

M. Katherine Gavin, Ph.D., &
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Using the Schoolwide Enrichment Model in Mathematics

A How-to Guide for Developing
Student Mathematicians



A **Prufrock Press** Book

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**Schoolwide
Enrichment
Model** in Mathematics



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A Note to Our Readers



We wrote this book just for you. We hope you will use it as the title indicates, as a how-to guide to create what we affectionately call the Three Es of learning—enjoyment, engagement, and enthusiasm—with your students in math class.

This book is not a math curriculum such as that which you would find in a typical grade-level textbook. Rather, it is a supplementary set of recommended teaching strategies and resources that will allow you to infuse various enrichment experiences and high-level content into any and all math curricula that may be prescribed by existing textbooks or lists of standards used in your school district. This book was designed to provide instructional strategies and curriculum resources that will enhance whatever math curriculum you have adopted.

Instruction provided in the math curriculum is the “raw material” of learning, and we can enhance prescribed curriculum through hands-on activities, selected enrichment resources, and an investigative mindset. Strengthening your students’ enjoyment, engagement, and enthusiasm for learning will make your classroom a more exciting place and help your students see the role and relevance of math instruction. We hope learning about the Enrichment Triad Model in math, along with views from the classroom that show how teachers put ideas into action, will inspire you.

We wish you and your students upcoming math classes filled with enjoyment, engagement, and enthusiasm.

—Kathy and Joe



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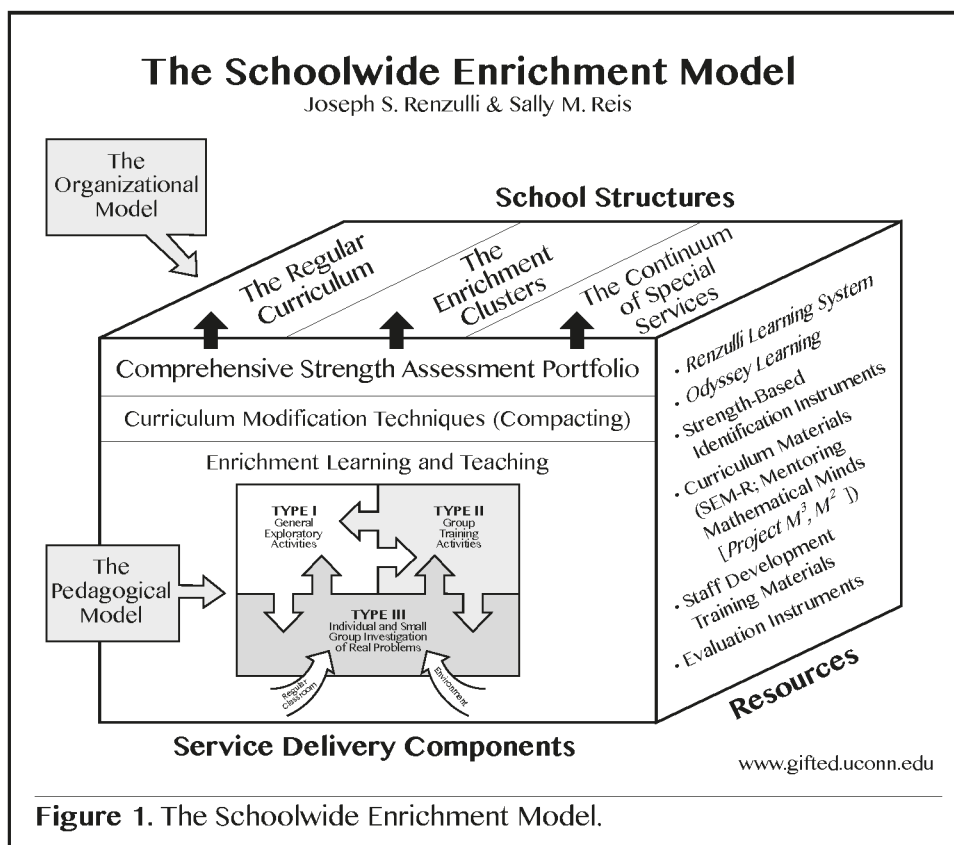
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Overview of the Schoolwide Enrichment Model

The Schoolwide Enrichment Model: A Focus on Student Strengths and Interests

Most of our work on the Schoolwide Enrichment Model (SEM) has been devoted to research and development on identification practices and teaching strategies for promoting gifted behaviors. Over the years, we realized that many students, in addition to those formally identified as gifted, could benefit from school experiences that are more enriching, engaging, and challenging. We also realized that in order to make changes in entire schools we needed to pay some attention to an organizational plan or model for the delivery of these strategies and the professional development that is guided by our theories and research. The SEM is not intended to replace existing gifted education programs. Rather, it is designed to infuse various types of enrichment into all aspects of the curriculum and to make certain types of enrichment activities available to the larger school population. We believe that a total talent development model must look at the mission, culture, and commitment of entire schools in addition to what goes on in special programs. This approach to applying the pedagogy of gifted education to total talent development is a departure from most traditional approaches that focus only on identified gifted students. Although there has been some criticism from persons representing more conservative positions in the field, national interests in both promoting 21st-century skills for all students and the need to



recognize talent potentials in underrepresented groups have resulted in a growing number of adoptions of the programming model presented in this book. The chapters that follow cover major components of the model as applied to the field of mathematics. An overview of the model is presented in Figure 1, and you will note here that the three major service delivery components of the SEM are brought to bear on three school organizational structures. These service delivery components, as they relate to mathematics, will be discussed in the chapters that follow.

In this chapter we provide an overview of the Schoolwide Enrichment Model and recommend that interested readers examine the third edition of *The Schoolwide Enrichment Model: A How-to Guide for Talent Development* (Renzulli & Reis, 2014) for detailed information about implementing the model. This book contains several instruments and planning guides that can be reproduced with permission for individual use.

What's a Model?

Before providing this overview of the SEM, it might be worthwhile to reflect for a moment about the meaning and purpose of this or any other plan that is designed to bring about selected changes in a school and the ways in which educators serve young people. The first consideration in answering the above question is the distinction between two categories of educational models. We will refer to the first category as administrative models and the second as theoretical models.

Administrative models consist of patterns of school organization and procedures for dealing with such issues as how educators group students, develop schedules, and allocate time, money, and human resources. Administrative models focus mainly on how educators group students and “move them around” and how they arrange for the delivery of services. Issues dealt with in administrative models might include homogeneous versus heterogeneous grouping, length of the school day or year, inclusion of special education students in regular classrooms, and whether or not educators should use a resource room or within-the-classroom program for the gifted.

Theoretical or pedagogical models, on the other hand, focus on the ways that educators provide the actual services to students, regardless of the manner in which they organize their schools or school schedules. Theoretical models consist of principles that guide the learning process and give direction to the content of the curriculum, the assessment and instructional strategies that teachers use, and ways in which educators evaluate the extent and quality of what their students have learned. Theoretical models focus on the actual outcomes of learning experiences that might take place within any given administrative pattern of organization. Theoretical models are influential in determining the quality of school experiences, whereas administrative models are more concerned with the efficiency and “smoothness” of the school's operation.

Although the SEM has certain implications for organizational patterns, we consider it a theoretically based model because it is guided by the Enrichment Triad Model (Renzulli, 1977a) that is based on: (1) a series of assumptions about individual differences in learners and the use of strength-based assessment, (2) research-based principles of learning, and (3) recommended practices that logically follow from these assumptions and principles. A crucial consideration in selecting this or any other model is whether or not there is a consensus among teachers, parents, and administrators about the assumptions, principles, and recommended practices. We have found that when such a consensus exists,

most schools easily accomplish the relatively small organizational or administrative changes necessary for implementing the model. Our experience has also shown that a theoretical model that infuses instructional practices into existing administrative patterns of organization has a higher probability of success and sustainability than an approach that tries to completely reorganize the school.

A Brief History of the Schoolwide Enrichment Model

How do we view and develop gifted behaviors in young people? How can we develop the potential of all children? What services should be provided to students who are identified for gifted and talented programs that would also provide some enrichment services to all students who can benefit from more engaging and challenging school experiences? Can enrichment programs for all students help to increase academic achievement scores? Can creative productivity be enhanced when students participate in enrichment or gifted education programs? How can we help children learn to think creatively and value opportunities for creative, self-selected work?

The Schoolwide Enrichment Model was developed to address these questions through a focus on developing gifted behaviors and creative productivity in young people. Although innovation is typically viewed as a process that always begins with a creative idea and ends with new or improved products, there are other factors that contribute to designing purposive tasks that can be organized into systematic plans for transforming ideas into tangible outcomes. In this chapter we discuss three interrelated components of creative productivity—above-average ability in a particular domain, creativity, and task commitment. The interaction between and among these components of the innovative process is necessary to provide the strategies for developing products or performances that can become audience- or consumer-valued products. Next, we discuss three types of educational services for promoting innovation in young people. These services consist of exposing students to areas of potential interest and task commitment, providing them with the methodological skills to pursue their interests in a professionally authentic manner, and providing the opportunities, resources, and encouragement to see their ideas through to fruition.

The two following examples of the model at work in mathematics will give you a practical understanding of the model in action.

As a 10th grader, Amber experimented with the mathematical equations used to calculate changes in space and time near black holes, and then shared this information via the Internet with physicists in other parts of the world. This talent for logical and insightful thinking was recognized and validated for Amber when scientists provided constructive feedback for her ideas, helping her to obtain new knowledge and skills as well as providing her with an outlet for synthesizing information about this topic. Several websites provide opportunities to “ask the experts.” With the help of scientists on the *Scientific American* website (<https://www.scientificamerican.com/section/ask-the-experts>), Amber became interested in specific calculations related to the black hole phenomenon. She entered her mathematical calculations about this topic in a local science fair competition and was awarded an honorable mention at the state level for her precision in and explanation of her calculations. As a result of her project, she also developed a website to teach other students about astronomy and space travel.

At age 17, Chris’s interest in engineering led to his acceptance in a competitive summer internship at a university well known for its faculty who conduct scientific research. Paired with a scientist in chemical engineering, Chris was assigned a project to investigate cancer cell growth. During the 7-week internship, he researched the topic, wrote a program in a computer language he had barely used before, solved the mathematical problem given to him about the growth of a particular cancer cell, and then crafted and presented a paper entitled “Computer Simulations and Cancer Research: A New Solution to a Complex Problem.”

The ability to develop innovative ideas can begin at an early age and evolve over time. These students have multiple characteristics in common, namely, the capacity to work intensely on a specific topic, to apply their natural inclinations and ability toward a specific activity, and to create something that they want to share with others, particularly with an audience who appreciates the topic. Although Amber is still establishing her knowledge base in the field of black holes, she is able to produce sound ideas and questions that have received pointed responses from international experts who are nurturing her understanding of astronomy and physics. Her science fair submission and her website represent outlets for sharing her work with others. Chris also created new products, which he disseminated to the public. Children and young adults have the ability to be more than consumers of information. They can be creative producers of high-quality products (Delcourt, 2008; Renzulli, 1986; Renzulli & Reis, 1985). To help youths achieve this goal, we need to recognize the characteristics that

lead to innovative behaviors and provide the type of environment that is a catalyst for these endeavors.

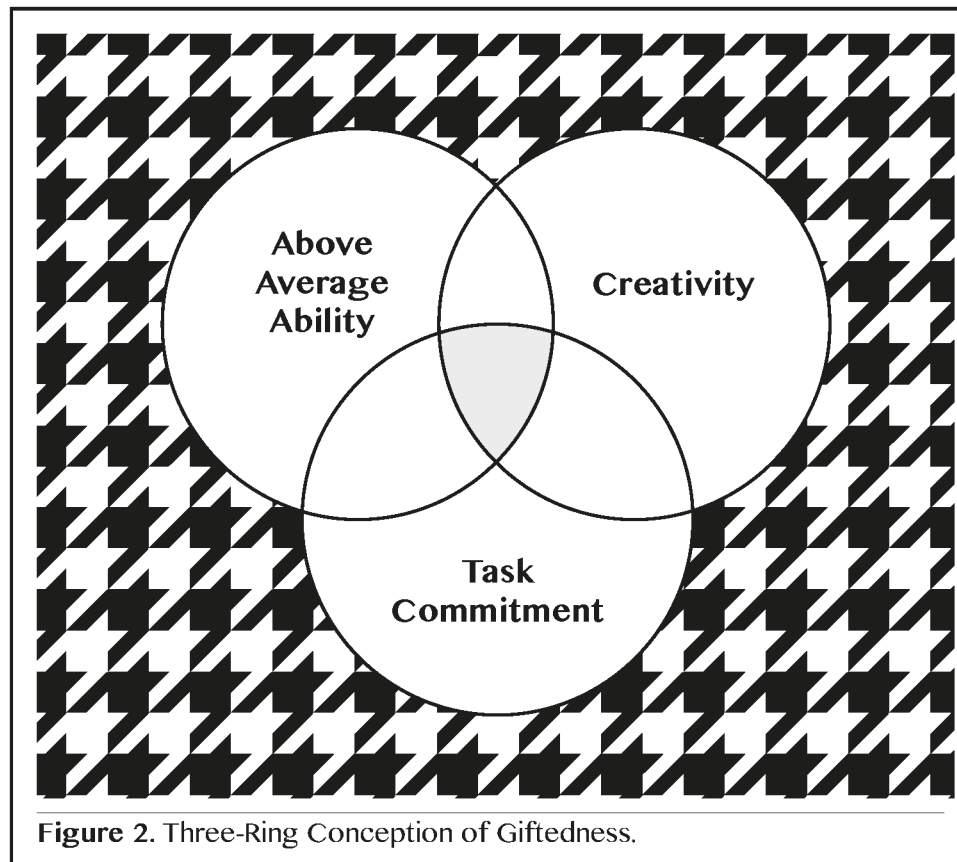
The Three-Ring Conception of Giftedness

The underlying theory about how we view and define gifted behaviors is called the Three-Ring Conception of Giftedness (Renzulli, 1978). Research on creative productive people has consistently shown that although no single criterion can be used to determine giftedness, individuals who have achieved recognition because of their unique accomplishments and creative contributions possess a relatively well-defined set of three interlocking clusters of traits. These clusters consist of above-average, although not necessarily superior, ability; task commitment; and creativity (see Figure 2). It is important to point out that no single cluster “makes giftedness.” Rather, it is the interaction among the three clusters that researchers have shown to be the necessary ingredient for creative productive accomplishment (Renzulli, 1978). This interaction is represented by the shaded portion in the center of Figure 2. It is also important to indicate that each cluster plays an important role in contributing to the display of gifted behaviors. This point is emphasized because one of the major errors that educators continue to make in identification procedures is to overemphasize superior abilities at the expense of the other two clusters of traits. The background of this diagram, referred to as a “houndstooth” pattern, represents the interaction between internal (personality) and external (environmental) characteristics that influence each individual.

Above-Average Ability

Above-average ability can be defined in two ways. One view includes the broad concept of overall capacity to think and perform, although the other perspective targets particular ways in which an individual understands, interprets, and reacts to information in a specialized domain (Csikszentmihalyi, 1990; Renzulli, 2005; Treffinger, 1998).

General ability consists of the capacity to process information, to integrate experiences that result in appropriate and adaptive responses in new situations, and the capacity to engage in abstract thinking. Examples of general ability are verbal and numerical reasoning, spatial relations, memory, and word fluency.



These abilities are usually measured by tests of general aptitude or intelligence, and are broadly applicable to a variety of traditional learning situations. The following are examples of general abilities (Renzulli & Reis, 2014):

- › High levels of abstract thinking, verbal and numerical reasoning, spatial relations, memory, and word fluency.
- › Adaption to and the shaping of novel situations encountered in the external environment.
- › The automatization of information processing. Rapid, accurate, and selective retrieval of information. (p. 23)

Specific abilities consist of the capacity to acquire knowledge, skill, or the capacity to perform in one or more activities of a specialized kind and within a restricted range, as indicated in the following examples (Renzulli & Reis, 2014):

- › The application of various combinations of the general abilities listed above to one or more specialized areas of knowledge or areas of human performance (e.g., the arts, leadership, administration).

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- › The capacity for acquiring and making appropriate use of advanced amounts of formal knowledge, tacit knowledge, technique, logistics, and strategy in the pursuit of particular problems or the manifestation of specialized areas of performance.
- › The capacity to sort out relevant and irrelevant information associated with a particular problem or area of study or performance. (p. 23)

These abilities are defined according to the ways in which human beings express themselves in real-life (i.e., nontest) situations. Examples of specific abilities are chemistry, ballet, mathematics, musical composition, sculpture, and photography. Specific abilities can be further subdivided into even more specific areas (e.g., portrait photography, astrophotography, photo journalism, etc.). Specific abilities in certain areas such as mathematics and chemistry have a strong relationship with general ability, and some indication of potential in these areas can therefore be determined from tests of general aptitude and intelligence. Achievement tests of specific aptitudes and actual examples of students' work—what is commonly referred to as performance-based assessment—can also help identify talent. Looking for behavioral characteristics that mimic the processes mathematicians and scientists use is also valuable in identifying potential in these areas. These characteristics can help identify students who have creative-productive potential, the ones who have the ability to contribute new theories and discoveries in mathematics and science. Many specific abilities, however, do not even have tests that are specifically designed to help measure potential or achievement associated with them.

Within this model, the term *above-average ability* is used to describe both general and specific abilities. *Above-average* should also be interpreted to mean the upper range of potential within any given area. Although it is difficult to assign numerical values to many specific areas of ability, “well-above-average ability” refers to persons who are capable of performance or the potential for performance that is representative of the top 15%–20% of any given area of human endeavor. This is the population typically represented in 4-year college enrollment.

Task Commitment

A second cluster of traits that has consistently been found in creative productive persons is a refined or focused form of motivation known as task commitment. Examples include the following (Renzulli & Reis, 2014):

- › The capacity for high levels of interest, enthusiasm, fascination, and involvement in a particular problem, area of study, or form of human expression.
- › The capacity for perseverance, endurance, determination, hard work, and dedicated practice.
- › Self-confidence, a strong ego, and a belief in one's ability to carry out important work, freedom from inferiority feelings, and drive to achieve.
- › The ability to identify significant problems within specialized areas. The ability to tune in to major channels of communication and new developments within given fields.
- › Setting high standards for one's work, maintaining openness to self- and external criticism, developing an aesthetic sense of taste, quality, and excellence about one's own work and the work of others. (p. 23)

Whereas motivation is usually defined in terms of a general energizing process that triggers responses in organisms, task commitment represents energy brought to bear on a particular problem (task) or specific performance area. The terms most frequently used to describe task commitment are perseverance or grit, endurance, hard work, dedicated practice, and self-confidence in one's ability to carry out important work.

Creativity

The third cluster of traits that characterize persons who display gifted behaviors consists of factors usually lumped together under the general heading of "creativity." The following are examples of creative abilities (Renzulli & Reis, 2014):

- › Fluency, flexibility, and originality of thought.
- › Openness to experience; receptiveness to that which is new and different (even irrational) in the thoughts, actions, and products of oneself and others.
- › Curious, speculative, adventurous, and "mentally playful." Willing to take risks in thought and action, even to the point of being uninhibited.
- › Sensitive to detail, aesthetic characteristics of ideas and things. Willing to act on and react to external stimulation and one's own ideas and feelings. (p. 23)

As one reviews the literature in this area, it becomes readily apparent that the words *gifted*, *genius*, *innovators*, and *eminent creators* or *highly creative persons*