

STANDARDS FOR PREPARING TEACHERS OF MATHEMATICS



ASSOCIATION OF MATHEMATICS TEACHER EDUCATORS

STANDARDS FOR PREPARING TEACHERS OF MATHEMATICS WRITING TEAM

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CONTENTS

TABLES	VI
FIGURES	VII
VIGNETTES	VIII
FOREWORD	IX
PREFACE	XI
Purpose	xiii
Audience	xiii
Organization of the Document	xiv
CHAPTER 1. INTRODUCTION	1
Assumptions about Mathematics Teacher Preparation	1
A Well-Prepared Beginning Teacher of Mathematics	4
CHAPTER 2. CANDIDATE KNOWLEDGE, SKILLS, AND DISPOSITIONS	5
Organization of This Chapter	5
What Should Well-Prepared Beginning Teachers of Mathematics Know and Be Able to Do, and What Dispositions Should They Develop?	7
Standard C.1. Mathematics Concepts, Practices, and Curriculum	8
Standard C.2. Pedagogical Knowledge and Practices for Teaching Mathematics	12
Standard C.3. Students as Learners of Mathematics	18
Standard C.4. Social Contexts of Mathematics Teaching and Learning	21
Closing Remarks	24
CHAPTER 3. PROGRAM CHARACTERISTICS TO DEVELOP CANDIDATE KNOWLEDGE, SKILLS, AND DISPOSITIONS	25
Organization of this Chapter	25
Standard P.1. Partnerships	27
Standard P.2. Opportunities to Learn Mathematics	29
Standard P.3. Opportunities to Learn to Teach Mathematics	33
Standard P.4. Opportunities to Learn in Clinical Settings	37
Standard P.5. Recruitment and Retention of Teacher Candidates	41
Closing Remarks	43
CHAPTER 4. ELABORATIONS OF THE STANDARDS FOR THE PREPARATION OF EARLY CHILDHOOD TEACHERS OF MATHEMATICS	45
Part 1. Elaborations of the Knowledge, Skills, and Dispositions Needed by Well-Prepared Beginning Early Childhood Teachers of Mathematics	48
EC.1. Deep Understanding of Early Mathematics	48
EC.2. Positive Attitudes Toward Mathematics and Productive Dispositions Toward Teaching Mathematics	53
EC.3. Mathematics Learning Trajectories: Paths for Excellence and Equity	54

EC.4. Tools, Tasks, and Talk as Essential Pedagogies for Meaningful Mathematics.....	58
EC.5 Understanding Young Children's Mathematical Thinking Informs Teaching	61
EC.6. Collaboration With Families Enhances Children's Mathematical Development.....	63
EC.7. Seeing Mathematics Through Children's Eyes.....	64
EC.8. Creating Positive Early Childhood Learning Environments.....	66
Part 2. Elaborations of the Characteristics Needed by Effective Programs Preparing Early Childhood Teachers of Mathematics	68
EC.9. Mathematics Content Preparation of Early Childhood Teachers.....	69
EC.10. Mathematics Methods Experiences for Early Childhood Teachers	70
EC.11. Clinical Experiences in Mathematics for Early Childhood Teachers	71
Closing Remarks.....	72

CHAPTER 5. ELABORATIONS OF THE STANDARDS FOR THE PREPARATION OF UPPER ELEMENTARY GRADES TEACHERS OF MATHEMATICS..... 73

Part 1. Elaborations of the Knowledge, Skills, and Dispositions Needed by Well-Prepared Beginning Teachers of Mathematics in the Upper Elementary Grades.....	75
UE.1. Mathematics Concepts and Connections to Mathematical Practices	75
UE.2. Pedagogical Knowledge and Teaching Practices	82
UE.3. Tools to Build Student Understanding.....	84
UE.4. Assessment to Promote Learning and Improve Instruction	85
UE.5. Students' Sense Making	86
UE.6. Ethical Advocates for Students.....	87
Part 2. Elaborations of the Characteristics Needed by Effective Programs Preparing Mathematics Teachers for Upper Elementary Grades.....	89
UE.7. Mathematical Content Preparation of Teachers of Mathematics in the Upper Elementary Grades	89
UE.8. Mathematics Methods Coursework for Upper Elementary Teachers of Mathematics.....	90
UE.9. Clinical Experiences for Upper Elementary Teachers of Mathematics	91
Closing Remarks.....	92

CHAPTER 6. ELABORATIONS OF THE STANDARDS FOR THE PREPARATION OF MIDDLE LEVEL TEACHERS OF MATHEMATICS 93

Part 1. Elaborations of the Knowledge, Skills, and Dispositions Needed by Well-Prepared Beginning Teachers of Mathematics at the Middle Level	95
ML.1. Essential Understandings of Mathematics Concepts and Practices	95
ML.2. Content Progressions for Middle Level Learners.....	102
ML.3. Strategies to Support Early Adolescents	103
ML.4. Meaningful and Interdisciplinary Contexts	104
ML.5. Mathematical Practices of Middle Level Learners.....	106
ML.6. Respond to the Needs of Early Adolescents	107
ML.7. Equitable Structures and Systems in Middle Schools.....	109
Part 2. Elaborations of the Characteristics Needed by Effective Programs Preparing Middle level Teachers.....	110
ML.8. Mathematics Content Preparation for Teachers of Mathematics at the Middle Level.....	111
ML.9 Pedagogical Preparation for Middle Level Teachers of Mathematics	113
ML.10. Clinical Experiences in Middle Level Settings	114
Closing Remarks.....	116

CHAPTER 7. ELABORATIONS OF THE STANDARDS FOR THE PREPARATION OF HIGH SCHOOL TEACHERS OF MATHEMATICS 117

Part 1. Elaborations of the Knowledge, Skills, and Dispositions Needed by Well-Prepared Beginning High School Mathematics Teachers	120
HS.1. Essential Understandings of Mathematics Concepts and Practices in High School Mathematics	122
HS.2. Use of Tools and Technology to Teach High School Mathematics	125
HS.3. Supporting Each and Every Student's Opportunity to Learn Mathematics	127
HS.4. Cultivating Positive Mathematical Identities in Each and Every Student	130
Part 2. Elaborations of the Characteristics Needed by Effective Programs Preparing High School Mathematics Teachers	133
HS.5. Effective Programs to Support Preparation of Teachers of Mathematics at the High School Level	133
HS.6. Partnerships to Support Preparation of Teachers of Mathematics at the High School Level	134
HS.7. Mathematical Content Preparation of Teachers of Mathematics at the High School Level	136
HS.8. Ethics and Values for Teaching Mathematics at the High School Level	138
HS.9. Mathematics Methods Experiences for Teachers of Mathematics at the High School Level	141
HS.10. Clinical Experiences for Teachers of Mathematics at the High School Level	142
Closing Remarks	144
Potential Pathways for Preparing High School Mathematics Teachers	145

CHAPTER 8. ASSESSING MATHEMATICS TEACHER PREPARATION 147

Features of Effective Assessments Used in Mathematics Teacher Preparation	151
Recommendation AF.1. Focus on Mathematics Teaching in Assessments	151
Recommendation AF.2. Promote Equity and Access in Assessments	152
Recommendation AF.3. Embody Openness in Assessments	152
Recommendation AF.4. Support Valid Inferences and Action Based on Assessments	152
Recommendation AF.5. Embody Coherence and Sustainability in Assessments	155
Assessing Quality of Mathematics Teacher Candidates	156
Recommendation AC.1. Assessment of Mathematical Knowledge Relevant to Teaching	156
Recommendation AC.2. Assessment of Mathematics Teaching Practice	157
Recommendation AC.3. Assessment of Dispositions	157
Assessing Quality of Mathematics Teacher Preparation Programs	158
Recommendation AP.1. Assessment of Stakeholder Engagement	158
Recommendation AP.2. Assessment of Program Curriculum and Instruction	159
Recommendation AP.3. Assessment of Effective Clinical Experiences	159
Recommendation AP.4. Assessment of Recruitment and Retention	160
Closing Remarks	162

CHAPTER 9. ENACTING EFFECTIVE PREPARATION OF TEACHERS OF MATHEMATICS 163

Improving the Preparation of Teachers of Mathematics	163
Process for Supporting Improvement of Mathematics Teacher Preparation	164
A Call to Action	166
Closing Remarks	167

REFERENCES..... 169

TABLES

Table 0.1. Standards and Reports Related To Preparing Teachers of Mathematics	xi
Table 2.1. Standards and Related Indicators For Well-Prepared Beginning Teachers of Mathematics.....	6
Table 2.2. Mathematics Teaching Practices	15
Table 3.1. Standards and Related Indicators for Effective Programs for Preparing Beginning Teachers of Mathematics	26
Table 3.2. Minimum Mathematics Content Preparation for Teacher Candidates.....	30
Table 4.1. Elaborations of Selected Candidate and Program Standards for Early Childhood Teachers of Mathematics	46
Table 4.2. Connections to <i>MET II</i> (CBMS, 2012) Related to Number and Numeration in Early Childhood.....	50
Table 4.3. Connections to <i>MET II</i> (CBMS, 2012) Related to Operations and Algebraic Thinking in Early Childhood.....	51
Table 4.4. Connections to <i>MET II</i> (CBMS, 2012) Related to Geometry, Measurement, and Data in Early Childhood	53
Table 4.5. Research-based Developmental Learning Trajectories.....	55
Table 5.1. Elaborations of Candidate and Program Standards for Teachers of Mathematics for Upper Elementary Grades	74
Table 5.2. Connections to <i>MET II</i> (CBMS, 2012) Related to Multiplicative Structures in the Upper Elementary Grades	77
Table 5.3. Connections to <i>MET II</i> (CBMS, 2012) Related to Fractions and Decimals in the Upper Elementary Grades	78
Table 5.4. Connections to <i>MET II</i> (CBMS, 2012) Related to Geometry and Measurement in the Upper Elementary Grades	80
Table 5.5. Connections to <i>MET II</i> (CBMS, 2012) Related to Algebraic Thinking in the Upper Elementary Grades	81
Table 6.1. Elaborations of Candidate and Program Standards for Middle Level Teachers of Mathematics.....	94
Table 6.2. Connections to <i>MET II</i> (CBMS, 2012) Related to Ratios and Proportional Reasoning at the Middle Level	96
Table 6.3. Connections to <i>MET II</i> (CBMS, 2012) Related to the Number System at the Middle Level.....	97
Table 6.4. Connections to <i>MET II</i> (CBMS, 2012) Related to Expressions, Equations, and Functions at the Middle Level	98
Table 6.5. Connections to <i>MET II</i> (CBMS, 2012) Related to Geometry at the Middle Level	98
Table 6.6. Connections to <i>MET II</i> (CBMS, 2012) and SET (Franklin et al., 2015) Related to Statistics and Probability at the Middle Level	99
Table 6.7. Interdisciplinary Modeling Task for the Middle Grades	105
Table 7.1. Elaborations of Candidate and Program Standards for High School Teachers of Mathematics	117
Table 8.1. Purposes for Assessment of Mathematics Teacher Preparation.....	147
Table 8.2. Recommendations About Assessing Mathematics Teacher Preparation	149
Table 8.3. Relation of Assessment Recommendations to the Candidate Standards	150
Table 8.4. Relation of Assessment Recommendations to Program Standards.....	150
Table 8.5. Mathematics Teacher Preparation Quality: Attributes and Evidence	154
Table 9.1. Assumptions About Preparing Teachers of Mathematics	163

FIGURES

Figure 1.1. The teacher development continuum	4
Figure 2.1. Fraction-bar representation of the problem $3 \div \frac{1}{5}$	8
Figure 2.2. Sample task for Pre-K–5 teacher candidates.	11
Figure 2.3. Sample task for middle level teacher candidates.	11
Figure 4.1. A learning trajectory for recognition of number and subitizing.	57
Figure 9.1. The ongoing and cyclic nature of improving mathematics teacher preparation programs.	165

VIGNETTES

Vignette 4.1. Examining Memories of Learning Mathematics	54
Vignette 4.2. Building From What Children Understand Mathematically	60
Vignette 4.3. Collaborative Sense Making of Children’s Mathematical Thinking.....	62
Vignette 4.4. Solving $21 + 32$	67
Vignette 5.1. Measuring Angles	79
Vignette 5.2. Students’ Use of a Key-Words Strategy to Solve Word Problems.....	83
Vignette 6.1. A Sequence of Course Activities Focused on Proportional Reasoning and Student Thinking	101
Vignette 6.2. Co-planning and Co-teaching to Support Every Student	104
Vignette 6.3. Attending to Algebra Content and Language in a Classroom With Emergent Multilingual Students.....	108
Vignette 7.1. Meaningful Algebraic Expressions.....	121
Vignette 7.2. Providing Opportunities to Learn	128
Vignette 7.3. Mathematical Identity	130
Vignette 7.4. A Student Teacher’s Revelation Related to Emergent Multilingual Students.....	131
Vignette 7.5. The Importance of Coherent and Consistent Program Components	135
Vignette 7.6. Helping Candidates Consider Ethical Decisions They May Face.....	140

FOREWORD

The Association of Mathematics Teacher Educators (AMTE), uniquely positioned as the lead organization for mathematics teacher education in the United States, puts forth these standards as a national guide for the preparation of prekindergarten through Grade 12 (Pre-K–12) teachers of mathematics. The mission of AMTE is to promote the improvement of mathematics teacher education Pre-K–12 with stated goals focused on effective mathematics teacher education programs and effective policies and practices related to mathematics teacher education at all levels. Over the 25-year history of AMTE, the organization has produced three standards documents: *Principles to Guide the Design and Implementation of Doctoral Programs in Mathematics Education* (2002), designed for institutions of higher education to guide review, revision, or creation of doctoral programs in mathematics education; *Standards for Elementary Mathematics Specialists* (first published 2009, updated 2013), designed to define and advocate for effective preparation of mathematics specialists; and, now, the *Standards for the Preparation of Teachers of Mathematics* (2017), created to address issues and challenges facing teacher preparation and articulate a national and comprehensive vision for the initial preparation of teachers of mathematics in Pre-K–12.

The AMTE Board of Directors' decision to initiate the development of standards was ignited by Dr. Nadine Bezuk of San Diego State University during her delivery of the Judith Jacobs Lecture (JJL) at the 2015 AMTE annual conference. The JJL was established in 2003 to honor Dr. Judith E. Jacobs, one of the founding members of AMTE. Since that time a renowned mathematics educator has been selected each year to deliver the JJL at the annual conference. The focus of Dr. Bezuk's lecture was the need for AMTE to provide leadership to the field by developing standards for preparing teachers to teach mathematics. She argued, "While there exist a number of documents that address various aspects of the initial preparation of mathematics teachers, there is no single, definitive document addressing the initial preparation of mathematics teachers more globally." Further, she noted an absence, within the mathematics teacher education community, of a shared vision of what preparing teachers entails and a limited understanding of the degree to which any such vision may be shared with other stakeholders involved in the initial preparation of mathematics teachers. Unequivocally, the AMTE Board of Directors was challenged by the lecture to develop and disseminate national standards, and as a result, the AMTE Board of Directors made the decision to develop and disseminate Pre-K–12 standards for mathematics teacher preparation.

In March 2015, the AMTE Board of Directors established a leadership team to develop the standards. The team includes Douglas H. Clements of the University of Denver as lead developer for the early childhood grades; Nadine Bezuk as chair and lead developer for upper elementary grades; Jennifer Bay-Williams of the University of Louisville as lead developer for the middle level; and W. Gary Martin of Auburn University for high school.

The leadership team began by establishing criteria for the expertise required to write the standards. The leadership team sought, as a collective body of knowledge across the members of the writing team, expertise that included experiences in the preparation of mathematics teachers at the early childhood, upper elementary, middle school, or high school level; the teaching of mathematics methods courses; the teaching of mathematics coursework and development of mathematical knowledge for teaching; the supervision of clinical placements and field experiences; the responsibility for recruiting and retaining students in mathematics teacher education programs; pedagogy to support emergent multilingual learners; pedagogy to support special needs students; advocacy for equity in mathematics teaching and learning; and the administration of mathematics teacher education policy and change agency. With these criteria in mind, the leadership team identified and invited mathematics educators and mathematicians to serve as members of the writing team. The members of the writing team are listed in the document. The AMTE Board of Directors extends sincere appreciation and gratitude to the members of the writing team. These individuals were dedicated and committed to the significance of this work and the potential of the influence of these standards for improving mathematics teaching.

Another noteworthy aspect of the process was the attention given and time invested in a coherent and comprehensive review of drafts. In the initial planning phase, the leadership team established and implemented a review process to ensure that a broad range of stakeholders would have opportunities to provide critical feedback throughout the development stages. As such, a group of mathematics educators was identified and invited to serve on a review team to facilitate or assist in development of the document while the writing group engaged in writing and refining chapters. The AMTE Board of Directors especially thanks the review team for providing feedback on the drafts of the chapters on an ongoing basis. The following were members of the review team:

- Robert Q. Berry, III, University of Virginia; President-elect, National Council of Teachers of Mathematics
- Francis (Skip) Fennell, McDaniel College; Past President, Association of Mathematics Teacher Educators; Past President, National Council of Teachers of Mathematics; Vice Chair, Council for the Accreditation of Educator Preparation Board of Directors
- Matt Larson, Lincoln (Nebraska) Public Schools and President, National Council of Teachers of Mathematics
- Dale Oliver, Humboldt State University (California)
- Margaret (Peg) Smith, University of Pittsburgh

Additionally, announcements and special requests for review of the draft standards went out to AMTE members, other key stakeholders, and professional organizations representing mathematics education and the mathematical sciences. The AMTE Board of Directors sincerely appreciates all individuals who provided feedback on the draft document and professional organizations for assembling teams to provide feedback on their behalf. We received feedback from the following organizations:

- American Mathematical Association of Two-Year Colleges (AMATYC)
- American Mathematical Society (AMS)
- American Statistical Association (ASA)
- Association for Middle Level Education (AMLE)
- Association for Women in Mathematics (AWM)
- Association of Mathematics Teacher Educators of Alabama (AMTEA)
- Association of State Supervisors of Mathematics (ASSM)
- Benjamin Banneker Association (BBA)
- Council of Chief State School Officers (CCSSO)
- Education Development Center (EDC)
- Hoosier Association of Mathematics Teacher Educators (HAMTE)
- Mathematical Association of America (MAA)
- National Association of Mathematicians (NAM)
- National Council of Supervisors of Mathematics (NCSM)
- National Council of Teachers of Mathematics (NCTM)
- Society for Industrial and Applied Mathematics (SIAM)
- TODOS: Mathematics for All

The *Standards for Preparing Teachers of Mathematics* are aspirational, advocating for practices that support candidates in becoming effective teachers of mathematics who guide student learning. These standards will guide the improvement of individual teacher preparation programs and promote national dialogue and action related to the preparation of teachers of mathematics.

Christine D. Thomas

AMTE President (2015–2017)

PREFACE

The future mathematical success of our nation's children is largely dependent on the teachers of mathematics they encounter from prekindergarten to Grade 12 (Pre-K–12). According to Tatto and Senk (2011), “If the quality of education for every child is to be improved, the education of teachers needs to be taken seriously” (p. 134). Those involved in preparing teachers of mathematics must ensure that all their candidates have the knowledge, skills, and dispositions to provide all students access to meaningful experiences with mathematics.

The Association of Mathematics Teacher Educators (AMTE) is the largest U.S. professional organization devoted to the preparation of teachers of mathematics. AMTE includes more than 1,000 members supporting preservice teacher education and professional development of teachers of mathematics at all levels from Pre-K–12. AMTE members include professors, researchers, teacher leaders, school-based and district mathematics supervisors and coordinators, policymakers, graduate students, and others. The Standards described in this document reflect AMTE's leadership in shaping the preparation of Pre-K–12 teachers of mathematics, including clearly articulated expectations for what *well-prepared beginning mathematics teachers* need to know and be able to do upon completion of a certification or licensing program and the characteristics such programs must have to support teachers' development.

Although the field continues to conduct research regarding effective practices for teacher preparation, we have a growing research base describing what teaching practices affect student learning and student experiences in mathematics classrooms. As an example, research indicates that focusing only on teachers' behaviors has a smaller effect on teachers' development and subsequently on their students' learning than does focusing on teachers' knowledge of the subject, on the curriculum, or on how students learn the subject (Carpenter, Fennema, Peterson, & Carey, 1988; Kennedy, 1998; Kwong et al., 2007; Philipp et al., 2007).

A number of recent documents address various aspects of the initial preparation of mathematics teachers.¹ Table O.1 summarizes their focus. Although all these documents inform mathematics teacher preparation, no single, comprehensive document addresses the initial preparation of mathematics teachers across Pre-K–12. AMTE's goal is for the standards in this document to provide a clear, comprehensive vision for initial preparation of teachers of mathematics. Building on the documents in Table O.1, we, in this document's standards, elaborate what beginning teachers of mathematics must know and be able to do as well as the dispositions they must have to increase equity, access, and opportunities for the mathematical success of each student. Given the challenges that teachers of mathematics face in preparing their students for future success, mathematics teacher educators must be guided by a well-articulated vision to prepare teachers of mathematics to meet those challenges. In this document, we take up that charge.

TABLE O.1. STANDARDS AND REPORTS RELATED TO PREPARING TEACHERS OF MATHEMATICS

Standards and Reports Specific to Mathematics Teachers

The Mathematical Education of Teachers II (MET II)

The *MET II* (Conference Board of Mathematical Sciences [CBMS], 2012) addressed the mathematical content knowledge well-prepared beginning teachers of mathematics should know at the elementary, middle and high school levels.

¹ For the purposes of this document, mathematics teacher preparation includes preparation to teach statistics, following common practice. However, we recognize that statistics and statistics education, although related to mathematics and mathematics education, are distinct. This document will use the term mathematics to encompass mathematics and statistics; when the distinction between mathematics and statistics is important to emphasize, statistics will be identified separately.

National Council of Teachers of Mathematics' Council for the Association of Educator Preparation (CAEP) Standards

The *National Council of Teachers of Mathematics (NCTM) CAEP Standards* (NCTM & CAEP, 2012a, 2012b) described what effective preservice teachers of secondary mathematics should know and be able to do, informing program reviews for middle and high school mathematics programs.

Statistical Education of Teachers (SET)

SET (Franklin et al., 2015) describes the statistical content knowledge well-prepared beginning teachers of mathematics should know at the elementary, middle and high school levels.

Teacher Education and Development Study in Mathematics (TEDS-M)

In the *TEDS-M*, researchers examined and discussed findings and challenges related to the mathematics education of future primary, middle, and high school teachers (Tatto & Senk, 2011).

Standards Not Specific to Mathematics That Also Apply to Teachers of Mathematics

Association for Middle Level Education (AMLE) Middle Level Teacher Preparation Standards with Rubrics and Supporting Explanations

AMLE (2012) describes five standards, including one on content, which addresses content, standards for students, and the interdisciplinary nature of knowledge.

Council for Exceptional Children (CEC) Initial Preparation Standards

The CEC (2012) standards require that beginning professionals understand and use mathematics concepts to individualize learning for students.

Council for the Accreditation of Educator Preparation (CAEP) Accreditation Standards

The CAEP (2013) describes candidate and program expectations that define quality programs. Among these is the expectation that candidates demonstrate content and pedagogical knowledge in their content domains.

National Association for the Education of Young Children (NAEYC) Standards for Initial and Advanced Early Childhood professional Preparation Programs

The NAEYC (2010) professional standards describe the importance of knowing mathematics and teaching it in ways that promote sense making and nurture positive development.

Standards for Experienced Teachers of Mathematics

Mathematics Specialists

In the *Standards for Elementary Mathematics Specialists* (2013), AMTE outlined “particular knowledge, skills, and dispositions” needed by elementary mathematics specialists who “teach and support others who teach mathematics at the elementary level.” (p. iv)

Interstate Teacher Assessment and Support Consortium (InTASC) Model Core Teaching Standards

The *InTASC Model Core Teaching Standards* (Council of Chief State School Officers [CCSSO], 2013) are used in states, school districts, professional organizations, and teacher education programs to support teachers.

National Board for Professional Teaching Standards (NBPTS) Standards

The NBPTS standards recognize accomplished teachers and included certifications for early childhood and elementary school generalists (2012a, 2012b), and middle school and high school mathematics teachers (2010).

PURPOSE

This document includes a set of comprehensive standards describing a national vision for the initial preparation of all teachers Pre-K–12 who teach mathematics. That is, in addition to early childhood and elementary school teachers who teach all disciplines, middle grade teachers, and high school mathematics teachers, these standards are also directed toward special education teachers, teachers of emergent multilingual students, and all others who have responsibility for aspects of student learning in mathematics.

These standards are intended to

- guide the improvement of individual teacher preparation programs,
- inform the accreditation process of such programs,
- influence policies related to preparation of teachers of mathematics, and
- promote national dialogue and action related to preparation of teachers of mathematics.

These standards are aspirational, advocating for mathematics teacher preparation practices that support candidates in becoming high-quality teachers who are ethical advocates for children and effectively guide student learning aligned with research and best practices, rather than describing minimum levels of competency needed by beginning teachers. The standards are intended both to build on existing research about mathematics teacher preparation and existing standards and to motivate researchers to investigate areas that are less well understood.

AUDIENCE

The audience for these standards includes all those involved in mathematics teacher preparation, including faculty and others involved in the initial preparation of mathematics teachers; classroom teachers and other Pre-K–12 school personnel who support student teachers and field placements; coordinators of mathematics teacher preparation programs; deans, provosts, and other program administrators who make decisions regarding content and funding of mathematics teacher preparation programs; CAEP, the largest accreditor of teacher education programs in the United States as well as state licensure or credentialing agencies/organizations; NCTM, the professional association responsible for setting standards for educator-preparation programs for preservice, middle, and high school mathematics; and other organizations, including specialized professional associations (e.g., NAEYC, CEC) and agencies focused on and involved in the preparation of mathematics teachers.

ORGANIZATION OF THE DOCUMENT

The AMTE Standards for Preparing Teachers of Mathematics are organized in the following way:

- Chapter 1 describes the overall framework, including a set of *assumptions* that underlie the recommendations made throughout the document.
- Chapter 2 provides *standards* for the professional knowledge, skills, and dispositions that well-prepared beginning teachers of mathematics need to possess related to content, teaching, learners and learning, and the social context of mathematics education. Each standard includes *indicators* describing what attainment of that standard by candidates entails.
- Chapter 3 describes *standards* for mathematics teacher preparation programs designed to develop the knowledge, skills, and dispositions of their teacher candidates described in Chapter 2. Again, each standard includes *indicators* of what attainment of that standard by a program entails.
- Chapters 4 through 7 provide specific *elaborations* of the standards in Chapters 2 and 3 to relate them to the specific needs for preparation of teachers of mathematics at different levels of instruction and discuss their alignment with other standards. These grade-bands include Prekindergarten to Grade 2 (early childhood), Grades 3 through 5 (upper elementary), Grades 6 through 8 (middle level), and Grades 9 through 12 (high school).
- Chapter 8 provides *recommendations* for effectively assessing candidates and programs in meeting the standards and elaborations.
- Chapter 9 provides advice on how the vision of this document can be attained, including *action steps* that those involved in mathematics teacher preparation.

CHAPTER 1. INTRODUCTION

As a professional community, mathematics teacher educators have begun to define, research, and refine the characteristics of effective teachers of mathematics and, in particular, the professional proficiencies of a well-prepared beginning teacher of mathematics. This document describes a set of proficiencies for well-prepared beginners and for programs preparing mathematics teachers. Although these proficiencies are grounded in available research, in many areas that research is not yet sufficient to determine the specific knowledge, skills, and dispositions that will enable beginning teachers to be highly effective in their first years of teaching. Hence, the standards presented in this document are intended to engage the mathematics teacher education community in continued research and discussion about what candidates must learn during their initial preparation as teachers of mathematics.

ASSUMPTIONS ABOUT MATHEMATICS TEACHER PREPARATION

The *Standards for Preparing Teachers of Mathematics* are centered on five foundational assumptions about mathematics teacher preparation. These assumptions reflect the emerging consensus of those involved in mathematics teacher preparation in response to the needs of both their teacher candidates and the students those candidates will teach. These assumptions underlie the standards presented in Chapters 2 and 3 as well as the grade-band elaborations in Chapters 4 through 7.

Assumption #1. Ensuring the success of each and every learner requires a deep, integrated focus on equity in every program that prepares teachers of mathematics.

Over the past decades, the need for a central focus on issues related to equity in mathematics education has become clear in reflecting on the uneven performance of students by various demographic factors (AMTE, 2015; NCTM, 2000, 2014a, 2014b). Although equity, diversity, and social-justice issues need to be specifically addressed as standards, they must also be embedded within *all* the standards. Addressing these issues solely within the context of “equity standards” might be misinterpreted to imply that these issues are not important within the other standards; conversely, if they are not directly addressed in standards addressing equity, their centrality to the mission of mathematics teacher preparation can be overlooked. Thus, we believe that equity must be both addressed in its own right and embedded within every standard. Every standard must be built on the premise that it applies to each and every student, recognizing that equity requires acknowledging the particular context, needs, and capabilities of each and every learner rather than providing identical opportunities to students.

Assumption #2. Teaching mathematics effectively requires career-long learning.

Experienced teachers reflecting on their first year of teaching mathematics have frequently described how much more they can now accomplish, given their current level of teaching competence and understanding of the mathematics and students they are teaching. Teachers improve through reflective experience and through intentional efforts to seek additional knowledge. They use that knowledge to build their understanding of the mathematics they teach and to support their improvement in supporting students’ learning of mathematics. This process must begin during their initial preparation and continue throughout their careers. Knowing that candidates will complete teacher preparation programs without the expertise they will later develop focuses attention on

priorities for beginning teachers. Those priorities become the knowledge, skills, and dispositions of a well-prepared beginner.

Assumption #3. Learning to teach mathematics requires a central focus on mathematics.

Teaching is often approached as a general craft that is independent of the content being taught. Effective mathematics teaching, however, requires not just general pedagogical skills but also content-specific knowledge, skills, and dispositions. To support student learning and develop positive dispositions toward mathematics, mathematics teachers at every level of instruction need deep and flexible knowledge of the mathematics they teach, of how students think about and learn mathematics, of instructional approaches that support mathematical learning, and of the societal context in which mathematics is taught and used in everyday life to effectively support student learning of, and positive dispositions toward, mathematics.

Assumption #4. Multiple stakeholders must be responsible for and invested in preparing teachers of mathematics.

Preparing teachers to teach in ways that ensure that each and every student learns important mathematics requires the concerted effort of everyone who holds a stake in students' future successes. Mathematics teacher educators and mathematicians; other teacher educators; program and school administrators; classroom teachers, including special education teachers; families and communities; policymakers and others in the educational system all play critical roles. When these groups send mixed messages about how mathematics is best taught and learned, beginning teachers receive incomplete and fragmented visions of how to enact effective mathematics learning environments for their students. Successful mathematics teacher preparation requires a shared vision of mathematics learning outcomes for students, of effective mathematics learning environments, and of the kinds of experiences that best support a mathematics teacher's continuing growth and development. Moreover, stakeholders must feel both included in the development of that vision and accountable for enacting that vision.

Assumption #5. Those involved in mathematics teacher preparation must be committed to improving their effectiveness in preparing future teachers of mathematics.

Mathematics teacher preparation program structures differ widely, as do the needs and backgrounds of teacher candidates. Additionally, mathematics teacher preparation occurs within a range of contexts; in the United States, hundreds of institutions as well as online and school district programs prepare teachers of mathematics, and each one is unique. Thus, program personnel need to discern how existing research might apply to their context and how they can respond to issues not yet addressed by research. Further, they must assess the relationship between their unique program and its effectiveness, sharing relevant findings with the broader mathematics teacher preparation community (e.g., through publications and presentations at conferences).

A WELL-PREPARED BEGINNING TEACHER OF MATHEMATICS

As stated in Assumption 2, the development of teachers' content and teaching knowledge, skills, and dispositions occurs over a career-long trajectory. For example, InTASC developed learning progressions to describe "a coherent continuum of expectations for teachers from beginning through accomplished practice" (Council of Chief State School Officers, 2013, p. 6). Figure 1.1 depicts the career-long continuum of teacher development.

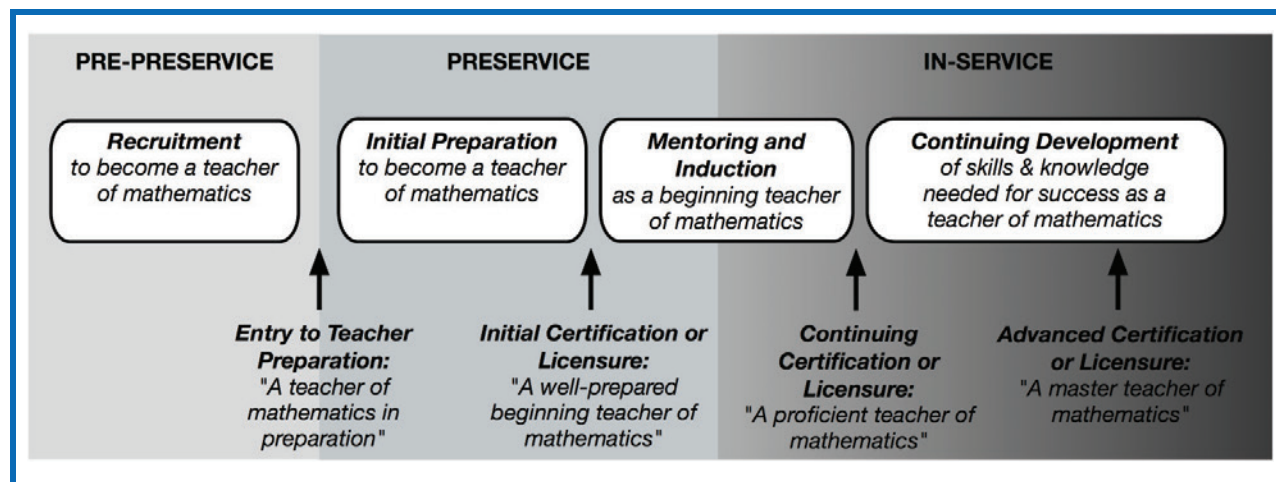


Figure 1.1. The teacher development continuum.

Note. Adapted from *Developing the Analytic Framework: A Tool for Supporting Innovation and Quality Design in the Preparation and Development of Science and Mathematics Teachers* (p. 9) by C. R. Coble, 2012. Washington, DC: Association of Public and Land-grant Universities. Copyright 2012 by APLU.

The standards in this document address primarily the initial preparation phase of the trajectory depicted in Figure 1.1, with some attention to the recruitment of teacher candidates. Chapter 2 provides clear expectations, based on the current knowledge base and national recommendations, for what a well-prepared beginning teacher of mathematics needs to know and be able to do as well as productive dispositions they need to develop, while Chapter 3 describes what programs need to provide to enable candidates to meet these expectations. Well-prepared beginning teachers of mathematics must be committed to supporting the mathematical success of each and every student, and with proper support from the mathematics teacher education community, they will continue to become more effective throughout their careers.