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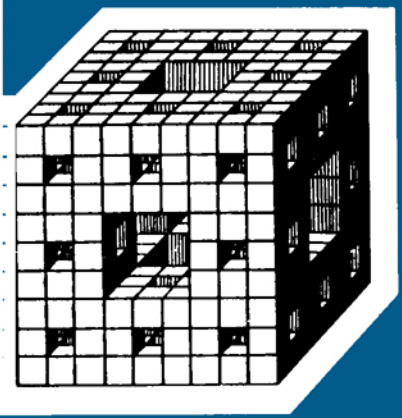
Stories about Sets

N.Ya.Vilenkin



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STORIES ABOUT SETS

N. Ya. Vilenkin

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Foreword

Professor Vilenkin has produced a small masterpiece which can be read with profit and delight by anybody, beginning with high school juniors and seniors. Slightly more than half of the book explores the notion of cardinality of sets and the remainder traces the evolution of some of the most important concepts of mathematics such as function, curve, surface, and dimension. The exposition combines informality with integrity of presentation and there is a wealth of unusual examples illustrating the paradoxical properties of curves and surfaces. It is safe to say that Professor Vilenkin's essay provides a royal road to the important concepts with which it is concerned.

A. SHENITZER

January, 1968
Adelphi University
Garden City, New York

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Preface

I first had occasion to hear of the theory of sets at a lecture conducted by I. M. Gel'fand for Moscow school children. He was then just beginning his teaching career, but is now a corresponding member of the Academy of Sciences of the USSR. During the course of two hours he told us about what seemed to us to be completely improbable things: that there are just as many natural numbers as there are rational numbers, and that there are just as many points in an interval as there are in a square.

My acquaintance with the theory of sets was further developed during my time as a student of mathematics and mechanics at the Moscow State University. In addition to the lectures and seminars, we had our own ways of learning, ways that our professors and lecturers probably did not even suspect. After class (and sometimes, I must confess, even during class, if the lecture was not especially interesting) we wandered through the corridors of the old building on Mokhovoi Street and discussed interesting problems, surprising examples, and clever proofs. In these conversations, for example, the first-year students learned from their more experienced fellows how to construct a curve which passes through all the points of a square, or how to find a function which has a derivative nowhere, and so forth.

Of course, the explanations given were, so to speak, "out of bounds," and it would be considered a mark of inexcusable frivolousness if you went to take an exam after having listened to these discussions. No, really, there was no talk of exams—according to the course of study, we would not be taking "real variables" for two more years. But, then, how this "corridor" preparation helped in taking exams and understanding lectures! For each of the theorems we could recall interesting problems which we had to solve earlier, perceptive juxtapositions, and intuitive examples.

I want to tell the reader about the theory of sets in the same way, in which I learned it, by following the "corridor" course of study. Thus, our attention will be focused mainly on giving clear presentations of problems, discussing unexpected or surprising examples, quite often giving contradictory "naive" discussions. We shall find that the theory of functions of a real variable is richly endowed with all these. And if, after he has read this book, a high-school or college student wants to study the theory of sets or the theory of functions of a real variable more deeply, the author will feel that his book has been a success.

Of the many standard presentations of these subjects, the following are recommended:

1. A. N. Kolmogorov and S. V. Fomin, "Measure, Lebesgue Integrals, and Hilbert Space" (Book II of "Elements of the Theory of Functions and Functional Analysis," translated by N. A. Brunswick and A. Jeffrey). Academic Press, New York, 1961.

2. I. P. Natanson, "Theory of Functions of a Real Variable," Vol. 1, edited by Leo F. Boron and Edwin Hewitt, 1955. Vol. 2, edited by Leo F. Boron, 1959. Ungar, New York.