# ROBUST ADAPTIVE CONTROL

Edited by

G. C. GOODWIN

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# ROBUST ADAPTIVE CONTROL

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Edited by

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# A Prospect of Adaptive Control

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Abstract. A brief commentary is made on progress in adaptive control over 40 years and it noted how irregular this progress has been. As an essentially practical topic progress has often been conditioned by the available technical means to implement schemes. It has proved convenient to divide progress into decades each with its own particular flavour.

It is concluded that in this business of mechanising adaptation we are slow learners.

Keywords. Adaptive control, self-tuning controllers, $H^{\infty}$ designs, least squares estimation.

## INTRODUCTION

The idea of adaptative control has been a gleam in the eye of control engineers for almost 40 years. The question arises as to why it has taken such an inordinately long time to bring it to practical effect. It is convenient to divide the intervening period into decades marking the stages in the development of the idea.

The first such stage was in the 1950's in what might be dubbed the knife and fork era of development when it was understandable that efforts were of a rather fumbling nature lacking as they were any well-worked out theoretical basis for design. Performance criteria were of an ad hoc nature, settings of the terms of a 3-term controller to give acceptable transient response or gain and phase margins.

A personal attempt at a practical adaptive controller at that time was described as a self-setting controller in an unpublished note dated December 1954 as a part of a consulting assignment. It was designed to automatically set the proportional and reset terms of a controller to accomodate changes in plant characteristics with change of flow rate in a continuous chemical process. Its action depend on measuring the amplitude and frequency of a limit cycle oscillation contrived to be of small excursion using describing function techniques. The two terms were set by exploiting the characteristics of the resulting Nyquist diagram. It was not in fact very practical for two reasons:

1) the technical means then available for adjusting the terms, involving analogue multipliers chopper stabilised for drift correction, were cumbersome, unreliable and expensive.

2) when more parameters had to be adjusted some better organised way of seeking an optimum was needed and added greatly to the complication of the hardware.

It was a very laboured approach to adaptive control and like many other proposals at the time was never used on an operating plant.

However the long-sighted of those times had noted the work of Wiener (1949) whose theory of filtering was based on the use of statistical spectral analysis. This seemed to represent the way forward. A modest extension of these ideas into the multivariable case Westcott (1958) takes up the theme, next to a paper by Kalman (1958) who employed correlation functions in the time domain in the design of a self optimising control system. It is the only paper by that distinguished author with photographs of equipment. It describes a mechine having an on-line least squares algorithm producing estimates of the ordinates of an impulse response. These estimates were then used at every sample instant to compute a dead-beat control law. A machine was certainly built but whether results rose to meet aspirations is not recorded.

More characteristic of the author were the conclusions:
"The task of the Control Engineer in the future will not be to design a specific system but to improve the principles on which machines of the type described here will operate. Unlike his predecessor the stock in trade of the new control systems engineer will not be graph paper, the slide rule or even the analog computer, but a form of deep-seated understanding of the fundamentals on which automatic control is based. The drugery of computing will be taken over by machines, but the challenge of thinking remains"

What we were looking for at that time was some way of adapting the optimum design technique as employed in Wiener's filter to apply to the design of closed loop systems. There ought to be a way of doing this by putting a constraint on the control variable excursions, as indeed there was using a Lagrange multiplier, Westcott (1954).

1960's The 1960's saw the flowering of a proper theoretical framework for adaptive control and represented the beginning of respectability for the topic. It was realised Westcott (1962) that three components were essential:

1) a performance index which took into account not only the departure of output from a desired path but also the "cost" of the control effort to achieve this and to use a variational principle to minimise the sum of the squares.

2) the inclusion of random disturbing influences whose underlying generators are fixed and can be known leading to a stochastic problem involving stationary time series.

3) a formulation of the dynamic state transition probability characteristics of the disturbed system in order that the transient effect of the disturbances on the performance index may be calculated.

For a single state dynamic system using a quadratic performance index we obtain a performance index state transition equation of the following form:

$$-\frac{\partial V}{\partial t} = (A+u)\,\frac{\partial V}{\partial x} + \frac{\sigma^2}{2}\cdot\frac{\partial^2 V}{\partial x^2}$$

| | | |
|---|---|---|
| 1st order dynamics drift | effect of control action | disturbance diffusion |

where $V = E[\int(x^2+u^2)dt]$

$\dot{x} = Ax + e \quad e = \text{random disturbance } R(0,\sigma).$

It was then further realised that if the underlying generator of the disturbances and the dynamic parameter A were not fixed but needed to be estimated then a different compromise had to be struck since to do this properly required excursions away from what one would otherwise regard as good control policy and one would have to weigh in balance the value of the knowledge gained against the value of seemingly better current control on the basis of possibly incorrect parameters (a technique known today as probing stochastic control).

Speculation was not restricted to these rather general ideas. Tangible results on how to tackle the problem were swiftly to follow. The seminal paper by Kalman and Bucy (1961) introduced a combination of two familiar ideas with rewarding results: the use of the state transition method to describe the dynamics with linear filtering regarded as orthogonal projection in Hilbert space. On the face of it this lead to the same results as Wiener filtering but instead of having to deal with the Wiener-Hopf integral equation directly a nonlinear differential equation of the Riccati type is obtained whose solution gives the covariance matrix of the minimum filtering error and hence the optimal filter.

This represented a considerable break through on several fronts. In the first place the dynamic equations were couched in state space form, the most basic and classical representation of dynamics in differential equations form leading directly to vector matrix analysis. Thus state transition matrices were used to describe the evolving dynamic behaviour. Furthermore the equations applied naturally to the multivariable case without any change. The estimation procedures employed were least squares and so of the most direct and simplest kind, but more importantly they were recursive and so ideally suited to tracking and utilising the improved knowledge of the system as data accumulated. This also meant that systems whose underlying statistical parameters were not constant but slowly time varying and whose statistics were therefore non-stationary could still be handled in practice.

A further bonus was provided by a duality in the equations which provided a link between the stochastic filtering case which is the normal format of the Kalman filter and the optimal control of a regulator. It seemed natural to ask whether the two versions of the equations could be exploited together; the filter to estimate the system and the regulator to provide the optimal controller. Provided certainty equivalence prevails which is a fairly demanding and technical requirement the two sets of equations do not get entangled. In practice it is often found to work well.

Almost immediately the role of the filter was extended to also allow estimation of system parameters and this also often works well in practice. So at this early date more then 20 years ago a very adequate theoretical framework existed for adaptive control. At this stage however theory was well ahead of what could readily be achieved in reliable hardware. Although computers were improving they were not reliable and programming them was a pain: they remained expensive and bulky. Their use in this role was restricted to prestige projects where their expense could be more readily justified.

1970's The 1970's brought a different approach to the problem of adaptive control. It was to be the era of the 'self-tuner' encouraged no doubt by the promise of better, smaller and cheaper computers, the onset of the revolution to be brought about by the microprocessor. In this new approach simplicity in the analysis was all. Whereever possible procedures were telescoped so that for example estimation of parameters of the plant model was entirely bypassed and controller parameters were estimated directly instead; the so-called implicit self-tuners. The first step in the development of the algorithm is thus to obtain a model of the system expressed in terms of the unknown controller parameters.

A clear treatment of how this is carried out is given in Astrom and Wittenmark (1973). It reveals an immediate difficulty of an insufficiency of equations, so that in every case one coefficient (usually the first coefficient of the control polynomial, $b_0$) has to the guessed to initiate the procedure. Unhappily the rate of convergence to steady estimates can depend on the skill (or luck) with which this is chosen. Sometimes convergence is very slow, sometimes it does not converge at all. Astrom was able to show that if does converge it always gives the correct answer. Ljung (1974) has explored the conditions for convergence and how to improve them.

Much work has been done in proving convergence for adaptive control for both deterministic and stochastic cases; see Goodwin et al (1981). It is particularly tricky in the case of implicit self-tuners where estimation is "wrapped into" the optimising of controller parameters. Thus as the optimum control parameters are approached the sensitivity to their values reduces due to non-uniqueness of the plant parameters for the particular solution.

## Least squares estimation: a suggestion

While least squares estimation has enormous attraction for its simplicity, it does have its aggravation as far as assurity of convergence is concerned. Is there any way round it? One way is to go to more sophisticated estimation techniques such as maximum likelihood estimates. These are non-linear and require the use of non-linear numerical optimisation techniques. To use full information maximum likelihood techniques (as has been justified in attempting to control a national economy) involves a horrendous volume of computing and even then can involve various stages of approximation in order to bring the computational load back to manageable properties. Such extremes are not normally justified in the context of engineering. So is there a way out of the convergence problems of least squares estimation? Here is a suggestion proposed by Burt (1985).

We take as an example the equation for a simple self-tuner whose coefficients are to be estimated:

$$A(q^{-1})y_n = q^{-1}B(q^{-1})u_n + C(q^{-1})e_n$$

where $q^{-1}$ is the backward shift operator

$$A = 1 + a_1q^{-1}$$

$$B = 1 + b_1q^{-1}$$

$$C = 1 + c_1q^{-1} + c_2q^{-1}$$

hence

$$y_n = -a_1q^{-1}y_{n-1} + (1 + b_1q^{-1})u_{n-1} + (1+c_1q^{-1}+c_2q^{-2})e_n$$

where $e_n$ is a random noise sequence and var(e) = 1.

The samples $y_n$ and $u_n$ are observable so we may estimate the following expectation values:

$$p_m = Ey_n\, y_{n-m} = p_{-m}$$

$$q_m = Eu_n\, u_{n-m} = q_{-m}$$

$$r_m = Ey_n\, u_{n-m}\,.$$

However the following expectations :

$$s_m = Ey_n\, e_{n-m}$$

and $\quad t_m = Eu_n\, e_{n-m}$

are one-sided (having no value for m $<$ 0) and are not observable since e is a random process. From the expression given for $y_n$ we can derive sets of equations for the only 3 expectation values that involve y (that is $p_m$, $r_m$ and $s_m$) for various values of the shift m in terms of the other expectations and the coefficients of A, B, and C. Below is shown how this works out for the example:

$$\begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ r_1 \\ r_2 \\ s_0 \\ s_1 \\ s_3 \end{bmatrix} \begin{array}{c} \\ \text{L.S.} \\ \\ = \\ \\ \\ \text{L.S.} \\ \\ \\ \end{array} \begin{bmatrix} p_1 & r_1 & r_2 & s_0 & s_1 & s_2 \\ p_0 & r_0 & r_1 & \cdot & s_0 & s_1 \\ p_1 & r_{-1} & r_0 & \cdot & \cdot & \cdot \\ p_2 & r_{-2} & r_{-1} & \cdot & \cdot & \cdot \\ r_0 & q_0 & q_1 & \cdot & t_0 & t_1 \\ r_1 & q_1 & q_0 & \cdot & \cdot & t_0 \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ s_0 & t_0 & \cdot & \cdot & s_0 & \cdot \\ s_1 & t_1 & t_0 & \cdot & \cdot & s_0 \end{bmatrix} \times \begin{bmatrix} -a_1 \\ 1 \\ b_1 \\ 1 \\ c_1 \\ c_2 \end{bmatrix}$$

There are many more relationships that can be obtained by using greater values of shift m, but with the set given we have nine linear equations for 4 system coefficients $a_1$, $b_1$, $c_1$ and $c_2$ to be found and with 5 unobservables $s_0$, $s_1$, $s_2$, $t_0$ and $t_1$ to be eliminated so we have sufficient linear equations for a solution. Since there is a choice in which equations to choose it has become our practice to choose the set that allows the a and b coefficients to be obtained without involving the unobservables and then using these known values to eliminate the unobservables from the remaining equation to give the values of the c's.

The equations are of course somewhat familiar containing within them all the relationship that are made use of in least squares estimation and also when c coefficients are acknowledged to be present in extended least squares. These sets of equations are indicated in the matrix equation, but there are also some additional equations which are the key to the success of the new method in avoiding difficulty with convergence. It is easy to see how this difficulty arises with these earlier methods since the unobservable components are unacknowledged and give rise to biassed estimates which biasses have to be worked out by iteration and may in fact take a very long time to do so.

Fig. 1 shows how much better the new method is (the curves labelled BR in the figure) at determining the two c coefficients than extended least squares (labelled ELS). However there is a price to be paid for these benefits. The set of least squares and extended least squares equations are symmetric in the matrix allowing the matrix inversion lemma to be used. Such is clearly not the case for the new method and so different numerical procedures need to be used for the inversion.

## 1980's

The current decade of the 1980's is producing another shift of interest in technique which will certainly have implications for the topic of adaptive control and it comes from a quite new direction. This decade will probably become known as the $H^{\infty}$ era. This is a method with new virtues some of which seemed improbable until recently. A very useful tutorial paper is Safonov et al (1987). The method is rooted in the idea of frequency functions that are maintained positive real. Controllers which preserve this characteristic within a closed loop can never be destabilished by unmodelled modes that are themselves dissipative.

As a frequency response method it makes extensive use of spectral factorisation following in the steps of Wiener in this respect. On the other hand it has been found possible to couch the whole analysis in state-space matrix formulation and so to utilise the state-space approach to infinity norm control. The solution involves the design of a feedback controller which generates internal stability and the $L^{\infty}$ norm of the closed loop transfer T (see fig. 2) less than or equal to unity. The meeting of this requirement can be viewed as a Hankel model reduction problem. The now classic paper by Glover (1984) shows how a state-space theory based on balanced realisations solves the multivariable model reduction problem. Truncated balanced realisations are particularly useful in model reduction because of their assurred infinity-norm error bounds. Both upper and lower error bounds are secured, the upper being pessimistic but the lower being realised for practical purposes. While the analytical complications are considerable the benefit in having such firm hooks on the bounds of error are very attractive indeed.

The manner in which the controller evolves is illustrated in the signal flow diagram of fig. 3. It will be observed that the controller is composed of two parts, a part labelled central LQG regulator to which is added a further feedback through Q. In the LQG regulator state feedback F and output injection H are at our disposal to ensure stabilisability through the use of F and detectability through the use of H involving the solving of a Riccati equation in each case. A particular choice of H and F allows the expression for $T = T_{11} + T_{12}QT_{21}$ in fig. 2 to take a form in which $T_{12}$ and $T_{21}$ are all-pass transfers. This allows the use of the Hankel model reduction procedure. It remains to choose Q via a minimisation :

$$\min_{Q \in H^{\infty}} \| T_{11} + T_{12} Q T_{21} \|_{\infty}$$

when $T_{12}$ and $T_{21}$ are both square matrices the solution falls out readily. The more general cases involve additional factorisations which are troublesome. It is here that breakthroughs are looked for and are on the way.

The method allows a specified degree of robustness to be built into a system by permitting constraints to be placed on the frequency characteristic of a closed loop system. Essentially it allows the roll-off characteristic at high frequency to be controlled in any desired way while guaranteeing stability. This has always been the aim of earlier cut and try methods but the new technique goes about it in a more systematic and scientific way. It is easy to see that given such a possibility the technique will be useful in handling unmodelled parameters or securing satisfactory performance in the face of sensor noise arising within the system and many other benefits.

Until recently the computational load involved was such as to dominate the reaction time of any system in which repeated updates were needed, so while explicit self-tuners were a theoretical possibility they were not very practical. Methods are now on the horizon which reduce this drastically. The real thing one would like to see from an adaptive control viewpoint would be some proper recursive formulation giving the method the possibility of handling non-stationary time series; more like Kalman and less like Wiener, but these developments are for the future. For all those looking for problems to tackle there are happy days ahead.

## CONCLUSIONS

The message of this account as we head for the decade of the 1990's is that progress in this topic is not steady, it often comes from unexpected developments and is still very slow and painful. We have only to look around in nature to see that when it comes to mechanising adaptation we are really only beginners.