then you would bring down a zero as the next number and place a decimal point in the answer.

Example 2

Divide 23 by 17

17	2	3

Firstly, divide the 17 into the first figure (i.e. the one on the left, which is 2). Obviously dividing 17 into 2 goes zero times or will not go. So, we then consider the next number (3) and ask the question how many times can 17 go into 23? Obviously, the answer is once; twice 17 would equal 34. So, the answer is 1.

17 into 23 goes 1
Multiply 1 × 17 =
Subtract the 17 from 23 =
So, the answer is 1 remainder 6

$$\begin{array}{r l} & 1 \\ 17 & \overline{)\,23} \\ & \,\underline{17} \\ & 6 \end{array}$$

This could also be expressed as 1 $\frac{6}{17}$, but we would calculate to 2 or more decimal places.

We can consider 23 being the same as 23.00000 therefore we can continue to divide the number:

$$\begin{array}{r l} & 1. \\ 17 & \overline{)\,23.0} \\ & \,\underline{17} \\ & 60 \end{array}$$

Bring down the zero and put decimal point in the answer

Then start the process again.

17 × 1 = 17
17 × 2 = 34
17 × 3 = 51
17 × 4 = 68 17 into 60 goes 3
17 × 5 = 85 Multiply 3 × 17 =
17 × 6 = 102 Subtract the 51 from 60 =
Bring down the next zero

$$\begin{array}{r l} & 1.3 \\ 17 & \overline{)\,23.00} \\ & \,\underline{17} \\ & 60 \\ & \underline{51} \\ & 9 \end{array}$$

And again repeat the process until there is no remainder or enough decimal places have been reached.

$17 \times 1 = 17$
$17 \times 2 = 34$
$17 \times 3 = 51$
$17 \times 4 = 68$
$17 \times 5 = 85$
$17 \times 6 = 102$

$$
\begin{array}{lrr|rrrrr}
 & & & & 1 & .3 & 5 & 2 \\ \cline{4-8}
 & 1 & 7 & 2 & 3 & .0 & 0 & 0 \\
 & & & \underline{1} & \underline{7} & & & \downarrow \\
 & & & & 6 & 0 & & \\
 & & & & \underline{5} & \underline{1} & & \\
 & & & & & 9 & 0 & \\
 & & & & & \underline{8} & \underline{5} & \\
\text{17 into 50 goes 2} & & & & & & 5 & 0 \\
\text{Multiply } 2 \times 17 = & & & & & & \underline{3} & \underline{4} \\
\text{Subtract the 34 from 50} = & & & & & & 1 & 6
\end{array}
$$

If we were working to 2 decimal points, then our answer would be 1.35 (see rounding of decimals later in the chapter).

RULES OF ARITHMETIC

Now that we have covered basic multiplication and division, in what order should perform an arithmetic sum?

Consider the sum: $3 + 4 \times 6$

- Do we add 3 and 4 together, and then multiply by 6, to give 42? or
- Do we multiply 4 by 6, and then add 3, to give 27?

There are two possible answers depending upon how you would solve the above sum – which one is right? The correct answer is 27.

Why is this? Rules were developed to ensure that everyone follows the same way to solve problems – known as rules for the order of operations.

Rules for the order of operations

The processes of adding (+), subtracting (-), multiplying (×), and dividing (/ or ÷) numbers are known as *operations*. When you have complicated sums to do, you have to follow simple rules known as the *order of operations*. Initially (a long time ago) people agreed on an order in which mathematical operations should be performed, and this has been universally adopted.

The acronym or word BODMAS is used to remember the correct order of *operations*: each letter stands for a common mathematical operation; the order of the letter matches the order of doing the mathematical operations.

Table 2.2 BEDMAS

B	stands for	'Brackets'	e.g. $(3 + 4)$
E or O	stands for	'Exponentials' or 'pOwers'	e.g. 2^3
D	stands for	'Division'	e.g. $6 \div 3$
M	stands for	'Multiplication'	e.g. 3×4
A	stands for	'Addition'	e.g. $3 + 4$
S	stands for	'Subtraction'	e.g. $4 - 3$

> The basic rule is to work from *left* to *right*

For the types of calculations that you would encounter every day – you need to remember division/multiplication are done before addition/subtraction.

Consider the following simple sum: $10 - 3 + 2$

Remember – work from *left* to *right*.

The first operation we come across is subtraction, so this is done first:

$$10 - 3 = 7$$

Then addition, so $7 + 2 = 9$ the right answer.

Now consider this sum: $3 + 5 \times 4$

Remember – work from *left* to *right*.

The first operation we come across is addition, but we notice that the next one is multiplication. Remembering the order of operations – multiplication is done before addition. So, multiplication is done before addition.

$$5 \times 4 = 20$$

Then addition, so $3 + 20 = 23$ the right answer.

To help you to remember the rules, you can remember the acronym BODMAS. You can even make up your own phrase to remember the correct order of operations.

Example 1

If we look at the example from calculating creatinine clearance, we can see that it is quite a complicated sum:

$$\text{CrCl}\left(\text{mL / min}\right) = \frac{1.23 \times \left(140 - 67\right) \times 72}{125} = 51.7$$

In the top line, the sum within the brackets is done first, i.e. (140 – 67), then multiply by 1.23 and then by 72.

Thus, (140 – 67) = 73, so the sum is 1.23 × 73 × 72 = 6,464.88

Then divide by 125 to give the answer of 51.7 to one decimal place.

> If there is a 'line' – then work out the top, then the bottom, and finally divide.

FRACTIONS AND DECIMALS

A basic knowledge of fractions and decimals is helpful since they are involved in most calculations. It is important to know how to multiply and divide fractions and decimals, as well as to be able to convert from a fraction to a decimal and vice versa. An understanding of fractions will also demonstrate numeracy skills.

Fractions

Before we look at fractions, a few points need to be defined to make explanations easier.

A fraction is *part of a whole number or one number divided by another.*

For example:

$\frac{2}{5}$ is a fraction and means 2 parts of 5 (where 5 is the whole).

The number above the 'line' is called the NUMERATOR, and the word is derived from the Latin verb to count – 'enumero'. It 'counts' or indicates the number of parts of the whole number that are being used (i.e. 2 in the above example).

The number below the 'line' is called the DENOMINATOR, and the word is derived from the Latin word for name – 'nomen'. It 'names' or indicates the value of the fraction and it indicates the number of parts into which the whole is divided (i.e. 5 in the above example).

Thus in the above example, the whole has been divided into 5 equal parts and you are dealing with 2 parts of the whole.

$$\frac{2}{5}\ \begin{matrix}\text{numerator}\\ \text{denominator}\end{matrix}$$

It can also be written as 2/5.

If the denominator (bottom number) is *less* or *smaller* than the numerator (top number) then the fraction is *greater* than 1.

If the denominator (bottom number) is *more* or *bigger* than the numerator (top number) then the fraction is *less* than 1.

Simplifying (reducing) fractions

When you haven't a calculator handy, it is often easier to work with fractions that have been 'simplified' or reduced to their lowest terms. We have encountered this before when dealing with long division.

To reduce a fraction, choose any number that divides exactly into the *numerator* (number on the top) and the *denominator* (number on the bottom).

A fraction is said to have been reduced to its lowest terms when it is no longer possible to divide the numerator and denominator by the same number. This process of converting or reducing fractions to its simplest form is called *cancellation*.

Remember – reducing or simplifying a fraction to its lowest terms does not change the value of the fraction.

Calculations with fractions (addition, subtraction, multiplication, or division) are made easier if the fraction is simplified first.

Examples

1). $$\frac{\overset{3}{\cancel{15}}}{\underset{5}{\cancel{25}}} = \frac{3}{5}$$

2). $$\frac{\overset{\overset{3}{\cancel{27}}}{\cancel{135}}}{\underset{\underset{7}{\cancel{63}}}{\cancel{315}}} = \frac{3}{7}$$

The 15 and 25 are divided by 5

a) The 135 and 315 are divided by 5
b) The 27 and 63 are divided by 9

Remember:
- any number that ends in 0 or 5 is divisible by 5
- any even number is divisible by 2
- there can be more than one step (see the example on the right)

If you have a calculator, then there is no need to reduce fractions to its lowest terms: the calculator does all the hard work for you!

Equivalent fractions

Consider the following fractions:

$$\frac{1}{2}\,\frac{3}{6}\,\frac{4}{8}\,\frac{12}{24}$$

Each of the above fractions has the same value: they are called *equivalent* fractions.

If you reduce them to their simplest form, you will notice that each is exactly a half.

Now consider the following fractions:

$$\frac{1}{3}\,\frac{1}{4}\,\frac{1}{6}$$

If you want to convert them to equivalent fractions with the *same* denominator, you have to find a common number that is divisible by all the individual denominators. For example, in the above case, multiply each denominator by 2, 3, 4, etc. until the smallest common number is found, as illustrated in the following table:

Table 2.3 Finding the common number

	3	*4*	*6*
× 2	6	8	12
× 3	9	12	18
× 4	12	16	24

In this case, the common denominator is 12. For each fraction, multiply the numbers above and below the line by the common multiple. So, for the first fraction, multiply the numbers above and below the line by 4; for the second multiply them by 3; and the third multiply them by 2. So the fractions become:

$$\frac{1\times4}{3\times4}=\frac{4}{12} \text{ and } \frac{1\times3}{4\times3}=\frac{3}{12} \text{ and } \frac{1\times2}{6\times2}=\frac{2}{12}$$

$$\frac{1}{3}\frac{1}{4}\frac{1}{6} \text{ equals } \frac{4}{12}\frac{3}{12}\frac{2}{12}$$

Adding and subtracting fractions

To add (or subtract) fractions with the *same* denominator, add (or subtract) the numerators and place the result over the common denominator, i.e.

$$\frac{14}{32}+\frac{7}{32}-\frac{4}{32}=\frac{14+7-4}{32}=\frac{17}{32}$$

To add (or subtract) fractions with the *different* denominators, first convert them to equivalent fractions with the same denominator, then add (or subtract) the numerators and place the result over the common denominator as before, i.e.

$$\frac{1}{4}-\frac{1}{6}+\frac{1}{3}=\frac{3}{12}-\frac{2}{12}+\frac{4}{12}=\frac{3-2+4}{12}=\frac{5}{12}$$

Sometimes, when converting to equivalent fractions may mean that the resultant fractions may have large numbers. To make them easier to deal with, see if you can simplify the fraction in any way.

Multiplying fractions

It is quite easy to multiply fractions. You simply multiply all the numbers 'above the line' (the *numerators*), and the numbers 'below the line' (the *denominators*).

For example:

$$\frac{2}{5}\times\frac{3}{7}=\frac{2\times3}{5\times7}=\frac{6}{35}$$

However, it may be possible to 'simplify' the fraction before multiplying, i.e.

$$\frac{\overset{3}{\cancel{9}}}{\underset{5}{\cancel{15}}} \times \frac{2}{5} = \frac{3 \times 2}{5 \times 5} = \frac{6}{25}$$

In this case, the first fraction has been reduced to its lowest terms by dividing both the numerator and denominator by 3.

You can also 'reduce' both fractions by dividing diagonally by a common number, i.e.

$$\frac{\overset{2}{\cancel{6}}}{7} \times \frac{5}{\underset{3}{\cancel{9}}} = \frac{2 \times 5}{7 \times 3} = \frac{10}{21}$$

In this case, in both fractions there is a number that is divisible by 3 (6 and 9).

Dividing fractions

Sometimes it may be necessary to divide fractions. You will probably encounter fractions expressed or written like this:

$\dfrac{\frac{2}{5}}{\frac{3}{7}}$ which is the same as $\dfrac{2}{5} \div \dfrac{3}{7}$

In this case, you simply invert the second fraction (or the bottom one) and multiply, i.e.

$$\frac{2}{5} \times \frac{7}{3} = \frac{2 \times 7}{5 \times 3} = \frac{14}{15}$$

If, after inverting, you see that reduction or cancellation is possible, you can do this before multiplying. For example:

$$\frac{5}{2} \div \frac{25}{8}$$

This becomes:

$$\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{4}{\cancel{8}}}{\underset{5}{\cancel{25}}} = \frac{1 \times 4}{1 \times 5} = \frac{4}{5}$$

> When doing any sum involving fractions; simplifying the fractions will make the calculation easier to do.

Converting fractions to decimals

This is quite easy to do. You simply divide the top number (*numerator*) by the bottom number (*denominator*).

If we use our original example:

$\frac{2}{5}$ which can be re-written as $2 \div 5$ or $5\overline{)2}$

$$\begin{array}{r|l} & 0\,.\,4 \\ \hline 5 & 2\,.\,0 \\ & 2\;\;\,0 \\ \hline & \;\;\;\;0 \end{array}$$

It is important to place the decimal point in the correct position, usually after the number that is being divided (in this case it is 2).

Decimals

Decimals, in the form of digital displays, may be encountered in a regular basis – for example with infusion and syringe pumps. Understanding decimals are therefore an important part of numeracy skills.

Pierce et al. (2008) highlighted that some nurses had difficulty interpreting decimal numbers as understanding of decimals was inadequate. For example, 4.63 is bigger than 4.8 (63 is bigger than 8). Problems with decimals can lead to medication errors. Lesar (2002) looked at medication errors over an 18-month period, 200 were ten-fold medication errors (61% were overdoses) due to a misplaced decimal point in 43.5% of cases.

Similarly, another study (Doherty and McDonnell, 2012) looked at ten-fold medication errors – this time over a 5-year period in a paediatric hospital. Errors of dose calculation due to an omitted or misplaced decimal points and confusion with zeros were frequent contributing factors.

Decimals describe 'tenths' of a number, i.e. in terms of 10. A decimal number consists of a decimal point and numbers both to the left and right of that decimal point. Just as whole numbers have positions for units, tens, hundreds etc., so do decimal numbers, but on *both* sides of the decimal point, i.e.

1 2 3 . 4 5 6

HUNDREDS TENS UNITS DECIMAL POINT TENTHS HUNDREDTHS THOUSANDTHS

Numbers to the *left* of the *decimal point* are *greater than one*

Numbers to the *right* of the *decimal point* are *less than one*

Thus

0.25 is a fraction of 1
1.25 is 1 plus a fraction of 1

Multiplying decimals

Decimals are multiplied in the same way as whole numbers except there is a decimal point to worry about.

If you are not using a calculator, remember to put the decimal point in the correct place in the answer.

Consider the sum: 0.65×0.75.

At first, it looks a bit daunting with the decimal points, but the principles covered earlier with long multiplication also apply here. You just have to be careful with the decimal point.

$$\begin{array}{ccccc} & 0 & . & 6 & 5 \\ \times & 0 & . & 7 & 5 \\ \hline \end{array}$$

In essence, you are multiplying '65' by '75'

$$\begin{array}{ccccc} & 0 & . & \mathbf{6} & \mathbf{5} \\ \times & 0 & . & \mathbf{7} & \mathbf{5} \\ \hline \end{array}$$

As before, multiply the top row first by the number after the decimal point (part of a unit), then by the units, tens etc., as appropriate.

First, multiply the top row by 5:

$$\begin{array}{ccccc} & 0 & . & 6 & 5 \\ \times & 0 & . & 7 & 5 \\ \hline & & & & 5 \\ & & & {\scriptstyle 2} & \end{array}$$

Five times five equals 25, write the five under the Units column and carry over the 2 to the Tens column

Continue as before until you have multiplied all the numbers in the top row.

$$\begin{array}{ccccc} & 0 & . & 6 & 5 \\ \times & 0 & . & 7 & 5 \\ \hline & 3 & & 2 & 5 \\ & & & {\scriptstyle 2} & \end{array}$$

Next, multiply the top row by 7 (don't forget to place a zero at the end of the second line):

$$\begin{array}{ccccc} & 0 & . & 6 & 5 \\ \times & 0 & . & 7 & 5 \\ \hline & 3 & & 2 & 5 \\ 4 & 5 & & 5 & \mathbf{0} \\ & {\scriptstyle 3} & & & \end{array}$$

Now add the two lines together:

$$
\begin{array}{cccccc}
 & 0 & . & 6 & 5 \\
\times & 0 & . & 7 & 5 \\
\hline
 & 3 & & 2 & 5 \\
4 & 5 & & 5 & 0 \\
\hline
4 & 8 & & 7 & 5
\end{array}
$$

Finally, we have to decide where to place the decimal point.

The decimal point is placed how many places to the *left* as there are numbers after it in the sum (i.e. in this case there are four).

$$
\begin{array}{c}
0\,.\,6\,5 \times 0\,.\,7\,5 \\
\boldsymbol{1\,2 \qquad 3\,4 = 4}
\end{array}
$$

Therefore, in the answer, the decimal point is moved 4 places to the *left*.

$$.\,4\,8\,7\,5 = 0.4875$$

Multiplying by multiples of 10

To multiply a decimal by multiples of 10 (100, 1000, etc.) you simply move the decimal point the number of places to the *right* as there are zeros in the number you are multiplying by.

For example:

Table 2.4 Multiplying by multiples of 10

Number to multipy by	*Number of zeros*	*Move the decimal point to the right*
10	1	1 place
100	2	2 places
1,000	3	3 places
10,000	4	4 places etc.

and so on.