AN IMPULSE AND EARTHQUAKE ENERGY BALANCE APPROACH IN NONLINEAR STRUCTURAL DYNAMICS

Izuru Takewaki and Kotaro Kojima

An Impulse and Earthquake Energy Balance Approach in Nonlinear Structural Dynamics

Taylor & Francis
Taylor & Francis Group
http://taylorandfrancis.com

An Impulse and Earthquake Energy Balance Approach in Nonlinear Structural Dynamics

Izuru Takewaki and Kotaro Kojima

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

First edition published 2021
by CRC Press
6000 Broken Sound Parkway NW, Suite 300, Boca Raton, FL 33487-2742

and by CRC Press
2 Park Square, Milton Park, Abingdon, Oxon, OX14 4RN

© 2021 Izuru Takewaki and Kotaro Kojima

CRC Press is an imprint of Taylor & Francis Group, LLC

The right of Izuru Takewaki and Kotaro Kojima to be identified as authors of this work has been asserted by them in accordance with sections 77 and 78 of the Copyright, Designs and Patents Act 1988.

Reasonable efforts have been made to publish reliable data and information, but the author and publisher cannot assume responsibility for the validity of all materials or the consequences of their use. The authors and publishers have attempted to trace the copyright holders of all material reproduced in this publication and apologize to copyright holders if permission to publish in this form has not been obtained. If any copyright material has not been acknowledged please write and let us know so we may rectify in any future reprint.

Except as permitted under U.S. Copyright Law, no part of this book may be reprinted, reproduced, transmitted, or utilized in any form by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying, microfilming, and recording, or in any information storage or retrieval system, without written permission from the publishers.

For permission to photocopy or use material electronically from this work, access www.copyright.com or contact the Copyright Clearance Center, Inc. (CCC), 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. For works that are not available on CCC please contact mpkbookspermissions@tandf.co.uk

Trademark notice: Product or corporate names may be trademarks or registered trademarks and are used only for identification and explanation without intent to infringe.

Library of Congress Cataloging-in-Publication Data
Names: Takewaki, Izuru, author. | Kojima, Kotaro, author.
Title: An impulse and earthquake energy balance approach in nonlinear
structural dynamics / Izuru Takewaki, Kotaro Kojima.
Description: First edition. | Boca Raton, FL : CRC Press, 2021. | Includes
bibliographical references and index.
Identifiers: LCCN 2021000740 (print) | LCCN 2021000741
(ebook) | ISBN 9780367681401 (hardcover) | ISBN 9780367681418
(paperback) | ISBN 9781003134435 (ebook)
Subjects: LCSH: Earthquake engineering. | Force and energy.
Classification: LCC TA654.6 .T3424 2021 (print) | LCC TA654.6 (ebook) |
DDC 624.1/762--dc23
LC record available at https://lccn.loc.gov/2021000740
LC ebook record available at https://lccn.loc.gov/2021000741

ISBN: 978-0-367-68140-1 (hbk)
ISBN: 978-0-367-68141-8 (pbk)
ISBN: 978-1-003-13443-5 (ebk)

Typeset in Sabon
by SPi Global, India

Contents

Preface

On April 14 and 16, 2016, two consecutive large earthquakes occurred in Kumamoto, Japan. In most countries, it is a common understanding that building structures are designed to resist once to the ground motion from a large earthquake, e.g., one with the intensity level 7 (the highest level on the Japan Meteorological Agency scale; approximately X–XII on the Mercalli scale), during its service life. However, in case of the Kumamoto earthquakes, a number of buildings were subjected to such large ground motions twice in a few days. This phenomenon was unpredictable, and the authors were convinced during and immediately after the earthquake that the critical excitation method is absolutely necessary for enhancing the earthquake resilience of building structures and engineering systems.

The senior author believed for a long time that near-fault ground motions have peculiar characteristics with a few simple waves (see Figure 0.1), and the response of buildings to such ground motions can be characterized by the response to such a simple wavelet. Furthermore, the response to such a simple wavelet can be substituted by the response to the equivalent impulse set (see Figure 0.2). The response to impulses can be described by the continuation of free vibration components, and this fact leads to the straightforward derivation of responses of even elastic-plastic structures.

In this monograph, the critical excitation problems for elastic-plastic structures under double and triple impulses are explained with the interval of impulses as a variable parameter (Kojima and Takewaki 2015a, b). Furthermore, the critical excitation problems for elastic-plastic structures under multiple impulses as a representative of long-period and long-duration ground motions are tackled with the interval of impulses as a variable parameter (Kojima and Takewaki 2015c, Kojima and Takewaki 2017). This approach can overcome the difficulty, called the nonlinear resonance, encountered first around 1960 in the field of nonlinear vibration, and the

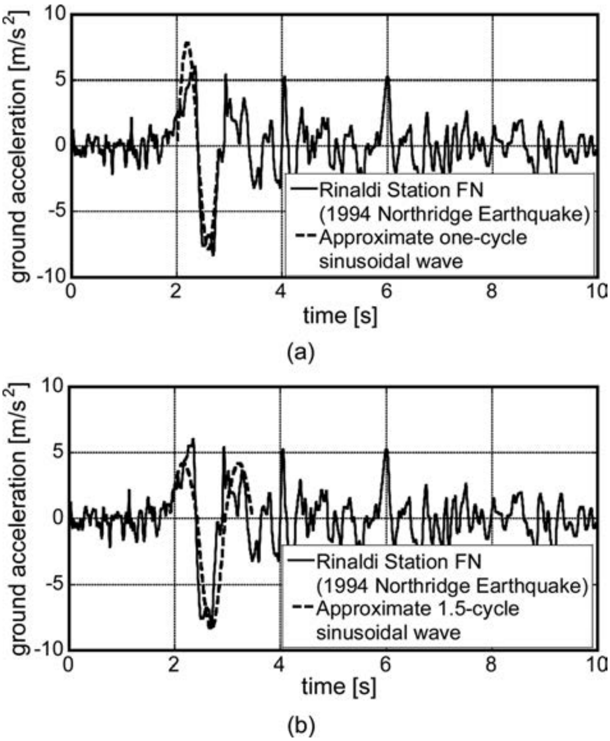

Figure 0.1 Simple modeling of Rinaldi Station fault-normal component (Northridge EQ. 1994) as representative of near-fault ground motion: (a) one-cycle sinusoidal wave modeling, (b) 1.5-cycle sinusoidal wave modeling.

critical excitation problems for elastic-plastic structures are tackled in a more direct way than the conventional methods including laborious computation (see Table 0.1). It can be said that the approach explained newly in this monograph opened the door for an innovative field of nonlinear dynamics.

In principle, the method explained in this monograph is based on the energy balance law, which is taught as part of a high school physics course. Therefore, undergraduate students can read and understand this work. The authors hope that this monograph is also useful for graduate students for research and structural designers/engineers for practice.

Izuru Takewaki
Kotaro Kojima
Kyoto, 2020

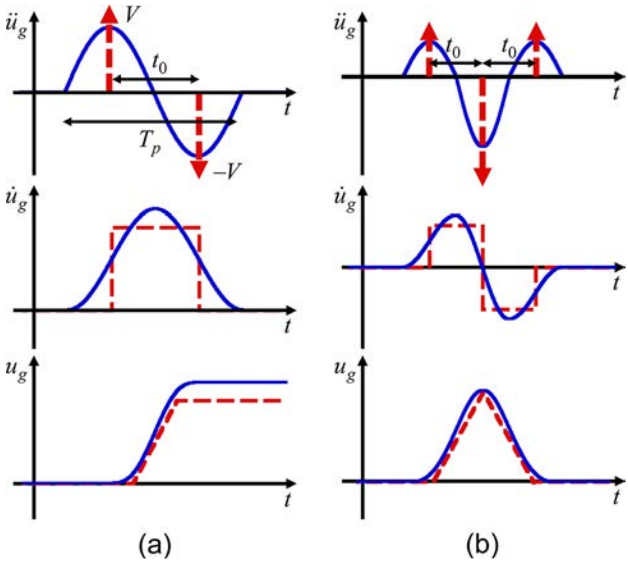

Figure 0.2 Impulse modeling of near-fault ground motion: (a) one-cycle sinusoidal wave and double impulse, (b) 1.5-cycle sinusoidal wave and triple impulse (Kojima and Takewaki 2015a).

Table 0.1 Conventional method and proposed method.

Conventional method *(1960s Caughey, Iwan)*	*Proposed method* *(2015 Kojima and Takewaki)*
① Steady-state ② Difficulty in elastic-perfectly plastic model ③ Inevitable repetition (equivalent parameters/resonant frequency)	① Transient and steady-state ② Possible even for elastic-perfectly plastic (any model) ③ No repetition required
<Proposed method enables> →Closed-form **critical** response of elastic-plastic structure →Derive **resonant frequency** (impulse interval) without repetition →Closed-form **noncritical** response of elastic-plastic structure based on closed-form **critical** response	

REFERENCES

K. Kojima and I. Takewaki (2015a). Critical earthquake response of elastic-plastic structures under near-fault ground motions (Part 1: Fling-step input), *Frontiers in Built Environment* (Specialty Section: Earthquake Engineering), Volume 1, Article 12.

K. Kojima and I. Takewaki (2015b). Critical earthquake response of elastic-plastic structures under near-fault ground motions (Part 2: Forward-directivity input), *Frontiers in Built Environment* (Specialty Section: Earthquake Engineering), Volume 1, Article 13.

K. Kojima and I. Takewaki (2015c). Critical input and response of elastic-plastic structures under long-duration earthquake ground motions, *Frontiers in Built Environment* (Specialty Section: Earthquake Engineering), Volume 1, Article 15.

K. Kojima and I. Takewaki (2017). Critical steady-state response of SDOF bilinear hysteretic system under multi impulse as substitute of long-duration ground motions, *Frontiers in Built Environment* (Specialty Section: Earthquake Engineering), Volume 3, Article 41.

Authors

Izuru Takewaki is a professor of building structures in Kyoto University and president of Architectural Institute of Japan (AIJ). He also serves as the field chief editor of *Frontiers in Built Environment* and has published over 200 international journal papers. He has been awarded numerous prizes, including the Research Prize of AIJ (2004), the 2008 Paper of the Year in *Journal of the Structural Design of Tall and Special Buildings*, and the Prize of AIJ for Book (2014).

Kotaro Kojima is an assistant professor of building structures in Kyoto Institute of Technology since 2018. He obtained a PhD from Kyoto University in 2018 on the theme of Critical Earthquake Excitation Method for Elastic-plastic Building Structures using Impulse Sequence and Energy Balance Law.

Taylor & Francis
Taylor & Francis Group
http://taylorandfrancis.com

Chapter 1

Introduction

1.1 MOTIVATION OF THE PROPOSED APPROACH

1.1.1 Simplification of near-fault pulse-type ground motion

The recording and documentation of earthquake ground motions started in the middle of the 20th century (PEER Center 2013). A classification of earthquake ground motions has been conducted (Abrahamson et al. 1998, Takewaki 1998, Bozorgnia and Campbell 2004), e.g., rock records, soil records, etc. Other classifications exist, namely (1) the near-fault ground motion and (2) the long-period, long-duration ground motion. While the former is well known and has been investigated for a long time, the latter is getting attention recently. In this book, both types are discussed.

Since the time typical earthquakes began to be recorded, the effects of near-fault ground motions on the structural responses have been investigated extensively (Bertero et al. 1978, Hall et al. 1995, Sasani and Bertero 2000, PEER Center et al. 2000, Mavroeidis et al. 2004, Alavi and Krawinkler 2004, Kalkan and Kunnath 2006, 2007, Xu et al. 2007, Rupakhety and Sigbjörnsson 2011, Yamamoto et al. 2011, Vafaei and Eskandari 2015, Khaloo et al. 2015, Kojima and Takewaki 2015). The concepts of fling-step and forward-directivity discussed in the field of engineering seismology are widely used to characterize such near-fault ground motions (Mavroeidis and Papageorgiou 2003, Bray and Rodriguez-Marek 2004, Kalkan and Kunnath 2006, Mukhopadhyay and Gupta 2013a, b, Zhai et al. 2013, Hayden et al. 2014, Yang and Zhou 2014, Kojima and Takewaki 2015). In particular, the San Fernando earthquake in 1971, the Northridge earthquake in 1994, the Hyogoken-Nanbu (Kobe) earthquake in 1995, and the Chi-Chi (Taiwan) earthquake in 1999 drew special attention to many earthquake structural engineers.

The fling-step (fault-parallel) and forward-directivity (fault-normal) inputs have been characterized by two or three wavelets. For this class of ground motions many useful researches have been conducted. Mavroeidis and Papageorgiou (2003) investigated the characteristics of this class of ground motions in detail and proposed some wavelets (for example, Gabor wavelet and Berlage wavelet). Xu et al. (2007) used a kind of Berlage wavelet and applied it to the evaluation of performances of passive energy dissipation systems. Takewaki and Tsujimoto (2011) employed the Xu's approach and proposed a method for scaling ground motions from the viewpoints of story drift and input energy demand. Takewaki et al. (2012) made use of a sinusoidal wave for simulating pulse-type waves. Kojima and Takewaki (2015) started a new approach using the double impulse (Kojima et al. 2015a) by picking up the principal properties of those ground motions. They focused on the intrinsic response characteristics by the near-fault ground motion (Kojima and Takewaki 2015).

Most of the previous works on the near-fault ground motions deal with the elastic response, because the number of parameters (e.g., duration and amplitude of a pulse, ratio of pulse frequency to structure natural frequency, change of equivalent natural frequency for the increased input level) to be considered is huge, and even the computation of elastic-plastic response is quite complicated.

To tackle such important but complicated problem, the double impulse input was introduced as a substitute of the fling-step near-fault ground motion, and a closed-form solution was derived of the elastic-plastic response of a structure by the "critical double impulse input" (Kojima and Takewaki 2015). It was shown that, since only the free-vibration is induced under such double impulse input, the energy balance approach plays an important role in the derivation of the closed-form solution of a complicated elastic-plastic response. It is also shown that the maximum inelastic deformation can occur either after the first impulse or after the second impulse depending on the input level of the double impulse. The validity and accuracy of the theory were investigated through the comparison with the response analysis result to the corresponding equivalent one-cycle sinusoidal input as a representative of the fling-step near-fault ground motion. The amplitude of the double impulse was modulated so that its maximum Fourier amplitude coincided with that of the corresponding one-cycle sinusoidal input. The validity and accuracy of the theory were also checked through the comparison with the elastic-plastic responses under the actual recorded near-fault ground motions.

1.1.2 Resonant response in nonlinear structural dynamics and earthquake-resistant design

The closed-form or nearly closed-form expressions for the elastic-plastic earthquake response have been obtained so far only for the steady-state response to sinusoidal input or the transient response to an extremely simple sinusoidal input (Caughey 1960a, b, Iwan 1961, 1965, Roberts and Spanos 1990, Liu 2000). In the approach explained in this book, the following motivation was drawn based on the observation that the main part of a

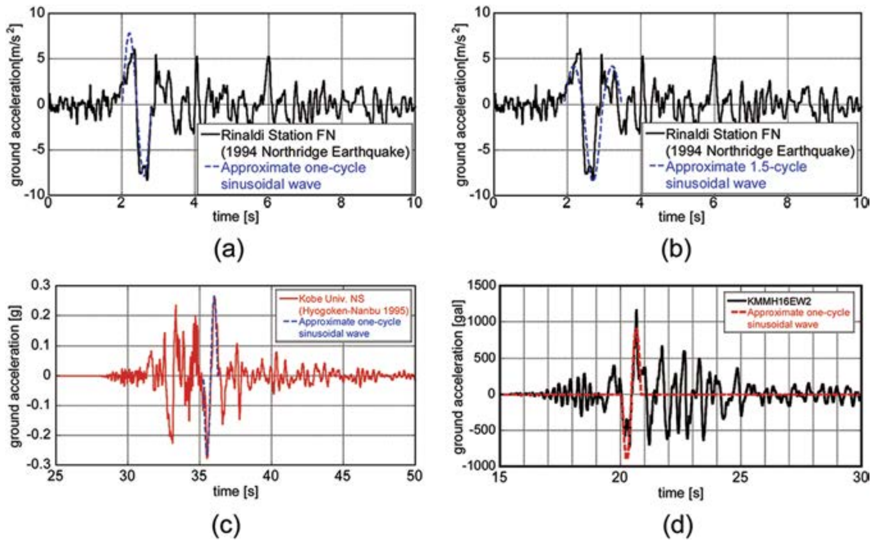

Figure 1.1 Recorded ground motions with pulse-type main parts modeled by simple sinusoidal wavelets, (a) Rinaldi Station FN (Northridge 1994) vs one-cycle sine wave, (b) Rinaldi Station FN (Northridge 1994) vs 1.5-cycle sine wave, (c) Kobe Univ. NS (Hyogoken-Nanbu 1995) vs one-cycle sine wave, (d) Mashiki EW (Kumamoto 2016, April 16) vs one-cycle sine wave.

near-fault ground motion is usually characterized by a one-cycle or a few-cycle sinusoidal wave as shown in Figure 1.1, and this part greatly influences the maximum deformation of building structures. If such one-cycle or a few-cycle sinusoidal wave as the main part of the near-fault ground motion can be represented by a double impulse as shown in Figure 1.2, the elastic-plastic response (continuation of free-vibrations) can be derived by using the full advantage of an energy balance approach without solving the equation of motion directly. The input of impulse is expressed by the instantaneous change of velocity of the structural mass leading to the instantaneous input of kinetic energy.

In the earthquake-resistant design, the resonance and the role of damping are two key issues, and they have been investigated extensively. In particular, since the resonance brings worse and critical effects to structures, it has been treated as a main theme in the earthquake-resistant design of structures. While the resonant equivalent frequency has to be computed for a specified input level by changing the excitation frequency in a parametric manner in case of treating the sinusoidal input (Caughey 1960a, b, Iwan 1961, 1965, Roberts and Spanos 1990, Liu 2000), no iteration is required in the method for the double impulse explained in this book. This is because the resonant equivalent frequency can be obtained directly without repetitive procedure.

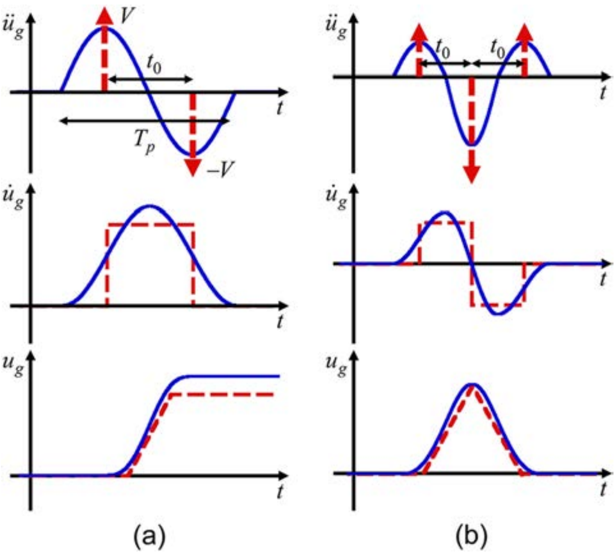

Figure 1.2 Simplification of ground motion (acceleration, velocity, displacement): (a) Fling-step input and double impulse, (b) Forward-directivity input and triple impulse (Kojima and Takewaki 2015).

In the double impulse, the analysis can be done without the input frequency (timing of impulses) before the second impulse is inputted. The resonance can be proved by using energy investigation, and the critical timing of the second impulse can be characterized as the time with zero restoring force. The maximum elastic-plastic response after impulse can be obtained by equating the initial kinetic energy computed by the initial velocity to the sum of hysteretic and elastic strain energies. It should be pointed out that only critical response (upper bound) is captured by the method explained in this book, and the critical resonant frequency can be obtained automatically for the increasing input level of the double impulse.

In the history of the seismic-resistant design of building structures, the earthquake input energy has played an important role together with deformation and acceleration (for example, Housner 1959, 1975, Berg and Thomaides 1960, Housner and Jennings 1975, Zahrah and Hall 1984, Akiyama 1985, Leger and Dussault 1992). While deformation and acceleration can predict and evaluate the local performance of a building structure mainly for serviceability, the energy can evaluate the global performance of a building structure mainly for safety. Especially energy is appropriate for describing the performance of building structures of different sizes in a unified manner because energy is a global index different from deformation and acceleration as local indices. In fact, in Japan, there are three criteria in parallel: force, deformation, and energy. In 1981, the force was introduced as a criterion for safety, and in 2000, the deformation was introduced as a criterion for safety. More recently, in 2005, the input energy evaluated from the design velocity response spectrum was used as a criterion. These three criteria are used now in parallel (BSL in Japan 1981, 2000, 2005).

A theory of earthquake input energy to building structures under single impulse was shown to be useful for disclosing the property of the energy transfer function (Takewaki 2004, 2007). This property means that the area of the energy transfer function is constant. The property of the energy transfer function similar to the case of a simple single-degree-of-freedom (SDOF) model has also been clarified for a swaying-rocking model. By using this property, the mechanism of earthquake input energy to the swaying-rocking model including the soil amplification has been made clear under the input of single impulse (Kojima et al. 2015b). However single impulse may be unrealistic because the frequency characteristic of input cannot be expressed by this input. In order to resolve such an issue, the double impulse is introduced in this book. Furthermore, because the elastic-plastic response is treated, the time-domain formulation is introduced in this book (Kojima and Takewaki 2015).

1.2 DOUBLE IMPULSE AND CORRESPONDING ONE-CYCLE SINE WAVE WITH THE SAME FREQUENCY AND SAME MAXIMUM FOURIER AMPLITUDE

The velocity amplitude V of the double impulse is related to the maximum velocity of the corresponding one-cycle sine wave with the same frequency (the period is twice the interval of the double impulse) so that the maximum Fourier amplitudes of both inputs coincide (Kojima 2018, Akehashi et al. 2018). The detail is explained in this section.

The double impulse is expressed by

$$\ddot{u}_g(t) = V\delta(t) - V\delta(t - t_0) \tag{1.1}$$

where V is the velocity amplitude of the double impulse and $\delta(t)$ is the Dirac delta function. The Fourier transform of Eq. (1.1) can be obtained as

$$\ddot{U}_g(\omega) = V\left(1 - \mathrm{e}^{-\mathrm{i}\omega t_0}\right) \tag{1.2}$$

Let A_p, T_p, $\omega_p = 2\pi/T_p$ denote the acceleration amplitude, the period and the circular frequency of the corresponding one-cycle sine wave, respectively. The acceleration wave $\ddot{u}_g^{\mathrm{SM}}$ of the corresponding one-cycle sine wave is expressed by

$$\ddot{u}_g^{\mathrm{SM}} = A_p \sin(\omega_p t) \quad \left(0 \le t \le T_p = 2t_0\right) \tag{1.3}$$

The time interval t_0 of two impulses in the double impulse is related to the period T_p of the corresponding one-cycle sine wave by $T_p = 2t_0$. Although the starting points of both inputs differ by $t_0/2$, the starting time of one-cycle sine wave does not affect the Fourier amplitude. For this reason, the starting time of the one-cycle sine wave will be adjusted so that the responses of both inputs correspond well. In this section, the relation of the velocity amplitude V of the double impulse with the acceleration amplitude A_p of the corresponding one-cycle sine wave is derived. The ratio a of A_p to V is introduced by

$$A_p = aV \tag{1.4}$$

The Fourier transform of $\ddot{u}_g^{\mathrm{SM}}$ in Eq. (1.3) is computed by

$$\ddot{U}_g^{\mathrm{SM}}(\omega) = \int_0^{2t_0} \left\{A_p \sin(\omega_p t)\right\} e^{-i\omega t} dt = \frac{\pi t_0 A_p}{\pi^2 - (\omega t_0)^2}\left(1 - e^{-2t_0\omega i}\right) \tag{1.5}$$

From Eqs. (1.2) and (1.5), the Fourier amplitudes of both inputs are expressed by

$$\left|\ddot{U}_g(\omega)\right| = V\sqrt{2 - 2\cos(\omega t_0)} \tag{1.6}$$

$$\left|\ddot{U}_g^{\mathrm{SW}}(\omega)\right| = A_p \left|\frac{2\pi t_0}{\pi^2 - (\omega t_0)^2}\sin(\omega t_0)\right| \tag{1.7}$$

The coefficient a can be derived from Eqs. (1.4), (1.6), (1.7), and the equivalence of the maximum Fourier amplitude $\left|\ddot{U}_g(\omega)\right|_{\max} = \left|\ddot{U}_g^{\mathrm{SW}}(\omega)\right|_{\max}$.

$$a(t_0) = \frac{A_p}{V} = \frac{\max\left|\sqrt{2 - 2\cos(\omega t_0)}\right|}{\max\left|\dfrac{2\pi t_0}{\pi^2 - (\omega t_0)^2}\sin(\omega t_0)\right|} \tag{1.8}$$

In the numerator of Eq. (1.8), $\max\left|\sqrt{2 - 2\cos(\omega t_0)}\right| = 2$ holds. The denominator $\max|2\pi t_0 \sin(\omega t_0)/\{\pi^2 - (\omega t_0)^2\}|$ in Eq. (1.8) will be evaluated next.

Let us define the function $f(x)$ given by

$$f(x) = \frac{1}{\pi^2 - x^2}\sin x \tag{1.9}$$

The maximum value $f_{\max}$ of $f(x)$ and the corresponding argument $x = x_0$ can be obtained as follows.

$$x_0 = 2.63099585\ldots \tag{1.10}$$

$$f_{\max} = f\left(x = x_0\right) = 0.165802809\ldots \tag{1.11}$$

The values in Eqs. (1.10) and (1.11) were obtained numerically. From Eqs. (1.8)–(1.11), the coefficient a is expressed as a function of the time interval t_0 of two impulses.

$$a\left(t_0\right) = 1/\left(\pi t_0 f_{\max}\right) \tag{1.12}$$

Figure 1.3(a) shows the relation between t_0 and a. Furthermore, Figure 1.3(b) presents examples of the Fourier amplitudes of both inputs with the same maximum Fourier amplitude. Since the Fourier amplitudes of both inputs differ greatly in larger frequencies, further investigation will be necessary in dealing with multi-degree-of-freedom models.

Consider next the ratio of the maximum velocity V_p of the one-cycle sine wave to the velocity amplitude V of the double impulse. The velocity $\dot{u}_g^{\mathrm{SW}}$ of the one-cycle acceleration sine wave is expressed by

$$\dot{u}_g^{\mathrm{SW}} = \int_0^t \ddot{u}_g^{\mathrm{SW}} dt = \int_0^t A_p \sin\left(\omega_p t\right) dt = \frac{A_p}{\omega_p}\left\{1 - \cos\left(\omega_p t\right)\right\} \tag{1.13}$$

From Eq. (1.13), the maximum velocity V_p of the one-cycle sine wave can be expressed by

$$V_p = 2A_p/\omega_p \tag{1.14}$$

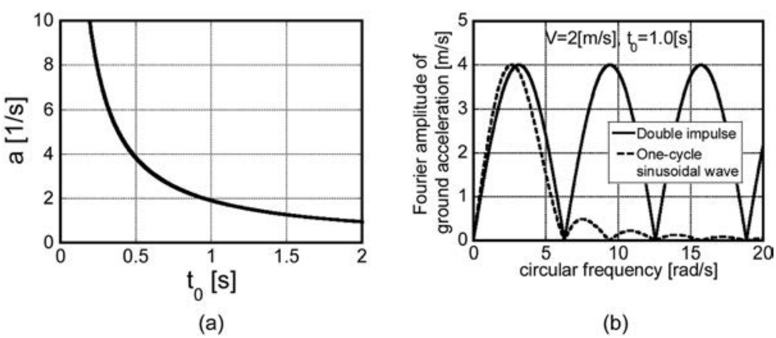

Figure 1.3 Relation of amplitudes between double impulse and one-cycle sine wave, (a) Coefficient a ($=A_p/V$) with respect to impulse timing t_0, (b) Fourier amplitude of double impulse and one-cycle sine wave (Akehashi et al. 2018).

Eqs. (1.4), (1.12), (1.14), and $\omega_p = \pi/t_0$ lead to the relation between V_p and V.

$$V_p = \left\{2/\left(\pi^2 f_{\max}\right)\right\} V \tag{1.15}$$

From Eqs. (1.11) and (1.15), V_p/V is expressed as

$$V_p/V = 2/\left(\pi^2 f_{\max}\right) = 1.22218898\ldots \tag{1.16}$$

It can be found from Eq. (1.16) that if the maximum Fourier amplitudes of both inputs are the same, the ratio of V_p to V becomes constant. The modulated one-cycle sine wave will be called "the corresponding one-cycle sine wave."

1.3 ENERGY BALANCE UNDER EARTHQUAKE GROUND MOTION AND IMPULSE

The energy balance under an earthquake ground motion and the corresponding impulse, which plays a central role in this book, is explained in this section.

1.3.1 Undamped model

Consider a single-degree-of-freedom (SDOF) model of mass m and stiffness k subjected to an earthquake ground acceleration $\ddot{u}_g(t)$ (see Figure 1.4(a)). Assume the initial condition $x(0) = \dot{x}(0) = 0$. Let $x(t)$ denote the displacement of the mass relative to the ground. The equation of motion of the model can be described by

$$m\ddot{x} + kx = -m\ddot{u}_g \tag{1.17}$$

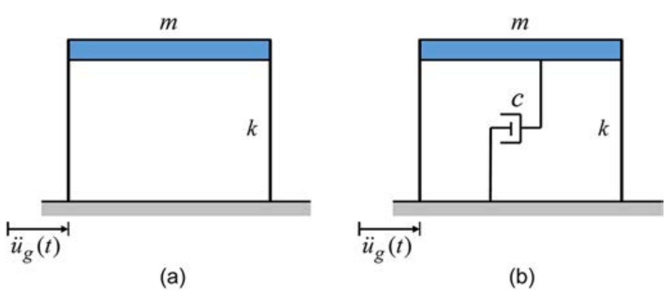

Figure 1.4 SDOF model, (a) Undamped model, (b) Damped model.

Let's consider the vibration of the model during $t = [0, t_0]$. Multiplication of the velocity $\dot{x}$ on both sides of Eq. (1.17) and integration over $t = [0, t_0]$ provide

$$\left[(1/2)m\dot{x}(t)^2\right]_0^{t_0} + \left[(1/2)kx(t)^2\right]_0^{t_0} = -\int_0^{t_0} m\ddot{u}_g(t)\dot{x}(t)dt \tag{1.18}$$

With the initial condition, Eq. (1.18) leads to

$$(1/2)m\dot{x}(t_0)^2 + (1/2)kx(t_0)^2 = -\int_0^{t_0} m\ddot{u}_g(t)\dot{x}(t)dt \tag{1.19}$$

The first term of the left-hand side of Eq. (1.19) indicates the kinetic energy and the second term is the strain energy. It is important to note that, only after the response $x(t)$ is computed numerically, the energy balance can be evaluated.

On the other hand, consider that the model is subjected to the single impulse $\ddot{u}_g(t) = V\delta(t)$. In this case, since $x(0) = 0, \dot{x}(0) = -V$, Eq. (1.18) can be expressed as

$$(1/2)m\dot{x}(t_0)^2 - (1/2)mV^2 + (1/2)kx(t_0)^2 = 0 \tag{1.20}$$

Assume that the maximum displacement occurs at $t = t_0$ (see Figure 1.5(a)). Since $\dot{x}(t_0) = 0$, Eq. (1.20) yields

$$(1/2)kx(t_0)^2 = (1/2)mV^2 \tag{1.21}$$

Eq. (1.21) means that the maximum displacement can be obtained from the energy balance law.

This principle can be applied to an elastic-plastic model. Let $f(x)$ denote the nonlinear restoring-force characteristic of the elastic-plastic model. For this model, Eq. (1.20) can be modified into

$$(1/2)m\dot{x}(t_0)^2 - (1/2)mV^2 + \int_0^{x_0} f(x)dx = 0 \tag{1.22}$$

where $x_0 = x(t_0)$. Assuming again that the maximum displacement occurs at $t = t_0$, Eq. (1.22) leads to

$$\int_0^{x_0} f(x)dx = (1/2)mV^2 \tag{1.23}$$

By specifying an explicit expression of $f(x)$, e.g., an elastic-perfectly plastic one (see Figure 1.5(b)) or a bilinear one, Eq. (1.23) provides the maximum displacement x_0.

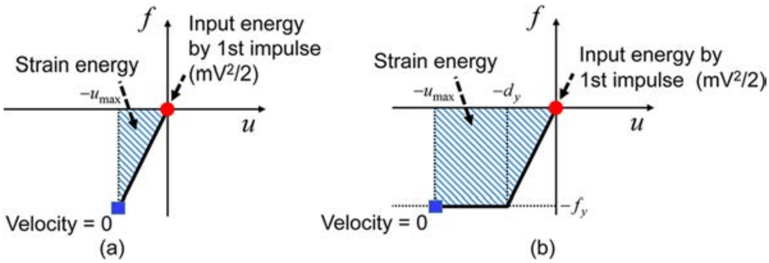

Figure 1.5 Input energy (kinetic energy) by impulse and strain energy, (a) Elastic model, (b) Elastic-plastic model.

1.3.2 Damped model

Consider next a SDOF model of mass m, stiffness k, viscous damping coefficient c subjected to an earthquake ground acceleration $\ddot{u}_g(t)$ (see Figure 1.4(b)). Assume again the initial condition $x(0) = \dot{x}(0) = 0$. The equation of motion of this model can be expressed by

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{u}_g \tag{1.24}$$

Consider the vibration of the model during $t = [0, t_0]$. Multiplication of the velocity $\dot{x}$ on both sides of Eq. (1.24) and integration over $t = [0, t_0]$ provide

$$\left[(1/2)m\dot{x}(t)^2\right]_0^{t_0} + \int_0^{t_0} c\dot{x}(t)^2\,dt + \left[(1/2)kx(t)^2\right]_0^{t_0} = -\int_0^{t_0} m\ddot{u}_g(t)\dot{x}(t)\,dt \tag{1.25}$$

Application of the initial condition to Eq. (1.25) leads to

$$(1/2)m\dot{x}(t_0)^2 + \int_0^{t_0} c\dot{x}(t)^2\,dt + (1/2)kx(t_0)^2 = -\int_0^{t_0} m\ddot{u}_g(t)\dot{x}(t)\,dt \tag{1.26}$$

The first term of the left-hand side of Eq. (1.26) indicates the kinetic energy and the third term is the strain energy. The second term is the energy dissipated by the viscous damping. As in the undamped case, it should be remarked that only after the response $x(t)$ is computed numerically can the energy balance be evaluated.

On the other hand, consider that the model is subjected to the single impulse $\ddot{u}_g(t) = V\delta(t)$. In this case, since $x(0) = 0, \dot{x}(0) = -V$, Eq. (1.26) can be expressed as

$$(1/2)m\dot{x}(t_0)^2 - (1/2)mV^2 + \int_0^{t_0} c\dot{x}(t)^2\,dt + (1/2)kx(t_0)^2 = 0 \tag{1.27}$$

Assume that the maximum displacement occurs at $t = t_0$. Since $\dot{x}(t_0) = 0$, Eq. (1.27) yields

$$(1/2)kx(t_0)^2 = (1/2)mV^2 - \int_0^{t_0} c\dot{x}(t)^2\, dt \tag{1.28}$$

Eq. (1.28) indicates that the maximum displacement can be obtained from the energy balance law. However, the energy dissipated by the viscous damping has to be evaluated in an appropriate manner in this case. This principle can also be applied to an elastic-plastic model. The left-hand side of Eq. (1.28) can be replaced by $\int_0^{x_0} f(x)dx$. If the second term of the right-hand side of Eq. (1.28) can be expressed approximately in terms of x_0, x_0 can be evaluated by Eq. (1.28) after replacing the left-hand side of Eq. (1.28) by the term $\int_0^{x_0} f(x)dx$.

It can be shown that the right-hand side of Eq. (1.25) expresses the total input energy to the model. The work done by the ground input on the SDOF model (see Figure 1.6) can be expressed by

$$\int_0^{x_0} m\{\ddot{u}_g(t) + \ddot{x}(t)\}dx = \int_0^{t_0} m\{\ddot{u}_g(t) + \ddot{x}(t)\}\dot{x}(t)dt \tag{1.29}$$

Integration by parts of Eq. (1.29) leads to

$$\int_0^{t_0} m\{\ddot{u}_g(t) + \ddot{x}(t)\}\dot{x}(t)dt = \left[(1/2)m\dot{x}\dot{u}_g\right]_0^{t_0} - \int_0^{t_0} m\dot{x}\ddot{u}_g dt + \left[(1/2)m\dot{u}_g{}^2\right]_0^{t_0} \tag{1.30}$$

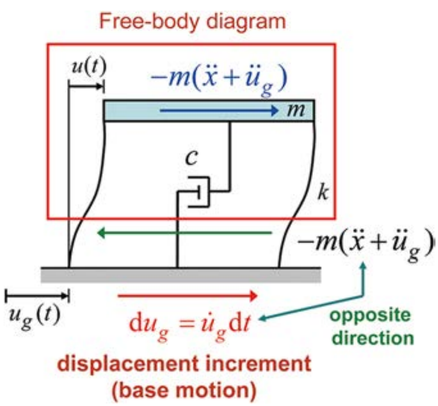

Figure 1.6 SDOF model subjected to earthquake ground motion (Kojima et al. 2015a).

The condition $\dot{u}_g(0) = \dot{u}_g(t_0) = 0$ provides

$$\int_0^{t_0} m\{\ddot{u}_g(t) + \ddot{x}(t)\}\dot{x}(t)dt = -\int_0^{t_0} m\dot{x}\ddot{u}_g dt \tag{1.31}$$

This means that, if t_0 indicates the final time of input ground motion, the right-hand side of Eq. (1.25) expresses the total input energy to the model.

1.4 CRITICAL INPUT TIMING OF SECOND IMPULSE IN DOUBLE IMPULSE

The energy balance approach explained in the above section is often used in the seismic-resistant design of structures. A more important aspect in the present approach is the critical timing of the second impulse in the double impulse input. The energy balance law explained in the above section (after the input of the first impulse until the maximum displacement) can also be used in a similar manner for the response process after the second impulse until the next (inverse-direction) maximum displacement. The initial velocity V just after the first impulse has to be changed to $v^*_{\max} + V$ where $v^*_{\max}$ is the velocity of mass just before the second impulse is applied (see Figure 1.7). Because the velocity attains the maximum at the point of zero restoring force, the subscript "max" is given. Since it can be proved that the critical timing of the second impulse is the time when the restoring force attains zero (see Chapter 2), the area of the nonlinear restoring-force characteristic from the zero restoring force (applied point of the second impulse) to the next maximum displacement $u_{\max2}$ can be evaluated without difficulty. A more detailed explanation will be provided in Chapter 2.

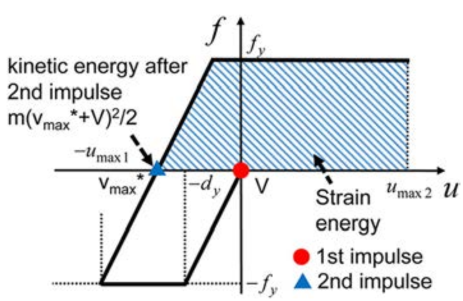

Figure 1.7 Critical input timing of second impulse attaining maximum value of $u_{\max2}$.

Table 1.1 Comparison of conventional methods and proposed method for nonlinear resonant analysis.

Conventional method 1960, 1961 Caughey, Iwan	**Proposed method 2015 Kojima and Takewaki**
① Steady state ② Impossible for elastic–perfectly plastic ③ Repetition required (equivalent parameters/ resonant frequency) afterward, stochastic linearization(transient response)	① Transient and steady state ② Possible even for elastic–perfectly plastic (any bilinear) ③ No repetition required <Proposed method enables> →Closed-form **critical** response of elastic-plastic structure →Derive **resonant frequency** (impulse interval) without repetition →Closed-form **noncritical** response of elastic-plastic structure based on closed-form **critical** response

1.5 COMPARISON OF CONVENTIONAL METHODS AND THE PROPOSED METHOD FOR NONLINEAR RESONANT ANALYSIS

Table 1.1 shows the comparison of conventional methods and the proposed method for nonlinear resonant analysis. In the conventional methods represented by Caughey (1960a, b) and Iwan (1965), a steady state was treated and the analysis of the elastic–perfectly plastic model was impossible because of the computational stability. In addition, repetition was required for obtaining the convergent values or solving the transcendental equations. On the other hand, in the proposed method, both transient and steady-state responses can be dealt with. In addition, the analysis of the elastic–perfectly plastic model is possible and no iteration is required. Furthermore, the proposed method enables (1) the derivation of closed-form critical response of elastic-plastic structures, (2) the capture of the resonant frequency without repetition, and (3) the derivation of closed-form expressions on the noncritical responses of elastic-plastic structures. The detail of these facts will be explained in subsequent chapters.

1.6 OUTLINE OF THIS BOOK

In Chapter 1, the motivation of the proposed earthquake energy balance approach using impulses is explained. The simplification of fling-step near-fault ground motions into the double impulse is explained and the earthquake energy balance approach is introduced for undamped elastic–perfectly plastic (EPP) single-degree-of-freedom (SDOF) models under the critical double impulse.

In Chapter 2, a closed-form expression is derived by using the energy balance law for the maximum response of an undamped EPP SDOF model under the critical double impulse input.

In Chapter 3, a closed-form expression is derived by using the energy balance law for the critical response of an undamped EPP SDOF model under the triple impulse input as a representative of forward-directivity near-fault ground motions. Complicated phenomena on the critical response under the triple impulse input are investigated. The existence of the third impulse brings such complicated phenomena on the critical response.

In Chapter 4, the multiple impulse input of equal time interval is introduced as a substitute of long-duration earthquake ground motions which is expressed in terms of harmonic waves. A closed-form expression is derived by using the energy balance law for the maximum response of an EPP SDOF model under the critical multiple impulse input.

In Chapter 5, the double impulse is introduced as a good substitute for the one-cycle sinusoidal wave in representing the main part of a near-fault ground motion. A closed-form expression is derived by using the energy balance law for the maximum deformation of an EPP SDOF model with viscous damping under the critical double impulse. It uses (1) a quadratic function to approximate the damping force-deformation relation, (2) the assumption that the zero restoring-force timing in the unloading process is the critical timing of the second impulse, and (3) the energy balance law for the elastic-plastic system with viscous damping.

In Chapter 6, the multi-impulse is introduced as a substitute of the long-duration ground motion and the closed-form expression is derived for the steady-state elastic-plastic response of a bilinear hysteretic SDOF system under the critical multi-impulse. While the computation of the resonant equivalent frequency of the elastic-plastic system is a tough task in the conventional method dealing directly with the sinusoidal wave (Iwan 1961), it is shown that the steady-state elastic-plastic response under the critical multi-impulse can be obtained in closed form (without repetition) by using the energy balance law and the critical time interval of the multi-impulse (the resonant frequency) can also be obtained in closed form for the increasing input level.

In Chapter 7, the double impulse is introduced as a substitute of the fling-step near-fault ground motion. A closed-form expression for the elastic-plastic response of an EPP SDOF model on the compliant (flexible) ground by the "critical double impulse" is derived based on the expression for the corresponding EPP SDOF model with a fixed base. It is shown that the closed-form expression for the critical elastic-plastic response of the superstructure enables the clarification of the relation of the critical elastic-plastic response of the superstructure with the ground stiffness.

In Chapter 8, the closed-form expression for the maximum elastic-plastic response of a bilinear hysteretic SDOF model under the critical double impulse (Kojima and Takewaki 2016a) is extended to a dynamic stability (collapse) problem of elastic-plastic SDOF models with negative post-yield stiffness. Negative post-yield stiffness is treated to consider the P-delta effect. The double impulse is used as a substitute for the fling-step near-fault ground motion. The dynamic stability (collapse) limit of the velocity level is obtained for the critical input case by using the energy balance law.

In Chapter 9, an explicit limit on the input velocity level of the double impulse as a representative of the principal part of a near-fault ground motion is derived for the overturning of a rigid block. The energy balance law and the conservation law of angular momenta of the rigid block are used for describing and determining the rocking response under the critical double impulse.

In Chapter 10, because the response of 2DOF elastic-plastic building structures is quite complicated due to the phase lag between two masses compared to SDOF models for which a closed-form critical response can be derived, the upper bound of the critical response is introduced by using the convex model. The accuracy of the upper bound is then investigated.

In Chapter 11, an innovative method for optimal viscous damper placement is explained for EPP multi-degree-of-freedom (MDOF) shear building structures subjected to the critical double impulse as a representative of near-fault ground motions. Simultaneous treatment of nonlinear MDOF structures and uncertainty in the selection of input is the most remarkable point that has never been overcome in the past.

In Chapter 12, some future directions are explained which include

1. Treatment of noncritical case
2. Extension to nonlinear viscous damper and hysteretic damper
3. Treatment of uncertain fault-rupture model and uncertain deep ground property
4. Application to passive control systems for practical tall buildings
5. Stopper system for pulse-type ground motion of extremely large amplitude
6. Repeated single impulse in the same direction for repetitive ground motion input
7. Robustness evaluation
8. Principles in seismic resistant design (constant energy law, constant displacement law, law for resonant case)

In particular, application of the proposed approach to more practical situations will certainly enhance the broad and profound significance of the proposed innovative approach to earthquake structural engineering in nonlinear structural dynamics.