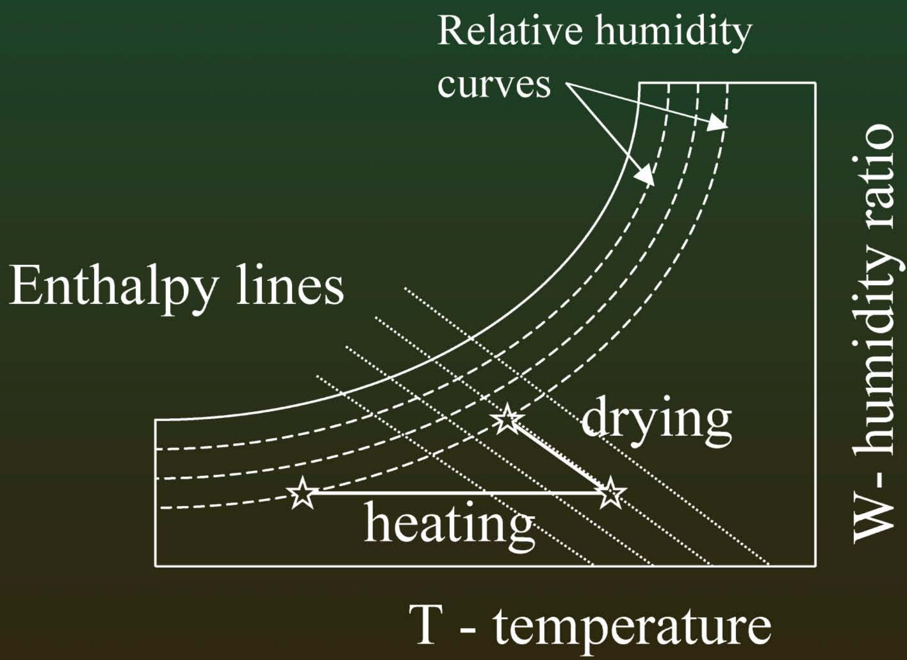
# Math Concepts *for* Food Engineering

## Second Edition

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# *Contents*

# Preface to Second Edition

For over 10 years now, we have been assessing the math skills of students entering the first food engineering course in our curriculum. There is a clear correlation between these skills and the student's ability to do well in this course. Students who enter with good math skills generally do well in engineering class. However, those students who need extra help with math can make up the gap through hard work and practice. This is the advantage that *Math Concepts for Food Engineering* brings to those students willing to work at improving their math skills.

For this second edition, we have incorporated some simple food engineering principles within the text. Without going into the detail of a food engineering textbook, some of the more important technical principles have been included relative to the learning outcomes for our food engineering class. We feel that this will give our students a better perspective of the importance of the math skills, and help them better relate these simple problems with the principles they are learning in class.

In this second edition, we have also made several other additions. First, we have incorporated various exercises throughout the text that use spreadsheets, a valuable tool for analyzing and manipulating data. The use of spreadsheets to create mathematical tools of practical use for some applications is developed in chapters 1 through 5, and these are used to help solve some of the examples in chapters 6 through 11, the second part of the text. The publisher will make the spreadsheet exercises seen in the book available on its Web site for those who purchase the book. We have also included a chapter on mass transfer, and added a simple units conversion page in the appendix. This offers a more complete reference for our students by providing complete coverage of basic balance and transport principles used in food engineering.

*Math Concepts for Food Engineering*, second edition, is still intended as a supplemental reference to a standard textbook for a food engineering class. Its purpose is to provide practice and experience in solving simple engineering problems so that students are better prepared to face the more rigorous problems presented in class.

**Richard W. Hartel**
**Robin K. Connelly**
**Terry A. Howell, Jr.**
**Douglas B. Hyslop**

# *The Authors*

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**Terry A. Howell, Jr.**, received his BS and MS from Texas A&M University in agricultural engineering, with a PhD from the University of Wisconsin–Madison in food engineering. He served on the University of Arkansas Food Science faculty for three years, and now works for McKee Foods Corp. (Collegedale, TN) as manager of research and new technology.

**Douglas B. Hyslop** received his PhD in food chemistry in 1978 from the University of Wisconsin–Madison with an emphasis on physical surface and colloid chemistry. Since joining the staff at Madison in 1980, he has taught numerous food science courses, including food analysis, food chemistry, food physical chemistry, food colloid chemistry, food processing, and food engineering.

# Introduction

Mathematical reasoning is an important skill for a food scientist, but it is a skill that is not shared by all students at the same level. Although all food science students are required to have a comprehensive course in calculus (both integral and differential), often, for one reason or another, mathematical reasoning skills have been lost or forgotten by the time a student reaches the food engineering course.

In the spirit of the Institute of Food Technologists education standards, it makes sense to assess a student's mathematical reasoning skills prior to starting the food engineering course. The intention of this book is to help with that assessment and then to provide assistance for those students who need to brush up on their mathematical skills.

The book is organized into sections that present different materials needed for developing mathematical reasoning skills. The first section (chapters 1–4) covers important mathematical skills needed by students in a food engineering class. These principles are primarily a review of previous math classes (algebra, calculus, etc.). This preliminary section is followed by an important chapter on problem solving (chapter 5). The remaining chapters (6–11) cover food engineering topics likely to be found in a food engineering course for food scientists.

In the back of the book is a short quiz, the screening test (appendix 3a), which can be used to assess mathematical reasoning skills prior to the start of a food engineering course. Students are asked to read through the introductory math sections to refresh their memory on the important concepts prior to taking the screening test. For those students who score low on the screening test or simply want to improve their quantitative skills, the problems provided in *Math Concepts for Food Engineering* are intended to build mathematical confidence, as well as to bridge the simplest math concepts and the more complex engineering principles. As the semester unfolds, students should progress through chapters 6–11 as each topic is covered in class. Students should study each worked problem to make sure they understand the mathematical (and engineering) principles being demonstrated, and then work independently on the accompanying practice problems. To help students apply the principles in *Math Concepts for Food Engineering* to food engineering class, this second edition has included a brief coverage of engineering principles important for the sample problems as a supplement to the main *Food Engineering* textbook.

The following approach is recommended for working through this book:

- Review chapters 1–4 to refresh your knowledge of some important math skills.
- Take the screening test at the back of the book in appendix 3a.
- Grade yourself with the answer key in appendix 3b (after you have completed the entire exam).
- Assess which mathematical skill areas you struggled with.
- Review appropriate materials in chapters 1–4 based on your skills assessment.
- Carefully read through the problem-solving approach in chapter 5. Consider your own approach to mathematical problem solving to see where you might find ways to improve. For example, did you generally follow the steps outlined in chapter 5 when you took the screening test? If not, what might you do differently?
- Work your way through chapters 6 to 11 as each section is covered in a food engineering class. We recommend that you work through the appropriate chapter during the first day or two as each topic is covered in class. In that way, you will be well prepared to solve the more complex problems required in class. If you get stuck on a practice problem, be sure to meet with your instructor for assistance. Although the answers for these practice problems are given in the back of the book, seeking help from your instructor can help you see where you are getting stuck.

In our experience, those students who score low on the screening test but work hard at mastering the mathematical principles covered in *Math Concepts for Food Engineering* go on to do well in food engineering classes. As with most things, the likelihood of success is enhanced when the student is willing to put lots of hard work into learning the material. We hope that *Math Concepts for Food Engineering* provides a resource for you to improve your mathematical reasoning skills and, thereby, to attain greater success in learning food engineering principles.

# *chapter one*

# *Algebra*

Perhaps the easiest way to begin a discussion about algebraic equations and their components is to use examples to point out the various terms. The following expressions will be used to demonstrate the different parts of an equation:

$$y = 3x - 7 \tag{1.1}$$

$$y = 2\mathrm{a}x + 3\mathrm{b} \tag{1.2}$$

An equation is a mathematical statement that can be read like a sentence. Equation (1.1) may be read as "$y$ equals 3 times $x$ minus 7."

Since both sides of the equation are equivalent, one must always be very careful that any arithmetic operation performed on one side of the equation be performed on the other side as well. This will be stressed in later portions of this chapter.

Equations consist of variables, constants, and arithmetic operators. Mathematically, an equation relates different variables and constants; however, equations become more vital as one understands how physical parameters are linked together through them.

## *1.1 Variables and constants*

### *1.1.1 Variables*

Variables are so named because they are allowed to "vary," and their value may assume different amounts at different times or situations. When one substitutes $x = 1$ into equation (1.1), $y$ is calculated to be −4; however, when $x = 3$ is plugged into equation (1.1), one calculates $y = 2$. One can see that $x$ and $y$ are variables in this equation. Variables may be further categorized into independent and dependent variables. One might be able to logically determine that the value for $y$ in equations (1.1) and (1.2) is *dependent* on the value of $x$ in the equations. That is, as $x$ is *independently* varied, different values for $y$ will be produced. In many situations, the manner in which the equation is written will dictate which variable is dependent and which is independent.

Usually, a term written by itself on the left side of the equation is the dependent variable, while variables on the right side of the equal sign are independent variables. Equation (1.1) can be rewritten by adding 7 to both sides and then dividing both sides by 3 to produce equation (1.3).

$$x = (1/3)y + (7/3) \tag{1.3}$$

Here, $y$ becomes the independent variable and $x$ the dependent variable.

Time is almost always an independent variable. Most biological processes or reactions depend on time. One should be able to recognize that time is the independent variable in virtually every equation in which it appears.

### 1.1.2 Constants

As the name implies, the values assigned to constants do not change. In equation (1.2) above, "a" and "b" are constants. Some common constants that scientists and engineers encounter include:

g (gravity): The acceleration due to gravity can almost always be considered constant. Its value in SI units is 9.8 m/s$^2$.

$N_A$ (Avogadro's number): This number, representing the number of atoms per mole of a substance, is constant at $6.02 \times 10^{23}$ atoms/mole.

$\pi$ (pi): A number used in equations for area, surface area, and volume of rounded objects; also used to convert from angular degrees to radians; approximately equal to 3.14159.

## 1.2 Equations

### 1.2.1 Functions

Functions are used to describe a mathematical relationship between an independent variable and one or more constants. They are often represented as $f$ (independent variable). Equation (1.2) may be rewritten as

$$y = f(x), \text{ where } f(x) = 2\mathrm{a}x + 3\mathrm{b}. \tag{1.4}$$

Equation (1.4) is read as, "$y$ is a function of $x$, where the function of $x$ is 2a$x$ plus 3b." In a similar fashion, $x$ is a function of $y$ in equation (1.3). In many cases, shorthand notation showing that a variable is a function of another variable is used. Equation (1.4) may also be written as

$$y(x) = 2\mathrm{a}x + 3\mathrm{b}. \tag{1.5}$$

This form is read identically to that of equation (1.4).

*Example 1.2.1*

$$y = 2x - 1 \tag{1.6}$$

| | |
|---|---|
| Which are variables? | $x, y$ |
| What is the independent variable? | $x$ |
| What is the dependent variable? | $y$ |
| Is x a function of a variable as this equation is written? | **no** |
| Is y a function of a variable as this equation is written? | **yes** |

*Example 1.2.2*

$$V(t) = (g/2)^*t + V_0 \tag{1.7}$$

This equation relates the velocity at time, $t$, to the initial velocity ($V_0$), acceleration due to gravity (g), and time ($t$).

| | |
|---|---|
| Which are constants? | **g, $V_0$** |
| Which are variables? | **t, V(t)** |
| What is the independent variable? | **t** |
| What is the dependent variable? | **V(t)** |
| Is V a function of any variable? | **Yes, t** |

### *1.2.2 Manipulation of equations*

As stated previously, an equation is a **true** statement, and both sides of the equation must be manipulated in the same way to maintain the validity of the equation. Once one understands the rules and procedures in rearranging equations, one can solve many food engineering problems. The following is an overly simplified example:

$$1 = 1 \tag{1.8}$$

If we want to multiply the left side by 3, we must do it to the right side also so that the statement is still true.

$$3^*1 = 3^*1$$

$$3 = 3 \tag{1.9}$$

Now, if we wanted to subtract 1 from the left side, we must perform the same operation on the right side of the equation.

$$3 - 1 = 3 - 1$$

$$2 = 2 \tag{1.10}$$

Now, let us move on to the manipulation of a more relevant equation to the food industry, the ideal gas law equation. One of its many forms is as follows:

$$PV = n\mathrm{R}T \tag{1.11}$$

where

$P$ = pressure (Pa = $N/m^2$)
$V$ = volume ($m^3$)
$n$ = moles of gas (g·mol)
R = gas law constant (8.314 N·m/K·mol)
$T$ = temperature (K)

Which are the variables and constants in this equation? $P$, $V$, $n$, and $T$ are variables, and R is a constant. This equation is very useful, but many times, one would like to isolate one variable from the rest. How would one go about rearranging this equation so that the temperature could be solved, knowing the other four quantities?

To isolate $T$, we must remove the $n$ and R from the right side of the equation. If we start with $n$, we must divide the right side by $n$ to remove it and, therefore, must also divide the left side by $n$. Our equation now looks like:

$$PV^*(1/n) = n\mathrm{R}T^*(1/n)$$

$$PV/n = \mathrm{R}T \tag{1.12}$$

The same procedure must be repeated with R so that the equation takes the final form as shown.

$$(PV/n)^*(1/\mathrm{R}) = \mathrm{R}T^*(1/\mathrm{R})$$

$$PV/n\mathrm{R} = T \tag{1.13}$$

If one were given the values for $P$, $V$, and $n$, and knowing the value for the ideal gas law constant, the value for $T$ can now be calculated.

---

## *Example 1.2.3*

Calculate the temperature for an ideal gas, given:

$P$ = 200 Pa
$n$ = 2 moles
$V$ = 30 $m^3$

From equation (1.13), $T = PV/n\mathrm{R}$

Substitute known values for $P$, $V$, $n$, and R.

$T = (200\ \mathrm{Pa})(30\ \mathrm{m}^3)/(2\ \mathrm{moles})(8.314\ \mathrm{N{\cdot}m/K\ mol})$
$T = 360.83$ K

---

---

*Example 1.2.4*

Rearrange the following equation to solve for $x$. If $y = 4$, find $x$.

$$y = x^2 - 5$$

First, add 5 to both sides.

$$y + 5 = x^2$$

Now, take the square root of both sides.

$$(y + 5)^{1/2} = x$$

Now solve for $y = 4$.

$$(4 + 5)^{1/2} = x$$

$$(9)^{1/2} = x$$

$$3 = x$$

$$x = 3$$

---

*Example 1.2.5*

Fourier's equation is used to determine heat transfer by conduction through a material. A simplified form looks like the following:

$$Q = k^*\Delta T/\Delta x \tag{1.14}$$

where

$Q$ = heat transferred per surface area ($W/m^2$)
$k$ = thermal conductivity of the material (W/m·K)
$\Delta T$ = change in $T$ across material (K)
$\Delta x$ = thickness of material (m)

You are given the heat transferred through a wall, $Q = 1000$ $W/m^2$; the change in temperature, $\Delta T = 20$ K; and thermal conductivity, $k = 50$ W/m·K. Calculate the thickness of wall required to produce this amount of heat transfer.

Multiply both sides by $\Delta x$.

$$Q^*\Delta x = k^*\Delta T$$

Divide both sides by $Q$:

$$\Delta x = k^*\Delta T/Q$$

Substitute known values for $Q$, $k$, and $\Delta T$:

$$\Delta x = (50\ \text{W/m·K}*20\ \text{K})/(1000\ \text{W/m}^2) = (1000\ \text{W/m})/(1000\ \text{W/m}^2) = 1\ \text{m}$$

### *1.2.3 Rules of equations applied to engineering units*

At this point in the text, it is appropriate to apply this knowledge of equations directly to food engineering concepts. As in all engineering disciplines, problems are solved by using equations that apply to the unique situation at hand. In the case of food engineering, the equations are typically related to mass, heat, and energy balances and transfer. The equations that govern these phenomena are covered in this and other texts. As has been seen in the previous examples of equations, food engineering equations contain constants and variables, both dependent and independent.

The concept of units sets apart engineering and scientific equations from basic algebra. Whereas algebra looks at equations and mathematics in a pure sense, science and engineering apply the concepts to real problems. Units are vital to add understanding to the results that come from solving an engineering problem. In fact, most engineering solutions are invalid without providing the units of measure for the answer. If a colleague asks you for the flow rate of corn syrup to a mixing vessel, there is a huge difference between 10 (meaningless without units), 10 liters/hour, and 10 gallons/hour.

When dealing with units, the food engineer must be careful to use a common system while solving problems. There are two main systems, and the reader most likely has some familiarity with both the English system (still used predominantly in the United States), and the Système International (SI or metric unit system, used more commonly worldwide). Usually, a problem and its solution is given in one of these unit systems, but occasionally, a problem may be presented with some variable(s) in SI and other(s) in English units. In addition, even within one of the systems, several different units can be used to describe the same measure (e.g., inches, feet, yards, etc., are all English units for length). The remainder of this section will detail how to easily convert from one system to another and within one system.

Starting with a simple example, recall that 1 ft is equivalent to 12 in. Writing this in an equation form yields

$$1\ \text{ft} = 12\ \text{in.} \tag{1.15}$$

By definition, the equation must be true, and it can be manipulated like any other. When a manipulation is performed, the units must be carried with the manipulation. So, to generate a relationship that shows how many inches are in one foot, equation (1.15) can be manipulated by dividing both sides by 1 ft.

$$(1\ \text{ft}/1\ \text{ft}) = 1 = (12\ \text{in}/1\ \text{ft}) \tag{1.16}$$

The left side of equation (1.16) reduces to 1 (with no units) because the units of feet are divided by units of feet (this is typically called "canceling units"). Thus, when unit relationships like that above are developed, they can be used as if multiplying by 1. Now, the relationship on the right side of equation (1.16) may be used to convert any length in feet to inches by simply multiplying with it and canceling units. Again, it is as if 1 is being multiplied by the length in feet.

---

### *Example 1.2.6*

1. Use equation (1.16) to convert 5 ft to units of inches.
2. Also, rearrange equation (1.15) to develop a relationship to convert from inches to feet, and then convert 42 in. to feet.
   1. Convert feet to inches.

$$5 \text{ ft} * (12 \text{ in.}/1 \text{ ft}) = \mathbf{60\ in.}$$

   2. Dividing both sides of equation (1.15) by 12 in. gives

$$(1 \text{ ft}/12 \text{ in.}) = (12 \text{ in.}/12 \text{ in.}) = 1 \tag{1.17}$$

$$42 \text{ in.} * (1 \text{ ft}/12 \text{ in.}) = \mathbf{3.5\ ft.}$$

These are obviously simple examples, but they show how the concept works. The concept can be applied to any units conversion that is required.

This example shows the technique to use when converting between primary units — those measuring the most basic of dimensions. Primary units describe things like length, mass, time, and temperature. Secondary units (or derived units) are used to describe quantities that are combinations of primary units. The units of velocity (distance divided by time) comprise a secondary unit, regardless of the actual primary units used to express it. When a scientist needs to convert secondary units from one system to another, one can use the method above for each primary unit to convert the derived unit. Appendix A contains a brief listing of some useful unit conversions in food engineering. Also, there are some very complete conversion tables available that will also show the direct relationship between secondary units in different systems (e.g., the units of energy, 1 Btu/s = 1.0545 kW).

---

### *Example 1.2.7*

The density of soybean oil is 920 $kg/m^3$. Convert this to English units ($lb_m/ft^3$). Use only primary units in one solution. Use a secondary-to-secondary conversion in another solution.

$$1 \text{ m} = 3.2808 \text{ ft}$$

$$1 \text{ lb}_m = 0.45359 \text{ kg}$$

$$920 \text{ kg/m}^3 * (1 \text{ m})^3/(3.2808 \text{ ft})^3 = 26.05 \text{ kg/ft}^3$$

$$26.05 \text{ kg/ft}^3 * (1 \text{ lb}_m/0.45359 \text{ kg}) = \mathbf{57.44 \text{ lb}_m/\text{ft}^3}$$

Notice that in converting $m^3$ to $ft^3$, the entire quantity had to be cubed to correctly make the conversion. Also, since the $m^3$ term was in the denominator in the original unit, the conversion factor needed the $m^3$ term in the numerator to be canceled properly. Understanding these two concepts will serve the scientist or engineer very well.

$$1 \text{ lb}_m/\text{ft}^3 = 16.019 \text{ kg/m}^3$$

$$920 \text{ kg/m}^3 * [1 \text{ lb}_m/\text{ft}^3/(16.019 \text{ kg/m}^3)] = \mathbf{57.43 \text{ lb}_m/\text{ft}^3}$$

---

One last technique that may help erase the confusion that sometimes occurs when converting units involves using a "table" method. The technique allows the user to see the conversions a little easier, thus ensuring that each unit is cancelled properly. It is demonstrated below using the previous example:

$$(920 \text{ kg/m}^3)\ [(1 \text{ m})^3/(3.2808 \text{ ft})^3]\ (1 \text{ lb}_m/0.45359 \text{ kg}) = \mathbf{57.44 \text{ lb}_m/\text{ft}^3}$$

Setting up units conversion problems like this allows the scientist to quickly ensure that the units will cancel properly and that everything is in order before actually solving the problem. Once the technique has been practiced, it will become second nature.

## *1.3 Linear and nonlinear equations*

The standard equation of a straight line is

$$y = mx + b \tag{1.18}$$

where

$y$ = any dependent variable
$x$ = any independent variable
$m$ = the slope (constant)
$b$ = the $y$-intercept (constant, takes the value of $y$ when $x$ equals zero).

If an equation fits this standard form, it may be considered linear. Note that occasionally the equation must be rearranged to fit this form. If an equation has a different form than this standard equation, it is nonlinear.

---

*Example 1.3.1*

$$y = 2x + 2$$

Is this equation linear? If so, find the slope and y-intercept.

The equation has the standard linear equation form. The **slope** is **2** and the ***y*-intercept** is **2**.

---

In many cases, a nonlinear equation can be manipulated to produce a pseudo-linear equation by replacing the "nonlinear" variable with a new variable that is linear. Example 1.3.2 illustrates this idea.

---

## *Example 1.3.2*

$$y = x^2 - 3$$

Is this a linear equation? If not, can you linearize it in another form? Determine the slope and $y$-intercept of the linear equation.

In this form, the equation is **nonlinear** in $x$; however, it can be linearized by creating a new variable. Let $u = x^2$. The equation now looks like $y = u - 3$, which has the linear equation format with respect to $u$. The **slope** is **1** and the ***y*-intercept** is **−3**.

---

## *Example 1.3.3*

The following equation is used to model the viscosity of a Herschel–Bulkley fluid, one that exhibits a yield stress and power law characteristics such as toothpaste.

$$\sigma = k\gamma^{n} + \sigma_{0}{}^{0} \tag{1.19}$$

where

$\sigma$ = shear stress in the fluid (Pa) = (N/m$^2$)
$k$ = fluid consistency index (Pa s$^n$)
$n$ = flow behavior index (no units)
$\gamma$ = shear rate (1/s)
$\sigma_0$ = yield stress (Pa).

What are the constants and variables in this equation? Is this a linear equation in $\sigma$? If not, can you linearize it?

The constants are $k$, $n$, and $\sigma_0$, while the shear stress and shear rate are variables.

Since the form of the equation does not fit the standard linear form, the equation is not linear. One method for linearizing this equation involves substituting $u = \gamma^n$ into the equation. The equation now has the form:

$$\sigma = ku + \sigma_0 \tag{1.20}$$

where equation (1.20) is linear in $\sigma$. Another technique involves taking the logarithm of both sides of equation (1.19), and this concept will be covered in section 3.3.