CD-404043

Middle/Upper Grades

Algebra II Practice

SUPPORTS NCTM STANDARDS

EXAMPLES & PRACTICE

$$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$$

$$f(x) = x^2 + 3x + 4$$

$$\frac{2y}{2} = \frac{15 - 9x}{2}$$

$$(\sqrt{5})^2 = 5$$

$$(c^2)^3 = c^{2 \cdot 3} = c^6$$

BY
DR. BARBARA SANDALL, ED. D.,
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AND TRAVIS OLSON, M.S.

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Dr. Sandall taught middle-school science for 18 years, a fifth-grade self-contained class for one year, as well as courses in elementary music and basic computer programming. She has also worked at Illinois State University in the Center for Mathematics, Science, and Technology Education; the Eisenhower Midwest Consortium for Mathematics and Science Education at the North Central Regional Educational Laboratory (NCREL); and the Teachers Academy for Mathematics and Science in Chicago.

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Melfried Olson—Ed.D., Oklahoma State University; MA, University of Arkansas; BS, Valley City State College—is currently a mathematical researcher at the Curriculum Research & Development Group at the University of Hawaii.

Dr. Olson taught mathematics content and methods courses for 18 years at Western Illinois University and 11 years at the University of Wyoming. He taught grades 7–12 for three years, teaching grade 7, grade 8, Algebra I, Geometry, Algebra II, and Pre-Calculus.

Dr. Olson has been active at the local, state, and national level in mathematics education through numerous publications, presentations, and grants. Dr. Olson has been an active consultant in mathematics teaching and learning. He directed a nationwide staff development project in geometry as well as extensive staff development efforts with teachers K–12. For three years, Dr. Olson served as a co-editor for the Problem Solvers section of the NCTM journal *Teaching Children Mathematics.* He served as president for the Research Council on Mathematics Learning and on the board of directors for the Illinois Council of Teachers of Mathematics and School Science and Mathematics.

Travis Olson has a master's degree in mathematics from Western Illinois University, has taught mathematics (intermediate algebra through calculus III) two years at Cottey College, and is currently a doctoral student in mathematics education at the University of Missouri—Columbia. He has assisted with numerous staff development activities for elementary and middle-school teachers of mathematics and has taught two years in the Johns Hopkins University Center for Talented Youth program.

Mathematical consultant: Joseph A. Kunicki received his BS (1986) and MS (1988) in mathematics from Youngstown State University in northeast Ohio as a non-traditional student. His pre-academic career included five years in the United States Marine Corps, working as an aviation electrician, and 15 years as a blue-collar worker, predominantly in the Bay Area of northern California. Dr. Kunicki finished his Ph.D. in mathematics education at the Ohio State University in 1994, where he concentrated on multi-variate statistical analyses, abstract algebra, and college-level mathematics pedagogy. He has substantive experience working with the National Council of Mathematics Teachers directed toward the Curriculum & Evaluation Standards and their alignment with various district and state mathematics standards. Dr. Kunicki is currently an associate professor of Mathematics at the University of Findlay in northwest Ohio. He resides in Arcadia, Ohio, with his wife Pat St. Onge, a graphic illustrator for Mark Twain Media, Inc., and their two Siberian Huskies, Star and Sherman.

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Table of Contents

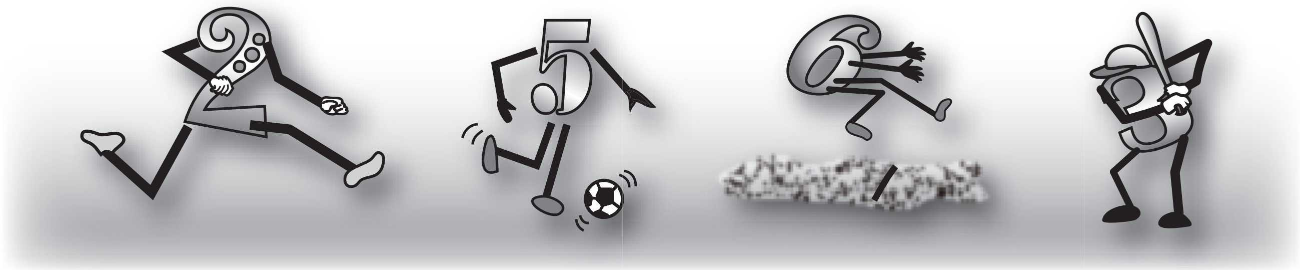
Introduction to the Math Practice Books Series

The *Math Practice Books Series* will introduce students in middle school and high school to the course topics of Pre-algebra, Algebra, Algebra II, and Geometry. All of the practice books are aligned with the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics.* (NCTM 2000)

This series is written for classroom teachers, parents, families, and students. The practice books in this series can be used as a full unit of study or as individual lessons to supplement textbooks or curriculum programs. Parents and students can use this series as an enhancement to what is being done in the classroom or as a tutorial at home. Students will be given a basic overview of the concepts, examples, practice problems, and challenge problems using the concepts introduced in the section. At the end of each section, there will be a set of problems to check progress on the concepts and a challenge set of problems over the whole section. At the end of the book, there will be problems for each section, which could be used for assessment.

According to the Mathematics Education Trust and NCTM, new technologies require that the fundamentals of algebra and algebraic thinking should be a part of the background for all citizens. These technologies also provide opportunities to generate numerical examples, graph data, analyze patterns, and make generalizations. An understanding of algebra is also important because business and industry require higher levels of thinking and problem solving. NCTM also suggests that understanding geometry, including the characteristics and properties of two and three-dimensional shapes, spatial relationships, symmetry, and the use of visualization and spatial reasoning, can also be used in solving problems.

The NCTM *Standards* suggest that content and vocabulary are necessary but of equal importance are the processes of mathematics. The process skills described in the Standards include: problem solving, reasoning, communication, and connections. The practice books in this series will address both the content and processes of algebra and algebraic thinking and geometry. This worktext, *Algebra II Practice*, will help students transition from Algebra to Algebra II.



Common Mathematics Symbols and Terms

Term	Symbol/Definition	Example
Addition sign	+	2 + 2 = 4
Subtraction sign	–	4 – 2 = 2
Multiplication sign	x or a dot • or 2 numbers or letters together or parentheses	3 x 2 2 • 2 2*x* 2(2)
Division sign	÷ or a slash mark (/) or a horizontal fraction bar	6 ÷ 2 4/2 $\frac{4}{2}$
Equals or is equal to	=	2 + 2 = 4
Does Not Equal	$\neq$	$5 \neq 1$
Parentheses – symbol for grouping numbers	( )	(2 x 5) + 3 =
Pi – a number that is approximately 22/7 or ≈ 3.14	π	3.1415926…
Negative number – to the left of zero on a number line	-	-3
Positive number – to the right of zero on a number line	+	+4
Less than	<	2 < 4
Greater than	>	4 > 2
Greater than or equal to	$\geq$	$2 + 3 \geq 4$; $2 \cdot 5 \geq 10$
Less than or equal to	$\leq$	$2 + 1 \leq 4$; $3 + 2 \leq 5$
Is approximately	$\approx$	$\pi \approx 3.14$
Radical sign	$\sqrt[n]{\ }$ $\sqrt{\ }$ *n* represents the index, which is assumed to be 2, the square root, when there is none shown.	$\sqrt{9}$ The square root of 9 $\sqrt[3]{27}$ The cube root of 27
The *n*th power of *a*	a^n	$3^2 = 9$

Common Mathematics Symbols and Terms (cont.)

Variables	Are letters used for unknown numbers	$x + 8 = 12$ x is the variable representing the unknown number
Mathematical Sentence	Contains two mathematical phrases joined by an equals (=) or an inequality {≠, <, >, ≤, ≥} sign	$2 + 3 = 5$ $9 – 3 > 5$ $3x + 8 = 20$ $4 + 2 \neq 5$
Equation	Mathematical sentence in which two phrases are connected with an equals (=) sign.	$5 + 7 = 12$ $3x = 12$ $1 = 1$
Mathematical Operations	Mathematics has four basic operations: addition, subtraction, multiplication, and division. Symbols are used for each operation.	+ sign indicates addition – sign indicates subtraction ÷ indicates division • or x indicates multiplication
Like Terms	Terms that contain the same variables with the same exponents and differ only in their coefficients	3, 4, 5 $3c$, $-5c$, $\frac{1}{2}c$ the variable is the same with the same exponent; they are like terms.
Unlike Terms	Terms with different variables, or terms with the same variable but different exponents	$5 + a$ Cannot be added because they are unlike terms $3x + 4y + 1z$ Cannot be added because the variables are different, so they are unlike terms
Coefficient	The number in front of the variable, that is, the numerical part of a term	$5x$ In this number, 5 is the coefficient
Identity Property of Addition	Any number or variable added to zero remains unchanged.	$0 + 5 = 5$ $-3 + 0 = -3$ $a + (4 – 4) = (a + 4) – 4 = a$
Identity Property of Multiplication	Any number or variable multiplied by one remains unchanged.	$12 \bullet 1 = 12$ $b \bullet 1 = b$ $3y \bullet \left(\frac{2}{2}\right) = \frac{6y}{2} = \left(\frac{1}{2}\right) \bullet 6y = 3y$

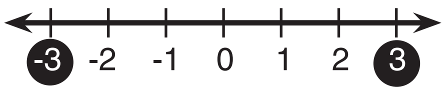
Common Mathematics Symbols and Terms (cont.)

Commutative Property of Addition	No matter the order in which you add two numbers, the sum is always the same.	$4 + 7 = 7 + 4$ $b + c = c + b$
Commutative Property of Multiplication	No matter the order in which you multiply two numbers, the answer is always the same.	$20 \text{ x } \frac{1}{2} = \frac{1}{2} \text{ x } 20$ $5 \bullet 3 = 3 \bullet 5$ $a \bullet b = b \bullet a$
Associative Property of Addition	When you add three numbers together, the sum will be the same no matter how you group the numbers.	$(5 + 6) + 7 = 5 + (6 + 7)$ $(a + b) + c = a + (b + \text{c})$
Associative Property of Multiplication	No matter how you group the numbers when you multiply, the answer will always be the same product.	$(5 \bullet 4) \bullet 8 = 5 \bullet (4 \bullet 8)$ $(a \bullet b) \bullet c = a \bullet (b \bullet c)$
Distributive Property of Multiplication Over Addition	Allows the choice of multiplication followed by addition or addition followed by multiplication.	$3(5 + 2) = 3 \bullet 5 + 3 \bullet 2$ $a(b + c) = a \bullet b + a \bullet c$
Inverse Operation	Operation that undoes another operation	Multiplication and division $5 \bullet x = 5x$ $\frac{5x}{5} = x$ Addition and Subtraction $n + 5 - 5 = n$
Reciprocal or Multiplicative Inverse Property	Two reciprocals are multiplied, and the product is 1.	For any non-zero number: $\text{Number x } \frac{1}{\text{Number}} = 1$ $\frac{1}{\text{Number}} \text{ x Number} = 1$ $a \bullet \frac{1}{a} = 1$ $5 \bullet \frac{1}{5} = 1$

Common Mathematics Symbols and Terms (cont.)

Exponents	Shorthand for repeated multiplication	$a^2 = a \bullet a$ $y^4 = y \bullet y \bullet y \bullet y$
Square Numbers	The result of multiplying a number or variable by itself	$4 \bullet 4 = 16$ $a \bullet a = a^2$
Square Roots	A square root indicated by the radical sign $\sqrt{\ }$ is the positive number multiplied by itself to get the radicand.	$\sqrt{9}$ What positive number multiplied by itself = 9? $3 \bullet 3 = 9$ So $\sqrt{9} = 3$
Radicand	Number under the radical	$\sqrt{9}$ 9 is the radicand
Index	Number inside the crook of the radical sign that tells which root of the radicand is being calculated. When it is not written, the index is assumed to be 2.	$\sqrt[3]{125}$ The index 3 tells us that the cube root or the third root is being sought. Since $5^3 = 125$, then $\sqrt[3]{125} = 5$.
Numerator	Top number of a fraction	$\frac{3}{5}$ In this fraction, 3 is the numerator.
Denominator	Bottom number of a fraction	$\frac{3}{5}$ In this fraction, 5 is the denominator.
Integers	Zero, the natural numbers, and their opposites—the negative integers	Set of integers: {…-3,-2,-1,0,1,2,3…}
Additive Inverse Property of Addition	The sum of an integer and its opposite integer will always be zero.	$a + (-a) = 0$ $-5 + 5 = 0$
Set	A well-defined collection of numbers, elements, or objects	Set of integers: {…-3,-2,-1,0,1,2,3…}

Common Mathematics Symbols and Terms (cont.)

Absolute Value	The absolute value of a number can be considered as the distance between the number and zero on the number line. The absolute value of every number will be either positive or zero. Real numbers come in paired opposites, *a* and -*a*, that are the same distance from the origin but in opposite directions. 	Absolute value of *a*: $\lvert a\rvert = a$ if *a* is positive $\lvert a\rvert = a$ if *a* is negative $\lvert a\rvert = 0$ if *a* is 0 With 0 as the **origin** on the number line, the absolute value of both -3 and +3 is equal to 3, because both numbers are 3 units in distance from the origin.
Expression	Any collection of numbers, variables, or terms with grouping symbols and mathematical operators.	$-3xy$ $2ab + b$ $2z + 4c + 2 - y$ $5[(x + 3)^2 - 4b] + 2h$
Monomial	A polynomial with one term	$-x^3$ $14x$ -0.2
Binomial	A polynomial with two terms	$2x^2 - 5x$ $x^3 + 2.4x$ $x + 3$ $2 - 7x$
Polynomial in One Variable	Is an expression containing the sum of a finite number of terms of the form ax^n, for any real number *a* and any whole number *n*	$-2x^2 + 6x + 3$ $x - \frac{3}{8}$ $x^3 + 2.4x - 1$ $3x^5 - x^4 + 5x^2 + 3x - 9$

Common Mathematics Symbols and Terms (cont.)

<table>
<tr><td>Function</td><td>A special type of relation in which no two ordered pairs have the same first coordinate and have a different second coordinate.

A function can be thought of as a rule that takes an input value from the domain and returns an output value in the range.

A function can be indicated by a set of ordered pairs or by a graph in the plane.</td><td>Example 1: $y = 0.5x + 2$
Example 2:
We choose a value of x and calculate the corresponding value for y. The function consists of the set of ordered pairs: $\{(x, 0.5x + 2)\}$.<table><tr><th>x</th><th>y</th></tr><tr><td>-1</td><td>0</td></tr><tr><td>5</td><td>6</td></tr><tr><td>0</td><td>1</td></tr><tr><td>7</td><td>8</td></tr></table></td></tr>
<tr><td>Ordered Pair</td><td>Describes a point in the coordinate plane. The first number of the pair tells the location relative to the x-axis, and the second tells the location of the point relative to the y-axis.</td><td>(3, 8) - three to the right of 0 on the x-axis (x-coordinate); 8 up on the y-axis (y-coordinate). The point is where these two intersect.</td></tr>
<tr><td>Relation</td><td>Defined as any set of ordered pairs $\{(x, y)\}$. A relation can be indicated by an equation in two variables or by a graph in the plane.</td><td>$x + y^2 = y + 2$
$xy = 1$
$\{(-2, 3), (0, 4), (3, 3), (0, -1)\}$
$y = -2x^2 + 6x + 3$</td></tr>
<tr><td>Domain</td><td>Set from which an input value can be chosen for the independent variable, that is, the first coordinate of an ordered pair in a relation</td><td>$\{(5, -6), (10, 7), (-2, 0)\}$
The set $\{5, 10, -2\}$ is the domain for this relation.</td></tr>
<tr><td>Range</td><td>Set of all second-coordinate values of the dependent variable used in the ordered pair in a relation</td><td>$\{(5, -6), (10, 7), (-2, 0)\}$
The set $\{-6, 7, 0\}$ is the range for this relation.</td></tr>
<tr><td>Function Notation</td><td>$f(x)$ is read as "f of x" or "f at x" and means "f evaluated at the value x."</td><td>If $f(x) = 2x^2 - 5x$, then "$f(4)$ means to substitute 4 into the function for x, that is,
$f(4) = 2 \bullet (4)^2 - 5 \bullet (4) = 12$</td></tr>
<tr><td>Comparison Property</td><td>For any two real numbers, a and b, exactly one of these statements is true. $a < b$ $a = b$ $a > b$</td><td>$9 < 11$ $7 = 7$ $-3 > -5$</td></tr>
</table>

Common Mathematics Symbols and Terms (cont.)

Transitive Property	If $a < b$ and $b < c$, then $a < c$.	$3 < 10 < 12$
Addition Property	If $a < b$, then $a + c < b + c$	$4 < 6$ $\quad 4 + 2 < 6 + 2$
Multiplication Property	If $a < b$ and c is a positive number, then $ac < bc$. If $a < b$ and c is a negative number, then $ac > bc$.	$4 < 6$ $\quad 4(2) < 6(2)$ $4 < 6$ $\quad 4(-2) > 6(-2)$
Conjunction	A sentence formed by joining two sentences with the word *and.* A conjunction is true only when both sentences are true.	$x > -2$ and $x < 3$
Disjunction	A sentence formed by joining two sentences with the word *or.* A disjunction is true if at least one of the sentences is true.	$x < 2$ or $x = 2$
Open Sentences	An equation or inequality that contains one or more variables.	$x < 9$ $x = 4 + y$
Solution Set	Set of all solutions for an open sentence	$x < 9$ Solution set: {8, 7, 6, …}
Constant	A number with a precise value	$-3, \frac{1}{2}, 0, \pi, \sqrt{5}$
Degree of a Variable	Number of times the variable is a factor	$3x^3$ Degree of x is 3 $2y^2$ Degree of y is 2
Degree of a Monomial	Sum of the exponents of all the variables in the monomial	$3ab^3$ is degree 4, since a is degree 1 and b^3 is degree 3
Similar Monomials Also called **Like Monomials**	Monomials with the same variables and the same exponents that may differ in their coefficients	ab^3 and $3ab^3$ are similar ab^2 and $3ab^3$ are not similar
Simplified Polynomial	Polynomial that has no two terms that are similar. The terms are usually arranged in order of decreasing degree of one of the variables.	$3x^3 + 2x^2 + 4x - 1$

Common Mathematics Symbols and Terms (cont.)

Degree of Polynomial	The greatest of the degrees of its terms after it has been simplified. The process of writing a polynomial as a product of prime coefficients, and prime monomials, and prime binomials, etc.	$3x^3 + 2x^2y^2 + 4x - 1$ $3x^3$ (degree is 3) + $2x^2y^2$ (degree is 4 because you add the 2 exponents) + $4x$ (degree is 1) – 1 (0 has no degree). The highest degree then is 4, so the degree of this polynomial is 4. $10x^2 - 90y^2$ $= 2 \bullet 5 \bullet (x - 3y \bullet (x + 3y)$
Prime Factorization	The process of writing a positive integer as a product of primes The process of writing a polynomial as a product of prime coeficients, prime monomials, and prime binomials, etc.	$24 = 2 \bullet 2 \bullet 2 \bullet 3$ $10x^2 - 90y^2$ $= 2 \bullet 5 \bullet (x - 3y) \bullet (x + 3y)$
Greatest Common Factor (GCF)	The largest positive integer that will divide each number from a given set of numbers. A monomial that divides every term of a given polynomial or other set of monomials	2 is the GCF of {6, -8, 12} $3x^2$ is the GCF of $15x^5 - 12x^3 + 3x^2$
Least Common Multiple (LCM)	The smallest positive integer that can be divided by each number from a given set of numbers The monomial of least degree with the smallest coefficient that can be divided by each monomial from a given set The polynomial of least degree with the smallest coefficient that can be divided by each polynomial from a given set	72 is the LCM of {6, -8, 12} $12a^3$ is the LCM of $\{2a, 4a^3, 6a^2\}$ $30x^3(2 - y)^2$ is the LCM of $\{3x(2 - y), 10x^3, 5x^2(2 - y)^2\}$
Factor Set	Set from which numbers or polynomials are chosen as factors	The factor set for 14 is {(1)(14), (-1)(-14), (2)(7), (2-)(-7)}
Prime Number	An integer greater than 1 whose only positive factors are 1 and itself.	2, 3, 5, 7, etc.

Common Mathematics Symbols and Terms (cont.)

Repeating Decimal	When a rational number has a denominator whose prime factors consist of numbers other than twos or fives, the decimal representation of the number repeats itself with the same block of digits infinitely.	$\frac{4}{11} = 0.3636363636...$ $\frac{37}{7} = 5.285714285714...$ $\frac{14}{75} = 0.18666666666...$
Terminating Decimal	When a rational number has a denominator whose prime factors consist of only twos or fives, the decimal representation of the number terminates.	$\frac{53}{20} = 2.65$ $\frac{3}{32} = 0.09375$
Infinite Non-repeating Decimal	Irrational numbers cannot be written as a quotient of two integers, but can be represented by a decimal that doesn't repeat and doesn't terminate.	$\sqrt[3]{5}$ π $4.404004000400004...$ $2.5 - \sqrt{10}$
Imaginary Numbers	Any multiple of is called an imaginary number, where $= \sqrt{-1}$	$-3 \ = 3\sqrt{-1}$
Complex Numbers	The set of all numbers, $a + b$, where a and b are real numbers. (Notice that real numbers are the subset of the complex numbers, such that $b = 0$ in the form $a + b$)	$5 -$ $-5 + \sqrt{-1}$ 5 5 $a + b\sqrt{-1}$
Real Numbers	The set of all numbers that corresponds to points on the number line. Important subsets of the real numbers include: natural numbers, whole numbers, integers, rational numbers, and irrational numbers.	1, 2, 3, 4, 5, 6,... 0, 1, 2, 3... 0, ±1, ±2, ±3... $\frac{1}{2}$, $\frac{45}{8}$, 7.825, $-14.55\overline{5}$, 275, ... $\sqrt[3]{77}$, π, $\sqrt{5}$
Rational Numbers	Real numbers that can be expressed as the quotient of two integers (denominator not equal to 0), and when expressed in decimal form, they either repeat infinitely or terminate.	$\frac{1}{2}$, $\frac{45}{8}$, 7.825, $-14.55\overline{5}$, 275, ...