# Basic GIS Coordinates 

## THIRD EDITION



## Jan Van Sickle

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## Preface

A fundamental change is coming. Two datums at the foundation of the current geographic information system (GIS) work will be replaced: the North American Datum 1983 and the North American Vertical Datum 1988. This book provides some of the information necessary to cope with that change. It is a book about coordinates.

Coordinates? Press a few buttons on a computer and they are automatically imported, exported, rotated, translated, collated, annotated, and served up in any format you choose with no trouble at all. There is really nothing to it. Why have a book about coordinates?
It is a good question really. Computers are astounding in their ability to make the mathematics behind coordinate manipulation transparent to the user. This book is not much about that sort of mathematics. However, it is about coordinates and coordinate systems. It is about understanding how these systems work and how they sometimes do not work. It is about how points that should be in New Jersey end up in the middle of the Atlantic Ocean even if the computer has done nothing wrong. And that is, I suppose, the answer to the question from my point of view. Computers are currently very good at repetition and very bad at interpretation. People are usually not so good at repetition. We tend to get bored. However, we can be very good indeed at interpretation, that is, if we have the information in our heads to understand what we are interpreting. This book is about providing some of that sort of information on the subject of coordinates and coordinate systems.
Coordinates are critical to GIS, cartography, and surveying. They are their foundation. Points, lines, and polygons form the geometry. Lines define the polygons, points define the lines, and coordinates define the points. Therefore, at bottom, it is the coordinates that tie the real world to their electronic image in the computer.

There are more than 1000 horizontal geodetic datums and over 3000 Cartesian coordinate systems sanctioned by governments around the world and currently in use to describe our planet electronically and on paper.

## Author

Jan Van Sickle has many years of experience in GIS, global positioning system (GPS), surveying, mapping, and imagery. He began working with GPS in the early 1980s when he supervised control work using the Macrometer, the first commercial GPS receiver. He created and led the GIS department at Qwest Communications, Denver, Colorado, for the company's 25,000-mile worldwide fiber optic network. He also led the team that built the GIS for natural gas gathering in the Barnett Shale. He has led nationwide seminars based on his three books: GPS for Land Surveyors, Basic GIS Coordinates, and Surveying Solved Problems. He led the team that collected, processed, and reported control positions for more than 120 cities around the world for the ortho-rectification of satellite imagery now utilized in a global web utility. He managed the creation of the worldwide test and evaluation (T\&E) sites for two major earth observation satellites that are used for frequent accuracy assessments. He created an imagery-based system of deriving road centerlines that meet the stringent Advanced Driver Assistance specifications and developed a method of forest inventory to help quantify that depleted resource in Armenia. He assisted the supervision of the first GPS survey of the Grand Canyon for the photogrammetric evaluation of sandbar erosion along the Colorado River. He has performed threedimensional mapping with terrestrial photogrammetry and light detection and ranging (LiDAR) as well as Building Information Modeling for major buildings in Washington, DC. He was a member of the team of authors for the Geospatial Technology Competency Model for the Department of Labor. He has provided technical assistance in the reconstruction of the geodetic network of Nigeria. He has managed the gravity/magnetic and hyperspectral/multispectral analysis of Borzon VII, a concession block in the South Gobi in Mongolia. He writes the From Scratch blog (http://www. sparpointgroup.com/jan-van-sickle) for the SPAR Point Group. He has contributed a chapter on satellite-based surveying technology to the update of the more than 30-year old American Society of Civil Engineers Engineering Surveying Manual of Practice and course work on global navigation satellite systems (GNSS) to the American Association for Geodetic Surveying Geodetic Certification Program. He has been a featured speaker at many conferences, including MAPPS, geospatial information and technology
association (GITA), and the Institute of Navigation Annual Meeting. He is a senior lecturer at Pennsylvania State University, State College, Pennsylvania, and teaches at the University of Colorado Denver, Denver, Colorado. He was formerly on the board of Rocky Mountain - American Society of Photogrammetry and Remote Sensing (RM-ASPRS), the vice-chairman of GIS in the Rockies, and the founding chairman of the U.S. West chapter of the Americas Petroleum Survey Group. Sickle earned a PhD in geospatial engineering at the University of Colorado. He has been a licensed professional land surveyor for 36 years and is a member of the Royal Institute of Chartered Surveyors and currently licensed in Colorado, California, Oregon, Texas, North Dakota, West Virginia, and Pennsylvania. His website is www.janvansickle.com.

## 1

## Foundation of a Coordinate System

## Uncertainty

Coordinates are slippery devils. A stake driven into the ground holds a clear position, but it is awfully hard for its coordinates to be so certain, even if the figures are precise. For example, a latitude of $40^{\circ} 25^{\prime} 33.504^{\prime \prime} \mathrm{N}$ with a longitude of $108^{\circ} 45^{\prime} 55.378^{\prime \prime} \mathrm{W}$ appears to be an accurate and unique coordinate. Actually, it could correctly apply to more than one place. An elevation, or height, of 2658.2 m seems unambiguous too, but it is not.

In fact, this latitude, longitude, and height once pinpointed a control point known as Youghall, but not anymore. Oh, Youghall still exists. It is a bronze disk cemented into a drill hole in an outcropping of bedrock on Tanks Peak in the Colorado Rocky Mountains. But its coordinates have not been nearly as stable as the monument. In 1937 the United States Coast and Geodetic Survey set Youghall at a latitude of $40^{\circ} 25^{\prime} 33.504^{\prime \prime} \mathrm{N}$ and longitude $108^{\circ} 45^{\prime} 55.378^{\prime \prime} \mathrm{W}$. You might think that was that, but in November of 1997 Youghall suddenly got a new coordinate, $40^{\circ} 25^{\prime} 33.39258^{\prime \prime} \mathrm{N}$ and $108^{\circ} 45^{\prime} 57.78374^{\prime \prime} \mathrm{W}$, that is more than $56 \mathrm{~m}, 185 \mathrm{ft}$, west and $3 \mathrm{~m}, 11 \mathrm{ft}$, south of where it started. Its elevation changed too. It was 2658.2 m in 1937. It is 2659.6 m today. It rose to $41 / 2 \mathrm{ft}$.

Of course, it did no such thing; the station is right where it has always been. The Earth shifted underneath it, well, nearly. It was the datum that changed. The 1937 latitude and longitude for Youghall was based on the North American Datum 1927 (NAD27). Sixty years later, in 1997 the basis of the coordinate of Youghall became the North American Datum 1983 (NAD83).

## Datums to the Rescue

Coordinates without a specified datum are vague. It means that questions like Height above what?, Where is the origin?, and On what surface do they lay? go unanswered. When that happens coordinates are of really no use. An origin, a starting place, is a necessity for them to be meaningful. Not only must they
have an origin, they must be on a clearly defined surface. These foundations constitute the datum.

Without a datum, coordinates are like checkers without a checkerboard, you can arrange them, analyze them, and move them around, but absent the framework, you never really know what you have got. In fact, datums very like checkerboards have been in use for a long time. They are generally called Cartesian.

## René Descartes

Cartesian systems got their name from René Descartes, a mathematician and philosopher. In the world of the seventeenth century he was also known by the Latin name Renatus Cartesius, which might explain why we have a whole category of coordinates known as Cartesian coordinates. Descartes did not really invent the things, despite a story of him watching a fly walk on his ceiling and then tracking the meandering path with this system of coordinates. Long before, around 250 BC or so the Greek mathematician, geographer, and astronomer, Eratosthenes, used a checkerboard-like grid to locate positions on the Earth and even he was not the first. Dicaearchus had come up with the same basic idea before him. Nevertheless, Descartes was probably the first to use graphs to plot and analyze mathematical functions. He set up the rules we use now for his particular version of a coordinate system in two dimensions defined on a flat plane by two axes.

## Cartesian Coordinates

Two-dimensional Cartesian coordinates are expressed in ordered pairs. Each element of the coordinate pair is the distance measured across the flat plane parallel with an axis. If the measurement is parallel with the $x$-axis, it is called the $x$-coordinate, and if the measurement is parallel with the $y$-axis, it is called the $y$-coordinate.
Figure 1.1 shows two axes perpendicular to each other labeled $x$ and $y$. This labeling is a custom established by Descartes. His idea was to symbolize unknown quantities with letters at the end of the alphabet: $x, y$, and $z$. This leaves letters at the beginning that are available for known values. Coordinates became so often used to solve for unknowns, the principle that Cartesian axes have the labels x and y was established. The fancy names for the axes are the abscissa, for x , and ordinate, for y. Surveyors, cartographers, and mappers call them north and east, but back to the story.
These axes need not be perpendicular with each other. They could intersect at any angle, though they would obviously be of no use if they were parallel. But so much convenience would be lost using anything other than a right angle, it has become the convention. Another convention is the idea that the units along the $x$-axis are identical with the units along the $y$-axis, even though there is no theoretical requirement that this be so. Finally, on


FIGURE 1.1
The Cartesian coordinate system.
the $x$-axis, any point to the west, that is left, of the origin is negative, and any point to the east, to the right, is positive. Similarly, on the $y$-axis, any point north of the origin is positive; and south, negative. If these principles are held, then the rules of Euclidean geometry are true and the off-the-shelf CAD and GIS software on your PC have no trouble at all working with these coordinates, a most practical benefit.
For example, the distance between these points can be calculated using the coordinate geometry you learned in high school. The x and y coordinates for the points in the illustration are the origin point, $\mathrm{P}_{1}(220,295)$ and point, $\mathrm{P}_{2}$ $(311,405)$ therefore where:

$$
\begin{aligned}
& \mathrm{x}_{1}=220 \\
& \mathrm{y}_{1}=295
\end{aligned}
$$

and

$$
\begin{aligned}
& \mathrm{x}_{2}=311 \\
& \mathrm{y}_{2}=405
\end{aligned}
$$

$$
\begin{aligned}
& \text { Distance }=\sqrt{\left[\left(\mathrm{x}_{1}\right)-\left(\mathrm{x}_{2}\right)\right]^{2}+\left[\left(\mathrm{y}_{1}\right)-\left(\mathrm{y}_{2}\right)\right]^{2}} \\
& \text { Distance }=\sqrt{[(220)-(311)]^{2}+[(295)-(405)]^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& \text { Distance }=\sqrt{(-91)^{2}+(-110)^{2}} \\
& \text { Distance }=\sqrt{8,281+12,100} \\
& \text { Distance }=\sqrt{20,381} \\
& \text { Distance }=142.76
\end{aligned}
$$

The system works. It is convenient. But it is not very helpful unless it has an attachment to something a bit more real than these unitless numbers, which brings up an important point about datums.

## Attachment to the Real World

The beauty of datums is that they are errorless, at least in the abstract. On a datum every point has a unique and accurate coordinate. There is no distortion. There is no ambiguity. For example, the position of any point on the datum can be stated exactly, and it can be accurately transformed into coordinates on another datum with no discrepancy whatsoever. All of these wonderful things are possible only as long as a datum has no connection to anything in the physical world. In that case it is perfectly accurate, and perfectly useless.

But suppose you wished to assign coordinates to objects on the floor of a very real rectangular room. A Cartesian coordinate system could work, if it is fixed to the room with a well-defined orientation. For example, you could put the origin at the southwest corner, stipulate that the walls of the room are oriented in cardinal directions, and use the floor as the reference plane.
With this datum you not only have the advantage that all of the coordinates are positive, but you can define the location of any object on the floor of the room. The coordinate pairs would consist of two distances: (1) the distance east and (2) the distance north from the origin in the corner. As long as everything stays on the floor, you are in business. In this case there is no error in the datum, of course, but errors in the coordinates are inevitable. These errors are due to the less than perfect flatness of the floor, the impossibility of perfect measurement from the origin to any object, the ambiguity of finding the precise center of any of the objects being assigned coordinates, and so on. In short, as soon as you bring in the real world, things get messy.

## Cartesian Coordinates and the Earth

Cartesian coordinates then are rectangular, or orthogonal if you prefer, defined by perpendicular axes from an origin, along a specifically oriented reference surface. These elements can define a framework, a foundation, and a datum for meaningful coordinates.

As a matter of fact, two-dimensional Cartesian coordinates are an important element in the vast majority of coordinate systems, the State Plane coordinates in the United States, the Universal Transverse Mercator (UTM) coordinate system, and others. The datums for these coordinate systems are well established. But there are also local Cartesian coordinate systems whose origins are often entirely arbitrary. For example, if surveying, mapping, or other work is done for the construction of a new building, there may be no reason for the coordinates used to have any fixed relation to any other coordinate systems. In that case a local datum may be chosen for the specific project with north and east fairly well defined and the origin moved far to the west and south of the project to ensure that there will be no negative coordinates. Such an arrangement is good for local work, but it does preclude any easy combination of such small independent systems. Large-scale Cartesian datums, on the other hand, are designed to include positions across significant portions of the Earth's surface into one system. Of course, these are also designed to represent our decidedly round planet on the flat Cartesian plane, no easy task (Figure 1.2).


FIGURE 1.2
Distortion increases over long lines and large areas.

But how would a flat Cartesian datum with two axes represent the Earth? There is obviously distortion inherent in the idea. If the planet were flat it would do nicely of course, and across small areas that very approximation, a flat Earth, works reasonably well. That means that even though the inevitable warping involved in representing the Earth on a flat plane cannot be eliminated, it can be kept within well-defined limits as long as the region covered is small and precisely defined. If the area covered becomes too large distortion does defeat it. So the question is, Why go to all the trouble to work with plane coordinates? Well, here is a short example.
It is certainly possible to calculate the distance from station Youghall to station Karns using latitude and longitude, also known as geographic coordinates, but it is easier for your computer, and for you, to use Cartesian coordinates. Here are the geographic coordinates for these two stations, Youghall at latitude $40^{\circ} 25^{\prime} 33.39258^{\prime \prime} \mathrm{N}$ and longitude $108^{\circ} 45^{\prime} 57.78374^{\prime \prime} \mathrm{W}$ and Karns at latitude $40^{\circ} 26^{\prime} 06.36758^{\prime \prime} \mathrm{N}$ and longitude $108^{\circ} 45^{\prime} 57.56925^{\prime \prime} \mathrm{W}$ in the North American Datum 1983 (NAD83). Here are the same two station's positions expressed in Cartesian coordinates.

Youghall

$$
\begin{aligned}
& \text { Northing }=y_{1}=1,414,754.47 \\
& \text { Easting }=x_{1}=2,090,924.62
\end{aligned}
$$

Karns

$$
\begin{array}{r}
\text { Northing }=y_{2}=1,418,088.47 \\
\text { Easting }=x_{2}=2,091,064.07
\end{array}
$$

The Cartesian system used here is called state plane coordinates in Colorado's North Zone, and the units are survey feet, more about those later. The important point is this; these coordinates are based on a simple two-axis Cartesian system operating across a flat reference plane.

As before the distance between these points using the plane coordinates is easy to calculate.

$$
\begin{aligned}
& \text { Distance }=\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}} \\
& \text { Distance }=\sqrt{(2,090,924.62-2,091,064.07)^{2}+(1,414,754.47-1,418,088.47)^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& \text { Distance }=\sqrt{(-139.45)^{2}+(-3334.00)^{2}} \\
& \text { Distance }=\sqrt{(19,445.3025)+(11,115,556.0000)} \\
& \text { Distance }=\sqrt{11,135,001.30} \\
& \text { Distance }=3336.91 \mathrm{ft}
\end{aligned}
$$

It is 3336.91 ft . The distance between these points calculated from their latitudes and longitudes is slightly different, it is 3337.05 ft . Both of these distances are the result of inverses, which means they were calculated between two positions from their coordinates (Figure 1.3).

When you compare the results between the methods you see a difference of about 0.14 ft , a bit more than a tenth of a foot. In other words, the spacing between stations would need to grow more than seven times, to about $41 / 2$ miles, before the difference would reach 1 ft . So part of the answer to the


FIGURE 1.3
Youghall to Karns.
question, Why go to all the trouble to work with plane coordinates? is this, they are easy to use and the distortion across small areas is not severe.
This rather straightforward idea is behind a good deal of the conversion work done with coordinates. Geographic coordinates are useful but somewhat cumbersome at least for conventional trigonometry. Cartesian coordinates on a flat plane are simple to manipulate but inevitably include distortion. Moving from one to the other it is possible to gain the best of both. The question is How do you project coordinates from the nearly spherical surface of the Earth to a flat plane? Well, first you need a good model of the Earth.

## The Shape of the Earth

People have been proposing theories about the shape and size of the planet for a long time. Despite the fact that local topography is obvious to an observer standing on the Earth, efforts to grasp the more general nature of the planet's shape and size have been occupying scientists for at least 2,300 years. There have, of course, been long intervening periods of unmitigated nonsense on the subject. Ever since 200 BC when Eratosthenes almost calculated the planet's circumference correctly, geodesy has been getting ever closer to expressing the actual shape of the Earth in numerical terms (Figure 1.4).
At noon the reflection of the midsummer sun was there in the water of a deep well at Syene. The sun was directly overhead. On the same day, measurement of the noon shadow was cast by a pillar at Alexandria. It showed that the sunbeam strikes the earth at an angle of $7.2^{\circ}$ off the vertical. Eratosthenes reasoned that the angle between Alexandria and Syene must be $7.2^{\circ}$-one fiftieth of the $360^{\circ}$ circle. Syene is 491 miles south of Alexandria and a great circle must therefore be 50 times 491 miles in length-24,550 miles. In fact, the circumference of the earth is around 24,900 miles.
A real breakthrough came in 1687 when Newton suggested that the Earth shape was ellipsoidal in the first edition of his Principia. The idea was not entirely without precedent. Years earlier in 1672, astronomer J. Richer found the closer he got to the equator, the more he had to alter the pendulum on his one-second clock. The clock lost about $21 / 2$ minutes every day because its pendulum swung more slowly in Cayenne, Guyana, and South America than it did in Paris. In South America he had to shorten the pendulum about 3 mm to bring it back into regulation. When Newton heard about it he speculated that the force of gravity was less in South America than in France. He explained the weaker gravity by the proposition that when it comes to the Earth there is simply more of it around the equator. He wrote, "The Earth is higher under the equator than at the poles, and that by an excess of about 17 miles" (Philosophiae naturalis principia mathematica, Book III, Proposition XX). He was pretty close to right; the actual distance is about 13 miles, only about 4 miles less than he thought.
Some supported Newton's idea that the planet bulges around the equator and is flattened at the poles, but others disagreed, for example, the director of

