## Derek Haylock

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Numeracy for Teaching

Derek Haylock is Senior Lecturer in Education at the University of East Anglia Norwich, where he is Co-Director of Primary Initial Teacher Training and responsible for the Mathematics components of the primary programmes. He has worked for 30 years in teacher education, both initial and in-service, but he also has considerable practical experience of teaching in primary classrooms. His work in mathematics education has taken him to Germany, Belgium, Lesotho, Kenya, Brunei and India. He is co-author (with Anne Cockburn) of Mathematics in the Lower Primary Years (Paul Chapman Publishing, 1997), co-author (with Doug McDougall) of Mathematics Every Elementary Teacher Should Know (Trifolium Books, Toronto, 1999), co-author (with Marcel D'Eon) of Helping Low Achievers Succeed at Mathematics (Trifolium Books, Toronto, 1999) and author of Teaching Mathematics to Low Attainers 8-12 (Paul Chapman Publishing, 1991). His other publications include seven books of Christian drama for young people and a Christmas musical (published by Church House/National Society), and frequent contributions to education journals.

# Numeracy for Teaching 

## Derek Haylock

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## Read this first

## The professional context of teaching

Numeracy for Teaching is a book that, in a way, I wish I did not have to write. Of course, I am in favour of the idea that teachers, in all phases and in all subject areas, should be able to make decisions and judgements based on numerical information with confidence and a reasonable level of accuracy. Indeed, for any group of professionals, this would be a laudable aim, the achievement of which would contribute to all of us having a greater sense of general security as we go about our daily business. I am always pleased to take any opportunity to contribute to raising confidence and reducing the widespread anxiety about working with mathematical ideas that all of us who work in teacher training continue to meet in every new group of trainees. I have worked in mathematics education long enough to understand where these anxieties come from and to realise that they need a serious and sympathetic response.

My misgivings arise from the current professional context of teaching. We now work in a field where those in control of educational policy have an obsession with reducing everything in education to numbers, tables, charts and graphs. There are people out there who actually believe that by improving the statistics you necessarily improve the quality of education. The educational 'newspeak' of today is littered with the language of standards, baseline assessment, target-setting, action plans, performance indicators, value-added data, levels of achievement, average points-scores, thresholds, quartiles and percentiles, Ofsted grades, audits, league tables, and so on. This, we are led to believe, is modernising the profession. Within the Department for Education and Skills (DfES, formerly the Department for Education and Employment) there is a powerful group called the Standards and Effectiveness Unit. Within that is the School Effectiveness Division and within that the Pupil Performance Team. I'm not making this up, honest! The main function of these groups seems to be to gather and disseminate huge quantities of statistical data about pupils and schools, mainly focused on levels of attainment in national tests and public examinations. As a consequence, educational achievement and practice are defined by sets of numbers. Each year this standards unit sends out to schools an 'autumn package' of statistical information. This is intended to enable schools to judge their performance against national standards and against the achievements of other similar schools. ‘Similar’ schools are determined by the proportions of pupils known to
be eligible for free school meals, which is apparently a reliable predictor of the mix of social groups from which the school draws its population. Comparisons with the achievements of these similar schools are key factors in the judgements made by Ofsted inspectors about how well a school is doing. In some cases I have known headteachers to actively seek out the one or two more pupils they need to move them into a different band for free school meals, where the comparisons will be more favourable! This is the kind of daft thing that happens when too much significance is given to numerical indicators in judgements about the quality of teaching and learning.

So, is this the brave new world of teaching? Of course, it isn't. The real world of teaching is still the delight of a teacher interacting with the hearts and minds of young people, the encouragement of seeing genuine learning taking place, and the occasional thrill when a pupil shows enthusiasm, flair and creativity. It continues to be about developing the skills you need to manage a class of 30 uncooperative individuals on a Friday afternoon in a mobile classroom and finding ways of making your material interesting and relevant to their needs and interests. These are the real joys and challenges of teaching.

But the other stuff won't go away. So we just need to make sure that we can handle the mathematics, that we can make sense of the numbers and that we are not being hoodwinked by those who credit the numerical data with unjustified reliability or validity. This book is intended to make a small contribution in this respect. If you have taken the trouble to read my ramblings so far, then you will appreciate why the occasional touch of cynicism will emerge in the material that follows. I find it helps to keep me sane in today's educational climate - and I would recommend a small dose from time to time, particularly when the latest batch of DfES documents arrive at your school.

## The QTS numeracy test

One of the consequences of all this is that the DfES has required the Teacher Training Agency (TTA) to introduce a Basic Skills Test in numeracy for all new entrants to the profession. It is no longer possible to achieve Qualified Teacher Status (QTS) without passing this test. The test focuses mainly on interpreting the kinds of statistics that occur in the autumn package, presumably as a way of raising awareness of this annual document within the profession. I suppose it must be a bit galling for the standards unit to keep churning out this stuff, knowing that in most schools it gets no further than a cupboard in the head-
teacher's office. So, one of the main purposes of my writing this book is to provide teacher-trainees with help in preparation for the QTS numeracy test.

## Artificial questions

I have tried as far as possi ble to set the material of this book in the professional context of teaching. But, of course, I cannot provide the genuine, meaningful context in which you will encounter the need to use the numeracy skills covered in this book, nor the purposefulness in the questions which would help you to make more immediate sense of them. Just as you will find in the QTS numeracy test, many of my questions will be rather artificial. I apologise now for this, because there is plenty of research to indicate that people are much more successful with mathematics when it is purposeful and embedded in a meaningful context. So, although I have tried to draw as much as possible on the professional context of teaching, please realise that I have had to design my examples with the purpose of explaining and discussing a particular skill or concept, rather than to reflect the reality of professional decision-making.

## Sources of data: a disclaimer

This book is not intended to be a source of reliable and accurate statistical data about education! As far as possible I have drawn on actual statistics and other data from individual schools, from the DfES, the QCA (Qualifications and Curriculum Authority), and their predecessor, SCAA (Schools Curriculum and Assessment Authority). I have made particular use of the data available on the DfES website. All the data drawn on in this way is therefore available in the public domain. But, occasionally, in order to make the data more accessible for teaching purposes, I have had to adapt it or prune it a little. In some cases, as will be obvious, the source of the data is my own imagination.

I should also apologise to readers in Wales, Scotland and Northern Ireland. I work in England and I have therefore drawn on data related to the context with which I am personally familiar. I have made a definite decision not to include a few token examples from other parts of the United Kingdom to give the impression that the book draws on the full range of educational contexts. I am confident that the material will nevertheless be useful to such readers.

## Feedback from trainees

I have trialled the material in this book with my own students and have been encouraged by their responses:

The material is very hel pful indeed as preparation for aspects of teaching and for the QTS numeracy test. The appropriateness of the educational setting of the subject matter of each check-up adds to its usefulness.

It is easy to follow and has clear and straightforward explanations. I found the summary of key points helpful as they are quick reference to reinforce what has just been read. It is bound to be beneficial in preparation for the QTS skills test.

The book is excellent. I think it will present students and others with a valuable resource, not only to help with the QTS test, but also for teaching. I could see myself using it on a dip-in-as-necessary basis.

Many thanks for letting me work through your sample material. I cannot begin to tell you how much more I have learnt! This has definitely made me feel more confident about passing the QTS numeracy skills test!

I have definitely demonstrated to myself from this material that my ability to complete calculations mentally has increased.

I passed the QTS numeracy test last week after working through this material!

## Numeracy

The mathematical material in this book focuses especially on weaknesses in numeracy that are often observed in adults in general, and in teacher-trainees in particular.

Many adults rely too much on using calculators or formal written methods to do simple calculations that could be done mentally. So in this book I emphasise especially the development of confidence in using informal and mental methods of calculation. I am also aware that, when they do have to use calculators, many people use them inefficiently. So, help is provided in this respect as well.

Many readers will have gained a mathematical qualification when they were 16 and not have done any formal mathematics for many years since. Understandably, they will have forgotten the meanings of some of the technical vocabulary of mathematics that is not used in everyday life, words such as
median or denominator. So I have taken into account that they will need to brush up on mathematical terminology, as well as revisiting many of the basic processes and skills in which they feel a bit rusty.

Adults are generally weak in handling the concepts of ratio and proportion. Again, many people tend to rely too much on unnecessarily formal procedures for handling problems in this area. So there is a lot of emphasis in this book on expressing proportions in fraction notation and in decimal notation, and on percentages. For example, you should be able to move freely between $\frac{3}{5}, 0.6$ and $60 \%$. This is an important facility when comparing the ratios and proportions that proliferate in the context of teaching.

The biggest concern for those facing the numeracy test is likely to be the interpretation of government statistics, particularly presented in various forms of tables and graphs. By the time you have worked through this book, you should be able to handle with confidence such things as: means and modes, medians and quartiles, inter-quartile ranges, percentiles, box-and-whisker diagrams, pie charts, stacked-column charts, two-way tables, weighted means, cumulative frequency graphs, and even the DfES's favourite - value-added data.

## How to use this book

This is not a book to read. It is a book to work through. You need pencil and paper to hand at all times, and, when suggested, a calculator.

It consists of 62 check-ups, each focusing on one numeracy skill or concept. Start by trying the check-up question. The answers will be over the page. If you find the check-up question insultingly easy, then give your back a pat and your confidence a boost and go on to something else. Otherwise, work through the discussion and explanation that follows the answers. This is followed by some 'see al so' suggestions. These will be other check-ups that might cover some of the prerequisite mathematics needed, or related areas, or extensions of the material being discussed. I then provide a summary of key points, for future reference and to highlight the main things to learn from the check-up. After this there will be one or two further practice questions. You will usually find it very helpful to work through these to reinforce and to assess your own understanding. Answers for these further practice questions are provided at the end of the book. It is important to look at these answers, because often I have included some substantial teaching points here.

This is not a systematic book and the material does not have to be worked through in the order provided. You will probably find it most useful to dip into it from time to time whenever you have a spare half an hour or so. I assume that you will have done most of the mathematics here before. What you need is probably to revisit and practise skills and concepts from your past, to meet them again in the professional context of teaching, and to be provided with a little more enlightenment here and there about what is going on when you are manipulating and interpreting numerical data.

## And finally...

In common with my other books, this one will not bear the kite-mark of the Teacher Training Agency for England and Wales. I continue to turn down all invitations to submit material for this scheme, being of the opinion that intelligent readers are quite capable of deciding for themselves whether or not this book is worth buying, without it first having received the approval of a government agency.

Derek Haylock
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## Checsup

## Mental calculations, changing proportions to percentages

Without using a calculator, rewrite these statements using percentages:
a) A quarter of the pupils in my class have free school meals.
b) Three-quarters of the pupils in my class do not have free school meals.
c) Seven out of eight pupils in primary schools like their teacher.
d) This year our school had to employ a supply teacher 17 days out of 20 .
e) Four-fifths of the lessons observed by Ofsted in our school were good or very good.
f) A total of 273 pupils out of 300 achieved at least one GCSE at grade C or above.

## Answers to check-up 1

a) $25 \%$ of the pupils... b) $75 \%$ of the pupils... c) $87.5 \%$ of primary pupils...
d) $85 \%$ of the days...
e) $80 \%$ of the lessons... f) $91 \%$ of the pupils...

## Discussion and explanation of check-up 1

Per cent (\%) means 'for each hundred'. For example, 27\% (27 per cent) means '27 in each hundred', or ' 27 out of a hundred'. Percentages are useful because they give us a standard way of expressing proportions. This makes it easy to compare different proportions of pupils with free school meals in two schools if they are expressed as percentages (e.g. $37 \%$ and $35 \%$ ). It is not so easy if all you have is the raw data (e.g. 170 out of 459 in one school and 238 out of 680 in the other).

Many simple proportions or fractions can be easily expressed as percentages, using mainly mental calculations. For example, the fraction one-half ( $\frac{1}{2}$ ) might represent the proportion of a set of secondary pupils who own a mobile telephone. Without knowing how many pupils there are in the set, we can still express this proportion as an equivalent percentage. One-half as a proportion means 'one out of every two pupils owns a mobile'. That's equivalent to 'fifty out of a hundred', or $50 \%$. Knowing this we can easily deduce percentage equivalents for some other common fractions. Since a quarter $\left(\frac{1}{4}\right)$ is 'half of a half', then it must be equivalent to half of $50 \%$, that is $25 \%$. And three-quarters $\left(\frac{3}{4}\right)$ will be three times this, which is $75 \%$.

Eighths are a bit trickier. One-eighth is 'half of a quarter', so expressed as a percentage it must be 'half of $25 \%$ ', which gives $12.5 \%$. Knowing this, you can then work out percentage equivalents for $\frac{3}{8}, \frac{5}{8}$ and $\frac{7}{8}$. Actually, in example (c) I found it easier to think 'one-eighth of the pupils do not like their teacher', which is $12.5 \%$, and then to subtract this from $100 \%$ to find the percentage who do like their teachers. This works because $100 \%$ represents the whole set of pupils, that is ' 100 out of 100 '.

In example (d) the ' 17 out of 20 ' is easily converted to an equivalent proportion out of a hundred, just by multiplying by 5 . This gives ' 85 out of 100 ', which is $85 \%$.

In example (e) we can think of $\frac{4}{5}$ as ' 4 out of 5 ', which is equivalent to 80 out of 100 , or $80 \%$.

In example (f) we simply divide 273 by 3 , to deduce that ' 273 out of 300 ' is equivalent to '91 out of 100 ', or $91 \%$.

## See also...

Check-up 2: Mental calculations, changing more proportions to percentages

## Summary of key ideas

- Per cent means 'for each hundred' (e.g. 35\% means '35 out of 100')
- A proportion can be written as a percentage, by working out the equivalent number out of a hundred (e.g. ' 7 out of 20 ' is ' 35 out of 100 ', which is $35 \%$; ' 240 out of 300 ' is ' 80 out of 100 ', which is $80 \%$ ).
- The percentage equivalents of many simple proportions (such as $\frac{1}{2}=50 \%, \frac{3}{4}=75 \%, \frac{4}{5}=80 \%$ ) should be memorised.


## Further practice

Do these without using a calculator.
1.1 Work out the equivalent percentages for the following fractions and then commit them all to memory!

$$
\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{20}, \frac{3}{20}, \frac{7}{20}, \frac{9}{20}
$$

1.2 A pupil scores 21 marks out of 25 in one test and 17 out of 20 in another. Express these marks as equivalent percentages, to decide which mark is the higher proportion of the total marks available.
1.3 In a school of 600 pupils, there are 126 pupils with English as an additional language. In another school the number is 104 out of 400 , Compare these proportions by expressing them as percentages.

## Checs-Up

## Mental calculations, changing more proportions to percentages

Do not use a calculator.
a) School $X$ has 248 pupils out of 400 living within a mile of the school; School Y has 135 out of 250 . Which has the larger proportion living within a mile of the school? To answer this, express the proportions as percentages.
b) A pupil scores 22 marks out of 40 for part A of a mathematics test and 39 out of 60 for part B. In terms of percentages, in which part did the pupil score higher marks?
c) Three schools expressed the proportion of pupils on free school meals in different ways.
School A: $\frac{1}{5}$ of the pupils have free school meals.
School B: 17\% of pupils have free school meals.
School C: 77 pupils out of 350 have free school meals.
Which school has the lowest and which school has the highest proportion of pupils on free school meals?

## Answers to check-up 2

a) School X: 62\%. School Y: 54\%. School X has the larger proportion.
b) Part A: $55 \%$. Part B: $65 \%$. Part B was the higher percentage mark.
c) The lowest is School B with $17 \%$, the highest is School C with $22 \%$; School A has $20 \%$.

## Discussion and explanation of check-up 2

Proportions such as these can be changed into equivalent proportions out of 100, by simple multiplications and divisions, especially doubling and halving where possible. They can then be expressed as percentages. My strategy is to look at the total number involved and ask how I can relate it to 100 . So, for example, given a proportion of 400 , I will halve and halve again to get an equivalent proportion of 100 . Given a proportion of 250 , I will double and double again to get to 1000 and then divide by 10 to get to 100 . Below I show how I reasoned for the proportions in these check-up questions. You may well have done these differently, which is fine, of course. I have used an arrow $(\rightarrow)$ to mean 'is equivalent to'.

248 out of $400 \rightarrow 124$ out of 200 [halving] $\rightarrow 62$ out of 100 [halving] $=62 \%$
135 out of $250 \rightarrow 270$ out of 500 [doubling] $\rightarrow 540$ out of 1000 [doubling] $\rightarrow 54$ out of 100 [dividing by 10] $=54 \%$.

22 out of $40 \rightarrow 11$ out of 20 [dividing by 2] $\rightarrow 55$ out of 100 [multiplying by 5$]=55 \%$.

39 out of $60 \rightarrow 13$ out of 20 [dividing by 3] $\rightarrow 65$ out of 100 [multiplying by 5$]=65 \%$
$\frac{1}{5} \rightarrow 1$ in $5 \rightarrow 2$ in 10 [doubling] $\rightarrow 20$ in 100 [multiplying by 10 ] $=20 \%$.
77 in $350 \rightarrow 11$ in 50 [dividing by 7] $\rightarrow 22$ in 100 [doubling] $=22 \%$.
Note that in all these questions the relationships between the numbers are such that it is not difficult to manipulate them mentally to obtain proportions out of 100 . In the last example it was easy to do this once I spotted that 7 divided exactly in 77 and 350. If, however, the proportion had been, say, 79 out of 350 or 77 out of 352 , this would have been far more difficult to handle by purely mental methods and it would make more sense to use a calculator.

## See also...

Check-up 18: Using a calculator to express a proportion as a percentage
Check-up 21: Mental calculations, multiplication strategies
Check-up 22: Mental calculations, division strategies

## Summary of key ideas

- Many proportions can be expressed as percentages by changing them to equivalent proportions out of 100 , by simple multiplications and divisions.
- Look for ways of using doubling, halving, multiplying by 5 or 10 , or using simple divisions to relate the total number involved to 100 .
- If there is no simple way of changing a proportion to an equivalent proportion out of 100 , then use a calculator to do this.


## Further practice

Do these without using a calculator.
2.1 A pupil scores 19 out of 25 in a spelling test and 42 out of 70 marks in a mathematics test. Express these marks as equivalent percentages.
2.2 In Ofsted inspections of a sample of 250 schools, 35 schools were graded as unsatisfactory, and 130 were graded as good, for leadership and management. Express these proportions as percentages. What percentage of schools were graded satisfactory (the only other grade awarded)?
2.3 Three primary schools reported the proportions of their Year 2 pupils achieving level 2 or above for reading as follows:

School P $\frac{3}{5}$ of the pupils achieved level 2 or above
School Q 74\% of the pupils achieved level 2 or above
School R 36 out of 45 pupils achieved level 2 or above
Put these proportions in order from the highest to the lowest.

## Decimals and percentages

a) A primary school calculates the average (mean) National Curriculum level for the core subjects for their pupils at age 11 to be as follows: English, 4.3; mathematics, 4.195; science, 4.28. Arrange these in order from the lowest to the highest.
b) In a secondary school, the proportions of Year 9 pupils achieving level 5 or above in the Key Stage 3 tests last year were 0.76 for mathematics, 0.8 for science and 0.675 for English. Without using a calculator, express these proportions as percentages.

## Answers to check-up 3

a) 4.195 (mathematics), 4.28 (science), 4.3 (English).
b) $76 \%$ for mathematics, $80 \%$ for science, $67.5 \%$ for English.

## Discussion and explanation of check-up 3

(a) To compare a set of numbers written in decimal notation (such as, 4.3, $4.195,4.28$ ) if can be helpful to rewrite them (in your head) with the same number of figures after the decimal point. Since one of the numbers in this set has three figures after the decimal point, I might think of them as: 4.300, 4.195 and 4.280. This helps to put them in order: 4.195, 4.280, 4.300. The inclusion of extra zeros after the last figure after a decimal point does not change the value of the number. So, for example, 4.28 means 4 units, 2 tenths and 8 hundredths; and 4.280 means 4 units, 2 tenths, 8 hundredths and no thousands. These are clearly the same. Note that you can mislead yourself by thinking that 4.28 is smaller than 4.195 , for example, because 28 is smaller than 195 . The misunderstanding here is not to realise that the 28 means ' 28 hundredths', whereas the 195 means ' 195 thousandths'. It is also helful to think of the position of these number on a number line diagram, as below. Notice that 4.28 comes between 4.2 and 4.3 , and that 4.195 comes between 4.19 and 4.20 (which is also written as 4.2).

(b) The decimal number 0.76 means 7 tenths and 6 hundredths, or 70 hundredths +6 hundredths, which is 76 hundredths. Another way of writing 76 hundredths is $76 \%$. So, it is really very easy to convert a decimal number with
two figures after the decimal point to a percentage, and vice versa: $0.76=76 \%$, $0.57=57 \%, 0.40=40 \%, 0.04=4 \%$, and so on.

If there is just one figure after the decimal point, then just mentally include an extra zero. For example, $0.8=0.80=80 \%$. Similarly, $0.2=20 \%, 0.4=40 \%$ and so on. If there are more than two figures after the decimal point, because it's the first two that tell you how many hundredths you have, you move them so they are in front of the decimal point, as follows: $0.675=67.5 \%$. Here are some other examples: $0.045=04.5 \%=4.5 \%, 0.1234=12.34 \%, 0.9005=90.05 \%$.

Sometimes percentages greater than $100 \%$ are used. These can also be written as decimals. Here are some examples: $125 \%=1.25,117.5 \%=1.175,250 \%=2.5$.

## See also...

Check-up 10: Fractions to decimals and vice versa

## Summary of key ideas

- To compare a set of numbers written in decimal notation it can be helped to rewrite them with the same number of figures after the decimal point.
- It is helpful to visualise numbers written in decimal notation in terms of their position on a number line.
- To change a decimal number to a percentage, move the digits two places to the left, relative to the decimal point; for example, $0.46=$ $46 \%$, $0.175=17.5 \%$
- To change a percentage to a decimal number, move the digits two places to the right, relative to the decimal point; for example, $99 \%$ $=0.99,150 \%=1.50$ or 1.5 .


## Further practice

3.1 A pupil labels some points on a number line in order, as follows: 1.7, 1.8, $1.9,1.10, \ldots$ what is the error here?
3.2 A school's target is that the proportion of pupils absent each day should be less than 0.08 . On which of the following days do they not achieve the target? The numbers in brackets are the proportions of pupils absent.

Monday (0.075), Tuesday (0.1), Wednesday (0.09), Thursday (0.079), Friday (0.009)
3.3 Without using a calculator, write the proportions given in the previous question as percentages.

## Chesk-Up

## Understanding data presented in tables

A primary school completed the following table using data provided by the DfES for national results and their own results in the Key Stage 2 science test.

Percentages of pupils attaining level 4 or above in the Key Stage 2 science test.

|  | National results | School results |
| :--- | :---: | :---: |
| All | 69 | 65 |
| Boys | 70 | 62 |
| Girls | 68 | 69 |

a) What information is represented by the 68 in this table? What is represented by the 62?
b) How well did the pupils in this school do in the science test compared to national results?

## Answers to check-up 4

a) $68 \%$ of girls nationally achieved level 5 or above in the Key Stage 2 science test. $62 \%$ of the boys in this school who took the Key Stage 2 science test achieved level 4 or above.
b) Overall, the proportion of pupils in this school achieving level 4 or above for science was 4 percentage points lower than the national proportion.

The proportion of the boys in the school who achieved level 4 or above was 8 percentage points lower than the proportion of boys nationally.

However, the proportion of girls in the school who achieved level 4 or above was 1 percentage point higher than the proportion of girls nationally.

## Discussion and explanation of check-up 4

A table like the one in this check-up question is made up of rows and columns. A row and a column intersect in a cell. The data written in a cell can be either numbers or labels.

The strategy for reading tables like this is first to be quite clear about what is being measured by the numbers written in the various cells. Technically, we could call this the variable and the numbers in the cells are values of the variable. In this case, the variable is 'the percentage of pupils achieving level 4 or above in the Key Stage 2 science test'. We can find this in the caption above the table. The table therefore provides us with the values of this variable for different groups of pupils. So, when we see the number 68 in the table it means $68 \%$ of some set of pupils achieved level 4 or above in this test. Then we look at the labels that act as headings for the rows and columns to identify quite clearly and specifically to what each row and each column refers. In this case, for example, the column headed 'National results' refers to the results obtained by pupils from all schools nationally; the column headed 'School results' refers to the results obtained by pupils in this particular school. The rows clearly distinguish between 'all pupils', 'boys' and 'girls'. We can then be quite clear about what each cell refers to. For example, where the 'National results' column meets the 'Girls' row we will find the percentage of girls in all schools nationally who achieved level 4 or above in the Key Stage 2 science test (68\%).

Tables like this can be particularly confusing when some of the labels are themselves numbers or percentages. Further Practice question 4.1 is an example of this.

