APPLIED ENGINEERING MATHEMATICS

Brian Vick

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Preface

This book is a practical approach to engineering mathematics with an emphasis on visualization and applications. This book is intended for undergraduate and introductory graduate courses in engineering mathematics and numerical analysis. It is aimed at students in all branches of engineering and science. This book contains a comprehensive blend of fundamental physics, applied science, mathematical analysis, numerical computation, and critical thinking. It contains both theory and application, with the applications interwoven with the theory throughout the text. The emphasis is visual rather than procedural.

This book covers some of the most important mathematical methods and tools used in applied engineering. After an introduction in Chapter 1, this book begins with a summary of the most important principles of physics in Chapter 2, followed by Chapter 3 dedicated to the proper mathematical modeling of physical processes. Then the basics of calculus are presented in Chapter 4, including a thorough treatment of numerical integration. Next the essentials of linear algebra are presented in Chapter 5. Then the topic of nonlinear algebra, with an emphasis on numerical methods, is presented in Chapter 6. The topic of the remaining five chapters is ordinary differential equations. An introduction is presented in Chapter 7, giving an overview and fundamental understanding of the origins and meaning of differential equations. Then the Laplace transform method is presented in Chapter 8. A thorough treatment of the numerical solution of ordinary differential equations is then described in Chapter 9. Chapters 10 and 11 are on first-order and second-order ordinary differential equations, respectively, and cover some important examples and characteristics of first- and second-order equations, including bifurcations. Although the chapters stand alone and can be studied in any order, the organization of this book is a logical sequence from mathematical modeling to solution methodology.

A distinctive characteristic of the text is that the visual approach is emphasized as opposed to excessive proofs and derivations. The reader will take away insight and deeper understanding with the visual images and thus have a better chance of remembering and using the mathematical methods. Many of the figures were created and computations performed with Mathematica, and the dynamic and interactive codes accompanying the examples are available for the reader to explore on their own.

My style has been developed from experience as a long-time teacher and researcher in a variety of engineering and mathematical courses. My background includes the areas of heat transfer, thermodynamics, engineering design, computer programming, numerical analysis, and system dynamics at both undergraduate and graduate levels. Also, my experiences in various research areas have motivated some of the specific topics and examples.

I would like to express thanks to my wife Linda and our children Kristen, Kelsey, Alison, and Everett for all the great times we have had and all your patience with me. I am so blessed and I love you all.

About the Author

Brian Vick received his bachelor's degree in 1976, master's degree in 1978, and doctoral degree in 1981, all in mechanical engineering, all from North Carolina State University. In 1982, he joined the Department of Mechanical Engineering at Virginia Tech as an assistant professor and was promoted to associate professor in 1989. Dr. Vick's main research and teaching activities have been in the areas of heat transfer, applied mathematics, numerical analysis, tribology, wave mechanics, nonlinear dynamics, and parameter estimation.

He has taught undergraduate courses in the areas of heat transfer, thermodynamics, thermal systems design, engineering design, computer programming, numerical analysis, and system dynamics. He has taught graduate level courses in the areas of conduction heat transfer, convection heat transfer, and advanced engineering mathematics.

Dr. Vick has conducted research projects in a variety of areas including the wave nature of heat transport, analysis of phase change phenomena, laminarization in highly accelerated flows, heat transfer augmentation in heat exchangers, modeling and analysis of thermal storage systems, thermal analysis of slip ring designs, development of boundary element methods, electro-thermo-mechanical analysis of shape memory alloy actuators, heat transfer in heterogeneous materials, and thermo-mechanical and thermionic emission studies in tribological processes.

He is currently working on research with NASA on the newest generation of earth radiation budget instruments. He also is actively working in the parameter estimation and inverse problem area with application to radiation instruments, estimation of blood perfusion in living tissue, and the dynamics of biofilm formation.

Chapter 1

Overview

CHAPTER OBJECTIVES

This chapter is an introduction to and overview of the educational philosophy employed in this book. The physical applications and mathematical methods are briefly summarized.

Specific objectives and topics covered are

- Objectives
- Educational philosophy
- Physical processes: conservation laws, rate equations, and property relations
- Mathematical models: algebraic equations, ordinary differential equations, and partial differential equations
- Solution methods for algebraic and differential equations
- Software

1.1 OBJECTIVES

This is a comprehensive book consisting of a blend of fundamental physics, applied science, mathematical analysis, numerical computation, and critical thinking. It contains both theory and application, with the applications interwoven with the theory throughout the text. The emphasis is visual rather than procedural.

The specific goals consist of the following.

- **Physical Processes**

  To gain a fundamental understanding of the physical processes, fundamental principles, and mathematical formulations of physical problems.
- **Mathematical Methods**

  To learn mathematical techniques for solving model equations.
- **Software**

  To learn the use of software packages, such as *Mathematica* or *MATLAB*®, to program solutions, perform calculations, and create graphics. Computational studies and graphics enhance insight into the effect of important parameters in addition to building a fundamental understanding of physical mechanisms.

- **Insight and Critical Thinking**
  To develop a sound foundation for problem-solving using critical thinking, interpretation, and reasoning skills. Applications are used extensively to foster *insight* and *intuition*.

1.2 EDUCATIONAL PHILOSOPHY

The following *guiding principles* **are** fundamental **to learning;**

- We learn by active participation, not by passive observation. That is, we learn by doing, not just by watching.
- A picture is worth a thousand words. The human brain is made to process *visual* information. More information can be assimilated in a few seconds by looking at graphics than by studying that same information for months from a printout of numerical values.
- We are all responsible for our own learning. You need to be self-motivated and have the desire to learn.

An important issue is *knowledge* **versus** *information*. There is an old saying:

Give me a fish and I'll eat for a day. Teach me to fish and I'll eat for a lifetime.

With today's information explosion resulting from the internet, this old saying is more relevant than ever. Knowledge and fundamental reasoning skills are giving way to an unmanageable amount of information consisting of seemingly unconnected facts and figures. Hopefully, a greater emphasis can be placed on knowledge as opposed to just raw information. True progress requires a balance between raw information and basic knowledge.

In this same vein, too much *coverage of material* at the expense of *depth of understanding* can be the enemy of learning and leads to memorization and frustration. Depth of understanding is considered to be far more important than coverage of more topics.

In addition to developing an appreciation for and mastering knowledge of the fundamental physical principles and mathematical techniques, a major goal of this course is to develop basic learning skills and strategies. These include

- Reasoning and interpretive skills—the foundation of problem-solving
- Pattern recognition skills
- Adaptability—learning to recognize abstract concepts from specific applications and conversely, learning to apply abstract concepts to specific applications
- Thinking for yourself
- Motivation, enthusiasm, and passion

Regardless of your potential, there is no substitute for hard work. One must persist and struggle with difficult concepts until they are understood. Learning is a lifelong activity and is the key to success. Let's make it fun and exciting!

1.3 PHYSICAL PROCESSES

Observations of the physical world indicate that all processes are governed by a small number of fundamental principles. These are *conservation principles*, which are supplemented by *rate equations* and *property relations*. Together, they form a complete description of nature. Although there are only a handful of principles, there are countless applications and special cases. These principles are summarized in the following and are described in more detail in Chapters 2 and 3.

- The ***conservation laws*** are:
  - Conservation of mass: continuity
  - Conservation of momentum: Newton's second law
  - Conservation of energy: first law of thermodynamics
  - Conservation of chemical species
  - Conservation of electrical charge

  These are general principles and are *independent* of the material.

- The ***rate equations*** supplement the conservation principles. The most important ones are:
  - Heat conduction: Fourier's law
  - Heat convection: Newton's law of cooling
  - Thermal radiation
  - Viscous fluid shear: Newton's viscosity law
  - Binary mass diffusion: Fick's law
  - Electrical conduction: Ohm's law
  - Stress-strain: Hooke's law

  These are constitutive relations and are *dependent* on the material.

- The ***property relationships*** are also needed to complete the mathematical model. A few such relationships are:
  - Constant properties
  - Density: $\rho = \rho(T, P)$
  - Viscosity: $\mu = \mu(T, P)$
  - Specific heat: $c = c(T, P)$
  - Thermal conductivity: $k = k(T, P)$

  These are material-dependent characteristics. Many times, it can be justified to assume constant properties.

1.4 MATHEMATICAL MODELS

The mathematical description of physical problems generally leads to an equation or set of equations involving either *algebraic expressions* or *derivatives* (i.e., differential equations). Differential equations that are a function of only one independent variable are referred to as *ordinary differential equations*. Those that depend on two or more independent variables are called *partial differential equations*. As displayed in the following figures, these equations can be classified according to the number of equations or dependent variables and whether they are linear or nonlinear.

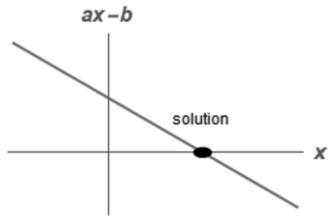
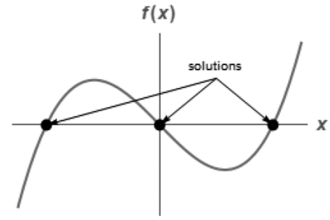
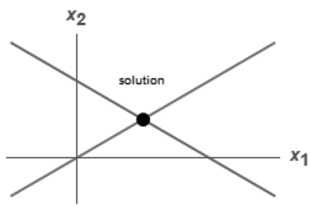
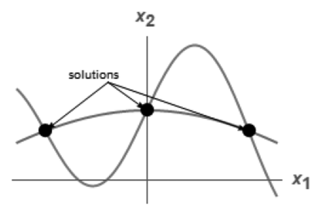
<table>
<tr><th>Number of Equations</th><th>Linear</th><th>Nonlinear</th></tr>
<tr><td>1</td><td>$a \cdot x = b$

</td><td>$f(x) = 0$

</td></tr>
<tr><td>2</td><td>$a_{11}x_1 + a_{12}x_2 = b_1$
$a_{21}x_1 + a_{22}x_2 = b_2$

</td><td>$f_1(x_1, x_2) = 0$
$f_2(x_1, x_2) = 0$

</td></tr>
<tr><td>n</td><td>$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$
$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$
$\vdots$
$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$</td><td>$f_1(x_1, x_2, \cdots, x_n) = 0$
$f_2(x_1, x_2, \cdots, x_n) = 0$
$\vdots$
$f_n(x_1, x_2, \cdots, x_n) = 0$</td></tr>
</table>

Figure 1.1 Classification of algebraic equations.

1.4.1 Algebraic Equations

Algebraic equations can be classified according to the characteristics shown in Figure 1.1. Linear algebraic equations can be solved with procedures such as Gaussian elimination. Many practical problems are modeled with a well-behaved set of linear equations with a unique solution. On the other hand, nonlinear equations can have multiple solutions or no solutions at all and can be tricky to solve.

1.4.2 Ordinary Differential Equations

Differential equations are used to model the dynamical behavior of physical systems. They are rich in application and meaning. A brief summary of ordinary differential equations (ODEs) and partial differential equations (PDEs) is presented in Figures 1.2 and 1.3, respectively.

ODEs can further be classified as *initial value problems* or *boundary value problems* depending on the auxiliary conditions. Initial value problems typically involve time as the independent variable and require starting values for the dependent variables. On the other hand, boundary value problems typically have position as the independent variable and require conditions on all the boundaries of the dependent variables.

1.4.3 Partial Differential Equations

Many types of PDEs exist, exhibiting a wide variety of characteristics. The basic elliptic, parabolic, and hyperbolic equations are displayed in Figure 1.3.

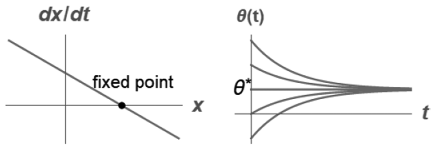
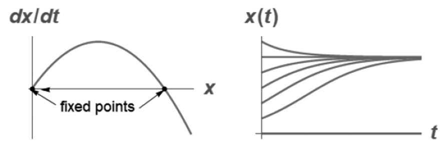
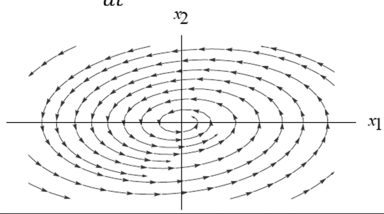
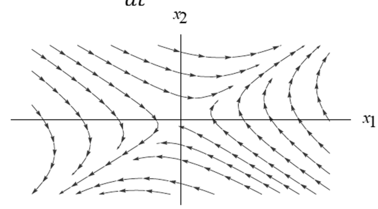
<table>
<tr><th>Dependent Variables</th><th>Linear</th><th>Nonlinear</th></tr>
<tr><td>1</td><td>$\frac{dx}{dt} = a \cdot x + b$

</td><td>$\frac{dx}{dt} = f(x)$

</td></tr>
<tr><td>2</td><td>$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2$
$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2$

</td><td>$\frac{dx_1}{dt} = f_1(x_1, x_2)$
$\frac{dx_2}{dt} = f_2(x_1, x_2)$

</td></tr>
<tr><td>n</td><td>$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + b_1$
$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + b_2$
$\vdots$
$\frac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + b_n$</td><td>$\frac{dx_1}{dt} = f_1(x_1, x_2, \cdots, x_n)$
$\frac{dx_2}{dt} = f_2(x_1, x_2, \cdots, x_n)$
$\vdots$
$\frac{dx_n}{dt} = f_n(x_1, x_2, \cdots, x_n)$</td></tr>
</table>

Figure 1.2 Classification of ordinary differential equations.

Many other variations of these basic PDEs can be formulated. In all cases, starting or initial conditions in time as well as boundary conditions in space are required.

1.5 SOLUTION METHODS

A wide variety of mathematical schemes have been proposed over the centuries to solve equations arising in engineering and applied physics. Some of the more popular and successful ones are summarized in the following.

Some commonly used methods for systems of ***linear algebraic equations*** are:

- Gaussian elimination
- *LU* decomposition
- Gauss–Seidel iteration

For ***nonlinear algebraic equations***, root find methods are employed. They include:

- Bisection
- False position
- Newton–Raphson
- Secant methods
- Golden search
- Gradient methods

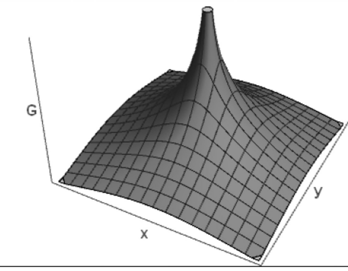
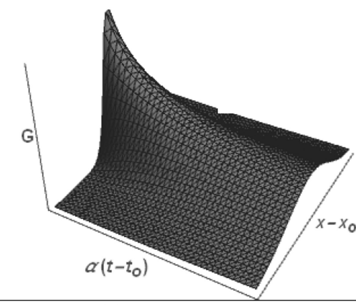
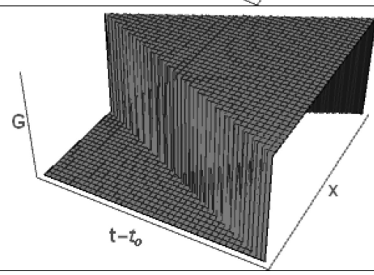
Type of Equation	Typical Equation	Fundamental Behavior
Elliptic or potential equation	$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$	
Parabolic or diffusion equation	$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$	
Hyperbolic or wave equation	$\frac{\partial^2 T}{\partial t^2} = C^2 \frac{\partial^2 T}{\partial x^2}$	

Figure 1.3 Classification of Partial differential equations.

Numerical methods are frequently used to solve ***differential equations***. Some of the most successful methods are:

- Runge–Kutta methods
- Finite difference methods
- Finite element methods
- Boundary element methods
- Cellular automata

Numerous ***analytical methods*** have been developed for ***differential equations***. Some of the most successful methods are:

- Fourier series, orthogonal function expansion, and separation of variables
- Fourier integrals and Fourier transforms
- Green's functions
- Laplace transforms
- Duhamel's method
- Integral methods
- Similarity methods

1.6 SOFTWARE

Some tremendous software packages are currently available. Two of the best and most popular choices are *Mathematica* and *MATLAB®*.

- *Mathematica* is a powerful software package and programming language, which combines numerical computations, symbolic manipulation, graphics, and text. Its symbolic manipulation capabilities are the most powerful ever developed. Mathematica is built on the powerful unifying idea that everything can be represented as a symbolic expression.
- *MATLAB®* is also a programming language that combines numerical computations with graphics. It also has symbolic manipulation capability. The basic data structure in MATLAB® (<u>Mat</u>rix <u>Lab</u>oratory) is the matrix.
- *Programming* sophisticated software packages can be powerful tools only if one has the necessary skill to program them. This requires logical and structured programming skills. These can only be achieved through tremendous study, practice, and patience.

The learning curve for a general, all-purpose, and powerful package such as Mathematica or MATLAB® can be steep. In the end, the rewards are well worth the effort required. Understanding and advancement of knowledge are greatly facilitated by the ability to program a computer to perform numerical computations, manipulate symbolic expressions, and visualize graphics.

Chapter 2

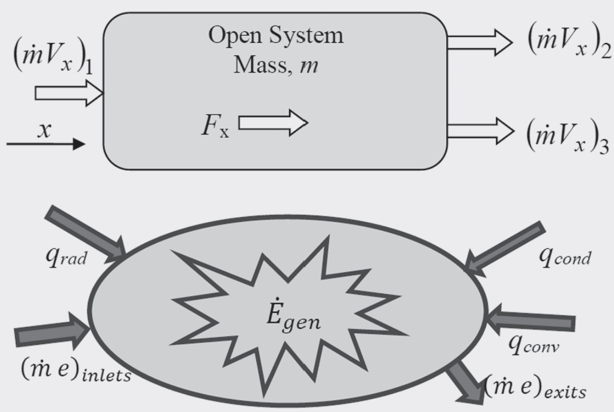
Physical Processes

CHAPTER OBJECTIVES

This chapter describes the basic principles of physics that govern processes in our world. These are postulates based on observation.

Specific objectives and topics covered are

- Physical phenomena
- Fundamental principles
- Conservation laws governing mass, momentum, and energy
- Rate equations relating potentials to flows for heat conduction, convection, radiation, viscous fluid shear, binary mass diffusion, electrical conduction, and stress
- Diffusion analogies



2.1 PHYSICAL PHENOMENA

Physical processes occurring in nature have been categorized in many different ways. Some of the more common categorizes are the following.

- Thermal
- Mechanical
- Chemical
- Electrical
- Biological

In addition, processes involving the coupling or interaction of two or more of the previous basic processes are referred to by names such as

- Thermomechanical
- Electromechanical
- Thermoelectric

Examples of coupled processes are the thermoelectric effect in thermocouples and thermomechanical effects that cause unusual behavior in shape memory alloys such as *Nitinol* (an alloy of approximately 50% nickel and 50% titanium).

Some important applications are tribology (friction, wear, and lubrication), smart materials, lasers, computers, nanostructures, aircraft design, and countless more. One of the fascinating aspects of applied mathematics is the rich and diverse number of applications, all stemming from a few basic principles.

In an attempt to understand our world, humans have classified various observed physical phenomena into these categories. Similarly, we have also departmentalized our universities and companies into categories: Mechanical Engineering Department, Chemical Engineering Department, Biological Systems Department, and so on. However, nature does not recognize these artificial divisions. During a physical process, heat and electricity flow, stresses form, friction and wear occur, and chemical reactions continually change the composition of the system. As a result, the mathematical modeling of real-life systems can be challenging. Engineers and physicists must use intuition and experience, in addition to mathematical procedures, in order to accurately model complex processes in the natural world. The process of mathematically modeling complex processes involves science and art, and perhaps a bit of luck.

2.2 FUNDAMENTAL PRINCIPLES

Observations of the physical world indicate that all processes are governed by a small number of fundamental principles. These are *conservation principles*, which are supplemented by *rate equations* and *property relations*. Together, they form a complete description of nature.

The *conservation laws* are based on the principle that the physical material making up the universe cannot be created or destroyed. The conservation laws are:

- Conservation of mass: Continuity
- Conservation of momentum: Newton's second law
- Conservation of energy: First law of thermodynamics
- Conservation of chemical species
- Conservation of electrical charge

These are general principles, *independent* of the material.

The *rate laws* relate the flow of a conserved quantity, like electric charge or energy, to a driving potential, like voltage or temperature. The rate equations are:

- Heat conduction: Fourier's law
- Heat convection: Newton's law of cooling
- Thermal radiation
- Viscous fluid shear: Newton's viscosity law
- Binary mass diffusion: Fick's law
- Electrical conduction: Ohm's law
- Stress-strain: Hooke's law