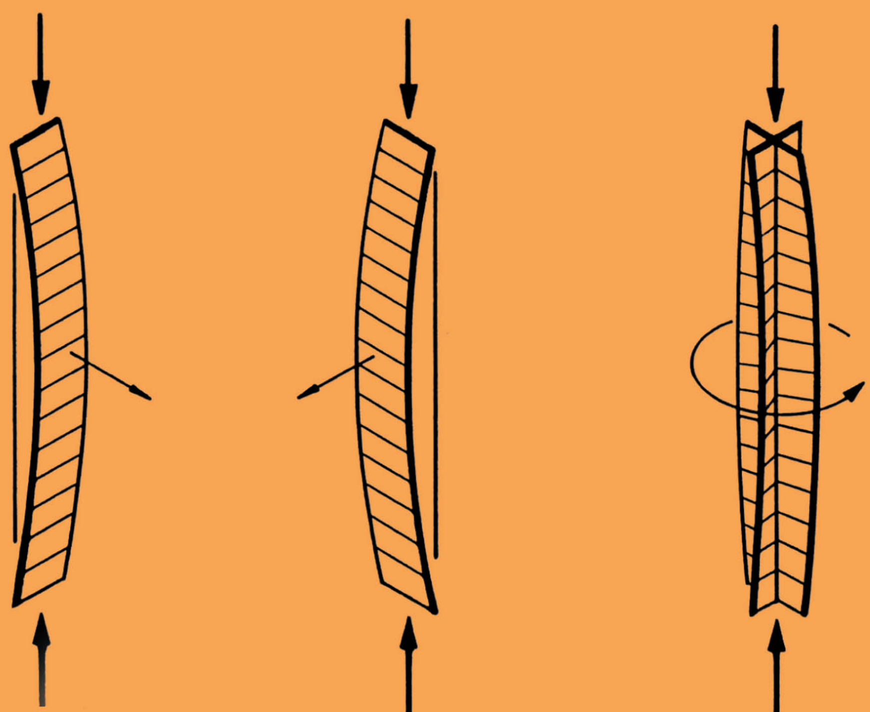
Flexural-Torsional Buckling of Structures

N.S. TRAHAIR



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E & FN SPON
An Imprint of Chapman & Hall
London · Glasgow · New York · Tokyo · Melbourne · Madras

Published by E & FN Spon, an imprint of
Chapman & Hall, 2–6 Boundary Row, London SE1 8HN

Chapman & Hall, 2–6 Boundary Row, London SE1 8HN, UK

Blackie Academic & Professional, Wester Cleddens Road, Bishopbriggs, Glasgow G64 2NZ, UK

Chapman & Hall Japan, Thomson Publishing Japan, Hirakawacho Nemoto Building, 6F, 1-7-11 Hirakawa-cho, Chiyoda-ku, Tokyo 102, Japan

Chapman & Hall Australia, Thomas Nelson Australia, 102 Dodds Street, South Melbourne, Victoria 3205, Australia

Chapman & Hall India, R. Seshadri, 32 Second Main Road, CIT East, Madras 600 035, India

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Published in the USA and Canada by CRC Press Inc., 2000 Corporate Boulevard NW, Boca Raton, Florida 33431, USA

First edition 1993

Typeset in 10/12pt Times by Thomson Press (India) Ltd, New Delhi

ISBN 0 419 17110 5

A catalogue record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data available

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Preface

Flexural-torsional buckling is a mode of structural failure in which one or more members of a frame suddenly deflect and twist out of the plane of loading. Because flexural-torsional buckling reduces the load-carrying capacity of the structure, designers must prevent it either by providing additional bracing, or by using larger members.

From the 1930s, the subject of flexural-torsional buckling has been dealt with in S.P. Timoshenko's widely used textbook *Theory of Elastic Stability*, last published (with J.M. Gere) in 1961, and later in F. Blelch's *Buckling Strength of Metal Structures*, published in 1952. These cover a wide range of structural stability topics, and so their coverage of flexural-torsional buckling is limited to a few chapters.

These books were both written before the advent of the electronic digital computer in the 1950s, and the subsequent explosion in published research on the subject. Consequently, more recent treatments of stability theory have been more limited either in their scope or in their depth. Strangely, there have been few, if any, recent books published which provide thorough treatments of flexural-torsional buckling.

This book is intended to provide both an up-to-date treatment of modern methods of analysing flexural-torsional buckling, and also to provide sufficiently detailed summaries of knowledge on flexural-torsional buckling that it can be used as a source book by both designers and researchers.

It may also be used for teaching purposes as a text or reference book. In advanced level undergraduate courses, the teacher will want to simplify and edit the material given. Such a course may introduce the theory of structural stability given in Chapter 2, and the hand and computer methods of analysis dealt with in Chapters 3 and 4. Subsequent material on column, beam, and beam-column buckling may be selected from Chapters 5, 7 and 11, and on design against flexural-torsional buckling from Chapter 15.

For normal level post-graduate courses, these topics may be presented in greater detail, and expanded with material on restrained buckling from Chapters 6, 8, 9, 10 and 12, and possibly with a treatment of inelastic buckling based on Chapter 14. For advanced courses, these topics would be studied more thoroughly, while additional topics may be introduced from Chapters 13 and 16.

It is not often these days that a researcher is allowed to develop a fascination for a subject and spend the amount of time on it that I have on flexural-torsional buckling, and I count myself as being privileged in this regard. My first introduction to the subject was through a Structures Honours course given to me as an undergraduate by the late J.W. Roderick in 1955. As a masters research student, a topic on the flexural-torsional buckling of beam-columns was suggested to me by one of my co-students, P.G. Lowe, and my work on this, which was supervised by Roderick, confirmed my interest in the subject. In early 1961 my curiosity on the lateral buckling of beams was stimulated by the British Standard BS 449:1959. This led me into work for the Standards Association of Australia for the development of an Australian code for the design of steel structures, and into my doctoral study of the flexural-torsional buckling of frame structures, again supervised by Roderick.

In 1968 I had the good fortune to spend a study leave at Washington University with T.V. Galambos, who inspired me with his own fascination for the subject. In the early 1970s, I learnt much with J.M. Anderson, S. Kitipornchai, P. Vacharajittiphan, S.T. Woolcock, T. Poowannachaikul and B.R. Mutton, an exceptional group of graduate students at the University of Sydney. My good fortune continued in 1974–75, when I spend another study leave, this time at the University of Sheffield where I collaborated with D.A. Nethercot. In the 1980s, I again had a number of outstanding students, M.A. Bradford, P.E. Cuk and J.P. Papangelis, while two of my colleagues at the University of Sydney, G.J. Hancock and N.L. Ings, collaborated with me on flexural-torsional buckling research, as did M.A. Bradford and S. Bild as post-doctoral fellows, and later Y.L. Pi.

In the early 1970s I began teaching the same Structures Honours course that Roderick taught me in 1955, and also a post-graduate course on Structural Stability. I gave a related stability course at the University of Alberta in 1985. The stimulus of preparing and developing these courses and of challenging and reacting to my students has done much to extend my own understanding of the subject, as well as to give me a broad outline for this book.

My studies of flexural-torsional buckling have not been limited to research, and my work for nearly 30 years with the Standards Association of Australia on the preparation of codes for the design of steel structures has given me a keen appreciation of the need to translate research findings on flexural-torsional buckling into forms that are easily understood and used by designers.

I have been greatly influenced in the preparation of this book by my teaching and research experiences in Australia at the University of Sydney, in the USA at Washington University, in the UK at the University of Sheffield, and in Canada at the University of Alberta. While a significant proportion of the material in this book has been developed by me and my colleagues and students, much of it is not original, but has been gathered from many sources. Unfortunately, it is very difficult or even impossible to acknowledge all individual sources, and so the references given in this book are restricted to those which the general reader may wish to consult for further information.

I would like to thank the School of Civil and Mining Engineering of the University of Sydney for the facilities that it has made available to assist in the preparation of this book. The manuscript was expertly typed by Jean Whittle and Cynthia Bautista, and the diagrams prepared by Ron Brew and Kim Pham.

Many valuable comments and suggestions were made by my colleagues and students at the University of Sydney, especially by K.J.R. Rasmussen.

Finally, I wish to acknowledge the unfailing help and support of my wife, Sally, without whom the writing of this book would not have been possible.

Nicholas Trahair

Units and conversion factors

Units

While most expressions and equations used in this book are arranged so that they are non-dimensional, there are a number of exceptions. In almost all of these, SI units are used which are derived from the basic units of kilogram (kg) for mass, metre (m) for length, and second (s) for time.

The SI unit of force is the newton (N), which is the force which causes a mass of 1 kg to have an acceleration of 1 m/s^2. The acceleration due to gravity is 9.807 m/s^2 approximately, and so the weight of a mass of 1 kg is 9.807 N.

The SI unit of stress is the pascal (Pa), which is the average stress exerted by a force of 1 N on an area of 1 m^2. The pascal is too small to be convenient in structural engineering, and it is common practice to use either the megapascal (1 MPa = 10^6 Pa) or the identical newton per square millimetre (1 $N/mm^2 = 10^6$ Pa). The megapascal (MPa) is used generally in this book.

Table of conversion factors

To Imperial (British) Units		To SI units	
1 kg	= 0.068 53 slug	1 slug	= 14.59 kg
1 m	= 3.281 ft	1 ft	= 0.304 8 m
	= 39.37 in.	1 in.	= 0.025 4 m
1 mm	= 0.003 281 ft	1 ft	= 304.8 mm
	= 0.039 37 in.	1 in.	= 25.4 mm
1 N	= 0.224 8 lb	1 lb	= 4.448 N
1 kN	= 0.224 8 kip	1 kip	= 4.448 kN
	= 0.100 36 ton	1 ton	= 9.964 kN
1 MPa*†	= 0.145 0 kip/in.^{2}(ksi)	1 kip/in.^{2}†	= 6.895 MPa
	= 0.064 75 ton/in.^{2}	1 ton/in.^{2}	= 15.44 MPa
1 kNm	= 0.737 6 kip ft	1 kip ft	= 1.356 kNm
	= 0.329 3 ton ft	1 ton ft	= 3.037 kNm

*1 MPa = 1 N/mm^2.

† There are a few dimensionally inconsistent equations used in this book which arise because a numerical value (in MPa or kip/in.^{2}) is substituted for the Young's modulus of elasticity E while the yield stress F_Y remains algebraic. The value of the yield stress F_Y used in these equations should therefore be expressed in either MPa or kip/in.^{2}, whichever is appropriate. Care should be used in converting these equations from SI to Imperial units, or vice versa.

Glossary of terms

Arch A member curved in the plane of loading.

Beam A member which supports transverse loads or moments only.

Beam-column A member which supports transverse loads or moments which cause bending and axial loads which cause compression.

Beam-tie A member which supports transverse loads or moments which cause bending and axial loads which cause tension.

Brace A secondary member which prevents or restrains deflection or twist rotation of a main member.

Braced beam A beam with a number of cross-sections braced against lateral deflection and twist rotation.

Buckling A mode of failure in which there is a sudden deformation in a direction or plane normal to that of the loads or moments acting.

Cantilever A member with an end which is unrestrained against lateral deflection and twist rotation.

Capacity factor A factor used to multiply the nominal capacity to obtain the design capacity.

Column A member which supports axial compression loads.

Conservation of energy A principle describing the conditions under which a structure and its loads may deform without any change in the total energy of the system.

Continuous beam A beam which is continuous over one or more supports.

Design capacity The capacity of the structure or element to resist the design loads. Obtained as the product of the nominal capacity and the capacity factor.

Design load The combination of factored nominal loads which the structure is required to resist.

Distortion A mode of deformation in which the cross-section of a member changes shape.

Effective length The length of an equivalent simply supported member which has the same elastic buckling load as the actual member.

Elastic behaviour Deformations without yielding.

Energy method A method of buckling analysis based on the principle of conservation of energy.

Finite element analysis A computer method of numerical analysis in which a complete structure is divided into a number of elements of finite size.

First-order analysis Elastic linear analysis in which equilibrium is formulated for the undeformed position of the structure, so that the moments caused by products of the loads and deflections are ignored.

First-yield moment The value of the bending moment which nominally causes the first yield of a cross-section.

Flexural-torsional buckling A mode of buckling in which a member deflects and twists.

Frame A skeletal structure consisting of a number of members connected together at joints.

Frame buckling A mode of buckling in which all the members of a frame participate.

Geometrical imperfections Initial crookedness or twist.

Inelastic behaviour Deformations accompanied by yielding.

In-plane behaviour The behaviour of a member which deforms only in the plane of the applied loads.

Lateral buckling Flexural-torsional buckling of beams.

Limit states design A method of design in which the performance of the structure is assessed by comparison with a number of limiting conditions of usefulness. The most common conditions are the strength limit state and the serviceability limit state.

Load and resistance factor design The limit states method of design in which the factored (reduced) resistance is compared with the factored (increased) loads.

Load factor A factor used to multiply a nominal load to obtain part of the design load.

Local buckling A mode of buckling which occurs locally (rather than generally) in a thin plate element of a member.

Member One-dimensional structural element which supports transverse or longitudinal loads or moments.

Member buckling A mode of buckling involving the complete length of a member.

Nominal capacity Capacity of a member or structure computed using the formulations of a design code or specification.

Nominal load Load magnitude determined from a loading code or specification.

Non-uniform torsion The general state of torsion in which the twist of the member varies non-uniformly.

Out-of-plane buckling The buckling of a member out of the plane of loading.

Plastic analysis A method of analysis in which the ultimate strength of a structure is computed by considering the conditions for which there are sufficient plastic hinges to transform the structure into a mechanism.

Plastic hinge A fully yielded cross-section of a member which allows the member portions on either side to rotate under constant moment (the plastic moment).

Plastic moment The value of the bending moment which will cause a section to become fully yielded.

Post-buckling behaviour Behaviour after buckling.

Potential energy Energy associated with height of a gravitational load above a datum.

Pre-buckling behaviour Behaviour before buckling.

Purlin A horizontal member between main beams which supports roof sheeting.

Reduced modulus The modulus of elasticity used to predict the buckling of inelastic members under constant applied load, so called because it is reduced below the elastic modulus.

Residual stresses The stresses in an unloaded member caused by uneven cooling after rolling, flame cutting, or welding.

Resistance Capacity.

Restraint An element which restrains the deflection or twisting of a member.

Second-order analysis Non-linear analysis in which equilibrium is formulated for the deformed position of the structure, so that the moments caused by products of the loads and deflections are included.

Shear centre The point in the cross-section of a beam through which the resultant transverse force must act if the beam is not to twist.

Shear modulus The initial modulus of elasticity for shear stresses.

Squash load The value of the compressive axial load which will cause yielding throughout a short member.

Strain energy Energy associated with the straining of a structure.

Strain-hardening A stress-strain state which occurs at stresses which are greater than the yield stress.

Strength limit state The state of collapse or loss of structural integrity.

Tangent modulus The slope of the inelastic stress-strain curve which is used to predict the buckling of inelastic members under increasing load.

Total potential The sum of the strain energy of a structure and the potential energy of the gravitational loads acting on it.

Uniform torque That part of the total torque which is associated with the rate of change of the angle of twist rotation of the member.

Uniform torsion The special state of torsion in which the twist of the member varies linearly.

Virtual work A principle used to assess whether a structure is in an equilibrium position.

Warping A mode of deformation in which plane cross-sections do not remain plane.

Warping torque The other part of the total torque (than the uniform torque), which only occurs during non-uniform torsion, and which is associated with changes in the warping of the cross-sections.

Work Energy transferred during the movement of a force.

Yield stress The average stress during yielding when significant straining takes place. Usually, the minimum yield stress in tension specified for the particular steel.

Young's modulus The initial modulus of elasticity for normal stresses.

Principal notation

The following is the principal notation used in this book. Usually, only one meaning is assigned to each symbol, but in those cases where more meanings than one are possible, then the correct one will be evident from the context in which it is used.

A	Cross-sectional area
$[A_L], [A_Q]$	Matrices for linear and quadratic potential energy contributions of $\{q\}$
$[A_{LQ}], [A_{QQ}]$	Matrices for linear and quadratic potential energy contributions of $\{Q\}$
B	Flange width, or Bimoment
$[B_i]$	Matrix for in-plane generalized strains
$[B_L], [B_Q]$	Matrices for linear and quadratic generalized strains
$[B_u], [B_v]$	Matrices for out-of-plane generalized strains
C_{bc}	Moment gradient factor for beam-columns (equation 11.31)
C_m	Moment gradient factor for beam-columns (equation 14.55)
$[C]$	Matrix for out-of-plane nodal deformations
$[C_i]$	Matrix for in-plane nodal deformations
D	Overall depth of cross-section
$\{D\}$	Vector of restraint point shear centre deformations
$[D]$	Generalized elasticity matrix
$[D_i]$	In-plane generalized elasticity matrix
$[D_u]$	Out-of-plane generalized elasticity matrix
$[D_v]$	Generalized initial stress matrix
E	Young's modulus of elasticity
E_r	Reduced modulus of elasticity
E_s	Strain-hardening modulus of elasticity
E_t	Tangent modulus of elasticity
F_T	Translational restraint force
F_u	Ultimate tensile strength
F_Y	Yield stress
G	Shear modulus of elasticity
G_A, G_B	Relative stiffnesses of beam restraints at ends A, B
G_s	Strain-hardening shear modulus of elasticity
G_t	Tangent shear modulus of elasticity

$[G]$	Global stability matrix
$[G_e]$	Transformed element stability matrix
I_B, I_T	Second moments of area of bottom and top flanges
I_e	Second moment of area of elastic core
I_P	Polar second moment of area $= (I_x + I_y)/A$
I_{Px}	Section property $= \int_A y(x^2 + y^2)\mathrm{d}A$
I_w	Warping section constant
I_x, I_y	Second moments of area about the x, y axes
I_{yc}	Second moment of area of compression flange
J	Torsion section constant
K	Torsion parameter $= \sqrt{(\pi^2 EI_w/GJL^2)}$
$\bar{K}$	Beam parameter $= \sqrt{(\pi^2 EI_y h^2/4GJL^2)}$
$[K]$	Global out-of-plane stiffness matrix
$[K_e]$	Transformed element out-of-plane stiffness matrix
$[K_i]$	Global in-plane stiffness matrix
$[K_{it}]$	Global in-plane tangent stiffness matrix
L	Length of member
L_e	Effective length
L_R	Length of restraining segment
M	Moment, or Concentrated mass
M^*	Design bending moment
M_B, M_T	Bottom and top flange minor axis end moments
M_b	Nominal member moment capacity
M_d	Distortion moment
M_E	Elastic buckling moment
M_f	Flange moment
M_I	Inelastic buckling moment
M_L	Limiting moment at first yield
M_m	Maximum value of M_x
M_P	Full plastic moment
M_{Rx}, M_{Ry}, M_{Rz}	Bending and torsional restraint moments
M_s	Nominal section moment capacity
M_u	Uniform torque, or Ultimate moment capacity, or Unbraced buckling moment
M_{u0}	Ultimate moment capacity for uniform bending
M_w	Warping torque
M_x, M_y	Bending moments about x, y axes
M_Y	Moment at nominal first yield
M_{yz}	Uniform bending buckling moment $= \sqrt{\{P_y GJ(1 + K^2)\}}$
M_z	Torque about the longitudinal axis
$[M]$	Out-of-plane matrix of powers of z/L, or Mass matrix

$[M_i]$	In-plane matrix of powers of z/L
N	Axial tension force
$[N]$	Matrix relating $\{u, v, w, \phi\}^T$ to $\{\delta\}$
$[N_Q], [N_q]$	Matrices relating $\{\theta\}_1^T, \{\theta_2^T\}^T, \{\theta\}$ to $\{\delta\}$
$[N_\sigma]$	Matrix relating $\{\Phi\}$ to $\{\delta\}$
P	Axial compression force
P^*	Design axial compression force
P_E	Elastic buckling load
P_F	Failure load
P_I	Inelastic buckling load
P_r	Reduced modulus buckling load
P_s	Strain-hardening buckling load
P_t	Tangent modulus buckling load, or Nominal tension capacity
P_u	Ultimate axial force capacity
P_x, P_y	Column flexural buckling loads $= \pi^2 EI_x/L^2, \pi^2 EI_y/L^2$
P_Y	Squash load
P_z	Column torsional buckling load $= (GJ + \pi^2 EI_w/L^2)/r_2^2$
Q	Concentrated load
Q_E	Elastic buckling load
$\{Q_i\}$	In-plane global nodal forces
Q_Y	Load at nominal first yield
Q_∞	Value of Q for elastic buckling with rigid torsional support restraints
R	Radius of curvature
$\{R\}$	Vector of discrete restraint actions
$[S]$	Matrix of cross-section coordinates of P
T	Flange thickness
$[T_e]$	Out-of-plane transformation matrix
$[T_{ie}]$	In-plane transformation matrix
U	Strain energy
U, V, W	Deflections in global X, Y, Z directions
U_e	Element out-of-plane strain energy
U_{fb}	Flange bending strain energy
U_i	In-plane strain energy
U_R	Discrete restraint strain energy
U_T	Total potential
U_t	Uniform torsion strain energy
V	Potential energy, or Shear force, or Volume
V_e	Element out-of-plane potential energy
V_i	In-plane potential energy
V_y	Shear force in y direction

W	Work done, or
	Wagner stress resultant
X, Y, Z	Global axes
Z_x, Z_y	Elastic section moduli about x, y axes
$\{Z\}$	Vector of four powers of z/L
$\{Z_w\}$	Vector of two powers of z/L
a	Load distance along beam
a_0	Distance from shear centre
$\{a\}$	Vector of coefficients of powers of z
b	Width of thin rectangular element, or
	Distributed bimoment per unit length
d	Depth of narrow rectangular section
$\{d\}$	Vector of shear centre deformations
f_t	Translational restraint force per unit length
$[g_e]$	Element stability matrix
h	Distance between flange centroids
k	Effective length factor
k_l	Load height effective length factor
k_{Ry}	Minor axis bending effective length factor
k_w	Warping effective length factor
$[k_e]$	Element out-of-plane stiffness matrix
$[k_{ie}]$	Element in-plane stiffness matrix
$[k_{iet}]$	Element in-plane tangent stiffness matrix
m	Moment factor
m_{rx}, m_{ry}, m_{rz}	Bending and torsional restraining moments per unit length
m_u	Uniform torque per unit length
m_w	Warping torque per unit length
n	Integer
q	Load per unit length
q_0	Value of q for an unrestrained beam
$\{q_{ie}\}$	Element in-plane distributed loads
r_x, r_y	Radii of gyration about x, y axes
r_0^2	$=(I_x + I_y)/A$
r_1^2	$= r_0^2 + y_0^2$
r_2^2	$= r_0^2 + x_0^2 + y_0^2$
$\{r\}$	Vector of continuous restraint actions
s	Distance along section mid-thickness line, or
	Distance between discrete restraints, or
	Distance along curved shear centre axis
t	Thickness of thin-walled section, or
	Thickness of web, or
	Time
t_p	Distance from mid-thickness surface
u	Shear centre deflection in X direction

u_b	Buckling component of u
u_B, u_T	Bottom and top flange deflections in X direction
u_0	Initial crookedness
u_P, v_P, w_P	Deflections of P in X, Y, Z directions
$\{u\}$	Vector of deformations
v	Shear centre deflection in Y direction
v_b	Buckling component of v
$\{v_i\}$	In-plane nodal deflections of cross section
w	$= w_0 - \omega_c \phi'$
w_b	Buckling component of w
w_c	Value of w_P at centroid
w_s	Shear centre deflection in Z direction
x, y	Principal centroidal axes
x_c, y_c	Coordinates of centre of buckling rotation
x_0, y_0	Coordinates of shear centre
x_R, y_R	Distances of discrete rotational restraints from centroid
x_r, y_r	Distances of continuous rotational restraints from centroid
x_T, y_T	Distances of discrete translational restraints from centroid
x_t, y_t	Distances of continuous translational restraints from centroid
$\bar{y}$	Distance to centroid
y_Q	Distance of concentrated load from centroid
y_q	Distance of distributed load from centroid
z	Longitudinal axis through centroid
α	Angle, or Beam torsional stiffness
α_c	Stiffness of critical segment
α_L	Limiting value of stiffness
α_m	Buckling factor for beams
α_R	Stiffnesses of flange minor axis rotational end restraint, or Stiffness of restraining segment
$\alpha_{Rx}, \alpha_{Ry}, \alpha_{Rz}$	Stiffnesses of discrete bending and torsional restraints
$\alpha_{rx}, \alpha_{ry}, \alpha_{rz}$	Stiffnesses of continuous bending and torsional restraints
α_s	Slenderness reduction factor
α_{Tx}, α_{Ty}	Stiffnesses of discrete translational restraints
α_{tx}, α_{ty}	Stiffnesses of continuous translational restraints
α_W	Stiffness of discrete warping restraint
α_w	Stiffness of continuous warping restraints
$[\alpha_B]$	Discrete restraint stiffness matrix
$[\alpha_b]$	Continuous restraint stiffness matrix
β	Ratio of end moments
β_x	Monosymmetry section constant $= I_{Px}/I_x - 2y_0$
β_1, β_2	Major and minor axis end restraint parameters
γ	Shear strain, or End moment coefficient
γ_F	Stiffness factor for moment distribution

γ_M	Stiffness factor for restraints at far end
γ_P	Shear strain at P
γ_y	Load height factor for continuously restrained beams (equation 8.25)
γ_α	Restraint factor for continuously restrained beams (equation 8.24)
$\delta, \delta_x, \delta_y$	Central or end deflections
δ_0	Initial central crookedness
$\{\Delta\}$	Global out-of-plane nodal deformations
$\{\Delta_i\}$	Global in-plane nodal deformations
$\{\delta_e\}$	Element out-of-plane nodal deformations
$\{\delta_{ie}\}$	Element in-plane nodal deformations
ε	Normal strain, or Dimensionless distance of load from centroid
ε_P	Normal strain at P
ε_r	Residual strain $= \sigma_r/E$
ε_s	Strain-hardening strain
ε_Y	Yield strain
$\{\varepsilon\}$	Generalized strain vector
$\{\varepsilon_i\}$	Generalized in-plane strain vector
$\{\varepsilon_u\}$	Generalized stiffness strain vector
$\{\varepsilon_v\}$	Generalized stability strain vector
η	Crookedness parameter, or Coefficient of viscosity
θ	Rotation
θ_0	Initial central twist rotation
θ_d	Distortional twist rotation of flange
θ_f	Warping rotation of flange
$\theta_X, \theta_Y, \theta_Z$	Rotations about global X, Y, Z axes
κ	Curvature
λ	Buckling load factor
λ_c	Zero interaction buckling load factor of critical segment
λ_n	Zero interaction buckling load factor of nth segment
λ_R	Zero interaction buckling load factor of restraining segment
μ	$= \sqrt{(P/EI_x)}$
ν	Poisson's ratio
ρ	Density, or $= I_{yc}/I_y$
ρ_0	Perpendicular distance from shear centre
σ	Normal stress
σ_a	Allowable working stress
σ_m	Maximum normal stress
σ_r	Residual stress
σ_P	Normal stress at P

σ_w	Warping normal stress
$\{\sigma\}$	Generalized out-of-plane stress vector
$\{\sigma_i\}$	Generalized in-plane stress vector
τ	Shear stress
τ_P	Shear stress at P
Φ	Rotation about global Z axis
ϕ	Twist rotation, or
	Capacity factor
ϕ_b	Buckling component of ϕ
ϕ_0	Initial twist
Ω	Nature frequency of vibrations
ω	Section warping function

1 Introduction

1.1 General

Thin-walled structural members may fail in a flexural-torsional buckling mode, in which the member suddenly deflects laterally and twists out of the plane of loading. This form of buckling may occur in a member which has low lateral bending and torsional stiffnesses compared with its stiffness in the plane of loading.

The most common form of flexural-torsional buckling is for I-section beams which are loaded in the planes of their webs, but which buckle by deflecting laterally and twisting, as shown in Figures 1.1 and 1.2a. Flexural-torsional buckling may also occur in concentrically loaded columns. This can be regarded as a general case, of which flexural buckling without twisting is one limiting example (Figure 1.2b). Some columns may buckle torsionally without bending (Figure 1.2c), which is the other limiting example of the flexural-torsional buckling of columns. Beam-columns bent in a plane of symmetry may also buckle in a flexural-torsional mode.

Flexural-torsional buckling is not confined to individual members, but also occurs in rigid-jointed structures, where continuity of rotations between adjacent members causes them to interact during buckling.

Flexural-torsional buckling is a primary consideration in the design of steel structures, as it may reduce the load-carrying capacity. Unless it is prevented by using either sufficient bracing or members which have adequate flexural and torsional stiffnesses, then larger members must be used to avoid premature failure. The determination of these larger members will be dominated by considerations of flexural-torsional buckling.

This chapter provides an introduction to flexural-torsional buckling. An historical survey is made in section 1.2, which is followed by general reviews of structural behaviour in section 1.3, of buckling in section 1.4, and of design against buckling in section 1.5.

Chapter 2 provides a general treatment of buckling with particular reference to flexural-torsional buckling, while Chapters 3 and 4 present hand and computer methods of predicting elastic flexural-torsional buckling.

The buckling of individual columns, beams, and beam-columns is described in Chapters 5–9 and 11, while the buckling of continuous beams, frames, and arches (Figure 1.3) and rings is discussed in Chapters 10, 12, and 13.

Inelastic buckling is dealt with in Chapter 14, while the use of flexural-torsional buckling predictions in the determination of design strength is described in Chapter 15. A number of special topics are briefly discussed in Chapter 16.

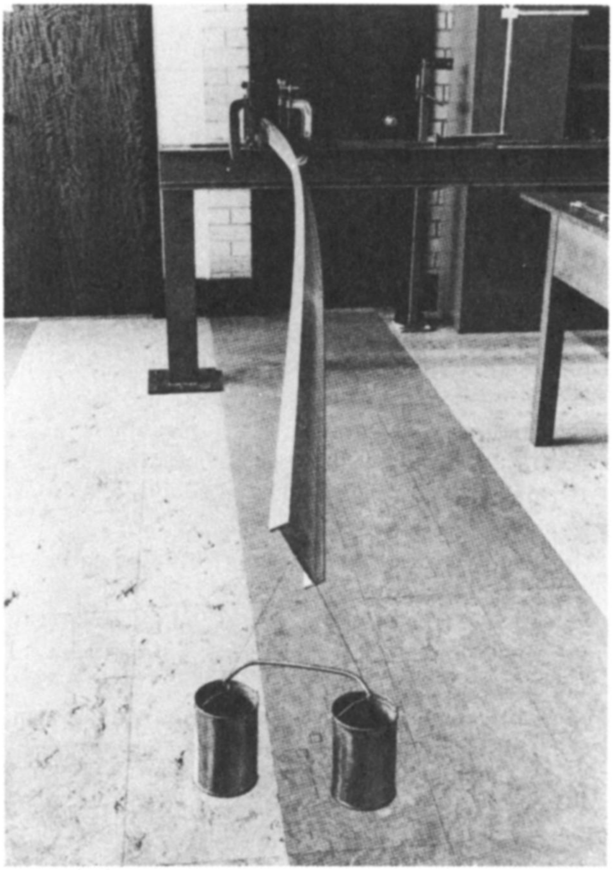

Figure 1.1 Flexural-torsional buckling of a cantilever.

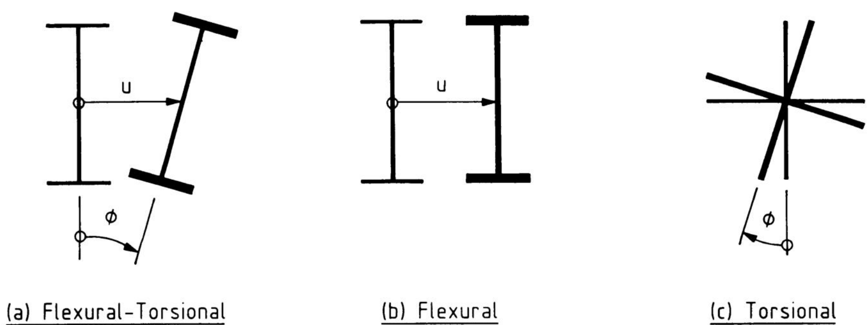

Figure 1.2 Forms of member buckling.

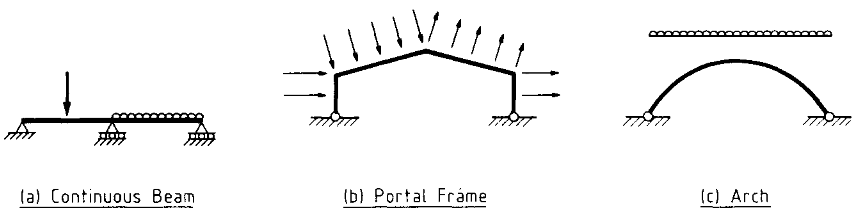

Figure 1.3 Some structural forms.

1.2 Historical development

1.2.1 ELASTIC BUCKLING THEORY

The initial theoretical research into elastic flexural-torsional buckling was preceded by Euler's 1759 treatise [1] on column flexural buckling (Figure 1.4a), which gave the first analytical method of predicting the reduced strengths of slender columns, and by Saint-Venant's 1855 memoir [2] on uniform torsion (Figure 1.4b), which gave the first reliable description of the twisting response of members to torsion.

However, it was not until 1899 that the first treatments were published of flexural-torsional buckling by Michell [3] and Prandtl [4], who considered the lateral buckling of beams of narrow rectangular cross-section. Their work was extended in 1905 by Timoshenko [5, 6] to include the effects of warping torsion in I-section beams.

Subsequent work in 1929 by Wagner [7] and later work by others led to the development of a general theory of flexural-torsional buckling, as stated by

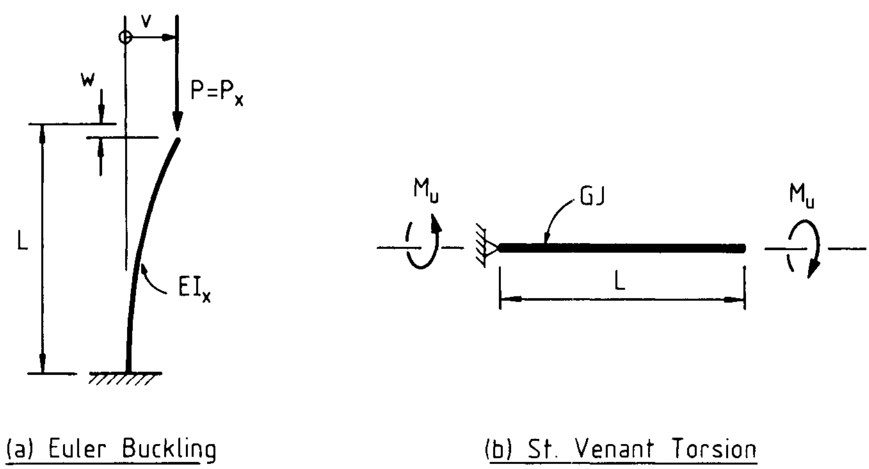

Figure 1.4 Euler buckling and St Venant torsion.

Timoshenko [8] and Vlasov [9], and incorporated in the textbooks of Timoshenko [10] and Bleich [11].

Specific studies of flexural-torsional buckling were made by many researchers, but prior to the 1960s, these were limited by the necessity to make extensive calculations by hand. Some of these are included in the 1960 survey by Lee [12].

This situation changed dramatically with the advent of the modern digital computer, and the 1960s saw an explosion in the amount of published research. As a result, the focus of research moved from the flexural-torsional buckling of isolated members under various loading conditions to the effects of end restraints exerted on a member of a rigid-jointed frame as a result of its continuity with adjacent members. Many of these studies are summarized in the 1971 survey of the Column Research Committee of Japan [13].

The extension of the general finite element method of structural analysis [14] to flexural-torsional buckling problems by Barsoum and Gallagher in 1970 [15] saw a further change, in that it was no longer necessary to publish comprehensive results of elastic flexural-torsional buckling studies, since almost any particular situation could now be analysed using a general purpose computer program. This development is similar to that which occurred in the in-plane analysis of plane rigid-jointed frames, in which the tabulations of solutions used in the 1930s were replaced by general purpose plane frame computer analysis programs.

Many of the developments of the theory of flexural-torsional buckling have been made by extensions of the previously accepted theories, as expressed either by the differential equations of elastic bending and torsion or by the energy equation for buckling. Not all of these extensions have received general acceptance, and so a number of attempts have been made through the 1980s to produce a generally acceptable theory of flexural-torsional buckling. This book includes such a general theory which is based on the use of the second-order relationships between the deformations and strains that take place during bending and torsion, the concept of the total potential, and the principles of virtual work and equilibrium, and of conservation of energy during buckling. This approach has been used, for example, to re-examine the flexural-torsional buckling of arches, early studies of which were reported by Vlasov [9] and Timoshenko [10].

1.2.2 STRENGTH AND DESIGN OF STEEL STRUCTURES

While the historical development of knowledge of flexural-torsional buckling undoubtedly was initiated by the need to prevent premature failure of steel structures in this mode, this is not well documented. It seems likely, however, that early design procedures for preventing the lateral buckling of steel beams followed and were closely related to those used for preventing the flexural failure of columns.

The need to be able to design against flexural-torsional buckling was the catalyst for the development of a theory for flexural-torsional buckling which would allow the successful prediction of failure. Early theoretical research was into the elastic buckling of perfectly straight members, some of which was verified experimentally. However, the very straight and slender members used for these experiments were unrepresentative of the real steel beams used in practice, tests of which showed that their strengths were reduced below those predicted solely by elastic buckling theory.

Theoretical research therefore extended from the elastic buckling of straight members to study the influences of crookedness, yielding, and residual stresses on the strengths of real steel beams, and to determine how to incorporate these into the procedures used in design. These developments tended to follow behind the corresponding developments from the elastic flexural buckling theory to the strengths of real steel columns. Early research on the inelastic lateral buckling of steel beams was carried out by Neal [16] and Galambos [17]. Flint [18] was one of the early researchers studying the effects of initial crookedness and twist [19] on the lateral buckling of beams and beam-columns.

Some of the early well-documented experiments on the lateral buckling of real steel beams were carried out by Hechtman, Hattrap, Styer, and Tiedmann [20]. Fukumoto and Kubo [21–23] reviewed and produced a data base of the experimental studies prior to 1977 on the lateral buckling of real steel beams.

Early rules foı designing steel beams against lateral buckling were generally transpositions of rules for designing columns against flexural buckling, with perhaps the first proposal based on flexural-torsional buckling being made in 1924 by Timoshenko [24]. The first modern treatment was probably given by Kerensky, Flint and Brown [25] as the basis for the British Standard BS153–1958 [26]. More recently, most countries have or are transforming their design standards into the limit states format [27]. Current design criteria are reviewed in [28–30].

1.3 Structural behaviour

1.3.1 ELASTIC BEHAVIOUR

1.3.1.1 Linear behaviour

The simplest and most widely used model of the behaviour of a structure under static loads assumes that all of the deformations are proportional to the magnitude of the load set acting on the structure, so that the relation between load and response is linear, as shown by Curve 1 in Figure 1.5.

For this linear model to be valid, the material itself must have a linear relationship between stress and strain. Such a material is usually described as elastic. (Strictly, elastic means perfect recovery on unloading, so that an elastic material may be non-linear. However, most elastic materials are linear.) Most structural steels are linear, at least for stresses less than the yield stress F_Y, as shown in Figure 1.6, while many other structural materials are regarded as being linear over most of the range of working load.

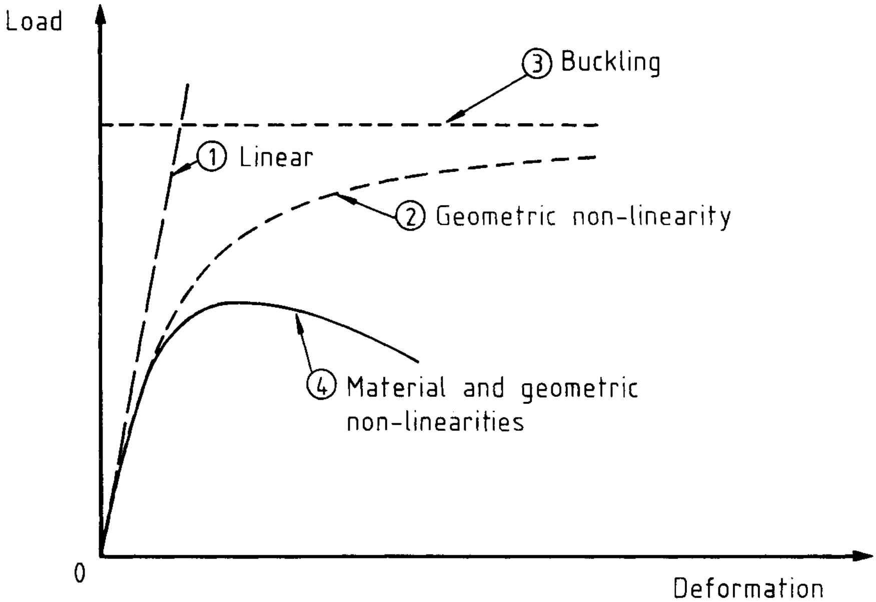

Figure 1.5 Structural behaviour.

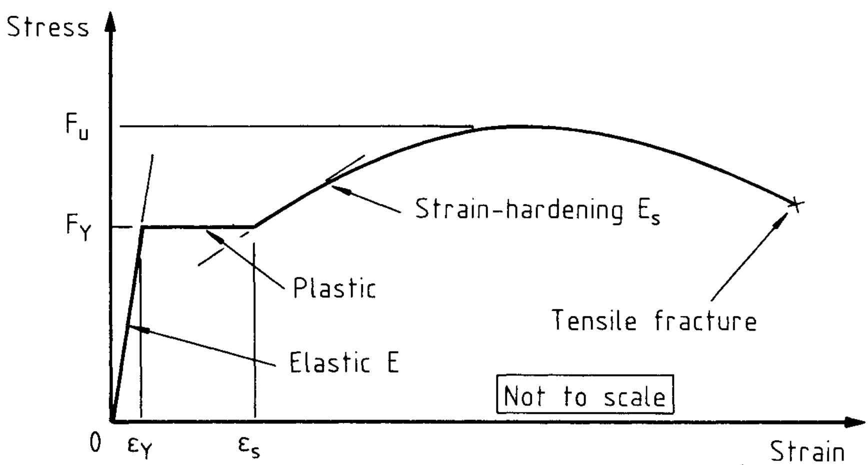

Figure 1.6 Idealized stress–strain relation for structural steel.

The structure itself must also behave linearly for the linear model to be valid. No structure is truly linear, but many are approximately so, provided the deflections are small.

The modern popularity of the linear elastic model of structural behaviour arises from the widespread availability of computer programs for linear elastic analysis [31]. These allow the deflections of the structure under load to be assessed for serviceability design under the working loads, and the member end actions to be approximated for strength design.

1.3.1.2 Non-linear behaviour

The linear elastic model of itself does not allow the strength of the structure to be assessed. For this purpose it is necessary to know of any material and structural non-linearities before the real behaviour of the structure and its maximum load-carrying capacity can be approximated.

Structural non-linearities cause the deformation response of the structure to load to become non-linear as shown by Curve 2 in Figure 1.5, even when the material remains linear. The most common structural non-linearities are associated with additional moments caused by the products of the loads and the transverse deflections of the structure or member as shown in Figure 1.7. Such effects are allowed for when equilibrium is formulated for the deformed geometry of the structure under load [32, 33], instead of the unloaded position, and so these non-linearities are usually described as being geometric, or second-order.

Geometric non-linearities may cause the load–deformation behaviour of an indefinitely elastic structure to asymptote towards a limit, as shown by Curve 2 in

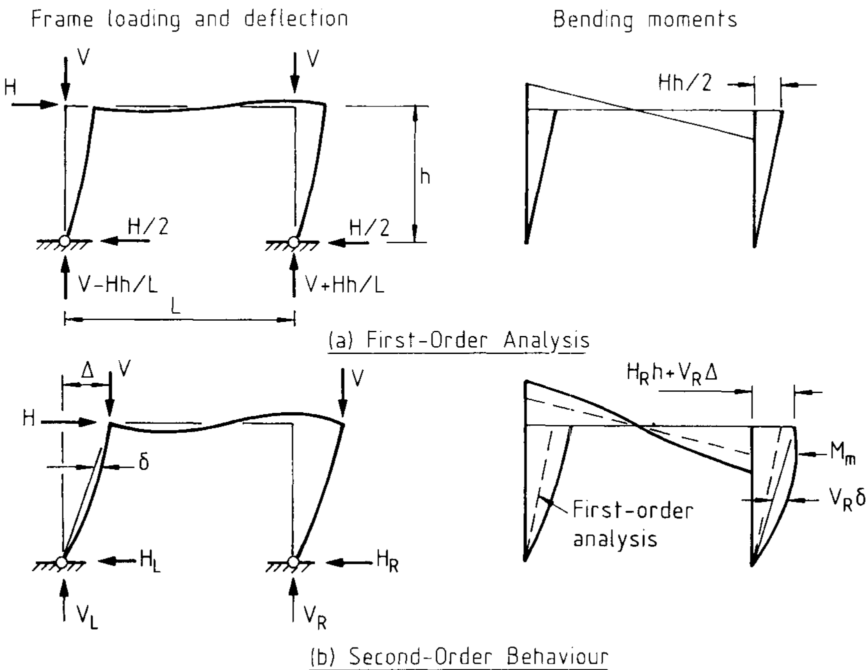

Figure 1.7 First-order analysis and second-order behaviour.

Figure 1.5. This limit (Curve 3 in Figure 1.5) is the elastic buckling load of the structure. Real structural behaviour will depart from this asymptotic behaviour when the material becomes non-linear as shown by Curve 4 in Figure 1.5, and a maximum load capacity will usually be reached, after which the load capacity will decrease.

The elastic buckling behaviour of a structure can be regarded as the limit of the elastic non-linear behaviour. In elastic buckling, the primary or pre-buckling response of the structure is in a different direction to the buckling response. For example, the pre-buckling response of the compression member shown in Figure 1.4a is due to longitudinal shortening w, while the buckling response is one of transverse bending deflections v. Thus the buckling response v remains zero until the buckling load P_x is reached, when the buckling response may initiate and continue indefinitely. For the buckling response to remain zero until the buckling load is reached, there must be no real or equivalent loads which would cause a primary response in the buckling direction.

A structure such as a concentrically loaded column or a beam loaded in the plane of the web may exhibit a real load–deformation response which differs only slightly from the idealized buckling response, in that the response in the buckling direction remains small until the buckling load is approached. Examples include straight concentrically loaded columns with small transverse loads, and columns with initial crookednesses which cause small transverse bending effects. For such members, the elastic buckling load may provide a quite accurate assessment of the strength, especially for slender members, for which small transverse loads or crookednesses are less important.

After the buckling load is reached, the post-buckling load–deformation curve may remain constant, or may rise or fall. This is caused by changes in the member stiffness that occur during buckling, which may lead to redistributions of the actions through the structure. Large deformations, for which there are gross changes in the chord lengths of some members and their rotations, may also affect the post-buckling behaviour.

1.3.2 INELASTIC BEHAVIOUR

1.3.2.1 Inelastic materials

All structural steels have a limited range over which the stress-strain behaviour is linear. Normal structural steels exhibit a horizontal yield plateau once the yield stress F_Y has been reached, as shown in Figure 1.6, followed by a slowly rising strain-hardening region. Cold-formed and stainless steels and aluminium all exhibit stress–strain curves which are rounded after a limit of proportionality is reached, as shown in Figure 1.8.

1.3.2.2 Inelastic stress distribution

The actual stress distribution at a member cross-section depends on the geometry of the section, its structural actions, the material stress–strain curve, and the

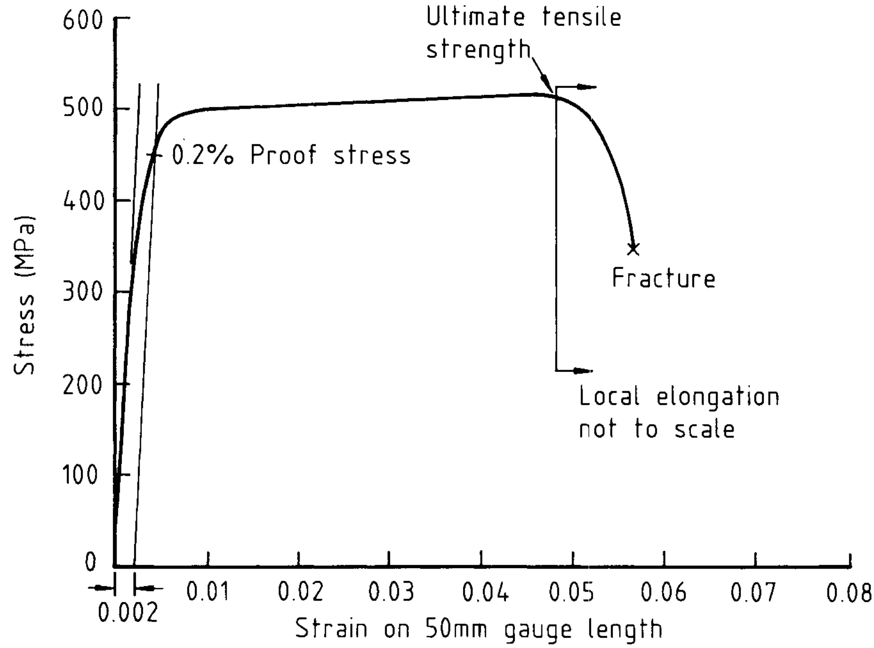

Figure 1.8 Stress–strain curve of cold reduced steel sheet.

residual stresses present before loading, such as those caused by the method of manufacture of the member.

When the stresses are low so that the member remains elastic, the stress distributions caused by axial force and bending actions are linear, as shown in Figure 1.9. Under bending actions, the maximum stresses occur at the extreme fibres, and when these reach the yield stress, a redistribution of the stresses commences. Useful structural limits (in the absence of local buckling effects) for structural steel members are provided by the moment M_P or axial force P_Y at which the cross-section becomes fully plastic.

The presence of residual stresses such as those caused by uneven cooling after hot-rolling or welding causes early initiation of yield, and generally affects the inelastic stress distribution. Because such residual stress distributions must be self-equilibrating so that they have zero axial force and bending actions in the unloaded member, they have no effect on the section full plastic capacities M_P and P_Y.

1.3.2.3 Inelastic members and structures

Inelastic effects on the behaviour of members and structures subject to buckling are best described separately in terms of their effects on the pre-buckling behaviour, on the buckling behaviour, and on the non-linear behaviour of members with geometrical imperfections such as initial crookedness or twist.

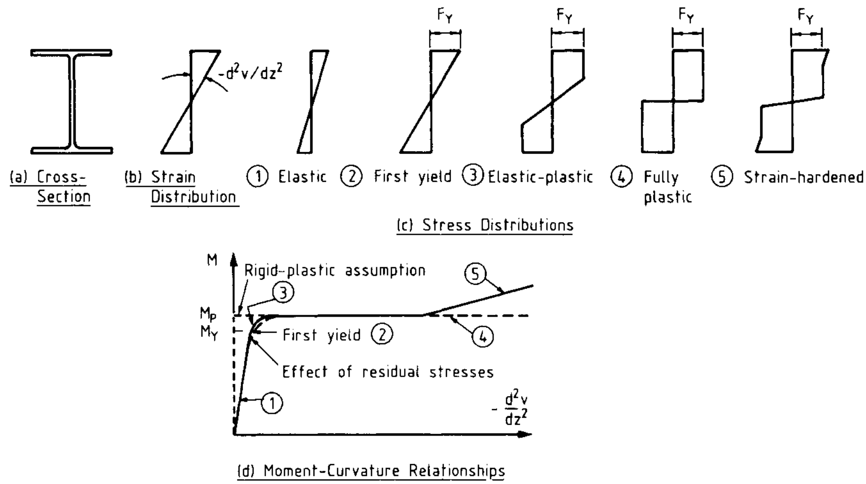

Figure 1.9 Moment–curvature relationships for steel beams.

Inelastic effects change the distributions of the bending moment, and to a lesser extent of the axial force, in an indeterminate structure before buckling. Since buckling depends on the pre-buckling distributions of these actions, these inelastic effects may be important.

Inelastic effects also increase the deflections. When there are significant geometric non-linear effects in the pre-buckling regime resulting from the pre-buckling deflections of the structure, these non-linear effects may be increased by additional deflections caused by inelastic behaviour. The advanced analysis of structures which accounts for geometric and material non-linearities including the effects of geometrical imperfections and residual stresses is described in [34].

While inelastic behaviour affects the buckling actions as described above, it also reduces the buckling resistance below the corresponding elastic resistance. Yielding causes local reductions in the cross-section stiffness which when aggregated over the complete member or structure may substantially reduce its buckling resistance. For example, one simple model of the inelastic flexural buckling of columns ignores the stiffness of any yielded regions of the column, so that its buckling resistance is based only on the regions of the column which remain elastic.

Inelastic behaviour also affects the strength of a member with small geometrical imperfections such as initial crookedness and twist. While the member remains elastic, its load–deformation behaviour asymptotes towards the elastic buckling behaviour of a perfectly straight member, as shown in Figure 1.5. The actual behaviour departs from this when the member first yields, and a maximum load is reached which is less than the elastic buckling load. This maximum load, which depends on the inelastic material properties, is sometimes approximated by the load at first yield.

1.4 Buckling

1.4.1 GENERAL

Buckling has already been described as the behaviour in which a structure or a structural element suddenly deforms in a (buckling) plane different to the original (pre-buckling) plane of loading and response. Member buckling (Figure 1.1) involves a single member, and may occur in flexural, torsional, or flexural-torsional modes. The half wave length of the buckle is of the same order as the member length.

Buckling may involve all the members of a frame, with interactions between the individual members. The buckle half wave length may be of the same order as a member length, or may be of the order of the frame size.

On the other hand, local buckling (Figure 1.10) usually takes place over a short length of a member of the same order as the cross-section width or depth. Distortional buckling (Figure 1.11) lies between member and local buckling, and is usually of a half wave length intermediate between the member and the cross-section dimensions.

These various forms of buckling are described in more detail in the following sub-sections.

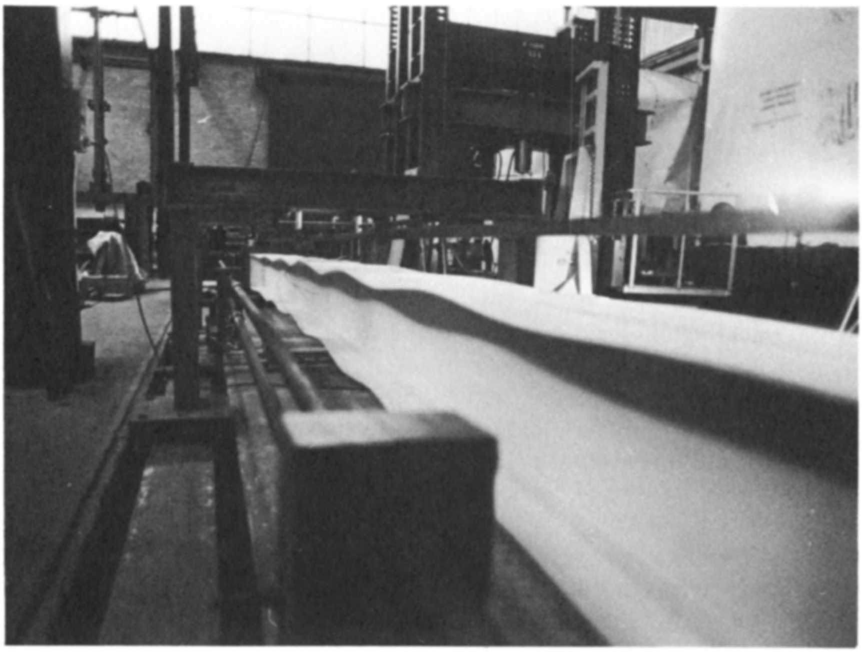

Figure 1.10 Local buckling of an I-section column.

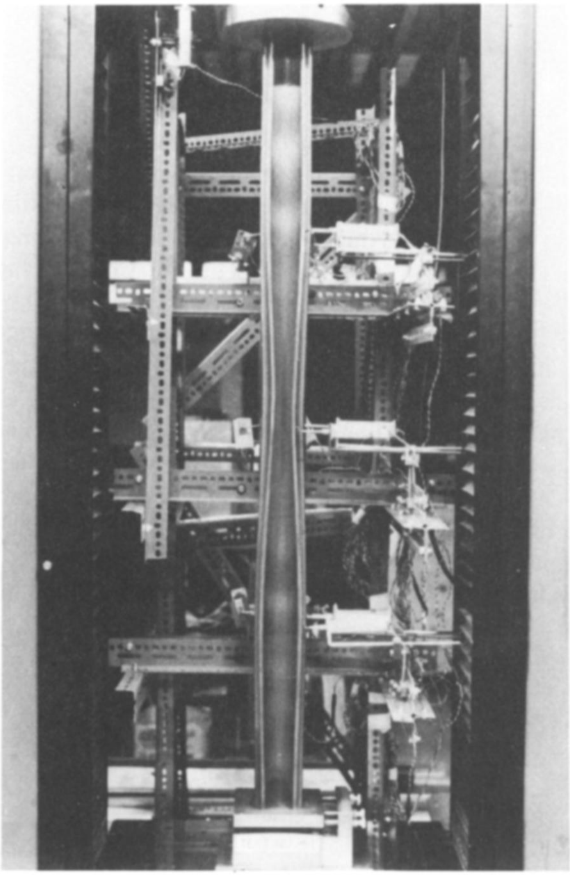

Figure 1.11 Distortional buckling of a channel section column.

1.4.2 FLEXURAL BUCKLING

Flexural buckling of a member (Figures 1.2b and 1.4a) may involve transverse displacements u or v of the member cross-sections, and is resisted by the flexural rigidity EI_y or EI_x of the member. It occurs when the second-order moments caused by the product of the axial compression force P with the displacements u or v are equal everywhere to the internal bending resistances $EI_y \mathrm{d}^2u/\mathrm{d}z^2$ or $-EI_x \mathrm{d}^2v/\mathrm{d}z^2$. Flexural buckling can be regarded as a limiting case of flexural-torsional buckling.

Flexural buckling may involve a single member, a group of members, or a complete frame. In braced frames, buckling is usually concentrated near one member, which is directly restrained by interactions with the adjacent members, and indirectly by the more remote members. In unbraced multi-storey structures, buckling occurs at one storey, and involves all the columns of that storey, which are restrained by the beams and columns of the adjacent storeys. Other unbraced frames may buckle in modes which directly involve many or all of the members.

1.4.3 TORSIONAL BUCKLING

Torsional buckling (Figure 1.2c) of a member involves twist rotations ϕ of the member cross-sections, and is resisted by the torsional rigidity GJ and the warping rigidity EI_w. It occurs when second-order torques $Pr_0^2 \mathrm{d}\phi/\mathrm{d}z$ caused by the axial compression force P and the twist $\mathrm{d}\phi/\mathrm{d}z$ are equal everywhere to the sum of the internal torsion resistances $GJ\,\mathrm{d}\phi/\mathrm{d}z$ and $-EI_w \mathrm{d}^3\phi/\mathrm{d}z^3$. Torsional buckling can be regarded as a limiting case of flexural-torsional buckling.

Torsional buckling may also occur in complete frames. Often the buckling resistance of these is dominated by the flexure of the individual members, as for example in tower frames whose horizontal cross-sections rotate.

1.4.4 FLEXURAL-TORSIONAL BUCKLING

Flexural-torsional buckling, which is the subject of this book, involves both displacements u, v and twist rotations ϕ, and is therefore resisted by combinations of the bending resistances $EI_y \mathrm{d}^2u/\mathrm{d}z$ and $-EI_x \mathrm{d}^2v/\mathrm{d}z^2$ and the torsional resistances $GJ\,\mathrm{d}\phi/\mathrm{d}z^2$ and $-EI_w \mathrm{d}^3\phi/\mathrm{d}z^3$.

While doubly symmetric columns whose centroidal and shear centre axes coincide buckle in either a flexural or a torsional mode, monosymmetric and asymmetric section columns may buckle in flexural-torsional modes. In these cases, the separation of the centroidal and shear centre axes causes these axes to become skew during buckling, so that the axial compression force acting along the centroidal axis has transverse components which create torques acting about the shear centre axis.

The flexural-torsional buckling of beams (Figures 1.1, 1.2a) involves lateral displacements u out of the plane of bending and twist rotations ϕ. In this case, the twist rotations ϕ cause the applied moments to have components acting out of the original plane of bending, while the lateral rotations $\mathrm{d}u/\mathrm{d}z$ cause the applied moments to have torque components about the axis of twist through the shear centre.

Beam-columns bent in a plane of symmetry may also buckle in flexural-torsional modes which combine those of columns and beams.

Flexural-torsional buckling may occur in frames (Figure 1.3b), where there are interactions between the adjacent members during buckling. In continuous or braced beams (Figure 1.3a), one span or segment is usually the most critical, and is restrained by the adjacent spans or segments. In three-dimensional frames, the members in each primary load-carrying plane interact during out-of-plane flexural-torsional buckling, and may be restrained by transverse members between adjacent primary frames.

Arches loaded in their plane (Figure 1.3c) may also buckle in a flexural-torsional mode by deflecting out of the plane and twisting.

1.4.5 LOCAL BUCKLING

Local buckling of a thin plate element (of thickness t) of a structural member involves deflections of the plate out of its original plane, as shown in Figure 1.10. Local buckling is resisted by the plate flexural rigidity $Et^3/12(1 - v^2)$, and occurs when the second-order actions caused by the in-plane compressions and the out-of-plane deflections are equal everywhere to the internal resistances of the plate element to bending and twisting.

Local buckling is usually concentrated near one particular cross-section of a member where the in-plane compressions of the plate elements are greatest, although multiple local buckles may occur in members whose stresses are constant along the member length. The half wave length of the buckle is of the order of the plate width. Local bucking effects may reduce the resistance of a member to flexural-torsional buckling.

Local buckling may occur in plate and shell structures, as well as in the structural members used in frame structures. Examples of plate structures include stiffened plate girders and rectangular and trapezoidal tanks, while shell structures include cylindrical and spherical containment structures. These may buckle locally in the more highly stressed regions, as well as in a more global fashion, involving larger regions of the structure.

1.4.6 DISTORTIONAL BUCKLING

Distortional buckling (Figure 1.11) describes a buckling mode intermediate between those of local and member buckling. In member buckling, the cross-section is assumed not to distort and buckling involves the whole member length, while local buckling involves relative displacements of the component plates over a short length of the member.

Distortional buckling often involves web flexure and corresponding rotations of the flanges which vary slowly along the member length, as shown in Figure 1.11. Distortional effects may reduce the flexural-torsional buckling resistances of thin-web beams.

1.5 Design against buckling

Methods of designing against flexural-torsional buckling are essentially of two types. For the first type, buckling is avoided, and the member's in-plane capacity is fully utilized. One way of achieving this is to use sections which are not susceptible to buckling. For example, closed sections have very much higher torsional rigidities GJ and higher flexural rigidities EI_y than corresponding open I-section members, and rarely buckle in a flexural-torsional mode. Less effective