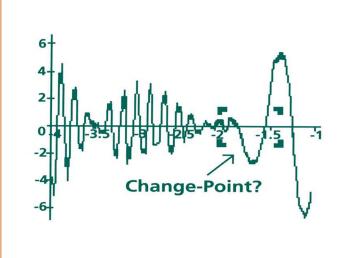
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# Applied Sequential Methodologies

**Real-World Examples with Data Analysis** 



edited by Nitis Mukhopadhyay Sujay Datta Saibal Chattopadhyay

## Applied Sequential Methodologies

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In celebration of the brilliant career of Professor Anis Mukhopadhyay, my elder brother and teacher, and in recognition of his recent retirement from the Indian Statistical Institute, Calcutta, this volume is presented to him with love and affection.

Nitis Mukhopadhyay

In the memory of my late father, in recognition of my mother's lifelong dedication to the well-being of her son, with deepest gratitude to my ever-inspiring wife, and in loving acknowledgement of many sweet distractions from my two-year old daughter.

Sujay Datta

In loving memories of my late sister and father, in admiration of the untiring effort of my mother toward my upbringing, and in recognition of the support and encouragement of my wife and children.

Saibal Chattopadhyay

### APPRECIATION

To those colleagues who most kindly offered to help and freely shared their expertise and vision at various junctures of editing this volume, the Co-Editors express their sincerest gratitude and appreciation.

Many colleagues helped tremendously in the editorial process by diligently sharing the burden of refereeing one or more articles. What a difference each individual has made! The Co-Editors thank each referee for showing unselfish dedication and unmistakable enthusiasm.

> The Co-Editors consider it a privilege on their part to mention all referees by name:

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## Preface

Since the publication of Abraham Wald's classic text, *Sequential Analysis*, in 1947, a particularly impressive list of monographs has appeared in this field. These have led to an enormous growth in research methodologies. Some monographs boldly charted the research-track and created an indelible mark of tradition that is so well-known in sequential analysis. It is a fitting tribute to the authors that these volumes continue to serve as flag-bearers and resource guides in this field. We cite some of these influential volumes here:

- Bechhofer, R.E., Kiefer, J. and Sobel, M. (1968). Sequential Identification and Ranking Procedures. University of Chicago Press: Chicago.
- Berry, D.A. and Fristedt, B. (1985). *Bandit Problems*. Chapman & Hall: New York.
- Chernoff, H. (1972). Sequential Analysis and Optimal Design. CBMS #8. SIAM: Philadelphia.
- Chow, Y.S., Robbins, H. and Siegmund, D. (1971). *Great Expectations: The Theory of Optimal Stopping*. Houghton Mifflin: *Boston*.
- Ghosh, B.K. (1970). Sequential Tests of Statistical Hypotheses. Addison-Wesley: Reading.
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- Ghosh, M., Mukhopadhyay, N. and Sen, P.K. (1997). Sequential Estimation. Wiley: New York.
- Govindarajulu, Z. (1981). The Sequential Statistical Analysis. American Sciences Press: Columbus.
- Gut, Allan (1988). Stopped Random Walks: Limit Theorems and Applications. Springer-Verlag: New York.
- Mukhopadhyay, N. and Solanky, T.K.S. (1994). Multistage Selection and Ranking Procedures. Marcel Dekker: New York.
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- Wald, A. (1947). Sequential Analysis. Wiley: New York.
- Wetherill, G.B. (1975). Sequential Methods in Statistics, 2<sup>nd</sup> ed. Chapman & Hall: London.
- Woodroofe, M. (1982). Nonlinear Renewal Theory in Sequential Analysis. CBMS #39. SIAM: Philadelphia.

While one continues to draw inspiration from these exclusive publications, we may add that some leading 'non-sequential' books have included important material from this area too. We cite, for example, the following monographs:

- Bechhofer, R.E., Santner, T.J. and Goldsman, D.M. (1995). Design and Analysis of Experiments for Statistical Selection, Screening, and Multiple Comparisons. Wiley: New York.
- Gibbons, J.D., Olkin, I. and Sobel, M. (1977). Selecting and Ordering Populations. Wiley: New York.
- Gupta, S.S. and Panchapakesan, S. (1979). *Multiple Decision Theory*. Wiley: New York.

Rao, C.R. (1973). *Linear Statistical Inference*, 2<sup>nd</sup> ed. Wiley: New York. Zacks, S. (1971). *The Theory of Statistical Inference*. Wiley: New York.

Wald's monograph was unique in its style in 1947 and in many ways it still remains unique largely because Wald's elegantly original mathematical and statistical contributions played a fundamental role in solving practical problems of real-life importance at the time. We surmise, however, that over the years these other volumes have pointed more toward theoretical advancements. Directly or indirectly, purely theoretical contributions have received more encouragement from many quarters and hence the theory of sequential analysis has indeed become very rich. Unfortunately, at the same time, real applications have taken serious hits.

We are personally convinced that this field can and should interface with every conceivable applied area of statistics. But since this field has not been accessible to practitioners for widespread real-world applications, we believe that its popularity among statisticians has dwindled. Real-life experimental data are rarely presented or discussed in sequential books and journal articles. This frustrating situation amounting to what may be viewed as a 'death sentence' has developed over many decades and sadly, this otherwise attractive field with such great promise has alienated itself nearly completely from most practitioners in statistical sciences.

One notable exception, in our view, is the area of clinical trials which has continued to be the major beneficiary of some of the basic research in sequential methodologies. Again, we cite some influential volumes in this area:

Armitage, P. (1975). *Sequential Medical Trials*, 2<sup>nd</sup> ed. Blackwell Scientific Publications: Oxford.

Jennison, C. and Turnbull, B.W. (1999). Group Sequential Methods with Applications in Clinical Trials. Chapman & Hall: London.

#### Preface

## Rosenberger, W.F. and Lachin, J.M. (2002). Randomization in Clinical Trials: Theory and Practice. Wiley: New York.

But, a specialized field such as sequential analysis cannot be expected to thrive solely on applications in just one area of statistics. It is time for everyone involved to join an aggressive pursuit of real applications of sequential methodologies in as many contemporary and interesting problems of statistics as possible. We believe that time is quickly running out for purely theoretical researchers in this field to continue building newer levels of ivory towers and living in them!

Together, we all must make sequential analysis accessible to all practitioners in statistics. The idea that it is all right for sequential analysis to remain esoteric since a practitioner can seek assistance from a sequential analyst whenever needed remains as far-fetched as ever. That attitude has not worked in the last fifty years and is certainly not about to work now. We urge sequential analysts to take the initiative to vigorously 'market' their methodologies themselves — someone else can hardly ever be expected to do that for us. The field has survived thus far largely by perpetuating the idea of potential applications in the sense that somebody else may eventually use sequential methodologies somewhere in solving real-life problems some day! But, when we look at the bigger picture today, it becomes abundantly clear that sequential analysis has nearly lost its deserving place in the realm of applied statistics. This field has been ignored by nearly every practicing statistician. This is why we strongly feel that it is incumbent on all researchers in sequential analysis to try to rebuild this field's image and market their products themselves. It may eventually mean the difference between 'life' and 'death' of our wonderful field.

We urge everyone to energetically engage in turning the situation around in a positive way because there is still a great deal of hope out there. We believe that the spectrum of applications of sequential methodologies is much broader than what one finds in some of the so-called mainstream statistical monographs and journals. A variety of interesting and important real applications already exist. We dare to dream that the present volume will help in narrowing the unhealthy gap that has existed far too long between the theory and practice of sequential methodologies in problems and issues of broader interest. For us it will indeed be a dream come true if this volume serves as a catalyst to raise the level of consciousness of all sequential analysts about the current status of the field and to inspire them to fine-tune the focus of their initiatives appropriately from time to time so that sequential analysis becomes more relevant to contemporary statistical applications.

The contributing authors for the present volume of collected papers

were earnestly requested to adhere to a set of general guidelines including the following:

"Every article should discuss clearly at least one substantive applied problem and the appropriate sequential method(s). Tangential references to potential applications are strongly discouraged. A specific application should remain in focus and guide throughout the development and/or implementation of a methodology. That is, each article should justify the relevance, importance, and usefulness of sequential methodology by highlighting an application and the associated gains with the help of real data. Theoretical developments, specific to a problem on hand, will be most welcome but their practical usefulness should be demonstrated.

Real applications are encouraged rather than potential applications. An article will preferably include the data or refer its readers to the source of the data or provide a web-site-address if appropriate. Each article will be anonymously referred.

The exposition should be such that any interested reader may readily appreciate the importance of the practical problem(s) discussed and the conclusions drawn. The idea is that the variety of applied problem(s) considered in this volume will ultimately entice readers to take a look at the methodologies even if they do not consider themselves as sequential analysts. We hope to demonstrate that mathematical sophistication and complexity need not deter enthusiastic practitioners to take a look at this field which has plenty to offer in terms of everyday statistics and as it turns out, sequential methodologies are indeed often essential for solving today's challenging practical problems. It is our belief that with sufficient care, the technical coverage can be judiciously blended with lucidity of presentation so that the volume may remain accessible to many users including graduate students and budding researchers, statisticians or otherwise, looking for exposure to this area. At the same time, some hard-core researchers in sequential analysis would be expected to benefit significantly from seeing real-world applications of our craft."

We had a modest set of goals. We clearly understood that we simply could not continue doing business as usual. We wanted to present the material in such a way that sequential analysts would get a taste of real-life problemsolving which could, in turn, inspire more methodological work in the near future. At the same time, we wanted to make sure that those scientists who were not thoroughly familiar with sequential analysis would also benefit from this volume by observing sequential methodologies at work in the real world. We thank the authors for trying their v ery best to address our seemingly unending list of demands like these and others.

In the early stages of planning, we invited a number of leading scientists in many substantive areas of applications including Agricultural Statistics, Animal Abundance, Bayesian Strategies, Biometry, Clinical Trials,

#### Preface

Computer Simulation, Data Mining, Ecology, Engineering, Finance, Fisheries, Genetics, Multiple Comparisons, Multivariate Analysis, Nonparametrics, Psychology, Sonar Detection, Tracking, and Time Series to contribute specially prepared articles. However, on account of tight deadlines set by us or due to other commitments, some invited authors could not participate in this project. We deeply regret this and their contributions are sorely missed. To those who have kindly participated in our crusade to revive our field's relevance and image in today's statistical world, we remain eternally grateful.

To our true delight, we report that this volume includes interesting methodological articles on:

- *passive acoustic detection of marine mammals* (Abraham)
- selecting the best component (Aoshima, Aoki, and Kai)
- randomization tests (Banerjee and Ghosh)
- multistate processes (Barón)
- adaptive designs for clinical trials with longitudinal responses (Biswas and Dewanji)
- *data mining* (Chang and Martinsek)
- approximations for moving sums of discrete random variables (Chen and Glaz)
- *measurement-error model* (Datta and Chattopadhyay)
- density estimation of wool fiber diameter (de Silva and Mukhopadhyay)
- financial applications of nonparametric curve estimation (Efromovich)
- *interim and terminal analyses of clinical trials* (Lai)
- *tests for target tracking* (Li and Solanky)
- multiple comparisons (Liu)
- *designing computer simulations* (Mukhopadhyay and Cicconetti)
- estimation in the agricultural sciences (Mulekar and Young)
- contrasting group-sequential and time-sequential interim analysis in clinical trials (Sen)
- change-point detection in multichannel and distributed systems (Tartakovsky and Veeravalli)
- two-stage multiple comparison procedures in Psychology (Wilcox)
- testing in the agricultural sciences (Young), and
- ordering genes (Zacks and Rogatko).

We believe that this is quite an impressive list indeed.

More than one colleague refereed each paper anonymously. The authors revised their manuscripts diligently by taking into account all constructive suggestions and criticisms from the referees. What one finds in this volume is a direct result of total commitment as well as unending patience and support from all parties involved. We remain indebted to this enthusiastic group of colleagues.

We admit that the twenty articles included here are not all written at the same level and personally, we view this disparity positively. Seeing this unevenness in some places, the readers will probably come to realize more that routine phrases such as "applied statistics" and "usefulness of a methodology" are also subject to interpretation.

It is our belief that many students, researchers, or practitioners will find in this volume some important and interesting material. The volume can be used both as reference material as well as a solo textbook. One can also use it as a companion with another book while offering a senior undergraduate or graduate level course in sequential methods. An experienced teacher may also discover a number of h idden or not-so-hidden ideas on conducting hands-on practical experiments to gather real or realistic data that would make a traditional offering of a course in sequential analysis more interesting, lively, and above all, relevant.

Even though this is a substantial volume in itself on applications of sequential methodologies, in no way do we claim that this represents all types of applications. To be truthful, it is far from it. We hope to have other opportunities in the future to be more inclusive and capture a greater diversity of applications. If the present volume makes readers realize that this is a field with a great deal of promise for both intra-disciplinary work in statistics as well as for all types of inter-disciplinary work, then that will be our most gratifying reward.

It has been a real pleasure to work with the editorial and production staff at Marcel Dekker in planning and completing this project. We specially mention Ms. Maria Allegra and Ms. Helen Paisner and thank them both. Without their patience and constant support, this project could not have reached this stage. We are also thankful to several technical experts who helped us at various stages of the compilation process, especially to Dr. Andrew A. Poe from the Department of Mathematics and Computer Science, Northern Michigan University and Professor Uttam Sarkar from the Indian Institute of Management Calcutta. In addition, two of us (Datta and Chattopadhyay) gratefully acknowledge the support received in the form of a faculty grant from Northern Michigan University and a research grant from the Center for Management Development Studies, Indian Institute of Management

#### Preface

Calcutta, respectively. One of us (Mukhopadhyay) gratefully acknowledges the support received through a sabbatical <u>bave</u> from the University of Connecticut in the fall semester 2003 and expresses heartfelt gratitude to Dr. Tom F. Babor, Professor and Head of the Department of Community Medicine, University of Connecticut Health Center, for gracious hospitalities and use of their facilities.

Both collectively and individually, we express indebtedness to our colleagues, students and staff at our home institutions. We must apologize, however, for not mentioning them by name. Mr. Ranjan Mukhopadhyay and Ms. Cathy Brown have rendered invaluable help during the final preparation of a camera-ready copy. Our sincerest thanks go to both Ranjan and Cathy.

Finally, we express our deepest sense of gratitude to our families for their never-ending encouragement and love that gave us the ultimate courage to shoulder this challenging project in the first place.

> Nitis Mukhopadhyay Sujay Datta Saibal Chattopadhyay



## Contents

Prefa	ace		v
Cont	ributi	ng Authors	xxi
1.		ive Acoustic Detection of Marine Mammals g Page's Test Douglas A. Abraham	1
	1.1	Introduction	1
	1.2	Acoustic Detection of Marine Mammals	3
	1.3	Methodology and Analysis	4
		1.3.1 Algorithm Specifics	8
		1.3.2 Detection Results	10
		1.3.3 Localization	11
	1.4	Concluding Remarks	16
	Refere	ences	16
2.	Two-	-Stage Procedures for Selecting the Best	
	Com	ponent of a Multivariate Distribution	19
		Makoto Aoshima, Mitsuru Aoki and Masaki Kai	
	2.1	Introduction	19
	2.2	A Specific Application	21
		2.2.1 The Rule $R_1$	22
		2.2.2 The Rule $R_2$	24
	2.3	Methodologies and Analysis	26
		2.3.1 Two-Stage Procedure for $R_1$	26
		2.3.2 Two-Stage Procedure for $R_2$	29

	2.4 Refe	Concluding Remarks rences	30 33
3.	Seq	uential Randomization Tests Tathagata Banerjee and Onkar Prosad Ghosh	35
	3.1	Motivating Background	35
	3.2		38
	3.3	Sequential Estimation of a p-Value	41
	3.4	An Illustrative Example	44
	3.5	Concluding Remarks	47
	Refe	rences	48
4.	Seq	uential Methods for Multistate Processes Michael I. Baron	53
	4.1	Introduction and the General Scheme	53
	4.2	Analysis and Prediction of Electricity Prices	56
	4.3	Methodology and Implementation	59
	4.4	Other Applications	63
	4.5	Concluding Remarks	64
	Refe	rences	65
5.	Sea	uential Adaptive Designs for Clinical	
21	-	als with Longitudinal Responses	69
		Atanu Biswas and Anup Dewanji	07
	5.1	Introduction	69
	5.2	Longitudinal Binary Responses	72
	5.3	Longitudinal Ordinal Responses	76
	5.4	Longitudinal Multivariate Ordinal Responses	77
	5.5	Incorporating Prognostic Factors	78
	5.6	Longitudinal Continuous Responses	79
	5.7	An Application	80
	5.8	Concluding Remarks	81
		rences	83
6.	Sea	uential Approaches to Data Mining	85
	1	Yuan-Chin Ivan Chang and Adam Martinsek	

xiv

6.1 Introduction 85

Con	tents		XV
		6.2.1 Clustering Tree	88
		6.2.2 Stratification	93
		6.2.3 Computational Complexity	95
	6.3	Sequential Fixed-Width Interval Estimation	96
	6.4	Concluding Remarks	100
	Refer	rences	101
7.	Арр	proximations and Bounds for Moving	
	Sum	<b>is of Discrete Random Variables</b> Jie Chen and Joseph Glaz	105
	7.1	Introduction	105
	7.2	The Distribution and Expectation of $W_{k,m}$	107
		7.2.1 Bernoulli Model	108
		7.2.2 Binomial and Poisson Models	110
	7.3	Numerical Results	112
	7.4	Concluding Remarks	113
	Refe	rences	119
8.		mation of the Slope in a Measurement-Erro	
	Mod	Iel Sujay Datta and Saibal Chattopadhyay	123
	0.1		100
	8.1	Introduction and Motivation	123
	8.2 8.3	Justification of Model and Literature Survey	126 128
	0.3	Methodologies and Analysis 8.3.1 A sketch of the Proof of Theorem 8.3.2	128
	8.4	Concluding Remarks	132
	8.5	Appendix	134
		rences	136
9.	Ker	nel Density Estimation of Wool Fiber	
		neter	141
	Diai	Basil M. de Silva and Nitis Mukhopadhyay	141
	9.1	Introduction	141
	9.2	The Formulation	143
	9.3	Literature Review and Specific Aims	146
		9.3.1 Specific Aims	147
	9.4	A Two-Stage Procedure	148
		941 Band-Width Selection	148

	9.5 9.6 9.7 9.8 Apper Refere	Application to Wool Fiber Diameter Data Concluding Remarks ndices	150 152 153 156 158 159 167
10.	Fina	ncial Applications of Sequential	
	Nonj	parametric Curve Estimation Sam Efromovich	171
	10.1	Introduction	171
		Capital Asset Price Model	173
	10.3	•	175
	10.4		180
	10.5	••	189
	10.6	Concluding Remarks	190
	Refer	•	191
11.	Tria	rim and Terminal Analyses of Clinical ls with Failure-Time Endpoints and ted Group Sequential Designs	193
	КСТА	Tze Leung Lai	175
	11.1	Introduction	193
	11.2	Interim and Terminal Analysis of BHAT Data	194
		11.2.1 Analysis by the Data and Safety	
		Monitoring Board	195
		11.2.2 Further Analysis of BHAT Data	196
	11.3	Design and Analysis of Group (or Time) Sequential	
		Trials	197
		11.3.1 Time-Sequential Rank Statistics and Their	
		Asymptotic Distributions	199
		11.3.2 Two Time Scales and Some Stopping Rules	202
		11.3.3 Theory of Group Sequential Boundaries	204
		11.3.4 Stochastic Curtailment	209
		11.3.5 Confidence Intervals in Group Sequential	
		Designs	210
	11.4 Dofor	Concluding Remarks	210 213

xvi

Contents			xvii
12.		lications of Sequential Tests to Target Eking by Multiple Models X. Rong Li and Tumulesh K. S. Solanky	219
	12.1	Introduction	219
	12.2	An Example	222
	12.3	12.3.1 Sequential Solutions for Problems Involving	223
		Two Model-Sets	224
		12.3.2 Sequential Solutions for Multihypothesis Problems	231
	12.4	Aircraft Motion Example: Evaluating Performance	237
	12.4	Appendix	243
	Refer		243
13.	A Se	quential Procedure that Controls Size and	
100		er in a Multiple Comparison Problem Wei Liu	249
	13.1	Introduction	249
	13.2		251
	13.3	6	254
	13.4	Concluding Remarks	255
	Refer		257
14.	How	Many Simulations Should One Run? Nitis Mukhopadhyay and Greg Cicconetti	261
	14.1	Introduction: One-Sample <i>t</i> -Test	261
	14.2	Formulation, Loss and Risk Functions, and Objective	263
		14.2.1 The Formulation	267
		14.2.2 A Relative Squared Error Loss Function	267
		14.2.3 The Objective: Prespecified Proportional Accuracy	269
	14.3	A Purely Sequential Methodology	270
	14.4	A Two-Stage Methodology	272
		14.4.1 The Motivation	273
	14.5	Computer Simulations	275
		14.5.1 The Purely Sequential Methodology	
		(14.3.2)-(14.3.3)	276
		14.5.2 The Two-Stage Methodology	
			A

/ii

xviii			Contents
	14.6	<ul> <li>14.5.3 Comparing the Two Methodologies and Conclusions</li> <li>Large-Sample Properties</li> <li>14.6.1 The Purely Sequential Methodology (14.3.2)</li> </ul>	281 284
		(14.3.3): Proof of Theorem 14.6.1 14.6.2 The Two-Stage Methodology (14.4.1)-(14.4	285 4.2):
	Refere	Proof of Theorem 14.6.1 ences	288 289
15.	Seau	ential Estimation in the Agricultural	
	Scie	•	293
	Sele	Madhuri S. Mulekar and Linda J. Young	<b>2</b> /3
	15.1	Introduction	293
	15.2		295
	15.3		297
	15.4	Concluding Remarks	312
	Refer		313
16.	Whi	ther Group-Sequential or	
	Tim	e-Sequential Interim Analysis in	
		ical Trials?	319
	0	Pranab K. Sen	• • • •
	16.1	Introduction	319
	16.2		321
	16.3	•	327
	16.4	Evolution of Time-Sequential Methods	330
	16.5	Concluding Remarks	334
	Refer	ences	336
17.		nge-Point Detection in Multichannel	
	and	Distributed Systems with Applications	339
		Alexander G. Tartakovsky and Venugopal V. Veeravalli	
	17.1	Introduction	339
	17.2	Multichannel-Change Detection: Theory	340
		17.2.1 Problem Formulation	340
		17.2.2 The Detection Procedure and False	
		A 1 D-4-	242

#### Contents

		17.2.3 Asymptotic Performance for Low FAR	343
		17.2.4 Composite Hypotheses: Adaptive Detection	
		Procedures	347
	17.3	Detection in Distributed Sensor Systems	349
	17.4	Applications and Experimental Results	353
		17.4.1 Target Detection and Tracking in Surveillance	
		Systems	353
		17.4.2 Rapid Attack/Intrusion Detection	357
		17.4.3 Decentralized Detection Example and	
		Simulation Results	359
	17.5	Concluding Remarks	362
	Apper		363
	Refer	ences	367
18.	Exte	nsion of Hochberg's Two-Stage	
	Mult	tiple Comparison Method	371
		Rand R. Wilcox	
	18.1	Introduction	371
	18.2		373
	18.3		374
	18.4		377
	18.5	Concluding Remarks	379
	Refer	ences	379
19.	Seau	ential Testing in the Agricultural	
	Scier	<b>e</b>	381
	20101	Linda J. Young	001
	19.1	Introduction	381
	19.2		382
	19.3		383
	19.4	The 2-SPRT	389
	19.5	Binomial Sampling Based on the Negative	
		Binomial	393
	19.6	Binomial Sampling Based on an Ecological	
		Model	397
	19.7	Concluding Remarks	399
	Refer	ences	403

20.	Bayesian Sequential Procedures for			
	Ordering Genes			
		Shelemyahu Zacks and André Rogatko		
	20.1	Introduction: The Gene Ordering Problem	411	
	20.2	Basic Model of Independence	413	
		20.2.1 Orders, Gametes and Recombination	413	
		20.2.2 The General Case	413	
	20.3	Bayesian Testing	416	
		20.3.1 Posterior Order Probabilities	416	
		20.3.2 Bayesian Sequential Testing	417	
		20.3.3 Bayesian Testing with Fixed Sample Sizes	423	
	20.4	Stepwise Ordering	424	
		20.4.1 Stepwise Search for Maximal Posterior		
		(SSMAP)	424	
	20.5	Applications in Genetic Counseling	428	
	Refer	ences	430	
Autl	nor Inc	dex	433	
Subject Index			443	

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#### xxiv

## Chapter 1

## Passive Acoustic Detection of Marine Mammals Using Page's Test

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#### 1.1 INTRODUCTION

Sonar signal processing is a subset of signal processing related to the analysis of acoustic signals measured underwater. Applications of sonar signal processing lie in diverse fields such as oil exploration, marine mammal study and naval warfare. The fundamental objectives in sonar signal processing are the detection, classification, and localization of sounds that are heard under water. Sonar systems can be either active or passive in their use of sound (Burdic (1984), Urick (1983)). Passive sonar systems only process signals that are recorded on underwater microphones called hydrophones. A typical sonar system will use many hydrophones that are held together physically in what is called an array. An active sonar system transmits a signal using an underwater loud speaker and processes the subsequently heard reflections.

Many of the signal processing algorithms that are used in sonar systems were developed using methodologies from statistical decision theory. Detectors and classifiers may be formulated as binary and multiple hypothesis tests. Localization is simply the estimation of the physical location of the sound emitter or reflector. Sequential methodologies have primarily been used for the detection of sounds, but have also seen use in localization procedures such as target tracking (Lerro and Bar-Shalom (1993)). This paper will focus on the application of the cumulative summation type of sequential procedure called Page's test. Page (1954) developed the procedure to determine the time at which a change in the distribution of sequentially obtained data had occurred. It has seen significant applications in areas such as fault detection (Basseville and Nikiforov (1993)), but has also been used for the detection of unknown but finite duration signals (Han et al. (1999)). It is this latter application that is useful in both active and passive sonar signal processing. Page's test, when it utilizes the log-likelihood ratio, is known to be optimal in the sense of minimizing the average delay before detection while constraining the average time between false alarms (Lorden (1971), Moustakides (1986)). Analysis of its performance at detecting finite duration signals is not trivial, though accurate approximations exist for simple configurations (Han et al. (1999)).

As an example of an active sonar signal processing application, Page's test has been used to detect target echoes when they are corrupted by propagation through a shallow water environment (Abraham and Willett (2002), Abraham (1996b)). In shallow water environments, the sound travels from the source to the target and then from the target to the receiving hydrophones through many paths. As the time it takes to traverse each path differs, the received signal appears to be spread over time. Owing to the difficulty in accurately modeling the ocean propagation, Page's test was applied to obtain adequate detection performance over a wide range of environmental conditions.

An example of a passive sonar signal processing application, and the focus of this paper, may be found in the detection of marine mammals by their acoustic emissions. Such non-invasive detection of marine mammals is necessary for the study of their habits as well as to aid in ensuring their absence prior to any potentially harmful activity such as oil exploration or naval testing. In this passive sonar application, the sounds generated by marine mammals are recorded on hydrophone arrays and must be discerned from all background noises. As marine mammals vary considerably in size and vocal characteristics, the detector must be quite flexible and able to detect both short and long duration signals with potentially widely varying frequency content. In the following sections the problem will be described in more detail along with the sequential detector that was implemented and some results from the analysis of real data. A more detailed description of the analysis may be found in the SACLANT Undersea Research Centre report (Abraham (2000)) from which most of this article is derived.

### 1.2 ACOUSTIC DETECTION OF MARINE MAMMALS

Detection of the presence of marine mammals is crucial in the vicinity of a research vessel carrying out operations involving the projection of acoustic energy into the local ocean environment. The following sections detail an analysis of passive acoustic data garnered from data recorded during the SACLANT Undersea Research Centre's SWAC4 sea-trial in Kyparissiakos Gulf off the coast of Greece during May 1996. Specifically, the automatic detection processing that was implemented will be described and examples of the data processing results presented.

The automatic detection of marine mammal acoustic emissions is a difficult task. The data available are in the form of passive sonar recordings from a hydrophone array that have been "beamformed" to emphasize signals coming from certain directions. This process increases the signal-to-noise ratio and aids in the localization of the sound source by dictating on which bearing it is heard. Unfortunately, it also means that the detection algorithm must be applied to the data from each beam (direction). The data from the SWAC4 sea-trial were beamformed to point to 120 directions spanning from forward to aft. Designing the detection algorithm is difficult owing to the wide variety of frequencies, bandwidths, and time characteristics of marine mammal acoustic emissions from different species. Take, for example, the sperm whale clicks shown in Figure 1.2.1. These data are in the frequency range of 750–1500 Hertz owing to the limitations imposed by the experimental conditions and acquisition systems. It is known that marine mammals emit sound over a much wider frequency band. Nevertheless, even with this limited band, it is seen that the spectrum is not necessarily flat and that the time series exhibits the effects of multipath propagation, spreading the signal energy in time. As other types of waveforms are expected (for example, whistles or sweeps) a detector is desired that is flexible both in the time duration of the waveform and its frequency content.

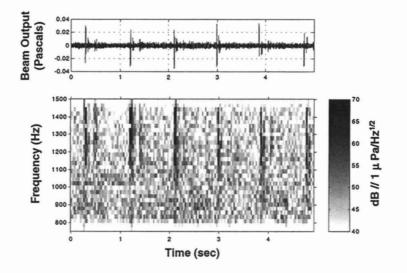


Figure 1.2.1: Time Series and Frequency Spectrum of Sperm Whale Click Train Limited to 750–1500 Hertz Frequency Band.

# 1.3 METHODOLOGY AND ANALYSIS

In hypothesis testing, there must be some difference between the null and alternative hypotheses in order for the Type-I and Type-II errors to be small. In terms of detector design, this translates into requiring that there be some means for distinguishing the signal to be detected from the background noise and any interferences. As this situation calls for a very general detector, not tuned to any specific characteristics of individual marine mammal emissions, the only distinction from the background or interferences is time duration. Thus, a detector is desired that finds short duration signals that are not similar to the more slowly changing background (for example, ambient ocean noise from the ocean surface or distant shipping) or interferences (for example, near-by surface vessels including the ship towing the hydrophone array). Additionally, the detector should be robust to varying signal duration and frequency content. A detector with these characteristics was proposed by Abraham and Stahl (1996) by combining Page's (1954) test with the power-law processor of Nuttall (1994) for the combination of discrete Fourier transform (DFT) bin outputs. A block diagram of this detector structure is shown in the left half of Figure 1.3.1. The time series data are transformed into the frequency domain by overlapping DFTs. The duration of the DFT should be near or less than the duration of the shortest signal that may be encountered. Define the magnitude squared of the DFT bins of interest for the  $k^{th}$  DFT as  $\{X_{k,1}, X_{k,2}, \ldots, X_{k,m}\}$ . For the data presented in this paper the bins of interest are those in the 750–1500 Hertz band. Let the estimated background power in the  $j^{th}$  DFT bin at time k be  $\lambda_{k,j}$  for  $j = 1, \ldots, m$ . As will soon be described, the background power levels in each DFT bin need to be estimated from previously observed data. These estimates are used to form normalized DFT bin data,

$$Y_{k,j} = \frac{X_{k,j}}{\lambda_{k,j}},\tag{1.3.1}$$

which are combined into a single test statistic by a power-law non-linearity,

$$Z_{k} = \left(\frac{1}{m} \sum_{j=1}^{m} Y_{k,j}^{p}\right)^{\frac{1}{p}}.$$
 (1.3.2)

The power-law non-linearity provides robustness against varying signal bandwidth or frequency structure and can have the effect of either picking the maximum DFT bin output (high power law) or summing all the DFT bins together (power law equal to one) as in an energy detector. This one statistic from each DFT is then used in Page's test to detect the onset of a signal. The update for Page's test has the form

$$W_{k+1} = \max\left\{0, W_k + Z_{k+1} - b_0\right\}$$
(1.3.3)

where  $b_0$  is a bias and a detection is declared when  $W_k$  exceeds a threshold. The bias is most easily chosen through Dyson's (1986) method which uses the average value of  $Z_k$  under the null and alternative hypotheses. In certain cases it is possible to choose the bias to optimize an asymptotic performance measure (Abraham (1996a)). The threshold may be chosen according to a desired average time between false alarms through the standard Wald or Siegmund approximations or quantization based approximations (Basseville and Nikiforov (1993), Brook and Evans (1972), Abraham and Stahl (1996)). Assuming perfect normalization, the performance of a power-law non-linearity feeding a Page

test is examined in terms of the average sample numbers (average time between false alarms and average delay before detection) in Abraham and Stahl (1996). A more relevant measure of detection performance is the probability of detection, which may be obtained from quantization based methods (Han et al. (1999)). Theoretical analyses such as those found in Abraham and Stahl (1996), Han et al. (1999) or Nuttall (1994) typically assume that the normalization is perfect and hence that the DFT bin data are exponentially distributed with unit mean when no signal is present. It is often assumed that the signal manifests itself as either a change in the scale or as a constant additive component to the complex DFT data, yielding a non-central chi-squared distribution for the DFT power data. Though these theoretical analyses help evaluate the performance of various detector structures under ideal conditions, the processing of real data introduces many difficulties ranging from data quality issues (for example, data glitches or drop-outs that can occur with varying frequency) to inadequate modeling (simplifications in the modeling that allow analysis but only approximately represent the real data). Thus, in practice, signal strength based parameters such as the bias in (1.3.3) are usually set for a minimum detectable signal level and thresholds are typically chosen so that the operator and any post-detection processing are not overloaded with false alarms.

In this application, Page's test provides robustness to the unknown duration of the signal compared with the use of a sliding fixed block detector that would be tuned to a single signal duration. Robustness in this sense means that the detector provides adequate, though perhaps sub-optimal, performance over a wide range of signal durations. As estimates of both the start and end times of the signal are desired, the alternating-hypothesis form of Page's test (Streit (1995), Abraham and Willett (2002)) must be implemented. The update structure for this form is similar to (1.3.3) and is described in the following section. In the alternating-hypothesis configuration, the start and end times of a detected event are estimated by the most recent reset of the Page test to its null state as described in Abraham (1997).

Necessary to the implementation of the detector is the estimation of the background noise and interference power at the output of each DFT bin. As these may be considered nuisance parameters (that is, parameters that need to be estimated but are not used to describe the signal), the scheme proposed in Abraham (1996c) which exploits the structure of the Page test to isolate data believed to be signal-free is appropriate. This detector structure uses data prior to the most recent

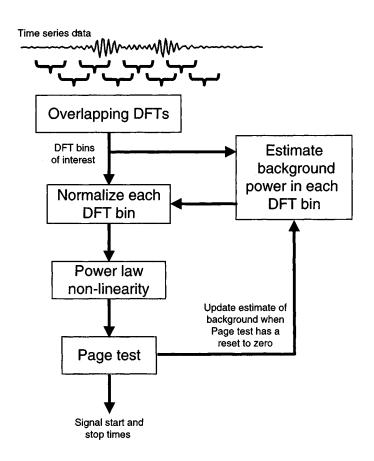


Figure 1.3.1: Block Diagram of Detection Scheme.

reset of the Page test to estimate the nuisance parameters. The form of the background estimator may be of a sliding block or exponentially averaged type. In this case, the latter is chosen because it is (marginally) easier to implement. The exponential averager simply applies a single pole filter to the time series; thus, updating the background estimate results in

$$\lambda_{k+1,j} = \alpha \lambda_{k,j} + (1-\alpha) X_{k,j} \tag{1.3.4}$$

where  $\alpha \in (0, 1)$  is usually near one to provide a long averaging time. Each time the Page test has a reset to zero, the background estimate is updated as in (1.3.4) but using only the non-signal data between the previous reset to zero and the current reset to zero (that is, if a signal was detected in between, those data would not be included). The estimates should be initialized with either a fixed block estimate or an exponential averager that has been running for a short time.

#### **Algorithm Specifics** 1.3.1

Following are the algorithm (in pseudocode) and definitions of the variables used to implement the detector along with their values when they require setting. The dependence of the variables on DFT snapshot k is suppressed as it is not necessary when implementing the algorithm.

#### **Detection Algorithm**

- (1) Initialization
  - Set  $W = 0, k = 1, i_0 = 1, i'_0 = 1$
  - Form initial estimate of  $\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$
- (2) Normalization and power-law
  - Form normalized DFT bin outputs  $Y_j = \frac{X_{k,j}}{\lambda_j} \text{ for } j = 1, \dots, m$ • Apply power-law to normalized data

$$Z = \left[\frac{1}{m}\sum_{j=1}^{m}Y_{j}^{p}\right]^{\frac{1}{p}}$$

- (3) If  $W < h_0$ ,
  - Set  $W = \max\{0, W + Z b_0\}$
  - If  $W \ge h_0$ ,
    - The leading edge of a signal has been detected
    - An estimate of the starting time index is  $i_0$
    - Set  $W = h_0 + h_1$  and  $i_1 = k$
  - Else if W = 0.
    - A reset to zero has occurred, update background estimate

for  $i = i'_0$  to k and  $j = 1, \ldots, m$ 

$$\lambda_j = \alpha \lambda_j + (1 - \alpha) X_{i,j}$$

$$\operatorname{end}$$

- Set  $i_0 = k$  and  $i'_0 = k$ 

- Set  $W = \min\{h_0 + h_1, W + Z b_1\}$
- If  $W \leq h_0$ ,
  - The lagging edge of a signal has been detected
  - An estimate of the stopping time index is  $i_1$
- Set W = 0,  $i_0 = k$ , and  $i'_0 = i_1$
- If  $W = h_0 + h_1$ , set  $i_1 = k$
- (5) Set k = k + 1 and go to (2)

## **Description of variables**

- p power law ( $p \ge 1, p = 1$  was used)
- $h_0$  threshold for signal onset detection ( $h_0 = 12$ )
- $b_0$  Page test bias for signal onset detection ( $b_0 = 2.5$ )
- $h_1$  threshold for signal termination detection ( $h_1 = 10$ )
- $b_1$  Page test bias for signal termination detection ( $b_1 = 5$ )
- $\alpha\,$  time constant for exponential averager (0 <  $\alpha$  < 1,  $\alpha\,=\,0.95~{\rm was}~{\rm used})$
- $N_{fft}$  size of DFT block ( $N_{fft} = 128$ )
- $N_{off}$  offset from one DFT block to next ( $N_{off} = 32$ )
- $W\,$  Page test statistic
- $i_0$  index to most recent reset to zero
- $i'_0$  index for updating background power estimates
- $i_1$  index to most recent reset to  $h_1 + h_0$  (signal present state)

It should be noted that the indices for the starting and stopping times are in terms of DFT blocks and must be converted to time samples based on the DFT size and amount of overlap. Also, the power-law was kept at unity because of the small bandwidth of the data being processed relative to the potential bandwidth of the marine mammal acoustic emissions. In general, data used for the acoustic detection of marine mammals would have a higher bandwidth and may exhibit tonal emissions. In this more common situation, a higher power law would improve detection performance.

## **1.3.2** Detection Results

The algorithm of Section 1.3.1 was applied to data from Run  $9^1$  of the SWAC4 data set to obtain a series of start and stop times for every detection. From these start and stop times, the total signal energy was estimated and tabulated along with the current average noise power estimate (here it is assumed that the detected signal is an energy signal and that the background noise is a power signal). Additionally, an estimate of the noise background after removal of the detected signals was formulated every 12 seconds. The detection processing results are then displayed in Figure 1.3.2 as the total energy-to-noise power ratio (ENR) detected on each beam over 6 second intervals for the nearly three hour Run 9. This display only shows the results of the detection processing for short duration signals. Thus, when detections occur over an extended period, as seen in the figure, then the time domain signal has persistent non-stationary components as illustrated by the sperm whale clicks in Figure 1.2.1. As previously mentioned, the beams span from forward to aft and are spaced equally in the cosine space of bearing so that there are more beams broadside to the hydrophone array (which was in the shape of a line) than near the forward or aft directions.

The detection results are grouped into events by visual association over beam and time, as indicated by the numbers in Figure 1.3.2. Event time series are then formed by choosing the beam containing the largest ENR over each 12 second period. These time series were then submitted to Prof. G. Pavan of the University of Pavia, Pavia, Italy for classification. Those shown in Figure 1.3.2 were all classified as sperm whale click trains. The detector also found many signals associated with surface vessels, particularly those from the Research Vessel Alliance (the ship towing the hydrophone array) in the forward beams, and a plethora of isolated detections that could be marine mammal, fish, man-made or false alarms. It is possible to associate some of the detections with surface vessels by overlaying the detection results on the estimated background noise, as shown in Figure 1.3.3 for the first 20 minutes of Run 9. In this figure the background power is shown in gray as indicated by the scale on the right. The ENRs of detected short time duration events are overlaid in black. The surface vessels are clearly visible in the background power estimate as slowly moving lines in the gray scale. Short duration signal detections that overlay

 $<sup>^{1}</sup>$ As will be seen in Section 1.3.3, Run 9 is unique in that the data allow for localization of two sperm whales.

the surface vessels in bearing and time are most likely originating there as well (though this is not necessarily so). The sperm whale click trains (events 1, 2 and 3) arrive on quiet beams (that is, there is no surface ship in the background), additionally supporting their classification as marine mammal. Event 21 was eventually classified as acoustic emissions from fish of unknown type. It may also be surmised that, of the two sperm whales detected during the first several minutes, event 1 is nearer than event 2, assuming they both produced approximately the same source levels. As will be discussed in the following section, the towed hydrophone array was completing a turn previously carried out by the RV Alliance so localization of these two events is possible, including resolution of the left/right ambiguity inherent in the array signal processing.

## **1.3.3** Localization

Passive listening of acoustic emissions inherently only provides bearing information. Additionally, owing to the straight line shape of the hydrophone array, there exists a cone of ambiguity; that is, the sound arriving at the array sounds the same if it arrives from anywhere on a cone axially aligned with the towed line array. All of the runs analyzed were such that the tow ship (RV Alliance) was on a constant bearing. Thus, triangulating detections observed over extended periods of time still results in an ambiguity to the left or right side of the array. However, during the first 10 minutes of Run 9, the array was still completing a turn the tow-ship had made prior to commencing the run. From the array heading information (which is quite noisy) it was possible to localize the two sperm whale click trains detected as shown in Figures 1.3.4 and 1.3.5. Lines along the bearing of the detected events from the position of the RV Alliance are shown for events 1, 4, 10 and 15 of Run 9 in Figure 1.3.4. These events are believed to originate from the same whale, though there is no proof of this other than approximate coincidence in space and time. Each line is 15 nautical miles long and when taken in conjunction with the others form a locus where the sperm whale might have been, effectively localizing the whale in range, bearing and resolving the left/right ambiguity. The lines from event 1 (the lighter ones) are shown assuming that the whale was on either the left or right side of the array. The diverging lines seen to the east of the track illustrate an incorrect localization. Figure 1.3.5 contains the localization of events 2, 3, 5, 6, 8, 12, 13 and 17 of Run 9. The ranging

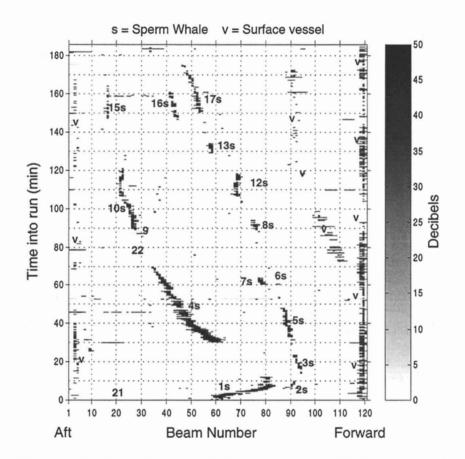


Figure 1.3.2: Signal Energy-to-Noise Power Ratio (ENR) of Detected Short Time Duration Events from Run 9, Combined Over Every 6 Seconds.

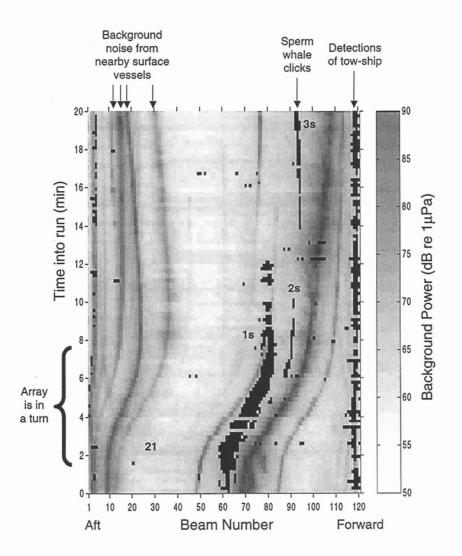


Figure 1.3.3: ENR of Detected Events from First 20 Minutes of Run 9 (Black) Overlaid on Background Noise and Interference Power Estimates (Gray Scale).

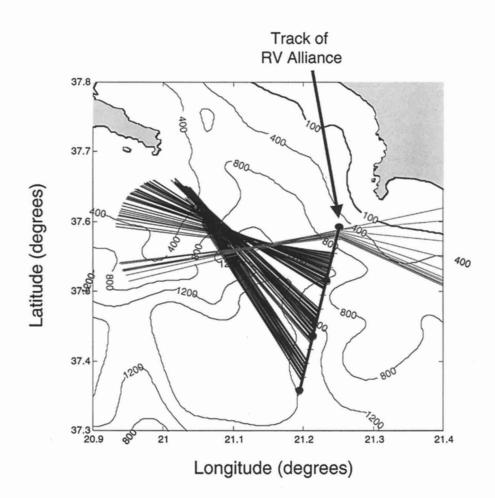


Figure 1.3.4: Lines Along Bearings of Detections for Events 1, 4, 10 and 15 of Run 9. Each Line Is 15 Nautical Miles in Length. The Lighter Lines Are from Event 1 and Illustrate Localization of a Sperm Whale to the West Side of the Track of RV Alliance.

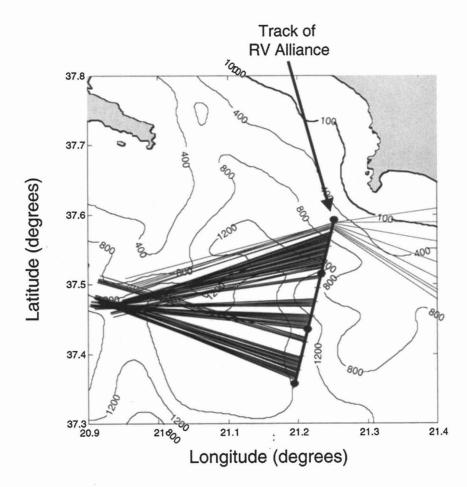


Figure 1.3.5: Lines Along Bearings of Detections for Events 2, 3, 5, 6, 8, 12, 13 and 17 of Run 9. Each Line Is 15 Nautical Miles in Length. The Lighter Lines Are from Event 2 and Illustrate Localization of a Sperm Whale to the West Side of the Track of RV Alliance.

information garnered from Figures 1.3.4 and 1.3.5, that event 2 is farther away than event 1, is corroborated by the ENR levels observed in Figure 1.3.2 where event 2 is weaker than event 1.

# 1.4 CONCLUDING REMARKS

This paper has illustrated the use of sequential methodologies in both active and passive sonar signal processing. It was shown in detail how Page's test may be applied to the detection of marine mammal acoustic emissions having unknown time and frequency structure. The sequential procedure allows for the detection and segmentation of signals with unknown and potentially widely varying time structure. The algorithm described was effective in estimating the background noise and interference power and then detecting departures from that norm, which in this case were either acoustic emissions from marine mammals or non-stationary acoustic emissions from nearby surface ships. In the data presented, sperm whale sounds were detected, along with numerous other detections not analyzed further owing to limited time and resources. Localization of some of the sperm whale sounds was possible through triangulation.

### ACKNOWLEDGMENT

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