Theories of Probability

An Examination of Foundations

TERRENCE L. FINE

ACADEMIC PRESS

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ACADEMIC PRESS 1973 New York and London

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ACADEMIC PRESS, INC.
111 Fifth Avenue, New York, New York 10003

United Kingdom Edition published by
ACADEMIC PRESS, INC. (LONDON) LTD.
24/28 Oval Road, London NW1

LIBRARY OF CONGRESS CATALOG CARD NUMBER: 72-84364

AMS (MOS) 1970 Subject Classifications: 60A05, 62A15, 62C05, 68A20

PRINTED IN THE UNITED STATES OF AMERICA

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Preface

My interest in the foundations of probability was aroused by difficulties I encountered when first faced with applying a conventional (Kolmogorov's axioms and a limit of a relative-frequency interpretation) theory of probability to actual electrical engineering problems. Efforts to understand, usefully formulate, and resolve the problems encountered in the design and analysis of inference and decision-making systems led, it now seems inexorably, to a study of the foundations of probability. As I gradually became aware of the issues and proposals that constitute the present subject of the foundations of probability, I found myself drawn to their consideration not only for the primary pragmatic reasons but also out of respect for the breadth, depth, and provocative originality of many of the contributions to this study. It is my hope that the reader will share this appreciation.

My aim is to address all who explicitly use some theory of probability but who may not be aware of the criticisms of their preferred theory or the claims of alternative theories. Many of the difficulties encountered by engineers, physical and social scientists, and philosophers are, perhaps, attributable to misapprehensions as to the nature of the concepts of probability on which they rely. It is expected that the reader has some knowledge of a particular theory of probability. The many well-written texts on probability discouraged the inclusion of enough material to make this book self-contained as an introduction to probability. It is recommended that proofs be ignored, except by those interested in research in specific areas. To underscore this, the proofs in Chapters II, V, and VII are appended to their respective chapters, and the few proofs in the other chapters are mainly worked into the discussion.

This book can at best describe aspects of the present stage in the nascent study of theories of probability. It should raise more questions than it answers. I will be well satisfied if the critical view of probability presented stimulates the reader to the thought and research necessary to lead us all eventually to a better understanding of the work so many of us are engaged in. It is my belief that an improved understanding of the foundations of probability will induce far reaching changes in engineering and scientific practice and not merely lead to an improved justification for what we presently do.

Some years ago I set out to write a survey paper on decision theory as practiced in electrical engineering. Feeling that some of the problems of decision theory could best be understood when referred to the problems with the underlying notion of probability, I wrote an introductory section on this issue. My dissatisfaction with each draft was only temporarily allayed by an expanded redraft. This book is in fact the still unsatisfactory introduction to that as yet unwritten paper. As to its publication, I take comfort in the words attributed to Cardinal Newman that "Nothing would be done at all if a man waited till he could do it so well that no one could find fault with it."

My great debt to the many contributors to the foundations of probability, and especially to A. N. Kolmogorov, R. von Mises, and L. J. Savage, is evident throughout the book. Less evident, but no less pervasive, is the influence of discussions I have had with Messrs. Max Black, Thomas M. Cover, Zoltan Domotor, Arthur Fine, Peter C. Fishburn, Michael A. Kaplan, R. Duncan Luce, Leonard J. Savage, Herbert Shank, Georg H. von Wright, and with my wife, Susan Woodward Fine.

Introduction

1A. Motivation

Formal uses for probability and its associated concepts are found in the construction of models of random phenomena, the design of inference and decision-making systems, statements and verifications of the applicability of scientific laws, and attempts to understand knowledge and induction. Informal uses for probability include the Butlerian view of probability as a guide to life and the frequent appearance of the words "probably" and "likely" in ordinary discourse. Notwithstanding the importance of probability in the explication of knowledge and induction, its roles in the verification of laws or "as a guide to life," or its prevalence in discourse, we leave the analyses of these issues to suitably trained philosophers. Our concern is primarily with those concepts of probability that are important for the modeling of random phenomena and the design of information-processing systems.

Methods for modeling the random phenomena of chance and uncertainty and the design of inference and decision-making systems are of great importance in fields as diverse as engineering and the physical and social sciences. In electrical engineering, areas such as communications, detection, pattern classification, and stochastic control owe their very formulation to concepts of probability theory. The fundamental work