

ARITHMETIC APPLIED MATHEMATICS



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International Series in Nonlinear Mathematics: Theory, Methods and
Applications

Volume 1

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Preface

In this book we will develop a computer, rather than a continuum, approach to the deterministic theories of particle mechanics. Thus, we will formulate and study new models of classical physical phenomena from both Newtonian and special relativistic mechanics by use only of arithmetic. At those points where Newton, Leibniz, and Einstein found it necessary to apply the analytical power of the calculus, we shall, instead, apply the computational power of modern digital computers. Most interestingly, our definitions of energy and momentum will be identical to those of continuum mechanics, and we will establish the very same laws of conservation and symmetry. The unifying concept will be that of the potential. In addition, the simplicity of our approach will yield simple models of complex physical phenomena and solvable dynamical equations for both linear and nonlinear behavior. The price we pay for such mathematical simplicity is that we must do our arithmetic at high speeds.

For their help in the preparation of this manuscript, I wish to thank John Collier, Robert LaBudde, and Judy Swenson. For their permission to quote freely from my monograph DISCRETE MODELS (Addison-Wesley, Reading, Mass., 1973), I wish to thank the editors of Addison-Wesley.

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