The Fundamentals of Mathematical Analysis

Volume 1

G.M. Fikhtengol'ts

Translation edited by I. N. Sneddon, Professor of Mathematics in the University of Glasgow

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FOREWORD

THIS book is planned as a textbook of analysis for first and second year mathematics students at Russian universities and consequently is divided into two volumes. In compiling the book I have made extensive use of my three-volume *Course of Differential and Integral Calculus*, revising and abridging it in order to adapt it to the official mathematical analysis programme and to make it meet the requirements of a lecture course.

The tasks I set myself and the points by which I was guided are as follows:

1. First and foremost to provide a systematic and, as far as possible, rigorous treatment of the fundamentals of mathematical analysis. I consider it obligatory for the contents of a textbook to be presented in a logical sequence, in order to achieve a clearly defined and systematic presentation of the facts.

This does not, however, prevent the lecturer from deviating from a strict systematic approach, but, perhaps, even helps him in this respect. In my own lecture courses, for example, I usually put aside for a while such difficult tasks for beginners as the theory of real numbers, the principle of convergence or the properties of continuous functions.

2. To uphold my own opinion that a course of mathematical analysis should not appear to students to be merely a long chain of "definitions" and "theorems", but that it should also serve as a guide to action. Students must be taught to apply the theorems in practice in order to assist them in mastering the computational apparatus of analysis. Although this can be achieved largely with the help of exercises, I have also included some examples in my treatment of the theoretical material. The total number of these examples is, out of necessity, small, but they have been selected in such a way as to prepare students for conscientious work on the exercises.

3. It is well known that mathematical analysis has diverse and remarkable applications both in mathematics itself and in related scientific fields. Whilst students will realize this more and more as time passes, it is essential that they should learn and get used to the relationship of mathematical analysis with other mathematical sciences and with the requirements of practical work whilst studying the fundamentals of analysis. For this very reason I have provided, wherever possible, examples of the application of analysis not only to geometry, but also to mechanics, physics and engineering.

4. The problem of completing analytic work up to numerical results is of both theoretical and practical importance. Since an "exact" or "closed form" solution of a problem in analysis is possible in the simplest cases only, it is important to acquaint students with the use of approximate methods. Some attention has been given to this within the pages of this book.

5. By way of a brief explanation of my treatment of the subject matter, I have first of all considered the concept of a limit which plays the principal role among the fundamental concepts of analysis and which crops up in diverse forms literally throughout the entire course. Hence arises the problem of establishing a unified form of all variations of the limit. This is not only important from the viewpoint of principles but also vital from a practical standpoint, to obviate the necessity of having to construct the theory of limits anew each time it arises. There are two ways of achieving this aim: we can either immediately give the general definition of the limit of "directed variable" (following, for example, Shatunovskii and Moore, or Smith), or we can reduce every limit to the simplest case of the limit of a variable ranging over an enumerated sequence of values. The first alternative is difficult for beginners, and I have, therefore, chosen the second method of approaching the problem. The definition of each new limit is given first by means of the limit of a sequence and only later on "in ε–δ language".

6. To indicate a second feature of my treatment of the subject matter I have in Volume II, when speaking of curvilinear and surface integrals, emphasized the difference between the curvilinear and surface "integrals of first kind" (the exact counterparts of the

ordinary and double integral over unoriented domains) and similar "integrals of second kind" (where the analogy partly vanishes). Experience has convinced me that this distinction not only leads to a better understanding of the material, but is also convenient in applications.

7. As a short appendix to the book I have included a brief account of elliptic integrals and in several cases I have presented problems with solutions involving elliptic integrals. This may help to destroy the harmful illusion, acquired by merely solving simple problems, that the results of analytic calculations must necessarily be "elementary".

8. In various places throughout the book the reader will come across remarks of an historical nature. Moreover, Volume I ends with a chapter entitled, "Historical survey of the development of the fundamental concepts of mathematical analysis" and Volume II concludes with "An outline of further developments in mathematical analysis". However, neither of these two "surveys" has been introduced to serve as a substitute for a complete history of mathematical analysis, which students meet with later in general courses on "the history of mathematics". The first survey touches upon the origin of the concepts, whilst the final chapter in Volume II aims at providing the reader with at least a general *idea of the chronology* of the most important events in the history of analysis.

At this point, and in connection with the preceding paragraph, I should like to give a warning to potential readers of this book. The sequence in which I have treated various topics is closely connected with modern demands for strict mathematical rigour—demands which have become more and more acute over the years.

Historically speaking, therefore, the development of mathematical analysis has not been followed as closely as it might have been.

Thus, Chapter 1 is devoted to "real numbers", Chapter 3 to the "theory of limits", and it is not until Chapter 5 that I have commenced to give a systematic account of the differential and integral calculus. The historical sequence of events was, of course, the complete reverse. The differential and integral calculus were founded in the seventeenth century and developed in the eighteenth century, being applied to numerous important problems; the theory of limits became the foundation-stone of mathematical analysis at the beginning of the nineteenth century and only in the second half of the nineteenth century did a clearly defined concept of real numbers come into being, which justified the most refined propositions of the theory of limits.

This book summarizes many years of experience in lecturing on mathematical analysis in Leningrad University.

G. M. Fikhtengol'ts