EXAMPLES OF THE Design of Reinforced Concrete Buildings TO BS 8110

FOURTH EDITION

Charles E. Reynolds and
James C. Steedman

Spon Press
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Preface

Like its predecessor, this edition of what is often colloquially known as the *Examples* book has been delayed repeatedly during the past few years. As with its sister publication, the *Reinforced Concrete Designer's Handbook*, since it last appeared it has passed through the ownership of two publishers before coming to rest in the safe hands of Spon.

Once again, sincere thanks must go to two groups of people. Firstly, I am grateful to the many editorial and production staff at E. & F. N. Spon, who are equally involved with myself in the production but whose names do not appear in the 'credits'. And secondly, I would like to thank Freda Reynolds and the other members of her family for their continued encouragement and support. All of us hope that this edition will prove as useful to those designers for whom it is intended as did its predecessors.

The present edition follows the same plan as earlier editions. Part One describes the various British Standard and Code of Practice requirements relating to the design of various parts of reinforced concrete buildings. Part Two consists of drawings and calculations for a reasonably typical six-storey framed building. Most of the dimensions of this structure are the same as of that designed to meet the requirements of CP110 in the previous edition, and the building is as close as possible to the previous imperial design. This affords an interesting and sometimes enlightening, although of course seldom typical, comparison between the designs that result from the use of BS8110 and those based on its predecessors, although it is becoming clear that more radical changes in the examples provided will be needed in the future.

Because of the differing nature of the two books, a far larger proportion of the present work has to be rewritten each time a new edition is prepared than is the case with subsequent editions of the *Reinforced Concrete Designer's Handbook*. Nevertheless, I have taken care to retain from the previous editions all Charles Reynolds's ideas and helpful advice where these are still relevant, while updating and supplementing this information where necessary.

Although it would be gratifying to discover otherwise, experience with both this and the *Reinforced Concrete Designer's Handbook* has shown that it is a practical impossibility entirely to eliminate errors in books of this nature, however hard one tries. I would therefore like to take this opportunity to apologize for such mistakes and to thank those readers who take the trouble to write and point them out. It is such people who help to make this book and its companion, the *Handbook*, the useful reference works that they are.

J.C.S.
Upper Beeding

The authors

Charles Edward Reynolds was born in 1900 and educated at Tiffin Boys School, Kingston upon Thames, and Battersea Polytechnic. After some years with Sir William Arroll, BRC and Simon Carves, he joined Leslie Turner and Partners, and later C. W. Glover and Partners. He was for some years Technical Editor of Concrete Publications Ltd and later became its Managing Editor, combining this post with private practice. In addition to the *Reinforced Concrete Designer's Handbook*, of which well over 150 000 copies have been sold since it first appeared in 1932, Charles Reynolds was the author of numerous other books, papers and articles concerning concrete and allied subjects. Among his various appointments, he served on the council of the Junior Institution of Engineers and was the Honorary Editor of its journal at his death on Christmas Day 1971.

The current author of *Examples of the Design of Reinforced Concrete Buildings to BS8110*, James Cyril Steedman, was educated at Varndean Grammar School and was first employed by British Rail, whom he joined in 1950 at the age of 16. In 1956 he commenced working for GKN Reinforcements Ltd and later moved to Malcolm Glover and Partners. His association with Charles Reynolds commenced when, following the publication of numerous articles in the magazine *Concrete and Constructional Engineering*, he took up an appointment as Technical Editor of Concrete Publications Ltd in 1961, a post he held for seven years. Since that time he has been engaged in private practice, combining work for the Publications Division of the Cement and Concrete Association with his own writing and other activities. In 1981 he established Jacys Computing Services, an organization specializing in the development of microcomputer software for reinforced concrete design, and much of his time since then has been devoted to this project. He is also the joint author, with Charles Reynolds, of the *Reinforced Concrete Designer's Handbook*.

Introduction to fourth edition

The data, calculations and designs in this edition conform to the recommendations of the current British Standard codes of practice for reinforced concrete, and in particular to Parts 1 to 3 of BS8110:1985 'Structural use of concrete', BS6399 'Design loading for building. Part 1: Code of practice for dead and imposed loads: 1984' and CP3 'Functional requirements of buildings. Chapter V: Loading. Part 2: Wind loads: 1972'. These codes have been prepared by the British Standards Institution and the writers thank the Institution for permission to refer in detail to their contents.

In Part One the recommendations of these codes and other supplementary data relating to the design of buildings are considered in the same sequence as in design calculations: namely, loads, bending moments, design strengths, resistance to ultimate limit-state, resistance to shearing and torsional forces, bond and serviceability limit-states. Structural parts are then considered in the order in which design proceeds: namely, slabs, beams, columns and load-bearing walls, stairs, basements and foundations. The overall stability of the structure is then considered. The final chapter in this part is devoted to a consideration of the fire resistance of reinforced concrete in general and in relation to the building in Part Two in particular.

The application of the recommendations of the codes is illustrated by designing the principal parts of a reinforced concrete building, many of the calculations and drawings for which are given in Part Two. Although the codes are here applied to a specific structure, the data and comments in Part One and the design procedure in Part Two are of general application. The writers' interpretation of the intention of the recommendations is given where there may be ambiguity; no doubt usage will in time eliminate uncertainties, and precedents will be established for matters now left to the discretion of the designer.

The building in Part Two has been planned to incorporate as many as possible of the matters dealt with by the codes. Although the structural design complies with the codes, the general planning of the building may not necessarily comply with the bylaws of all local authorities or with other mandatory regulations. Alternative designs are given for some structural parts of buildings: namely, floors of slab-and-beam construction, flat slabs, hollow-tile slabs and precast slabs, columns with and without taking into account the effects of wind, and so on. Alternative designs of column bases are also indicated, as well as some designs for other simple types of foundations not necessarily related to the building in Part Two.

Note that current UK practice does not favour the use of inclined bars to resist shear or to provide top steel over supports, bars with hooked or bobbed ends, and so on. However, the use of these techniques is still discussed in detail and illustrated in the example forming Part Two of this book, and elsewhere. This is done because the adoption of such procedures still remains valid in situations where different economic circumstances prevail.

Readers who knew the original editions of this book will observe that this edition includes many more charts, graphs and similar design aids than its predecessors. Although many of the aids contained in the current edition of the *Reinforced Concrete Designer's Handbook* were specially devised to facilitate rapid design according to BS8110, for various reasons it was not possible to include all the design aids that were desirable. Therefore, in preparing the present book, the opportunity has been taken to incorporate as many of these as possible. For example, more than twenty charts are provided to simplify the determination of deflections by the rigorous analytical procedure set out in Part 2 of BS8110.

It should be emphasized that the information and material provided on the following pages are intended to supplement and not to supplant that given in companion publications dealing with the same subject. In addition to the *Reinforced Concrete Designer's Handbook* by C. E. Reynolds and J. C. Steedman, these publications include the *Handbook to British Standard BS8110:1985* by R. E. Rowe *et al.*, and *Reinforced Concrete Design to BS8110 – Simply Explained* by A. H. Allen. For brevity these books, which are mentioned frequently on later pages, are there referred to as *RCDH*, *Code Handbook* and *Allen* respectively.

Where, to solve a particular problem, it has been possible to devise alternative but equally valid graphical aids to those that are given in the *Reinforced Concrete Designer's Handbook*, this has been done. As with methods of structural analysis, often one method will appeal more to one designer and another more to the next. However, in the few cases where one form of chart or method appears distinctly superior to its rivals it is included here, even if this has meant reproducing a limited amount of material in a near-identical form to that in the *Reinforced Concrete Designer's Handbook*. Thus, while access to the above-mentioned publications (particularly the *Reinforced Concrete Designer's Handbook*) is desirable, it is certainly not essential, as the present book is self-contained. Reference to a copy of Part 1 of BS8110 itself is, however, important. (Note that specific table references to the *Reinforced Concrete Designer's Handbook* refer to the tenth edition; these numbers may not necessarily remain unchanged in subsequent editions.)

BS8110 permits two different bases to be employed when designing reinforced concrete sections at the ultimate limit-state. Of these two rigorous methods, design charts corresponding to that requiring the use of a so-called parabolic-rectangular concrete stress-block form Part 3 of BS8110. The other rigorous method involves the employment of a uniform rectangular concrete stress-block, and some design formulae based on this assumption are provided in Part 1 of the Code. The basis of both methods and the derivation of these formulae are discussed in Chapters 5 and 14, and the design of slab, beam and column sections in the calculations in Part II and elsewhere in this book is undertaken using design charts based on the uniform rectangular concrete stress-block. Owing to limitations on space, only a single design chart is provided for the design of beam and slab sections, but it is hoped to publish a comprehensive series of charts separately soon.

In accordance with the recommendations of BS3921 'Clay bricks and blocks', the format (i.e. the so-called standard designated size) of a metric brick (including the joint) is taken as 225 mm by 112.5 mm by 75 mm. Where dimensions are controlled by brick widths, this has unfortunately led to the need to introduce such cumbersome dimensions as 5.3375 m, for example, but it was thought that rounding such values to the nearest millimetre might obscure their derivation.

Notation

The notation employed in this book is based on that used in BS8110. This in turn takes as its basis the internationally agreed procedure for preparing notations produced by the European Concrete Committee (CEB) and the American Concrete Institute, which was approved at the 14th biennial meeting of the CEB in 1971 and was outlined in Appendix F of CP110. In the following list, terms specifically defined and used in BS8110 are indicated in bold type. Only the principal symbols are listed here; all others are defined where they appear.

A	Area
$\boldsymbol{A_c}$	Area of concrete
$\boldsymbol{A_s}$	Area of tension reinforcement
$\boldsymbol{A'_s}$	Area of compression reinforcement
A'_{s1}	Area of reinforcement near more highly compressed face
A_{s2}	Area of reinforcement near less highly compressed face
$\boldsymbol{A_{sc}}$	Total area of longitudinal reinforcement (in columns)
A_{sl}	Area of longitudinal reinforcement provided for torsion
$\boldsymbol{A_{s\,req}}$	Area of tension reinforcement required
$\boldsymbol{A_{sv}}$	Cross-sectional area of two legs of link reinforcement
a	Dimension (as defined); deflection
$\boldsymbol{a_b}$	Distance between bars
$\boldsymbol{a_{cr}}$	Distance between point at which crack width is evaluated and face of nearest longitudinal bar
$\boldsymbol{a_u}$	Deflection of column at ultimate limit-state
$\boldsymbol{a'}$	Distance between compression face and point at which crack width is evaluated
$\boldsymbol{b}$	Width of section; dimension (as defined)
$\boldsymbol{b_e}$	Effective breadth of strip of flat slab transferring moment to column
$\boldsymbol{b_t}$	Breadth of section at level of tension reinforcement
$\boldsymbol{b_w}$	Breadth of web or rib of member
$\boldsymbol{C}$	Torsional constant
C_f	Force coefficient when evaluating wind loading
C_{pe}	External pressure coefficient when evaluating wind loading
C_{pi}	Internal force coefficient when evaluating wind loading
c	Cover to reinforcement; column width
$\boldsymbol{c_{min}}$	Minimum cover to reinforcement
$\boldsymbol{d}$	Effective depth to tension reinforcement
$\boldsymbol{d'}$	Depth to compression reinforcement
d_{min}	Minimum effective depth that may be provided
$\boldsymbol{E_c}$	Short-term modulus of elasticity of concrete
$\boldsymbol{E_n}$	Nominal earth load
$\boldsymbol{E_s}$	Modulus of elasticity of steel
$\boldsymbol{e}$	Eccentricity
$\boldsymbol{e_a}$	Additional eccentricity due to deflection of column
$\boldsymbol{e_x}$	Resultant eccentricity of load at right angles to plane of wall
$\boldsymbol{e_{x1}}$	Resultant eccentricity calculated at top of wall
$\boldsymbol{e_{x2}}$	Resultant eccentricity calculated at bottom of wall
$\boldsymbol{F}$	Total design ultimate load
FEM	Fixed-end moment
F_s	Force due to ultimate load in bar or group of bars
$\boldsymbol{f_{bu}}$	Anchorage-bond stress due to ultimate load
f_c	Actual compressive stress in concrete (deflection analysis)
f_{ct}	Tensile stress in concrete at centroid of tension steel (deflection analysis)
$\boldsymbol{f_{cu}}$	Characteristic cube strength of concrete
$\boldsymbol{f_s}$	Service stress in reinforcement
$\boldsymbol{f_y}$	Characteristic strength of reinforcement
f_{yd}	Maximum design stress in reinforcement
f_{yd1}	Actual design stress in compression reinforcement
f_{yd2}	Actual design stress in tension reinforcement
f_{yl}	Characteristic strength of longitudinal torsional reinforcement
$\boldsymbol{f_{yv}}$	Characteristic strength of shearing reinforcement
$\boldsymbol{G}$	Shear modulus
$\boldsymbol{G_k}$	Characteristic dead load
g	Distributed dead load
$\boldsymbol{g_k}$	Characteristic dead load per unit area

h	Overall depth or diameter of section
h_c	Diameter of column head in flat-slab design
h_f	Thickness of flange
I	Second moment of area
I_e	Transformed second moment of area of cracked section (in concrete units)
I_g	Transformed second moment of area of uncracked section (in concrete units)
K	A constant; stiffness of member
K_u	Link-reinforcement factor
k	A constant
k_1, k_2, k_3	Factors determining shape of parabolic-rectangular concrete stress-block
l	Span
l_e	Effective span or effective height of member
l_{ex}	Effective height for bending about major axis
l_{ey}	Effective height for bending about major axis
l_0	Clear height of column between end restraints
l_x	Length of shorter side of rectangular slab
l_y	Length of longer side of rectangular slab
l_1	Length of flat-slab panel in direction of span measured between column centres
l_2	Width of flat-slab panel measured between column centres
M	Bending moment due to ultimate loads
M_{ds}	Design bending moments in flat slabs
M_i	Maximum initial moment in column due to ultimate load
M_{ix}	Initial moment about major axis of slender column due to ultimate load
M_{iy}	Initial moment about minor axis of slender column due to ultimate load
M_{sx}, M_{sy}	Bending moments at midspan on strips of unit width and of spans l_x and l_y respectively
M_t	Total moment on column due to ultimate load
M_{tx}	Total moment about major axis of slender column due to ultimate load
M_{ty}	Total moment about minor axis of slender column due to ultimate load
M_u	Design ultimate moment of resistance of section
M_{ux}	Maximum moment capacity of short column under action of ultimate load N and bending about major axis only
M_{uy}	Maximum moment capacity of short column under action of ultimate load N and bending about minor axis only
M_x, M_y	Moments about major and minor axes of short column due to ultimate load
M_1	Smaller initial end moment in column due to ultimate load
M_2	Larger initial end moment in column due to ultimate load
N	Ultimate axial load
N_{bal}	Ultimate axial load giving rise to balanced condition in column
N_{uz}	Ultimate resistance of section to pure axial load
n	Total distributed ultimate load per unit area ($= 1.4g_k + 1.6q_k$)
n_0	Number of storeys
Q_k	Characteristic imposed load
q	Distributed imposed load
q_k	Characteristic imposed load per unit area
r	Radius; internal radius of bend of bar
$1/r_{cs}$	Curvature due to shrinkage
$1/r_{ip}$	Instantaneous curvature due to permanent load
$1/r_{it}$	Instantaneous curvature due to total load
$1/r_{tp}$	Long-term curvature due to permanent load
$1/r_{tt}$	Long-term curvature due to total load
s_b	Spacing of bars
s_v	Spacing of links
S_1, S_2, S_3	Non-dimensional factors for evaluating wind loading
T	Torsional moment due to ultimate loads
u	Effective length of shear perimeter
u_{crit}	Length of critical perimeter
u_s	Effective perimeter of reinforcing bar
V	Design shearing force due to ultimate loads; basic wind speed
V_b	Total shearing resistance provided by inclined bars
V_s	Characteristic wind speed
v	Shearing stress on section due to ultimate loads
v_c	Ultimate shearing resistance per unit area provided by concrete alone
v_{max}	Limiting ultimate shearing resistance per unit area when shearing reinforcement is provided
v_t	Shearing stress due to torsion
$v_{t\,min}$	Ultimate torsional resistance per unit area provided by concrete alone
v_{tu}	Limiting ultimate combined resistance (i.e. shear torsion) per unit area when torsional reinforcement is provided
W_k	Characteristic wind load
w_k	Characteristic wind load per unit area
x	Depth to neutral axis
x_1	Lesser dimension of a link
y_1	Greater dimension of a link
z	Lever-arm
$\alpha, \beta, \zeta, \psi$	Factors or coefficients
α_e	Modular ratio (for serviceability calculations)
γ_f	Partial safety factor for loads

γ_m	Partial safety factor for materials
ϵ_{cs}	Free shrinkage strain in concrete
ε_h	Average surface strain at tension face (crack-width analysis)
ε_m	Adjusted surface strain (crack-width analysis)
ε_{mh}	Adjusted average surface strain at tension face (crack-width analysis)
ϵ_1	Strain at level considered (crack-width analysis)
ϱ	Proportion of tension reinforcement ($= A_s/bd$)
ϱ'	Proportion of compression reinforcement ($= A_s'/bd$)
ϱ_1	Proportion of total reinforcement in terms of gross section ($= A_s/bd$ or A_{sc}/bh)
ϕ	Bar size; continuity factor for precise moment distribution; creep coefficient
θ	Angle

Part One

Design of Reinforced Concrete Buildings

Chapter 1
Introduction to limit-state theory

More than twenty years have elapsed since the appearance of the preliminary version of CP110, the first Code of Practice for concrete wholly based on limit-state principles, caused shock waves to pass through the structural engineering profession. Yet for the generation of designers who have come into the profession since 1969 it is probably difficult to comprehend what all the fuss was about, since the principle on which CP110 was conceived, the so-called limit-state method of design, is in many respects simply a reworking and extension of principles that had already been embodied in such codes for more than thirty years. As early as 1924 George Manning (ref. 1) had suggested that, because of the discrepancies between the behaviour predicted by elastic analysis and that occurring in practice, the only logical theory to employ for reinforced concrete design was one based on the conditions existing in an actual structure when it had just reached its ultimate load. In 1934 the Code of Practice published by the Department of Scientific and Industrial Research (DSIR) permitted axially loaded columns to be designed by summing the individual resistances of the concrete and the reinforcement (i.e. ignoring the differences between the strains in the two adjacent materials). However, to conform to the basis adopted in the rest of the document, suitable factors of safety were incorporated solely by specifying low permissible material stresses, rather than working on actual ultimate loads and strengths.

A further step in this direction came with the appearance of the 1957 version of CP114 (the predecessor to CP110), where the concept of load-factor design was specifically stated and now extended to beams, slabs and eccentrically loaded columns. However, as before, design by elastic-strain (i.e. modular-ratio) principles was permitted in the same Code. Therefore it was thought necessary, to avoid any possibility of confusion arising due to the use of working (i.e. service) loads and strengths, and ultimate loads and strengths in the same document, to modify the load-factor method in such a way that the calculations were undertaken in terms of working loads and stresses. Unfortunately such an approach has led to some confusion in the minds of those using the Code as to exactly what their calculations were predicting.

The implementation of the limit-state design method presented in CP110 avoids such confusion. In addition it extends the logic of load-factor design, by permitting the relative uncertainty by which each individual type of load and material strength can be assessed to be considered individually, instead of needing to adopt a single global factor of safety to cater for all the possible uncertainties. As pointed out in the introduction to BS8110, an immediately apparent advantage of such a procedure occurs when a critical situation is brought about by a combination of loads, such that one load is at its maximum while the other is at a minimum. This happens, for example, where vertical load on a frame is combined with lateral wind forces. In such a case the greatest likelihood of overturning is when the least vertical load is combined with the greatest wind force. However, the use of a single global loading factor causes both loads to be increased.

Since CP110 was first published in 1972, there has been a gradual acceptance by the majority of engineers of the principles embodied in this document. When BS8110 appeared in 1985, it contained no basic changes in principle from its predecessor, although many minor modifications were introduced and it was considerably rewritten. BS8110 states that the redrafting and alterations were made in the light of experience of the practical convenience in using CP110, and that they were also undertaken to meet the criticisms of engineers preferring the form of CP114. Although going some way to achieve this aim, the rewording and rearrangement have sometimes introduced confusion as to whether a particular change is merely cosmetic or whether it indicates a definite change of policy.

Fortunately, there are two publications that help to resolve some of these doubts. As with CP110, the authors of the Code have produced a *Handbook to British Standard BS8110:1985*, which explains in detail the basis of many Code requirements; on later pages this is referred to as the *Code Handbook* for brevity. In addition Arthur Allen, who has for many years lectured on Cement and Concrete Association design courses dealing with CP110 and BS8110 and who has had long and detailed discussions with the BS8110 authors, has produced an invaluable book entitled *Reinforced Concrete Design to BS8110 – Simply Explained*, to which the present author is greatly indebted; it is referred to here as *Allen*.

Two other publications should be mentioned. In October 1985 a joint committee formed by the Institutions of Civil and Structural Engineers published the *Manual for the Design of Reinforced Concrete Building Structures*, which deals with those aspects of BS8110 of chief interest to reinforced concrete designers and detailers. The advice contained in this document, which generally but not always corresponds to the Code requirements, is presented concisely in a different form from that in BS8110 (and one clearly favoured by many designers); elsewhere in this book it is referred to as the *Joint Institutions Design Manual*. The *Standard Method of Detailing Structural Concrete* (ref. 2) is the product of another joint committee, this time of the Institution of Structural Engineers and the Concrete Society, and the drawings for the example that forms Part Two of this book have been prepared in accordance with the proposals put forward in this important publication.

In accordance with the current policy of the British Standards Institution, the present document BS8110:1985 is designated as a British Standard whereas its predecessor CP110 was a BS Code of Practice; however, this is not intended to indicate any change of status. The formal subtitles of Parts 1 and 2 of BS8110 are 'Code of practice for design and construction' and Code of practice for special circumstances', respectively.

Although many engineers have accepted the limit-state design philosophy, a vociferous body known as the Campaign for Practical Codes of Practice (CPCP) has fought long and hard to retain a revised version of CP114 as an alternative document based on permissible-stress principles and having similar status to BS8110. This group instigated a referendum of members of the Institution of Structural Engineers in 1987 and successfully obtained support for its proposal that the Institution should produce a draft document for public comment on its value as a code of practice. The final document (*Recommendations for the Permissible-Stress Design of Reinforced Concrete Building Structures*: ref. 3) has now been published.

At the same time, Part 1 of Eurocode 2 (EC2) for concrete structures has been made available in draft form to enable member states of the CEB (European Concrete Committee) to familiarize themselves with its requirements. Like BS8110, this is firmly rooted in limit-state principles. The current aim is to publish the document as an ENV (or Euronorm) during 1991. Such a document would then have an equivalent status to a BSI Draft for Development, with validity for three years and the possibility of an extension for a further two. By such time (the mid 1990s) it will have been reassessed and revised for fully operational use as a possible replacement for BS8110. For further information, see ref. 4.

1.1 LIMIT-STATE DESIGN

The limit-state concept is the rational outcome of a rethinking of the fundamental purpose of structural design, which is to produce structures that are safe, serviceable and economic (ref. 5). When used correctly the method gives a clearer idea than previous design procedures of the actual factors of safety employed, and enables these to be adjusted to cater for the degrees of uncertainty involved in the analysis and the seriousness of any resulting failure, taking account of variations that may occur in the loadings and material strengths, and inadequacies in the analytical methods and qualities of construction. The aim is to produce a structure that will not become unfit for its intended purpose during its planned lifetime.

A structure will become unfit for use, of course, if part or all of it collapses, but it will also become unfit if it deflects too much, if large cracks form, or if vibration is so great that discomfort or alarm is caused to the occupants or the operation of machinery is interfered with. Similar factors that affect a structure are fatigue, lack of durability and so on. Such conditions which cause unfitness are classified as *limit-states*; the limit-state at which collapse occurs is known as the ultimate limit-state. The three principal considerations that together constitute the limit-states of serviceability are the prevention of excessive deflection, the prevention of excessive cracking and the prevention of excessive vibration. Special types of structure may also be subject to additional limit-states.

As well as the foregoing limit-states, other phenomena may require consideration during structural design. For example, for a structure such as a machine foundation that is subjected to cyclic loading, the effects of fatigue on the materials may require consideration. Although the Code prescriptions are designed to meet normal durability requirements, it may also be necessary to take additional measures to combat exceptional exposure conditions such as those encountered when substances that are injurious to concrete are to be stored. In such cases reference should be made to specialist literature. A further consideration is that of fire resistance; details of the Code requirements in this respect are set out in Chapter 19. The robustness and stability of the entire structure must also be considered: see Chapter 18.

1.2 CHARACTERISTIC LOADS AND STRENGTHS

Limit-state design is carried out in terms of characteristic loads and characteristic strengths of materials. In theory, a characteristic load is obtained by adding to the mean load the product of the standard deviation from the mean and a factor K. The value of K is chosen to ensure that the probability of the characteristic load actually being exceeded is remote. In practice, however, it is not yet possible to specify dead and imposed loads in such statistical terms, and therefore BS8110 states that such loads should be taken as the dead load and imposed load, respectively, defined in Part 1 of BS6399. In the case of wind loads, however, the values given in Part 2 of CP3, Chapter V, incorporate a multiplying factor S_3 which takes account of the probability of the basic wind speed being exceeded during the specified life of the structure and ensures that the resulting loads are characteristic values.

The characteristic strength of each principal material is similarly theoretically found by subtracting from the mean strength of the material the product of the standard deviation and a factor K_1. The value of K_1 presently adopted is 1.64, which ensures that not more than one test result in twenty will fall below the characteristic value. For concrete, the method of specification is designed to achieve the correct characteristic strength; for reinforcement, the characteristic strength is taken as the minimum yield-point stress specified in the appropriate British Standard. Further information is given in Chapter 4.

Although the characteristic values adopted for loading take account of anticipated variations, they do not allow for loads that differ significantly from those assumed, for employing inadequate analytical methods or imprecise calculations during design, or for errors made during construction, such as incorrectly positioning bars or making minor errors in the sizes and spacing of members. To cater for these variations from the idealized design model, partial safety factors are employed to give so-called *design loads*. The requisite design load is obtained by multiplying the characteristic load by the appropriate partial safety factor for load γ_f. The actual partial safety factors prescribed in BS8110 depend on the particular limit-state being considered, and so take some account of the seriousness of this limit-state being attained or exceeded.

In a similar manner, since the quality of the materials actually used will probably differ from those tested and their performance may deteriorate during their lifetimes, partial safety factors are also employed to convert the characteristic strengths of the materials to design strengths. Here, of course, the design strength is obtained by dividing the characteristic strength by the appropriate partial safety factor for materials γ_m. Furthermore, since different values of γ_m can be employed for concrete and steel, it may be arranged that the mode of collapse that would theoretically result if the structure were loaded to failure is an acceptable one. This occurs, for example, when a beam fails due to yielding of the tension reinforcement, since such an action is preceded by excessive deflection, giving ample warning of imminent collapse, rather than by sudden explosive crushing of the concrete in compression. Again, BS8110 suggests the adoption of different values of γ_m for different limit-states. It should be noted, however, that many design expressions, such as the formulæ for the design of sections given in clause 3.4.4.4 of Part 1 of BS8110 itself, incorporate the correct values of γ_m for the appropriate limit-state concerned, so that calculations are carried out using the characteristic strengths for materials f_y and f_{cu}.

The values of γ_f and γ_m specified in the Code and the resulting design loads and strengths are set out in *Data Sheet 1*. At present the overall (i.e. so-called 'global') safety factor can be obtained by simply multiplying γ_f by γ_m. The values adopted for γ_f are so chosen that the resulting global safety factor corresponds to the possibility of failure occurring being acceptably low; however, since different values of γ_m are adopted for steel and for concrete, the actual overall safety factor determines the critical condition. In future it may be considered advisable to introduce additional individual partial safety factors relating to the nature of the structure (i.e. whether imminent failure would be indicated), the behaviour of the material (i.e. whether brittle or ductile) and the economic or social consequences of collapse. In addition, the partial safety factor for concrete could be varied depending on the standard of workmanship adopted when mixing or placing the material. A principal advantage of the limit-state method adopted in BS8110 is that it enables such developments to be introduced in the future without the need for any fundamental changes in the basic design procedure.

The critical ultimate forces and moments required for designing the members forming the structure are calculated as described in Chapter 3. The normal design procedure is then first to design each member to withstand the bending moments and shearing forces at the ultimate limit-state using the formulæ developed in Chapters 5 and 6. When this has been done, each section should be checked to confirm that it also satisfies the serviceability requirements of BS8110, particularly the prevention of excessive cracking and deflection. As explained in detail in Chapter 8, to avoid much unnecessary repetitive calculation, sets of rules have been developed and are presented in the Code. If these rules are complied with, the general serviceability requirements specified in BS8110 are automatically observed. However, the designer has the option of not complying with these simplified rules if he so wishes, provided that he can show by calculation that the general requirements regarding serviceability set out in the Code are still met. As regards deflection, the simplified rules consist of establishing limiting span/effective-depth ratios which are then modified according to the section shape and resistance provided. Cracking is controlled by such rules by limiting the spacing of the tension steel.

It is also necessary to ensure that the reinforcing bars are bonded sufficiently tightly to the surrounding concrete for the applied forces to be transferred between the two materials without slipping occurring. The requirements of the Code in this respect are discussed in Chapter 7. For certain members it may also be necessary to consider the effects of torsion (i.e. the rotation of the member about its longitudinal axis), and the appropriate requirements of BS8110 are dealt with in Chapter 6.

Chapter 2
Loads

As explained in the previous chapter, although ideally the dead and imposed loads employed in limit-state design should be expressed in statistical terms, this is not yet possible. Therefore the characteristic values of dead and imposed loading should at present be taken as the values of dead and imposed load recommended in BS6399, Part 1. In the case of wind loading, the pressures given in Part 2 of CP3, Chapter V, are already expressed as characteristic values.

2.1 IMPOSED LOADS ON BEAMS AND SLABS

Part 1 of BS6399:1984 specifies ten types of occupancy upon which the imposed loadings that must be considered are based. There are three types of residential property, namely self-contained dwellings, buildings such as boarding or guest houses, and hotels or motels. The remaining types are institutional or educational premises, public assembly buildings, offices, retail premises, industrial buildings, warehouses and similar stores and structures supporting vehicles. For each type of occupancy the Standard specifies a load per unit area, together with an alternative minimum concentrated load. The principal loadings recommended are set out in *Data Sheet 2*, where they are presented in a different arrangement to that adopted in the Code. For certain storage areas BS6399 specifies a uniform load per metre of height, on some occasions together with a minimum total uniform load per unit area; the most important of these cases are listed near the foot of *Data Sheet 2*. The alternative imposed concentrated load is assumed to be applied over an area 300 mm by 300 mm. This concentrated load need not be considered where the floor slab is capable of effectively distributing the load laterally, as would be the case with a solid reinforced concrete floor. Where no concentrated load is specified (marked 'nil' in the second column on the data sheet), the uniform load is considered adequate for design purposes.

If one span of a beam of a floor that is not used for storage purposes supports at least 40 m^2 of floor at one level, the intensity of the imposed load may be reduced by 5% for each 40 m^2 of floor supported. The greatest reduction permitted is 25%; i.e. no further reduction is permitted if the area of the floor supported by one span exceeds 240 m^2. Thus if q_k is the imposed load per unit area acting on an area A, the reduced total imposed load Q_k that must be considered is given by the expression $Q_k = (1.05 - A/800)q_kA$, where A must be between 40 and 240. If A exceeds 240 m^2, the corresponding expression becomes $0.75q_kA$. These reduced loads should only be adopted if the designer is assured that it is unlikely that the entire floor will be fully loaded.

Beams spaced at not more than 900 mm apart may be designed as floor slabs.

The intensities of the ordinary imposed loads are minimum values and should be increased if the specified load seems likely to be exceeded. The normal imposed loads include the ordinary effects of impact and acceleration, but not extraordinary loads.

The weight of heavy equipment should be allowed for in the design of floors carrying machinery and the like. Heavy computing or data-processing equipment, for example, should be considered independently of the recommended imposed load, and provision should be made in the design for moving the equipment into position. Large safes are similar items requiring special consideration. In the case of moving loads, the effects of vibration, impact, acceleration and deceleration must be taken into account. In cases such as the supports of lifts, cranes and similar items, the static loads should be increased by the amounts specified in BS6399 and BS2655, or by the makers, to allow for these effects.

2.2 IMPOSED LOADS ON GARAGE FLOORS

The floors of garages are classified in Part 1 of BS6399 as those for parking passenger vehicles and light cars not exceeding 2500 kg in gross weight, and those for parking heavier vehicles and which act as repair workshops for vehicles of all kinds. The weights of large commercial vehicles are such that suspended floors for garages catering for them would have to be designed to withstand loads comparable with those specified for highway bridges in order to allow for the possibility of the garage being occupied by loaded vehicles. It is therefore generally advisable for the floors of garages for such heavy vehicles to be laid directly on the ground where possible.

For vehicles not exceeding 2500 kg in weight, the uniform loading specified in BS6399 is 2.5 kN/m^2. The alternative concentrated load prescribed is 9 kN on a 300 mm square, but since the reinforced concrete garage-floor slab would be capable of effectively distributing the load laterally, this need not normally be considered. These same loadings also apply to ramps and driveways leading to such garages.

For heavier vehicles it may be necessary to determine the actual maximum loading that may be applied due to the wheel loads, but in no circumstances may a load of less than 5 kN/m^2 be considered. The alternative concentrated load specified is again that resulting from the worst possible combination of wheel loads. The gross weight of most cars does not exceed 2500 kg, but the fully laden weight of a few of the largest private cars may be slightly greater. Thus if a garage is to provide parking for all normal types of private and commercial vehicle it must be designed to withstand the greater loading prescribed in BS6399. In a multistorey garage it is theoretically possible to reserve the lower floor or floors for the heaviest vehicles and to use the upper ones to store light cars only, the floors then being designed accordingly. In assessing the greatest wheel load caused by vehicles where the floors are designed to support actual vehicle loads, it is necessary to consider the possibility of vehicles being stored while fully laden. Private cars are unlikely to impose a wheel load exceeding 7.5 kN. The maximum wheel load of a vehicle having a gross weight not exceeding 4 tonnes (which was the upper limit of the class 150 vehicles in a predecessor to BS6399, and is a reasonable value for a 'normal' commercial vehicle) is likely to be about 12.5 kN, and this load is used in the example in section 12.1.1 and is discussed at the end of this chapter.

Although not stated in the Code, a concentrated wheel load may presumably be spread over a certain area to allow for 45° dispersion through the slab. Thus, assuming a contact area of not less than 100 mm by 100 mm say, a concentrated wheel load of F kN would be spread over a width of $2h + 100$ mm, where h is the slab thickness in millimetres. To assist in designing garage floors, *Data Sheet 3* enables the equivalent uniformly distributed load per unit area of slab to be determined, in order to calculate the bending moment due to a concentrated load F/b per unit width on a span l. The curves given are for freely supported slabs, for slabs fixed at both ends and for slabs fixed at one end and freely supported at the other. Intermediate conditions of continuity can be interpolated.

The chart is used as follows. Assume a slab thickness h, and divide F by $2h + 100$ in order to obtain the load per unit width. Also $\alpha = (2h + 100)/l$. Read off the appropriate value (or values) of K corresponding to the value of α and the fixity. Then the equivalent uniform load per unit width is given by $n = KF/bl$ and, by substituting these values of n in the formulæ given against the curves on *Data Sheet 3*, the required moments can be obtained. For example, with a 150 mm fully fixed slab spanning 2.5 m and supporting a wheel load of 12.5 kN, the equivalent uniform load for the span moment, since $b = 0.4$ m and $\alpha = 0.4/2.5 = 0.16$, is $2.54 \times 12.5/(0.4 \times 2.5) = 31.8$ kN/m^2, and the equivalent uniform load for the support moment is $1.49 \times 12.5/(0.4 \times 2.5) = 18.6$ kN/m^2. The resulting ultimate span and support moments are thus $1.6 \times 31.8 \times 2.5^2/24 = 13.23$ kN m and $1.6 \times 18.6 \times 2.5^2/12 = 15.52$ kN m per metre width respectively.

The curves in *Data Sheet 3* apply to one wheel load in such a position on a span that the maximum bending moment is produced. One wheel load usually applies to small spans, as only on large spans is it possible to arrange the wheels of vehicles so that two or more loads act on one span in such a way that the bending moment exceeds that due to a single wheel. Also for large spans the minimum uniform imposed load of 5 kN/m^2 is often the critical load for a slab. Similarly, a beam is only likely to be subjected to one wheel load, as assumed in *Data Sheet 3*, if the span is small. Beams having larger spans therefore require special consideration, as described in Chapter 12. Concentrated loads on one-way slabs are discussed again in section 9.4 and two-way slabs in section 10.7.

2.3 IMPOSED LOADS ON STAIRS, LANDINGS, CORRIDORS ETC.

2.3.1 Stairs and landings

For stairs and landings in self-contained dwellings, BS6399 specifies a uniform load of 1.5 kN/m^2, with an alternative concentrated load of 1.4 kN. For flats etc. the specified loadings are 3 kN/m^2 and 4.5 kN respectively. For all other types of floor (although BS6399 does not specify values for stairs and landings in storage structures) a uniform load of 4 kN/m^2 or an alternative concentrated load of 4.5 kN should be considered. Note that in certain types of stair design the alternative concentrated load is the critical factor.

2.3.2 Corridors, balconies etc.

The corridor loadings specified in BS6399 for self-contained dwellings and flats are identical to those specified for stairs and landings. For hotels, retail premises and buildings storing vehicles, the prescribed loadings are 4 kN/m^2 (uniform) and 4.5 kN (concentrated). For all other structures, loadings of 5 kN/m^2 or 4.5 kN must be considered. For industrial structures, BS6399 differentiates between corridors which may be used by wheeled vehicles such as trolleys, where a 5 kN/m^2 uniform load or an alternative concentrated load of 4.5 kN should be considered, and other situations, where a 4 kN/m^2 uniform load is applicable, although the alternative concentrated load remains the same.

All balconies should be designed for the same loadings as the floor areas to which they give access, but as an alternative a concentrated load of 1.5 kN per metre run along the edge is specified.

2.3.3 Footpaths, terraces and plazas

Where no positive obstruction to vehicular traffic is provided, footpaths, terraces and plazas leading from ground-floor level should be designed for a uniform load of 5 kN/m^2 or an alternative concentrated load of 9 kN. Where access is definitely restricted to pedestrians only, these loads may be reduced to 4 kN/m^2 (uniform) and 4.5 kN (concentrated).

2.3.4 Balustrades and parapets

The balustrades and parapets of 'light-access' stairs and gangways not more than 600 mm wide should be designed to resist a horizontal load of 220 N per metre run acting at a height of 1100 mm above the foot of the balustrade or parapet, irrespective of the height of the handrail or coping level. For balustrades to other stairs in residential buildings and for balconies, ramps, landings and floors serving only an individual dwelling, the corresponding load should be 360 N per metre, while for structures of all other types except those designed for public assembly this load is increased to 740 N per metre. For stairs etc. provided for individual dwellings, BS6399 states that alternative horizontal loadings of 0.5 kN/m^2 distributed uniformly or a concentrated load of 0.25 kN acting on the infilling must be considered. For other stairs in residential buildings (except light-access stairs etc. as specified above) and stairs, landings etc. in all non-public assembly buildings, the alternative horizontal loadings specified are 1 kN/m^2 and 0.5 kN respectively. For structures designed for public assembly, BS6399 specifies horizontal loads on parapets and balustrades of 3 kN per metre at handrail or coping level, with alternative horizontal loads on the infill of 1.5 kN/m^2 (uniform) and 1.5 kN (concentrated). Where fixed seating is provided in balconies and stands to within 530 mm of the barrier concerned, BS6399 relaxes these values to 1.5 kN/m, 1.5 kN/m^2 and 1.5 kN respectively. Where the parapets are to footways or pavements adjoining access roads or similar areas, values of only 1 kN/m, 1 kN/m^2 and 1 kN need be considered.

2.4 IMPOSED LOADS ON ROOFS AND PARAPETS

Roofs are classified in BS6399 as flat and sloping. The imposed loads discussed in this section are those due to snow, access etc.; wind loads are considered in section 2.6.

For both flat roofs and roofs sloping at less than 45°, in order to cater for loads that may occur during maintenance, all roof coverings (other than glazing) must be able to support a load of 900 kN on a square 125 mm by 125 mm in plan.

2.4.1 Flat roofs

A roof is considered to be flat if the slope does not exceed 10°, i.e. if it is not steeper than about 1 in 5.7. The ordinary imposed load, excluding wind, on slabs and beams to be considered is 1.5 kN/m^2 of plan area supported, if general access to the roof is provided in addition to access for cleaning and repairing; the alternative concentrated load is 1.8 kN. If no general access is provided, the imposed load may be reduced to 0.75 kN/m^2 and the equivalent concentrated load to 0.9 kN.

2.4.2 Sloping roofs

For a roof inclined at more than 10° but not more than 30°, the minimum imposed load excluding wind should be 0.75 kN/m^2 of plan area. If the slope exceeds 75° no imposed load need be considered, but for a slope of between 30° and 75° (i.e. about 1 in 1.73 and 1 in 0.27) the imposed load may be obtained by linear interpolation between 0.75 kN/m^2 and zero.

When designing the slabs and possibly the beams also, it is necessary to know the load acting at right angles to the slope. The effect of the wind is specified in Part 2 of CP 3, Chapter V, in this manner, but the imposed loads are specified in Part 1 of BS6399 per unit area of plan. One of the curves on *Data Sheet 4* gives the imposed load on a unit area of sloping slab. The self-weight of the slab should also be converted into a force per unit area at right angles to the slab; and the dead-load factors, by which the weights of a unit area of slab and finishes should be multiplied to give this force, are also given in *Data Sheet 4*. The total load per unit area of slab acting at right angles to the slab is therefore the converted imposed load, the wind pressure and the converted dead load.

2.5 IMPOSED LOADS ON COLUMNS, PIERS, WALLS AND FOUNDATIONS

The imposed loads on columns and similar members are the same as the ordinary uniform loads for the floors that they support. However, for columns, piers, walls and other similar supporting members, and their foundations, which support several floors that are not garages, warehouses or floors used for storage or filing purposes, the imposed loads may be reduced in accordance with *Data Sheet 4*, the greatest reduction being 50%. The load on a column is generally the product of the area of the floor A supported by the column in question and the total imposed load (q_k per unit area) for the type of floor concerned. The reduction factors K in *Data Sheet 4* enable the total imposed load (in kN) on a column supporting two or more floors, if A is the same for each floor, to be calculated from Kq_kA. A smaller reduction is recommended for floors of factories and workshops designed for imposed loads of 5 kN/m^2 or more; the recommendation is that the reductions for non-storage floors can be adopted but the reduced load should not be less than that which occurs assuming all floors to be loaded at 5 kN/m^2; see *Data Sheet 4*.

Since it is generally easier to calculate the load on a column commencing from the top of a building, the reduction of the load on the floors varies as each storey is considered. To enable the total load to be calculated as on *Calculation Sheets 17*, the difference between successive factors K denotes the equivalent increment of imposed total load at each storey.

The reduced load that may be assumed if a single span of beams supports more than 40 m^2 of floor (see section 2.1) may be used in the design of columns and similar supporting members in place of the reductions described above if the resulting reduction proves to be greater.

2.6 WIND FORCES ON BUILDINGS

The characteristic wind pressure per unit area w_k depends on various factors including the locality, degree of exposure and height of the structure concerned. In Part 2 of CP 3, Chapter V, two principal methods of determining wind forces are described. The general procedure is first to determine the characteristic wind speed V_s. This is done by multiplying V, the basic wind speed which depends only on the locality and may be read from the map on *Data Sheet 5*, by three non-dimensional factors S_1, S_2 and S_3.

Factor S_1 relates to the topography of the site. In the majority of instances it should be taken as 1.0, although a value of 1.1 is recommended on exposed hills or in narrowing valleys, and the factor may be reduced to a minimum of 0.9 for an enclosed valley. Factor S_3 results from the statistical concept, and depends on the anticipated life of the structure and the probability of V being exceeded during that period. For practical use a value of unity is recommended; this corresponds to a probability of 0.63 that V will be exceeded once in 50 years. Factor S_2 relates the terrain, the plan size (or overall height) of the area considered and the height above ground to the top of the section involved. Appropriate values of S_2 can be read from the curves on *Data Sheet 6*. At heights below the general level of obstructions (i.e. to the left of the chain lines) the figures given should be treated with caution because of the likelihood that higher speeds may result from eddies due to the wind funnelling between the buildings.

When V_s has been determined, it is then converted into the corresponding characteristic wind pressure w_k by employing the expression w_k (in N/m^2) $= 0.613V_s^2$, where V_s is in metres per second. This conversion can conveniently be done by reading from the scale on *Data Sheet 6*. The next step is to determine the appropriate external pressure coefficient C_{pe} for a building of the given shape; the required coefficients for rectangular buildings with flat roofs are set out in *Data Sheet 5*.

The total wind force on a given area is then obtained by multiplying together the characteristic wind pressure, the external pressure coefficient and the area concerned. Thus to find the total wind force F acting on part of a rectangular building presenting an area A frontal to the wind, the appropriate expression is $F = w_kA(C_{pe1} - C_{pe2})$, where C_{pe1} and C_{pe2} are the external pressure coefficients for the windward and leeward faces respectively. Normally, however, except when stability is being considered, the designer wishes to know the forces acting on a particular face of a structure in order to design the structural members: then $F = w_kAC_{pe}$. Note from *Data Sheet 5* that the maximum suction to which a surface is subjected occurs when the wind is blowing parallel to the face in question.

In the case of cladding, the total force F on an element of area A is given by $F = w_kA(C_{pe} - C_{pi})$, where C_{pi} is the appropriate internal pressure coefficient. Typical values of C_{pi} are given on *Data Sheet 5*; for additional information on selecting suitable values, reference should be made to the Code itself.

The foregoing method may also be used to obtain the total wind force acting on a building by dividing it into component areas, determining the force on each area in turn, and then vectorially summing the results. An alternative method of obtaining the total wind force (in order, for example, to investigate the stability of the building) is to use the force coefficients C_f also provided in CP 3, Chapter V:Part 2. If the characteristic wind pressure w_k is found as described above, the total force $F = w_kAC_f$, where A is the frontal area presented by the building and C_f is the appropriate force coefficient. Some values of C_f for rectangular buildings are given in *Data Sheet 5*; values of C_f for structures of many other shapes are included in the Code itself, and some are reproduced in *RCDH*. The use of a value of w_k appropriate to the top of the building in this calculation corresponds to the assumption of a constant pressure over the entire height with the total force acting at a centroid of one-half of this height, thus overestimating the total force and also the overturning moment. A more accurate calculation may be made by dividing the height into a series of convenient lengths (usually corresponding to the storey heights) and employing values of S_2 corresponding to the height of the top of each length (see section 14.10).

The frame of the building should be designed to resist the wind pressures that act on the faces of the structure and on the roof. In the previous version of CP 3, Chapter V, it was considered unnecessary to so design such a frame if the building, the height of which did not exceed twice the width, was sufficiently stiffened by walls or by walls and floors. This concession has been eliminated from the current version of CP 3, however. In the building considered in Part Two it is assumed that sufficient longitudinal resistance to wind is provided for the whole building by the partition walls and staircase shafts.

2.6.1 Roofs

According to Part 2 of CP 3, Chapter V, the wind pressure coefficients for a roof depend on the part of the roof concerned, the angle of the wind and the slope of the roof. The Code gives general coefficients for the windward and leeward areas of roof, together with local coefficients for designing individual cladding components. Examination of these coefficients indicates that the resulting general wind pressures are negative (i.e. suctions occur) with flat roofs and on the leeward sides or halves of pitched roofs, and also on the windward sides or halves of pitched roofs sloping at not more than about 30°. All of the local coefficients given are also negative. As it is most unlikely that, on a roof sloping at less than 30°, no wind pressure ever occurs under any circumstances, it might seem that the present single values should be replaced by alternative maximum values of pressure and suction for roofs of any inclination.

If allowance must be made for wind pressure, the effect is to cause bending moments and shearing forces on the columns, and additional bending moments on the beams in line with the direction of the wind. A simple method of calculating the bending moments and shearing forces at any floor level, that complies with the requirements of BS8110, is to first calculate the total horizontal wind force on one bay above the level of the floor being considered. This gives the total horizontal shearing force on a single row of columns at the level considered, and should be so divided between the columns that each external column resists one-half of the shearing force resisted by an internal column. The bending moment on a column is then one-half of the product of the storey height and this shearing force, and the additional bending moment on the floor beam is the sum of the bending moments on the columns above and below the floor being considered. This method and similar methods of taking lateral forces into account when designing columns are discussed in more detail in section 14.10.

Part 2 of CP 3, Chapter V, gives much information on the determination of wind forces on tall structures of various shapes, including sheeted towers and unclad structures of a similar nature (but not chimneys, for which a BSI Draft for Development is in preparation). However, such structures are outside the scope of this book.

2.7 DEAD LOADS

The primary dead or permanent load of a reinforced concrete structure is the weight of the structural members, of finishes on walls, floors, ceilings, stairs and elsewhere, of brickwork, masonry, steelwork, partitions, fixed tanks and other permanent construction supported by the structural members. BS6399 refers the designer to BS648 for the weights of building materials. This standard gives the weight of plain broken-brick concrete as 19.6 kN/m^3, plain ballast concrete as 22.6 kN/m^3, and concrete containing about 2% of reinforcement as between 23.1 kN/m^3 and 24.7 kN/m^3. The nominal weight of reinforced concrete generally is given in BS648 as 23.6 kN/m^3; this value is employed in the examples in the following chapters. However, a more convenient and frequently used value is 24 kN/m^3, and this is the value adopted in the calculations in Part Two. For high percentages of reinforcement the weight may increase to as much as 25.6 kN/m^3 according to BS648.

2.7.1 Partitions

According to BS8110 the loads on floors due to partitions should be included in the dead load. Where the weights and positions of the partitions are known, the floor should be designed to support them. If the positions are not known, an addition should be made to the dead load of the floor to allow for the partitions, the addition being, according to BS6399, Part 1, a uniform load on each square metre of floor of one-third of the weight of a 1 m length of finished partition, but in the case of offices, of not less than 1 kN/m^2. This latter condition is normally not restrictive unless only timber or similar light partitions are to be used, as brick or clinker-block partitions demand a greater allowance. For example, for a partition 3 m high and plastered on both faces, an allowance of 2.5 kN/m^2 would be required if the partition were of 115 mm brickwork, and 1.2 kN/m^2 if of 75 mm clinker blocks.

2.8 DESIGN LOADS FOR THE BUILDING IN PART TWO

It is now possible to consider how the foregoing rules apply to the building designed in Part Two (*Drawings 1 and 2*).

2.8.1 Roofs

The slabs and beams of the main roof, to which there is access, should be designed for an imposed load of 1.5 kN/m^2. The small flat roof slabs over the stair and lift wells and the tank room may be designed for 0.75 kN/m^2 as there is no general access, but it is advisable to consider that people may walk on such flat roofs and there is little to be gained by designing them for less than 1.5 kN/m^2.

2.8.2 Upper floors

The offices on the second, third and fourth floors are intended for general use and could be designed for an imposed load of 2.5 kN/m^2 with an additional load of 1 kN/m^2 for lightweight partitions. However, it is not advisable to restrict storage and filing rooms to any particular part of an office floor, and it would therefore be preferable to design these floors for an imposed load of 5 kN/m^2 with an additional load of 1 kN/m^2 for lightweight partitions, as is done on *Calculation Sheets 1*.

Partitions other than lightweight partitions must be over the beams. In the design of the flat-slab (i.e. beamless) floor in Chapter 11 there are no beams, so an extra allowance must be made for heavy partitions, say an additional 1 kN/m^2 as allowed for on *Calculation Sheets 1*. The showrooms on the first floor may be considered as shop floors, and therefore designed for an imposed load of 4 kN/m^2. The residential flats on the top floor may be designed to carry 1.5 kN/m^2. Since it might be required to convert the showroom floor and residential floor into offices in the future it is, however, proposed to design both of these floors for the same load as the office floors. The flat roof or terrace on the same level as the residential floor could be designed to carry 1.5 kN/m^2, but it is not worth while making any reduction for such a small area.

The canopy cantilevering from the first floor should be considered as a flat roof to which there is access and designed for an imposed load of not less than 1.5 kN/m^2. The parapet of the terrace on the level of the top floor should be designed for a horizontal load of 740 N per metre acting at a height of 1.1 m.

Each of the ordinary main and secondary beams supports less than 40 m^2 of floor and therefore no reduction in the imposed load is permissible.

2.8.3 Ground floor

Part of the ground floor is to be used as a garage for vehicles not exceeding 4 tonnes in weight. The greatest load from a rear wheel of such a vehicle may be about 12.5 kN. Since each span of the slab is continuous over both supports, the conditions at midspan are intermediate between a freely supported slab and a slab fixed at both supports. Therefore for a concentrated load of 12.5 kN on a 150 mm slab, the equivalent uniformly distributed load for the bending moment at midspan is, from *Data Sheet 3*, about 27.4 kN/m^2. The conditions at the supports are slightly less rigid than fixity at both supports, and the equivalent uniform load for the bending moment at the support is therefore about 22.0 kN/m^2. These design loads are therefore in excess of the minimum uniform load of 5 kN/m^2 specified for such floors, and the slabs are designed for the greater loads as described in section 12.1.1.

The uniform load equivalent to a wheel load of 12.5 kN on a length of 0.4 m of a fully continuous secondary beam having a span of 6 m is, by a consideration similar to that for slabs, about 4.94 kN per metre for the bending moment at midspan and about 3.71 kN per metre for the bending moment at the support. The minimum imposed load of 5 kN/m^2 results in a load of $2.5 \times 5.0 = 12.5$ kN per metre of secondary beam, and each secondary beam should be designed for a load of not less than this amount. It is necessary to consider, as is done in section 12.1.2, whether two or more wheels acting on a beam produce an ultimate bending moment greater than does a uniform load of 12.5 kN per metre. The load on the main beams is considered in the same way (section 12.1.3).

The front part of the ground floor is occupied by shops, and the beams and slabs for this area could be designed for an imposed load of 4 kN/m^2 as in the alternative comparative designs in sections 12.2.1 to 12.2.3. It is nevertheless worth while considering designing the entire ground floor for the garage load so that it may be used without restriction in the same way as all the upper floors and would be capable of carrying a greater load than is likely at the time of the first occupancy. Such measures enhance the value of a property without adding much to the initial cost.

2.8.4 Stairs and landings

Since the front stairs above the ground floor serve offices, they should be designed for an imposed load of 4 kN/m^2. The back stairs to the basement should be designed for 5 kN/m^2 as they would probably be used for delivering goods. The design of the front stairs is dealt with in section 16.1.

2.8.5 Columns

The columns are designed to support the same imposed loads as the roof and floors that they carry, but the reductions given on *Data Sheet 4* are applicable because the building is not intended for storage purposes. The application of the loading reductions on *Data Sheet 4* is illustrated on *Calculation Sheets 17 to 20*.

2.8.6 Wind

The height of the building above ground level in *Drawing 1* is about 23 m to the top of the parapet. If the degree of exposure is assumed to be condition 4 in the Code and the building is to be located in Plymouth, taking S_1 and S_3 as unity gives values of $V = 44$ and thus, at the top of the structure, $V_s = 44 \times 0.85 = 37.4$ m/s for the cladding and $44 \times 0.8 = 35.2$ m/s for the face of the building as a whole, since the total value of C_{pe} for the longitudinal faces of the building (with $h/a = 1.5$ and $b/a = 1.87$) is $0.7 + 0.3 = 1.0$; the resulting values of w_k are then 858 N/m^2 and 760 N/m^2 respectively. If it is assumed for simplicity that these loads apply over the entire building, the total wind force on any storey can then be obtained by multiplying the surface area of the storey concerned by this value of w_k.

Some reduction in these forces can be achieved by determining the appropriate value of S_2, and hence w_k, at each individual floor level (see section 14.10). However, inspection of the calculations involving the use of these wind forces (i.e. those on *Calculation Sheets 20*) shows that in the present example the additional refinement obtained by summing the forces at individual floor levels is not justified, since the wind forces obtained by the approximate method are insufficient to affect the amounts of reinforcement required. Nevertheless, in more-critical cases there are clear advantages in using the more detailed method.

It can be assumed that the floors act as horizontal beams transferring the lateral force from the wind on the longitudinal faces of the building to end shear-walls of reinforced or plain concrete. If instead, the end walls are constructed as reinforced concrete frames with infilling brick panels, the resulting columns and beams must be designed to resist the wind loading. Both of these possibilities are considered in the appropriate sections of the book (see section 14.10). The forces acting longitudinally (i.e. at right angles to the end walls) must be resisted by the walls surrounding the stairs and the partitions. All exterior wall panels, whether of reinforced concrete or some other material, should be designed to resist an inward or outward pressure of about $1.2 \times 0.858 = 1.03$ kN/m^2.

The wind pressure on the parapet is 0.76 kN/m^2. If the parapet is about 1.2 m high, the alternative load of 0.74 kN acting at the top, as previously described, produces the greater bending moment.

The flat roofs are assumed to be subjected to a maximum suction of 0.76 kN/m^2 on the windward half and 0.46 kN/m^2 on the leeward half, since the corresponding coefficients of C_{pe} given in the Code are -1.0 and -0.6 respectively. These are the general wind loads on the roof, but on strips along the edge of the roof that are 0.15 times the width (or length) of the building a suction of 1.52 kN/m^2 should be considered, since $C_{pe} = -2.0$ for local effects in these areas. However, the self-weight of the roof slab acting downwards is far greater than these upward-acting forces, so that they can be ignored when designing the roof slab.

The projections above the general roof level are the lift-motor room and the tank room. The height of the roof of the motor room above the ground is about 25.8 m, the corresponding basic wind pressure for which only slightly exceeds 0.86 kN/m^2, but the weight of the slab far exceeds the equivalent upward pressure.

Chapter 3 Bending moments on structural members

When the characteristic loads for which a building is to be designed have been established (as described in the previous chapter), the next design stage is to determine the appropriate bending moments induced in the slabs and beams comprising the floors and roof. In a reinforced concrete building the beams and slabs are often formed monolithically with each other and with the supports; i.e. they are designed as continuous over the supports. In some cases the beams are also designed to act monolithically with the supporting columns, the structural frame then being analysed as a whole. Nowadays, however, it is quite common for precast reinforced or prestressed concrete units to be utilized in the construction of multistorey buildings, particularly to form the slab areas; alternatively a proprietary flooring system may be employed.

The bending moments on simple beams and slabs that are freely supported on two supports or cantilevered can be calculated from simple statics. The bending moments on continuous slabs, beams and frames are statically indeterminate and their determination is discussed in this chapter. Slabs spanning in two directions are considered in Chapter 10, flat slabs in Chapter 11 and other types of floor in Chapter 12. Bending moments due to concentrated loads on solid slabs are discussed in sections 9.4 and 10.7.

3.1 CONTINUOUS BEAMS AND SLABS

BS8110 specifies, in effect, three different methods of calculating the bending moments on beams and slabs that span in one direction and are continuous over several supports. If there are three or more nearly equal spans (for beams the Code states that the maximum permissible difference is 15% of the greatest span) supporting predominantly uniform loads, and in the case of beams the imposed load does not exceed the dead load, simple approximate expressions utilizing coefficients given in BS8110 may be used. The corresponding Code coefficients for slabs only apply where the area of the bay comprising the slab considered exceeds 30 m^2, and where the imposed load does not exceed the dead load by more than 25% and is not greater than 5 kN/m^2 (excluding partitions). Alternatively, for both beams and slabs, a theoretical analysis may be made assuming that the members are free to rotate about their supports (i.e. the frequently adopted assumption of knife-edge support). Otherwise, the members may be considered to be part of a monolithic frame and analysed as such. Each of these methods is now discussed in turn.

When calculating the maximum bending moments in the spans and at the supports, BS8110 states that the spans should be loaded as shown in *Figure 3.1a*. The arrangement of the imposed load prescribed to obtain the support bending moments does not give the theoretical maximum moments at the supports, to obtain which it is necessary to arrange the imposed loads as indicated in *Figure 3.1b*. The difference between the maximum negative bending moments resulting from the two sequences of loading is sometimes significant; for example, for a theoretically infinite number of spans and a uniform load, the loading specified in BS8110 gives bending moments at the internal supports that are 74% of the maximum values that result when the critical loading condition illustrated in *Figure 3.1b* is applied.

As shown in *Figure 3.1a*, BS8110 requires the analysis to be made considering a maximum dead load of $1.4G_k$ and a minimum dead load of $1.0G_K$, these loads being so arranged as to induce maximum moments. This requirement may be dealt with conveniently by considering instead a total 'imposed load' of $0.4G_k + 1.6Q_k$ and a dead load of $1.0G_k$, as shown in *Figure 3.1c* for the loading arrangement giving the maximum support moment.

For the purpose of calculating bending moments, BS8110 defines the effective span of a continuous member as the distance between the centres of the supports or from the face of a cantilever to the centre of its support.

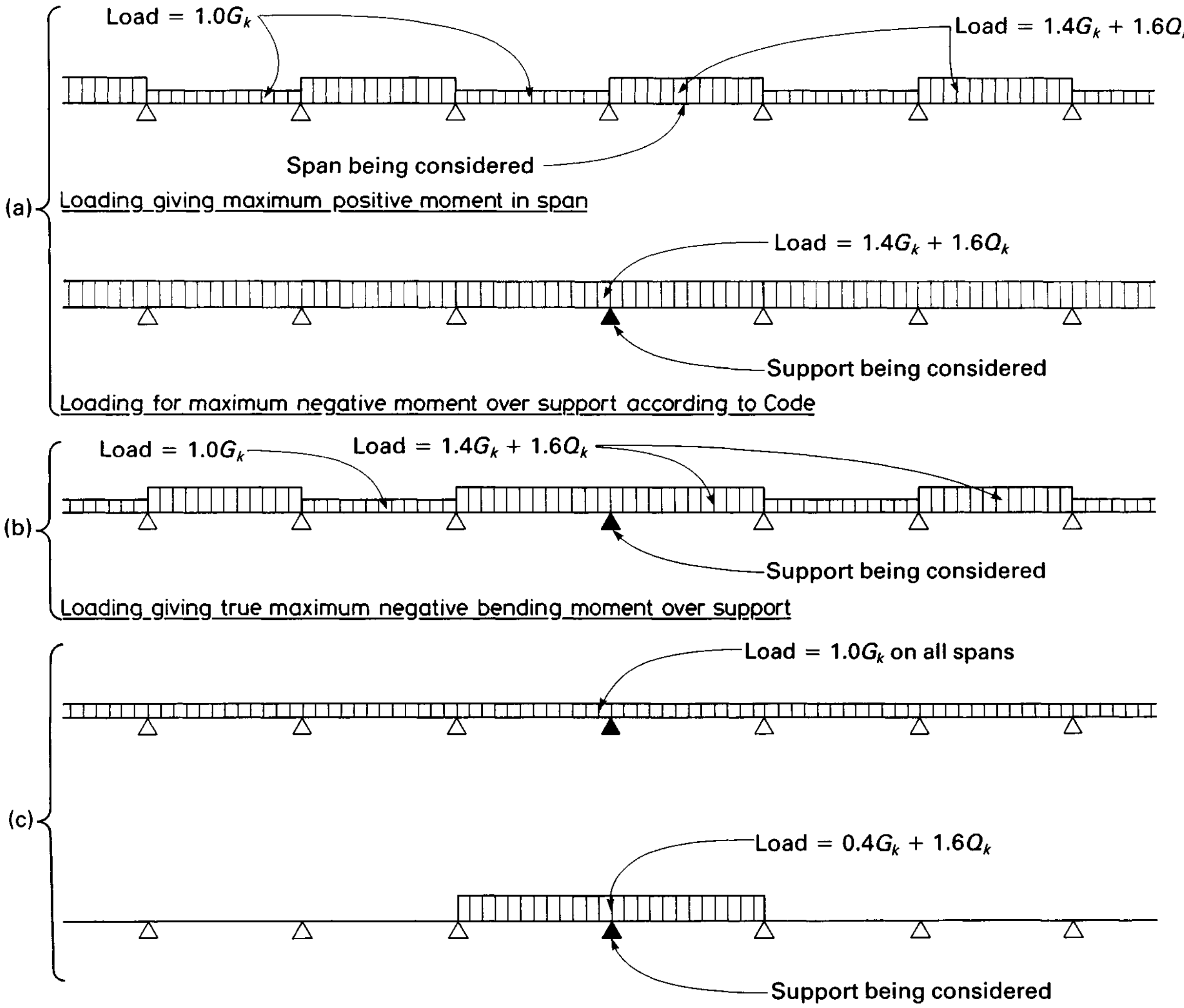


Figure 3.1 Loading arrangements for maximum moments.

For freely supported members, however, either the distance between the centres of the supports or the clear distance between the supports plus the effective depth, whichever is the lesser, may be adopted.

3.2 EQUAL SPANS: APPROXIMATE METHOD

For slabs spanning in one direction only, and for beams continuous over three or more approximately equal spans and carrying substantially uniform loads, where the ratio of characteristic imposed load Q_k to characteristic dead load G_k is not greater than 1 (for beams) and 1.25 (for slabs), and where for slabs the span exceeds 30/(breadth of building), and the imposed load is not greater than $5\,\text{kN/m}^2$ (excluding partitions), BS8110 gives the following approximate coefficients for the calculation of the ultimate bending moments at the critical sections:

	For beams	For slabs
At end support	0	0
In end span: near midspan	+0.09	+0.086
At penultimate support	−0.11	−0.086
In interior spans: near midspan	+0.07	+0.063
At internal supports	−0.08	−0.063

To obtain the ultimate bending moments, the above coefficients must be multiplied by the product of the intensity of the total ultimate load (i.e. $1.4G_k + 1.6Q_k$) and the span. Consecutive spans may be considered to be equal if the difference in length does not exceed 15% of the longer span. These coefficients appear to be based on the assumptions that the characteristic dead load and characteristic imposed load are equal and that the member is freely supported on the outer supports. Since this latter condition is rarely obtained in the case of solid slabs, and since the torsional resistance of the supporting beams is generally effective in restraining a slab from acting freely as if it were supported on knife-edge supports, it is questionable whether it is necessary to be so precise in the estimation of the bending moments on solid slabs as is indicated by the Code coefficients. Perhaps the well-known coefficients of one-tenth for the end spans and the penultimate supports and one-twelfth for all other spans and supports, which have been widely adopted for many years, are sufficiently realistic for such slabs.

A comparison of the ultimate bending moments calculated on the assumption of equal spans when the inequality is less than 15%, and by a more accurate method, is given in section 9.5.1 when considering the secondary beams in the building in Part Two.

3.3 THEORETICAL ANALYSIS: KNIFE-EDGE SUPPORTS

If the spans are unequal, if the loads are not distributed uniformly, if the limiting ratio of Q_k/G_k is exceeded, or if two or more of these conditions apply, the ultimate bending moments can be calculated by means of one of the exact theoretical methods, such as the theorem of three moments, slope deflection, least work etc. Such a procedure is often complex, even when permissible simplifying assumptions are made; in such cases iterative methods such as the well-known Hardy Cross moment-distribution method and its variants are simpler and sufficiently accurate. The application of one of these variants of moment distribution is described later in this chapter.

When a strict theoretical analysis is employed, BS8110 permits any section to be designed for a resistance moment that is not less than 70% of the bending moment obtained at that point from an elastic bending-moment analysis taking account of all appropriate combinations of dead and imposed load, provided that two further conditions are met. These are that the internal and external forces are in equilibrium, and that the maximum depth to the neutral axis assumed when designing the concrete section at that point is related to the percentage of moment redistribution that has been adopted at the section concerned. The latter condition results in a maximum neutral-axis depth of $0.6d$ (where d is the effective depth to the tension steel) if no redistribution is employed, and $0.3d$ when the maximum 30% redistribution is adopted.

The requirements relating to moment redistribution in the Code have been framed to ensure that, since the positions of the points of contraflexure are normally altered by the redistribution process, sufficient reinforcement is provided at these locations (which would otherwise theoretically only require nominal reinforcement, of course) to limit the formation of cracks due to the moments that arise at these points from service loads.

An important point to note is that BS8110 places no corresponding limit on the maximum percentage *increase* in moment that may be adopted.

Moment redistribution is discussed in some detail in the *Code Handbook*. The justification for redistributing bending moments in this way is that, as the actual moments occurring in a member reach the values which the critical sections can withstand, so-called 'plastic hinges' form which permit rotation to occur at these points without any increase in the moment-carrying capacity. Thus further increases in the load on the member are resisted by increases in the moments elsewhere, leading to the formation of further plastic hinges. Finally the system fails when the last plastic hinge to form renders the system unstable. The resulting collapse-moment diagram can be produced by evaluating an elastic-moment diagram using ultimate loads and then redistributing the moments as permitted by BS8110. For further details, reference should be made to the *Code Handbook*.

The application of moment redistribution to beams and slabs that are continuous over a number of equal spans is discussed in detail below, but the redistribution procedure is, of course, equally applicable to any continuous system. The major practical benefit of redistribution is that it enables the congestion of reinforcement that would otherwise occur at the supports, i.e. at the intersections of the beams and columns in an ordinary building, to be reduced. It is also convenient, especially in solid slabs, to have the same bending moment at the support as at midspan; and, although absolute uniformity of maximum ultimate bending moments can clearly not be obtained simultaneously for all types of load and at all the critical sections, advantage can be taken of using the permitted redistribution to keep the bending moments within allowable limits, with a view to reducing the inequality between peak moments. In flanged beams, however, it is generally an advantage to reduce the support moments as much as possible, since the area of concrete in compression in the rib at the support is so much less than that in the flange at midspan.

As explained above, the maximum percentage redistribution that may be undertaken is related to the adopted ratio of x/d by the expression $x/d \leq (\beta_b - 0.4)$, where β_b is the ratio of the bending moment after redistribution to that before redistribution, at the section considered. Thus if large redistributions of moment are contemplated it must be remembered that there will be a corresponding restriction on the maximum value of x/d that may be employed; if the ratio of d'/d is high, this may limit f_{yd1} to less than its maximum possible value. However, it must be remembered that the limit of 70% applies to *each particular combination* of load separately. Consequently, with imposed loads both the maximum span and maximum support moments may normally be reduced as they usually arise as a result of different loading conditions. Thus the overall adjustment is normally much less than the 30% limit, especially when taking the moments due to dead load into account as well. For example, considering the three-span beam carrying central concentrated loads which is examined in detail in the next section, if $g = q$ and making the full 30% reduction at the supports:

maximum moment in end span before adjustment

$$= 0.175 \times 1.0gl^2 + 0.213(1.6 + 0.4)ql^2 = 0.600gl^2$$

maximum moment in end span after adjustment

$$= 0.198 \times 1.0gl^2 + 0.198(1.6 + 0.4)ql^2 = 0.594gl^2$$

Thus the percentage adjustment made near midspan is only $(0.600 - 0.594) \times 100/0.6 = 1\%$ and the corresponding maximum value of x/d is 0.59. However, the support moment has been reduced by the full 30% and so the maximum permissible ratio of x/d here is only 0.3.

3.4 EQUAL SPANS AND UNIFORM MOMENTS OF INERTIA

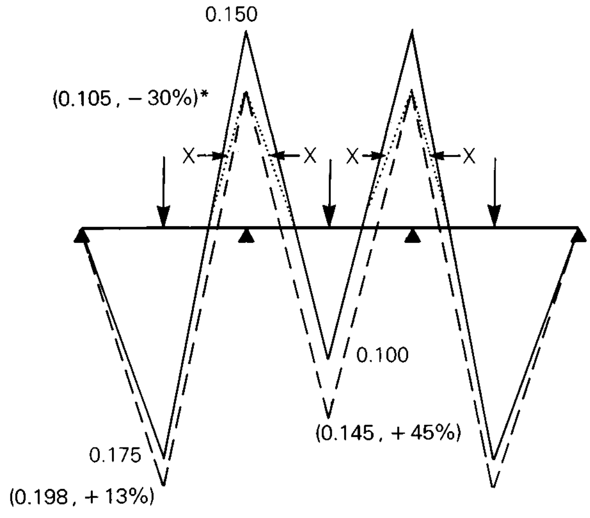
For members that are continuous over two or more equal spans, which have a uniform second moment of area throughout all spans and are freely supported at end supports, formulæ giving the critical bending moments at each support and near the middle of each span are given on *Data Sheets 7* and *8*, for uniform loads and for the loading transferred from two-way slabs according to BS8110 respectively. However, for the common case of such beams and slabs also supporting equal loads, it is worth while also tabulating the limiting bending moment coefficients. On *Data Sheet 9* such coefficients are given for two, three and a theoretically infinite number of spans with a uniform load extending over the entire span, a uniform load extending over the central 75% of the span, and a central concentrated load. Both the condition of all spans loaded (e.g. as in the case of dead load) and the various conditions of incidental (e.g. imposed) load producing the greatest bending moments are considered. If the coefficients are calculated by a so-called 'exact' method (as is the case with those on *Data Sheet 9*), the maximum values may be reduced to not less than 70% of the original values as described below. *Data Sheet 9* also give the coefficients for the positive bending moments at the supports and the negative bending moments on the spans which result from some conditions of imposed load; these enable the relevant bending-moment diagrams to be sketched and thus the reinforcing bar stopping-off points to be estimated.

The method of calculating the basic and adjusted coefficients, such as those tabulated on *Data Sheet 9*, is illustrated by the example shown in *Figure 3.2* of a beam that is continuous over three spans and is loaded with dead and imposed loads concentrated at midspan only. Although this is not a very practical example, as in real life there would always be some uniform load due to the self-weight of the beam, it does illustrate particularly clearly the numerical procedures and adjustments involved. The theoretical bending moments are calculated for all spans loaded (i.e. dead load) in *Figure 3.2a*, and for each of the three cases of imposed load that produce maximum bending moments according to BS8110: i.e. in *Figure 3.2b* for the middle of the central span (positive), in *Figure 3.2c* for the middle of an end span (positive), and in *Figure 3.2d* for a support (negative). (As explained earlier, for simplicity BS8110 permits the assumption to be made that the maximum negative moments at the supports occur when all the spans are loaded. In reality, the true maximum negative moment at any support occurs when the two spans adjoining the support are loaded, together with all alternate spans, and the true maximum positive moment at any support occurs when the two adjoining spans and all other alternate spans are unloaded, all remaining spans being loaded. The Code simplification means that for a continuous system of n spans, to determine the maximum support moments only a single loading condition needs to be considered rather than $2(n-1)$ conditions.)

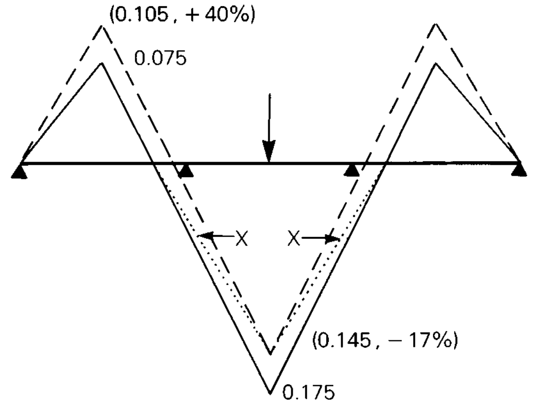
For each case in *Figures 3.2a–d* the theoretical bending-moment diagram is adjusted as follows. For the diagram of maximum negative support moments, the theoretical negative bending moments at the supports are reduced by 30% and the corresponding positive bending moments in the spans are increased accordingly. Then for the respective diagrams of maximum positive bending moments in the spans, the theoretical positive bending moments are reduced by 30% provided that the corresponding negative moments at the supports are not, as a result, increased to values greater than those obtained by making a 30% reduction in the maximum values of negative moment. If this would occur, the percentage reduction of positive moment made is limited to that which makes the corresponding increased negative support moments equal to the reduced negative support moments corresponding to the loading that produces the maximum negative moments. For example, it is only possible to decrease the span moments in *Figure 3.2c* by 7% without increasing the support-moment coefficient to more than -0.105, which is the value obtained by reducing the basic maximum support-moment coefficient of -0.150 by 30%. Similarly, the maximum reduction in the span moments that can be made in *Figure 3.2d* is 17%.

Figure 3.2e shows the resulting envelope of maximum bending moments due to imposed load only, both before and after a reduction of 30% has been applied to the support moments. Two points resulting from such an adjustment are worth noting. Firstly, the maximum negative moments throughout the spans are increased considerably. These moments are normally of little practical importance, however, since they act in opposition to the moments resulting from dead loading, but they may become more important with high ratios of imposed load to dead load. Secondly, the lines defining the areas of the bending-moment envelope after redistribution in the vicinity of the points of contraflexure need to be adjusted to meet the requirement in BS8110 that at least 70% of the elastic-moment values must be considered at all points. The adjusted lines, marked X on *Figures 3.2a–d*, influence the areas shaded on *Figures 3.2e* (and *3.2f*). When designing reinforcement for the lengths within a distance of about one-quarter of the span from any support, remember that the maximum bending moment and shear force generally result from a combination of only partial load on the span in question with full loading on others, a condition not considered in the Code.

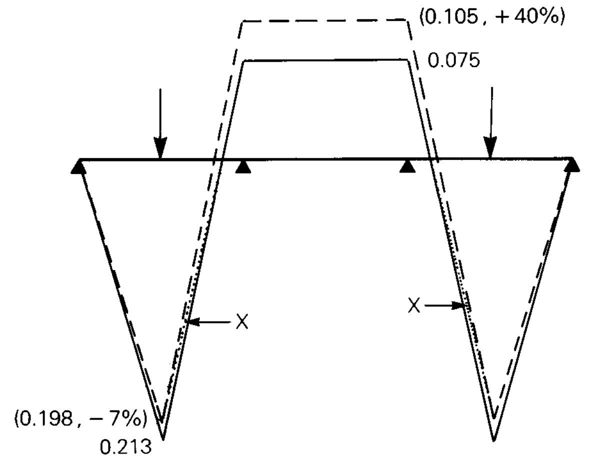
In preparing the foregoing tabulated coefficients it has been considered of prime importance to reduce the support moments by as much as possible. Although this is true in normal cases of frame construction for the reasons



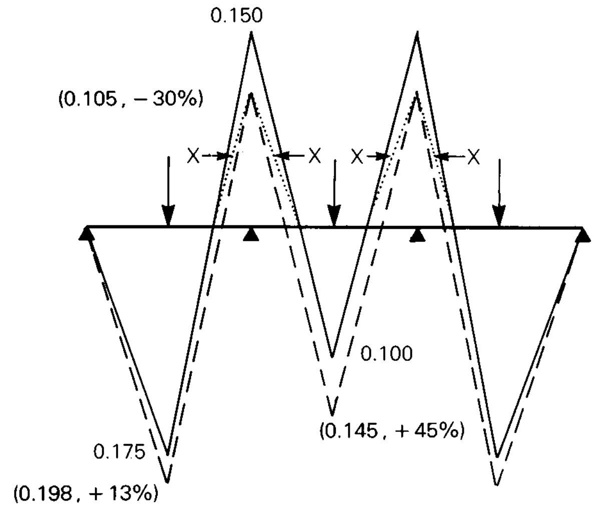
(a) dead load only.



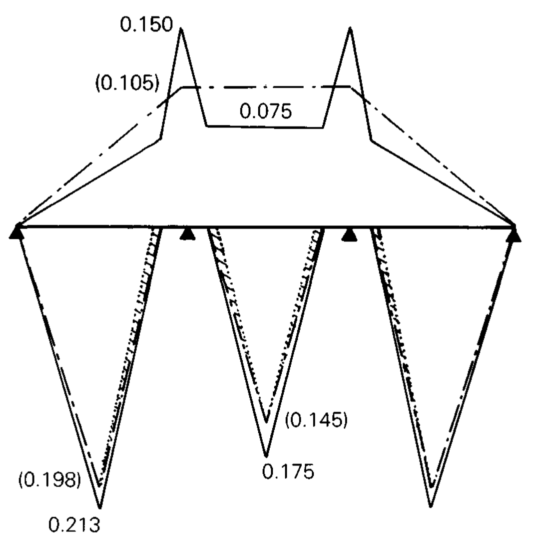
(b) maximum negative moment in end spans, maximum positive moment in central span.



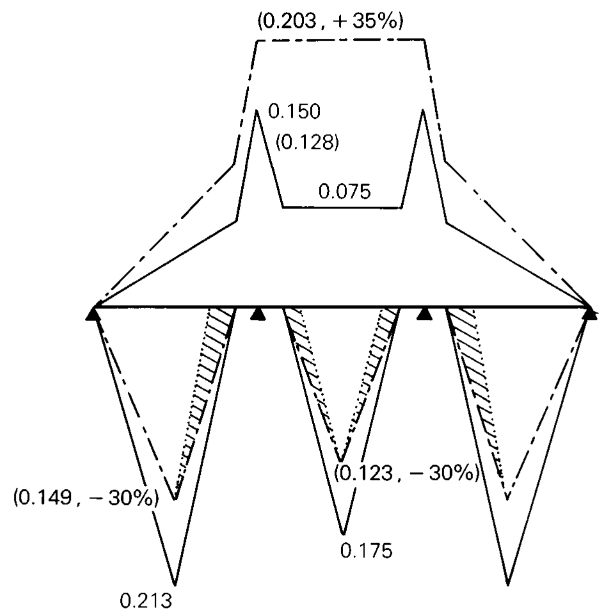
(c) maximum positive moment in end spans, maximum negative moment in central span.



(d) maximum negative moment at support (as BS8110).



(e) with maximum reduction of negative moments.



(f) with maximum reduction of positive moments.

Figure 3.2 (a)–(d) Maximum bending-moment diagrams before redistribution (full lines) and after (broken lines); (e)–(f) combined imposed-load envelopes before redistribution (full lines) and after (chain lines).

* Figures within brackets correspond to redistributed moments and percentage adjustments made to adjoining values.

explained in section 3.3, it is sometimes advantageous to reduce the span moments to the fullest extent (perhaps if upstand beams are employed) or to partially equalize the span and support moments. *Figure 3.2f* illustrates the bending-moment envelope obtained when all the span moments are reduced to 70% of their original values and the support moments are increased accordingly.

The position and magnitude of the maximum positive bending moments are readily calculated for beams carrying concentrated loads. For a beam such as LR of span l in *Figure 3.3*, carrying a uniformly distributed load of intensity n and subjected to theoretical negative support moments M_{LR} at L and M_{RL} at R, the position and magnitude of the maximum positive bending moment are given by

$$X = \tfrac{1}{2}l + \frac{M_{LR} - M_{RL}}{nl}$$

$$M_{max} = \frac{n}{2}\left(\frac{M_{LR} - M_{RL}}{nl} + \frac{l}{2}\right)^2 - M_{LR}$$

If the negative bending moments at the supports are each increased by x%, the modified formulae are

$$X' = \tfrac{1}{2}l + \frac{1 + 0.01x}{nl}(M_{LR} - M_{RL})$$

$$M'_{max} = \frac{n}{2}\left[\frac{1 + 0.01x}{nl}(M_{LR} - M_{RL}) + \frac{l}{2}\right]^2 - (1 + 0.01x)\,M_{LR}$$

The coefficients for the maximum positive bending moments tabulated in *Data Sheet 9* have been calculated from these expressions by taking $l = 1$ and $n = 1$.

Unless the support moments M_{LR} and M_{RL} are greatly dissimilar, it is generally sufficiently accurate to determine the maximum positive moment by subtracting the mean of the two support moments from the free moment.

In calculating the foregoing bending-moment coefficients it is assumed that the beams are freely supported at the outer supports. In the cases of beams framing into columns and of slabs which are monolithic with large supporting beams, however, negative bending moments occur at the end supports. These bending moments must be resisted and allowance should be made for their effect on the bending moments on adjoining spans. *Data Sheet 14* indicates the effect of a unit bending moment applied at one end, or at both ends simultaneously, of a series of continuous equal spans. The corresponding bending-moment diagrams should be superimposed upon the normal diagrams resulting from the case of free end support. It is seen from the coefficients in *Data Sheet 14* that a bending moment which is applied at an end support affects to any extensive degree only the bending moment at the penultimate support; other than in exceptional conditions, the effect on other supports and spans (beyond the first and second) may be ignored. Since the effect is to reduce the negative bending moment at the penultimate support, the bending moment applied at the end must not be overcalculated when making this reduction. The magnitude of the bending moment applied at the end may be calculated from the formulæ given in BS8110 for bending on exterior columns (see section 14.2) or from considering the beam as being a member of a monolithic frame (see section 3.8), or it may be produced by a cantilever extending beyond the end support.

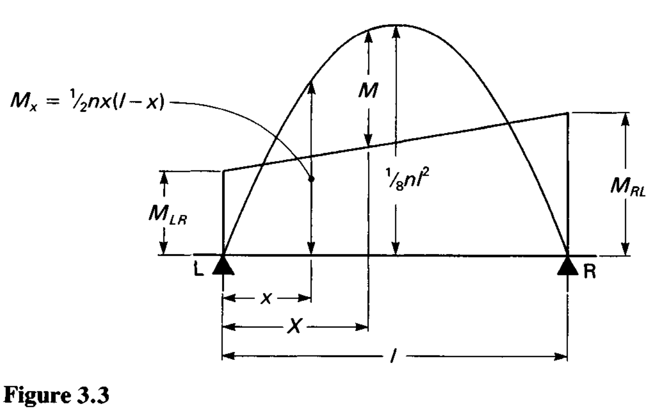


Figure 3.3

3.5 FORMULÆ FOR UNIFORM LOADS AND EQUAL SPANS

The use of tabulated coefficients is most convenient when the loads are concentrated and the resulting maximum moments occur beneath the loads. With distributed loads, however, it is more difficult to combine the maximum values that occur due to dead and imposed loads (since they occur in slightly different positions) and to sketch the resulting overall envelope of bending moments. In such cases it may be simpler to calculate the critical total moments produced by the combined dead and imposed loads from bending-moment formulæ. The expressions given in *Data Sheets 7 and 8* enable the moments at the supports and near midspan for beam and slab systems that are continuous over two, three or a theoretically infinite number of spans to be determined when loaded uniformly. Separate expressions are given for the moment at each point due to each arrangement of loading; these enable the bending-moment envelope to be sketched to a fair degree of accuracy if required. For many purposes, however, only a knowledge of the maximum values is necessary, and the appropriate formulæ for this purpose are enclosed in boxes on *Data Sheets 7 and 8*. If g is the characteristic dead load and $n = g + q$, the resulting moments are the normal elastic values. However, if g is taken as $1.0g$ and $n = 1.4g + 1.6q$, the resulting moments are the critical ultimate values required for design to BS8110.

The use of formulæ catering directly for the combined moments due to dead and imposed loads, such as those given on *Data Sheets 7 and 8*, is less convenient if moment redistribution is to take place. For this reason, *Data Sheet 9* gives the critical moment coefficients for various continuous-beam systems supporting uniform or central concentration loads for three conditions, namely: (a) with no moment redistribution; (b) with 10% redistribution; and (c) with a maximum redistribution of 30%. When using *Data Sheet 9* it is necessary to determine the moments due to dead loads of $1.0G_k$ and 'imposed' loads of $0.4G_k$ separately (taking into account the degree of redistribution required), and then to sum the resulting values.

3.6 ANALYSIS OF CONTINUOUS SYSTEMS

If the beams or slabs are continuous over several unequal spans, if the moment of inertia differs from span to span, or if the applied loading is not one of the three arrangements considered above, the system must be analysed by means of one of the so-called exact methods. These methods may be divided into two basic groups. The first category, which includes such methods as the theorem of three moments, slope deflection, least work etc., consists basically of setting up and solving a series of simultaneous equations to obtain exact results. Such equations are tedious to solve when a solution has to be obtained by hand methods, but are ideal when there is convenient access to a computer. If, however, the values obtained from a preliminary design require adjustment, the analysis must usually be repeated in full; this recycling process is continued until the resulting design is satisfactory, and it may therefore be impracticable if long delays occur between submitting the values for computation and receiving the results. In such cases it may well be advantageous to prepare an approximate design by trial and adjustment, using one of the methods described below, and only to employ the computer to check the design finally chosen and to calculate the exact bending moments for which the sections must be designed.

The other category of analytical methods consists of various iterative processes including relaxation and moment distribution. These methods involve a cyclic procedure whereby the final solution is approached in stages, each successive adjustment bringing the interim results progressively closer. If a computer is not available, these methods have two basic advantages over those involving simultaneous equations. The first is that the actual computational procedure is usually extremely simple. For example, in the well-known method of moment distribution the moments at each end of a span are first calculated on the assumption that the ends of each individual span are fully fixed. The ends of the members meeting at each particular intersection in turn are then assumed to be released, and the resulting out-of-balance 'fixed-end' moments at that joint are distributed between them in direct proportion to their stiffnesses. The next step is to 'carry over' a proportion (one-half in the case of prismatic members) of the distributed moments to the opposite ends of the members. These carried-over moments are then again distributed between the intersecting members at each individual joint, a further carry-over operation takes place, and so on. As the differences between the moments at the ends of the members meeting at a particular intersection become less and less, so the values of unbalanced moments to be distributed and then to be carried over become progressively smaller; thus a summation of the total moment at the end of a particular member more nearly approaches its true value. In theory, the cyclic procedure should be repeated until the moments being distributed and carried over are negligible. However, in practice two, or at the most three, complete cycles are usually sufficient with prismatic members to obtain results that are within a few per cent of their exact values, an accuracy that is quite sufficient in view of the uncertainties made in the basic assumptions regarding the stiffnesses of the members etc.

The second basic advantage of iterative methods is that it is often fairly clear, even after only one distribution cycle, whether or not the final values will be acceptable. If they are not, the analysis need not be continued further, thus saving much unnecessary work.

3.7 PRECISE MOMENT DISTRIBUTION

The original Hardy Cross method of moment distribution is too well known to warrant a detailed description here, since there are already many books dealing specifically with the subject. For the generation of reinforced concrete designers that came to structural analysis after the method was first conceived in the early 1930s and before the widespread use of computers, it became the most popular method of analysing series of continuous beams, and its continued popularity has led to the introduction of numerous developments and extensions of the original concept.

One such variant is a hybrid, combining features of both of the foregoing types of solution. Known most commonly as precise moment distribution, although it has also been referred to as the coefficient-of-restraint method, it is closely related to the method of fixed points and the degree-of-fixity method. The analytical procedure is extremely similar to and only slightly less simple than normal moment distribution, but the distribution and carry-over factors are so adjusted that an exact solution is obtained after only a single distribution in each direction. The method thus retains the advantage, when using hand computation, of eliminating the need to decide when to terminate the successive approximation procedure. Since it is perhaps less widely known than should be the case, its use to analyse series of continuous spans formed of prismatic members is now described. Owing to considerations of space, details of the theoretical basis of the method are kept to a minimum, since these are available elsewhere (refs 6–8). The few formulæ that are required are easy to memorize and the use of graphs or nomograms is not necessary, although the analysis may be undertaken even more quickly if they are employed. Alternatively, programming the formulæ for a