Analytic Computational Complexity

Edited by

J.F. Traub

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PREFACE

These Proceedings contain texts of all invited papers presented at a Symposium on Analytic Computational Complexity held by the Computer Science Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania, on April 7-8, 1975. Abstracts of contributed papers are also included.

The decision to have a symposium in April, 1975 was made very informally. A number of the major international figures in analytic complexity planned to be at Carnegie-Mellon University for periods of time ranging from a month to a year. The intersection of these visits was in April. One easy way for the researchers to let each other know about their work was to have them make formal presentations. From there it was just a small step to inviting a few additional speakers and making it public. The proceedings seem a good way to show present progress and future directions in analytic complexity.

The research in the papers by R.P. Brent, B. Kacewicz, H.T. Kung, R. Meersman, J.F. Traub, and H. Woźniakowski was supported in part by the National Science Foundation under Grant GJ-32111 and the Office of Naval Research under Contract N00014-67-A-0314-0010, NR 044-422.

J.F. Traub

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INTRODUCTION

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I believe there has been more progress made in analytic computational complexity in the last two years than since the beginning of the subject around 1960. Perhaps this Symposium helped serve as a forcing function in this progress. In this introduction I would like to summarize what I believe are some of the reasons for studying complexity in general and analytic computational complexity in particular. Then I will briefly overview the invited papers which are presented in these Proceedings.

Some of the reasons for studying complexity (a partial list):

1. The selection of algorithms is a central issue in much of computer science and applied mathematics. The selection of algorithms is a multi-dimensional optimization problem. One of these dimensions is the complexity of the algorithms.

2. The literature contains countless papers giving conditions for the convergence of an infinite process. A process has to be more than convergent in order for it to be computationally interesting. We must also be able to bound (preferably a priori) its cost. One central issue of analytic computational complexity is what additional conditions must be imposed on a problem such that the cost of its solution can be a priori bound.

3. Complexity results help give structure to a field. For example we now know that the maximal order of an iterative process depends only on the information used by the iterative algorithm. We can therefore classify algorithms by the information they use.

4. Lower bounds on problem complexity give us a natural hierarchy based on the intrinsic difficulty of the problems.

5. Complexity leads to a mathematically interesting and satisfying theory. There seem to be numerous, deep questions.

I now turn to an overview of the papers presented in these proceedings.

Winograd

A general adversary principle is enunciated by Winograd and established as a primary technique for proving lower bounds. Winograd applies this principle and shows how lower bound results can be obtained in a number of problem areas.

Traub and Woźniakowski

An early and valid criticism of traditional iterative complexity theory has been that the theory is asymptotic whereas in practice only a finite and indeed often a small number of iterations are used. In this paper a non-asymptotic theory is developed with strict upper and lower bounds on complexity.

Kung

Iterative computational complexity has always been a local theory which assumes that a sufficiently good initial approximation is given and a solution is then calculated. Clearly, the right approach is: given an operator equation with certain properties, bound the complexity of finding the solution. Kung shows that if the operator satisfies conditions similar to those of the Newton-Kantorovich Theorem, then a starting approximation for Newton iteration which falls within the Kantorovich ball can be guaranteed and the complexity of the total process can be bounded.

Brent (Optimal-Order)

Algorithms for calculating zeros of a scalar function f are introduced for the case that f' is cheaper than f. The existence of algorithms of order 2ν which use one evaluation of f and ν evaluations of f' at each step is established. Meersman (these Proceedings) shows these algorithms have optimal order. Optimal non-linear Runge-Kutta methods are also defined.