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Adrian S. Scarlat

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Approximate Methods in Structural Seismic Design

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Dedicated to
My wife Evemina
My sons Alex, Dan and Yuval

Contents

Preface

Three distinct stages may be defined in the structural design of buildings:

1. selection of the structural solution and assessment of the dimensions of the main elements;
2. detailed computation of stresses and displacements;
3. checking of the main results.

Note that computerized analysis suits only stage 2, whereas approximate methods have to be used for stages 1 and 3; this points out the exceptional importance of approximate methods in structural design.

This book deals with the most difficult problem arising in the design of multi-storey buildings and similar structures: the effect of horizontal forces, mainly seismic loads and similar loads such as wind pressure. In fact, the general stability of a structure capable of resisting horizontal forces is also ensured for vertical loads; possible accidents are generally only local. Unfortunately, the most difficult task is to define clearly the concept of approximate methods. Their reverse, the accurate methods, also elude definition: in order to apply them, several simplifying assumptions are required. Hence we are obliged to accept a rather vague definition: an approximate method in structural analysis is a method that permits the assessment of stresses or displacements in a much shorter time than the commonly used design methods.

We shall refer in the following to two different types of approximate methods: (1) methods designed to determine the stresses and displacements of a given structure by using substitute structures; (2) methods based on 'global parameters' (seismic coefficients, total area of structural walls, etc.). The advent of computerized structural analysis and, in particular, of finite element programs, has opened up vast possibilities for a fundamental reappraisal of existing approximate methods, including a more accurate definition of their scope and limits. We have taken advantage of these possibilities, have examined several 'classical' approximate methods, and have proposed a number of new techniques intended to complement the existing ones.

This book is the result of many years spent in structural design and teaching. One of its main sources is a course in structural dynamics taught by Professor P. Mazilu at the Institute of Civil Engineering of Bucharest, Romania, which the author was privileged to attend.

The target audience of the book is first of all design engineers, but it should be of use for non-specialist engineers too; it is assumed that it will serve also as a teaching aid for undergraduate students as well as for advanced high-rise buildings courses. It is hoped, by citing Mozart (*toute proportion gardée*), that 'here and there are sections that only connoisseurs will enjoy, but these sections have been written so that even a layman will have to enjoy them, albeit without knowing it.'

Adrian S. Scarlat

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1 Multi-storey building frames

1.1 Introduction

In this chapter we propose to assess the bending moments and the deflections of a multi-storey building frame acted upon by lateral forces (Figure 1.1a), by using approximate methods.

As will be shown in the next section (1.2), the approximate analysis of this frame may be performed on a 'substitute' (equivalent) one-bay, symmetrical frame (Figure 1.1b). Therefore we shall deal mainly with this latter frame, as a first step in analysing the actual multi-storey, multi-bay frame.

In our analysis, we assume several hypotheses aimed at simplifying the computation. These are as follows.

- It is assumed that all the horizontal loads are concentrated at floor levels.
- The effect of the shear forces (V) on the deformations is neglected. This hypothesis is acceptable as long as we deal with the usual systems of bars. The analysis of structural walls, where this effect is important, will be dealt with separately (Chapter 2).
- The effect of axial forces (N) on the deformations is neglected. This hypothesis, too is acceptable as long as the total length of the multi-bay frame (L) is not small with respect to its total height (H). In most practical cases this assumption is justified. We point out, in order to be consistent, that we also have to neglect the effect of axial forces in the analysis of the substitute frame. This means that we have to assume cross-sectional areas

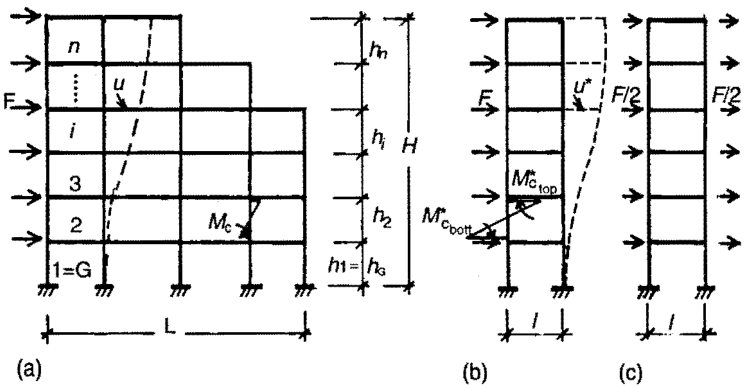

Figure 1.1

$A \to \infty$ (independently of the ratio L/H). Consequently, the asymmetrical load of Figure 1.1b may be replaced by the anti-symmetrical load shown in Figure 1.1c.

In the following, we shall deal mainly with columns fixed in the foundations. The case of pin-supported frames will be considered separately (section 1.2.4). The effect of soil deformability is dealt with separately, too (section 1.3.3).

1.2 Multi-storey, one-bay frames

1.2.1 GENERAL APPROACH

The approximate analysis of multi-storey, one-bay, symmetrical frames subjected to horizontal loads is performed by one of the following procedures:

1. the zero moment point procedure;
2. the continuum procedure (replacement of the beams by a continuous medium). This latter technique is only suitable for multi-storey frames with a large degree of uniformity.

In the case of very irregular substitute frames (from the aspect of both geometry and rigidity), it is advisable to perform the analysis by computer (the 'approximation' will stem from the conversion of the substitute frame to the actual one). We have to assume in this case, too, that the effect of axial forces is negligible (by assuming $A \to \infty$).

1.2.2 THE ZERO MOMENT POINT (ZMP) PROCEDURE

Let us consider the one-bay symmetrical frame shown in Figure 1.2. It is assumed that we know the position of the point where the moment diagram M intersects the column's axis (**zero moment point**, ZMP), i.e. the height h_0. The problem becomes statically determinate:

$$M_{\text{bott}} = \frac{V\,h_0}{2}; \qquad M_{\text{top}} = \frac{V(h_0 - h)}{2} \tag{1.1}$$

where V represents the sum of horizontal forces above the given column.

The position of ZMP may be defined by the ratio

$$\varepsilon = \frac{h_0}{h} = \frac{1}{1 - (M_{\text{top}}/M_{\text{bott}})} \tag{1.2a}$$

(M_{bott} and M_{top} have the same sign if they tension the same fibre).

It therefore follows:

$$M_{\text{bott}} = \frac{V\varepsilon h}{2}$$

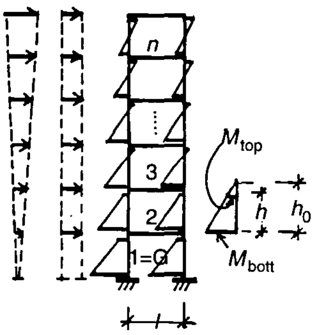

Figure 1.2

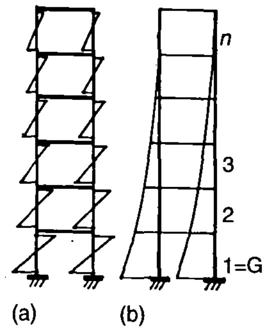

Figure 1.3

$$M_{top} = \frac{-V(1-\varepsilon)h}{2} \tag{1.2b}$$

For a frame of given geometry and for a given storey i, ε depends mainly on the ratio

$$v = \frac{k_{b_i}}{k_{c_i}} \tag{1.3}$$

where $k_{b_i} = I_{b_i}/l; k_{c_i} = I_{c_i}/h$; and $I_b(I_c)$ are moments of inertia of beam i (column i). We note that in most practical cases the ratio v lies between 0.1 and 5.

The type of loading has some effect on the position of ZMP, but we may safely neglect this effect and assume that for any laterally distributed load ZMP is invariant.

Figure 1.3 presents diagrams for two extreme situations. Figure 1.3a shows extremely stiff beams ($v \to \infty$), where ZMP lies at the mid-height of each storey ($\varepsilon = 0{\cdot}5$). Figure 1.3b illustrates extremely flexible beams ($v \to 0$); the diagram of bending moments M is of the **cantilever type**. In the case of stiff beams ZMP lies within the given storey. In the case of flexible beams ZMP may be positioned above the given storey.

We shall now determine the approximate position of ZMP by referring to two cases: uniform and non-uniform frames.

(a) Uniform frames

Uniform frames have equal heights, $I_c = \text{constant}$ and $I_b = \text{constant}$ at each storey.

Analyses were performed for $n = 6, 10$ and 15 storeys subjected to uniform and inverted triangular loads (Figure 1.4); for each type of frame and loading, eight ratios $v = k_b/k_c$ were considered: 0·01, 0·1, 0·5, 1, 2, 5, 10, 1000 (a total of 48 cases).

Ratios ε were computed at three levels: $\varepsilon_1 = \varepsilon_G$ (at the ground floor), ε_2 (one floor above) and ε_m (at the mid-height of the structure). The ratio ε at the top floor is not significant, as the corresponding moments are usually very small.

Average curves for ε versus v are shown in Figure 1.5 for $\varepsilon_1 = \varepsilon_G$, ε_2 and ε_m:

- ε_G is close to 0.5–0.6 for $v > 2$ and greater than 1 for $v < 0.2$.
- ε_m remains close to 0.5 for $v > 0.5$.

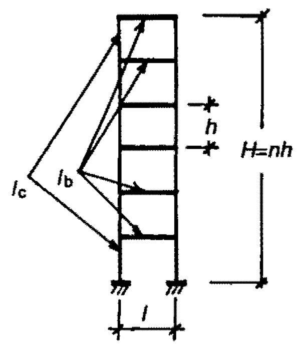

Figure 1.4

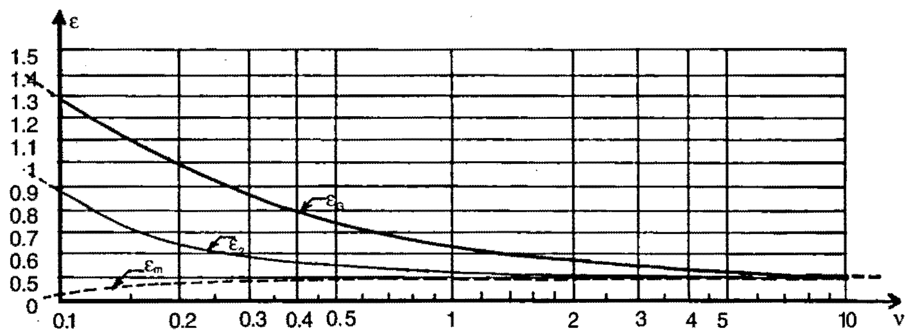

Figure 1.5

We note that even for reinforced concrete structures the column reinforcement is usually uniform along the height of the storey. In cases where $\varepsilon_m \neq 0.5$ (either $\varepsilon_m > 0.5$ or $\varepsilon_m < 0.5$), the maximum moment is greater than the moment we have computed on the assumption that $\varepsilon_m = 0.5$.

Therefore, in design it is advisable to consider the bending moments around the mid-height of the frame, increased by 10–20% with respect to the moments based on $\varepsilon_m = 0.5$.

ε_2 lies between ε_G and ε_m; for $\nu = 0.1–0.3$, ε_2 varies between 0·5 and 0·9.

For ratios $\nu < 0.1$, the spread of results is too wide to permit a reasonable average value to be accepted. In such cases, it is more convenient to relate the maximum moment acting on the columns (M_{max} above the foundations) directly to the maximum moment acting on a cantilever (M_{cant}), due to loads $F/2$; see Figure 1.6.

From the frames we have analysed we obtain:

$$\left.\begin{array}{llr} \nu = k_b/k_c = 0{\cdot}001 & M_{max}/M_{cant} = & 0{\cdot}8–0{\cdot}9 \\ \quad\quad\quad\quad\; 0{\cdot}01 & & 0{\cdot}4–0{\cdot}6 \\ \quad\quad\quad\quad\; 0{\cdot}1 & & 0{\cdot}15–0{\cdot}3 \end{array}\right\} \quad (1.4)$$

We note that even for very flexible beams ($\nu = 0{\cdot}01$) the effect of the beams remains significant (it ensures a decrease of the maximum moments acting on the columns by $\sim 50\%$ with respect to the cantilever moments).

(b) Non-uniform frames

The position of ZMP for several cases of 10-storey frames has been examined. The results are as follows.

- The moments of inertia of the columns vary along the height of the frame (Figure 1.7), the beams having constant moments of inertia (various ratios ν between 0.1 and 10 have been considered). The computations show that,

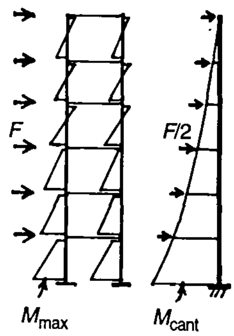

Figure 1.6

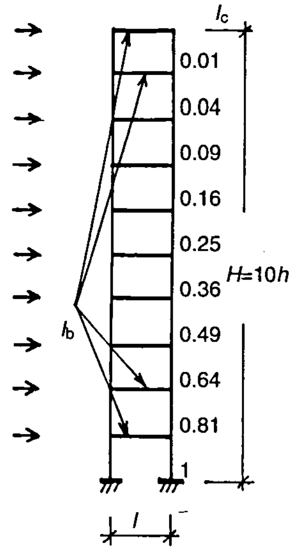

Figure 1.7

if we denote

$$\nu = k_b / k_{c_{max}} \tag{1.5}$$

then we may use the curves shown in Figure 1.5.

- The height of the ground floor (h_G) is greater than the height of the remaining floors (h): see Figure 1.8. The moment of inertia is assumed constant (I_c), so that $k_{c_G} = I_c/h_G < k_c = I_c/h$. Ground-floor heights $h_G = 1.5h$–$2h$ have been taken into account.

Ratios ν between 0·1 and 10 have been checked.

The results show that we may use the curves of Figure 1.5 on condition that

for ε_G, we refer to

$$\nu_G = \frac{k_b}{k_{c_G}} = \frac{I_b/l}{I_{c_G}/h_G} \tag{1.6}$$

for ε_2, we refer to

$$\nu = \frac{k_b}{k_c} = \frac{I_b/l}{I_c/h} \tag{1.7}$$

More accurate results may be obtained in this latter case by using the continuum approach (see section 1.2.3).

In the case of variable heights and moments of inertia, the results obtained by the ZMP procedure are only reliable for the ground floor.

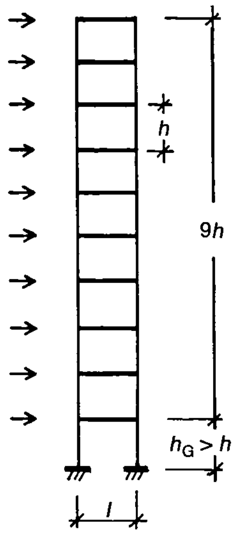

Figure 1.8

□ Numerical example 1.1

Consider the uniform one-bay, 10-storey frame shown in Figure 1.9, loaded by horizontal identical concentrated loads $F = 1$ kN. $I_c = 1$, i.e. $k_c = 0{\cdot}333$. Assume $I_b = 1{\cdot}333$, i.e. $k_b = 1{\cdot}333/4 = 0{\cdot}333$; $\nu = k_b/k_c = 1$.

The accurate moments are shown in Figure 1.9b (within the brackets). To compute the same moments approximately, we use the curves given in Figure 1.5: $\nu = 1$, $\varepsilon_G = 0{\cdot}66$, $\varepsilon_2 = 0{\cdot}53$, $\varepsilon_m = 0{\cdot}50$. The corresponding moments are shown in Figure 1.9b (outside the brackets).

For instance, at the ground floor:

$$M_{\text{bott}} = \frac{1}{2}\sum_{1}^{10} F \cdot \varepsilon_G \cdot h_G = \frac{10}{2} \times 0{\cdot}66 \times 3{\cdot}0 = 9.9\,\text{kN m} \quad \text{(accurate: } 9{\cdot}5\,\text{kN m).}$$

$$M_{\text{top}} = -\frac{1}{2}\sum_{1}^{10} F \cdot (1 - \varepsilon_G) \cdot h_G = -\left(\frac{10}{2}\right) \times 0{\cdot}34 \times 3{\cdot}0 = -5{\cdot}1\,\text{kN m}$$

$$\text{(accurate: } -5{\cdot}5\,\text{kN m).}$$

By assuming a different ratio, $\nu = k_b/k_c = 0{\cdot}1$, the accurate moments are as shown in Figure 1.9c. From the curves in Figure 1.5, $\varepsilon_G = 1{\cdot}30, \varepsilon_2 = 0{\cdot}88$; ε_m is uncertain. The approximate moments are shown outside the brackets.

At the ground floor:

$$M = \frac{10}{2} \times 1{\cdot}30 \times 3{\cdot}0 = 19{\cdot}5\,\text{kN m} \quad \text{(accurate: } 18{\cdot}9\,\text{kN m)}$$

$$M = -\frac{10}{2} \times (1 - 1{\cdot}3) \times 3{\cdot}0 = 4{\cdot}5\,\text{kN m} \quad \text{(accurate: } 4{\cdot}5\,\text{kN m)}$$

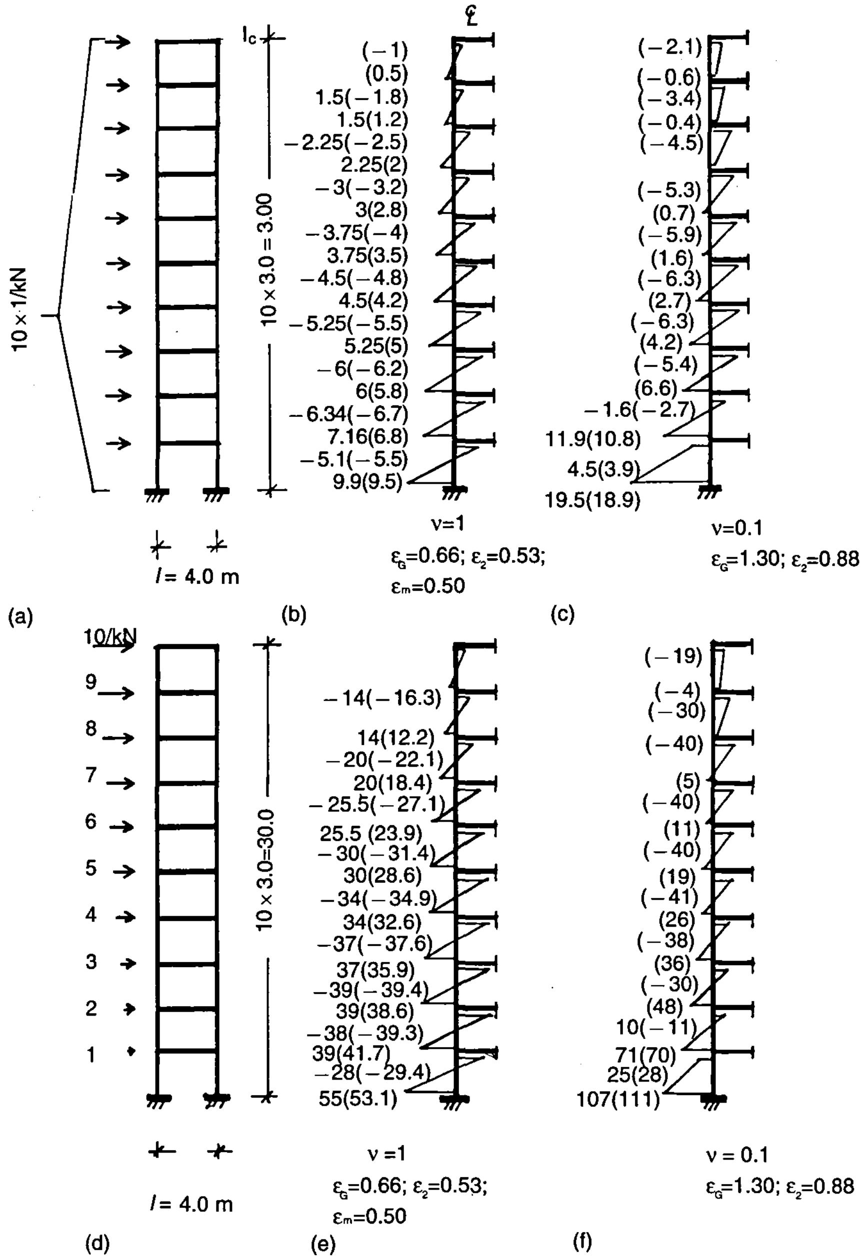
10 × 1/kN
$l = 4.0$ m
I_c
10 × 3.0 = 3.00
(a)
(−1)
(0.5)
1.5(−1.8)
1.5(1.2)
−2.25(−2.5)
2.25(2)
−3(−3.2)
3(2.8)
−3.75(−4)
3.75(3.5)
−4.5(−4.8)
4.5(4.2)
−5.25(−5.5)
5.25(5)
−6(−6.2)
6(5.8)
−6.34(−6.7)
7.16(6.8)
−5.1(−5.5)
9.9(9.5)
ν=1
ε_G=0.66; ε_2=0.53;
ε_m=0.50
(b)
(−2.1)
(−0.6)
(−3.4)
(−0.4)
(−4.5)
(−5.3)
(0.7)
(−5.9)
(1.6)
(−6.3)
(2.7)
(−6.3)
(4.2)
(−5.4)
(6.6)
−1.6(−2.7)
11.9(10.8)
4.5(3.9)
19.5(18.9)
ν=0.1
ε_G=1.30; ε_2=0.88
(c)
10/kN
9
8
7
6
5
4
3
2
1
10 × 3.0=30.0
$l = 4.0$ m
(d)
−14(−16.3)
14(12.2)
−20(−22.1)
20(18.4)
−25.5(−27.1)
25.5 (23.9)
−30(−31.4)
30(28.6)
−34(−34.9)
34(32.6)
−37(−37.6)
37(35.9)
−39(−39.4)
39(38.6)
−38(−39.3)
39(41.7)
−28(−29.4)
55(53.1)
ν =1
ε_G=0.66; ε_2=0.53;
ε_m=0.50
(e)
(−19)
(−4)
(−30)
(−40)
(5)
(−40)
(11)
(−40)
(19)
(−41)
(26)
(−38)
(36)
(−30)
(48)
10(−11)
71(70)
25(28)
107(111)
ν = 0.1
ε_G=1.30; ε_2=0.88
(f)

Figure 1.9

If we refer to the **cantilever moment**:

$$M_{\text{cant}} = \frac{10}{2} \times 3{\cdot}0 \times (1 + 2 + \cdots + 10) = 82{\cdot}5$$

yielding:

$$\frac{M_{\max}}{M_{\text{cant}}} = \frac{18{\cdot}9}{82{\cdot}5} = 0{\cdot}23$$

Similar cases are shown in Figure 1.9d–f, but for an inverted triangular load: the same ratios v as considered in the first case (uniformly distributed loads) have been assumed.

Referring to the cantilever moment:

$$M_{\text{cant}} = \left(\frac{3{\cdot}0}{2}\right) \times (1{\cdot}1 + 2{\cdot}2 + \cdots + 10{\cdot}10) = 577{\cdot}5$$

yielding:

$$\frac{M_{\max}}{M_{\text{cant}}} = \frac{111}{577{\cdot}5} = 0{\cdot}19$$ □

□ Numerical example 1.2

The frame shown in Figure 1.10 has variable heights, as well as variable moments of inertia. The approximate moments for the ground floor only are computed as follows.

$$v_G = \frac{k_{b_G}}{k_{c_G}} = \frac{1{\cdot}2/3{\cdot}0}{1{\cdot}5/4{\cdot}5} = 1{\cdot}2$$

From Figure 1·5: $\varepsilon_G = 0{\cdot}63$

$$M_{\text{bott}} = \frac{10}{2} \times 0{\cdot}63 \times 4{\cdot}5 = 14{\cdot}2\,\text{kN m} \quad \text{(accurate: 14·1)}$$

$$M_{\text{top}} = -\frac{10}{2} \times 0{\cdot}37 \times 4{\cdot}5 = -8{\cdot}3\,\text{kN m} \quad \text{(accurate: } -8{\cdot}4)$$ □

1.2.3 CONTINUUM APPROACH

This procedure is based on replacing the beams of the one-bay, multi-storey building frame by a continuous medium. It yields satisfactory results provided the frame is uniform, or nearly uniform: i.e. identical moments of inertia of columns, I_c; identical moments of inertia of beams, I_b (except the top slab, where $I'_b = I_b/2$); and not very different heights of columns ($h_i/h_j = 2/3\ldots3/2$).

Let us consider a completely uniform frame subjected to a laterally distributed load of intensity p_x (Figure 1.11a). The beams are replaced by a continuous medium formed by an infinite number of very thin horizontal laminae at

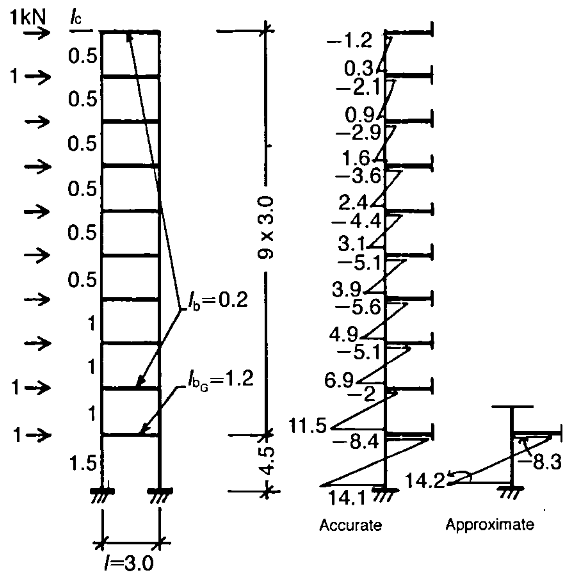

Figure 1.10

distances dx, having the moment of inertia (Figure 1.11b):

$$I'_{\mathrm{b}} = \frac{I_{\mathrm{b}}\mathrm{d}x}{h} \tag{1.8}$$

By using the flexibility method, we may formulate the differential equation of compatibility of displacements and subsequently determine the couples C acting on the columns at each floor (Figure 1.11c).

For a uniformly distributed load:

$$p = \frac{\sum F}{H} \tag{1.9}$$

we obtain (Csonka, 1962a):

$$\left.\begin{aligned} C_i &= \left(\frac{p}{\alpha^2}\right)\cdot \sinh\left(\frac{\alpha h}{2}\right)\cdot [\theta\cdot\cosh(\alpha x_i) - \sinh(\alpha x_i)] + \frac{phx_i}{2} \\ &\qquad (i = 1, 2, \ldots, n-1) \\ C_{\mathrm{top}} &= \left(\frac{p}{2\alpha^2}\right)\cdot \left[\theta \sinh\left(\frac{\alpha h}{2}\right) - \cosh\left(\frac{\alpha h}{2}\right) + 1\right] + \frac{ph^2}{16} \end{aligned}\right\} \tag{1.10}$$

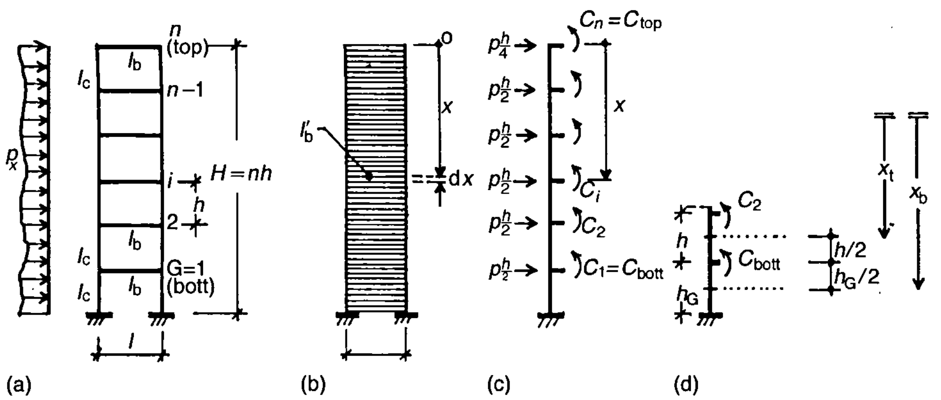

Figure 1.11

where:

$$\alpha = \sqrt{\left(\frac{6I_b}{I_c \cdot h \cdot l}\right)}\ \mathrm{m}^{-1}; \qquad \theta = \frac{\sinh(\alpha H) - \alpha H}{\cosh(\alpha H)} \quad (\text{for } H > 20,\ \alpha = 1). \quad (1.10)$$

We note that the coordinates x_i originate at the top.

Each column is now acted upon by the couples C_i and the given loads $p/2\,\mathrm{kN\,m^{-1}}$, i.e. $ph/2\,\mathrm{kN}$ at each joint, except the top joint, where $ph/4$ and C_{top} are acting (Figure 1.11c). The problem is statically determinate, and we may compute the bending moments:

$$M_i = \frac{M_{p_i}}{2} + M_{c_i} \qquad (1.11)$$

$$\text{where } M_{p_i} = \frac{px_i^2}{2}; \qquad M_{c_i} = \sum_1^i C \qquad (1.11)$$

In the case of a different height at ground floor ($h_G \neq h$), the corresponding couple to be considered is (Figure 1.11d)

$$C_{bott} = \left(\frac{p}{2\alpha^2}\right) \cdot \{\theta[\sinh(\alpha x_B) - \sinh(\alpha x_T)] - (\cosh(\alpha x_B) - \cosh(\alpha x_T)\} + \frac{p(x_B^2 - x_T^2)}{4} \qquad (1.12)$$

The results in this case are not very accurate.

In fact, we may use the results obtained for the uniformly distributed loads p (namely, the positions of the ZMPs) for an approximate analysis of the frame subjected to any lateral distributed load by assuming that the ZMPs are the same.

As such, we solve the problem in two stages.

1. We determine the positions of the ZMP at each storey as for a uniformly distributed load.
2. We compute for the given load the bending moments acting on the columns corresponding to the ZMPs obtained in stage 1.

An alternative procedure has been proposed, where the differential equation of compatibility of displacements is replaced by a difference equation. This procedure entails an excessive volume of computations and requires a very high degree of precision. It is therefore seldom used in design.

□ Numerical example 1.3

Let us refer to the frame shown in Figure 1.12 (a nearly uniform frame): $n = 10$; $l = 3{\cdot}0$ m; $h = 3{\cdot}0$ m; $h_G = 4{\cdot}5$ m; $H = 31{\cdot}5$ m; $I_b/I_c = 1{\cdot}5$.

$$p = \frac{1{\cdot}0}{3{\cdot}0} = 0{\cdot}333\,\mathrm{kN\,m^{-1}} \text{ (or: } F_i = 1\,\mathrm{kN};\ F_{top} = 0{\cdot}5\,\mathrm{kN};\ F_{bott} = 1{\cdot}25\,\mathrm{kN}).$$

$$\alpha = \sqrt{\left(\frac{6 \times 1{\cdot}5}{1 \times 3 \times 2}\right)} = 1; \qquad \theta = 1$$

$$C_i = \left(\frac{p}{\alpha^2}\right).\sinh\left(\frac{\alpha}{2}\right).\,[\theta{\cdot}\cosh(\alpha x_i) - \sinh(\alpha x_i)] + \frac{phx_i}{2} = \cdots$$

$$= 0{\cdot}709 \cdot [\cosh(\alpha x_i) - \sinh(\alpha x_i)] + 0{\cdot}5\,x_i$$

$i = 2$	$x_i = 24$ m	$C_i = 11{\cdot}99$ kN m
3	21	10·49
4	18	8·99
5	15	7·50
6	12	6·00
7	9	4·50
8	6	3·00
9	3	1·53

$$C_{top} = \left(\frac{p}{2\alpha^2}\right)\cdot\left[\theta\sinh\left(\frac{\alpha h}{2}\right) - \cosh(\alpha h/2) + 1\right] + \frac{p\cdot h^2}{16}$$

$$= \left(\frac{0{\cdot}333}{2 \times 1^2}\right)\cdot(1 \times 2{\cdot}1293 - 2{\cdot}3524 + 1) + \frac{0{\cdot}333 \times 3^2}{16} = 0.32\,\mathrm{kN\,m}$$

$$C_{bott} = \left(\frac{p}{2\alpha^2}\right)\cdot\{\theta\cdot[\sinh(\alpha x_B) - \sinh(\alpha x_T)] - (\cosh(\alpha x_B) - \cosh(\alpha x_T)\}$$

$$+ \frac{p(x_B^2 - x_T^2)}{4}$$

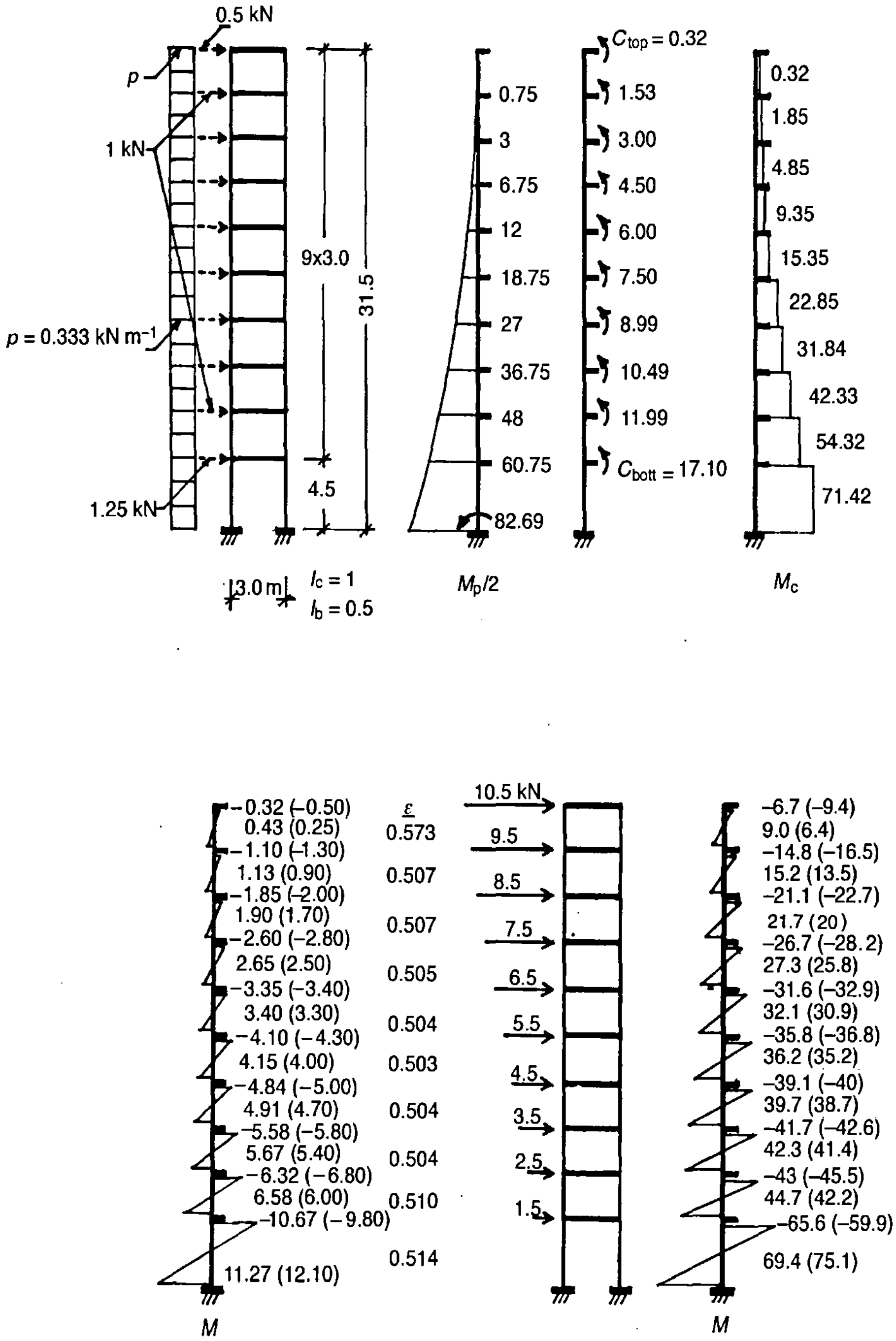
0.5 kN
p
1 kN
p = 0.333 kN m⁻¹
1.25 kN
9x3.0
31.5
4.5
3.0 m
$I_c = 1$
$I_b = 0.5$
0.75
3
6.75
12
18.75
27
36.75
48
60.75
82.69
$M_p/2$
$C_{top} = 0.32$
1.53
3.00
4.50
6.00
7.50
8.99
10.49
11.99
$C_{bott} = 17.10$
0.32
1.85
4.85
9.35
15.35
22.85
31.84
42.33
54.32
71.42
M_c
−0.32 (−0.50)
0.43 (0.25)
−1.10 (−1.30)
1.13 (0.90)
−1.85 (−2.00)
1.90 (1.70)
−2.60 (−2.80)
2.65 (2.50)
−3.35 (−3.40)
3.40 (3.30)
−4.10 (−4.30)
4.15 (4.00)
−4.84 (−5.00)
4.91 (4.70)
−5.58 (−5.80)
5.67 (5.40)
−6.32 (−6.80)
6.58 (6.00)
−10.67 (−9.80)
11.27 (12.10)
M
ε
0.573
0.507
0.507
0.505
0.504
0.503
0.504
0.504
0.510
0.514
10.5 kN
9.5
8.5
7.5
6.5
5.5
4.5
3.5
2.5
1.5
−6.7 (−9.4)
9.0 (6.4)
−14.8 (−16.5)
15.2 (13.5)
−21.1 (−22.7)
21.7 (20)
−26.7 (−28.2)
27.3 (25.8)
−31.6 (−32.9)
32.1 (30.9)
−35.8 (−36.8)
36.2 (35.2)
−39.1 (−40)
39.7 (38.7)
−41.7 (−42.6)
42.3 (41.4)
−43 (−45.5)
44.7 (42.2)
−65.6 (−59.9)
69.4 (75.1)
M

Figure 1.12

$$x_B = 27 + \frac{4\cdot 5}{2} = 29\cdot 25\,\text{m}; \qquad x_T = 27 - \frac{3\cdot 0}{2} = 25\cdot 5\,\text{m}$$

$$C_{\text{bott}} = \left(\frac{0\cdot 333}{2 \times 1^2}\right) \times \{1 \times [\sinh(1 \times 29\cdot 25) - \sinh(1 \times 25\cdot 5)]$$

$$-(\cosh(1 \times 29\cdot 25) - \cosh(1 \times 25\cdot 5)\} + \frac{0\cdot 333(29\cdot 25^2 - 25\cdot 5^2)}{4} \cong 0 + 17\cdot 10$$

$$= 17\cdot 10\,\text{kN m}$$

$$M_{(p_i/2)} = \frac{0\cdot 333\,x_i^2}{4}; \qquad M_{c_i} = \sum_1^i M_c; \qquad M_i = M_{(p_i/2)} + M_{c_i}.$$

From the diagram M_i we deduce the ratios ε.

By considering the same frame loaded with inverted triangular forces and taking into account the ratios ε, we determine the final moments ($M_i = \sum_1^i F\varepsilon h/2$). The accurate values are given within the brackets and the approximate ones outside the brackets. □

1.2.4 PIN-SUPPORTED FRAMES

Multi-storey building frames with pinned supports are seldom used in design, especially if the columns are of reinforced concrete. The actual supports are elastic, but designers usually consider them as fixed. If we want to take into account the elasticity of the supports against rotation in the frame of an approximate analysis we have to perform two separate analyses, the first by considering fixed supports (Figure 1.13a) and the second by considering pinned supports (Figure 1.13b), and then interpolate the results. In the case of slender frames, the effect of the high vertical reactions will further increase the deflections (see section 1.3.3).

The pin-supported frame has two important features.

- Its reactions are statically determinate (the assumption of the axial indeformability of the beams allows the replacement of the actual loads F with pairs of anti-symmetrical loads $F/2$ and leads to anti-symmetrical reactions). As a result (Figure 1.14):

$$M_G = \sum F \cdot \frac{h_G}{2}$$

- Very flexible beams ($\nu = k_b/k_c \rightarrow 0$) bring the structure close to a mechanism and excessive deflections are to be expected. The corresponding moment diagram is shown in Figure 1.15a; it is noteworthy that its shape is similar to the shape of a moment diagram of a frame obtained by superimposing a series of three-hinged frames (Figure 1.15b).

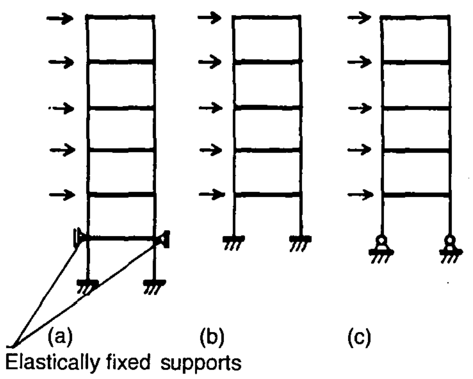

Figure 1.13

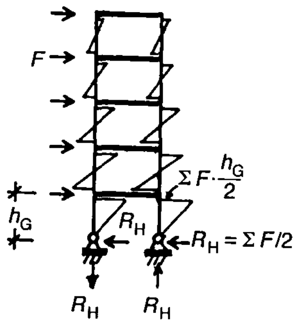

Figure 1.14

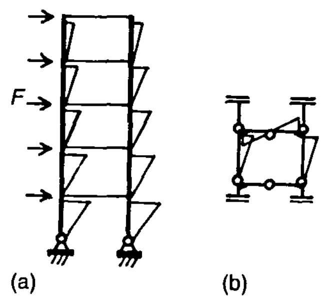

Figure 1.15