

# Adaptive Backstepping Control of Uncertain Systems with Actuator Failures, Subsystem Interactions, and Nonsmooth Nonlinearities



Wei Wang · Changyun Wen · Jing Zhou



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**CRC Press**

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CRC Press  
Taylor & Francis Group  
6000 Broken Sound Parkway NW, Suite 300  
Boca Raton, FL 33487-2742

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Printed on acid-free paper  
Version Date: 20170718

International Standard Book Number-13: 978-1-4987-7643-1 (Hardback)

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# Preface

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In practice, actuators may inevitably undergo failures and various nonsmooth nonlinearities such as backlash, hysteresis, and dead-zone, which will influence its effectiveness in executing the control commands. These actuator imperfections, often uncertain in time, pattern and values, can cause deteriorated performance or even instability of the system if they are not well handled. If the system parameters are poorly known, the compensation problem will become more complicated. Though adaptive control has been proved to be a promising tool to solve the problem, several important issues, such as guaranteeing transient performance of adaptive failure compensation control system and accommodating intermittent type of failures, remain unexplored.

Due to the increasing complexity of large scale systems, subsystems are often interconnected, whereas the interactions between any two subsystems are difficult to be identified or measured. Decentralized adaptive control technique is an efficient and practical strategy to be employed for many reasons such as ease of design and familiarity. It is aimed to design a local controller for each subsystem using only local information while guaranteeing the stability and performance of the overall system. However, simplicity of the design makes the analysis of the overall system quite a challenge, especially when adaptive control approaches are employed to handle system uncertainties. On the other hand, advances in communication techniques enable information exchanges among distinct subsystems so that certain collective objectives, such as consensus and formation control, can be achieved via carefully designed subsystem interactions.

In this book, a series of innovative technologies for designing and analyzing adaptive backstepping control systems involving treatment on actuator failures, subsystem interactions and nonsmooth nonlinearities are presented. Compared with the existing literature, the novel solutions by adopting backstepping design tool to a number of hotspot and challenging problems in the area of adaptive control are provided.

In [Section I](#), three different backstepping based adaptive actuator failure compensation methods will be introduced for solving the problems of relaxing

relative degree condition with respect to redundant inputs (Chapter 3), guaranteeing transient performance (Chapter 4) and tolerating intermittent failures (Chapter 5).

In Section II, some advances in decentralized adaptive backstepping control of uncertain interconnected systems are presented. Issues including decentralized adaptive stabilization despite the presence of dynamic interactions depending on subsystem inputs and outputs (Chapter 6), decentralized adaptive stabilization with backlash-like hysteresis (Chapter 7), decentralized adaptive output tracking (Chapter 8), decentralized adaptive output tracking with delay and dead-zone input (Chapter 9) are discussed in detail. Note that the subsystem interactions in these chapters are uncertain in structure and strength. Their effects need to be handled with care, otherwise the entire closed-loop system may be destabilized. In Chapter 10, our recent result on backstepping based distributed adaptive coordinated control of uncertain multi-agent systems is presented. Different from Chapters 6-9, this chapter is aimed to achieve output consensus tracking of all the subsystems by carefully designing the subsystem interactions.

Discussion remarks are provided in each chapter highlighting new approaches and contributions to emphasize the novelty of the presented design and analysis methods. Besides, simulation results are given in each chapter, sometimes in a comparative manner, to show the effectiveness of these methods.

Some undergraduate-level mathematical background on calculus, linear algebra and undergraduate-level knowledge on linear systems and feedback control are needed in reading this book. This book enables readers to establish an overall perspective and understanding of typical adaptive accommodation solutions to different issues. It can be used as a reference book or a textbook on advanced adaptive control theory and applications for students with some background in feedback control systems. Researchers and engineers in the field of control theory and applications to electrical engineering, mechanical engineering, aerospace engineering and others will also benefit from this book.

We are grateful to Beihang University (China), Nanyang Technological University (Singapore) and University of Agder (Norway) for providing plenty of resources for our research work. Wei Wang appreciates and acknowledges National Natural Science Foundation of China for their support with Grants 61673035 and 61203068. We express our deep sense of gratitude to our beloved families who have made us capable enough to write this book. Wei Wang is very grateful to her parents, Xiaolie Wang and Minna Suo, her husband, Qiang Wu, and her daughter, Huanxin Wu, for their care, understanding and constant encouragement. Changyun Wen is greatly indebted to his wife, Xiu Zhou and his children Wen Wen, Wendy Wen, Qingyun Wen and Qinghao Wen for their constant invaluable support and assistance throughout these years. Jing Zhou is greatly indebted to her parents, Feng Zhou and Lingfang Ma, and her husband, Xiaozhong Shen, her children Zhile Shen, Arvid Zhiyue Shen and Lily Yuxin Shen for their constant support throughout these years.

Finally, we thank the entire team of CRC Press for their cooperation and great efforts in transforming the raw manuscript into a book.

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# *Chapter 1*

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## **Introduction**

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To stabilize a system and achieve other objectives such as desired output tracking by using adaptive control methodology, a controller is normally constructed to involve adjustable parameters generated by a parameter estimator. Both the controller and parameter estimator are designed on the basis of the mathematical representation of the plant. Adaptive control is one of the most promising techniques to handle uncertainties on system parameters, structures, external disturbances and so on. Since the backstepping technique was proposed and utilized in designing adaptive controllers, numerous results on adaptive control of linear systems had been extended to certain classes of nonlinear systems not based solely on feedback linearization. In contrast to conventional adaptive control design methods, adaptive backstepping control can easily remove relative degree limitations and provide improved transient performance by tuning the design parameters. Although there are a large number of results developed in the area of adaptive backstepping control, some interesting issues such as adaptive compensation for actuator failures, subsystem interactions and nonsmooth nonlinearities still have not been extensively explored.

### **1.1 Adaptive Control**

Adaptive control is a design idea of self-tuning the control parameters based on the performance error related information to better fit the environment. Thus a variety of objectives such as system stability, desired output tracking with guaranteed steady-state accuracy and transient performance can be achieved. Since it was conceived in the early 1950s, it has been a research area of great theoretical and practical significance. The design of autopilots for high performance aircraft was one of the primary motivations for active research in adaptive control [67]. During nearly six

decades of its development, a good number of adaptive control design approaches have been proposed for different classes of systems to solve various problems. Model reference adaptive control (MRAC) [110, 123, 187], system and parameter identification based schemes [8, 124], adaptive pole placement control [44, 45] are some commonly used conventional adaptive control methods. In the 1980s, several modification techniques such as normalization [120, 129], dead-zone [51, 88], switching  $\sigma$ -modification [66] and parameter projection [113, 180, 196] were developed to improve the robustness of the adaptive controllers against unmodeled dynamics, disturbances or other modeling errors. In the early 1990s, adaptive backstepping control [90] was presented to control certain classes of nonlinear plants with unknown parameters. The tuning functions concept provides improved transient performance of the adaptive control system. The results listed above are only a part of remarkable breakthroughs in the development of adaptive control, more detailed literature reviews of conventional adaptive control can be found in [9, 52, 67, 114, 157] and other related textbooks or survey papers.

The prominent feature of adaptive control in handling systems with unknown parameters constitutes one of the reasons for the rapid development of this technique. An adaptive controller is normally designed by combining parameter update law and control law. The former one is also known as parameter estimator providing the adaptation law for the adjustable parameters of the controller at each time instant [157].

Adaptive control techniques used to be classified into direct and indirect ones according to the procedure of obtaining the controller parameters. The methods of computing the controller parameters based on the estimated system parameters are referred to as indirect adaptive control, while the controller parameters are estimated (directly) without intermediate calculation in direct adaptive control. The common principle of conventional adaptive control techniques, no matter direct or indirect, is certainty equivalence principle. This means the controller structure is designed as if all estimated parameters were true, to achieve desired performances.

## 1.2 Adaptive Backstepping Control

Adaptive control approaches can also be classified into Lyapunov-based and estimation-based ones according to the type of parameter update law and the corresponding stability analysis. In the former design procedure, the adaptive law and the synthesis of the control law are carried out simultaneously based on Lyapunov stability theory. However, in estimation-based design, the construction of adaptive law and control law are treated as separate modules. The adaptive law can be chosen by following gradient, least-squares or other optimization algorithms.

To deal with linear systems, traditional Lyapunov-based adaptive control is only applicable to the plants with relative degree no more than two. Such relative degree limitation is translated to another structure obstacle on the “level of uncertainty” in the nonlinear parametric state-feedback case, where the “level of uncertainty” refers to the number of integrators between the control input and the unknown

parameter [81]. The structure restrictions in linear and nonlinear cases can be removed by a recursive design procedure known as backstepping. The technique is comprehensively addressed in [90], where a brief review of its development can also be found. Tuning functions and modular design are the two main design approaches presented in the book. The former approach is proposed to solve an over-parameterization problem existed in previous results on Lyapunov-based adaptive backstepping control. It can keep the number of parameter estimates equal to the number of unknown parameters and help simplify the implementation. In the latter design approach, the estimation-based type adaptive laws can be selected to update controller parameters by synthesizing a controller with the aid of nonlinear damping terms to achieve input-to-state stability properties of the error system. Such an approach is known as modular design since a significant level of modularity of the controller-estimator pair is achieved.

Both tuning functions and modular design approaches can provide a systematic procedure to design the stabilizing controllers and parameter estimators. Moreover, the adaptive backstepping control technique has other advantages such as avoiding cancelation of useful nonlinearities, and improving transient performance of the system by tuning the design parameters.

## 1.3 Motivation

In this book, a series of novel adaptive control methods based on backstepping technique are presented to handle the issues of actuator failures, subsystem interactions and nonsmooth nonlinearities. The state-of-art of related research areas and motivation of our work are elaborated from the following three aspects.

### 1.3.1 Adaptive Actuator Failure Compensation

In a control system, an actuator is a mechanism representing the link between the controller and the controlled plant. It performs the control command generated from the controller on the plant, for the purposes of stabilizing the closed-loop system and achieving other desired objectives. In practice, an actuator is not guaranteed to work normally all the time. Instead, it may undergo certain failures which will influence its effectiveness in executing the control law. These failures may cause deteriorated performance or even instability of the system. Accommodating such failures is important to ensure the safety of the systems, especially for life-critical systems such as aircrafts, spacecrafts, nuclear power plants and so on. Recently, increasing demands for safety and reliability in modern industrial systems with large complexity have motivated more and more researchers to concentrate on the investigation of proposing control design methods to tolerant actuator failures and related areas.

Several effective control design approaches have been developed to address the actuator failure accommodation problem for both linear [20, 21, 32, 75, 95, 100, 158,



160, 169, 191, 212] and nonlinear systems [14, 18, 40, 74, 86, 105, 126, 153, 154, 156, 202, 204]. They can be roughly classified into two categories, i.e., passive and active approaches. Typical passive approaches aim at achieving insensitivity of the system to certain presumed failures by adopting robust control techniques, see for instance in [14, 95, 117, 169, 191, 212]. Since fixed controllers are used throughout failure/failure-free cases and failure detection/diagnostic (FDD) is not required in these results, the design methods are computationally attractive. However, they have the drawback that the designed controllers are often conservative for large failure pattern changes. This is because the achieved system performance based on worst-case failures may not be satisfactory for each failure scenario. In contrast to the passive methods, the structures and/or the parameters of the controllers are adjustable in real time when active design approaches are utilized. Furthermore, FDD is often required in active approaches and provide the estimated failure information to the controller design. Therefore, the adverse effects brought by the actuator failures, even if large failure pattern changes are involved, can be compensated for and the system stability is maintained. A number of active schemes have been presented, such as pseudo-inverse method [49], eigenstructure assignment [7, 75], multiple model [18, 20, 21, 103], model predictive control [80], neural networks/fuzzy logic based scheme [40, 126, 202, 204] and sliding mode control based scheme [32]. Different from the ideas of redesigning the nominal controllers for the post-failure plants in these schemes, virtual actuator method [136, 137] hides the effects of the failures from the nominal controller to preserve the nominal controller in the loop.

Apart from these, adaptive control is also an active method well suited for actuator failure compensation [3, 17, 86, 100] because of its prominent adapting ability to the structural, parametric uncertainties and variations in the systems. As opposed to most of the active approaches, many adaptive control design schemes can be applied with neither control restructuring nor FDD processing. Moreover, not only are the uncertainties caused by the failures, but also the unknown system parameters are estimated online for updating the controller parameters adaptively. In [158, 160], Tao *et al.* proposed a class of adaptive control methods for linear systems with total loss of effectiveness (TLOE) type of actuator failures. It is known that the backstepping technique [90] has been widely used to design adaptive controllers for uncertain nonlinear systems due to its prominent advantages on relaxing relative degree limitation and improving transient performance. The results in [158, 160] have been successfully extended to nonlinear systems in [153, 154, 156, 208] by adopting the backstepping technique. In [209], a robust adaptive output feedback controller was designed based on the backstepping technique to stabilize nonlinear systems with uncertain TLOE failures involving parameterizable and unparameterizable time varying terms. In fact, adaptive control also serves as an assisting tool for other methods as in [18, 20, 21, 40, 100, 126, 192, 202, 204]. For example, a reconfigurable controller is designed by combining neural networks and adaptive backstepping technique to accommodate the incipient actuator failures for a class of single-input single-output (SISO) nonlinear systems in [202]. In [192], the actuator failure tolerance for linear systems with known system parameters is achieved by proposing a control scheme combining linear matrix inequality (LMI) and adaptive control.

In addition to the actuators, unexpected failures may occur on other components such as the sensors in control systems. The research area of accommodating these failures to improve the system reliability is also referred to as fault tolerant control (FTC). More complete survey of the concepts and methods in fault tolerant control could be found in [15, 16, 82, 127, 147, 207].

Although fruitful results have been reported on adaptive actuator failure compensation control, some challenging problems still exist that deserve further investigation. For example, there is a common structural condition assumed in most representative results, such as [153, 154, 156, 158, 160]. That is, only two actuators, to which the corresponding relative degrees with respect to the inputs are the same, can be redundant for each other. The condition is restrictive in many practical situations such as to control a system with two rolling carts connected by a spring and a damper for regulating one of the carts at a specified position; see Section 3.1.1 for details. Suppose that there are two motors generating external forces for distinct carts, respectively. One of them can be considered to be redundant for the other in case that it is blocked with the output stuck at an unknown value. The relative degrees corresponding to the two actuators are different. Moreover, an elevator and a stabilizer may compensate for each other in an aircraft control system, of which the relative degree condition is also hard to be satisfied.

It is well known that the backstepping technique [90] can provide a promising way to improve the transient performance of adaptive systems in terms of  $L_2$  and  $L_\infty$  norms of the tracking error in failure-free case if certain trajectory initialization can be performed. Some adaptive backstepping based failure compensation methods have been developed [153, 154, 156, 208, 209]. Nevertheless, there are limited results available on characterizing and improving the transient performance of the systems in the presence of uncertain actuator failures. This is mainly because the trajectory initialization is difficult to perform when the failures are uncertain in time, pattern and value.

In most of the existing results on adaptive control of systems with actuator failures, only the cases with finite number of failures are considered. It is assumed that one actuator may only fail once and the failure mode does not change afterwards. This implies that there exists a finite time  $T_r$  such that no further failure occurs on the system after  $T_r$ . However, it is possible that some actuator failures occur intermittently in practice. Thus the actuators may unknowingly change from a failure mode to a normally working mode or another different failure mode infinitely many times. For example, poor electrical contact can cause repeated unknown breaking down failures on the actuators in some control systems. Clearly, the actuator failures cannot be restricted to occur only before a finite time in such a case. Moreover, the idea of stability analysis based on Lyapunov function for the case with finite number of failures cannot be directly extended to the case with infinite number of failures, because the possible increase of the Lyapunov function cannot be ensured bounded automatically when the parameters may experience an infinite number of jumps.

### **1.3.2 Decentralized Adaptive Control with Nonsmooth Nonlinearities**

Nowadays, interconnected systems quite commonly exist in practice. Power networks, urban traffic networks, digital communication networks, ecological processes and economic systems are some typical examples of such systems. They normally consist of a number of subsystems which are separated distantly. Due to the lack of centralized information and computing capability, decentralized control strategy was proposed and has been proved effective to control these systems. Even though the local controllers are designed independently for each subsystem by using only the local available signals in a perfectly decentralized control scheme, to stabilize such large scale systems and achieve individual tracking objectives for each subsystem cannot be straightforwardly extended from the results for single loop systems. This is because the subsystems are often interconnected and the interactions between any two subsystems may be difficult to be identified or measured. Besides, the interconnected systems often face poor knowledge on the plant parameters and external disturbances. In such cases, the problem of compensating the effects of the uncertain subsystem interactions and other variety of uncertainties is quite complicated.

Adaptive control is one of the most promising tools to accommodate parametric and structural uncertainties. Thus, this technique is also an appropriate strategy to be employed for developing decentralized control methods. Based on a conventional adaptive approaches, several results on global stability and steady state tracking were reported; see for examples [38, 53, 63, 65, 119, 181, 182]. In [65], a class of linear interconnected systems with bounded external disturbances, unmodeled interactions and singular perturbations are considered. A direct MRAC based decentralized control scheme is proposed with the fixed  $\sigma$ -modification performed on the adaptive laws. Sufficient conditions are obtained which guarantee the existence of a region of attraction for boundedness and exponential convergence of the state errors to a small residual set. The related extension work could be found in [66] where nonlinearities are included. The relative degree corresponding to the decoupled subsystems are constrained no more than two due to the use of Kalman-Yakubovich (KY) lemma. An indirect pole assignment based decentralized adaptive control approach is developed to control a class of linear discrete-time interconnected systems in [181]. The minimum phase and relative degree assumptions in [63, 65] are not required. By using the projection operation technique in constructing the gradient parameter estimator, the parameter estimates can be constrained in a known convex compact region. Global boundedness of all states in the closed adaptive system for any bounded initial conditions, set points and external disturbances are ensured if unmodeled dynamics and interactions are sufficiently weak. The results are extended to continuous-time interconnected systems in [179].

The backstepping technique was firstly adopted in decentralize adaptive control by Wen in [178], where a class of linear interconnected systems involving nonlinear interactions were considered. In contrast to previous results by utilizing conventional direct adaptive control based methods, the restrictions on subsystem relative degrees

were removed by following a step-by-step algorithm. Thus the interconnected system to be regulated consists of  $N$  subsystems, each of which can have arbitrarily relative degrees. By using the backstepping technique, more results have been reported on decentralized adaptive control [70, 76, 96, 99, 183, 206]. Compared to [178], a more general class of systems with the consideration of unmodeled dynamics is studied in [183, 206]. In [70, 76], nonlinear interconnected systems are addressed. In [76, 99], decentralized adaptive stabilization for nonlinear systems with dynamic interactions depending on subsystem outputs or unmodeled dynamics is studied. In [96], the results for stochastic nonlinear interconnected systems are established.

Except for [76, 183, 206], all the decentralized adaptive control results mentioned above are only applicable to systems with interaction effects bounded by static functions of subsystem outputs. This is restrictive as it is a kind of matching condition in the sense that the effects of all the unmodeled interactions to a local subsystem must be in the range space of the output of this subsystem. In practice, an interconnected system unavoidably has dynamic interactions involving both subsystem inputs and outputs. Especially, dynamic interactions directly depending on subsystem inputs commonly exist. The results reported to control systems with interactions directly depending on subsystem inputs even for the case of static input interactions by using the backstepping technique are very limited. This is due to the challenge of handling the input variables and their derivatives of all subsystems during the recursive design steps.

A limited number of results have been obtained in solving tracking problems for interconnected systems. The main challenge is how to compensate the effects of all the subsystem reference inputs through interactions to the other local tracking errors, the equations of which are key state equations used in backstepping adaptive controller design. References [183] and [76] are two representative results reported in this area. In [183], decentralized adaptive tracking for linear systems are considered and local parameter estimators are designed using the gradient type of approaches. In [76], decentralized adaptive tracking of nonlinear systems is addressed. To handle the effects of reference inputs, two critical assumptions are imposed. One is that the interaction functions are known exactly, which is difficult to be satisfied in practice, especially in the context of adaptive control. To cancel the effects of reference inputs, the interactions must also satisfy global Lipschitz condition. The other is that the designed filters are partially decentralized in the sense that the reference signals from other subsystems are used in local filters. It means that all the controllers share prior information about the reference signals. Therefore, the proposed controllers are partially decentralized.

Nonsmooth nonlinearities such as dead-zone [93, 140], backlash [149, 162], hysteresis [2, 121] and saturation [46, 199] can be commonly encountered in industrial control systems. For example, dead-zone is a static input-output characteristic which often appears in mechanical connections, hydraulic servo valves, piezoelectric translators and electric servomotors. Hysteresis can be represented by both dynamic input-output and static constitutive relationships, which exists in a wide range of physical systems and devices. Such nonlinearities, which are usually poorly known and vary with time, often limit system performance. A

desirable control design approach should be able to accommodate the uncertainties. The need for effective control methods to deal with nonsmooth systems has motivated growing research activities in adaptive control of systems with such common practical nonsmooth nonlinearities [163, 164]. Various design methods based on different control objectives and system conditions have been developed and verified in theory and practice. Adaptive control schemes have been used to cope with actuator dead-zone [26, 30, 131, 166], backlash [2, 151, 162], hysteresis [121, 144, 157, 161] and saturation [5, 23, 24, 46, 83, 116]. Other schemes to handle such nonlinearities have included neural networks control in [87, 125, 139, 140], fuzzy logic control in [71, 85, 92, 93], variable structure control in [10, 30, 31, 33, 62, 149, 175], pole placement control in [23, 46, 199] and recursive least square algorithm in [188].

Besides, stabilization and control problem for time-delay systems have also received much attention; see for example [72, 101, 190]. The Lyapunov-Krasovskii method and Lyapunov-Razumikhin method are normally employed. The results are often obtained via linear matrix inequalities. However, little attention has been focused on nonlinear time-delay large-scale systems. References [78] and [189] considered the control problem of the class of time-invariant large-scale interconnected systems subject to constant delays. In [27], a decentralized model reference adaptive variable structure controller was proposed for a large-scale time-delay system, where the time-delay function is known and linear. In [60], the robust output feedback control problem was considered for a class of nonlinear time-varying delay systems, where the nonlinear time-delay functions are bounded by known functions. In [145], a decentralized state-feedback variable structure controller was proposed for large-scale systems with time delay and dead-zone nonlinearity. However, in [145], the time delay is constant and the parameters of the dead-zone are known. Due to state feedback, no filter is required for state estimation. Furthermore, only the stabilization problem was considered.

### ***1.3.3 Distributed Adaptive Consensus Control***

Because of its widespread potential applications in various fields such as mobile robot networks, intelligent transportation management, surveillance and monitoring, distributed coordination of multiple dynamic subsystems (also known as multi-agent systems) has achieved rapid development during the past decades. Consensus is one of the most popular topics in this area, which has received significant attention by numerous researchers. It is often aimed to achieve an agreement for certain variables of the subsystems in a group. A large number of effective control approaches have been proposed to solve the consensus problems; see [6, 11, 12, 57, 69, 111, 132, 133] for instance. According to whether the desired consensus values are determined by exogenous inputs, which are sometimes regarded as virtual leaders, these approaches are often classified as leaderless consensus and leader-following consensus solutions; see [79, 115, 146, 201] and the references therein. Besides, many of the early works were established for systems with first-order dynamics, whereas more results have been reported in recent years such as [115, 135, 141, 198] for systems with second or higher-order dynamics. A comprehensive overview of the state-of-the-

art in consensus control can be found in [134], in which the results on some other interesting topics including finite-time consensus and consensus under limited communication conditions including time delays, asynchronization and quantization are also discussed.

It is worth mentioning that except for [79], all the aforementioned results are developed based on the assumptions that the considered model precisely represents the actual system and is exactly known. However, such assumptions are rather restrictive since model uncertainties, regardless of their forms, inevitably exist in almost all the control problems. Motivated by this fact, the intrinsic model uncertainty has become a new hot-spot issue in the area of consensus control. In [59, 97, 194], robust control techniques are adopted in consensus protocols to address the intrinsic uncertainties including unknown parameters, unmodeled dynamics and exogenous disturbances. In addition, adaptive control has also been proved as a promising tool in dealing with such an issue. In [79], a group of linear subsystems with unknown parameters are considered and a distributed model reference adaptive control (MRAC) strategy is proposed. Different from [97] where  $H_\infty$  control is investigated, the bounds of the unknown parameters are not required *a priori* by using adaptive control. However, the result is only applicable to the case that the control coefficient vectors of all the subsystems are the same and known. In [118], adaptive consensus tracking controllers are designed for Euler-Lagrange swarm systems with nonidentical dynamics, unknown parameters and communication delays. However, it is assumed that the exact knowledge of the desired trajectory is accessible for all the subsystems. In [36], a distributed neural adaptive control protocol is proposed for multiple first-order nonlinear subsystems with unknown nonlinear dynamics and disturbances. The state of the reference system is only available to a subset of the subsystems. Based on the condition that the basis neural network (NN) activation functions and the reference system dynamics are bounded, the convergence of the consensus errors to a bound can be ensured if the local control gains are chosen to be sufficiently large. The results are extended to a more general class of systems with second and higher-order dynamics in [37] and [200]. In [197], distributed adaptive control on first-order systems with similar structures to those in [36] is investigated. By introducing extra information exchange of local consensus errors among the linked agents, the assumptions on boundedness of inherent nonlinear functions can be relaxed. Apart from these, there are also some other results on distributed adaptive control of multi-agent systems, for instance [58, 104, 150, 210]. Nevertheless, to the best of our knowledge, results on distributed adaptive consensus control of more general multiple high-order nonlinear systems are still limited. In [177], output consensus tracking problem for nonlinear subsystems in the presence of mismatched unknown parameters is investigated. By designing an estimator whose dynamics is governed by a chain of  $n$  integrators for the desired trajectory in each subsystem, bounded output consensus tracking for the overall system can be achieved. However, it is not easy to check whether the derived sufficient condition in the form of LMI is satisfied by choosing the design parameters properly. Moreover, transmissions of online parameter estimates among the neighbors are required, which may increase communication burden and also cause some other potential problems such as those related to network security.

## 1.4 Objectives

In this book, innovative technologies for designing and analyzing adaptive backstepping control systems involving treatment on actuator failures, subsystem interactions and nonsmooth nonlinearities are presented. Compared with the existing literature, the novel solutions by adopting a backstepping design tool to a number of hot-spot and challenging problems in the area of adaptive control are provided.

In the first part of this book, three different backstepping based adaptive actuator failure compensation methods will be introduced for solving the problems of relaxing relative degree condition with respect to redundant inputs (Chapter 3), guaranteeing transient performance (Chapter 4) and tolerating intermittent failures (Chapter 5). Chapters 3-4 employ a tuning function design scheme, whereas Chapter 5 adopts a modular design method.

In the second part of this book, some advances in decentralized adaptive backstepping control of uncertain interconnected systems are presented. Issues including decentralized adaptive stabilization despite the presence of dynamic interactions depending on subsystem inputs and outputs (Chapter 6), decentralized adaptive stabilization with backlash-like hysteresis (Chapter 7), decentralized adaptive output tracking (Chapter 8), decentralized adaptive output tracking with delay and dead-zone input (Chapter 9) are discussed in detail. Note that the subsystem interactions in these chapters are uncertain in structure and strength. Their effects need be handled with care, otherwise the entire closed-loop system may be destabilized. In Chapter 10, our recent result on backstepping based distributed adaptive coordinated control of uncertain multi-agent systems is presented. Different from Chapters 6-9, Chapter 10 is aimed to achieve output consensus tracking of all the subsystems by carefully designing the subsystem interactions.

## 1.5 Preview of Chapters

This book is composed of 11 chapters. Chapters 2-11 are previewed below.

In Chapter 2, the concepts of adaptive backstepping control design and related analysis, as the basic tool of new contributions achieved in the remaining chapters are given.

In Chapter 3, by introducing a pre-filter before each actuator in designing output-feedback controllers for the systems with TLOE type of failures, the relative degree restriction corresponding to the redundant actuators will be relaxed. To illustrate the design idea, we will firstly consider a set-point regulation problem for linear systems and then extend the results to tracking control of nonlinear systems.

In Chapter 4, transient performance of the adaptive systems in failure cases, when the existing backstepping based compensation control method is utilized, will be analyzed. A new adaptive backstepping based failure compensation scheme will be proposed to guarantee a prescribed transient performance of the tracking error, no matter when actuator failures occur.

In Chapter 5, a modular design based adaptive backstepping control scheme will be presented with the aid of projection operation technique to ensure system stability



in the presence of intermittent actuator failures. It will be shown that the tracking error can be small in the mean square sense when the failure pattern changes are infrequent and asymptotic tracking in the case with finite number of failures can be ensured.

In [Chapter 6](#), a decentralized control method, by using the standard adaptive backstepping technique without any modification, will be proposed for a class of interconnected systems with dynamic interconnections and unmodeled dynamics depending on subsystem inputs as well as outputs. It will be shown that the overall interconnected system can be globally stabilized and the output regulation of each subsystem can be achieved. The relationship between the transient performance of the adaptive system and the design parameters will also be established. The results on linear interconnected systems will be presented firstly and then be extended to nonlinear interconnected systems.

In [Chapter 7](#), two decentralized output feedback adaptive backstepping control schemes are presented to achieve stabilization of unknown interconnected systems with hysteresis. In Scheme I, the term multiplying the control and the system parameters are not assumed to be within known intervals. Two new terms are added in the parameter updating law, compared to the standard backstepping approach. In Scheme II, uncertain parameters are assumed inside known compact sets. Thus projection operation is adopted in the adaptive laws. With Scheme II, the strengths are allowed arbitrary strong provided that their upper bounds are available.

In [Chapter 8](#), a solution of designing decentralized adaptive controllers is provided for achieving output tracking of nonlinear interconnected systems in the presence of external disturbances. The subsystem interactions are unknown and allowed to satisfy a high-order nonlinear bound. A new smooth function is proposed to compensate the effects of unknown interactions and the reference inputs. Apart from global stability ensured with the designed local controllers, a root mean square type of bound for the tracking error is obtained as a function of design parameters.

In [Chapter 9](#), a decentralized adaptive tracking control scheme is presented for a class of interconnected systems with unknown time-varying delays and with the input of each loop preceded by unknown dead-zone nonlinearity.

In [Chapter 10](#), output consensus tracking for a group of nonlinear subsystems in parametric strict feedback form is discussed under the condition of directed communication graph. A distributed adaptive control approach based on backstepping technique is presented to achieve asymptotically consensus tracking. Then the design strategy is successfully applied to solve a formation control problem for multiple nonholonomic mobile robots.

Finally, the entire book is concluded in [Chapter 11](#) by summarizing the main approaches, contributions and discussing some promising open problems in the areas of adaptive failure compensation, decentralized adaptive control and distributed adaptive coordinated control.





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## *Chapter 2*

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# **Adaptive Backstepping Control**

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Backstepping technique is a powerful tool to stabilize nonlinear systems with relaxed matching conditions. It was initiated in the early 1990s and was comprehensively discussed by Krstic, Kanellakopoulos and Kokotovic [90]. “Backstepping” vividly describes a step-by-step procedure to generate the control command for achieving system stabilization and certain specific output regulation properties for a higher-order system, while starting with the first scalar differential equation. In those immediate steps, some state variables are selected as virtual controls and stabilizing functions are designed correspondingly.

To handle systems with parametric uncertainties, adaptive backstepping controllers are designed by incorporating the estimated parameters. Similar to traditional adaptive control methods, the adaptive backstepping control systems can be constructed either directly or indirectly [67]. In direct adaptive backstepping control, parameter estimators are designed at the same time with controllers based on the Lyapunov functions augmented by the squared terms of parameter estimation errors. By combining tuning function technique, the over-parametrization problem can be solved and the cost for implementing the adaptive control scheme can be reduced. However, in indirect adaptive backstepping control, parameter estimators are treated as separate modules from the control modules, thus they are often designed as gradient or least-squares types.

In this chapter, the concepts of integrator backstepping and adaptive backstepping control will be firstly introduced. The procedures to design adaptive controllers by incorporating the tuning functions and modular design schemes are then presented. In the second part, a class of parametric strict-feedback nonlinear systems is considered and stability analysis for the two schemes are also provided briefly.

## 2.1 Some Basics

### 2.1.1 Integrator Backstepping

Consider the system

$$\dot{x} = f(x) + g(x)u, \quad f(0) = 0, \quad (2.1)$$

where  $x \in \mathbb{R}^n$  and  $u \in \mathbb{R}$  are the state and control input, respectively. To illustrate the concept of integrator backstepping, an assumption on (2.1) is firstly made.

**Assumption 2.1.1** *There exists a continuously differentiable feedback control law*

$$u = \alpha(x) \quad (2.2)$$

and a smooth, positive definite, radially unbounded function  $V: \mathbb{R}^n \rightarrow \mathbb{R}_+$  such that

$$\frac{\partial V}{\partial x}(x)[f(x) + g(x)\alpha(x)] \leq -W(x) \leq 0, \quad \forall x \in \mathbb{R}^n, \quad (2.3)$$

where  $W: \mathbb{R}^n \rightarrow \mathbb{R}$  is positive semidefinite.

We then consider a system that is (2.1) augmented by an integrator,

$$\dot{x} = f(x) + g(x)\xi \quad (2.4)$$

$$\dot{\xi} = u, \quad (2.5)$$

where  $\xi \in \mathbb{R}$  is an additional state,  $u \in \mathbb{R}$  is the control input. Based on [Assumption 2.1.1](#), the control law for  $u$  will be generated in the following two steps.

**Step 1.** We stabilize (2.4) by treating  $\xi$  as a virtual control variable. According to [Assumption 2.1.1](#),  $\alpha(x)$  is a “desired value” of  $\xi$ . We define an error variable  $z$  as the difference between the “desired value”  $\alpha(x)$  and the actual value of  $\xi$ , i.e.,

$$z = \xi - \alpha(x). \quad (2.6)$$

Rewrite the first equation (2.4) by considering the definition of  $z$  and differentiate  $z$  with respect to time,

$$\dot{x} = f(x) + g(x)(\alpha(x) + z) \quad (2.7)$$

$$\dot{z} = \dot{\xi} - \dot{\alpha}(x) = u - \frac{\partial \alpha(x)}{\partial x} [f(x) + g(x)(\alpha(x) + z)]. \quad (2.8)$$

**Step 2.** We define a positive definite function  $V_a(x, z)$  by augmenting  $V(x)$  in [Assumption 2.1.1](#) as

$$V_a(x, z) = V(x) + \frac{1}{2}z^2. \quad (2.9)$$

Computing the time derivative of  $V_a(x, z)$  along with (2.3), (2.7) and (2.8), we have

$$\begin{aligned} \dot{V}_a(x, z) &= \dot{V}(x) + z\dot{z} \\ &= \frac{\partial V}{\partial x}(f + g\alpha + gz) + z \left[ u - \frac{\partial \alpha}{\partial x}(f + g\alpha + gz) \right] \\ &= \frac{\partial V}{\partial x}(f + g\alpha) + z \left[ \frac{\partial V}{\partial x}g + u - \frac{\partial \alpha}{\partial x}(f + g\alpha + gz) \right] \\ &\leq -W(x) + z \left[ \frac{\partial V}{\partial x}g + u - \frac{\partial \alpha}{\partial x}(f + g\alpha + gz) \right], \end{aligned} \quad (2.10)$$

where the argument ( $x$ ) has been omitted for simplicity. By observing (2.10), we may choose  $u$  as

$$u = -cz + \frac{\partial \alpha}{\partial x}(f + g\alpha + gz) - \frac{\partial V}{\partial x}g, \quad (2.11)$$

where  $c$  is a positive constant. Thus

$$\dot{V}_a \leq -W(x) - cz^2 \triangleq -W_a(x, z). \quad (2.12)$$

Thus global boundedness of all signals can be ensured. If  $W(x)$  is positive definite,  $W_a$  can also be rendered positive definite. According to the LaSalle-Yoshizawa Theorem given in Appendix B, the globally asymptotic stability of  $x = 0, z = 0$  is guaranteed. If  $\alpha(0) = 0$ , then from (2.6), the equilibrium  $x = 0, \xi = 0$  of (2.4)-(2.5) is also globally asymptotically stable.

The idea of integrator backstepping is further illustrated by the following example.

**Example 2.1.1** Consider the following second order system

$$\dot{x} = x^2 + x\xi \quad (2.13)$$

$$\dot{\xi} = u. \quad (2.14)$$

Comparing (2.13)-(2.14) with (2.4)-(2.5), we see that  $x \in \mathbb{R}$ ,  $f(x) = x^2$  and  $g(x) = x$ . To stabilize (2.13) with  $\xi$  as the input, we define  $V(x) = \frac{1}{2}x^2$ . By choosing the desired value of  $\xi$  as

$$\alpha(x) = -x - 1, \quad (2.15)$$

we have

$$\dot{V} = x(x^2 + x\alpha) = -x^2. \quad (2.16)$$

Thus the error variable is

$$z = \xi - \alpha = \xi + x + 1. \quad (2.17)$$

Substituting  $\xi = z - x - 1$  into (2.13) and computing the derivative of  $z$ , we obtain

$$\dot{x} = xz - x \quad (2.18)$$

$$\dot{z} = u + xz - x. \quad (2.19)$$

We then define  $V_a = \frac{1}{2}x^2 + \frac{1}{2}z^2$ , of which the derivative is computed as

$$\dot{V}_a = -x^2 + x^2z + z(u + xz - x). \quad (2.20)$$

Thus the control

$$u = -z - xz + x - x^2 \quad (2.21)$$

can render  $\dot{V}_a = -x^2 - z^2 < 0$ . From the LaSalle-Yoshizawa Theorem, global uniform boundedness of  $x, z$  is achieved and  $\lim_{t \rightarrow \infty} x(t) = \lim_{t \rightarrow \infty} z(t) = 0$ . From (2.15),  $\xi = z - x - 1$  and (2.21), we have  $\alpha, \xi$  and the control  $u$  are also globally bounded.

### 2.1.2 Adaptive Backstepping Control

To illustrate the idea of adaptive backstepping control, we consider the following second order system as an example, in which the parametric uncertainty enters the system one integrator before the control  $u$  does.

$$\dot{x}_1 = x_2 + \varphi^T(x_1)\theta \quad (2.22)$$

$$\dot{x}_2 = u, \quad (2.23)$$

where the states  $x_1, x_2$  are measurable,  $\varphi(x_1) \in \mathbb{R}^p$  is a known vector of nonlinear functions and  $\theta \in \mathbb{R}^p$  is an unknown constant vector. The control objective is to stabilize the system and regulate  $x_1$  to zero asymptotically.

We firstly present the design procedure of controller *if  $\theta$  is known*. Introduce the change of coordinates as

$$z_1 = x_1 \quad (2.24)$$

$$z_2 = x_2 - \alpha_1, \quad (2.25)$$

where  $\alpha_1$  is a function designed as a “desired value” of the virtual control  $x_2$  to stabilize (2.22) and

$$\alpha_1 = -c_1 x_1 - \varphi^T \theta, \quad c_1 > 0. \quad (2.26)$$

Define the control Lyapunov function as

$$V = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2, \quad (2.27)$$

whose derivative is computed as

$$\begin{aligned} \dot{V} &= z_1(z_2 - c_1 z_1) + z_2 \left[ u - \frac{\partial \alpha_1}{\partial x_1} (x_2 + \varphi^T \theta) \right] \\ &= -c_1 z_1^2 + z_2 \left[ z_1 + u - \frac{\partial \alpha_1}{\partial x_1} (x_2 + \varphi^T \theta) \right]. \end{aligned} \quad (2.28)$$

By choosing the control input as

$$u = -z_1 - c_2 z_2 + \frac{\partial \alpha_1}{\partial x_1} (x_2 + \varphi^T \theta), \quad c_2 > 0 \quad (2.29)$$

Eqn. (2.28) becomes

$$\dot{V} = -c_1 z_1^2 - c_2 z_2^2 < 0. \quad (2.30)$$

From the LaSalle-Yoshizawa Theorem,  $z_1$  and  $z_2$  are ensured globally asymptotically stable. Since  $x_1 = z_1$ , we obtain that  $\lim_{t \rightarrow \infty} x_1(t) = 0$ . From (2.26) and (2.25), we have  $\alpha_1, x_2$  are also globally bounded. From (2.29), we conclude that the control  $u$  is also bounded.

*However,  $\theta$  is actually unknown.* To ensure the stabilizing function  $\alpha_1$  is implementable, (2.26) can be modified by replacing  $\theta$  with its estimated parameter