AN
ACADEMIC PRESS
REPLICA REPRINT

RADIATION AND
PROPAGATION OF
ELECTROMAGNETIC WAVES

ELECTRICAL SCIENCE

A Series of Monographs and Texts

Edited by

Henry G. Booker
UNIVERSITY OF CALIFORNIA AT SAN DIEGO
LA JOLLA, CALIFORNIA

Nicholas DeClaris
UNIVERSITY OF MARYLAND
COLLEGE PARK, MARYLAND

JOSEPH E. ROWE. Nonlinear Electron-Wave Interaction Phenomena. 1965

MAX J. O. STRUTT. Semiconductor Devices: Volume I. Semiconductors and Semiconductor Diodes. 1966

AUSTIN BLAQUIERE. Nonlinear System Analysis. 1966

VICTOR RUMSEY. Frequency Independent Antennas. 1966

CHARLES K. BIRDSALL AND WILLIAM B. BRIDGES. Electron Dynamics of Diode Regions. 1966

A. D. KUZ'MIN AND A. E. SALOMONOVICH. Radioastronomical Methods of Antenna Measurements. 1966

CHARLES COOK AND MARVIN BERNFELD. Radar Signals: An Introduction to Theory and Application. 1967

J. W. CRISPIN, JR., AND K. M. SIEGEL (eds.). Methods of Radar Cross Section Analysis. 1968

GIUSEPPE BIORCI (ed.). Network and Switching Theory. 1968

ERNEST C. OKRESS (ed.). Microwave Power Engineering:
Volume 1. Generation, Transmission, Rectification. 1968
Volume 2. Applications. 1968

T. R. BASHKOW (ed.). Engineering Applications of Digital Computers. 1968

JULIUS T. TOU (ed.). Applied Automata Theory. 1968

ROBERT LYON-CAEN. Diodes, Transistors, and Integrated Circuits for Switching Systems. 1969

M. RONALD WOHLERS. Lumped and Distributed Passive Networks. 1969

MICHEL CUENOD AND ALLEN E. DURLING. A Discrete-Time Approach for System Analysis. 1969

K. KUROKAWA. An Introduction to the Theory of Microwave Circuits. 1969

H. K. MESSERLE. Energy Conversion Statics. 1969

GEORGE TYRAS. Radiation and Propagation of Electromagnetic Waves. 1969

GEORGES METZGER AND JEAN-PAUL VABRE. Transmission Lines with Pulse Excitation. 1969

C. L. SHENG. Threshold Logic. 1969

RADIATION AND PROPAGATION OF ELECTROMAGNETIC WAVES

George Tyras

Department of Electrical Engineering
Cullen College of Engineering
University of Houston
Houston, Texas

AN
ACADEMIC PRESS
REPLICA REPRINT

ACADEMIC PRESS 1969

A Subsidiary of Harcourt Brace Jovanovich, Publishers

New York London Toronto Sydney San Francisco

ACADEMIC PRESS, INC.
111 Fifth Avenue, New York, New York 10003

United Kingdom Edition published by
ACADEMIC PRESS, INC. (LONDON) LTD.
24/28 Oval Road, London NW1

LIBRARY OF CONGRESS CATALOG CARD NUMBER: 69-18341

PRINTED IN THE UNITED STATES OF AMERICA

81 82 9 8 7 6 5 4 3 2

To my children
Wanda Jean and Daryl George

This page intentionally left blank

CONTENTS

8. Green's Function Method of Solution

9. Axial Currents and Cylindrical Boundaries

10. Diffraction by Cylindrical Structures

11. Apertures on Cylindrical Structures

PREFACE

The period of the last twenty years has witnessed a great amount of research activity and many notable accomplishments both in the scope and in the understanding of the field of electromagnetics. Through the application of more modern and better mathematical methods it became possible to achieve a clearer exposition of the subject as well as a more systematic and logical treatment of various boundary value problems.

Unfortunately, not many of these accomplishments have found their way into the graduate teaching of the subject of electromagnetics in most schools. This phenomenon is due primarily to the fact that the development of modern textbooks on this subject has not, generally, kept pace with the research advances. Specialized monographs, devoted either to a single topic or to a special class of problems, have appeared from time to time. Such monographs are generally difficult to use in a classroom, especially if several of them are necessary in a single course.

The underlying philosophy of this graduate text is to narrow the existing gap between graduate teaching and modern research in electromagnetics. It is designed for a two-semester course in electrical engineering or electrophysics for students with only undergraduate preparation in electromagnetic theory. A practicing engineer should find this book useful for self-study as well as for a reference toward a better understanding of the current literature.

This book represents the author's viewpoint concerning the subject of an electrical engineer's graduate training in electromagnetic fields. A reader will find in this book a number of basic electromagnetic boundary value problems discussed in sufficient depth to enable subsequent serious work in more specialized branches of electromagnetic engineering, such as antennas, radiowave propagation, microwaves, and electrooptics. Among the topics covered here are the following: plane waves in anisotropic media, plane waves in inhomogeneous media, spectral representation of elementary sources, field of a dipole in a stratified medium, radiation in anisotropic plasma, axial currents and cylindrical boundaries, diffraction by cylindrical structures, and apertures on cylindrical structures.

To treat the subject in sufficient depth, it has been necessary to introduce, as the material demanded, certain mathematical techniques which, even though well known to the researchers in the field, are ordinarily not available in a proper form for classroom presentation. These techniques, which certainly are not beyond the grasp of an average graduate student of today, usually represent a major obstacle to the understanding of modern literature for those who have been exposed only to the "conventional" treatment of the subject as it is still being presented in many graduate schools today. The mathematical techniques used in the treatment of the various topics include the Fourier transform technique, Green's function and its spectral representations, contour integration, asymptotic expansions via the method of the steepest descents, the WKB method including the expansion about a turning point, the Lebedev-Kontorovich transform, and the Wiener–Hopf technique.

Clarity of presentation was the guiding thought in the writing of this book and originality was considered to be only of secondary importance. Consequently, some ideas have been borrowed from other books, monographs, and published papers. A list of references which most closely resemble the treatment of a particular topic is included with each chapter. A sufficient number of problems is also included at the end of each chapter. These problems are intended to serve the purpose of additional drill as well as wider applications of the mathematical techniques presented.

No special attempt has been made to make this book self-sufficient in its mathematical aspects. Because of the prevailing trends in graduate engineering education, it is reasonable to assume that either prior to or concurrently with taking the course for which this book is intended, the student has acquired mathematical competence at the level of, e.g., G. Arfken: "Mathematical Methods for Physicists" (Academic Press, 1966). In addition, the readily available "Handbook of Mathematical Functions," edited by M. Abramowitz and I. A. Stegun (Natl. Bur. Std. Appl. Math. Ser. 55), contains almost all of the mathematical formulas referred to in this text. In exceptional cases, when not readily available, the necessary mathematical formulas are developed as they become needed.

GEORGE TYRAS

Houston, Texas
August 1969

ACKNOWLEDGMENTS

Many contributors throughout the years have influenced my thinking concerning the material presented in the various sections of this book. In addition, many of my graduate students, former and present, who have struggled with me through the early editions of this material, by their enthusiasm and constructive criticism, have helped me to achieve a clearer exposition of the subject. To all of these people I wish to express my sincere thanks. Special thanks are due to Professor Leopold B. Felsen of the Polytechnic Institute of Brooklyn for his reading of the portions of the manuscript and his constructive criticism. To Professor Henry G. Booker of the University of California at San Diego, I wish to express my sincere appreciation for his advice and encouragement during the writing of the final draft. To Dean C. V. Kirkpatrick and Professor H. S. Hayre, Cullen College of Engineering, University of Houston, I wish to express my appreciation for lending me support and encouragement in the preparation of the final draft. The help of Mr. Richard G. Schell, who read the galley proof and corrected many errors, is gratefully acknowledged. Finally, very special thanks are due to my wife, Rosemary Louise, without whose help, understanding, and forbearance throughout the years, this book would never materialize.

This page intentionally left blank

RADIATION AND PROPAGATION OF ELECTROMAGNETIC WAVES

This page intentionally left blank

CHAPTER 1

ELECTROMAGNETIC THEORY

In this chapter we shall review the basic laws of electromagnetic theory, which were formulated in the early nineteenth century and combined by Maxwell in 1864 into a consistent set of equations. The reader is assumed to be familiar with the basic experiments that lead logically to the formulation of Maxwell's equations. To the best of our knowledge, Maxwell's equations correctly describe the large-scale (macroscopic) electromagnetic phenomena that occur in nature. Ample experimental evidence is available to support this view, including the classical experiments of Coulomb, Ampère, and Faraday, which provided the original motivation for Maxwell to postulate the concept of the "electromagnetic theory."

In what follows, we shall review the basic equations and derive a number of general formulas without specific reference to any physical problem. In the succeeding chapters, we shall undertake the task of solving specific boundary value problems in which these formulas will prove useful.

1.1 Maxwell's Equations

The electromagnetic fields arise from stationary or moving charges called currents. The electromagnetic theory is concerned with the average behavior of the moving charges, the effect of which can be measured. Thus, we are not concerned with the behavior of an individual charge, and when such a term is used in this book, it should be understood to mean an "average" behavior of an ensemble of charges.

The *current density* is defined by

$$\mathbf{J} = \rho \mathbf{v} \tag{1.1}$$

where ρ is the average charge density and $\mathbf{v}$ is the average velocity. The direction of the current density vector will coincide with the direction of the charge velocity if the current is produced by the motion of the positive charges, and it will be opposite to the velocity vector if the current is produced by the motion of electrons. The scalar quantity I, the *current*, is defined by

$$I = \int_S \mathbf{J} \cdot \mathbf{n}\, ds \tag{1.2}$$

where $\mathbf{n}$ is a unit vector normal to the surface S.

The electric field intensity vector $\mathbf{E}$ is defined as the force $\mathbf{F}$ acting on a stationary test charge $\mathbf{q}$ in the limit as the magnitude of the test charge becomes infinitesimally small so that it does not perturb the field; i.e.,

$$\mathbf{E} = \lim_{q \to 0} \mathbf{F}/q. \tag{1.3}$$

When a charge is in motion, there may be an additional force acting on it, even though the electric field $\mathbf{E} = 0$. This force is written as

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}. \tag{1.4}$$

Equation (1.4) may be considered as defining a new quantity, the *magnetic flux density* $\mathbf{B}$.

In addition to the two field vectors defined above, it will be convenient to define two more vectors, $\mathbf{D}$ and $\mathbf{H}$, as follows. The field vector $\mathbf{D}$, the *electric flux density*, will be thought of as a measure of the number of lines of force produced by a charge. The vector $\mathbf{H}$, the *magnetic intensity*, when measured on a closed contour, will relate the magnitude of the total current enclosed.

The basic postulate relating to the four defined vector field quantities is that they satisfy Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1.5a}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}, \tag{1.5b}$$

$$\nabla \cdot \mathbf{D} = \rho, \tag{1.5c}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{1.5d}$$

where all the necessary conditions for the continuity and existence of the derivatives at all points in space are tacitly implied.

The *equation of continuity*

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0 \tag{1.6}$$

is implied by Maxwell's equations. It can be derived by taking a divergence of (1.5b) and subsequently using (1.5c).

Maxwell's equations can also be written in an integral form. Using

Stokes's theorem

$$\int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, ds = \oint_C \mathbf{F} \cdot \boldsymbol{\tau}\, dc \tag{1.7}$$

where $\mathbf{n}$ is a unit vector normal to the area element ds of the surface S and $\boldsymbol{\tau}$ is a unit vector tangent to the element dc of the curve C, as shown in Fig. 1.1,

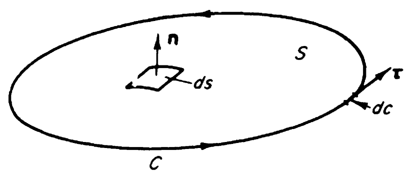

Fig. 1.1. Relevant relations for the application of Stokes's theorem.

the two curl equations in (1.5) can be written

$$\oint_C \mathbf{E} \cdot \boldsymbol{\tau}\, dc = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n}\, ds, \tag{1.8a}$$

$$\oint_C \mathbf{H} \cdot \boldsymbol{\tau}\, dc = I + \frac{\partial}{\partial t} \int_S \mathbf{D} \cdot \mathbf{n}\, ds \tag{1.8b}$$

where we also used the definition of the current in (1.2). The time derivative is taken outside the integral sign with the understanding that the surface S does not vary with time. Moreover, using the divergence (Gauss) theorem

$$\int_V \nabla \cdot \mathbf{F}\, dv = \oint_S \mathbf{F} \cdot \mathbf{n}\, ds \tag{1.9}$$

where S denotes a closed surface enclosing the volume V and $\mathbf{n}$ is a unit outward normal, as shown in Fig. 1.2, we obtain for the remaining two

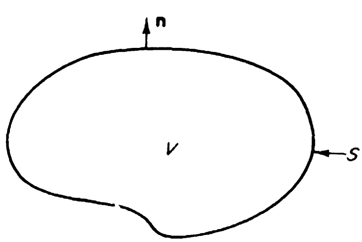

Fig. 1.2. Relevant relations for the application of the divergence (Gauss) theorem.

equations in (1.5) and (1.6)

$$\oint_S \mathbf{D} \cdot \mathbf{n}\, ds = \int_V \rho\, dv, \tag{1.10a}$$