Experimental Modelling in Engineering

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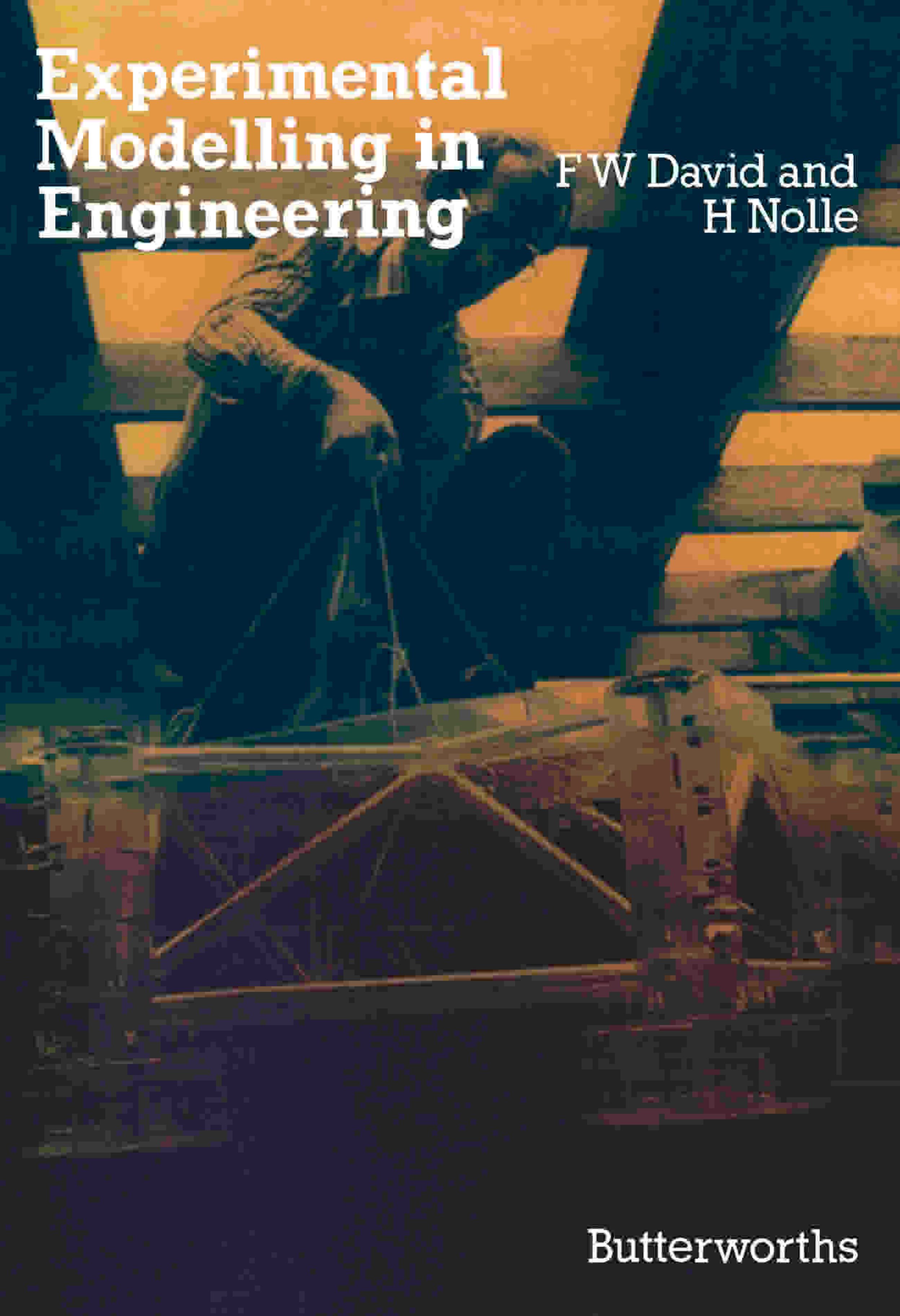

Butterworths

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The cover picture shows a 1 : 100 scale model of the hotel rig Alexander L Kielland, constructed from perspex. The model is designed to accurately represent the ballasting and buoyancy configurations of the actual vessel for the purposes of devising a method for uprighting the capsized vessel in Norway.

The method was implemented by the Structural Dynamics Group of Southampton at the end of 1980 in a salvage attempt that was later halted.

Tank testing was used extensively in devising a ballasting sequence and in the course of the actual operation in Norway. The model was also gimbal mounted and used as a training device for diver instruction.

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PREFACE

Experiments using models have been carried out for many years in a quest to discover and understand the behaviour and the properties of physical systems. Until the turn of this century, experimental technique followed largely an ad hoc procedure. This produced results of varying success, but usually the experiments were performed with narrow specific objectives in mind, and consequently the results would turn out to have very limited generality.

The origins of the more systematic approach to modelling and model testing may be traced back to scientific inquiry and the study of natural phenomena, particularly in the field of fluid flow. The practical advantages of modelling under these so called "conditions of similarity" were recognized soon after and put to use in a number of fields of engineering. From these origins the art and science of experimental modelling have continued to evolve and today form an integral part of practically all branches of Physics and Engineering.

Fourier, in the early part of the 19th century, and Rayleigh some 50 years later were the first to formally establish the principles for similarity of physical systems, conditions which are now derived by the well-known process called "dimensional analysis". On these foundations further theoretical refinements have been attempted, from time to time; however, these contributions, mainly of an abstract nature, have not added substantially to the existing knowledge. On the other hand, the basic theories of dimensional analysis and similarity have found ever increasing application to cover the needs of the fast developing engineering activities in two ways: firstly, they have been applied to experimental techniques, and secondly, they have been used to assist in analytical investigations of physical phenomena generally.

The history of scientific records reveals that the seed for the fundamental theorem for similarity —— the now well-known π-theorem —— was sown by a Frenchman A. Vaschy in about 1896. This Theorem was subsequently re-defined by a number of other writers, but it was not till Buckingham's formulation was published in 1914 that the Theorem and its practical significance became more widely known and accepted by the engineering community. Since then a great deal of literature has been accumulated, mostly concerned with specialist's analyses and procedures, and with some theoretical formulations. Among the latter, two concepts have been advanced that have, however, very limited, if any, practical application. They are the *directional* or *vector* properties of variables and the *dual* role of mass. Both concepts are briefly discussed in this text, and some worked examples of the first are used to illustrate the severe limitations of its usefulness for experimental modelling.

In this volume the principles of experimental modelling are presented methodically and in such a generalized way that they may lend themselves to application in practically all fields of technology.

Though only the basic concepts and procedures are developed, these are copiously illustrated by examples taken from a wide field of practical applications. By careful selection, fully worked examples from disciplines in which modelling is well established —— a 'tradition', so to speak —— are balanced by others taken from areas of technology in which the modelling technique is relatively new, yet potentially powerful, e.g. in the performance prediction of all-terrain vehicle-soil systems.

Chapters 1 to 4 present a compact yet comprehensive review of ideas pertaining to physical units and dimensionality and the theory of non-dimensional formulations. Commencing in Chapters 5 and 6 a range of commonly encountered questions and problems are examined, arising in the course of dimensional analysis and during the early stages of model design. In Chapter 7 the fundamental principle of similarity of physical systems is defined, and a number of essential concepts are discussed: these are the 'scaling factor', 'homology' and the 'characteristic variable'. Further, in Chapter 7, a set of guidelines to procedure in modelling are developed. These include also the modelling of possible input functions; compatibility conditions for them are examined which will ensure that the linear relations in homology are preserved. Further, the subject of "distortion" is considered and particular care is taken in defining the meaning and significance of model *distortion*, a term often rather loosely used and much abused. The concept of distortion is illustrated by detailed examples. The Chapter concludes with a treatise of the commonly encountered similarity conditions which provide a foundation knowledge to most of the modelling problems in engineering practice.

Scale effects —— the errors and uncertainties arising from scale-dependence of physical interactions, or from incomplete knowledge or understanding of the phenomenon of the system under investigation —— is the final topic in the theory of modelling, and is presented in Chapter 8.

In Chapter 9, theory and learning from past experiences have been applied in a series of fully worked cases involving model testing. These are taken from recently published technical literature, and all have been chosen for their general interest in their respective fields. In most instances the reader is shown experimental data which are used to assess the model behaviour and relevant scaling laws. Where appropriate, the results are subjected to a critical appraisal, and limitations of their applicability are pointed out. In a sense, the examples constitute precedents in modelling and experimental procedure, and as such should be of particular interest to those engaged in engineering and the applied sciences. But the practical bounds of the modelling techniques are determined to a large extent by the degree of competence, experience and imagination of the experimenter, and it is hoped that the selected range of applications in this Chapter will serve also as a guide in respect of procedural matters and as a source of inspiration and ideas for new and as yet unexplored problems.

Chapter 10 provides further material for exercises of varying difficulty, covering a wide field of technological and physical situations.

The modelling conditions for input functions discussed in Chapter 7 are dealt with in greater detail in Chapter 12; as well, further information is given on some fundamental relationships between physical quantities used in electromagnetic systems, and some definitions of such physical quantities.

In Chapter 13, solutions to the exercise problems have been given, with accompanying comment, where appropriate, to enhance the teaching value of the exercise.

The practice of experimental modelling continues to gain increasing acceptance at all levels of research, development and industrial design activity; and new applications for the technique are developed to this day. It would be outside the objectives of this fundamental treatise to provide a general listing of original works dealing with modelling and model testing. Such references number many hundreds and have already been subject to comprehensive literature searches and listings of titles in many specialist areas of technology. At the conclusion of this book the reader is referred to the various sources where such listings appear. The short list of journal titles represents a selection which usually carries articles on the subject of modelling, and their latest issues should provide a useful starting point in a search of the most up to date publications.

In the numerous footnotes references are given to papers from which the various examples in the text were evolved. These papers usually include a bibliography that would be of interest to readers who wish to study in greater detail some of the special problems under discussion.

The origin of this book stems from a lecture series presented at Monash University as part of a final year undergraduate unit called Engineering Morphology. In these lectures, the principles of similarity were established, to serve as a foundation for a broader study of 'shape' and 'form' in Nature and in man-made systems, and the forces that have influenced their evolution towards optimality —— and, not infrequently, beyond optimality and onwards to the final stage in the evolutionary cycle: extinction. In the course of these developments, the authors were inspired by the classic work of D'Arcy Thompson, "On Growth and Form"*, which is as fresh today as it was in 1917 when it was first published.

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* Cambridge University Press, 1966.

From there, the contents of some of the lecture notes themselves evolved into deeper, more specialized study of systems encountered specifically in Physics and Engineering.

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This text is written for undergraduate and graduate students in Engineering or in the Sciences, and for practising professionals who wish to familiarize themselves in some depth with the theory and application of experimental modelling techniques, or who wish to refresh their knowledge in these areas. The reader is expected to possess some knowledge of applied mathematics and a working knowledge of Physics or Applied Mechanics usually offered in the first year courses of tertiary institutions.

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