Fiber-Optic-Based Sensing Systems

Lazo M. Manojlović

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Contents

Synopsis

Since the revival of human civilization, there was an inevitable need for some sort of simple measurements of basic physical quantities such as length, time, and mass. In order to be able to exchange goods, the first traders established some primordial measurement standards. The earliest recorded systems of weights and measures originated in the 3rd or 4th millennium BC. Therefore, one can estimate that the history of measurements and consequently the history of measurement instruments is 5 to 6 millennia old. Early measurement standard units probably have been used only in a single community or small region, where every region developed its own standards for the physical quantity of interests, such as length, area, volume, and mass.

Starting from the first industrial revolution and subsequent development of manufacturing technologies, and consequently the growing importance of trade across the globe, standardized weights and measures became critical. Hence, since the 18th century, modernized, simplified, and uniform systems of weights and measures were developed. During the last three centuries, the fundamental units were defined by ever more precise methods in the science of metrology. Today's state-of-the-art measurement systems and instruments are capable of measuring different physical quantities with the resolution and speed that was unthinkable several decades ago.

Although optical fibers are almost exclusively used in the telecommunication networks for the high capacity data transfer, there is also another specific use of the optical fibers. Namely, having in mind that optical interferometry is one of the most sensitive measuring techniques that are capable of high-precision and high-speed measurements of different physical quantities, a large number of different sensors have been developed that are based on the optical fibers.

—Dr. Lazo M. Manojlović

Abbreviations

AC	alternating current
A/D	analog/digital
APD	avalanche photodiode
BER	bit error rate
CATV	common-antenna television
CCD	charge-coupled device
CIE	Commission Internationale de l'Eclairage
CIPM	Comité International des Poids et Mesures
CW	continuous-wave
DC	direct current
DWDM	dense wavelength division multiplexing
FDDI	fiber distributed data interface
FFT	fast fourier transform
FOC	fiber-optic coupler
FWHM	full width at half maximum
HDTV	high-definition television
IMG	index matching gel
IR	infrared
LAN	local area network
LCLS	low-coherence light source
LCS	low-coherence source
LED	light emitting diode
MAN	metropolitan area networks
NA	numerical aperture
OCT	optical coherence tomography
OS	optical spectrometer
OSA	optical spectrum analyzer
PC	personnel computer
PD	photodetector
PDF	probability density function
PMD	polarization mode dispersion
PMT	photomultiplier
PSD	power spectral density

QPD	quadrant photodiode
SLD	superluminescent diode
SNR	signal-to-noise ratio
TE	transverse electric
TIA	transimpedance amplifier
TM	transverse magnetic
TV	television
UV	ultraviolet
WDM	wavelength division multiplexed
WLS	white light Source
YAG	yttrium aluminum garnet

Preface

The book *Fiber-Optic-Based Sensing Systems* deals with the applicative aspects of using optical fibers as the sensing medium as well as the medium for transmitting the corresponding optical signals to the receiving unit. Basic optical phenomena with their main emphasis on applying the optical knowledge onto solving the real engineering problems are also treated within the book. The book is aimed toward the undergraduate and graduate students who want to broaden their knowledge of fiber-optic sensing system applications to the real-life engineering problems as well as to the engineers who want to acquire the basic principles of optics, especially fiber.

The book is arranged in a way that leads the reader toward better understanding of fiber-optical phenomena and their sensing applications. Basic tools and concepts are presented in the earlier chapters, which are then developed in more detail in the later chapters. The book is organized in seven chapters covering broad range of fiber-optical sensing phenomena.

Chapter 1 gives a brief historical overview of optical phenomena perception. Although the light and light-related phenomena have always been a source of immense curiosity for ancient people, one had to wait until the late 18th century and the Maxwell's classical electromagnetic theory in order to conceive most principles of modern optics.

Chapter 2 introduces the radiometry as the substantial part in the field of optical measurement and engineering together with its measurement techniques for measuring optical power and its spectral content.

Chapter 3 shows how the light can be detected. Although in the past times, the human eye was used exclusively as an optical detector, in order to objectively measure the intensity of a light in modern optical systems, a solid-state detector has been usually used.

Chapter 4 takes into account the wave nature of light, thus introducing the corresponding interference phenomena. Being inherently short wavelength electromagnetic wave, the interference of light gives a rise of highly sensitive measurements of many physical quantities, which are also presented in this chapter.

Today's state-of-the-art communication and sensing systems are unimaginable without optical fibers. Therefore, Chapter 5 is devoted to the optical fibers and their use in communication as well as in the highly sensitive versatile measurement systems.

Due to the their possibility of absolute optical paths difference measurements, a special attention is given in Chapter 6 to the low-coherence fiber-optic sensors principle of operation, where we have treated typical algorithms for extracting the value of the optical paths difference of the sensing interferometer.

Finally, Chapter 7 presents case studies that show how the concept of low-coherence fiber-optic-based sensing systems can be used for measuring different physical parameters ranging from simple measurement of the optical paths difference to the more complicated such as surface roughness distribution estimation and high-resolution optical coherence tomography.

CHAPTER 1

The Properties and the Nature of Light

ABSTRACT

The properties and nature of light intriguing the mankind since the beginning of human civilization. Today, passing more than 25 centuries from the first known philosophical treatment of the optical phenomena, which the early Greek philosopher Pythagoras have been made, the scientist are still astonished by the dual nature of light. Maxwell's classical electromagnetic theory perfectly describes the propagation of light, whereas the quantum theory treats the interaction between light and matter or the absorption and emission of light. So, if someone ask "What is light?," there is no a simple answer to this simple questions.

1.1 THE BRIEF HISTORY OF LIGHT PHENOMENA PERCEPTION

Since the dawn of human civilization the man was astonished and puzzled by the everyday interplay between the daylight and the nightdark caused by the Sun. Being unable to understand the origins of observed optical phenomena but understanding the importance of such phenomena on everyday life, the early men ascribed to the Sun the divine imprint. For a long period of human history, the Sun, the Moon, and the stars were the only sources of light in which periodic temporal behaviors were well understood and interweaved into the human perception of time. Sometimes, this perfectly tempered clock was interrupted by the solar and Moon eclipses as well as by the lightning and the polar light that confused the early man perception of light. The first turning point in understanding and controlling the optical phenomena and being able for the first time to make a man controlled light source was the discovery of fire control. For the first time, the primordial man was able not only to heat his home but also to illuminate it.

The curious mind of the ancient Greek philosophers was the first one who tries to get a deeper insight in the nature of light. The early Greek philosopher Pythagoras (c. 582–c. 497 B.C.) believed that light, which originates from the visible objects, is carried by the tiny particles of light. Empedocles (fifth-century B.C.) believed that light came from the illuminating objects but also believed that the light rays came out from our eyes. He was the first one who postulated that light travels at a finite speed. The famous Greek mathematician Euclid (c. 325–c. 265 B.C.) thought that the eyes transmit rays of light that cause the sensation of vision. Mirrors were also studied by Euclid, where in his book, entitled *Catoptrics* and written in the third-century B.C., the law of reflection was established.

The rectilinear propagation of rays of light was the main reason for Isaac Newton in his *Treatise on Opticks* to adopt the corpuscular nature of light, more than 20 centuries after Euclid. He believed that light consists of very small bodies emanating from the shining bodies. The shadow formation behind the illuminated objects as well as the law of mirror reflections goes in the favor of corpuscular nature of light. In contrast to Newton's corpuscular theory, Christian Huygens developed a different theory where light is considered to be a wave motion that spreads from the source uniformly in all directions. The optical phenomena such as interference and diffraction, which were inexplicable in the frame of Newton's corpuscular theory, perfectly fit the wave nature of light.

In the mid–19th century, one of the milestones in understanding the nature of light was reached. Namely, due to the genius work of James Clerk Maxwell that was published in his famous paper *A Dynamical Theory of the Electromagnetic Field*, the light can be described as an electromagnetic wave. However, the beginning of 20th century brings new dilemmas. The outstanding experimental results, first announced by German physicist Philipp Lenard in 1902, showed that the energy of electrons emitted from the illuminated surface increased with the frequency of the light. The classical electromagnetic theory crashes in trying to explain this phenomenon. Therefore, a new approach was necessary. This new approach came on the wings of new quantum theory where light has been considered to consist of discrete wave packets—photons.

The presented brief history of optical phenomena perception showed that throughout the entire history of human preoccupation with the light, there was back and forth movement between the wave and corpuscular nature of light. Maxwell's classical electromagnetic theory perfectly describes the propagation of light, whereas the quantum theory treats the interaction between light and matter or the absorption and emission of light. So, if someone ask "*What is light?,*" there is no a simple answer to this simple questions.

1.2 THE WAVE NATURE OF LIGHT

Maxwell's equations together with the Lorentz force law represent the complete classical theory of the electromagnetic fields. All the phenomena where the interaction between the electromagnetic field and matter occurs can be successfully explained within the frame of classical electromagnetic theory except in those cases where the quantum effects are distinct. The set of Maxwell's equations can be written in the following forms:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \tag{1.1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{1.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1.3}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \tag{1.4}$$

where **E** is the electric field vector (or electric field strength), **B** is the magnetic induction, ρ is the electric charge density, **J** is the electric current density, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m is the vacuum permittivity, and $\mu_0 = 4\pi \times 10^{-7}$ H/m is the vacuum permeability. The force **f** per unit volume acting on the electric charge density ρ and the electric current density **J** is given by the Lorentz force law:

$$\mathbf{f} = \rho\mathbf{E} + \mathbf{J} \times \mathbf{B}. \tag{1.5}$$

If the electromagnetic field is observed in the regions where there are no electric charges ($\rho = 0$) and no electric currents ($\mathbf{J} = 0$), such as in a vacuum, Maxwell's equations reduce to:

$$\nabla \cdot \mathbf{E} = 0, \tag{1.6}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{1.7}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1.8}$$

$$\nabla \times \mathbf{B} = \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}. \tag{1.9}$$

By taking curl ($\nabla\times$) of the curl equations and by using the identity ($\nabla\times$ ($\nabla \times \mathbf{X}) = \nabla(\nabla \cdot \mathbf{X}) - \nabla^2\mathbf{X}$, one can obtain the following wave equations:

$$\nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}. \tag{1.10}$$

$$\nabla^2 \mathbf{B} = \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}, \tag{1.11}$$

where c is the speed of light in vacuum identified as follows:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299{,}792{,}458 \mathrm{m/s}. \tag{1.12}$$

Although the speed of light in vacuum has the exact value, the speed of light v in matter depends on the medium characteristics such as relative permittivity ε_r and relative permeability μ_r in the following way:

$$v = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_r \mu_0}} = \frac{c}{n}, \tag{1.13}$$

where $n = \sqrt{\varepsilon_r \mu_r}$, is the material index of refraction. The electric field **E** and magnetic induction **B** of an electromagnetic wave are mutually perpendicular to each other and to the direction of wave propagation and are in phase with each other, that is, the wave propagates in the direction of $\mathbf{E} \times \mathbf{B}$ as it is presented in Figure 1.1.

If we take into consideration the Cartesian coordinate system and resolve the vector wave eqs 1.10 and 1.11 into components, one can observe that both components **E** and **B** satisfy the general scalar wave equation:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2}, \tag{1.14}$$

where U represents any of the electric field components E_x, E_y, and E_z, as well as any of the magnetic induction components E_x, E_y, and E_z. Let further consider the simplest case, where the spatial variation of U occurs only along some particular direction, for example, along the z-axis, that is, $U = U(z, t)$. In this particular case, eq 1.14 simplifies to the one-dimensional wave equation:

$$\frac{\partial^2 U}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2} \qquad (1.15)$$

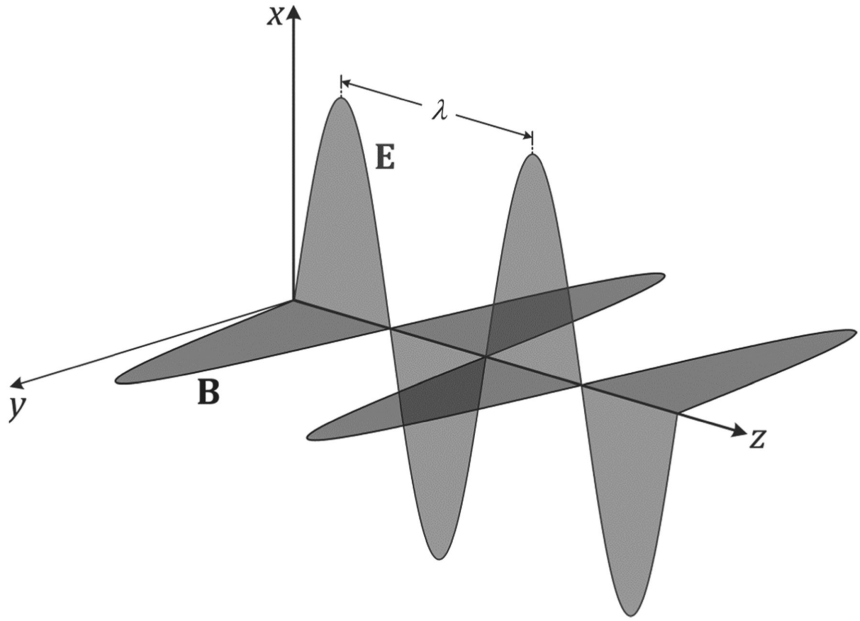

FIGURE 1.1 The electric field **E** and the magnetic induction **B** are mutually perpendicular to each other and to the direction of wave propagation and are in phase with each other.

A simple harmonic solution of this one-dimensional wave equation can be obtained by the variable separation in the following form:

$$U(z, t) = U_0 \cos(\omega t - kz), \qquad (1.16)$$

where U_0 is the corresponding electromagnetic wave amplitude and where wave number k and angular frequency ω are related as:

$$v = \frac{\omega}{k}, \qquad (1.17)$$

where the ratio of the angular frequency ω and the wave number k represents the phase velocity v. The presented particular solution, given by eq 1.16, is fundamental to the study of optics and represents the plane harmonic wave. The wavelength λ is defined as the distance, measured along the direction of the wave propagation, where the wave function completes one whole cycle, that is, the distance between two maximums (or minimums) of the wave function. The wavelength is determined by the wave number as:

$$\lambda = \frac{2\pi}{k}. \tag{1.18}$$

If we make a step back to the three-dimensional wave eq 1.14, it is easy to show that a similar simple harmonic solution is one of the particular solutions of this wave equation. The three-dimensional plane harmonic wave function is given by the following equation:

$$U(x, y, z, t) = U_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r}), \tag{1.19}$$

where the position vector r is defined as follows:

$$\mathbf{r} = \hat{\mathbf{i}}x + \hat{\mathbf{j}}y + \hat{\mathbf{k}}z, \tag{1.20}$$

where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the unit vectors ($|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$) along x-, y-, and z-axis, respectively. The propagation vector or the wave vector $\mathbf{k}$ can be also given in terms of its corresponding components as follows:

$$\mathbf{k} = \hat{\mathbf{i}}k_x + \hat{\mathbf{j}}k_y + \hat{\mathbf{k}}k_z, \tag{1.21}$$

where k_x, k_y, and k_z are the corresponding wave vector components along x-, y-, and z-axis, respectively. The magnitude of the wave vector is equal to the wave number, which is given in terms of the corresponding wave vector components as follows:

$$|\mathbf{k}| = k = \sqrt{k_x^2 + k_y^2 + k_z^2}. \tag{1.22}$$

If we further inspect eq 1.19 and its cosine function argument $\omega t - \mathbf{k} \cdot \mathbf{r}$, one can observe that the constant values of this argument define a set of planes in space named surfaces of constant phase (equiphase surfaces) given by the following equation:

$$\omega t - \mathbf{k} \cdot \mathbf{r} = \omega t - (k_x x + k_y y + k_z z) = \text{const}. \tag{1.23}$$

By interpreting eq 1.23, it is obvious that wave vector $\mathbf{k}$ is normal to the surfaces of constant phase as shown in Figure 1.2. Moreover, the constant-phase surfaces are moving in the direction of the wave vector with the velocity equal to the phase velocity:

$$v = \frac{\omega}{k} = \frac{\omega}{\sqrt{k_x^2 + k_y^2 + k_z^2}}. \tag{1.23}$$

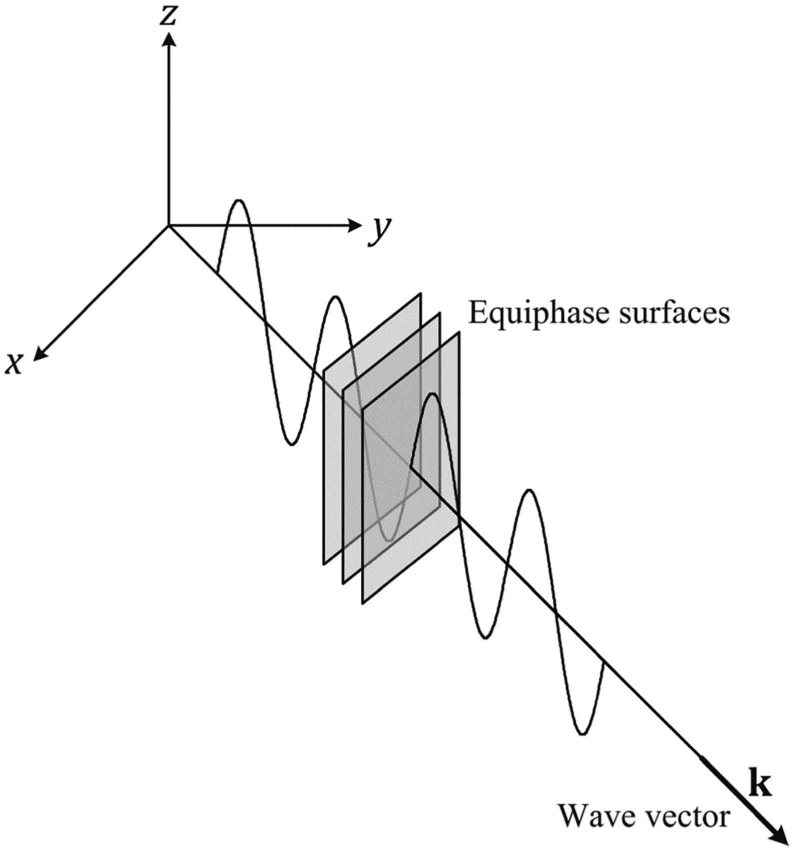

FIGURE 1.2 Equiphase surfaces of a plane wave.

Electromagnetic waves are produced by oscillating electric charged particles, so the frequency of oscillation determines the type of emitted radiation. Such produced electromagnetic disturbance propagates through space as a monochromatic wave, which is characterized by a single frequency, or polychromatic wave, which consists of many frequencies, either discrete or in a continuum. The energetic content of an electromagnetic wave, which is distributed among the various constituent waves, is called the spectrum of the radiation. Typically, to the different regions of the electromagnetic spectrum, specific names are given, such as radio waves, microwaves, optical waves (infrared, visible, and ultraviolet light), X-rays, gamma rays, and cosmic rays due to the different way of its production or detection. The common names, descriptions as well as typical frequency ranges are presented in Figure 1.3 where the electromagnetic spectrum is given in both terms frequency v and wavelength λ. One can recall that these two quantities, frequency and wavelength, are linked as follows:

$$v\lambda = c. \tag{1.24}$$

In the case when all the charges oscillate unison, for the emitted electromagnetic radiation, it is said to be coherent. In the case when all the charges oscillate independently and/or randomly, for the emitted electromagnetic radiation, it is said to be incoherent. Typical radiation sources in the optical domain such as flames, incandescent light bulbs, fluorescent lamps, and light-emitting diodes (LED) are incoherent light sources. In contrast to these incoherent sources of radiation, typical man-made radiation sources in the low-frequency range such as radio waves and microwaves are coherent. The only light source in optical domain that emits coherent radiation is laser.

Sometimes, it is convenient to present the wave function in the complex domain by using Euler representation:

$$U(x,y,z,t) = U_0 \exp\left[j(\omega t - \mathbf{k}\cdot\mathbf{r})\right]. \tag{1.25}$$

The real part of the complex wave function represents the actual physical quantity, where the real part is equal to the actual wave function given by eq 1.19. The reason why the complex representation of the wave function is often used is the simplicity of the algebra with the complex representation than with the trigonometric representation of the wave function. Besides simple harmonic solution of the plain waves that satisfies the wave equation in Cartesian coordinate system, given by eq 1.14, there is also a particular solution in the form of the spherical waves, which can be easily obtained if we rearrange the wave equation given by eqs 1.10 and 1.11 in the polar coordinate system. The wave function of the spherical waves is given by the following equation:

$$U(r,t) = \frac{U_0}{r}\cos(\omega t - kr) \text{ or } U(r,t) = \frac{U_0}{r}\exp\left[j(\omega t - kr)\right]. \tag{1.26}$$

where r is the radial distance from the radiation source to any particular point in the space.

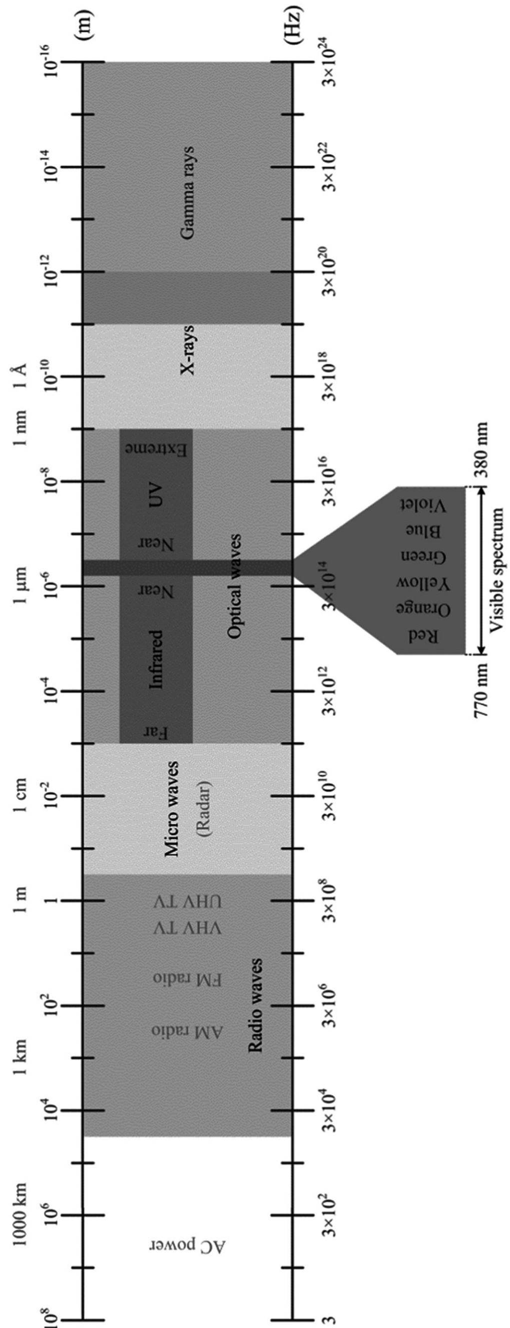

FIGURE 1.3 Electromagnetic spectrum arranged by wavelength in meters (m) and frequency in hertz (Hz).

As it was mentioned earlier, the phase velocity is the rate at which the wave phase propagates in space and it is defined as $v_p = \omega/k$. On the other side, the group velocity is defined as the rate at which the wave envelope propagates through space. To find the relation for the group velocity, we will observe two waves with the angular frequencies $\omega + \mathrm{d}\omega$ and $\omega - \mathrm{d}\omega$ and wave numbers $k + \mathrm{d}k$ and $k - \mathrm{d}k$. In a simple case, which will be considered here, we will assume without losing the generality of the analysis that both waves have the same amplitude U_0 and propagate in the same direction, let say along the z-axis. The superposition of both waves will result in a wave in which complex wave function is given by the following equation:

$$\begin{aligned} U &= U_0 \exp\left\{j\left[(\omega + \mathrm{d}\omega)t - (k + \mathrm{d}k)r\right]\right\} \\ &+ U_0 \exp\left\{j\left[(\omega - \mathrm{d}\omega)t - (k - \mathrm{d}k)r\right]\right\}. \end{aligned} \tag{1.27}$$

After the rearrangement of eq 1.27, we have:

$$U = 2U_0 \exp\left[j(\omega t - kr)\right]\cos(t\mathrm{d}\omega - r\mathrm{d}k). \tag{1.28}$$

The last equation can be interpreted as a single wave with the wave function $2U_0\exp\,[j(\omega t - kr)]$ that is modulated by the envelope function $\cos(t\mathrm{d}\omega - r\mathrm{d}k)$. The envelope travels not with the phase velocity but with the group velocity equal to:

$$v_{\mathrm{g}} = \frac{\mathrm{d}\omega}{\mathrm{d}k}. \tag{1.29}$$

Electromagnetic wave when propagates throughout the space transfers a certain amount of energy and momentum. Therefore, there is a certain electromagnetic energy that is stored in the space through which wave travels. The energy density, stored into the free space, is given by the following equation:

$$w_E = \frac{\varepsilon_0}{2}E^2, \tag{1.30}$$

$$w_B = \frac{1}{2\mu_0}B^2, \tag{1.31}$$

where w_E and w_B are the energy densities of electric and magnetic fields, respectively. As $E = cB$ is fulfilled, we have $w_E = w$. So, the total energy density is given by the following equation:

$$w = w_E + w_B = \varepsilon_0 E^2 = \frac{1}{\mu_0} B^2 \tag{1.32}$$

When propagates, electromagnetic wave fills the space with the energetic content. If we observe an electromagnetic wave for a very short period of time Δt, the electromagnetic energy that is brought into the observed elementary volume is equal to:

$$W = cAw\Delta t, \tag{1.33}$$

where A is the cross section of the elementary volume. By inspecting eq 1.33, a particular physical quantity can be defined that represents the energy transfer through the unit area and unit time as:

$$S = \frac{W}{A\Delta t} = cw. \tag{1.34}$$

This physical quantity represents the magnitude of the Poynting vector, which is defined as follows:

$$S = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}. \tag{1.35}$$

Besides the energy transfer, electromagnetic radiation carries also the momentum, so when electromagnetic wave impinges the surface of some object, it exerts force. As Maxwell showed, the radiation pressure is equal to the energy density of the electromagnetic wave. The radiation pressure exerted on the object that perfectly absorbs the radiation can be found by using virtual work principle. Therefore, we will observe the electromagnetic radiation that impinges the perfectly absorbing body, as it is presented in Figure 1.4. If we allow force, which is generated by the radiation, to virtually move the object for an infinitesimally short distance δz, the work done by this force will, according to the first principle, be equal to the change in stored electromagnetic energy. Thus, the energy conservation law gives:

$$\mathrm{PA}\,\delta z = wA\,\delta z. \tag{1.36}$$

where P is the radiation pressure and A is the cross section of the observed elementary volume. From eq 1.36, it is simply obtained $P = w$.

Referring further to Figure 1.5, where it is presented the radiation that impinges the perfectly absorbing object, the total momentum of the electromagnetic radiation that is stored in the elementary volume with the cross section A and thickness Δz will be transferred to the object in period of time equal to $\Delta t = \Delta z/c$. The overall momentum M that will be transferred to the object is equal to:

$$M = p'A\Delta z, \tag{1.37}$$

where p' is the volume density of electromagnetic momentum. According to the third Newton's law of motion, we have:

$$PA = M\Delta t. \tag{1.38}$$

that is, the force exerted on the object is caused by the overall momentum transfer during the period of time Δt. Therefore, the electromagnetic momentum volume density is given by the following equation:

$$p' = \frac{w}{c} = \frac{S}{c^2}. \tag{1.39}$$

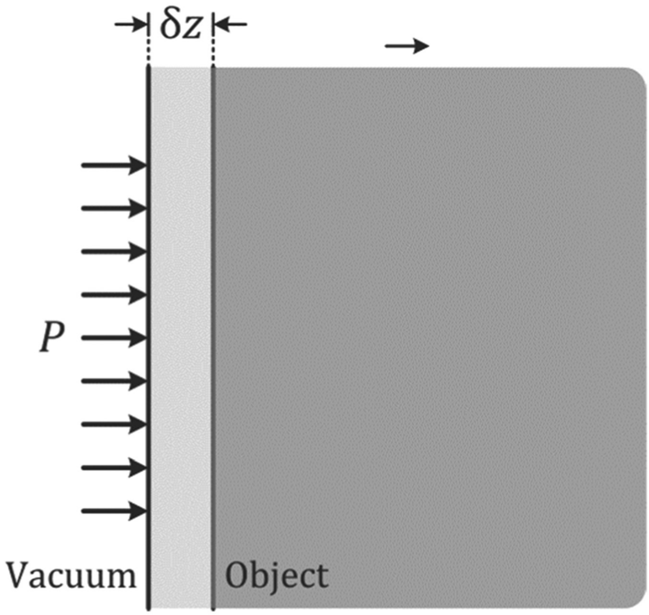

FIGURE 1.4 Electromagnetic radiation impinges the perfectly absorbing object.

One of the important physical quantities, especially in the radiometry, is the time average of the energy that is received by the unit area and unit time and is named irradiance. By definition, we can write:

$$I = \langle S \rangle, \tag{1.40}$$

where $\langle \bullet \rangle$ denotes the time average operator. So, the irradiance can be interpreted as the time-averaged Poynting vector.

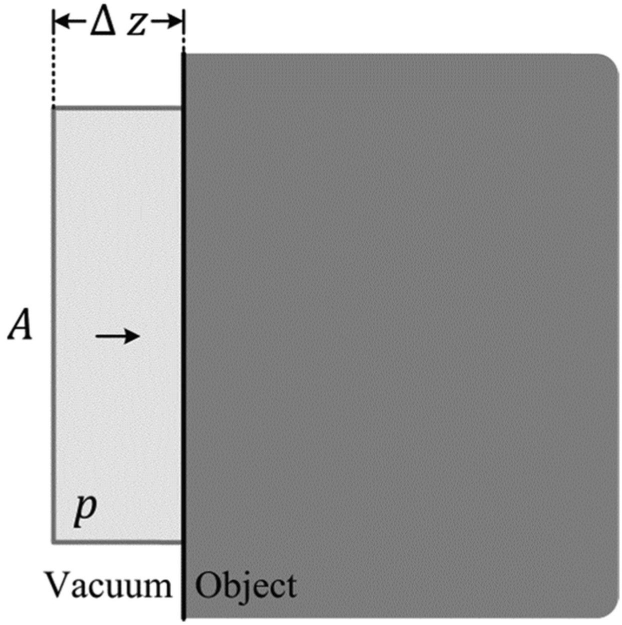

FIGURE 1.5 Electromagnetic radiation impinges the perfectly absorbing object and transfers the momentum.

If a monochromatic source of electromagnetic radiation is moving uniformly with a constant velocity toward the motionless observer and simultaneously transmits the electromagnetic waves with the particular frequency ν, then, in the observer's frame of reference, according to the relativistic Doppler effect, there will be a change in the received radiation frequency in the following way:

$$\nu' = \nu \sqrt{\frac{c \mp u}{c \pm u}}, \tag{1.41}$$

where ν' is the received frequency, u is the source velocity relative to the observer, and where upper signs stand for the case when the source is moving away from the observer and lower signs when the source is approaching the observer. Equation 1.41 is obtained by taking into consideration the Lorentz transformation, that is, eq 1.41 is valid for any velocity. Typically, the source (or the observer) velocity is much smaller than speed of light ($u \ll c$), so eq 1.41 simplifies to:

$$\nu' \approx \nu\left(1 \mp \frac{u}{c}\right) \tag{1.42}$$

1.3 THE CORPUSCULAR NATURE OF LIGHT

As mentioned earlier, Lenard's experiment had the far-reaching consequences on the deeper understanding of the nature of light. The experiment showed that the energy of emitted electrons is proportional to the frequency of light. The classical electromagnetic theory failed in this case. According to the classical theory, the energy of an emitted electron must be proportional to the intensity of the light but not to its frequency. Moreover, a very dim light would be sufficient to trigger the electron emission that was in opposite to the observed frequency threshold, that is, the minimal frequency where photoelectric effects start. Below the threshold, electrons aren't emitted regardless the light intensity or the exposure time. To bridge this gap, Albert Einstein suggested that a light beam is not a wave that propagates through space, but rather a set of discrete wave packets (photons), each with energy $\varepsilon = h\nu$, with $h = 6.626 \times 10^{-34}$Js being the Planck constant. There has been passed almost two decades before Einstein's light quanta hypothesis had been accepted. The period of transition between the hypothesis rejection and acceptance would undoubtedly have been longer if Compton's experiment had been less striking. Based on the thermodynamic approach to the black-body radiation, Einstein showed that monochromatic black-body radiation in Wien's law spectral region behaves with respect to thermal phenomena as if it consists of independently moving particles or quanta of radiant energy each with the energy proportional to its frequency.

There is also an alternative way to find the relation between the photon energy and the relevant physical quantities of the macroscopic electromagnetic wave. In the analysis shown next, the only assumption that will be taken into account is that electromagnetic radiation is quantized and the photon is the quantum of such an electromagnetic radiation. Further, it will be taken into consideration that an arbitrary monochromatic wave motion $u(\mathbf{r},t)$ at a particular point in space, defined by its position vector $\mathbf{r}$, is fully determined by its amplitude $U(\mathbf{r})$, wave number $\mathbf{k}$, and angular frequency ω as:

$$u(\mathbf{r},t)=U(\mathbf{r})\cos(\omega t-\mathbf{k}\cdot\mathbf{r}+\varphi), \tag{1.43}$$

where, without losing the generality of the analysis, the influence of the initial wave phase φ will be neglected, we will assume that the photon, as an integral part of the electromagnetic wave (radiation), has an average energy ε that is dependent on these three abovementioned parameters, as well as potentially dependent on some other physical parameters $\mathbf{x} = [x_1\ x_2\ \ldots\ x_n]$, that is, $\varepsilon = \varepsilon(U,\mathbf{k}, \omega,\mathbf{x})$, where $x_i \neq x_i(U,\mathbf{k},\omega)$ $(i = 1,2, \ldots, n)$ must be satisfied. In the opposite case, where we have $x_i = x_i(U,\mathbf{k},\omega)$ fulfilled, the average photon energy will be dependent only on the electromagnetic wave parameters and not on this particular physical quantity. Also, if the initial phase affects the average photon energy, the average photon energy will be dependent on the choice of the initial time instant, which is in the collision with the energy conservation law and thus absurd. For the wave number, we have $|\mathbf{k}| = 2\pi/\lambda$, where λ is the wavelength of the monochromatic electromagnetic wave in vacuum and $\lambda = 2\pi c/\omega$, is valid, where c is the speed of light in vacuum, so the wave number is only dependent on the frequency $\mathbf{k} = \mathbf{k}\ (\omega)$. Therefore, for the photon energy, we have $\varepsilon = \varepsilon(U,\omega,\mathbf{x})$.

To find out what is the dependence of the photon average energy on the wave amplitude, we will observe the unvarying, static, and monochromatic sources S of the electromagnetic radiation, as it is depicted in Figure 1.6. Looking at the distance z_1 from the source within a small solid angle $\Delta\Omega$ ($\Delta\Omega \ll 1$), the amplitude of the radiation is equal to U_1 whereas at the distance z_2 from the source, the amplitude is equal to U_2. Since $z_1 \neq z_2$, we have fulfilled $U_1 \neq U_2$. Taking into consideration that the source is unvarying, it emits a constant radiant flux $\Delta\Phi$ of the electromagnetic radiation within the observed solid angle $\Delta\Omega$, or equivalently, it emits a constant number of photons per unit time q within the corresponding solid angle having the average photon energy $\varepsilon = \varepsilon(U,\mathbf{k}, \omega,\mathbf{x})$. The photon velocity has been considered to be exactly the same as the speed of light in vacuum. If we consider the case where the photon velocity is smaller than the speed of light in vacuum, some parts of the electromagnetic wave that was first broadcasted by the transmitter will be not quantized at large distances from the transmitter and the others will be, which is naturally absurd. In the opposite case, where we consider the photon velocity to be greater than the speed of light in vacuum, we have also absurd situation since it is in the collision with the basic principle of the special relativity. Therefore, after the time interval $t_1 = z_1/c$ from the moment of their emission, photons reach the distance z_1 from the source S. The total number of photons, contained in a thin spherical shell, that is, located within the slab $A_1B_1C_1D_1$, having the solid angle $\Delta\Omega$, cross-section area ΔS_1 and with the thickness Δz_1 at the distance z_1, whereby $\Delta z_1 \ll \Delta z_1$ is satisfied, is $N_1 = q\ \Delta z_1$, where $\Delta t_1 = \Delta z_1/c$, with the assumption that there are no photons absorbed, scattered, and/or generated on their way since they are propagating in vacuum. Similarly, after time interval $t_2 = z_2/c$ from the moment of their emission, photons reach the distance z_2 from the source. Also, the total number of photons, contained in a thin spherical shell, that is, located within the slab $A_2B_2C_2D_2$, having the solid angle $\Delta\Omega$, cross-section area ΔS_2 and with the thickness Δz_2 at the distance z_2, whereby $\Delta z_2 \ll \Delta z_2$ is satisfied, is $N_2 = q\ \Delta t_2$ where $\Delta t_2 = \Delta z_2/c$, with the same assumption that there are no photons absorption, scattering, and/or generation on their way.

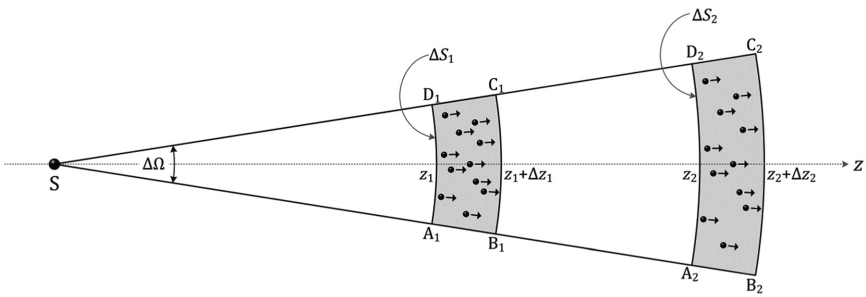

FIGURE 1.6 Unvarying, static, and monochromatic sources S of the electromagnetic radiation.

According to the continuity equation for $\Delta t_1 = \Delta t_2 = \Delta t$, the following is obviously valid $\Delta z_1 = \Delta z_2 = \Delta z$ and $N_1 = N_2 = N$, or due to the invariability of the velocity of photons, all photons contained in the slab $A_1B_1C_1D_1$, specified by the distances $[z_1 = \Delta z_1 = \Delta z_1]$ after time interval $\Delta t_{12} = (z_2 - z_1)/c$ are contained in the slab $A_2B_2C_2D_2$, specified by the distances $[z_2, z_2 + \Delta z_2]$, where $\Delta t = \Delta z/c$ is valid. The energy stored in the slab $A_1B_1C_1D_1$ or the energy of all photons that are located in the slab $A_1B_1C_1D_1$ at the distance z_1 is $W_1 = N\varepsilon(U_1, \omega, \mathbf{x}_1)$, while the energy stored in the slab $A_2B_2C_2D_2$ or the energy of all photons that are located in the slab $A_2B_2C_2D_2$ at a distance z_2 is $W_2 = N\varepsilon(U_2, \omega, \mathbf{x}_2)$, where $\mathbf{x}_1$ and $\mathbf{x}_2$ are the corresponding other physical parameters' values that may influence the photon energy at the position of $A_1B_1C_1D_1$ and $A_2B_2C_2D_2$ slabs, respectively. Further, if we simply apply the energy conservation law to the photons contained within the cone

$A_1A_2D_1D_2$ during their movement and having in mind that there are no external forces exerted on the slab surfaces A_1D_1 (B_1C_1) and A_2D_2 (B_2C_2) and thus there is no mechanical work done over the distances Δz_1 and Δz_2, the energy stored in the cone $A_1A_2D_1D_2$ must be the same as the energy stored in the cone $B_1B_2C_1C_2$. Furthermore, this leads to the conclusion that the energy content of the slab $A_1B_1C_1D_1$ is the same as the energy content of the slab $A_2B_2C_2D_2$, so we have $W_1 = W_2$ fulfilled, or equivalently:

$$\varepsilon(U_1, \omega, \mathbf{x_1}) = \varepsilon(U_2, \omega, \mathbf{x_2}). \tag{1.44}$$

Now, if we, without losing the generality of the analysis, assume that these two slabs are close enough, that is, infinitesimally close where $z_1 = z$ and $z_2 = z + \mathrm{d}z$ are fulfilled, the corresponding amplitudes of the electromagnetic wave have the following values $U_1 = U$ and $U_2 = U + \mathrm{d}U$, within the slabs $A_1B_1C_1D_1$ and $A_2B_2C_2D_2$, respectively, with $\mathrm{d}U$ being the infinitesimal change of the electromagnetic wave amplitude between these two slabs, and where the other influencing physical parameters have the following values $\mathbf{x_1} = \mathbf{x}$ and $\mathbf{x_2} = \mathbf{x} + \mathbf{dx}$, with $\mathbf{dx} = [\mathrm{d}x_1\, \mathrm{d}x_2 \dots \mathrm{d}x_n]$ being the infinitesimal change of the physical parameters' values between these two slabs. Having this in mind, eq 1.44 becomes:

$$\varepsilon\left(U_2, \omega, \mathbf{x_2}\right) - \varepsilon\left(U_1, \omega, \mathbf{x_1}\right) = \mathrm{d}\varepsilon\left(U, \omega, \mathbf{x}\right) = \frac{\partial \varepsilon}{\partial U}\mathrm{d}U + \sum\nolimits_{i=1}^{n} \frac{\partial \varepsilon}{\partial x_i}\mathrm{d}x_i = 0. \tag{1.45}$$

with $\mathrm{d}\varepsilon(U, \omega, \mathbf{x})$ being the total derivative of the photon energy. After rearranging eq 1.45, one obtains:

$$\frac{\partial \varepsilon}{\partial U} = -\sum\nolimits_{i=1}^{n} \frac{\partial \varepsilon}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}U}. \tag{1.46}$$

Since $x_i \neq x_i(U, \mathbf{k}, \omega)$, where $i = 1,2, \dots, n$ is valid, one obtains $\mathrm{d}x_i / \mathrm{d}U \equiv 0$. Therefore, from eq 1.46, we have:

$$\frac{\partial \varepsilon}{\partial U} \equiv 0 \rightarrow \varepsilon \neq \varepsilon\left(U\right), \tag{1.47}$$

or the photon energy is not dependent on the electromagnetic wave amplitude, so one has $\varepsilon = \varepsilon(\omega, \mathbf{x})$, that is, the photon energy potentially depends on the angular frequency of the electromagnetic radiation and potentially on various other physical parameters.

To obtain the mathematical relation between the photon energy and the angular frequency of the electromagnetic wave, as well as between the photon energy and other physical parameters that may influence photon energy, one will observe the moving emitter that emits plane, unvarying, and monochromatic electromagnetic waves in the direction of the motionless absorber, as shown in Figure 1.7. The moving emitter in its inertial frame of reference emits the total radiant flux of Φ. Since the emitted flux is constant, the photon rate is also constant and equals q in the moving emitter frame of reference. The measured flux and the photon rate are related by the following relation:

$$\Phi = q\varepsilon(\omega, \mathbf{x}). \tag{1.48}$$

Now, we will consider the thought experiment where the perfect, plane, motionless absorber, in which surface is parallel to the wavefront, absorbs each photon that is emitted by the moving emitter, as it is presented in Figure 1.7. Due to the Doppler effect, in the case of the motionless absorber, the absorbed radiant flux and angular frequency will differ from case of moving emitter emitted radiant flux and angular frequency. In the motionless absorber inertial frame of reference, the absorbed radiant flux and the angular frequency are $\Phi'$and ω', respectively.

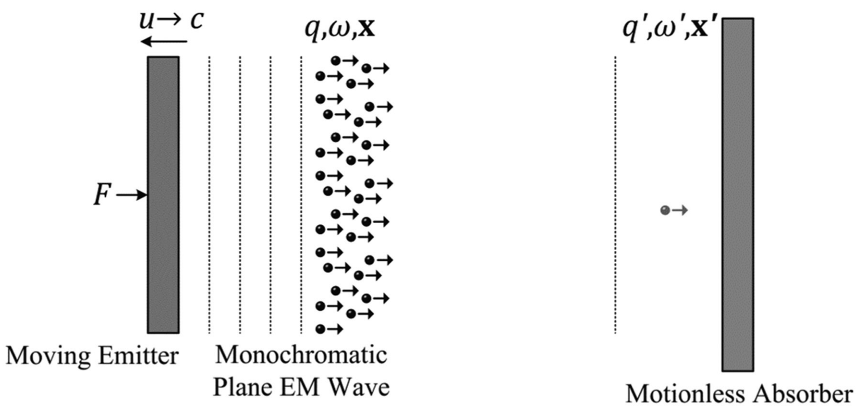

FIGURE 1.7 Moving emitter emits planar, unvarying, and monochromatic electromagnetic waves in the direction of the motionless absorber.

Due to the time dilation, caused by the motion of the moving emitter in the motionless absorber frame of reference, the number of photons per unit time that reaches the motionless absorber is q', where $q' \neq q$. In this case, the average time between observing two photons is $\tau = 1/q$, while the average time between absorbing two photons is $\tau' = 1/q'$, where $\tau' \neq \tau$

is also valid. The abovementioned three parameters Φ', q', and ω' that correspond to the motionless absorber are related by the following relation Φ', $= q'\varepsilon(\omega',\mathbf{x}')$. Based on the Lorentz transformations, the following is valid for the angular frequency ω' of the electromagnetic wave and the number of photons per unit time q' that have been absorbed:

$$\omega' = \omega\sqrt{\frac{c-u}{c+u}} \tag{1.49}$$

$$\tau' = \frac{\tau}{\sqrt{1-\left(u^2/c^2\right)}} \Rightarrow q' = q\sqrt{1-\frac{u^2}{c^2}} \tag{1.50}$$

According to the energy conservation law in the motionless absorber frame of reference, we have:

$$\Phi' = \Phi - Fu, \tag{1.51}$$

where F is the braking force that must be applied to the moving emitter in the opposite direction of its motion to provide the uniform motion of the emitter with constant velocity, or the emitter must not accelerate due to the repulsive force of the electromagnetic radiation that it captures. Further, eq 1.51 becomes:

$$q'\varepsilon(\omega',\mathbf{x}') = q\varepsilon(\omega,\mathbf{x}) - Fu. \tag{1.52}$$

By combining eqs 1.49, 1.50, and 1.52, we have:

$$q\sqrt{1-\frac{u^2}{c^2}}\varepsilon\left(\omega\sqrt{\frac{c-u}{c+u}},\mathbf{x}'\right) = q\varepsilon\left(\omega,\mathbf{x}\right) - Fu. \tag{1.53}$$

If the moving emitter velocity is very close to the speed of light in vacuum, that is, $u \to c$, according to eq 1.50, it is valid $q' \to 0$, that is, the average time between absorbing two consecutive photons tends to infinity $\tau' \to +\infty$ and $\varepsilon(\omega', x') \to \varepsilon(0, x')$, so regardless the value of $\varepsilon(0, x')$, eq 1.53 becomes:

$$F = q\frac{\varepsilon\left(\omega,\mathbf{x}\right)}{c}, \tag{1.54}$$

where it is assumed $\varepsilon(0, x') < +\infty$, while otherwise an indefinitely large energy would be needed to establish a time invariant electromagnetic field even in a limited volume. Based on the third Newton's law, the force F is also the repulsive force of the electromagnetic radiation that acts on the emitter. According to eq 1.54, the photon momentum $p(\omega, x)$ is given by

$p(\omega, x) = \varepsilon(\omega, x)/c$, which is the well-known relationship for the photon momentum.

The goal of the above-presented analyses was to find the photon momentum as a function of its energy. In the next step of the analysis, we will consider the case when the emitter is moving steadily toward the motionless absorber with constant velocity u, as shown in Figure 1.8. To overcome the repulsive force of the emitted electromagnetic radiation and to ensure uniform motion of the emitter, the force F must be exerted on the moving emitter in the direction of its motion.

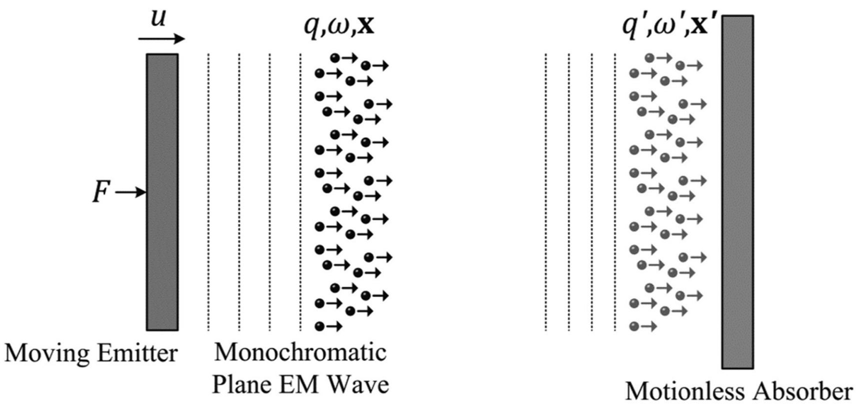

FIGURE 1.8 The moving emitter is moving uniformly toward the motionless absorber with a constant velocity.

According to the energy conservation law in the motionless absorber frame of reference, the following is valid:

$$\Phi' = \Phi - Fu. \tag{1.55}$$

Further, taking into consideration eq 1.54, the following is valid for eq 1.55:

$$q'\varepsilon(\omega', \mathbf{x}') = q\varepsilon(\omega, \mathbf{x}) + q\varepsilon(\omega, \mathbf{x})\frac{u}{c}. \tag{1.56}$$

By combining eqs 1.49, 1.50, and 1.56, we obtain the following functional equation:

$$\varepsilon\left(\omega\sqrt{\frac{c+u}{c-u}}, \mathbf{x}'\right)\sqrt{1-\frac{u^2}{c^2}} = \varepsilon(\omega, \mathbf{x}) + \varepsilon(\omega, \mathbf{x})\frac{u}{c}. \tag{1.57}$$