

Second Edition

CREATING A
**LANGUAGE-
RICH**
MATH CLASS

Strategies and
Activities for
Building Conceptual
Understanding

An Eye On Education Book

SANDRA L. ATKINS

Creating a Language-Rich Math Class

What meanings do your students have for key mathematics concepts? What meanings do you wish them to have? *Creating a Language-Rich Math Class* offers practical approaches for developing conceptual understandings by connecting concrete, pictorial, verbal, and symbolic representations. The focus is on making mathematics memorable instead of on memorizing.

You'll learn strategies for introducing students to math language that gives meaning to the terms and symbols they use every day; for building flexibility and precision in students' use of math language; and for structuring activities to make them more language-rich. This second edition also provides strategies for helping students to at times be quiet and listen to their peers; for purposefully using language to introduce students to more complex mathematical symbolism and algebraic properties; and for using writing prompts to zoom in on the meanings that individual students have given to the language-rich experiences.

Appropriate for elementary teachers and instructional coaches, the book also includes features such as Investigations to Try and Questions for Reflection to help you incorporate these ideas into your practice. In addition, there are Blackline masters of game cards and puzzles, which can also be found on our website for free download at www.routledge.com/9780367759957.

Sandra L. Atkins is committed to finding those "AHA moments" when mathematical connections are made by teachers and students. She currently works with school districts across the United States and with International Independent Schools through her company, Creating AHAs.

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Creating a Language-Rich Math Class

Strategies and Activities for Building
Conceptual Understanding

Second Edition

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Dedicated to

All of the teachers, students, and colleagues who have taught me
throughout my career.



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Support Material

The Blackline Masters from the book are also available on our website so you can print and use them in your own classroom. The material includes the following:

- ◆ Ten-Frame Dominos
- ◆ Sample Concentration Game Cards
- ◆ Tangram Piece Master
- ◆ Sample Tangram Puzzles
- ◆ Sample “I Have . . . Who Has?” Cards
- ◆ Word Problem Graphic Organizer
- ◆ Sample Word Problems

To download those items, go to the book product page, www.routledge.com/9780367759957. Then click on the tab that says “Support Material,” and select the files. They will begin downloading to your computer.

About the Author

Dr. Sandy Atkins is the owner and executive director of Creating AHAs, LLC. An inspiring speaker, Dr. Atkins is committed to finding those “aha moments” when mathematical connections are made by teachers and students. Her sessions are thought-provoking and practical.

Sandy is passionate about helping *all* students understand mathematics. Sandy works closely with teachers across the United States and in International Schools to ensure that all students are high achievers in mathematics. Dr. Atkins enjoys opportunities to provide keynote talks, workshops, seminars, and presentations. But her favorite professional development activities involve building relationships with teachers through ongoing site-based projects that include getting into classrooms and learning from students. Sandy loves hearing, “Oh I get it! Why wasn’t I taught this way?” AHA!

Dr. Atkins is also the author of *Creating Fraction and Decimal AHAs*, a resource that provides a systematic approach for helping students understand and even enjoy learning about fractions and decimals.

Please contact Sandy Atkins, Ph.D., at Creating AHAs for additional information.

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Thank you!

Last but not least, thank you for reading this long acknowledgment. As you read through the chapters of this book may your head hurt . . . in a good way.

Introduction

Language? It's Mathematics!

I was going to visit a friend who had just moved to Pittsburgh. She had emailed directions to her new house. About a third of the way through it said, "Continue down the windy road for about a mile." Now when you read that I bet you thought of a road with a lot of curves. Not me. I'll claim that I was tired. When I read it I wondered how she knew there would be wind on the road that day . . . as in gusts of . . . wind tunnel. Not one of my finer moments. It did make me laugh at the time. It still makes me smile and shake my head.

Language is tricky. Think about bough, dough, rough, and tough. What about "read" being either past or present tense depending on the pronunciation? I could go on but you get my point. Mathematical words are also tricky. A single word can have multiple meanings depending on context. A "second" is a unit of time but it is also a position in line. Some words have everyday meanings and we give them mathematics-specific meanings. "Similar" is just one example. Its everyday meaning describes things that are almost alike but not exactly alike. In mathematics it means proportional. Then there are mathematics-specific words such as minus or subtract and words that describe the materials we use. Whew!

The sad thing is that many people do not think of mathematics as being language rich, but it is. We cannot discuss mathematics without using words. Story problems require an ability to read and comprehend language, especially language used in math contexts. Context helps us know if "windy" is a curvy road or if it's a great day to fly a kite. Constructing viable arguments and critiquing the reasoning of others is quite challenging without language. Most importantly, language provides us with a window into the understandings children have about key mathematical concepts.

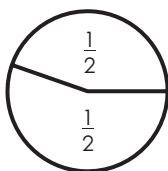
A language-rich mathematics class is an exciting place. Students are describing their thinking, defending their answers, and discussing mathematical ideas. Children who struggle with memorizing facts and procedures have a way to make sense of mathematics. They are able to connect mathematical symbols to things they can touch, things they can draw, and language. Learning mathematics shifts from memorizing to engaging in activities that are memorable.

What are we doing? We're finding out what our students think, purposefully designing tasks, using conceptual language, and encouraging students

to talk with one another. What is the most important thing we'll do to create a language-rich mathematics class? Learn to be quiet and listen. After all, who needs the most practice talking?

In this new edition we examine how to help students also learn to be quiet and listen. We know that once the answer is given, the thinking and discussion stops. How can students critique the reasoning of others if they aren't quiet and listening to their peers?

I'll warn you though. It may not all be smooth sailing. I had been asked to do a lesson in a first-grade classroom. I decided to do something on fractions. I wanted the children to understand that if I divided a whole into two pieces, they both needed to be the same size to be called halves. We were seated in a circle on the floor. I had drawn a large circle on a large sheet of paper. I cut it into two pieces but the pieces weren't the same size. I labeled each piece " $\frac{1}{2}$."



I placed the drawing in the center of our circle and asked the children what they thought. Nobody said anything. I asked, "If this were a cake and I cut it this way, what would you think?" Silence. I took scissors and cut the "cake" into the two pieces. I gave the large piece to one student and the small piece to another. I again asked, "If this were cake or pie, what would you think?" Nothing. I took the pieces back and cut them again into uneven pieces. Distributed the pieces and asked my question again. They just sat quietly, waiting patiently.

Now this was supposed to be a language-rich lesson, the focus of this book. But they weren't saying anything. There were other teachers watching and I was getting nowhere. I'm sure you can predict how I was feeling. I stuck with it though. I took back pieces and several more times cut the "cake" into smaller pieces and gave the pieces to children who didn't have a "slice" yet. Each time I asked what they thought. Same reaction. Polite silence. Finally every child had a piece of this "cake." Some had nice large pieces and some very small pitiful slices. I asked one last time.

Finally the first little girl I had given the large "half" to raised her hand. I was hopeful. I asked again. She looked at me and said, "Mrs. Hughes says it is better to have something than nothing and you should be happy you have anything at all."

Are you kidding me? I turned and looked at Mrs. Hughes and the other teachers and they just started laughing. No wonder I couldn't get them to discuss the fairness of the size of the pieces. Mrs. Hughes had done a

wonderful job of getting children to stop bickering when they might not have the exact amount as their neighbor. They were being extremely polite for their guest teacher. It sure did affect what I thought was going to be a great language-rich discussion.

Throughout the book we'll examine techniques for building conceptual understanding and creating a language-rich mathematics class. As you'll see, it takes time to develop mathematical language. It requires us to sit down, be quiet, be curious, and listen to what the children are saying. Don't be surprised if some of your other "class lessons" creep into your math conversations . . . "happy to have anything at all." Really! One of the teachers tried a similar lesson with her class. After the first cut, she asked a student which piece she would like. The student selected the smaller piece and said that she was trying to watch her fat content. They had just finished a nutrition unit.

As you try the activities for creating a language-rich mathematics class described in this book, remember to give children many opportunities to test out the language of mathematics. Let them share what they think and allow them to make missteps. If we listen carefully to what they say instead of what we hope to hear, we will learn about their mathematical understandings. Sometimes we may even smile. We may smile because of their sophistication in explaining their thinking. We may smile at their creativity in solving problems and making us think. We might even smile at the "windy-windy" confusion they have or laugh aloud as they foil a great launch to a mathematically language-rich discussion. After all, it is better to have something than nothing.

The additional chapters in this new edition include other aspects of a language-rich math class. I've mentioned students learning to give their peers the gift of time to think, to at times be quiet, and to listen to their peers (The Power of the Forced Mute). I've extended purposefully choosing materials to purposefully choosing images that build the language of the Associative and Distributive Properties (Purposefully Use Representations to Build the Language of Properties). Our investigation begins with images for introducing the property at the fact level. We then extend the use of the properties and related language to build computational fluency with fractions and decimals.

As the math class becomes more language-rich with students sharing their thinking and mental strategies for solving problems, we connect conceptual language to precise and more sophisticated mathematical symbolism (Introducing Mathematical Language as We Record Student Thinking). Conceptual language translations for the operations and symbols provide students with access to more complex symbolism before expecting them to use the symbolism. You'll be amazed as some students begin to use property

names and language as they describe their thinking. In a language-rich math class, you will witness wonderful mathematics discussions by your young mathematicians. More students will have opportunities to talk and to share their thinking. At times you'll need to pause and zoom in on the meaning that each individual student has taken from the discussions (The Importance of Writing in a Language-Rich Mathematics Class). Impromptu writing prompts as well as planned exit slips, journal entries, concept maps, and writing about important ideas, gives us a glimpse into each student's thinking. I hope you'll find these new chapters helpful in creating a language-rich math class.

In each chapter I've included *Investigations to Try* and *Questions for Reflection*. I hope you will try these investigations and reflect on the questions. I also hope this is not an independent journey. Talk with your peers about your thinking, your questions, your wonderings, and your AHAs. As I hope you'll see, a language-rich experience is very powerful for us all.

Question for Reflection

What stories came to mind regarding your students' struggles with language, in particular mathematical language?



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