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Building Support for Scholarly Practices in Mathematics Methods

Edited by Signe E. Kastberg, Andrew M. Tyminski, Alyson E. Lischka, and Wendy B. Sanchez

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Christine Browning, *Series Editor*

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For mathematics teacher educators
whose creativity, commitment, and care
inspire scholarly inquiry and practice
in mathematics teacher education.

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FOREWORD

Uniquely positioned as the lead organization and public voice for mathematics teacher education, the Association of Mathematics Teacher Educators (AMTE) established the production of a professional book series. This book, *Building Support for Scholarly Practices in Mathematics Methods*, is the third book of the series. Its focus on the practices of mathematics teacher educators (MTEs) furthers the mission of AMTE to improve mathematics teacher education as well as promoting AMTE's goals. Although all goals of AMTE's are advanced in this work, four goals are particularly prominent: research and scholarly endeavors in mathematics teacher education, equitable practices, effective mathematics teacher education programs and practices, and communication and collaboration among MTEs.

The work leading to the development of *Building Support for Scholarly Practices in Mathematics Methods* is described by the editors, Signe Kastberg, Andrew Tyminski, Alyson Lischka, and Wendy Sanchez, as an outgrowth of 5 years of ongoing scholarly inquiry centered on the investigation of MTEs' practices in mathematics methods courses. The development was launched in 2012 in a session presentation at the AMTE annual conference. This session focused on frameworks and activities used in methods courses. As a result of this session, the participants encouraged the editors to create a working group within the North American chapter of the Psychology of Mathematics Education (PME-NA) to continue collaboration of MTEs engaged in the exploration of mathematics methods. Thus, a PME-NA working group was established in 2012, and as a result, participating MTEs were now situated within a well-organized structure designed to sustain the ongoing collaboration. Further, they expanded the focus to include the study of

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residue—meaning that the focus of the scholarly inquiry now included the study of the "impact of an experience beyond methods courses."

The following two events are noteworthy given the influence of these events leading to conception of this book. In 2013, the editors presented a session at the AMTE annual conference entitled *Building a Theoretically Grounded Practice of Methods Instruction*. In 2015, the editors held a conference, Scholarly Inquiry and Practices Conference for Mathematics Education Methods (SIP; Sanchez, Kastberg, Tyminski, & Lischka, 2015), funded by the National Science Foundation. SIP was designed with an emphasis to support MTEs' engagement in conversations on theoretical perspectives. Further, the editors posit the significance of SIP leading to the organization and contents of this book. The editors employed the ongoing conversations on theoretical perspectives to set the stage of this book and noted that the conversations at SIP culminated in images of the variation in mathematics methods represented in the chapters in this book.

It is also pertinent to note that during the development of this book, AMTE was in the process of writing the newly released *Standards for Preparing Teachers of Mathematics* (SPTM). Although both SPTM and this book were in developmental stages at the same time, drafts of the AMTE Standards were available for review and influenced the elaboration of what "well-prepared beginning mathematics teachers" need to know and be able to do as presented within the pages of *Building Support for Scholarly Practices in Mathematics Methods*. The SPTM is a set of comprehensive standards describing a national vision for the initial preparation of all teachers, prekindergarten through grade 12, who teach mathematics. The standards advocate for practices that support candidates in becoming effective teachers of mathematics who guide student learning. In particular, one of the four standards, "Candidate Knowledge, Skills and Dispositions," focuses on the social contexts of mathematics teaching and learning. *Building Support for Scholarly Practices in Mathematics Methods* offers several examples of practices that promote equity and access in diverse classroom settings and help beginning teachers make connections with their students.

Collectively, the chapters in this book provide an initial work of the enactment of the SPTM as well as many other standards and prove to be an excellent resource, inspiring others to engage in examining their practices, share and collaborate with others, and continue to learn. This book is invaluable in highlighting the work of MTEs engaged in examining and researching their own practices as they focus on the development of beginning teachers of mathematics.

—Christine D. Thomas
Georgia State University
AMTE President 2015–2017

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PREFACE

This book is intended for mathematics teacher educators (MTEs) who teach prospective teachers (PTs) in mathematics methods courses. Through stories of practice and reports of research, it provides a focus on scholarly inquiry and practice (Lee & Mewborn, 2009) in mathematics methods courses for PTs. The chapters in this book arose from the work of the Scholarly Inquiry and Practices Conference on Mathematics Methods,[1] held in the fall of 2015 in Atlanta, Georgia. Over 50 MTEs were assembled to discuss ways in which theoretical perspectives influence teaching and research in mathematics methods courses.

Methods courses, in many ways, are the heart of teacher preparation. It is in these courses that PTs are asked to think about whom, how, and what they teach in the context of society. The content of mathematics methods courses has been shown to vary substantially across institutions (Taylor & Ronau, 2006). What is taught in mathematics methods courses is of interest to all stakeholders in mathematics education. Moreover, what is learned is even more important. What do PTs have the opportunity to learn through their mathematics methods courses, what do they learn, and what do they carry with them into their teaching practice? Further, how can MTEs build scholarly inquiry and practice (Lee & Mewborn, 2009) that explores this variation across mathematics methods courses in order to learn from each other? This book explores these questions by unpacking the ways in which MTEs use theoretical perspectives to inform their construction of goals, activities designed to address those goals, facilitation of activities, and ways in which MTEs make sense of experiences PTs have as a result.

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The 22 chapters in the book are organized in seven sections that highlight how MTEs' theoretical perspectives inform their scholarly inquiry and practice (Lee & Mewborn, 2009). The final section provides insight as we look backward to reflect, and forward with excitement, moving with the strength of the variation we found in our stories and the feeling of solidarity that results in our understandings of purposes for and insight into teaching mathematics methods. This work reflects the efforts of the Scholarly Inquiry and Practices Conference participants. We appreciate their willingness to share stories of practice and embark upon research inquiry that extended the conversations from the conference. In particular, we thank Rochelle Gutiérrez, Elham Kazemi, and Martin Simon for anchoring the discussions about perspectives that launched the conference. Thank you to the University of Washington College of Education for the wonderful book cover photo. Christine Browning, the series editor, has been instrumental in guiding this book to publication. We are most grateful to Fran Arbaugh who saw the power in the work of MTEs at the Scholarly Inquiry and Practices Conference and encouraged us to continue our efforts to create this book.

—Signe E. Kastberg
Andrew M. Tyminski
Alyson E. Lischka
Wendy B. Sanchez

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1. This material is based upon work supported by the National Science Foundation under Grant No. 1503358. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

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SECTION I

PERSPECTIVES AND MATHEMATICS METHODS COURSES

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CHAPTER 1

SETTING THE STAGE

Explorations of Mathematics Teacher Educator Practices

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When I (Signe) was assigned to teach a mathematics methods course for the first time, I reached out to Wendy Sanchez, whom I knew to be deeply committed to work with prospective mathematics teachers (PTs). As we talked, I wondered about the content of methods. My experiences teaching mathematics learners involved supporting their constructions of mathematics

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content, notions of self as a mathematics learner, and views of mathematics communities. Yet as I faced the prospect of working with PTs to support their constructions of mathematics teaching and learning, I wondered: What do I teach, if I am not teaching mathematics? *This probably gives away too much and puts me at risk for judgments about what I should have known or done, yet I am willing to take the risk to situate the work of authors in this book.*

Wendy shared ideas and resources, notably AMTE monographs (for example, Lee & Mewborn, 2009). Yet my efforts to create activities and ask questions seemed like a patchwork. I wanted to be intentional in developing curriculum and practices and then to reconstruct the curriculum and practices in engagement with PTs. Coherence among activities and emergence of productive pedagogies were elusive. These challenges were coupled with limited access to "exemplars of practice" (LaBoskey, 2007) containing images of ways of mathematics teacher educator (MTE) knowing in research literature (with notable exceptions such as Jaworksi, 2008).

Andrew Tyminski, Wendy Sanchez, Kelly Edenfield, and I began an enquiry (Schwab, Westbury, & Wilkof, 1978) of "the content of methods" through AMTE conference presentations and PME-NA working groups (Kastberg, Sanchez, Edenfield, Tyminski, & Stump, 2012; Kastberg, Sanchez, Tyminski, Lischka, & Lim, 2013). Discussions stemmed from AMTE monographs that challenged MTEs to engage in scholarly inquiry (Lee & Mewborn, 2009), explorations of "issues and practices through systematic data collection and analysis that yields theoretically-grounded and empirically-based findings" (p. 3), and the construction of scholarly practice and use of findings from "empirical studies of the teaching and learning of mathematics and the preparation of mathematics teachers" (p. 3).

The collaborations and research findings (Kastberg, Tyminski, & Sanchez, in press) motivated us to advocate for conscious use of theoretical perspectives in the development of curriculum and pedagogy for mathematics methods. We further sought explorations of MTEs' experiences and practices using methodologies such as autoethnography, self-study, and narrative inquiry in mathematics education research. To gain perspective, we searched for ways to enquire into and communicate about experiences of mathematics methods with MTE scholars. Scholarly Inquiry and Practices Conference for Mathematics Education Methods (SIP) (Sanchez, Kastberg, Tyminski, & Lischka, 2015), a conference dedicated to discussions of scholarly inquiry and practice (Lee & Mewborn, 2009), was the result of our efforts to create opportunities for enquiry and dialogue.

SIP was structured to support conversations of theoretical perspectives and associated *learning goals* for mathematics methods. These goals were then used to inform the construction of *activities* for mathematics methods, defined as situations MTEs provide for PTs to support development toward learning goals (Mewborn, 2000) through *experiences* or "ways in which the

preservice teachers internalized those activities" (p. 31). Three broad and commonly used theoretical perspectives (sociopolitical, cognitive, and situative) were selected to frame conference discussions. Our meanings for these perspectives during and after SIP evolved through use of the perspectives as lenses to reinterpret constructing curriculum and pedagogy with PTs in mathematics methods courses. The conference participants taught us that these perspectives are not absolutes, but are situated, interpreted, and operationalized in different ways by MTEs. Use of theoretical perspectives alone or in concert served to enrich participants' discussions of curriculum and pedagogy of mathematics methods. Conversations at SIP culminated in images of the variation in mathematics methods represented in the chapters in this book.

In the two remaining sections, we first discuss variation in mathematics methods as a critical strength of MTEs' work. This discussion is undertaken in the time of accountability and standards that necessitates understanding that just as one size cannot fit all for mathematics learners, MTEs' practices and curriculum are derived with and for PTs and the contexts in which they will teach and live. We conclude with an overview of the book sections and brief introductions of the chapters.

VARIATION IN MATHEMATICS METHODS

Research has demonstrated mathematics methods courses vary in content, activities, and goals (Harder & Talbot, 1997; Kastberg et al., in press; Otten, Yee, & Taylor, 2015; Taylor & Ronau, 2006; Watanabe & Yarnevich, 1999). In response to such findings, Taylor and Ronau (2006) suggested "establishing a common framework offers the possibility of developing shared sets of lenses and a common language, allowing us to conduct a broad-based and open discussion about syllabi and about mathematics methods courses in general" (p. 15). The draft version of the AMTE Standards for Mathematics Teacher Preparation (Bezuk et al., 2016), too, have called for consistency in outcomes of mathematics teacher education programs.

> AMTE's goal is for the standards in this document to provide a clear, comprehensive vision for initial preparation of teachers of mathematics. . . . [W]e, in this document's standards, elaborate what beginning teachers of mathematics must know and be able to do as well as the dispositions they must have to increase equity, access, and opportunities for the mathematical success of each student. (p. 14)

AMTE's vision, however, does not require conformity and should not define how these outcomes are achieved within teacher preparation programs. It is important for our field to understand that outcome alignment should not

come at the cost of variation and autonomy for MTEs. Variation exists because MTEs' work is psychological, social, temporal, and contextual and results in the evolution of curriculum and pedagogy of mathematics methods.

To understand why variation has been reported, take, for example, Marshall and Chao's chapter on mathematics autobiographies (Chapter 18). A quick review of Marshall and Chao's mathematics methods courses would reveal that both MTEs engage PTs in the construction of mathematics autobiographies and utilize Drake's (2006) autobiography story assignment. Yet a longer look would reveal Marshall's choice to invoke a journaling approach and Chao's use of photovoice (Wang & Burris, 1997). These choices help illustrate how their sociopolitical perspectives play out in relation to their personal histories and those of their students within the institutional and community contexts of their work. In short, the origin of variation in this case is human understanding of context and interpretation of needs, goals, and aims. When activities share the same name, such as rehearsal (see Arbaugh, Adams, Teuscher, Van Zoestin, & Wieman, Chapter 9) or clinical interviews (see Chao, Hale, & Behm Cross, Chapter 8), similarities are suggested and, looking broadly, common elements exist among them. Yet, as these authors' descriptions reveal, looking more closely, variation in focus, goals, and implementation also exists. Chapters in this book illustrate that reported variation in mathematics methods is the result of drawing from perspectives, conducting scholarly inquiry, and constructing scholarly practices identified by Lee and Mewborn (2009).

ORIENTATION AND OVERVIEW

The authors of chapters in this book have provided common elements to enable readers to build insight about MTEs' practices. Each chapter includes a description of theoretical perspectives used to inform authors' work. In addition, the terms *learning goal*, *activity*, and *experiences* described earlier in this chapter are used consistently throughout the chapters unless otherwise specified (for example, see Gutierrez, Gerardo, Vargas, & Irving, Chapter 10). The book is comprised of stories and reports of studies across institutional contexts that represent the teaching of mathematics methods "as a complex intellectual endeavor that unfolds in an equally complex sociocultural context" (Borko, Liston, & Whitcomb, 2007, p. 5).

Organization of the Book

The book is organized in seven sections. The first section sets the stage for the remaining sections by describing three theoretical perspectives

(sociopolitical, cognitive, and situative) that structured discussions at the SIP conference. The three keynote speakers at the conference were selected based on their expertise and work within a particular theoretical perspective, and each contributed a chapter drawn from their presentation. Gutiérrez (Chapter 2) shares a view of the sociopolitical perspective; Simon (Chapter 3) describes challenges in mathematics teacher education from a cognitive perspective; and Kazemi (Chapter 4) describes and illustrates her view of the situative perspective. These chapters serve to orient readers to an interpretation of each perspective, rather than providing the only possible interpretation. These perspectives are representative, rather than exhaustive, of those that have been and can be used to frame the work of MTEs.

Chapters in Section II describe the affordances and constraints of utilizing perspectives in the design of scholarly inquiry and practice. Weston (Chapter 5) explores her use of the knowledge quartet (Rowland, 2008) while the work of Earnest and Amador (Chapter 6) and Harper, Herbel-Eisenmann, and McCloskey (Chapter 7) provide additional insights regarding the way in which perspectives inform scholarly inquiry and practice. Section III provides examples of learning goals and how MTEs' scholarly practices are developed in connection to these goals. Included in this section are contributions by Chao, Hale, and Cross (Chapter 8) exploring clinical interviews and Arbaugh et al. (Chapter 9) exploring rehearsals. In addition, Gutiérrez, Gerardo, Vargas, and Irving (Chapter 10) explore rehearsals for the development of political knowledge for teaching (Gutiérrez, 2013) and Kinach, Bismark, and Salem (Chapter 11) draw from a cognitive perspective to describe approaches to conceptual-development teaching. Chapters in Section IV provide descriptions of MTEs' work in the development, refinement, or adaptation of a mathematics methods course activity. Wessman-Enzinger and Salem (Chapter 12) illustrate one approach to designing activities for PTs using the context of integer operations. Lawler, LaRochelle, and Thompson (Chapter 13) describe and illustrate how activities used in methods courses can be revised to include learning goals from a sociopolitical perspective. The final chapter in this section, Ward (Chapter 14), illustrates how MTEs' perspectives coupled with PTs' experiences inform revisions of an activity. Chapters in Section V describe MTEs' enactments and course contexts and attend to how these factors influence PTs' learning outcomes. Virmani, Taylor, Rumsey, and colleagues (Chapter 15) examine a variety of ways to embed mathematics methods courses within the context of the K–8 classroom. Singletary, de Araujo, and Conner (Chapter 16), draw from the situative perspective in their discussion of PTs' reflections on transcripts of their teaching as opportunities for learning. The section also includes the aforementioned chapter on autobiography by Marshall and Chao (Chapter 18), along with an exploration by Harper, Sanchez, and Herbel-Eisenmann (Chapter 17) that examines

the role of language across two university settings through a mathematical activity with sociopolitical underpinnings. Chapters in Section VI explore MTEs' use of perspectives in teaching and self-evaluation. Casey, Fox, and Lischka (Chapter 19) explore potential connections between theoretical perspectives and the ways MTEs evaluate video as a curricular resource. In Chapter 20, Smith, Taylor, and Shin explore MTEs' theoretical perspectives, learning goals, and activities, as well as alignment between perspectives, learning, and activities. The final entry in the section, by McCloskey, Lawler, and Chao (Chapter 21), describes how MTEs might explore progress toward their teaching goals using the metaphor of a mirror derived from the work of Guitérrez (2016). Section VII concludes with the closing chapter by Richard Kitchen, which provides insight as we look backward to reflect, and forward with excitement, moving with the strength of the variation we found in our stories and the feeling of solidarity that results in our understandings of purposes for and insight into teaching mathematics methods. Kitchen's discussion draws from his insights on the chapters to identify relationship building as a key structure critical to MTEs in mathematics methods.

The editors would like to thank all of the SIP conference participants and authors for their hard work and dedication in writing and refining their chapters through cycles of reviews and revisions. We invite readers to consider the ideas and issues raised by the authors and build upon them in their own work as we move forward in developing scholarly practices for teaching mathematics methods courses. We thank the National Science Foundation[1] for the funding the SIP conference that motivated the book. Finally, we thank Dr. Denise Spangler, who was the external evaluator on the grant but more importantly was a mentor to each of us at various points in our thinking before, during, and after the SIP conference.

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CHAPTER 2

POLITICAL CONOCIMIENTO FOR TEACHING MATHEMATICS

Why Teachers Need It and How to Develop It[1]

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Contrary to popular belief and research, addressing equity in mathematics education will not simply come once teachers understand the content they are to teach; when they find accessible, quality, or motivating activities and instructional strategies to use with students; or even when they develop meaningful relationships with students. Many teachers find their biggest struggle lies in understanding and negotiating the politics in their everyday practice. This is particularly true in mathematics, where teachers may expect their work to be straightforward—universal and culture free (Martin, 1997; Powell & Frankenstein, 1997). Teachers have not been trained to negotiate their local politics. Even teachers who have shown substantial

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success with students, especially ones who historically have been excluded from mathematics, suggest their knowledge of content, pedagogy, and students is not enough to maintain that success. Politics get in the way, their work is undermined, or they leave the profession.

Imagine if teachers were trained with as much skill and practice in dealing with the politics of teaching as they were with lesson planning, assessment, strategic instructional decisions, classroom management, connecting topics within mathematics, and relating to students. Instead of just carrying out local practices that are valued or have been in place for years, they might question whether those practices are in the best interest of students. They might be more inclined to engage in dialogue and influence others to consider new perspectives. Rather than stand by while new policies are being created that go against their sense of justice, they might advocate for their students or themselves, and perhaps more talented teachers might stay in the profession longer. In this chapter, I will argue (a) mathematics teaching is political, (b) mathematics teachers need political knowledge, (c) teacher education programs can develop political knowledge with teachers through particular activities, and (d) when mathematics teachers have opportunities to understand and deal with the politics of teaching, they are able to use that knowledge in their practice.

POLITICS OF TEACHING MATHEMATICS

All Teaching Is Political

Teaching has always been political, but we seem to be at an extreme point in history. We see talented and committed individuals reconsidering whether teaching will allow them to be the kinds of people they wanted to be when they entered this profession (Natale, 2014). As teachers are robbed of their ability to use professional judgment, even award-winning teachers are counseling the next generation of students to rethink teaching as a profession (Klein, 2014). Private and charter schools may be able to remain competitive because they can ask poor-performing students to leave or because they can simply close their doors if their school is no longer profitable (Seattle Education, 2015). Public school teachers know they must work with every student who walks through their doors. As such, part of teachers' work is creating a counternarrative to stories of students not having enough "grit" (Tough, 2016) or the view that teachers are slackers (Rosemond, 2004).

More and more, corporate America and billionaires with no expertise in education seek to control our schools. In 2015, Eli Broad and his foundation announced they are moving forward with a $490,000,000 plan to

privatize the Los Angeles public schools (Blume, 2015). The goal is to create 260 new schools in the next 7 years and to launch a massive marketing campaign that will get families and the general public to embrace the idea that charter schools are the next great innovation for the nation. The Bill and Melinda Gates Foundation, as well as the Walton family, show similar interests in public education. The emphasis on charter schools is likely to intensify with the multimillionaire Betsy DeVos, a leader in the school choice movement, as the new Secretary of Education. Curriculum development corporations like Pearson have capitalized on the standards movement to expand to student assessments and all of the related products to support districts (Persson, 2015). With teachers' salaries and positions partly dependent on student test scores, Pearson is, in a very real sense, controlling who is allowed to stay in the profession.

Corporations are making huge profits by promoting new standards and ways of assessing them, yet the benefits to the public, and to students in particular, are not so clear. The Common Core State Standards are little more than the *Adding It Up* report (National Research Council, 2001) combined with the National Council for Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (NCTM, 2000), documents we already had in our professional community. In fact, the Common Core State Standards are a move away from the "equity principle," one of six key components of previous standards (NCTM, 2000) and a departure from the equity position statement (NCTM, 2008) that suggested teachers need to connect mathematics with students' cultural roots and history. Equity has been the focus of more NCTM presidents' messages than any other topic (Gojak, 2012), yet there is no mention of equity in the Common Core State Standards, and accommodations for "English/Language learners" are in an appendix, something only the tenacious teacher would find.

Content-specific education professors have always evaluated the work of prospective teachers (PTs) and helped decide who is qualified to become a teacher. Now, for-profit corporations control those decisions. Thirty-five states and the District of Columbia have adopted the edTPA, a teacher performance assessment managed by Pearson. Under this new paradigm, PTs pay $300 to upload evidence of planning, instruction, and assessment in hopes of being positively evaluated to become a teacher. As part of the process, they are required to document the kinds of textbooks used in the schools where they are student teaching, important information for a corporation that is seeking to market its products to those not already using them. So, in some ways, our PTs have become data collection agents for a for-profit corporation.

It is not always easy for PTs to understand both the upsides and downsides of new reforms. Take, for example, the Partnership for Assessment of Readiness for College and Careers (PARCC), one of two new national tests

given to measure student learning and growth. The PARCC test seeks to better support students by offering a national standard and holding schools accountable for reaching it, thereby making it easier for parents anywhere in the country to judge the ability level of their students, regardless of the state or neighborhood in which they reside. There are many upsides to ensuring all students are held to high standards, as some fear our nation relies too heavily on social promotion (Balingit & St. George, 2016). However, most PTs do not realize that because the PARCC test was never normed on a national population before requiring states to use it, the test is not a valid measure of learning.[2] In fact, some educators have argued that schools are paying a corporation to norm the tests on the backs of their students (Gaines, 2015; Strauss, 2014) and are relinquishing upwards of 6 weeks of instruction to administer such tests. The first set of scores received by students was incredibly low, thereby justifying the need for states and districts to purchase additional materials from Pearson to raise those scores. The cycle often continues with more tests for students, little useful information for teachers about their students' learning, and more profit for corporations. I served on the PARCC item review committee at the high school level. When I raised the issue with Pearson officials in 2013 about consistently low student test scores across the nation and what this meant for students' futures, I was told that Pearson could not be held accountable for any decisions that school administrators made or what the public did with the test; Pearson was "just the people who make the tests." Their goal at that time was for the PARCC test to replace the ACT so that they would gain market share in testing for college. To some extent, their goal is already being realized, as colleges in Delaware, Kentucky, New Jersey, and Colorado are using PARCC scores in admission decisions and entry-level credit for courses. And, although the state of Illinois has recently stopped using the PARCC test (Rado, 2016), most states are still spending millions of dollars on Pearson-related products for PARCC testing. Where corporations might have had market share in textbook adoption, now they are poised to gain market share in college testing. Moreover, Pearson has recently expanded its markets to countries such as the Philippines with Affordable Private Education Center (APEC) secondary schools (Kamenetz, 2016) and intends to impact more than 200 million students worldwide by 2025 (Pearson.com). The increased influence of corporate America, high-stakes testing, and the deprofessionalization of teachers are all signs of an extreme point in the history of public education.

There is so much happening in the public sphere that it would be hard for a PT to keep track of it all or know how to make sense of it without guidance. Most teachers cannot understand how corporations or "philanthropists" could make money off of public schools. I list here just a few things that I have shared with my PTs. Pearson has a $32 million contract to

administer tests with the state of New York and $500 million in Texas (Otterman, 2011; Phillips, 2014). California is spending $900 million on Common Core. PARCC and Smarter Balanced received $330 million from the U.S. Department of Education. There is a long history of errors in scoring or delays in reporting scores, design flaws, insufficient memory in systems for testing, and untimely reporting of scores. Students' test scores influence not only teacher's salaries but also students' chances of getting into the next level of schooling. For-profit corporations are in control of not only tests used to decide who stays in teaching but also who becomes a teacher in the first place (e.g., edTPA) and are collecting data about textbook use through this process. Pearson's EnVisionMATH has been found to exaggerate claims of impact and generalizability to students of all ability levels, while grossing a minimum of $320 million per year on this one product, with a potential revenue stream of $2 billion/year (Singer, 2014). Fueled by Race to the Top money, charter schools are popping up everywhere. (KIPP and other charter schools play by their own rules. Pearson owns Connections Academy, a group of virtual charter schools.) Corporations encourage new standards and new products for districts (yet little new content). Pearson places gag rules in test contracts to prevent teachers from raising questions about the tests. Pearson has been caught monitoring kids' social media to stop testing leaks (Strauss, 2015). Students who take PARCC mathematics and reading language arts tests will spend more time testing than aspiring lawyers who sit for the bar exam. And they will get nothing in return. Pearson was implicated in an FBI investigation for unfair bidding practices in a $1.3 billion deal to provide curriculum via iPads to students in the Los Angeles Unified School District (Singer, 2014).

Fortunately, there is a movement of growing resistance from parents, teachers, students, and journalists who are bringing together visions of education that move beyond testing and to highlight the lack of transparency and the attacks on public education. Researchers crunching large data are providing a picture of what is happening in public education, noting that, in the past decade, we have had nearly 2,500 charter schools that have received a total of $3.7 billion in federal funding but have closed or never opened their doors (Persson, 2015).

As the influence of corporate America intensifies, individuals are joining forces with others to reclaim this profession of ours. Their response is not that education should give up all testing. National tests have helped us understand which populations of students are being served well by the school system and which are not. Rather, individuals are finding resources on the internet such as Fair Test, Change the Stakes, New York Core, Saving Our Schools, Creating Balance in an Unjust World, Rethinking Schools, Teachers for Social Justice, TODOS Mathematics for All, and many local groups who are fighting for a definition of education that moves beyond standardized

tests. Some teachers and principals are taking matters into their own hands by writing blogs to help distribute information to help families opt out of high-stakes tests (LaReviere, 2015). Others are writing letters to their students or to public officials that can create a wider public debate about not just testing but the nature of education and its place in our society (Goosetree, 2015; Lifshitz, 2015; Look-Ainsworth, 2015; Vilson, 2012).

Although these politics affect everybody, inner-city schools that lack the infrastructure or resources to carry out newer assessments or whose students need more support to reach learning goals based on new standards are more severely impacted. With edTPA and its associated text-heavy forms of evaluation, we may be discouraging or preventing individuals whose first language is not English from entering teaching. Given such politics, it is hard to imagine that we will be able to recruit and retain a large cadre of teachers of color into the profession. Regardless of where they work, PTs and mathematics teacher educators (MTEs) alike will need support to deepen their knowledge of the sociopolitical context of mathematics teaching and learning so that they can make informed decisions about their work (Association of Mathematics Teacher Educators [AMTE], 2017).

All Mathematics Teaching Is Political

How do the aforementioned politics relate to mathematics education in particular? I take as an example two schools—Railside and Union. Railside is a school in Northern California so noted for its success in mathematics that it has been studied by various researchers (Boaler, 2006; Boaler & Staples, 2008; Horn, 2004; Jilk, 2010; Nasir, Cabana, Shreve, Woodbury, & Louie, 2014); Union is a school in Chicago, also noted for its success (Gutiérrez, 1999, 2002a, 2014). Both schools serve low-income, largely Latin@/x[3] populations; both have had teachers who underwent extensive professional development for students to develop conceptual understanding over mere procedures; both have created a departmental community that held a common vision for advancement and a commitment to all students; both have used the Interactive Mathematics Program (Alper, Fendel, Fraser, & Resek, 1997) and showed clear signs of success. Their students have demonstrated the ability to make conjectures and defend their arguments publicly, attained higher test scores than peers in other schools, demonstrated higher classroom engagement overall, and produced a unimodal distribution of engagement from adolescents of different backgrounds. Students have worked in two languages, and a higher percentage of students took calculus (over 40% of the senior class at Union in the 1998–1999 school year).

Yet the efforts of both of these high school mathematics departments were derailed by district politics—a back-to-basic-skills movement in

Chicago and a teaching-to-the-test movement in Northern California. In both locations, highly successful teachers were demoralized and either succumbed to district mandates that went against their professional judgments or left their school or the profession altogether. These schools are not alone. We see pockets of success every day where teachers are working hard and are getting historically excluded students to see themselves as doers of mathematics and to perform well in coursework and on tests. So although the public and many mathematics education researchers seem to believe that the most difficult part about addressing issues of equity is how to get teachers to develop deep and flexible knowledge of mathematics or to adopt particular pedagogical practices, addressing equity is not a technical problem with a technical solution. Values, morals, and judgments all come into play, and these are the heart of politics.

Is it just mathematics teaching that is political, or is there actually something about mathematics as a discipline that is political? A number of researchers across the globe have begun to highlight the ways in which knowledge, power, and identity are interwoven with mathematics, something called the "sociopolitical turn" (Gutiérrez, 2010/2013;[4] Stinson & Bullock, 2015). Early examples that highlighted how power, identity, and knowledge relate to teaching, learning, and teacher education named these as "sociopolitical dimensions of mathematics education" (Valero & Zevenbergen, 2004); a "socio-political orientation" (Chronaki, 1999, p. 19); or simply "power" in mathematics education (Walkerdine, 1988; Walshaw, 2001). For example, Chronaki (1999) suggested that a "political view on mathematics education" should focus on "fostering of citizenship" (p. 19). In general, one distinction is that sociocultural dimensions tend to have enculturation as their goal, whereas sociopolitical dimensions concern themselves with emancipation. In writing about the sociopolitical turn, I chose not to hyphenate the word because I did not believe the social (issues of identity in particular) and the political (issues of power in particular) could be extracted from each other—there is no social without political and vice versa. In fact, sometime after 2010, when the sociopolitical turn was published, most researchers seem to have adopted the term sociopolitical instead of socio-political.

The way mathematics operates in our world and the politics that mathematics brings are important for MTEs to consider. On many levels, mathematics itself operates as Whiteness. Who gets credit for doing and developing mathematics, who is capable in mathematics, and who is seen as part of the mathematical community is generally viewed as White. School mathematics curricula emphasizing terms like Pythagorean theorem and pi perpetuate a perception that mathematics was largely developed by Greeks and other Europeans. Perhaps more importantly, mathematics operates with unearned privilege in society, just like Whiteness. Mathematics is viewed as so pure that

it has become the discipline by which we measure other disciplines. See for example, the XKCD comic (n.d.) that depicts mathematicians so far removed from other disciplines that they hardly recognize other scientists.

We treat mathematics as if it is a natural reflection of the universe. When we identify mathematics in the world around us (e.g., Fibonacci sequences in pinecones, fractals in snowflakes), we convince ourselves that mathematics occurs outside of human influence. Rather than recognizing that we may see patterns we want to see (because we set the rules for finding them), we instead feel mathematics is a way of encoding the universe with eternal truths, a natural order of things that should not be questioned. And so mathematics is viewed as a version of the world that is proper, separate from humans, where no emotions or agendas take place.

Because of its perceived purity, we assume mathematics should be the basis for how we think about the world and what is important. Currently, mathematics operates as a proxy for intelligence. Society perpetuates the myth that there are some people who are good at mathematics and some who are not (Mighton, 2004). If you tell someone you are a mathematician or mathematics educator, often you are met with two reactions: confession (e.g., "I was never really good at mathematics") or adulation ("You must be really smart!"). As MTEs, we need to ask ourselves whether we are challenging that adulation or simply accepting it because we enjoy the benefits of increased status and economic gains. Are we really smart just because we do mathematics? As researchers, are we more deserving of large grants because we focus on mathematics education and not social studies or English? Is there something inherent in mathematics as a discipline and human activity that merits higher prestige and higher paychecks?

When we combine the belief that mathematics operates with no values, no judgments, no agenda, with the idea that it properly confers intelligence and importance in society, it can impact how one thinks of oneself. Beyond how well students do in mathematics courses or whether they can imagine themselves pursuing a STEM-based career, they are influenced by this notion of what counts as intelligent. If one is not viewed as mathematical, there will always be a sense of inferiority that can be summoned, especially because the average citizen will not necessarily question the role of mathematics in society. The effects are lasting. So many people are walking around in society who have experienced trauma, microaggressions from participating in math classrooms where the idea of being a successful person, being an intelligent person, is removing oneself from the context, not involving emotions, not involving the body, and being judged by whether one can reason abstractly. Those are all messages that we can unknowingly transmit. It is not just that teaching is political; mathematics is also political. Therefore, whether we recognize it or not, mathematics teaching is a highly political activity.

All Mathematics Teachers Need Political Knowledge to Be Successful

When we acknowledge a sociopolitical perspective on mathematics education, it raises questions about whether PTs are receiving the kinds of knowledge and skills they need. Many are being prepared as if once they develop "ambitious" teaching practices (Lampert et al., 2013) they will be rewarded for their efforts and their students will learn. As we saw in the cases of Union and Railside High, this reality does not exist. High-stakes education, Response to Intervention initiatives, Race to the Top campaigns, and the latest packaged reforms can keep us from acting on what is in the best interest of our students and their learning. In terms of preparing teachers to become professionals, there is nothing in edTPA that will assess whether PTs can successfully deconstruct the deficit messages about teachers, students, or public education in movies like *Waiting for Superman* (Guggenheim & Kimball, 2010) or *Won't Back Down* (Barnes & Hill, 2012). Nor can the edTPA identify teachers who can see limitations in the latest reform movements like "growth mindset" or "grit." On the surface, these movies and reforms address equity by helping students get a better education. However, the savvy educator understands that these movies have the best interests of charter schools and corporate America in mind, instilling the idea that public schools need a hostile takeover. An effective teacher can realize that growth mindset and grit, although important characteristics for students, situate the problem of learning in individual student motivation and ignore broader institutional and systemic inequities. If teachers are unable to deconstruct the deficit messages circulating in society about themselves, their students, or public education, they cannot successfully advocate for policies and practices that are research-based or ethically just.

The majority of professional development that PTs and practicing teachers receive from teacher education programs, their districts, and professional societies like NCTM do not focus on helping teachers understand or negotiate the politics they regularly face. Though we have made many advances in such things as how to appropriately use technology or how to build upon the linguistic and cultural resources that students bring to school, most programs in teacher education still work largely from the same set of assumptions about the kinds of knowledge bases teachers of mathematics need, which were developed in the late 1980s. Whether we call it pedagogical content knowledge (Shulman, 1986) or mathematical knowledge for teaching (Hill et al., 2008), teachers are expected to become fluent in content knowledge, pedagogical knowledge, and knowledge of students.

POLITICAL CONOCIMIENTO FOR TEACHING

I am arguing for a fourth kind of knowledge—political knowledge for teaching. I refer to this knowledge as *political conocimiento,* and I explain more thoroughly what that means in other papers (Gutiérrez, 2012, 2013b). What is important to understand here is that although the Spanish term *conocimiento* translates to "knowledge" in English, I am borrowing a version from Anzaldúa (1987) that acknowledges that all knowledge is relational. Things cannot be known objectively; they must be known subjectively. This is comparable in English to when we say, "Do you know that restaurant?" We are not expecting that knowledge to be a universal objective set of facts. Instead, the speaker is getting at your relationship with that restaurant: Are you familiar with it? What experiences do you have with it? Your knowledge of that restaurant may overlap with the knowledge that others have of it, but it will not be the same. For our purposes, key features of *conocimiento* are subjectivity, solidarity with others, and interdependence.

For mathematics teachers, political *conocimiento* is the kind of knowledge that helps you deconstruct and negotiate the world of high-stakes testing and standardization. It helps you connect and explain your mathematics to community members and district officials. It buffers you from a system or helps you reinvent or reinterpret systems so that you can be an advocate for your students. In essence, political *conocimiento* is the kind of knowledge that allows you to see how politics permeates everything we do, in education in general and mathematics in particular, and affects how we are connected to each other today and how we might envision a different, more humane connection for the future.

The key difference in this model versus other models is the idea that knowledge is *with* students and communities, not knowledge *of* them or *for* them (see Figure 2.1). We come to "know" students not in some kind of

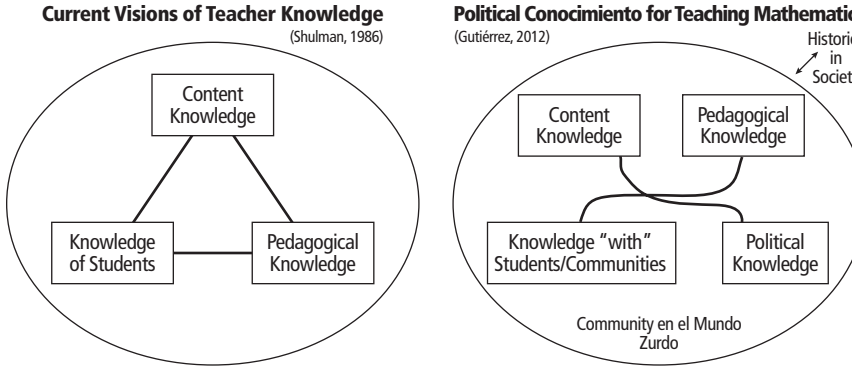

Figure 2.1 Teacher knowledge.

objectified way (Gutiérrez, 2009) but rather by standing alongside them, committed to being interdependent with them. All of this work is done not as individual teachers but in a supportive community with others. The term *el mundo zurdo* recognizes this community as the left-handed world of solidarity among people of color, people who are queer, historically looted, physically challenged, and resisting various forms of colonization (Moraga & Anzaldúa, 1981). The presence of the term *histories in society* recognizes that mathematics has been and is being practiced in different ways throughout the world. We are in a particular moment in time, not just in terms of modern mathematics, but in terms of what is happening with respect to mathematics and education today.

PTs who have developed political *conocimiento*—that useful knowledge that helps them deconstruct deficit narratives in society about students, teachers, or public education—are better prepared to question the world around them and to use their professional judgment when making decisions about the kinds of learning opportunities students need. They can see the benefits of using achievement data as a first step to identify who is not being served well by the school system, but they recognize the limitations of defining equity around such things as "closing the achievement gap." They understand that, more than just getting all kids to perform better or the same on tests of achievement, we should be invested in helping students become the kinds of people they want to be, fulfilling goals they have defined for themselves, which can mean different, not same outcomes (Gutiérrez, 2002b). Teachers with political *conocimiento* are able to question authority when outside entities come in and tell us that we need to focus on "bubble kids"or that we need to develop a "growth mindset" in our students. If we are telling students that it is really important for them to develop perseverance and grit or grow new dendrites to get smarter, but the system remains stacked against them, is that really a healthy perspective to promote? From the point of view of students of color and historically looted students,[5] does that just sound like a new version of "pull yourself up by your bootstraps"? When PTs and practicing teachers lack political *conocimiento*, they can unknowingly adopt simplistic reform packages and slogans that make them feel they are effectively addressing equity and social justice.

Creative Insubordination

When PTs are developing political *conocimiento*, they often feel a desire to do something to address the injustices they witness. This is where creative insubordination comes into play. *Creative insubordination* is a term grounded in the 1980s, a term I heard growing up in an activist family, a term used on a regular basis in my community. I later learned that creative insubordination

was published in literature on principal leadership because some principals were found to stand up to the establishment to protect their teachers when decisions were being made that did not seem fair (Crowson & Morris, 1985). I find it extremely helpful for naming the work that community leaders and exceptional teachers do as a matter of their everyday practice (Gutiérrez, 2013a, 2015a, 2015b; Gutiérrez & Gregson, 2013; Gutiérrez, Irving, & Gerardo, 2013). Creative insubordination recognizes innovative work that individuals, in collaboration with others, do when they need to get a job done but when doing so will be met with resistance from those protecting the status quo. Teachers who are creatively insubordinate learn to bend rules and interpret things in ways that rely on a higher ethical standard. Rather than simply following what others around them are doing or telling them to do, they reflect deeply and base their decisions on professional judgment guided by doing right by students. I emphasize the creative part to highlight the fact that this work is not done foolishly or naively. It is done in a way that keeps teachers from being fired. In this sense, like any other professional knowledge, it requires skill and precision.

Teacher Education Programs Can Develop Political Knowledge

One set of issues in which mathematics teachers need to be able to reinterpret or bend rules is equity. When PTs enter classrooms for observation or student teaching, they receive strong messages that equity is about the achievement gap; equity is about growth mindset; equity is about grit and other things. So before they enter those sites, I try to help them grapple with a more sophisticated notion of equity. I present for them four dimensions of equity/learning (Figure 2.2) that they should consider: access,

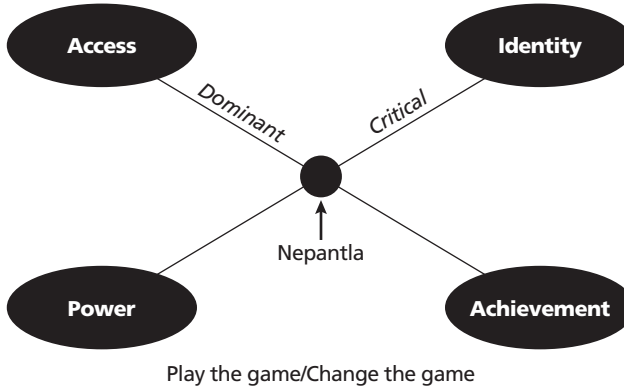

Figure 2.2 Dimensions of equity/learning.

achievement, identity, and power (Gutiérrez, 2007, 2009) and get them to identify particular scenarios as being more or less about particular dimensions (Gutiérrez, 2006).

In doing so, they come to recognize the complexity and tensions that play out in our work as mathematics teachers (Gutiérrez, 2009, 2015a, 2015b). That is, our work can remain neat and tidy, aligned with most administrators and policymakers as well as the general public, including many parents, if we adhere to a mainstream definition of equity that concerns itself only with access (e.g., students having equal opportunities to learn, loaded terms like "quality" teachers and "rigorous" curriculum) and achievement (e.g., equal outcomes on standardized tests, equal numbers of mathematics courses taken, equal representation in the STEM pipeline). This is what I refer to collectively as the *dominant axis* of equity because it dominates the beliefs held by most educators, parents, and policymakers. But we might ask ourselves, is this definition of equity/learning adequate if we also care about the kinds of identities that students develop inside and outside of our classrooms? Does this definition of equity reflect justice if, in order to be seen as legitimate participants in mathematics, students can only follow the "standard algorithm" or speak English while doing mathematics? Does this definition of equity make sense if students never come to understand the historical and cultural aspects of mathematics as a human practice? Does this definition of equity encourage teachers to model how mathematics can be used as a lens to identify inequities in society and to then address those inequities in one's home community? Or is it simply concerned with students getting good grades and access to college?

What I aim for in my teacher education courses is that PTs will walk away asking themselves, "For any given definition of equity, who benefits?" When given the opportunity to think deeply about definitions of equity and learning that circulate in society and in coursework, most PTs are able to understand the importance of identity and power, which is the *critical axis* on the diagram. Here, I mean critical not as in fundamental or key, but as in a critique of the status quo. This axis considers what will be meaningful from students' perspectives. Whenever we think of equity, we always ask, "equity for what purpose and from whose point of view?"

The four dimensions of equity/learning are a useful taxonomy and mapping space. Rather than being a definition that PTs will adopt uncritically, the four dimensions provide language for discussing more nuanced situations that arise in the teaching of mathematics, something that terms like *mathematics for all, closing the achievement gap,* or simply *equity* do not easily capture. This language also helps PTs recognize that part of their job may involve helping students to "play the game" of mathematics, as in do well on standardized tests and develop proficiency in the eight Standards for Mathematical Practice (National Governors Association, 2010). Not to attend to