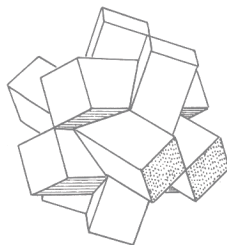
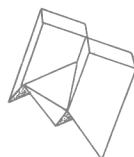
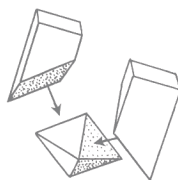
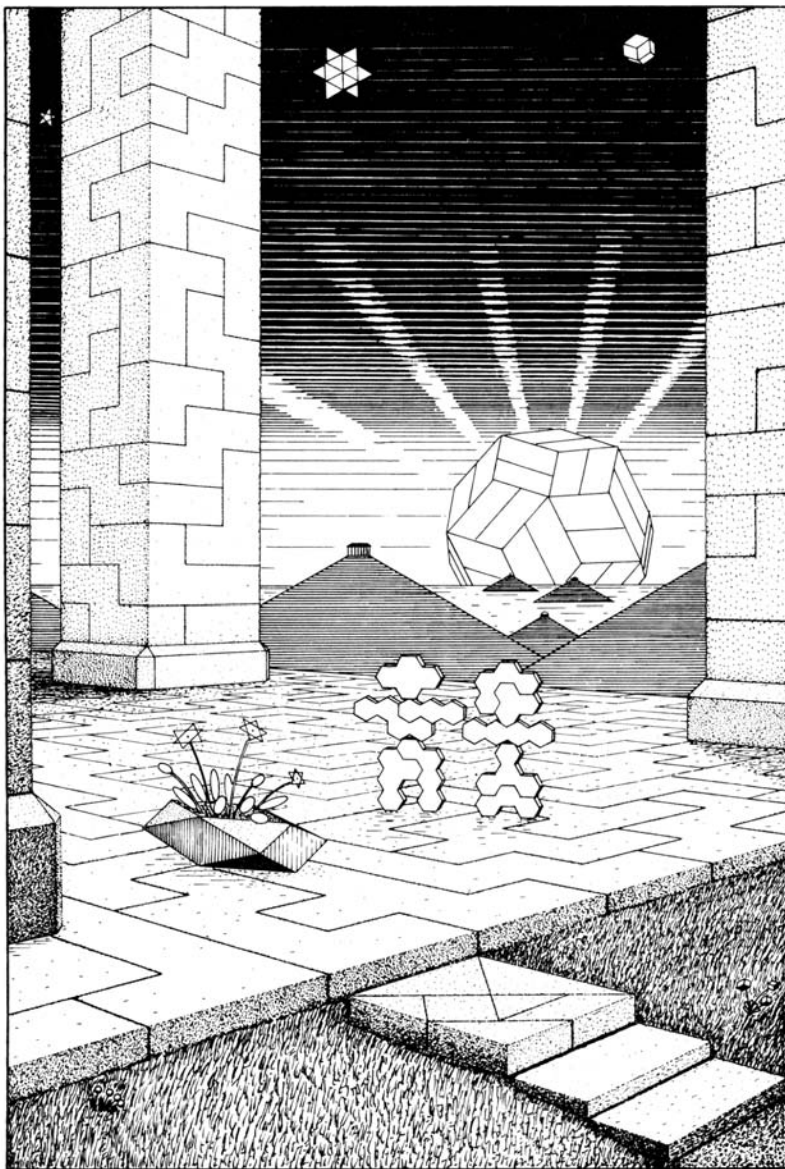

Geometric Puzzle Design



Stewart
Coffin

Geometric Puzzle Design



*"The most beautiful thing we can experience is the mysterious.
It is the source of all true art and science."*

—Albert Einstein, *What I believe* (1930)

Geometric Puzzle Design

Stewart Coffin



A K Peters, Ltd.
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Preface

This book had its conception in 1974, when my small cottage industry of designing geometric puzzles and handcrafting them in wood was then in its fourth year. It began as a newsletter of limited circulation having to do with mechanical puzzles in general, especially those that could be made in the classroom or workshop. It was intended for persons who enjoy designing puzzles, making them, collecting them, or just solving them. In 1978, the various issues were assembled into a booklet. Later more chapters were added, and it became a book of sorts called *Puzzle Craft*, bound and published right in my workshop. A revised and improved edition came out in 1985. Then, in 1990, an entirely new book based on the same material was published by Oxford University Press as *The Puzzling World of Polyhedral Dissections*—part of their Recreations in Mathematics series. With this book we continue along our convoluted path of discovery in this fascinating land of puzzledom. Much new material has been added, representing recent puzzle creations from 1990 up to the present, including some from the far corners of the world. All of the previous material has been thoroughly edited for possible improvements, and some of the fat has been trimmed off. Perhaps most important of all, the illustrations have been greatly enhanced, thanks to the expert work of John Rausch, the book's graphics editor. Indeed, without his enthusiasm and untiring efforts, probably none of this would have happened. For publishing this and all those other fine books on mathematical recreations, thanks to Klaus Peters of A K Peters, a friend in need if ever there was one. And for their patience in fitting all the parts together, thanks to the publisher's editorial and technical staff of Charlotte Henderson, Erica Schultz, and Larissa Zaretsky. Jerry Slocum has provided expert help on the history of puzzles. Assisting in the workshop and office in bygone

days were my late wife Jane and the three little elves—Abbie, Tammi, and Margie. Kathy Jones proofread an early version of the manuscript and offered helpful tips. And finally, heartfelt thanks to Mary Dow for not only tolerating the workshop dust and noise but also steadfastly offering encouragement and support whenever they were needed.

Stewart Coffin
Andover, Massachusetts

Introduction

Nearly everyone must have had at least a few amusements among his or her childhood treasures based on the simple principle of taking things apart and fitting them back together again. Indeed, many infants show a natural inclination to do this almost from birth. Constructing things out of wooden sticks or blocks of stone must surely be one of the most primitive and deeply rooted instincts of mankind. How many budding engineers do you suppose have been boosted gently along toward their careers by the everlasting fascination of a mechanical construction set? I know I certainly was. Even after life starts to become more complicated and most childhood amusements have long since been left by the way-side, the irrepressible urge to join things together never dies out.

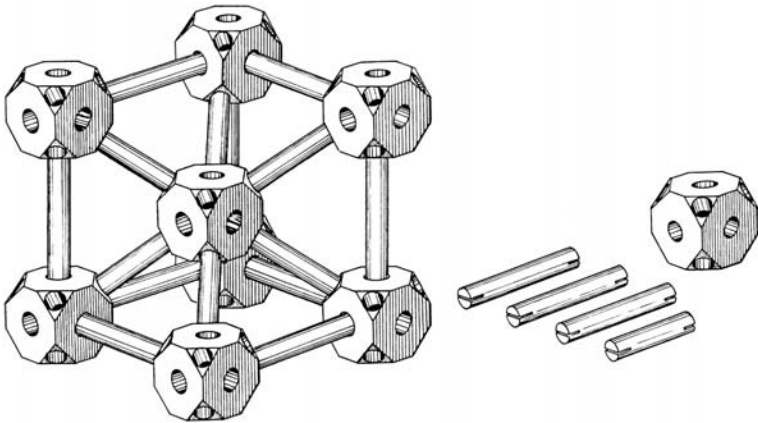


Figure 1. "He who wonders discovers that this is in itself a wonder."
—M. C. Escher

Construction pastimes in the form of geometric assembly puzzles have a universal appeal that transcends all cultural boundaries and practically all age levels. Young children catch on to them most quickly. One of the puzzle designs included in this book was the inspiration of an eight-year-old, and children younger than that have solved many of them. So much, then, for the presumptuous practice of rating the difficulty of puzzles according to age level, with adults of course always placing *themselves* at the top! Likewise, almost anyone from elementary school student to retiree having access to basic workshop facilities should be able to fabricate many of the puzzles to be described on the following pages.

On the other hand, this book is intended to be more than simply a collection of puzzle designs, plans, and instructions. This is a puzzle designer's guidebook. Some of the most rewarding recreations are neither in simply solving puzzles, nor in making them, but rather in discovering new ideas and crafting them into a form that others may enjoy too. Equally satisfying is discovering surprises long overlooked in traditional puzzles. It is amazing how many of these lie scattered about just beneath the surface waiting to be uncovered. Keep in mind that the systematic investigation of many types of problems covered in this book has taken place only within the last few decades. Throughout these pages, unsolved problems are mentioned, or at least implied, that should keep mathematicians and analysts, tinkers and inventors occupied for a long time to come. A few gems have even been purposely reburied so that the reader may have the joy of unearthing them again. But watch out for traps!

Life in general is a puzzle, is it not? Examples abound: trying to fathom the mysterious rules of English grammar and wondering if the spelling of some words was someone's idea of a joke! The engineer who dreamed up the assembly procedure for my car's transmission passed up a promising career as a puzzle inventor. Anyone who writes poetry or composes music knows the satisfaction that comes when all of the parts finally fit together properly, or the frustrations when they decline to do so. Almost any undertaking may become turned into a puzzle, intentionally or otherwise.

This book is devoted mostly to a broad and vaguely defined classification of geometric recreations that might be described as burrs and polyhedral dissections. Polyhedra are by definition any solids bounded by plane surfaces. One often associates the term polyhedra with the iso-

metrically symmetrical Platonic solids and their relatives. It is used here in a broader sense to include practically any solid or assemblage of parts having some sort of symmetry, including burrs. In puzzle nomenclature, burrs are assemblies of interlocking notched sticks. They are traditionally square sticks, but all sorts will be considered here. And for good measure, some puzzles will be included in this edition that do not fit in either category.



Figure 2. Burrs.

For convenience, the term *puzzle* is used throughout this book to include just about any sort of geometric recreation having pieces (actual or imagined) that come apart and fit back together again. Probably many readers associate the word puzzle with some task that is purposely confusing or difficult. That notion may be rather misleading when applied to many of the recreations described in this book. I much prefer to regard them as being fascinating and intriguing. Discovering the myriad amazing ways in which geometric solids fit together in space is in itself a marvelous revelation. If they also have the potential for challenging puzzle problems, so much the better. But let us not make the common mistake of assuming that the more satisfactory puzzle is one that is fiendishly difficult or complicated—a tendency more often than not counterproductive in any creative endeavor.

A proper treatise on geometric puzzles should probably begin with a historical overview. Here we have a problem. If you search long enough, you can usually find at least a brief written history on just about any possible subject, but apparently not so for geometric puzzles. Likewise, a search through the major anthropological museums of the world turns up practically nothing of ancient origin. (Added note: That could change. As we go to press, the definitive Slocum Puzzle Collection is being installed at Indiana University, Bloomington.) The conclusion to be

drawn from this is that most geometric puzzle designs must not be very old—not many over 200 years old. A popular marketing ploy of puzzle manufacturers is to invent stories of their ancient origins. One favorite theme is that they came down to us from the Orient. Some authors have called the six-piece burr the *Chinese Cross* puzzle. Conversely, perhaps puzzles sold in China are touted as products of Yankee ingenuity, and if so, they may be just as close to the truth.

Patent files are one of our most important historical resources on puzzles. As of 1980, there were about 1,000 patents of bona fide puzzles filed in the U.S. Patent Office and about the same number in the British Patent Office. The oldest U.S. patent is dated 1863. If the filing of patents is any accurate indication, then many of the classic designs familiar to us today, including various burrs and dissected blocks, date from the late 1800s. Starting around 1920, there is a decline in puzzle interest and patent activity (which, by the way, just happens to coincide with the phenomenal rise in popularity of the automobile). Puzzle interest picks up again after World War II and has been going strong ever since.

Many games and pastimes are known to be quite ancient, so why not three-dimensional puzzles too? We can only speculate, but here is one thought: of all three-dimensional puzzles, the so-called burr or notched square stick types are certainly the most familiar, the easiest to make, and probably the earliest to have become popular. To be entirely satisfactory, such puzzles should be made to close tolerances, and the only practical mass-production method is with specialized power woodworking machinery and suitable jigs. Power woodworking tools did not come into common use until the mid-nineteenth century. Note that most ancient games and pastimes use pebbles, beans, scratch marks on the ground, and other such things readily at hand.

To say that most geometric puzzles are less than 200 years old requires qualification. They are all based on mathematical principles known ages ago, which in turn have roots going even further back, finally fading away into the unknown of the past. To give credit where it is most due, the fascinating world of geometric dissections, and indeed of mathematical recreations in general, is utterly and profoundly Greek in origin. Behind every geometric model illustrated in this book, the shadow of the Acropolis looms dimly in the background, and within every tortuous puzzle solution lurks the ghost of the fabulous labyrinth of King Minos, brooding over its next victim!

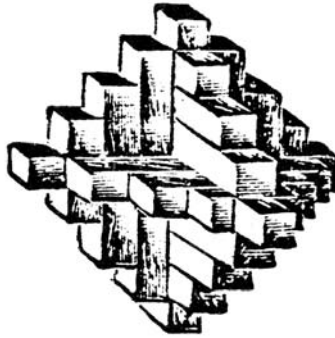


Figure 3. The Platonic solids.

The term *mathematical recreation* is in itself rather a misnomer, for every geometric puzzle worthy of consideration has non-mathematical aspects that are just as important if not more so. Most of the puzzle ideas described in this book were conceived by someone who was not a mathematician by either training or profession, but rather more of an inventor and craftsman, with perhaps a whimsical or artistic bent. Conversely, many creative endeavors that we certainly do not regard as geometric puzzles involve essentially the fitting together of discrete parts artistically into a logical and harmonious interlocking whole. The aspiring puzzle inventor seeking inspiration in the art of invention may be just as likely to find it in the classical arts as in mathematical textbooks.

Except for this edition's predecessors, *Puzzle Craft* and *The Puzzling World of Polyhedral Dissections*, there have been virtually no books published specifically on geometric puzzles. Many books on mathematical recreations have touched on the subject. There have been several compendia of mechanical puzzles in general that have included some burrs and geometric dissections. Likewise, a few woodworking books have included a chapter or two on puzzles. The closely related subjects such as polyhedra, symmetry, combinatorial theory, and design science all have extensive literature. Perhaps it is inherent in the very nature of dissection puzzles that even their literature is thus so scattered in bits and pieces. Trying to fit all of them together for the first time was quite a puzzle in itself!

Until recently, puzzles were regarded as little more than novelties and certainly not as a subject worthy of university-level study or museum exhibits. Before World War II, many wooden puzzles were mass-produced in the Orient, using the same few simple designs year after year. Typical were those found in the illustrious Johnson Smith & Co. mail-order catalog of the 1930s (Figure 4), priced at 10 cents or 15 cents postpaid! Then, cheap plastic versions in injection-molded styrene started flooding



Wood Puzzle

A unique wood puzzle imported from the orient. Will test your skill and patience. Each bar is a separate piece of wood and they can all be taken apart. Your job is to take them apart and put them back together again.
No. 3880. Price Postpaid . . . 15c

Figure 4. Product from the Johnson Smith & Co. catalog.

the market, perpetuating the image of puzzles as expendable toys and trinkets. But all that is changing. There is a growing interest in geometric recreations at all levels, from educational materials for preschoolers to university courses and seminars, arts and crafts exhibits, articles in scientific journals, and hopefully even a few good books!

One reason that geometric dissections have so much potential for recreation is the wide range of skills and talents that may be brought into play, from the theoretical to the practical and from the mathematical to the artistic. At the practical level, a complex interlocking puzzle well crafted in fine wood can be a challenging and rewarding project for the skilled woodworker. On another level, some persons are more intrigued by the geometric shapes themselves, and a sort of Greek renaissance sub-culture has sprung up in the field of architecture and decorative design having to do with the adoration of polyhedra. On yet another level, there is what I call, for lack of a better term, the psycho-aesthetics of puzzle design. This gets into the puzzling question of what it is that makes certain puzzles appeal to certain persons but not others. So far as I know, almost nothing has previously been written on this pregnant subject.

Most of the designs described in this book are for puzzles that can, in theory at least, be made in wood. Directions and helpful hints for doing so are given. Some are much easier to make than others. You can start with the easy ones and gradually work upward, depending upon your woodworking skills and workshop facilities. But what about the reader with no such inclination or no workshop? Do not despair. Many of the designs have been or are being produced commercially, and probably many more will be in the future. Furthermore, the reader with good spatial perception ought to be able to solve many of them visually or on paper, without the need for physical models.

We might carry this notion a step further and suggest that the essence of an intriguing geometric puzzle is really the idea behind it. The physical model of the puzzle then becomes more of a tool to aid the thinking process and help convey the idea. Crude models may suffice for this purpose. As you become more adept with these skills, you may find that the actual models assume less importance than the principles involved. Some designers and solvers of geometric puzzles work almost entirely in the abstract, using pencil and paper or a computer, plus the amazing imaginative powers of the human mind. Consider all the advantages: the parts always fit perfectly and, unlike their wooden counterparts, never swell or shrink, crack or break. And for the apartment dweller with limited space, just think how many designs can be created and stored inside the recesses of one's head, using spaces that might otherwise have remained vacant!

Most of those who invent puzzles like to be given credit when their ideas are published, and some even hope to profit from them. Mention is made of the originators or patent grantees for a few of the puzzles described in this book when known, especially for some of the older classics. Well over half of all the designs included in this book were conceived and published only within the past 35 years. Although the origins of most of them are known to the author, credit is purposely omitted for these reasons: Some of the ideas are so obvious that they probably have been discovered independently by more than one person. Others may be just minor variations of someone else's ideas. For example, one of the puzzles described in this book is the author's variation on a design picked up from a now deceased puzzle craftsman in Florida, who reported getting the idea from someone in California, who in turn reports getting it from a puzzle company in Europe. But the idea is said to have originated

in Japan, although it too is but a variation on a familiar theme. An analysis of its solutions came to me from yet another source, and he reports learning that someone else had done it independently. Trying to unravel something like that would perplex even a patent attorney. So, some of the puzzles in this book are in the public domain, some are patented, some are copyrighted, and some are none of these. But the author cannot say in all cases which are which, so will avoid misunderstandings by not trying to define all origins. Anyone planning to manufacture or publish any of them should undertake the research necessary to make certain that no one's rights or sense of pride are being overlooked.

Chapter 1

Two-Dimensional Dissections

Most of the designs described in this book can be thought of as dissections of some sort. By way of introduction, we will first consider some simple two-dimensional geometric dissections, which in their physical embodiment become assembly puzzles.

Jigsaw Puzzles

To dissect means literally to cut into pieces. Just about any chunk of material cut into pieces becomes a sort of dissection puzzle. If sawn freely perpendicular to the surface of a sheet of plywood (or die-cut of cardboard), the result is the familiar jigsaw puzzle. Most jigsaw puzzles are not designed to exercise or perplex the mind, at least in the sense that other types of puzzles do, and it is perhaps stretching the definition a bit to even call them puzzles. The definition given in the dictionary for the noun *puzzle* seems to have been purposely broadened so as to include what are really pastimes of pattern recognition, memory, and patience. The definition given for the verb *to puzzle* contains no such connotation.

Jigsaw puzzles have been popular for over 200 years, longer than most other types of puzzles. Although their relationship to burrs and polyhedral dissections may appear to be remote, they probably serve as an important historical root. The ancestry of inventions in general must be an incredibly complex web of ideas branching backward in time into just about every nook and cranny of human culture. Puzzles are certainly no exception, and jigsaw puzzles, by their sheer numbers and long history, must play at least a minor role in the evolution of many present-day geometric puzzles and recreations. How many of us played with jigsaw

puzzles at one time and then began to ponder, perhaps subconsciously, variations along logical and mathematical lines?

Various schemes have been employed to make jigsaw puzzles more clever, such as sawing on two different faces of a rectangular block or along multiple axes of a sphere (Figure 5). Some of these are quite entertaining, but still they are essentially non-geometric in principle.

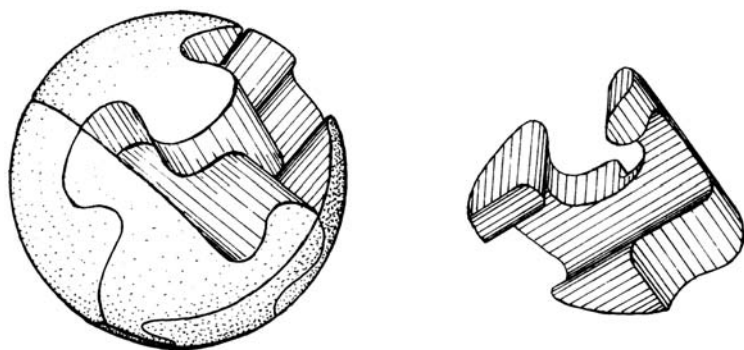


Figure 5.

Tangram

If, instead of cutting freely, the dissection is done according to some simple geometric plan, an entirely different type of puzzle results. Many fewer pieces are required to create interesting puzzle problems. Three characteristics of such puzzles are that they nearly always use straight line cuts, they usually assemble into many different puzzle shapes, and the problem shapes often have more than one solution.

Of the types of puzzles covered in this book, the oldest known is the popular seven-piece dissection of the square known as *Tangram*. It was at one time thought to be thousands of years old, but is now known to have originated in China sometime around 1800. (A quite similar Japanese seven-piece square dissection has been dated back to 1742.) *Tangram* became popular throughout Europe and America in the nineteenth century and continues to be so to this day. It is made and sold in many different materials. Thousands of problem shapes have been published for it over the years, and it is mentioned in many books. For more back-

ground information on *Tangram* and many similar puzzles, the reader is referred to *The Tangram Book* by Jerry Slocum (see bibliography). Here we will discuss some of the curious mathematical aspects of the puzzle not often mentioned in the literature. The dissection is shown in Figure 6.

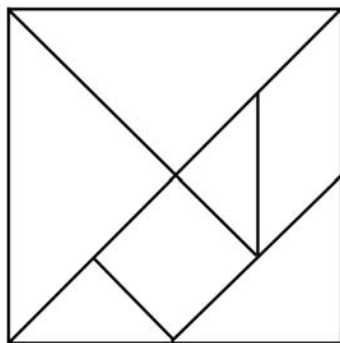


Figure 6.

In designing dissection puzzles of this type, the idea is to divide the whole according to some simple geometric plan so that the pieces will fit together many different ways. The way this is accomplished in *Tangram* is shown in Figure 7. A diagonal square grid is superimposed onto the square whole such that the diagonal of the square measures four units and the area is eight square units. The only lines of dissection allowed are those that follow the grid or diagonals of the grid. To put it another way, the basic structural unit is an isosceles right-angled triangle made by bisecting a grid square, and all larger puzzle pieces are composed of these unit triangles joined together different ways. In *Tangram*, there are two of the unit triangles alone, three pieces made up of two unit triangles joined all possible ways, and two large triangles made up of four unit triangles, for a total of 16 unit triangles. The relative lengths of all edges are thus powers of $\sqrt{2}$.

The first *Tangram* problem is to scatter the pieces and then reassemble the square. Note that it has only one solution, usually a mark of good design. (Rotations and reflections are not counted as separate solutions.) For the countless other problem shapes, you can try to solve the published ones found in many books and magazines or you can invent

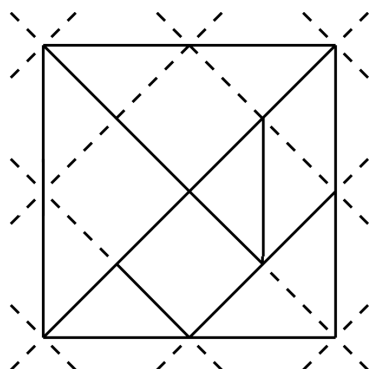


Figure 7.

your own. The easiest way to discover *Tangram* patterns is just by playing around with the pieces. Start by trying to make the simplest and most obvious geometric shapes—triangle, rectangle, trapezoid parallelogram, and so on—always using all of the pieces. An alternate method is to draw some simple shape on graph paper, following the rules already given and having an area of eight squares, and then try to solve it. Which of the examples shown in Figure 8 are possible to construct?

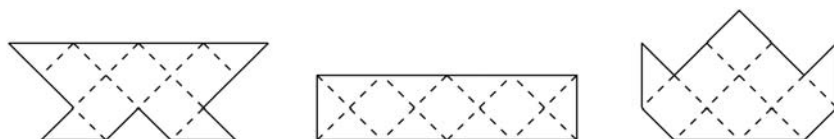


Figure 8.

Published *Tangram* patterns range all the way from the geometric shapes shown in Figure 8 to the other extreme of animated figures created by arranging the pieces artistically. This range is represented by the row of figures shown in Figure 9, reading left to right. Only those solutions that conform to a regular grid can be considered true geometric constructions. Careful inspection will show those to be the three on the left. The others may be very artistic and imaginative, but they are not within the province of this book.

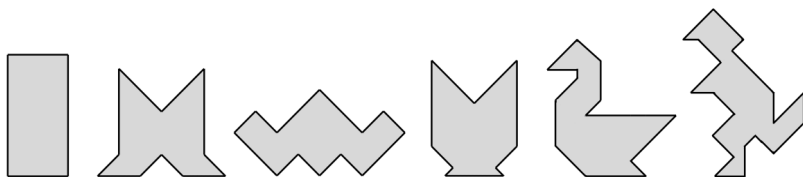


Figure 9.

The theme of discrete rather than random or incommensurable ratios of dimensions is one that plays continuously in the background throughout this book. In the case of *Tangram*-like dissection puzzles, it is easy to see that they cannot be made to work properly any other way. Beyond that, though, there must be something inherently appealing to our aesthetic sensibilities in simple, discrete ratios. They are, after all, the foundation of all music, although probably no one understands exactly why.

Figure 10 shows 13 convex *Tangram* pattern problems. A *convex* pattern is one that can be cut out with a paper cutter straightaway, i.e. with no holes or inside corners. They are all possible to construct. Are any others possible?

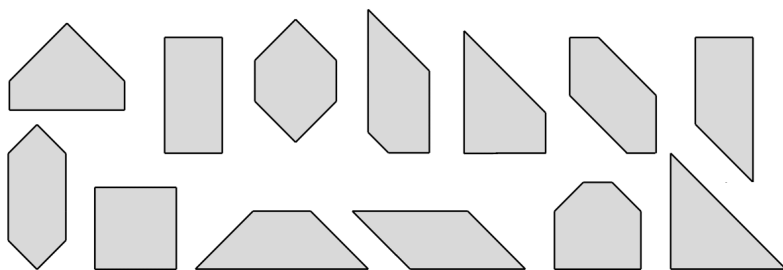


Figure 10.

For a slight change of pace from the usual *Tangram* problem, consider the following puzzler, which by the way is based more or less on an actual happenstance: Karl Essley made two *Tangram* sets as gifts, one to be sent to his sister and the other to his brother. The instructions were simply to assemble all the pieces into a square. Karl's sister brought hers back

and declared (correctly) that the solution was impossible. Examining her set, they discovered that Karl had made a mistake in packing and had accidentally put two pieces into the wrong box, so one person got a set of five pieces and the other got nine. Embarrassed, Karl suggested that they phone their brother and explain the mistake. But his sister reflected for a moment and then said, “No that won’t be necessary; he can make a square with his set.” Can you tell who got the two extra pieces and what shape or shapes they were? Answer later, but be careful. This puzzler contains a nasty trap!

In a similar vein to the above puzzler, note the pairs of figures shown in Figure 11. In each pair, one figure appears to be complete and the other appears to have a piece missing; yet they both use all seven pieces, as all *Tangram* figures must. Can you discover the common characteristic that all such confusing pairs have? (Answers later.) What other such pairs can you discover?



Figure 11.

In order to be entirely satisfactory, especially considering the examples just given, even simple puzzles such as this one should be accurately made of stable materials. If sawn directly out of a square of plywood, there will be noticeable errors introduced by the saw’s kerf. A more accurate way is to lay it out on cardboard, cut the cardboard with scissors, and then use the cardboard pieces as patterns.

Throughout this book, unscaled drawings are given for puzzle constructions. There are always a few readers who will report being unable to use such drawings, having been indoctrinated in woodworking class with the notion that nothing can be made out of wood without standard workshop blueprints with dimensions. Dimensions are omitted for the following reasons:

1. They are unnecessary. It should be obvious for example that in *Tangram* all of the angles are 45 or 90 degrees.