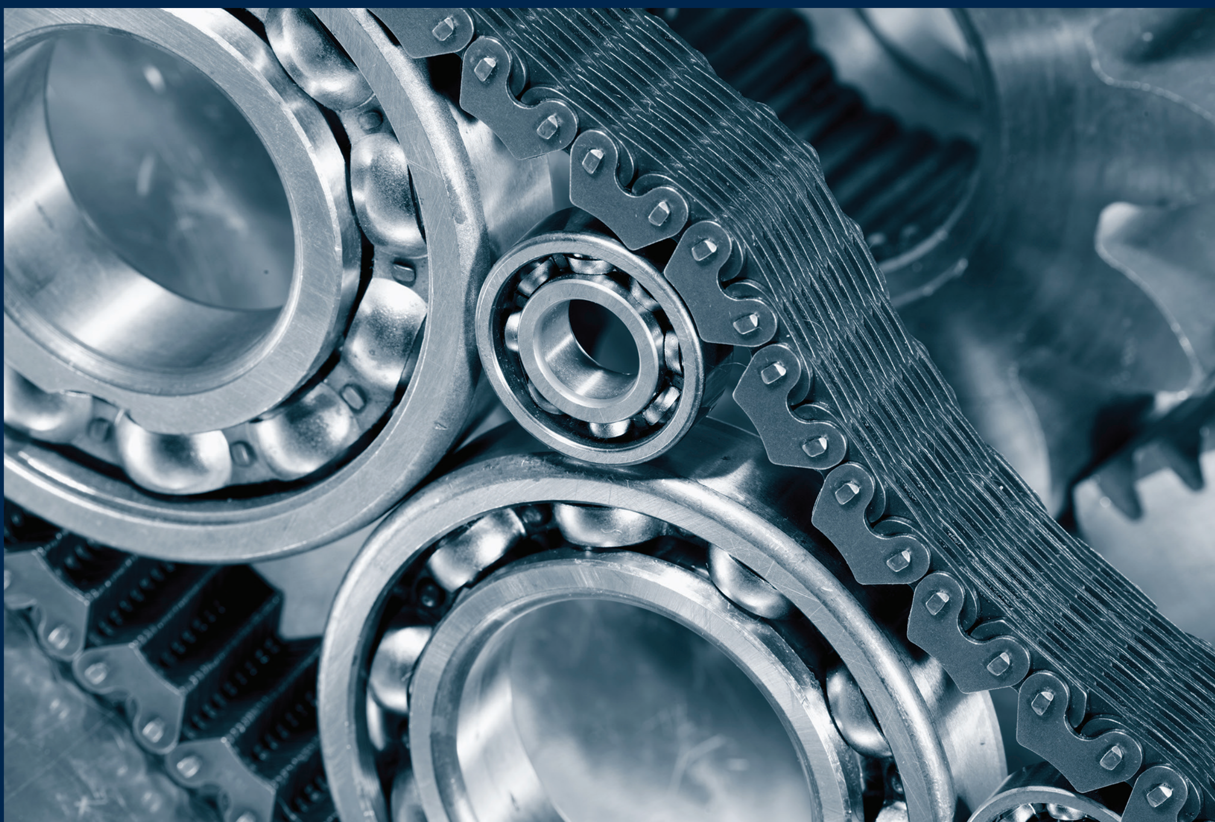
CRC Press
Taylor & Francis Group

Fundamentals of Machine Elements

SI Version

Third Edition



Steven R. Schmid
Bernard J. Hamrock
Bo O. Jacobson

Fundamentals of Machine Elements

SI Version

Third Edition

This page intentionally left blank

Fundamentals of

Machine Elements

SI Version

Third Edition

Steven R. Schmid
Bernard J. Hamrock
Bo O. Jacobson

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

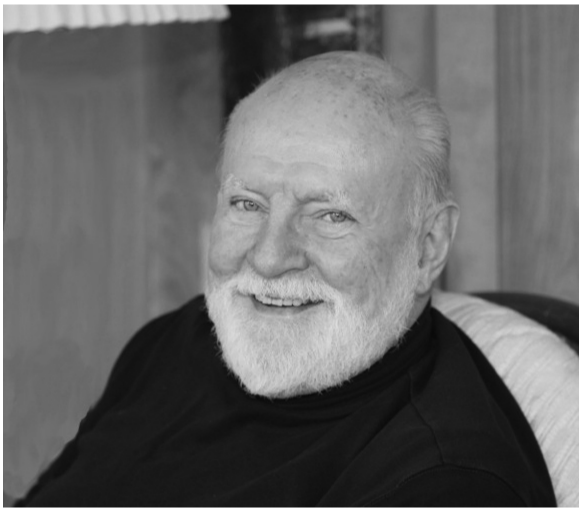
Version Date: 20140501

International Standard Book Number-13: 978-1-4822-4750-3 (eBook - PDF)

Visit the Taylor & Francis Web site at
http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com

Dedication



This book is dedicated to Professor Bernard J. Hamrock, a great friend and mentor. Those who have had the pleasure of knowing him understand that his is a rare intellect: a world-class researcher who fundamentally changed machine design with his contributions to contact mechanics and lubrication theory; a gifted instructor and research advisor; a prolific author of exceptional papers and books; and a valuable colleague to all who have come to know him.

Professor Hamrock's professional accomplishments are exceeded only by his personal ones: A beloved husband, his love for his wife, Rosemary, is unwavering, as is his dedication as a father and grandfather; friendly to all, and a trusted friend when needed. He is by no means the stereotypical bookish professor. A football athlete in his youth, he maintains a love of the Buckeyes, of his world travels and his wine sommeliering. Those who know Bernie are grateful for the experience.

Steven R. Schmid
Notre Dame, Indiana

Contents

Preface

The nature of the engineering profession is changing. It was once commonplace that students had significant machinery exposure before studying mechanical engineering, and it always was assumed that students would receive practical experience in internships or some form of co-operative employment during their college years, if not sooner. Students were historically drawn from much less diverse groups than today; students from a few decades ago (such as the authors) naturally gained experience with machinery from working on their car or tractor, and this experience was especially helpful for courses in design of machine elements. The demographics have changed, permanently and irrevocably, and the characteristics of incoming students have also changed. This has been exacerbated by the advances in technology that make maintenance of most machinery a discipline for only the specially trained. However, with a broad perspective, it has become clear that the demographics change has been an extremely positive development for the profession.

Design presents a number of challenges and opportunities to instructors. As a topic of study it is exciting because of its breadth and unending ability to provide fascinating opportunities for research, analysis, and creativity. Literally every discipline and sub-discipline in engineering has strong ties to design, and most universities have used design and manufacturing as the basis of a capstone course that culminates a mechanical engineering bachelor's degree. To students of engineering, it is, at first, an intimidating field so enormous that any semester or academic year sequence in machine design can do nothing but scratch the surface of the subject. This perception is absolutely true; like so many other areas of specialization within engineering, design truly is an area where lifelong learning is necessary.

Machine design is a challenge to both instructors and students. There are a number of courses, such as statics, dynamics, solid and fluid mechanics, etc., where topics for study are broken down into small portions and where closed-form, quantitative problems are routinely solved by students and by faculty during lectures. Such problems are important for learning concepts, and they give students a sense of security in that absolute answers can be determined. Too often, machine design is presented in a similar fashion. While, in practice, such closed-form solutions do exist, they are relatively rare. Usually, multiple disciplines are blended, and the information available is insufficient to truly optimize a desired outcome. In practice, engineers need to apply good judgment after they have researched a problem as best they can, given budgetary and time restrictions. They must then state or decide upon a solution, if not an answer. These difficult open-ended problems are much more demanding than closed-form solutions, and require a different mindset. Instead of considering a number as valid or invalid (usually by checking against the answer provided in the book or by the instructor), an open-ended problem can be evaluated only with respect to whether the result is reasonable and if good scientific methods were used. As experimental philosophers, design engineers should not hesitate proving their designs with prototypes or demonstrations. Of course, many students are taught that three weeks of modeling can save a day in the laboratory. (Sadly, this statement is not always recognized as ironic.)

This book is intended to provide the undergraduate student with a clear and thorough understanding of both the theory and application of the fundamentals of machine elements. It is expected that this book will also be used as a reference by practicing engineers. The book is not directed toward lower level undergraduate students — familiarity with differential and integral calculus is often needed to comprehend the material presented. The design of machine elements involves a great deal of geometry as well. Therefore, the ability to sketch the various configurations that arise, as well as to draw a free-body diagram of the loads acting on a component, are also needed. The material covered in this text is appropriate as a third- or fourth-year engineering course for students who have studied basic engineering sciences, including physics, engineering mechanics, and materials and manufacturing processes.

The book is divided into two parts. Part I (Chapters 1 to 8) presents the fundamentals, and Part II (Chapters 9 to 19) uses the fundamentals in considering the design of various machine elements. The material in Part I is sequential; material presented in early chapters is needed in subsequent chapters. This building-block approach provides the foundation necessary to design the various machine elements considered in Part II.

Learning Tools

The following pedagogical devices are used in each chapter to improve understanding and motivate the student:

- Each chapter will open with a photograph that clearly depicts the machine elements or topics covered in the chapter. Chapters will also have an opening quotation that is related to the chapter; the goal is to pique the reader's interest in the subject matter and start each chapter with a positive and entertaining feature to draw the students into the topic.

- In the margin to the side of the illustration, the contents, examples, case studies, and design procedures present in the chapter are listed.

- After the illustration, each chapter has a brief abstract that indicates the contents at a very high level. Part of this abstract will include a list of machine elements covered in the chapter, the typical applications of the machine elements in the chapter, and the alternate machine elements that can be considered by designers.

- A list of symbols and subscripts is then presented to help students with nomenclature as they read the chapter.

- Figures and tables have been redrawn in this edition to use modern graphical procedures of three-dimensional sketches, thick boundary lines, and sans-serif fonts in illustrations.

- Examples are printed with a light gray background to differentiate them from the text. Examples demonstrate the mathematical procedures covered and are useful for students performing quantitative problems.

- Design procedures are printed with a light color background to differentiate them from the text and examples. The design procedures are useful guides to common design problems and aide students with all levels of Bloom's taxonomy of learning.

- Case studies are printed with a light color background and are placed just before the chapter-ending summary. Case studies are mostly qualitative descriptions of important modern applications of the chapter's machine elements, but at a depth that requires an understanding of the chapter material. Case studies are intended to reference the chapter's subject matter and place it in the proper design framework so that students have no doubt that the chapter is relevant and important.

- After the summary, the chapter has a list of key words that the student can use for study or to help with jargon when necessary.

- A summary of equations is contained after the key words, and is intended to help students as they work on chapter-ending problems. The summary of equations is also a useful handout for instructors to copy and give to the students for exams.

- Every chapter includes lists of recommended readings consisting of modern as well as classic books and other resources that are especially timeless and relevant.

- The styles of the chapter-ending problems have been designed to cover every stage in modern learning taxonomies. Chapter-ending problems are organized as:

  1. Questions. These address the "remembering" task of learning taxonomies.
  2. Qualitative Problems. These are carefully designed to take an understanding of machine elements gleaned from the book and lecture and applying them to a new situation.
  3. Quantitative Problems. These problems focus on numerical analysis, with some extension to evaluating designs and results. Historically, machine element texts have provided only such analysis problems. Answers to the majority of quantitative problems are given. Solutions to the homework problems can be found in the Instructor's Solutions Manual, available to instructors who adopt the text. In addition, most problems have worksheets, where a partial solution is provided.
  4. Synthesis, Design, and Projects. These are open-ended, often team-based exercises that require creation of new designs or principles and that go beyond normal analysis problems.

Engineering educators will recognize that the end-of-chapter problems are designed to accommodate taxonomies of learning, allowing students of all backgrounds to develop an understanding, familiarity, and mastery of the subject matter.

The qualitative problems and synthesis, design, and projects class of problems also promote a useful method of active learning. In addition to conventional lecture format classes, an instructor can incorporate these problems in "seminar" sessions, active learning, or else for group projects. The authors have found this approach to be very useful and appreciated by students.

Certain users will recognize a consistent approach and pedagogy as the textbook *Manufacturing Engineering and Technology*, and will find that the texts complement each other. This is by intent, and it is hoped that the engineering student will realize quickly that *to do manufacturing or design, one needs to know both*.

Web Site

A web site containing other book-related resources can be found at www.crcpress.com/product/isbn/9781482247480. The web site provides reported errata, web links to related sites of interest, password-protected solutions to homework problems for instructors, a bulletin board, and information about ordering books and supplements. The web site also contains presentation files for instructors and students, using full-color graphics whenever possible.

Contents

Chapter 1 introduces machine design and machine elements and covers a number of topics, such as safety factors, statistics, units, unit checks, and significant figures. In designing a machine element it is important to evaluate the kinematics, loads, and stresses at the critical section. Chapter 2 describes the applied loads (normal, torsional, bending, and transverse shear) acting on a machine element with respect to time, the area over which the load is applied, and the location and method of application. The importance of support reaction, application of static force and moment equilibrium, and proper use of free-body diagrams is highlighted. Shear and moment diagrams applied to beams for various types of singularity function are also considered. Chapter 2 then describes stress and strain separately.

Chapter 3 focuses on the properties of solid engineering materials, such as the modulus of elasticity. (Appendix A gives properties of ferrous and nonferrous metals, ceramics, polymers, and natural rubbers. Appendix B explores the stress-strain relationships for uniaxial, biaxial, and triaxial stress states.) Chapter 4 describes the stresses and strains that result from the types of load described in Chapter 2, while making use of the general Hooke's law relationship developed in Appendix B. Chapter 4 also considers straight and curved members under these four types of load.

Certainly, ensuring that the design stress is less than the yield stress for ductile materials and less than the ultimate stress for brittle materials is important for a safe design. However, attention must also be paid to displacement (deformation) since a machine element can fail by excessive elastic deformation. Chapter 5 attempts to quantify the deformation that might occur in a variety of machine elements. Some approaches investigated are the integral method, the singularity function, the method of superposition, and Castigliano's theorem. These methods are applicable for distributed loads.

Stress raisers, stress concentrations, and stress concentration factors are investigated in Chapter 6. An important cause of machine element failure is cracks within the microstructure. Therefore, Chapter 6 covers stress levels, crack-producing flaws, and crack propagation mechanisms and also presents failure prediction theories for both uniaxial and multiaxial stress states. The loading throughout Chapter 6 is assumed to be static (i.e., load is gradually applied and equilibrium is reached in a relatively short time). However, most machine element failures involve loading conditions that fluctuate with time. Fluctuating loads induce fluctuating stresses that often result in failure by means of cumulative damage. These topics, along with impact loading, are considered in Chapter 7.

Chapter 8 covers lubrication, friction, and wear. Not only must the design stress be less than the allowable stress and the deformation not exceed some maximum value, but also lubrication, friction, and wear (tribological considerations) must be properly understood for machine elements to be successfully designed. Stresses and deformations for concentrated loads, such as those that occur in rolling-element bearings and gears, are also determined in Chapter 8. Simple expressions are developed for the deformation at the center of the contact as well as for the maximum stress. Chapter 8 also describes the properties of fluid film lubricants used in a number of machine elements. Viscosity is an important parameter for establishing the load-carrying capacity and performance of fluid-film lubricated machine elements. Fluid viscosity is greatly affected by temperature, pressure, and shear rate. Chapter 8 considers not only lubricant viscosity, but also pour point and oxidation stability, greases and gases, and oils.

Part II (Chapters 9 to 20) relates the fundamentals to various machine elements. Chapter 9 deals with columns, which receive special consideration because yielding and excessive deformation do not accurately predict the failure of long columns. Because of their shape (length much larger than radius) columns tend to deform laterally upon loading, and if deflection becomes critical, they fail catastrophically. Chapter 9 establishes failure criteria for concentrically and eccentrically loaded columns.

Chapter 10 considers cylinders, which are used in many engineering applications. The chapter covers tolerancing of cylinders; stresses and deformations of thin-walled, thick-walled, internally pressurized, externally pressurized, and rotating cylinders; and press and shrink fits.

Chapter 11 considers shafting and associated parts, such as keys, snap rings, flywheels, and couplings. A shaft design procedure is applied to static and cyclic loading; thus, the material presented in Chapters 6 and 7 is directly applied to shafting. Chapter 11 also considers critical speeds of rotating shafts.

Chapter 12 presents the design of hydrodynamic bearings — both thrust and journal configurations — as well as design procedures for the two most commonly used slider bearings. The procedures provide an optimum pad configuration and describe performance parameters, such as normal applied load, coefficient of friction, power loss, and lubricant flow through the bearing. Similar design information is given for plain and nonplain journal bearings. The chapter also considers squeeze film and hydrostatic bearings, which use different pressure-generating mechanisms.

Rolling-element bearings are presented in Chapter 13. Statically loaded radial, thrust, and preloaded bearings are considered, as well as loaded and lubricated rolling-element bearings, fatigue life, and dynamic analysis. The use of the elastohydrodynamic lubrication film thickness is integrated with the rolling-element bearing ideas developed in this chapter.

Chapter 14 covers general gear theory and the design of spur gears. Stress failures are also considered. The transmitted load is used to establish the design bending stress in a gear tooth, which is then compared with an allowable stress to establish whether failure will occur. Chapter 14 also considers fatigue failures. The Hertzian contact stress with modification factors is used to establish the design stress, which is then compared with an allowable stress to determine whether fatigue failure will occur. If an adequate protective elastohydrodynamic lubrication film exists, gear life is greatly extended.

Chapter 15 extends the discussion of gears beyond spur gears as addressed in Chapter 14 to include helical, bevel, and worm gears. Advantages and disadvantages of the various types of gears are presented.

Chapter 16 covers threaded, riveted, welded, and adhesive joining of members, as well as power screws. Riveted and threaded fasteners in shear are treated alike in design and failure analysis. Four failure modes are presented: bending of member, shear of rivet, tensile failure of member, and compressive bearing failure. Fillet welds are highlighted, since they are the most frequently used type of weld. A brief stress analysis for lap and scarf adhesively bonded joints is also given.

Chapter 17 treats the design of springs, especially helical compression springs. Because spring loading is most often continuously fluctuating, Chapter 16 considers the design allowance that must be made for fatigue and stress concentration. Helical extension springs are also covered in Chapter 16. The chapter ends with a discussion of torsional and leaf springs.

Brakes and clutches are covered in Chapter 18. The brake analysis focuses on the actuating force, the torque transmitted, and the reaction forces in the hinge pin. Two theories relating to clutches are studied: the uniform pressure model and the uniform wear model.

Chapter 19 deals with flexible machine elements. Flat belts and V-belts, ropes, and chains are covered. Methods of effectively transferring power from one shaft to another while using belts, ropes, and chains are also presented. Failure modes of these flexible machine elements are considered.

What's New in This Edition

This third edition represents a major revision from the second edition. In addition to the pedagogy enhancements mentioned above, the contents have been greatly expanded and organized to aide students of all levels in design synthesis and analysis approaches. Design synthesis is generally taught or expected of students only after a machine elements course in most college curricula. This book attempts to provide guidance through design procedures for synthesis issues, but it also exposes the reader to a wide variety of machine elements.

Users of the second edition will immediately recognize that this third edition has been completely re-typeset using a space-saving, two-column approach, and all figures redrawn to match the new column widths. The space-saving typesetting format has saved over 300 pages from the previous edition, while the content has been expanded considerably. This was, in fact, a goal: too many textbooks are difficult to use because they give the impression of completeness, but this is often illusory. Large margins and gaps between topics artificially produce heavy tomes. Our goal was to create a book with good coverage that can be more easily carried by students.

In every chapter opening box, the reader is directed toward other machine elements that can serve the same purpose, which can also help in synthesis. As an example, a student designing a gear set for power transmission between two shafts may thus be reminded that a belt drive is perhaps an alternative worthy of consideration.

The book has been designed to compliment the well-known manufacturing textbooks *Manufacturing Processes for Engineering Materials* and *Manufacturing Engineering and Technology* by Kalpakjian and Schmid. Students who use both texts in their engineering studies will recognize similarities in organization, graphical styles, and, it is hoped, clarity.

The classes of chapter-ending problems have been introduced above, but they have been carefully designed to aid students to develop a deep understanding of each chapter's subject matter. They have been developed using learning taxonomies that require ever-sophisticated cognitive effort. That is, students are required to remember (Questions), apply knowledge to fairly simple and straightforward questions (Qualitative Problems), extend the knowledge to analytical problems (Quantitative Problems), and finally asked to extend their analytical abilities to open-ended and synthesis problems requiring creativity in their solution (Synthesis and Design Problems).

A major effort has been made to expand coverage in all areas. Specific changes to this edition include:

- In Chapter 1, additional design considerations have been listed in Section 1.4, additional examples and case studies have been added, and life cycle engineering has been included.
- Chapter 3 now includes a description of hardness and common hardness tests used for metals; this clarifies the use of these concepts in gear design. In addition, the manufacturing discussion has been expanded.
- The use of retaining rings in Chapter 11 necessitated the inclusion of flat groove stress concentration factors in Chapter 6.
- Chapter 7, on fatigue design, has been significantly expanded. The staircase method for determining endurance limits has been added in Design Procedure 7.2, the fatigue strength concentration factor descriptions are longer with more mathematical models, and Haigh diagrams are included to show the effects of mean stress. Additional material data has been included for the fracture mechanics approaches to fatigue design.
- In Chapter 8, a streamlined discussion of typical surface finishes in machine elements, and manufacturing processes used to produce them, has been prepared. In addition, a discussion of the commonly used bearing materials has been added.
- Chapter 11 has been expanded considerably. In addition to an expanded discussion of keys and set screws, the chapter presents new treatment of spline, pin, and retaining ring design, and has a new section on the design of shaft couplings.
- Hydrodynamic bearings are increasingly important because of their widespread use in transportation and power industries; while the discussion of thrust and journal bearings has been retained, the analysis is simplified and more straightforward. The discussion of squeeze film and hydrostatic bearings has been expanded.
- Chapter 13 has been extensively rewritten to reflect the latest International Standards Organization standards that unify the approach used to design rolling element bearings. This has allowed a simplification of bearing selection and analysis, as will be readily apparent. Further, this remains the only machine element book that accurately depicts the wide variety of bearings available. This treatment now includes the topic of toroidal bearings, a novel design that is now widely available, and leads to compact and high load carrying designs. Life adjustment factors and effects of variable loading have been expanded, and an industrially relevant case study on windmill bearings has been exhaustively researched and included in the chapter.
- The treatment of spur gear design in Chapter 14 has been modified to reflect the latest advances in materials, including powder metal materials that have become extremely popular for automotive applications. The importance of lubrication in gears has been emphasized. Further, a design synthesis approach for spur gear design has been included in Section 14.14.
- Geometry factors for bevel gears in Chapter 15 have been simplified without loss in accuracy. Also, a design synthesis approach for worm gears has been included.
- The discussion of fasteners and welds in Chapter 16 has been expanded considerably. The importance of the heat affected zone for weld quality is discussed, and the classes of welds and their analysis methods are described. This includes the treatment of modern welding approaches such as friction stir welding as well as laser and electron beam welding.
- Gas springs and wave springs have been added to the discussion of Chapter 17.
- Chapter 18 has been reorganized, starting with fundamental principles that apply to all brake and clutch systems, especially thermal effects. Additional automotive examples have been added.
- Chapter 19 has been essentially rewritten to reflect the latest standards and manufacturer's recommendations on belt design, chains, and wire ropes. In addition, silent chains have been included into the chain discussion.
- The appendices have been expanded to provide the student with a wide variety of material properties, geometry factors for fracture analysis, and new summaries of beam deflection. While it is recognized that modern students have such information readily available via the Internet, making such material available in the textbook is useful for reference purposes.

This text has been under preparation for over four years, and required meticulous efforts at maintaining a consistent approach, careful statement of design procedures wherever they were useful, and expansion of chapter-ending problems. We hope the student of machine element design will enjoy and benefit from this text.

Steven R. Schmid
The University of Notre Dame

Bernard J. Hamrock
The Ohio State University

Bo O. Jacobson
Lund University

Acknowledgments

Many people helped to produce this textbook. Jonathan Plant, who was the editor of the first edition, was extremely supportive during the preparation of this edition. Professor Serope Kalpakjian (ret.) of the Illinois Institute of Technology made numerous comments and helpful suggestions, and also assisted in review of the manuscript. Professor William Dornfeld kindly used draft versions of this textbook in a number of semesters and made very constructive comments and suggestions. Dr. Michael Kotsalas of the Timken company was of great assistance in modernizing the chapters in bearing design. A number of faculty members from the mechanical engineering departments at Notre Dame and Ohio State University provided comments about the text. In particular, the authors would like to thank Professors John Renaud, Anthony Luscher, Si Lee, Necip Berme, and Bharat Bhushan. Special thanks are also due to Triodyne, Inc. personnel, especially Ralph L. Barnett, whose insights were invaluable. A number of resources from Triodyne were kindly made available in the preparation of the text.

Many reviewers have been associated with this project. The authors would like to acknowledge them for their time, expertise, and advice. The list of reviewers are Timothy Rodts, Michael Giordano, Matthew Prygoski, and Amy Libardi, University of Notre Dame; Miguel Angel Sellés Cantó, Escola Politècnica Superior d' Alcoi, Universitat Politẽcnica de València; Peder Klit, Technical University of Denmark; Robert W. Ellis, Lawrence Technological University; James Adams, Metal Powder Industries Federation; Thomas Kurfess, Georgia Institute of Technology; Paul K. Wright, University of California at Berkeley; Jian Cao, Northwestern University; K. Scott Smith, University of North Carolina at Charlotte; William G. Ovens, Rose-Hulman Institute of Technology; John D. Reid, University of Nebraska–Lincoln; Steven Y. Liang, Georgia Institute of Technology; Steven A. Velinsky, University of California–Davis; Thierry Blanchet, Rensselaer Polytechnic Institute; K.V.C. Rao, Michigan Technological University; Alexander G. Liniecki, San Jose State University; Jesa Kreiner, California State University–Fullerton; Clarence Maday, North Carolina State University; J. Darrell Gibson, Rose-Hulman Institute of Technology; Terry F. Lehnhoff, University of Missouri–Rolla; Yu Michael Wang, University of Maryland; B.K. Rao, Idaho State University; W. Brent Hall, University of Illinois–Urbana-Champaign; N. Duke Perreira, Lehigh University; Gordon Pennock, Purdue University; Richard E. Dippery, Kettering University; Anthony Luscher, Ohio State University; Michael Peterson, Colorado State University; Gary McDonald, The University of Tennessee at Chattanooga; Wayne D. Milestone, University of Wisconsin–Madison; and Dean Taylor, Cornell University.

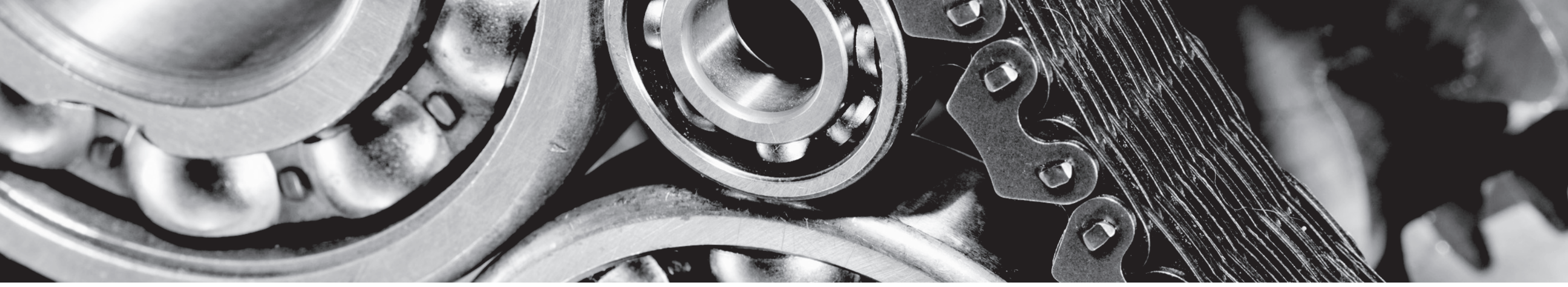
The authors also acknowledge the use of tables and illustrations from the following publishers: American Gear Manufacturers Association, American Society of Mechanical Engineers, American Society for Testing and Materials, BHRA Fluid Engineering, Buttersworths, Elsevier Science Publishing Company, Engineering Sciences Data Unit, Ltd., Heinemann (London), Hemisphere Publishing Corporation, Macmillan Publishing Company, Inc., Mechanical Technology Incorporated, Metal Powder Industries Federation, Non-Ferrous Founders Society, Oxford University Press, Inc., Penton Publishing Inc., Society of Automotive Engineers, Society of Tribologists and Lubrication Engineers, VCH Publishers, John Wiley & Sons, and Wykeham Publications (London), Ltd. The specific sources are identified in the text.

About the Authors

Steven R. Schmid received his B.S. degree in Mechanical Engineering from the Illinois Institute of Technology in 1986. He then joined Triodyne, Inc., where his duties included investigation of machinery failures and consultation in machine design. He earned his Master's degree from Northwestern University in 1989 and his Ph.D. in 1993, both in mechanical engineering. In 1993 he joined the faculty at the University of Notre Dame, where he is currently a Professor of Aerospace and Mechanical Engineering and teaches and conducts research in the fields of design and manufacturing. Dr. Schmid received the American Society of Mechanical Engineers Newkirk Award and the Society of Manufacturing Engineers Parsons Award in 2000. He was also awarded the Kaneb Center Teaching Award in 2000, 2003, and 2010, and served as a Kaneb fellow in 2003. Dr. Schmid holds professional engineering (P.E.) and certified manufacturing engineer (C.Mfg.E.) licenses. He is co-author (with S. Kalpakjian) of *Manufacturing Engineering and Technology* (2014) and *Manufacturing Processes for Engineering Materials* (2008), both published by Prentice Hall. He was awarded the ASME Foundation Swanson Fellowship in 2012, and is a Fellow of the American Society of Mechanical Engineers.

Bernard J. Hamrock joined the staff of Ohio State University as a professor of Mechanical Engineering in 1985 and is now Professor Emeritus. Prior to joining Ohio State University he spent 18 years as a research consultant in the Tribology Branch of the NASA Lewis Research Center in Cleveland, Ohio. He received his Ph.D. and Doctor of Engineering degrees from the University of Leeds, Leeds, England. Professor Hamrock's research has resulted in a book with Duncan Dowson, *Ball Bearing Lubrication*, published in 1982 by Wiley Interscience, three separate chapters for handbooks, and over 150 archival publications. His second book, *Fundamentals of Fluid Film Lubrication* was published in 1993 by McGraw-Hill, and a second edition, with co-authors Steven Schmid and Bo Jacobson, in 2004. His awards include the 1976 Melville Medal from the American Society of Mechanical Engineers, the NASA Exceptional Achievement Medal in 1984, the 1998 Jacob Wallenberg Award given by The Royal Swedish Academy of Engineering Sciences, and the 2000 Mayo D. Hersey Award from the American Society of Mechanical Engineers.

Bo O. Jacobson received his Ph.D. and D.Sc. degrees from Lund University in Sweden. From 1973 until 1987 he was Professor of Machine Elements at Luleå Technology University in Sweden. In 1987 he joined SKF Engineering & Research Centre in the Netherlands, while retaining a professorship at Chalmers University from 1987 to 1991 and at Luleå Technical University from 1992 to 1997. In 1997 he was appointed Professor of Machine Elements at Lund University, Sweden. Professor Jacobson was a NRC Research Fellow at NASA Lewis Research Center from 1981 to 1982. He has published four compendia used at Swedish universities. His text *Rheology and Elastohydrodynamic Lubrication* was published by Elsevier in 1991, and his book *Rolling Contact Phenomena* (with J. Kalker) was published in 2000 by Springer. Professor Jacobson has more than 100 archival publications. His awards include the prestigious Gold Medal given by the Institution of Mechanical Engineers, England, and the Wallenberg Award in 1984.



Part I — Fundamentals

Outline

This page intentionally left blank

Chapter 1

Introduction

The i8 concept car, a hybrid sports car requiring three liters per 100 km and acceleration from 0 to 100 km/hr in under five seconds. *Source:* Courtesy of BMW.

The invention all admir'd, and each, how he
To be th' inventor miss'd; so easy it seem'd,
Once found, which yet unfound most would have thought
Impossible

John Milton

Contents

Examples

Design Procedures

Case Studies

Design is arguably the most important specialization in modern industrial society. Integral to most engineering curricula, and also making up its own specialization in many schools, design is critical for wealth generation, development of economic activity, and the creation of jobs. This chapter introduces design synthesis, where a new machine or system is produced to address a need or to satisfy customer requirements. Design analysis is also discussed, which involves the use of engineering disciplines to determine critical dimensions, select acceptable materials, and even optimize designs. To bring high-quality products to market quickly, it is important to integrate multiple disciplines early in the design process, including solid and fluid mechanics, materials selection, marketing, manufacturing, safety, and environmental concerns. Many times, constraints are applied to mechanical designs, such as those mandated by governmental codes or industrial standards. It has been observed that simultaneous higher quality and lower costs arise when manufacturing can exploit automation, but automation can only be justified for larger production runs than most products can justify. One method of achieving the benefits of large-scale manufacture is to use standard sizes and types of machine elements in design, but this requires some sophistication with respect to significant figures, measurement units, and specification of dimensions.

Symbols

n_s	safety factor
n_{sx}	safety factor involving quality of materials, control over applied load, and accuracy of stress analysis
n_{sy}	safety factor involving danger to personnel and economic impact
σ_{all}	allowable normal stress, Pa
σ_d	design normal stress, Pa

1.1 What is Design?

Design means different things to different people. A clothing manufacturer believes that incorporating different materials or colors into a new dress constitutes design. A potter paints designs onto china to complement its surroundings. An architect designs ornamental facades for residences. An engineer chooses a bearing from a catalog and incorporates it into a speed-reducer assembly. These design activities, although they appear to be fundamentally different, share a common thread: they all require significant creativity, practice, and vision to be done well.

"Engineering," wrote Thomas Tredgold, "is the art of directing the great sources of power in nature for the use and convenience of man" [Florman 1987]. It is indeed significant that this definition of engineering is more than 60 years old — few people now use the words "engineering" and "art" in the same sentence, let alone in a definition. However, many products are successful for nontechnical reasons, reasons that cannot be proved mathematically. On the other hand, many problems are mathematically tractable, but usually because they have been inherently overconstrained. Design problems are, almost without exception, open-ended problems combining hard science and creativity. Engineering is indeed an art, even though *parts* of engineering problems lend themselves well to analysis.

For the purposes of this textbook, **design** is the transformation of concepts and ideas into useful machinery. A **machine** is a combination of mechanisms and other components that transforms, transmits, or uses energy, load, or motion for a specific purpose. If Tredgold's definition of engineering is accepted, design of machinery is the fundamental practice in engineering.

A machine comprises several different machine elements properly designed and arranged to work together as a whole. Fundamental decisions regarding loading, kinematics, and the choice of materials must be made during the design of a machine. Other factors, such as strength, reliability, deformation, tribology (friction, wear, and lubrication), cost, and space requirements also need to be considered. The objective is to produce a machine that not only is sufficiently rugged to function properly for a reasonable time, but also is economically feasible. Further, nonengineering decisions regarding marketability, product liability, ethics, politics, etc. must be integrated early in the design process. Since few people have the necessary tools to make all these decisions, machine design in practice is a discipline-blending human endeavor.

This textbook emphasizes one of the disciplines necessary in design — mechanical engineering. It therefore involves calculation and consideration of forces, energies, temperatures, etc., — concepts instilled into an engineer's psyche.

To "direct the great sources of power in nature" in machine design, the engineer must recognize the functions of the various machine elements and the types of load they transmit. A **machine element** may **function** as a normal load transmitter, a torque transmitter, an energy absorber, or a seal. Some common load transmitters are rolling-element bearings, hydrodynamic bearings, and rubbing bearings. Some torque transmitters are gears, shafts, chains, and belts. Brakes and dampers are energy absorbers. All the machine elements in Part II can be grouped into one of these classifications.

Engineers must produce safe, workable, good designs, as stated in the first fundamental canon in the *Code of Ethics for Engineers* [ASME 2012]:

> Engineers shall hold paramount the safety, health, and welfare of the public in the performance of their professional duties.

Designing reasonably safe products involves many design challenges to ensure that components are large enough, strong enough, or tough enough to survive the loading environment. One subtle concept, but of huge importance, is that the engineer has a duty to protect the welfare of the general public. Welfare includes economic well-being, and it is well known that successful engineering innovations lead to wealth and job creation. However, products that are too expensive are certain to fail in a competitive marketplace. Similarly, products that do not perform their function well will fail. Economics and functionality are always pressing concerns, and good design inherently means safe, economical, and functional design.

1.2 Design of Mechanical Systems

A **mechanical system** is a synergistic collection of machine elements. It is synergistic because as a design it represents an idea or concept greater than the sum of the individual parts. For example, a mechanical clock, although merely a collection of gears, springs, and cams, also represents the physical realization of a time-measuring device. Mechanical system design requires considerable flexibility and creativity to obtain good solutions. Creativity seems to be aided by familiarity with known successful designs, and mechanical systems are often collections of well-designed components from a finite number of proven classes.

Designing a mechanical system is a different type of problem than selecting a component. Often, the demands of the system make evident the functional requirements of a component. However, designing a large mechanical system, potentially comprising thousands or even millions of machine elements, is a much more open, unconstrained problem.

To design superior mechanical systems, an engineer must have a certain sophistication and experience regarding machine elements. Studying the design and selection of machine elements affords an appreciation for the strengths and limitations of classes of components. They can then be more easily and appropriately incorporated into a system. For example, a mechanical system cannot incorporate a worm gear or a Belleville spring if the designer does not realize that these devices exist.

A toolbox analogy of problem solving can be succinctly stated as, "If your only tool is a hammer, then every problem is a nail." The purpose of studying machine element design is to fill the toolbox so that problem solving and design synthesis activities can be flexible and unconstrained.

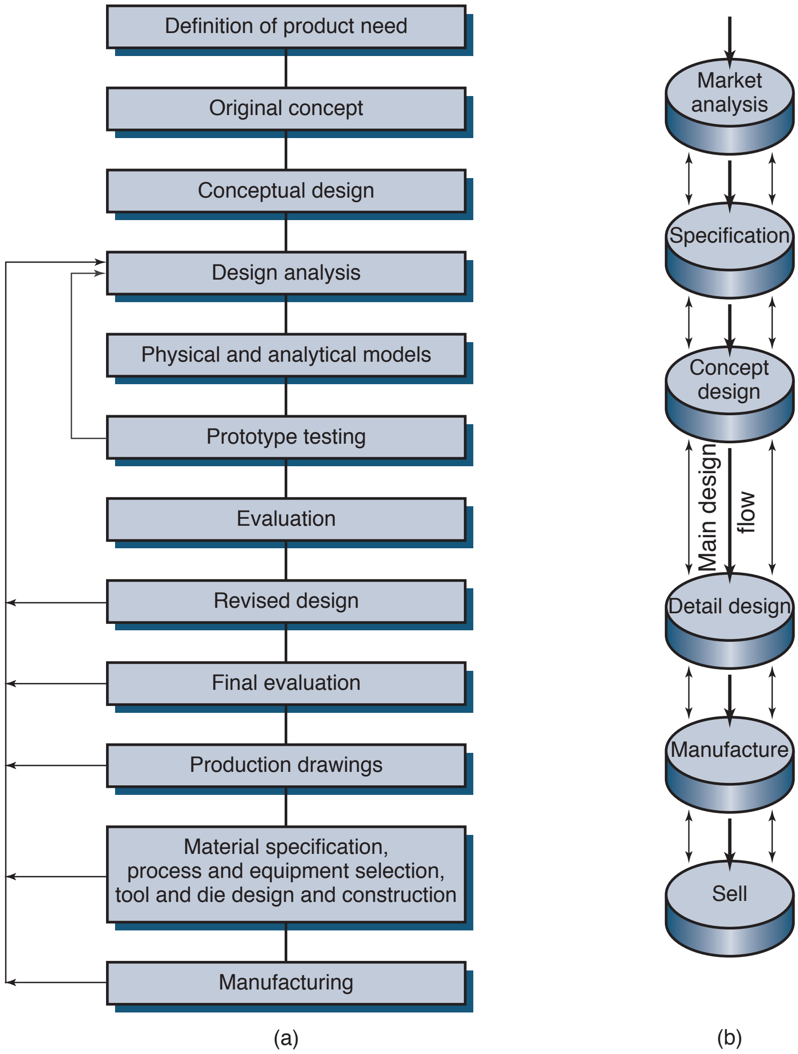


Figure 1.1: Approaches to product development. (a) Classic approach, with large design iterations typical of the over-the-wall engineering approach. *Source:* Adapted from Kalpakjian and Schmid [2003]. (b) A more modern approach, showing a main design flow with minor iterations representing concurrent engineering inputs. *Source:* Adapted from Pugh [1996].

1.3 Design as a Multidisciplinary Endeavor

The quality revolution that transformed the manufacturing sector in the early 1980s has forever changed the approach companies and engineers take toward product development. A typical design process of the recent past (Fig. 1.1a) shows that the skills involved in machine element design played an essential role in the process. This approach was commonly used in the United States in the post-World War II era.

The term *over-the-wall engineering* (OTW) has been used to describe this design approach. Basically, someone would apply a particular skill and then send the product "over the wall" to the next step in development. A product design could sometimes flow smoothly from one step to the next and into the marketplace within weeks or months. This was rarely the case, however, as usually a problem would be discovered. For example, a manufacturing engineer might ask that workpieces be more easily clamped into a milling machine fixture. The design engineer would then alter the design and send the product back downstream. A materials scientist might then point out that the material chosen had drawbacks and suggest a different choice. The design engineer would make the alteration and resubmit the design. This process could continue *ad infinitum*, with the result that the product would take a long time to develop. It is not surprising that modifications to this approach started being developed in the 1970s and 1980s.

Figure 1.1b shows a more modern design approach. Here, there is still the recognized general flow of information from product conception through introduction into the marketplace, but there is immediate involvement of many disciplines in the design stage. Different disciplines are involved simultaneously instead of sequentially as with the OTW approach. Some tasks are extremely technical, such as design analysis (the main focus of this book) or manufacturing. Others are nontechnical, such as market analysis. **Concurrent engineering** is the philosophy of involving many disciplines from the beginning of a design effort and keeping them involved throughout product development. Thus, redundant efforts are minimized and higher quality products are developed more quickly. Although design iterations still occur, the iteration loops are smaller and incur much less wasted time, effort, and expense. Also, design shortcomings can be corrected before they are incorporated. For example, service personnel can inform design engineers of excessive component failures in previous designs during the conceptual design stage for a new model, and shortcomings can be corrected instead of persisting. No such mechanism for correcting design shortcomings ever existed in conventional design approaches or management structures.

Another important concept in design is the time to market. Bringing high-quality products to market quickly is normal practice in the consumer electronics industry, where rapid change shortened useful market life to a few months. Minimizing time to market is now recognized as essential for controlling development costs. Further, new products introduced before their competitors' products usually enjoy a larger share of the market and profits. Thus, manufacturers who could ship products weeks or even days faster than their competitors had a distinct sales advantage. Saving development time through concurrent engineering made companies much more competitive in the global marketplace.

Concurrent engineering has profoundly affected design engineers. They can no longer work alone and must participate in group discussions and design reviews. They need good communications skills. Designing machinery has become a cooperative endeavor.

Clearly, many disciplines now play a role in product development, but design engineers cannot merely focus on their discipline and rely on experts for the rest. They need familiarity with other disciplines, at least from a linguistics standpoint, to integrate them into the design process. Thus, modern engineers may need to speak the language of materials science, law, marketing, etc., even if they are not experts in these fields.

1.4 Design of Machine Elements

Specifying a mechanical system is only the beginning of the design synthesis process. Particular machine element classes need to be chosen, possibly leading to further design iterations. Designing a proper machine element usually involves the following steps:

1. Selecting a suitable type of machine element from consideration of its function
2. Estimating the size of the machine element that is likely to be satisfactory
3. Evaluating the machine element's performance against design requirements or constraints
4. Modifying the design and dimensions until the performance is near to whichever optimum is considered most important

The last two steps in the process can be handled fairly easily by someone who is trained in analytical methods and understands the fundamental principles of the subject. The first two steps, however, require some creative decisions and, for many, represent the most difficult part of design.

After a suitable type of machine element has been selected for the required function, the specific machine element is designed by analyzing kinematics, load, and stress. These analyses, coupled with proper material selection, will enable a stress-strain-strength evaluation in terms of a safety factor (as discussed in Section 1.4.1). A primary question in designing any machine element is whether it will fail in service. Most people, including engineers, commonly associate **failure** with the actual breaking of a machine element. Although breaking is one type of failure, a design engineer must have a broader understanding of what really determines whether a part has failed.

A machine element is considered to have failed:

1. When it becomes completely inoperable
2. When it is still operable but is unable to perform its intended function satisfactorily
3. When serious deterioration has made it unreliable or unsafe for continued use, necessitating its immediate removal from service for repair or replacement

The role of the design engineer is to predict the circumstances under which failure is likely to occur. These circumstances are stress-strain-strength relationships involving the bulk of the solid members and such surface phenomena as friction, wear, lubrication, and environmental deterioration.

The principles of design are universal. An analysis is equally valid regardless of the size, material, and loading. However, an analysis by itself should not be looked on as an absolute and final truth. An analysis is limited by the assumptions imposed and by its range of applicability. Thus, designers often must check and verify if they have addressed considerations such as:

- Have all alternative designs been thoroughly investigated?
- Can the design be simplified and the number of its components minimized without adversely affecting its intended functions and performance?
- Can the design be made smaller and lighter?
- Are there unnecessary features to the product or some of its components, and if so, can they be eliminated or combined with other features?
- Have modular design and building-block concepts been considered for a family of similar products and for servicing and repair, upgrading, and installing options?
- Are the specified dimensional tolerances and surface finish unnecessarily tight, thereby significantly increasing product cost, and can they be relaxed without any adverse effects?
- Will the product be difficult or excessively time consuming to assemble and disassemble for maintenance, servicing, or recycling of some or all of its components?
- Is the use of fasteners minimized, including their quantity and variety?
- Have environmental considerations been taken into account and incorporated into product design, as well as material and process selection?
- Have green design and life-cycle engineering principles been applied, including recycling considerations?

Design analysis attempts to predict the strength or deformation of a machine element so that it can safely carry the imposed loads for as long as required. Assumptions have to be made about the material properties under different loading types (axial, bending, torsion, and transverse shear, as well as various combinations) and classes (static, sustained, impact, or cyclic). These loading constraints may vary throughout the machine as they relate to different machine elements, an important factor for the design engineer to keep in mind.

1.4.1 Safety in Mechanical Design

The code of Hammurabi, a Babylonian doctrine over 3000 years old, had this requirement:

> If a builder build a house for a man and do not make its construction firm, and the house which he has built collapse and cause the death of the owner of the house, that builder shall be put to death.

It could be argued that engineers are getting off a lot easier these days. Modern legal doctrines do not call for the death of manufacturers of unsafe products or of the engineers who designed them. Regardless of the penalty, however, engineers have a moral and legal obligation to produce reasonably safe products. A number of fundamental concepts and tools are available to assist them in meeting this challenge.

Safety Factor

If 500 tension tests are performed on a specimen of one material, 500 different yield strengths will be obtained if the precision and accuracy of measurement are high enough. With some materials, a wide range of strengths can be achieved; in others, a reasonable guaranteed minimum strength can be found. However, this strength does not usually represent the stress that engineers apply in design.

Using results from small-scale tension tests, a design engineer prescribes a stress somewhat less than the semi-empirical strength of a material. The **safety factor** can be expressed as

$$n_s = \frac{\sigma_{\text{all}}}{\sigma_d} \tag{1.1}$$

where σ_{all} is the allowable normal stress and σ_d is the design normal stress. If $n_s > 1$, the design is adequate. The larger n_s, the safer the design. If $n_s < 1$, the design may be inadequate and redesign may be necessary. In later chapters, especially Chapter 6, more will be said about σ_{all} and σ_d. The rest of this section focuses on the left side of Eq. (1.1).

It is difficult to accurately evaluate the various factors involved in engineering design problems. One factor is the shape of a part. For an irregularly shaped part, there may be no design equations available for accurate stress computation. Sometimes the load is uncertain. For example, the loading applied to a bicycle seat and frame depends on the size of the rider, speed, and size of bumps encountered. Another factor is the consequences of part failure; life-threatening consequences require more consideration than non-life-threatening consequences.

Engineers use a safety factor to ensure against such uncertain or unknown conditions. The engineering student is often asked, What safety factor was used in the design, and which value should be used? Safety factors are sometimes prescribed by code, but usually they are rooted in design experience. That is, design engineers have established through a product's performance that a safety factor is sufficient. Future designs are often based on safety factors found adequate in previous products for similar applications.

Particular design experience for specific applications does not form a basis for the rational discussion of illustrative examples or for the guidance of engineering students. The Pugsley [1966] method for determining the safety factor is a potential approach for obtaining safety factors in design, although the reader should again be warned that safety factor selection is somewhat nebulous in the real world and the Puglsey method can be unconservative; that is, it predicts safety factors that are too low for real applications. Pugsley

Table 1.1: Safety factor characteristics A, B, and C.

Characteristic[a]		B			
A	C	vg	g	f	p
vg	vg	1.1	1.3	1.5	1.7
	g	1.2	1.45	1.7	1.95
	f	1.3	1.6	1.9	2.2
	p	1.4	1.75	2.1	2.45
g	vg	1.3	1.55	1.8	2.05
	g	1.45	1.75	2.05	2.35
	f	1.6	1.95	2.3	2.65
	p	1.75	2.15	2.55	2.95
f	vg	1.5	1.8	2.1	2.4
	g	1.7	2.05	2.4	2.75
	f	1.9	2.3	2.7	3.1
	p	2.1	2.55	3.0	3.45
p	vg	1.7	2.15	2.4	2.75
	g	1.95	2.35	2.75	3.15
	f	2.2	2.65	3.1	3.55
	p	2.45	2.95	3.45	3.95

[a] vg = very good, g = good, f = fair, and p = poor.
A = quality of materials, workmanship, maintenance, and inspection.
B = control over load applied to part.
C = accuracy of stress analysis, experimental data or experience with similar parts.

Table 1.2: Safety factor characteristics D and E.

Characteristic E[a]	D		
	ns	s	vs
ns	1.0	1.2	1.4
s	1.0	1.3	1.5
vs	1.2	1.4	1.6

[a] vs = very serious, s = serious, and ns = not serious
D = danger to personnel
E = economic impact

systematically determined the safety factor from

$$n_s = n_{sx} n_{sy} \tag{1.2}$$

where
- n_{sx} = safety factor involving characteristics A, B, and C
- A = quality of materials, workmanship, maintenance, and inspection
- B = control over load applied to part
- C = accuracy of stress analysis, experimental data, or experience with similar devices
- n_{sy} = safety factor involving characteristics D and E
- D = danger to personnel
- E = economic impact

Table 1.1 gives n_{sx} values for various A, B, and C conditions. To use this table, estimate each characteristic for a particular application as being very good (vg), good (g), fair (f), or poor (p). Table 1.2 gives n_{sy} values for various D and E conditions. To use this table, estimate each characteristic for a particular application as being very serious (vs), serious (s), or not serious (ns). Substituting the values of n_{sx} and n_{sy} into Eq. (1.2) yields a proposed safety factor.

Although a simple procedure to obtain safety factors, the Pugsley method illustrates the concerns present in safety factor selection. Many parameters, such as material strength and applied loads, may not be well known, and confidence in the engineering analysis may be suspect. For these reasons the safety factor has sometimes been called an "ignorance factor," as it compensates for ignorance of the total environment, a situation all design engineers encounter to some extent. Also, the Pugsley method is merely a guideline and is not especially conservative; most engineering safety factors are much higher than those resulting from Eq. (1.2), as illustrated in Example 1.1.

Example 1.1: Safety Factor of Wire Rope in an Elevator

Given: A wire rope is used on an elevator transporting people to the 20th floor of a building. The design of the elevator can be 50% overloaded before the safety switch shuts off the motor.

Find: What safety factor should be used?

Solution: The following values are assigned:

A = vg, because life threatening
B = f to p, since large overloads are possible
C = vg, due to being highly regulated
D = vs, people could die if the elevator fell from the 20th floor
E = vs, possible lawsuits

From Tables 1.1 and 1.2 the safety factor is

$$n_s = n_{sx}n_{sy} = (1.6)(1.6) = 2.56$$

Note that the value of $n_{sx} = 1.6$ was obtained by interpolation from values in Table 1.1. By improving factors over which there is some control, n_{sx} can be reduced from 1.6 to 1.0 according to the Pugsley method, thus reducing the required safety factor to 1.6.

Just for illustrative purposes, the safety factor for this situation is prescribed by an industry standard [ANSI 2010] and cannot be lower than 7.6 and may need to be as high as 11.9. The importance of industry standards is discussed in Section 1.4.2, but it is clear that the Pugsley method should be used only with great caution.

Product Liability

When bringing a product to the market, it is probable that safety will be a primary consideration. A design engineer must consider the **hazards**, or injury producers, and the **risk**, or likelihood of obtaining an injury from a hazard, when evaluating the safety of a system. Unfortunately, this is mostly a qualitative evaluation, and combinations of hazard and risk can be judged acceptable or unacceptable.

The ethical responsibilities of engineers to provide safe products are clear, but the legal system also enforces societal expectations through a number of legal theories that apply to designers and manufacturers of products. Some of the more common legal theories are the following:

- **Caveat Emptor**. Translated as "Let the buyer beware," this is a doctrine founded on Roman laws. In the case of a defective product or dangerous design, the purchaser or user of the product has no legal recourse to recover losses. In a modern society, such a philosophy is incompatible with global trade and high-quality products, and is mentioned here only for historical significance.

- **Negligence.** In negligence, a party is liable for damages if they failed to act as a reasonable and prudent party would have done under like or similar circumstances. For negligence theory to apply, the injured party, or *plaintiff*, must demonstrate:

    1. That a standard of care was violated by the accused party, or *defendant*.
    2. That this violation was the *proximate cause* of the accident.
    3. That no contributory negligence of the plaintiff caused the misfortune.

- **Strict liability**. Under the strict liability doctrine, the actions of the plaintiff are not an issue; the emphasis is placed on the machine. To recover damages under the strict liability legal doctrine, the plaintiff must prove that:

    1. The product contained a defect that rendered it unreasonably dangerous. (For example, an inadequately sized or cracked bolt fastening a brake stud to a machine frame.)
    2. The defect existed at the time the machine left the control of the manufacturer. (The manufacturer used the cracked bolt.)
    3. The defect was a proximate cause of the accident. (The bolt broke, the brake stud fell off the machine, the machine's brake didn't stop the machine, resulting in an accident.) Note that the plaintiff does not need to demonstrate that the defect was the proximate cause; the actions of the plaintiff that contribute to his or her own accident are not considered under strict liability.

- **Comparative fault.** Used increasingly in courts throughout the United States, juries are asked to assess the relative contributions that different parties had in relation to an accident. For example, a jury may decide that a plaintiff was 75% responsible for an accident, and reduce the monetary award by that amount.

- **Assumption of risk.** Although rarely recognized, the *assumption of risk* doctrine states that a plaintiff has limited recourse for recovery of loss if they purposefully, knowingly, and intentionally conducted an unsafe act.

One important requirement for engineers is that their products must be reasonably safe for their intended uses as well as their *reasonably foreseeable misuses*. For example, a chair must be made structurally sound and stable enough for people to sit on (this is the intended use). In addition, a chair should be stable enough so that someone can stand on the chair to change a light bulb, for example. It could be argued that chairs are designed to be sat upon, and that standing on a chair is a misuse. This may be true, but represents a reasonably foreseeable misuse of the chair, and must therefore be considered by designers. In the vast majority of states, misuses of a product that are not reasonably foreseeable do not have to be considered by the manufacturers.

The legal doctrines and ethical requirements that designers produce safe products are usually consistent. Sometimes, the legal system does result in requirements that engineers cannot meet. For example, in the famous *Barker vs. Lull* case in New Jersey, the court ruled that product manufacturers have a nondelegable duty to warn of the unknowable.

Liability proofing is the practice of incorporating design features with the intent of limiting product liability exposure without other benefits. This can reduce the safety of machinery. For example, one approach to liability proofing is to place a very large number of warnings onto a machine, with the unfortunate result that all of the warnings are ignored by machine operators. The few hazards that are not obvious and can be effectively warned against are then “lost in the noise” and a compromise of machine safety can occur.

Case Study 1.1: *Mason v. Caterpillar Tractor Co.*

Wilma Mason brought action under negligence theory against Caterpillar Tractor Company and Patton Industries for damages after her husband received fatal injuries while trying to repair a track shoe on a Caterpillar tractor. Mr. Mason was repairing the track shoe with a large sledgehammer, when a small piece of metal from the track shoe shot out, striking him, and causing fatal injuries. The plaintiff alleged that the tractor track was defective because the defendants failed to use reasonable methods of heat treatment, failed to use a sufficient amount of carbon in the steel, and failed to warn the decedent of “impending danger.”

The Trial and Appellate courts both granted summary judgements in favor of the defendants. They ruled that the plaintiff failed to show evidence of a product defect that existed when the machine left the control of the manufacturer. Mr. Mason used a large, 10-kg sledgehammer with a full swing, striking a raised portion of the track shoe. There was no evidence that the defendants were even aware that the track shoes were being repaired or reassembled by sledgehammers. It was also noted by the court that the decedent wore safety glasses, indicating his awareness of the risk of injury.

Safety Hierarchy

A design rule that is widely accepted in general is the **safety hierarchy**, which describes the steps that a manufacturer or designer should use when addressing hazards. The safety hierarchy is given in Design Procedure 1.1. Eliminating hazards through design can imply a number of different approaches. For example, a mechanical part that is designed so that its failure is not reasonably foreseeable is one method of eliminating a hazard or risk of injury. However, design of a system that eliminates injury producers or moves them away from people also represents a reasonable approach.

This book emphasizes mechanical analysis and design of parts to reduce or eliminate the likelihood of failure. As such, it should be recognized that this approach is one of the fundamental, necessary skills required by engineers to provide reasonably safe products.

Design Procedure 1.1: The Safety Hierarchy

A designer should attempt the following, in order, in attempting to achieve reasonable levels of safety:

1. Eliminate hazards through design.
2. Reduce the risk or eliminate the hazard through safeguarding technology.
3. Provide warnings.
4. Train and instruct.
5. Provide personal protective equipment.

There is a general understanding that primary steps are more efficient in improving safety than later steps. That is, it is more effective to eliminate hazards through design than to use guards, which are more effective than warnings, etc. Clearly, the importance of effective design cannot be overstated.

Failure Mode and Effects Analysis and Fault Trees

Some common tools available to design engineers are **failure mode and effects analysis** (FMEA) and **fault tree analysis**. FMEA addresses component failure effects on the entire system. It forces the design engineer to exhaustively consider reasonably foreseeable failure modes for every component and its alternatives.

FMEA is flexible, allowing spreadsheets to be tailored for particular applications. For example, an FMEA can also be performed on the steps taken in assembling components to identify critical needs for training and/or warning.

In fault tree analysis, statistical data are incorporated into the failure mode analysis to help identify the most likely (as opposed to possible) failure modes. Often, hard data are not available, and the engineer’s judgment qualitatively identifies likely failure modes.

As discussed above, machine designers are legally required to provide reasonably safe products and to consider the product’s intended uses as well as foreseeable misuses. FMEA and fault tree analysis help identify unforeseeable misuses as well. For example, an aircraft designer may identify aircraft-meteorite collision as a possible loading of the structure. However, because no aircraft accidents have resulted from meteorite collisions and the probability of such occurrences is extremely low, the design engineer ignores such hypotheses, recognizing they are not reasonably foreseeable.

Load Redistribution, Redundancy, Fail Safe, and the Doctrine of Manifest Danger

One potential benefit of failure mode and effects analysis and fault tree analysis is that they force the design engineer to think of minimizing the effects of individual component failures. A common goal is that the failure of a single component should not result in a catastrophic accident. The design engineer can ensure this by designing the system so that, upon a component failure, loads are redistributed to other components without exceeding their nominal strengths — a philosophy known as **redundancy** in design. For example, a goose or other large bird sucked into an aircraft engine may cause several components to fail and shut down the engine. This type of accident is not unheard of and is certainly reasonably foreseeable. Thus, modern aircraft are designed with sufficient redundancy to allow a plane to fly and land safely with one or more engines shut down.

Many designs incorporate redundancy. Redundant designs can be *active* (where two or more components are in use but only one is needed) or *passive* (where one component is inactive until the first component fails). An example of an active redundant design is the use of two deadbolt locks on a door: both bolts serve to keep the door locked. A passive redundant design example would entail adding a chain lock on a door having a deadbolt lock: if the deadbolt lock fails, the chain will keep the door closed.

An often-used philosophy is to design machinery with **fail-safe** features. For example, a brake system (see Chapter 18) can be designed so that a pneumatic cylinder pushes the brake pads or shoes against a disk or drum, respectively. Alternatively, a spring could maintain pressure against the disk or drum and a pneumatic system could work against the spring to release the brake. If the pressurized air supply were interrupted, such a design would force brake actuation and prevent machinery motion. This alternative design is fail safe as long as the spring is far more reliable than the pneumatic system.

The **doctrine of manifest danger** is a powerful tool used by machinery designers to prevent catastrophic losses. If danger becomes manifest, troubleshooting is straightforward and repairs can be quickly made. Thus, if a system can be designed so that imminent failure is detectable or so that single-component failure is detectable before other elements fail in turn, a safer design results. A classic application of the doctrine of manifest danger is in the design of automotive braking systems, where the brake shoe consists of a friction material held onto a metal backing plate by rivets. By making the rivets long enough, an audible and tactile indication is given to the car driver when the brake system needs service. That is, if the friction material has worn, the rivets will contact the disk or drum, indicating through noise and vibration that maintenance is required, and this occurs long before braking performance is compromised.

Reliability

Safety factors are a way of compensating for variations in loading and material properties. Another approach that can be extremely successful in certain circumstances is the application of **reliability** methods.

As an example, consider the process of characterizing a material's strength through tension tests (see Section 3.4). Manufacturing multiple tension test specimens from the same extruded billet of aluminum would result in little difference in measured strength from one test specimen to another. Thus, aluminum in general (as well as most metals) is a *deterministic* material, and deterministic methods can be used in designing aluminum structures if the load is known. For example, in a few hundred tensile tests, a guaranteed minimum strength can be defined that is below the strength of any test specimen and that would not vary much from one test population to another. This guaranteed minimum strength is then used as *the* strength for design analysis. Such deterministic methods are used in most solid mechanics and mechanics courses. That is, all specimens of a given material have a single strength and the loading is always well defined.

Most ceramics, however, would have a significant range of any given material property, including strength. Thus, ceramics are *probabilistic*, and an attempt to define a minimum strength for a population of ceramic test specimens would be an exercise in futility. There would not necessarily be a guaranteed minimum strength. One can only treat ceramics in terms of a likelihood or probability of strength exceeding a given value. There are many such probabilistic materials in engineering practice.

Some loadings, on the other hand, are well known and never vary much. Examples are the stresses inside intravenous (IV) bags during sterilization, the load supported by counterweight springs, and the load on bearings supporting centrifugal fans. Other loads can vary significantly, such as the force exerted on automotive shock absorbers (depends on the size of the pothole and the speed at impact) or on wooden pins holding a chair together (depends on the weight of the seated person or persons) or the impact force on the head of a golf club.

For situations where a reasonable worst-case scenario cannot be defined, reliability methods are sometimes a reasonable design approach. In reliability design methods, the goal is to achieve a reasonable likelihood of survival under the loading conditions during the intended design life. This approach has its difficulties as well, including the following:

1. To use statistical methods, a reasonable approximation of an infinite test population must be defined. That is, mean values and standard deviations about the mean, and even the nature of the distribution about the mean, must be known. However, they are not usually very well characterized after only a few tests. After all, if only a few tests were needed to quantify a distribution, deterministic methods would be a reasonable, proper, and less mathematically intensive approach. Thus, characterization can be expensive and time consuming, since many experiments are needed.

2. Even if strengths and loadings are known well enough to quantify their statistical distributions, defining a desired reliability is as nebulous a problem as defining a desired safety factor. A reliability of 99% might seem acceptable, unless that were the reliability of an elevator you happened to be occupying. A reliability of 100% is not achievable, or else deterministic methods would be used. A reliability of 99.9999...% should be recognized as an extremely expensive affair, and as indicative of overdesign as a safety factor of 2000.

3. The mathematical description of the data has an effect on reliability calculations. A quantity may be best described by a Gaussian or normal distribution, a lognormal distribution, a binary distribution, a Weibull distribution, etc. Often, one cannot know beforehand which distribution is best. Some statisticians recommend using a normal distribution until it is proved ineffective.

The implications are obvious: Reliability design is a complicated matter and even when applied does not necessarily result in the desired reliability if calculated from insufficient or improperly reduced data.

This textbook will emphasize deterministic methods for the most part. The exceptions are the treatments of rolling-element bearings and gears and reliability in fatigue design. For more information on reliability design, refer to the excellent text by Lewis [1995] among others.

1.4.2 Government Codes and Industry Standards

In many cases, engineers must rely on government codes and industry-promulgated standards for design criteria. Some of the most common sources for industry standards are:

1. ANSI, the American National Standards Institute
2. ASME, the American Society of Mechanical Engineers
3. ASTM, the American Society for Testing and Materials
4. AGMA, the American Gear Manufacturers Association
5. AISI, the American Iron and Steel Institute
6. AISC, the American Institute of Steel Construction
7. ISO, the International Standards Organization
8. NFPA, the National Fire Protection Association
9. UL, Underwriters Laboratories

Government codes are published annually in the Code of Federal Regulations (CFR) and periodically in the Federal Register (FR) at the national level. States and local municipalities have codes as well, although most relate to building standards and fire prevention.

Code compliance is important for many reasons, some of which have already been stated. However, one essential goal of industry standards is conformability. For example, bolt geometries are defined in ANSI standards so that bolts have fixed thread dimensions and bolt diameters. Therefore, bolts can be mass produced, resulting in inexpensive, high-quality threaded fasteners. Also, maintenance is simplified in that standard bolts can be purchased anywhere, making replacement parts readily available.

1.4.3 Manufacturing

Design and manufacturing are difficult to consider apart from one another. The tenet of "form follows function" suggests that shapes are derived only from applied loads in the design environment. However, this is not always the case, and the shapes of products are often natural progressions from arbitrary beginnings.

Design for manufacturability (DFM) is a well-established and important tool for design engineers. Manufacturability plays a huge role in the success of commercial products. After all, a brilliant concept that cannot be manufactured cannot be a successful design (per the definition in Section 1.1). Also, because most manufacturing costs are determined by decisions made early in the design process, market success depends on early consideration of a complete product lifecycle, including manufacturing. Individual components should be designed to be easily fabricated, assembled, and constructed (*design for assembly*, DFA). Although manufacturing and assembly are outside the scope of this text, Fig. 1.2 shows their effect on design.

Engineers must wear many hats. Some predominant concerns of a design engineer have been discussed, but many more exist, including:

1. *Environmental or sustainable design:* This issue addresses whether products can be produced that are less harmful to the environment. Biodegradable or easily recycled materials may need to be selected to satisfy this concern.

2. *Economics:* Deciding whether a product will be profitable is of utmost concern.

3. *Legal considerations* - Violating patents and placing unreasonably dangerous products into the marketplace are not only ethically wrong but have legal ramifications as well.

4. *Marketing:* The features of a product that attract consumers and the product's presentation to the marketplace play a significant role in a product's success.

5. *Serviceability:* If a part breaks, can repairs be done in the field, or must customers send the product back to the manufacturer at excessive expense? Unless such concerns are incorporated into design, long-term customer loyalty is compromised.

6. *Quality:* Approaches such as *total quality engineering* and *Taguchi methods* have been successfully applied to make certain that no defects are shipped.

These are merely a few of the concerns faced by design engineers.

The design process may appear so elaborate and involved that no one can master it. In actuality, one important skill makes the design process flow smoothly: effective communication. Communication between diverse disciplines involved in product design ensures that all voices are heard and all design constraints are satisfied early, before significant costs are incurred. Effective communication skills, written and oral, are the most important trait of a good engineer. Although this text emphasizes the more analytical and technical sides of design, it is important to remember that design is not merely an analytical effort but one of human interaction.

1.4.4 Life Cycle Engineering

Life cycle engineering (LCE) involves consecutive and interlinked stages of a product or a service, from the very beginning to its disposal or recycling; it includes the following:

1. Extraction of natural resources

2. Processing of raw materials

3. Manufacturing of products

4. Transportation and distribution of the product to the customer

5. Use, maintenance, and reuse of the product

6. Recovery, recycling, reuse, or disposal of its components

All of these factors are applicable to any type of product. Each product can have its own metallic and nonmetallic materials, processed into individual components and assembled; thus, each product has its own life cycle. Moreover, (a) some products are intentionally made to be disposable, particularly those made of paper, cardboard, inexpensive plastic, and glass, but nonetheless are all recyclable, and (b) numerous other products are completely reusable.

A major aim of LCE is to consider reusing and recycling the components of a product, beginning with the earliest stage of product design. This is also called **green design** or **green engineering**. These considerations also include environmental factors, optimization, and numerous technical factors regarding each component of a product.

As is now universally acknowledged, the natural resources on Earth are limited, thus clearly necessitating the need and urgency to conserve materials and energy. The concept of **sustainable design** emphasizes the need for conserving resources, particularly through proper maintenance and reuse. While profitability is important to an organization, sustainable design is meant to meet purposes such as (a) increase the life cycle of products, (b) eliminate harm to the environment and the ecosystem, and (c) ensure our collective well-being, especially for the benefit of future generations.

Case Study 1.2: Sustainable Manufacturing in the Production of Nike Athletic Shoes

Among numerous examples from industry, the production of Nike shoes clearly has indicated the benefits of sustainable manufacturing. The athletic shoes are assembled using adhesives. Up to around 1990, the adhesives used contained petroleum-based solvents, which pose health hazards

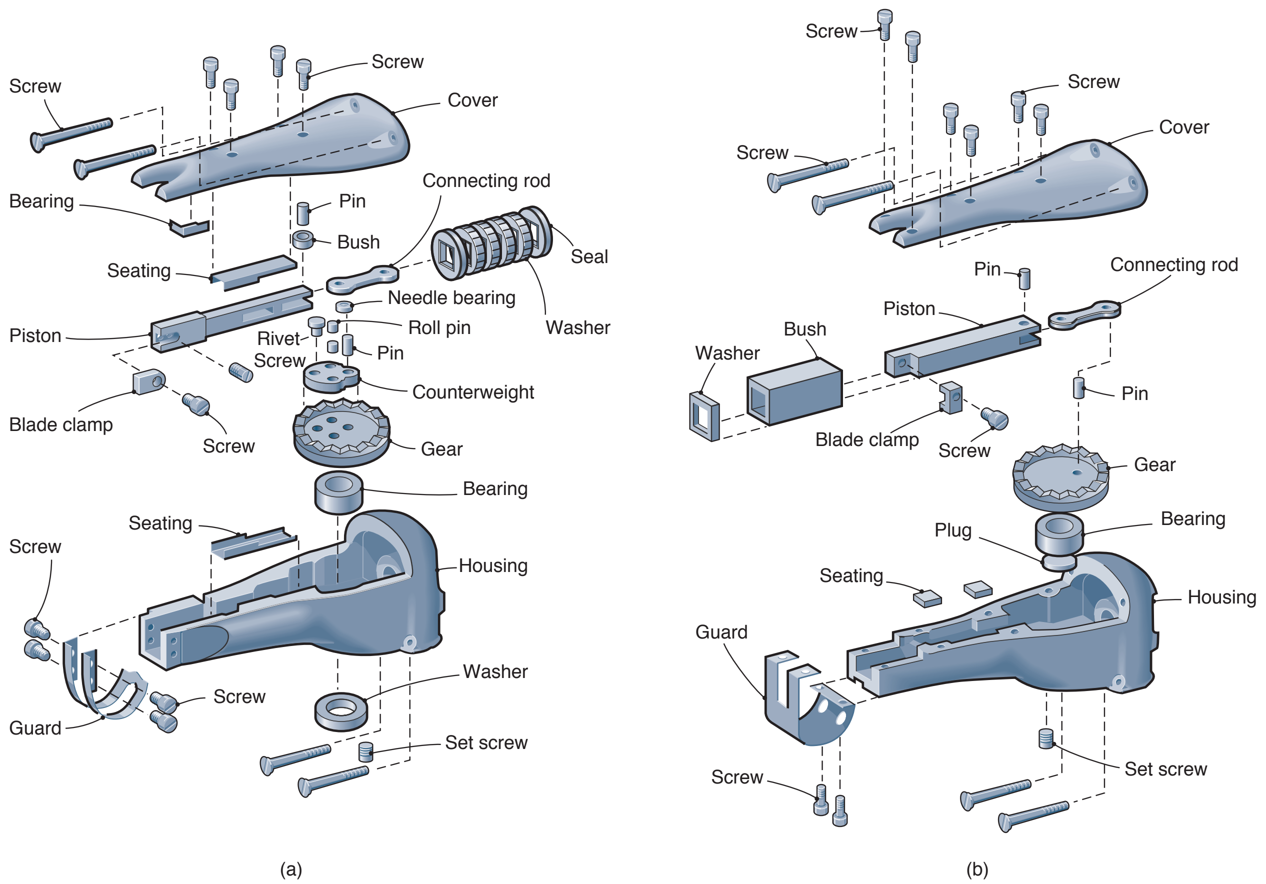


Figure 1.2: Effect of manufacturing and assembly considerations on the design of a reciprocating power saw. (a) Original design, with 41 parts and 6.37-min assembly time; (b) modified design, with 29 parts and 2.58-min assembly time. *Source:* Adapted from Boothroyd [1992].

and contribute to petrochemical smog. The company cooperated with chemical suppliers to successfully develop a water-based adhesive technology, now used in the majority of shoe-assembly operations. As a result, solvent use in all manufacturing processes in the subcontracted facilities in Asia has greatly been reduced.

Regarding another component of the shoe, the rubber outsoles are made by a process that results in significant amounts of extra rubber around the periphery of the sole (called *flashing*). With about 40 factories using thousands of molds and producing over a million outsoles a day, the flashing constitutes the largest chunk of waste in manufacturing the shoes. In order to reduce this waste, the company developed a technology that grinds the flashing into 500-μm rubber powder, which is then added back into the rubber mixture needed to make the outsole. With this approach, waste was reduced by 40%. Moreover, it was found that the mixed rubber had better abrasion resistance and durability, and its overall performance was higher than the best premium rubber.

Source: Adapted from Kalpakjian and Schmid [2010].

1.5 Computers in Design

Computer-aided design (CAD) also means different things to different people, but in this text it is the application of computer technology to planning, performing, and implementing the design process. Computers allow virtual integration of all phases of the design process, whether technical or managerial activities. With sophisticated hardware and software, manufacturers can now minimize design costs, maximize efficiency, improve quality, reduce development time, and maintain an edge in domestic and international markets.

CAD allows the design engineer to visualize geometries without making costly models, iterations, or prototypes. These systems can now analyze designs from simple brackets to complex structures quickly and easily. Designs can be optimized and modified directly and easily at any time. Information stored by computer can be accessed and retrieved from anywhere within the organization.

Whereas some restrict the term “CAD” to drafting activities, others will generically group all computer-assisted functions arbitrarily as CAD. **Artificial intelligence** (AI) attempts to duplicate how the human mind works and apply it to processes on the computer. Sometimes, AI is used to describe the cases where computers are used as more than mere drafting tools and actually help in the intellectual design tasks. **Expert systems** are rule-based computer programs that solve specialized problems and provide problem-solving skills to the design engineer. For example, an expert system could analyze a part drafted on a computer system for ease in manufacturing. If an excessively small tolerance is found, the expert system warns the engineer that manufacturing difficulties will ensue and suggests easing the tolerance. Similarly, an expert system could analyze a design to standardize parts (e.g., to make sure that an assembly uses only one bolt size instead of the optimum size for each location, thereby easing inventory and maintenance difficulties). Artificial intelligence is a more elaborate form of an expert system; AI is sometimes restricted to systems that can learn new information.

Rapid prototyping, also called *3D printing* or *additive manufacturing*, is another computer-driven technology that produces parts from geometry data files in hours or even minutes. Rapid prototyping has been especially helpful in design visualization and rapid detection of design errors. For example, a casting with an excessively thin wall is easily detected when a solid model is held in the hand, a subtlety that is difficult to discern when viewing a part drawing on a computer screen. Significant developments have occurred in rapid prototyping in recent years. Currently, a wide variety of polymers can be used, as well as metals and ceramics.

Finite element analysis (FEA) is the most prevalent computational method for solid and fluid mechanics analysis, as well as heat transfer. The finite element computational method solves complex shapes, such as those found in machinery, and replaces the complex shape with a set of simple elements interconnected at a finite set of node points. In FEA, a part geometry is sectioned into many subsections or *elements*. The stiffness of each element is known and is expressed in terms of a stiffness matrix for that element. By combining all the stiffness matrices, applying kinematic and stress boundary conditions, and solving for unknown stresses or displacements, complicated geometries and loading conditions can be easily analyzed.

1.6 Catalogs and Vendors

Manufacturing concerns are inseparable from design. Clearly, many machine elements are mass produced because there is an economic justification for large production runs using hard automation. Hard automation generally results in higher quality, tighter tolerance parts than soft automation or hand manufacture, and usually results in less expensive parts as well. In fact, many industry standards mentioned in Section 1.4.2 exist to prescribe geometries that can be mass produced in order to achieve quality and cost benefits. For example, a centerless grinder can produce many high-quality 15-mm-diameter bushings, whereas a single 15-mm bushing is difficult to manufacture and would be very expensive by comparison. Therefore, the practice of machine design often involves selecting mass-produced components from suppliers, often as summarized in catalogs or web sites.

Mechanical designers know the importance of good vendor identification and readily available and up-to-date catalog information. The Internet has brought a huge variety of machine element catalogs to every designer's desktop, and it will be assumed that students are well-aware of Internet search tools and can quickly retrieve product catalogs if desired. Often in this textbook, portions of a manufacturer's catalog will be provided so that data are convenient for problem solving, but it should be recognized that the complete product portfolios are usually much larger than the abstracted data presented.

1.7 Units

The solutions to engineering problems must be given in specific and consistent units that correspond to the specific parameter being evaluated. Two systems of units are generally used in practice:

1. Systéme International d'Unites (SI units): Force is measured in newtons, length in meters (sometimes millimeters are more convenient for certain applications), time in seconds, mass in kilograms, and temperature in degrees Celsius. In addition, absolute temperature is measured in degrees Kelvin, where the temperature in Kelvin is the temperature in Celsius plus 273.15°.

2. English units: Force is measured in pounds force, length in inches, time in seconds, mass in pounds mass, and temperature in degrees Fahrenheit.

In Chapter 8 an additional measure, viscosity, is given in the centimeter-gram-seconds (cgs) system.

The SI units, prefixes, and symbols used throughout the text are shown in Table 1.3 as well as inside the front cover. The primary units of this text are SI.

Basic SI units, some definitions, and fundamental and other useful conversion factors are given in Table 1.4, which is also inside the front cover. Note that many units can be quite confusing. For example, a ton in the United States and Canada refers to a weight of 2000 lb, while in the United Kingdom it is a term for weight or mass equivalent to 2240 lb. Sometimes, a weight is reported in short tons (2000 lb) or long tons (2240 lb). The metric equivalent, called a metric ton or a tonne, is 1000 kg. As another example, a horsepower in English units is 550 ft-lb/s or 746 W. However, in metric countries, a horsepower is defined as 736 W. Keeping track of units is a necessary task for design analysis, as illustrated in Case Study 1.3.

Example 1.2: Length of Electrical Connections in a Supercomputer

Given: A supercomputer has a calculation speed of 1 gigaflop $= 10^9$ floating-point operations per second. Performance can be limited if the electrical connections within the supercomputer are so long that electron travel times are greater than the calculation's speed.

Find: Determine the critical length of electrical wire for such connections if the electron speed for coaxial cables is 0.9 times the speed of light (3×10^8 m/s).

Solution: If the speed is determined only by the cable length,

$$l = \frac{(0.9)\left(3 \times 10^8\right)}{10^9} = 0.27 \text{ m} = 27 \text{ cm}$$

The mean cable length must be less than 27 cm.

Example 1.3: Astronomical Distances

Given: The distance from Earth to α-Centauri is 4 light-years.

Find: How many terameters away is α-Centauri? Note that the speed of light is 3×10^8 m/s.

Solution: Note that 1 year $= (365)(24)(3600)$ s $= \left(3.1536 \times 10^7\right)$ s. The distance from Earth to α-Centauri is

$$(4)(3.1536 \times 10^7 \text{ s})(3 \times 10^8) \text{ m/s} = 3.784 \times 10^{16} \text{ m}$$

From Table 1.3b, 1T = 10^{12}. Therefore, the distance is 37,840 Tm.

Table 1.3: SI units and prefixes.

(a) SI units

Quantity	Unit	SI symbol	Formula
SI base units			
Length	meter	m	-
Mass	kilogram	kg	-
Time	second	s	-
Temperature	kelvin	K	-
SI supplementary unit			
Plane angle	radian	rad	-
SI derived units			
Energy	joule	J	N-m
Force	newton	N	kg-m/s^2
Power	watt	W	J/s
Pressure	pascal	Pa	N/m^2
Work	joule	J	N-m

(b) SI prefixes

Multiplication factor	Prefix	SI symbol for prefix
1,000,000,000,000 = 10^{12}	tera	T
1,000,000,000 = 10^9	giga	G
1,000,000 = 10^6	mega	M
1000 = 10^3	kilo	k
100 = 10^2	hecto	h
10 = 10^1	deka	da
0.1 = 10^{-1}	deci	d
0.01 = 10^{-2}	centi	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p

Table 1.4: Conversion factors and definitions.

Definitions	
Acceleration of gravity	1 g = 9.8066 m/s^2 (32.174 ft/s^2)
Energy	Btu (British thermal unit) = amount of energy required to raise 1 lbm of water 1°F (1 Btu = 778.2 ft-lb) kilocalorie = amount of energy required to raise 1 kg of water 1K (1 kcal = 4187 J)
Length	1 mile = 5280 ft 1 nautical mile = 6076.1 ft
Power	1 horsepower = 550 ft-lb/s
Pressure	1 bar = 10^5 Pa
Temperature	Fahrenheit: $t_F = \frac{9}{5}t_C + 32$ Rankine: $t_R = t_F + 459.67$ Kelvin: $t_K = t_C + 273.15$ (exact)
Kinematic viscosity	1 poise = 0.1 kg/m-s 1 stoke = 0.0001 m^2/s
Volume	1 cubic foot = 7.48 gal
Useful conversion factors	
	1 in. = 0.254 m = 25.4 mm
	1 lbm = 0.4536 kg
	1° R = $\frac{5}{9}$K
	1 ft = 0.3048 m
	1 lb = 4.448 N
	1 lb = 386.1 lbm-in./s^2
	1 ton = 2000 lb (shortton) or 2240 lb (long ton)
	1 tonne = 1000 kg (metric ton)
	1 kgf = 9.807 N
	1 lb/in.^{2} = 6895 Pa
	1 ksi = 6.895 MPa
	1 Btu = 1055 J
	1 ft-lb = 1.356 J
	1 hp = 746 W = 2545 Btu/hr [a]
	1 kW = 3413 Btu/hr
	1 quart = 0.000946 m^3 = 0.946 liter
	1 kcal =3.968 Btu

[a] Note that in countries using the metric system, a horsepower is defined as 75 kpm/s, or 736 W.

Case Study 1.3: Loss of the Mars Climate Orbiter

On December 11, 1998, the Mars Climate Orbiter was launched to start its nearly 10-month journey to Mars. The Mars Climate Orbiter was a $125 million satellite intended to orbit Mars and measure the atmospheric conditions on that planet over a planetary year. It was also intended to serve as a communications relay for the Mars Climate Lander, which was due to reach Mars in December 1999. The Mars Climate Orbiter was destroyed on September 23, 1999 as it was maneuvering into orbit.

The cause for the failure was quickly determined: the manufacturer, Lockheed Martin, programmed the entry software in English measurements. However, the navigation team at NASA's Jet Propulsion Laboratory in Pasadena, California assumed the readings were in metric units. As a result, trajectory errors were magnified instead of corrected by midcourse thruster firings. This painful lesson demonstrated the importance of maintaining and reporting units with all calculations.

1.8 Unit Checks

Unit checks should always be performed during engineering calculations to make sure that each term of an equation is in the same system of units. The importance of knowing the units of the various parameters used in an equation cannot be overemphasized. In this text, a symbol list giving the units of each parameter is provided at the beginning of each chapter. If no units are given for a particular phenomenon, it is dimensionless. This symbol list can be used as a partial check during algebraic manipulations of an equation.

Design Procedure 1.2: Procedure for Unit Checks

It is generally advisable to carry units throughout calculations. However, an expression can generally be evaluated by:

1. Establish units of specific terms of an equation while making use of Table 1.3a.
2. Place units of terms into both sides of an equation and reduce.
3. The unit check is complete if both sides of an equation have the same units.

Example 1.4: Unit Checks

Given: The centrifugal force, P, acting on a car going through a curve with a radius, r, at a velocity, v, is $m_a v^2/r$, where m_a is the mass of the car. Assume a 1.3-tonne car drives at 100 km/hr through a 100-m-radius bend.

Find: Calculate the centrifugal force.

Solution: Rewriting using metric units gives

$$m_a = 1.3 \text{ tonne} = 1300 \text{ kg}$$

$$v = 100\ \text{km/hr} = \frac{(100\ \text{m})(1000)}{3600\ \text{s}} = 27.78\ \text{m/s}$$

The centrifugal force is

$$P = \frac{m_a v^2}{r} = \frac{(1300\ \text{kg})(27.78\ \text{m/s})^2}{100\ \text{m}}$$

This results in $P = 10,030$ kg-m/s^2 or $10,030$ N.

(a) (b)

Figure 1.3: The Invisalign® product. (a) An example of an Aligner; (b) a comparison of conventional orthodontic braces and a transparent Aligner. *Source:* Courtesy of Align Technology, Inc.

1.9 Significant Figures

The accuracy of a number is specified by how many significant figures it contains. Throughout this text, four significant figures will be used unless otherwise limited. For example, 8201 and 30.51 each have four significant figures. When numbers begin or end with a zero, however, it is difficult to tell how many significant figures there are. To clarify this situation, the number should be reported by using *scientific notation* involving powers of 10. Thus, the number 8200 can be expressed as 8.200×10^3 to represent four significant figures. Also, 0.005012 can be expressed as 5.012×10^{-3} to represent four significant figures.

Example 1.5: Significant Figures

Given: A car with a mass of 1502 kg is accelerated by a force of 14.0 N.

Find: Calculate the acceleration with the proper number of significant figures.

Solution: Newton's equation gives that acceleration equals the force divided by the mass

$$a = \frac{P}{m_a} = \frac{14.0}{1502} = 0.00932091\ \text{m/s}^2$$

Since force is accurate to three figures, the acceleration can only be calculated with the accuracy of

$$\pm\frac{0.5}{140} = \pm 0.004 = \pm 0.4\%$$

Therefore, the acceleration is 0.00932 m/s^2.

Case Study 1.4: Design and Manufacture of the Invisalign Orthodontic Product

Widespread healthcare and improved diet and living habits have greatly extended the expected lifetime of people within the last century. Modern expectations are not only that life will be extended, but also that the *quality* of life will be maintained late in life. One important area where this concern manifests itself is with teeth; straight teeth lead to a healthy bite with low tooth stresses, and they also lend themselves to easier cleaning and therefore are more resistant to decay. Thus, straight teeth, in general, last longer with less pain. Of course, there are aesthetic reasons that people wish to have straight teeth as well.

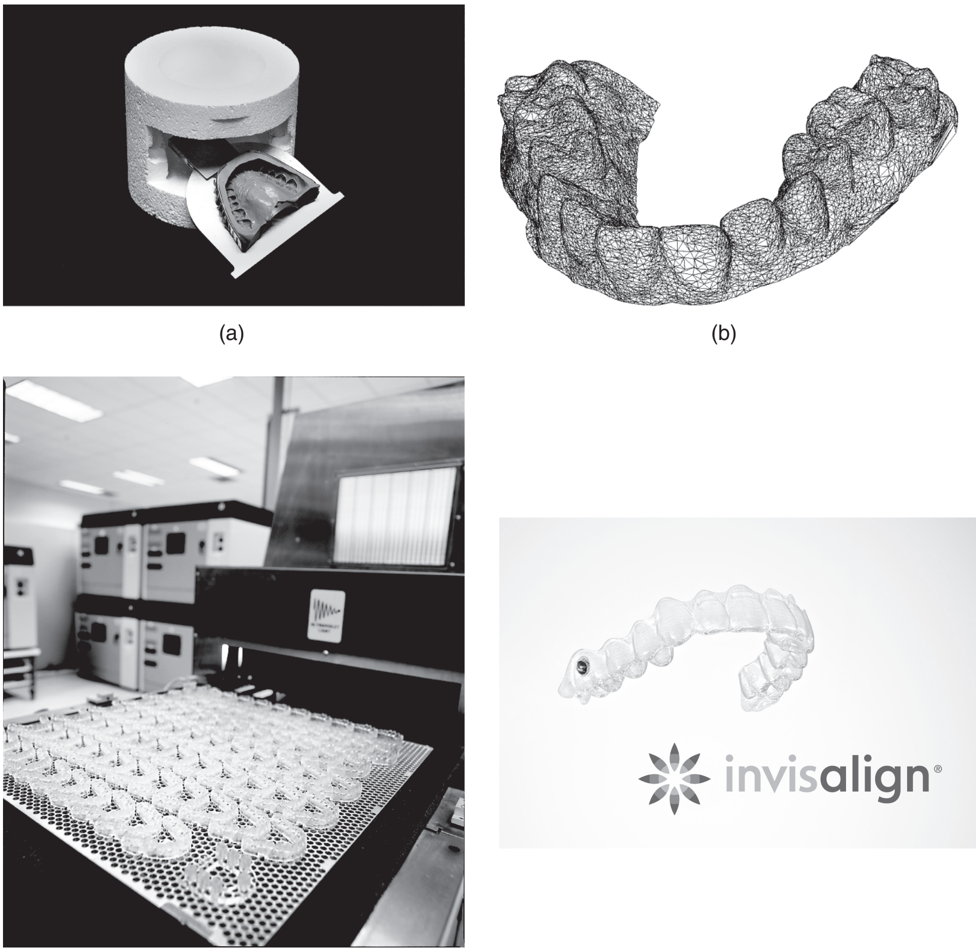
Orthodontic braces have been available to straighten teeth for over 50 years. These involve metal, ceramic, or plastic brackets that are adhesively bonded to teeth, with fixtures for attachment to a wire that then forces compliance on the teeth and straightens them to the desired shape within a few years. Conventional orthodontic braces are a well-known and wholly successful approach to long-term dental health. However, there are many drawbacks to conventional braces, including:

- They are aesthetically unappealing.
- The sharp wires and brackets can cause painful oral irritation to the teeth and gums.
- They trap food, leading to premature tooth decay.
- Brushing and flossing of teeth are far more difficult with braces in place, and therefore they are less effective for most individuals.
- Certain foods must be avoided because they will damage the braces.

One innovative solution is the Invisalign product produced by Align Technology. Invisalign consists of a series of Aligners, each of which the patient wears for approximately two weeks. Each Aligner (see Fig. 1.3) consists of a precise geometry which incrementally moves teeth to their desired positions. Because they are inserts that can be removed for eating, brushing, and flossing, most of the drawbacks of conventional braces are eliminated. Further, since they are produced from transparent plastic, they do not seriously affect the patient's appearance.

The Invisalign product uses an impressive combination of advanced technologies, and the production process is shown in Fig. 1.4. The treatment begins with an orthodontist creating a polymer impression of the patient's teeth or a direct digital image of the teeth using a 3D intra oral scanner (Fig. 1.4a). In case of physical impressions, the impressions are then used to create a three-dimensional CAD representation of the patient's teeth, as shown in Fig. 1.4b. Proprietary computer-aided design software then assists in the development of a treatment strategy for moving the teeth in optimal fashion.

Specially produced software called ClinCheck then produces a digital video of the incremental movements which can be reviewed by the treating orthodontist and modified if necessary.



(a) (b) (c) (d)

Figure 1.4: The process used in application of Invisalign orthodontic treatment. (a) Impressions are made of the patient's teeth by the orthodontist and shipped to Align Technology, Inc. These are used to make plaster models of the patient's teeth. (b) High-resolution, three-dimensional representations of the teeth are produced from the plaster models. The correction plan is then developed using computer tools. (c) Rapid-prototyped molds of the teeth at incremental positions are produced through stereolithography. (d) An Aligner is produced by molding a transparent plastic over the stereolithography part. Each Aligner is used for approximately two weeks. The patient is left with a beautiful smile. *Source:* Courtesy of Align Technology, Inc.

Once a treatment plan has been designed, the computer-based information needs to be used to produce the Aligners. This is done through a novel application of rapid prototyping technology. Stereolithography is a process that uses a focused laser to cure a liquid photopolymer. The laser only cures a small depth of the polymer, so a part can be built on a tray that is progressively lowered into a vat of photopolymer as layers or slices of the desired geometry are traced and rastered by the laser.

A number of materials are available for stereolithography, but these have a characteristic yellow-brown shade to them and are therefore unsuitable for direct application as an orthodontic product. Instead, the stereolithography machine produces patterns of the desired incremental positions of the teeth (Fig. 1.4c). A sheet of clear polymer is then molded over these patterns to produce the Aligners. These are sent to the treating orthodontist and new Aligners are given to the patient as needed, usually every two weeks or so.

The Invisalign product has proven to be very popular for patients who wish to have straight teeth without almost anyone knowing they are in treatment. It depends on advanced engineering technologies, precise force delivery through custom engineered shapes, CAD and manufacturing, and rapid prototyping and advanced polymer manufacturing processes.

Summary

This chapter introduced the concept of design as it applies to machines and machine elements. The most important goal of the design process is to ensure that the design does not fail. To avoid failure, the design engineer must predict the circumstances under which failure is first likely to occur. These circumstances or criteria can involve the material properties and applied loads, as well as surface phenomena, including friction, wear, lubrication, and environmental deterioration.

The concept of failure was quantified by using a safety factor, which is the ratio of the allowable stress established for the material to the maximum design stress that will occur. Besides the simple safety factor, other failure models, such as failure mode and effects analysis and fault tree analysis, among many others, were presented. The ethical requirements in producing safe designs were stated, along with strategies for achieving this constraint, including the Safety Hierarchy and Doctrine of Manifest Danger. Design was found to be a cooperative endeavor where multidisciplinary approaches are invaluable.

Key Words

artificial intelligence (AI) attempts to duplicate how the human mind works in computer processes

computer-aided design (CAD) application of computer technology to planning, performing, and implementing the design process

concurrent engineering design approach wherein all disciplines involved with a product are in the development process from beginning to end

design transformation of concepts and ideas into useful machinery

English units system of units where:

force is measured in pounds force (lbf)

length in inches (in.)

time in seconds (s)

mass in pounds mass (lbm)

temperature in degrees Fahrenheit (°F)

expert systems computer programs that solve specialized problems on an expert level

fail-safe design approach where no catastrophic loss can occur as a result of a component failure

failure the condition of a machine element when it is completely inoperable, cannot perform its intended function adequately, or is unreliable for continued safe use

failure mode and effects analysis (FMEA) systematic consideration of component failure effects on the entire system

fault tree analysis statistical data used to identify the most likely failure modes

finite element analysis (FEA) computational method used for solving for stress, strain, temperature, etc. in complex shapes, such as those found in machinery; replaces the complex shape with a set of simple elements interconnected at a finite set of node points

machine combination of mechanisms and other components that transform, transmit, or use energy, load, or motion for a specific purpose

machine element function normal load transmitter, torque transmitter, energy absorber, or seal

manifest danger design approach where needed service is made apparent before catastrophic failure

mechanical system synergistic collection of machine elements

rapid prototyping parts produced quickly from computer geometry description files

redundancy additional capacity or incorporation of backup systems so that a component failure does not lead to catastrophic loss

safety factor ratio of allowable stress to design stress

SI units system of units where:

force is measured in newtons (N)

length in meters (m)

time in seconds (s)

mass in kilograms (kg)

temperature in degrees Kelvin (K)

Recommended Readings

General Engineering

Florman, S.C. (1976) *The Existential Pleasures of Engineering*, St. Martin's Press.
Petroski, H. (1992) *To Engineer Is Human*, Vintage Books.

General Design

Haik, Y., (2003) *Engineering Design Process*, Thomson.
Hyman, B. (2003) *Fundamentals of Engineering Design*, 2nd ed., Prentice-Hall.
Lindbeck, J.R., (1995) *Product Design and Manufacture*, Prentice-Hall.
Otto, K., and Wood, K., (2000) *Product Design: Techniques in Reverse Engineering and New Product Development*, Prentice-Hall.
Ullman, D.G., (2009) *The Mechanical Design Process*, McGraw-Hill.
Vogel, C.M., and Cagan, J. (2012) *Creating Breakthrough Products*, 2nd ed., Prentice-Hall.

Manufacturing/Design for Manufacture

Boothroyd, G., Dewhurst, P., and Knight, W. (2010) *Product Design for Manufacture and Assembly*, 3rd ed., Taylor & Francis.
Boothroyd, G. (2005) *Assembly Automation and Product Design*, 2nd ed., Taylor & Francis.
DeGarmo, E.P., Black, J.T., and Kohser, R.A. (2011) *DeGarmo's Materials and Processes in Manufacturing*, 9th ed., Prentice-Hall.
Dieter, G.E. and Schmidt, L. (2008) *Engineering Design*, 4th ed., McGraw-Hill.
Kalpakjian, S., and Schmid, S.R. (2008) *Manufacturing Processes for Engineering Materials*, 5th ed., Prentice-Hall.
Kalpakjian, S., and Schmid, S.R. (2010) *Manufacturing Engineering and Technology*, 6th ed., Pearson.
Wright, P.K. (2001) *21st Century Manufacturing*, Prentice-Hall.

Concurrent Engineering

Anderson, D.M. (2010) *Design for Manufacturability & Concurrent Engineering*, CIM Press.
Nevins, J.L., and Whitney, D.E. (Eds.) (1989) *Concurrent Design of Products and Processes*, McGraw-Hill.
Prasad, B. (1996) *Concurrent Engineering Fundamentals*, Prentice-Hall.
Pugh, S. (1996) *Creating Innovative Products Using Total Design*, Addison-Wesley.
Pugh, S. (1991) *Total Design*, Addison-Wesley.

References

ANSI (2010) A17.1 "Minimum Safety Requirements for Passenger Elevators," American National Standards Institute.

ASME (2012) *Code of Ethics for Engineers*, Board on Professional Practice and Ethics, American Society of Mechanical Engineers.

Boothroyd, G. (1992) *Assembly Automation and Product Design*, Marcel Dekker.

Florman, S.C. (1987) *The Civilized Engineer*, St. Martin's Press.

Kalpakjian, S. and Schmid, S.R. (2003) *Manufacturing Processes for Engineering Materials*, 4th ed., Prentice-Hall.

Kalpakjian, S., and Schmid, S.R. (2010) *Manufacturing Engineering and Technology*, 6th ed., Pearson.

Lewis, E.E. (1995) *Introduction to Reliability Engineering*, 2nd ed., Wiley.

Petroski, H. (1992) *To Engineer Is Human*, Vintage Books.

Pugh, S. (1996) *Creating Innovative Products Using Total Design*, Addison-Wesley.

Pugsley, A.G. (1966) *The Safety of Structures*, Edward Arnold.

Questions

1.1 What is design?

1.2 What is *over-the-wall engineering*?

1.3 What is failure?

1.4 Define *safety factor*.

1.5 Explain the terms "product liability," "negligence," and "strict liability."

1.6 What is the Safety Hierarchy?

1.7 Give two examples of standards promulgating bodies.

1.8 What is a life cycle?

1.9 How do you define a product's life cycle?

1.10 Name two unit systems.

Qualitative Problems

1.11 Describe the differences between a safety factor and reliability.

1.12 Explain why it is said that design casts the largest shadow.

1.13 List factors that you feel should be considered when selecting a safety factor.

1.14 List some of the concerns that must be considered by a product designer.

1.15 What are the advantages and disadvantages of the Pugsley method for estimating safety factor?

1.16 Journal bearings on train boxcars in the early 19th century used a "stink additive" in their lubricant. If the bearing got too hot, it would attain a noticeable odor, and an oiler would give the bearing a squirt of lubricant at the next train stop. What design philosophy does this illustrate? Explain.

1.17 Explain why engineers must work with other disciplines, using specific product examples.

1.18 A car is being driven at 150 km/hr on a mountain road where the posted speed limit is 100 km/hr. At a tight turn, one of the tires fails (a blowout) causing the driver to lose control and results in an accident involving property losses and injuries but no loss of life. Afterward, the driver decides to file a lawsuit against the tire manufacturer. Explain which legal theories give him a viable argument to make a claim.

1.19 Give three examples of fail-safe and three examples of fail-unsafe products.

1.20 List three measures that are known within (*a*) one; (*b*) two; (*c*) three; and (*d*) more than three significant figures.

Quantitative Problems

1.21 A hand-held drilling machine has a bearing to take up radial and thrust load from the drill. Depending on the number of hours the drill is expected to be used before it is scrapped, different bearing arrangements will be chosen. A rubbing bushing has a 50-hr life. A small ball bearing has a 300-hr life. A two-bearing combination of a ball bearing and a cylindrical roller bearing has a 10,000-hr life. The cost ratios for the bearing arrangements are 1:5:20. What is the optimum bearing type for a simple drill, a semiprofessional drill, and a professional drill?

1.22 Using the hand-held drill described in Problem 1.21, if the solution with the small ball bearing was chosen for a semiprofessional drill, the bearing life could be estimated to be 300 hr until the first spall forms in the race. The time from first spall to when the whole rolling-contact surface is covered with spalls is 200 hr, and the time from then until a ball cracks is 100 hr. What is the bearing life

(a) If high precision is required?

(b) If vibrations are irrelevant?

(c) If an accident can happen when a ball breaks?

1.23 The dimensions of skis used for downhill competition need to be determined. The maximum force transmitted from one foot to the ski is 2500 N, but the snow conditions are not known in advance, so the bending moment acting on the skis is not known. Estimate the safety factor needed.

1.24 A crane has a loading hook that is hanging in a steel wire. The allowable normal tensile stress in the wire gives an allowable force of 100,000 N. Estimate the safety factor that should be used.

(a) If the wire material is not controlled, the load can cause impact, and fastening the hook in the wire causes stress concentrations. (If the wire breaks, people can be seriously hurt and expensive equipment can be destroyed.)

(b) If the wire material is extremely well controlled, no impact loads are applied and the hook is fastened in the wire without stress concentrations. (If the wire breaks, no people or expensive equipment can be damaged.)

1.25 Calculate the following:

(a) The velocity of hair growth in meters per second, assuming hair grows 0.75 in. in one month.

(b) The weight of a 1-in. diameter steel ball bearing in meganewtons.

(c) The mass of a 1-kg object on the surface of the moon.

(d) The equivalent rate of work in watts of 4 horsepower.

1.26 The unit for dynamic viscosity in the SI system is newton-seconds per square meter, or pascal-seconds (N-s/m^2 = Pa-s). How can that unit be rewritten using the basic relationships described by Newton's law for force and acceleration?

1.27 The unit for dynamic viscosity in Problem 1.26 is newton-seconds per square meter (N-s/m^2) and the kinematic viscosity is defined as the dynamic viscosity divided by the fluid density. Find at least one unit for kinematic viscosity.

1.28 A square surface has sides 1 m long. The sides can be split into decimeters, centimeters, or millimeters, where 1 m = 10 dm, 1 dm = 10 cm, and 1 cm = 10 mm. How many millimeters, centimeters, and decimeters equal 1 m? Also, how many square millimeters, square centimeters , and square decimeters equal a square meter?

1.29 A volume is 1 tera (mm^3) large. Calculate how long the sides of a cube must be to contain that volume.

1.30 A ray of light travels at a speed of 300,000 km/s = 3×10^8 m/s. How far will it travel in 1 ps, 1 ns, and 1 μs?

1.31 Two smooth flat surfaces are separated by a 10-μm-thick lubricant film. The viscosity of the lubricant is 0.100 Pa-s. One surface has an area of 1 dm^2 and slides over the plane surface with a velocity of 1 km/hr. Determine the friction force due to shearing of the lubricant film. Assume the friction force is the viscosity times the surface area times the velocity of the moving surface and divided by the lubricant film thickness.

1.32 A firefighter sprays water on a house. The nozzle diameter is small relative to the hose diameter, so the force on the nozzle from the water is

$$F = v\frac{dm_a}{dt}$$

where v is the water velocity and dm_a/dt is the water mass flow per unit time. Calculate the force the firefighter needs to hold the nozzle if the water mass flow is 3 tons/hr and the water velocity is 100 km/hr.

1.33 The mass of a car is 1346 kg. The four passengers in the car weigh 643 N, 738 N, 870 N, and 896 N. It is raining and the additional mass due to the water on the car is 1.349 kg. Calculate the total weight and mass of the car, including the passengers and water, using four significant figures.

1.34 During an acceleration test of a car the acceleration was measured to be 1.4363 m/s^2. Because slush and mud adhered to the bottom of the car, the mass was estimated to be 1400 $\pm$ 100 kg. Calculate the force driving the car and indicate the accuracy.

Design and Projects

1.35 Design transport containers for milk in 1- and 4-liter sizes.

1.36 Design a kit of tools for campers so they can prepare and eat meals. The kit should have all of the implements needed, and be lightweight and compact.

1.37 An acid container will damage the environment and people around it if it leaks. The cost of the container is proportional to the container wall thickness. The safety can be increased either by making the container wall thicker or by mounting a reserve tray under the container to collect the leaking acid. The reserve tray costs 10% of the thick-walled container cost. Which is less costly, to increase the wall thickness or to mount a reserve tray under the container?

Chapter 2

Load, Stress, and Strain

Collapse of the Tacoma Narrows bridge in 1940. *Source:* AP Photos.

The careful text-books measure
(Let all who build beware!)
The load, the shock, the pressure
Material can bear.
So when the buckled girder
Lets down the grinding span,
The blame of loss, or murder,
Is laid upon the man.
Not on the stuff - The Man!
Rudyard Kipling, *Hymn of Breaking Strain*

This chapter addresses fundamental problems essential to design: determining the location in a part that is likely to fail, and how to analyze stresses and strains that occur at the critical location. The concept of the critical section is discussed, and the terminology of different loads is defined. The concepts of equilibrium and free-body diagrams are then presented, leading to the production of shear and bending moment diagrams for beams. There are numerous methods of producing such diagrams, and three of the most common and powerful techniques are presented. Stress and strain are discussed next, with an emphasis that they are tensors. The common circumstances of plane stress and plane strain are defined. The ability to determine stress states based on orientation is demonstrated through stress transformation equations and Mohr's circle diagrams, and the procedure for finding principal stresses for a generalized three-dimensional stress state is given. The useful concept of octahedral stresses is presented, and the chapter ends by briefly describing the use of strain gages and rosettes to experimentally determine strains.

Contents

Examples

Design Procedures

Symbols

A	area, m^2
d	diameter, m
g	gravitational acceleration, 9.807 m/s^2
l	length, m
M	moment, N-m
m_a	mass, kg
n	any integer
P	force, N
q	load intensity function, N/m
R	reaction force, N
r	radius of Mohr's circle, m
$\mathbf{S}$	stress tensor
$\mathbf{S}'$	principal stress tensor
$\mathbf{T}$	strain tensor
T	torque, N-m
V	transverse shear force, N
W	normal applied load, N
w_o	load per unit length, N/m
x, y, z	Cartesian coordinate system, m
x', y', z'	rotated Cartesian coordinate system, m
γ	shear strain
δ	elongation, m
ϵ	normal strain
θ	deviation from initial right angle or angle of force application, deg
μ	coefficient of friction
σ	normal stress, Pa
τ	shear stress, Pa
$\tau_{1/2}$, $\tau_{2/3}$, $\tau_{1/3}$	principal shear stresses in triaxial stress state, Pa
ϕ	angle of oblique plane, deg

Subscripts

a	axial
b	biaxial stress
c	center
e	von Mises
r	roller
t	triaxial stress; transverse
x, y, z	Cartesian coordinates
x', y', z'	rotated Cartesian coordinates
θ	angle representing deviation from initial right angle
σ	normal stress
τ	shear stress
ϕ	angle of oblique plane
1, 2, 3	principal axes

2.1 Introduction

The focus of this text is the design and analysis of machines and machine elements. Since machine elements carry **loads**, it follows that an analysis of loads is essential in machine element design. Proper selection of a machine element often is a simple matter of calculating the stresses or deformations expected in service and then choosing a proper size so that critical stresses or deformations are not exceeded. The first step in calculating the stress or deformation of a machine element is to accurately determine the load. Load, stress, and strain in all its forms are the foci of this chapter, and the information developed here is used throughout the text.

2.2 Critical Section

To determine when a machine element will fail, the designer evaluates the stress, strain, and strength at the critical section. The **critical section**, or the location in the design where the largest internal load is developed and failure is most likely, is often not intuitively known beforehand. Design Procedure 2.1 lists the common steps in determining the critical section and loading. The first and second steps arise from system design. The third step is quite challenging and may require analysis of a number of locations or failure modes before the most critical is found. For example, a beam subjected to a distributed load might conceivably exceed the maximum deflection at a number of locations; thus, the beam deflection would need to be calculated at more than one position.

In general, the critical section will often occur at locations of geometric nonuniformity, such as where a shaft changes its diameter along a fillet, or at an interface between two different materials. Also, locations where load is applied or transferred are often critical locations. Finally, areas where the geometry is most critical are candidates for analysis. This topic will be expanded upon in Chapter 6.

Design Procedure 2.1: Critical Section and Loading

To establish the critical section and the critical loading, the designer:

1. Considers the external loads applied to a machine (e.g., a gyroscope)
2. Considers the external loads applied to an element within the machine (e.g., a ball bearing)
3. Locates the critical section within the machine element (e.g., the inner race)
4. Determines the loading at the critical section (e.g., contact stresses)

Example 2.1: Critical Section of a Simple Crane

Given: A simple crane, shown in Fig. 2.1a, consists of a horizontal beam loaded vertically at one end with a load of 10 kN. The beam is pinned at the other end. The force at the pin and roller must not be larger than 30 kN to satisfy other design constraints.

Find: The location of the critical section and also whether the load of 10 kN can be applied without damage to the crane.

Solution: The forces acting on the horizontal beam are shown in Fig. 2.1b. Summation of moments about the pin (at $x = 0$) gives

$$(1.0)P = (0.25)W_r,$$

so that W_r is found to be 40 kN. Summation of vertical forces gives

$$-W_p + W_r - 10\text{ kN} = 0,$$

which results in W_p = 30 kN. The critical section is at the roller, since $W_r > W_p$. Also, since $W_r > W_{\text{all}}$, failure will occur. To avoid failure, the load at the end of the horizontal beam must be reduced.

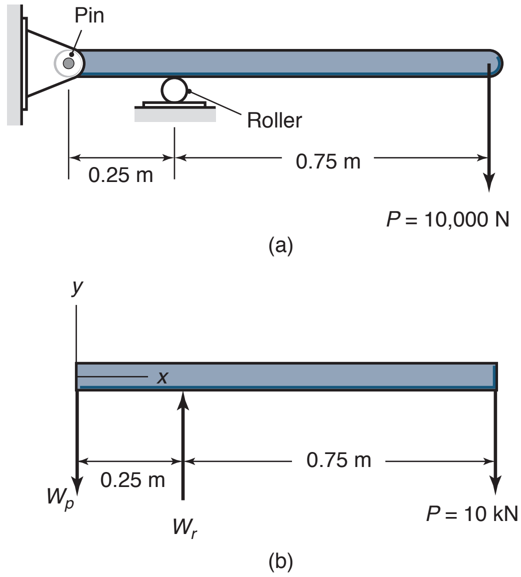


Figure 2.1: A schematic of a simple crane and applied forces considered in Example 2.1. (a) Assembly drawing; (b) free-body diagram of forces acting on the beam.

2.3 Load Classification and Sign Convention

Any applied load can be classified with respect to time in the following ways:

1. *Static load* — Load is gradually applied and equilibrium is reached in a relatively short time. The structure experiences no dynamic effects.

2. *Sustained load* — Load, such as the weight of a structure, is constant over a long time.

3. *Impact load* — Load is rapidly applied. An impact load is usually attributed to an energy imparted to a system.

4. *Cyclic load* — Load can vary and even reverse its direction and has a characteristic period with respect to time.

The load can also be classified with respect to the area over which it is applied:

1. *Concentrated load* — Load is applied to an area much smaller than the loaded member, such as presented for nonconformal surfaces in Section 8.4. An example would be the contact between a caster and a support beam on a mechanical crane, where the contact area is around 100 times smaller than the surface area of the caster. For these cases, the applied force can be considered to act at a point on the surface.

2. *Distributed load* — Load is spread along a large area. An example would be the weight of books on a bookshelf.

Loads can be further classified with respect to location and method of application. Also, the coordinate direction must be determined before the sign of the loading can be established:

1. *Normal load* — The load passes through the centroid of the resisting section. Normal loads may be tensile (Fig. 2.2a) or compressive (Fig. 2.2b). The established sign convention has tensile loads being positive and compressive loads being negative.

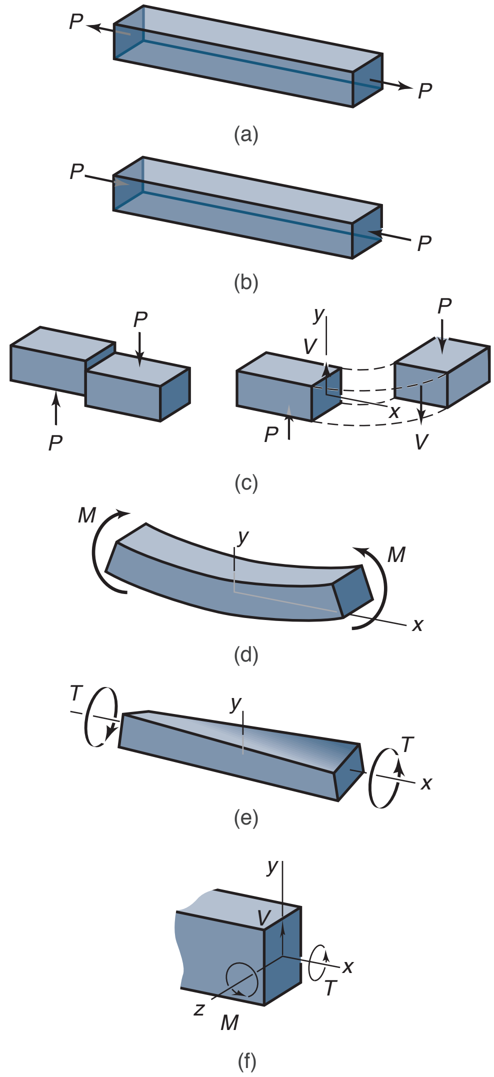


Figure 2.2: Load classified as to location and method of application. (a) Normal, tensile; (b) normal, compressive; (c) shear; (d) bending; (e) torsion; (f) combined.

2. *Shear load* — The separated bar in Fig. 2.2c illustrates the action of positive shearing. The figure has been redrawn to show the surface of interest on the right side. A shear force is positive if the force direction and the normal direction are both positive or both negative. The shear force, V, shown on the left surface of Fig. 2.2c is in the positive y-direction, which is upward, and the normal to the surface is in the positive x-direction. Thus, the shear force is positive. On the right surface of Fig. 2.2c the shear force is also positive, since the direction of the shear force and the normal to the surface are both negative. A shear force is negative if the force direction and the normal direction have different signs. If the positive y-coordinate had been chosen to be upward (negative) rather than downward (positive) in Fig. 2.2c, the shear force would be negative rather than positive. Thus, to establish whether a shear force is positive or negative, the positive x- and y-coordinates must be designated.

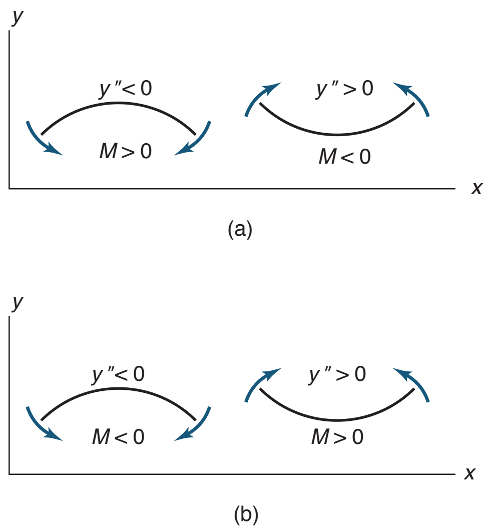


Figure 2.3: Sign conventions used in bending. (a) Positive moment leads to a tensile stress in the positive y-direction; (b) positive moment acts in a positive direction on a positive face. The sign convention shown in (b) will be used in this book.

3. *Bending load* — This commonly occurs when load is applied transversely to the longitudinal axis of the member. Figure 2.2d shows a member that is subject to equal and opposite moments applied at its ends. The moment results in normal stresses in a cross section transverse to the normal axis of the member, as described further in Section 4.5.2.

   The sign convention used in bending stress analysis should be briefly discussed. Two common sign conventions are used in engineering practice, as illustrated in Fig. 2.3. The difference between these two sign conventions is in the sign of the moment applied, and each sign convention has its proponents and critics. The proponents of the sign convention shown in Fig. 2.3a prefer that the stresses that arise in the beam follow the rule that, for a positive moment, a positive distance from the neutral axis results in a positive (tensile) stress. On the other hand, the sign convention shown in Fig. 2.3b allows certain mnemonic methods for its memorization, such as a positive moment results in a deformed shape that "holds water" or has a positive second derivative. Perhaps the best reason for using the sign convention in Fig. 2.3b is that the convention for bending moments is the same as for applied shear forces — that a positive force or moment acting on a face with a positive outward pointing normal acts in a positive direction when using a right-handed coordinate system.

   It should be recognized that sign conventions are arbitrary, and correct answers can be obtained for problems using any sign convention, as long as the sign convention is applied consistently within a problem. In this book, the bending sign convention of Fig. 2.3b will be used, but this should not be interpreted as mandatory for solution of problems.

4. *Torsion load* — Such a load subjects a member to twisting motion, as shown in Fig. 2.2e. The twist results in a distribution of shear stresses on the transverse cross section of the member. Positive torsion occurs in Fig. 2.2e. The right-hand rule is applicable here.

5. *Combined load* — Figure 2.2f shows a combination of two or more of the previously defined loads (e.g., shear, bending, and torsion acting on a member). Note that positive shear, bending, and torsion occur in this figure.

Example 2.2: Classification of Load Types

Given: A diver jumping on a diving board.

Find:
a) The load type when the diver lands on the diving board
b) The load type when the diver stands motionless waiting for the signal to jump
c) The load type on the diving board just as the diver jumps
d) The load type of the diving board assembly against the ground when no dynamic loads are acting

Solution:
a) Impact load — as the diver makes contact with the diving board.
b) Static load — when the diver is motionless.
c) Cyclic load — when the diving board swings up and down just after the dive
d) Sustained load — when gravity acts on the diving board structure, pressing it against the ground

Example 2.3: Loads on a Lever Assembly

Given: The lever assembly shown in Fig 2.4a.

Find: The normal, shear, bending, and torsional loads acting at section B.

Solution: Figure 2.4b shows the various loads acting on the lever, all in the positive direction. To the right of the figure, expressions are given for the loading at section B of the lever shown in Fig. 2.4a.

2.4 Support Reactions

Reactions are forces developed at supports. For two-dimensional problems (i.e., bodies subjected to coplanar force systems), the types of support most commonly encountered, along with the corresponding reactions, are shown in Table 2.1. (Note the direction of the forces on each type of support and the reaction they exert on the attached member.) One way to determine the support reaction is to imagine the attached member as being translated or rotated in a particular direction. If the support prevents translation in a given direction, a force is developed on the member in that direction. Likewise, if the support prevents rotation, a moment is applied to the member. For example, a roller prevents translation only in the contact direction, perpendicular (or normal) to the surface; thus, the roller cannot develop a coupled moment on the member at the point of contact.

2.5 Static Equilibrium

Equilibrium of a body requires both a balance of forces, to prevent the body from translating (moving) along a straight or curved path, and a balance of moments, to prevent the body from rotating. From statics, it is customary to present

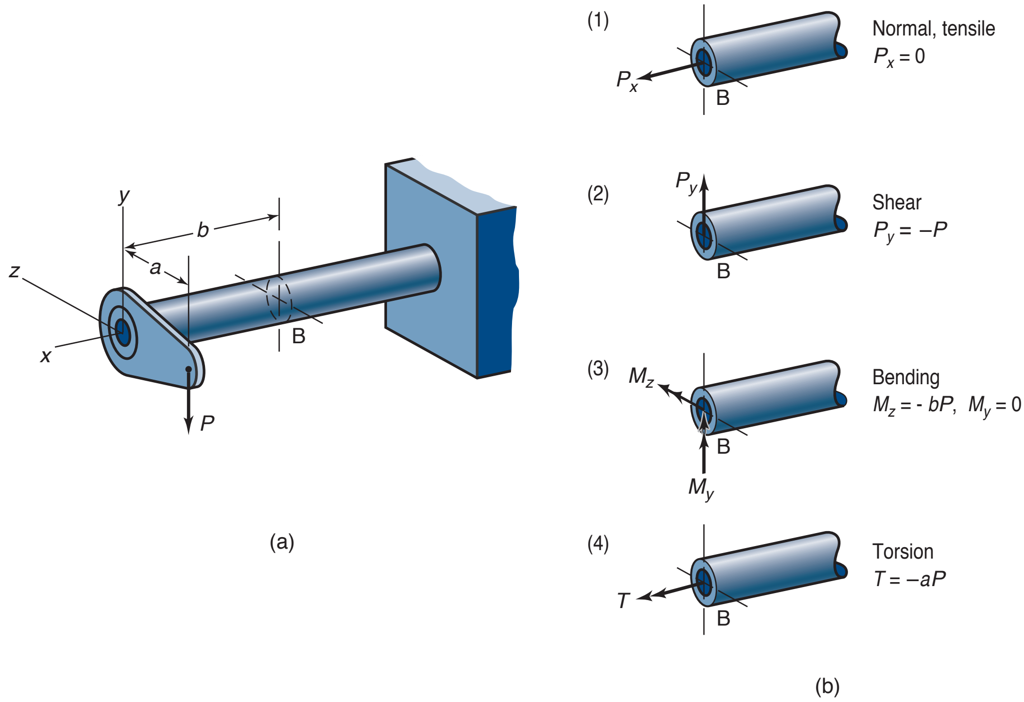
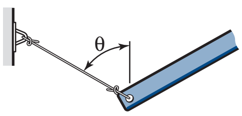
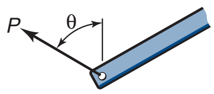
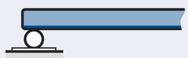
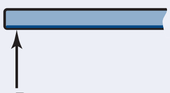
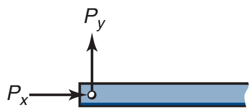
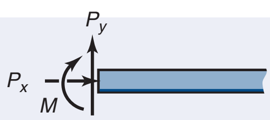


Figure 2.4: Lever assembly and results. (a) Lever assembly; (b) results showing (1) normal, tensile, (2) shear, (3) bending, and (4) torsion on section B of lever assembly.

Table 2.1: Four types of support with their corresponding reactions.

Type of support	Reaction
 Cable	
 Roller	
 Pin	
 Fixed support	

these equations as

$$\sum P_x = 0, \qquad \sum P_y = 0, \qquad \sum P_z = 0, \tag{2.1}$$

$$\sum M_x = 0, \qquad \sum M_y = 0, \qquad \sum M_z = 0. \tag{2.2}$$

Often, in engineering practice, the loading on a body can be represented as a system of coplanar forces. If this is the case, and the forces lie in the x-y plane, the equilibrium conditions of the body can be specified by only three equations:

$$\sum P_x = 0, \qquad \sum P_y = 0, \qquad \sum M_z = 0. \tag{2.3}$$

Note that the moment, M_z, is a vector perpendicular to the plane that contains the forces. Successful application of the equilibrium equations requires complete specification of all the known and unknown forces acting on the body.

Example 2.4: Static Equilibrium of a Ladder

Given: A painter stands on a ladder that leans against the wall of a house. Assume the painter is at the midheight of the ladder. The ladder stands on a horizontal surface with a coefficient of friction of 0.3 and leans at an angle of 20° against the house, which also has a coefficient of friction of 0.3.

Find: Whether the painter and ladder are in static equilibrium and what critical coefficient of friction, μ_{cr}, will not provide static equilibrium.

Solution: Figure 2.5 shows a diagram of the forces acting on the ladder due to the weight of the painter as well as the weight of the ladder. The mass of the ladder is m_l and the mass of the painter is m_p. If the ladder starts to slide, the friction force will counteract the motion.

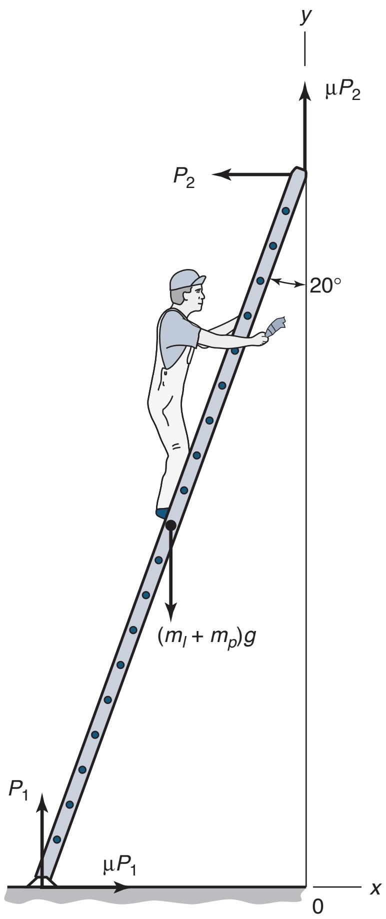


Figure 2.5: Ladder in contact with a house and the ground while having a painter on the ladder.

Summation of horizontal forces gives

$$\sum P_x = \mu_{\text{cr}} P_1 - P_2 = 0, \qquad (a)$$

or $P_2 = \mu_{\text{cr}} P_1$. Summation of vertical forces results in

$$P_1 \left(1 + \mu_{\text{cr}}^2\right) = (m_l + m_p)\, g.$$

Therefore,

$$P_1 = \frac{(m_l + m_p)\, g}{1 + \mu_{cr}^2}. \qquad (b)$$

Making use of Eq. (*a*) gives

$$P_2 = \frac{\mu_{cr}\,(m_l + m_p)\, g}{1 + \mu_{cr}^2}. \qquad (c)$$

Applying moment equilibrium about point 0 results in

$$P_1 l \sin 20^\circ - P_2 l \cos 20^\circ - (m_l + m_p)\, g \frac{l}{2} \sin 20^\circ = 0, \qquad (d)$$

where l is the ladder length. Substituting Eqs. (*b*) and (*c*) into Eq. (*d*) gives

$$0 = \frac{(m_l + m_p)\, gl \sin 20^\circ}{1 + \mu_{cr}^2} - \frac{\mu_{cr}\,(m_l + m_p)\, gl \cos 20^\circ}{1 + \mu_{cr}^2}$$

$$- (m_l + m_p)\, g \frac{l}{2} \sin 20^\circ,$$

or

$$0 = \frac{1}{1 + \mu_{cr}^2} - \frac{\mu_{cr}}{\tan 20^\circ \left(1 + \mu_{cr}^2\right)} - \frac{1}{2}.$$

Through algebraic manipulation,

$$0.5 = \frac{\tan 20^\circ - \mu_{cr}}{\tan 20^\circ \left(1 + \mu_{cr}^2\right)}$$

$$0.5 \tan 20^\circ + 0.5 \mu_{cr}^2 \tan 20^\circ = \tan 20^\circ - \mu_{cr}$$

$$\mu_{cr}^2 + \frac{\mu_{cr}}{0.5 \tan 20^\circ} - 1 = 0$$

so that

$$\mu_{cr} = 0.1763.$$

Since μ is given as 0.3, the ladder will not move, so that the painter and ladder are in static equilibrium. The critical coefficient where the ladder starts to slide is 0.1763.

2.6 Free-Body Diagram

An entire machine, any individual machine element, or any part of a machine element can be represented as a free body. Static equilibrium is assumed at each level. The best way to account for the forces and moments in the equilibrium equations is to draw the free-body diagram. For the equilibrium equations to be correctly applied, the effects of all the applied forces and moments must be represented in the free-body diagram.

A **free-body diagram** is a sketch of a machine, a machine element, or part of a machine element that shows all acting forces, such as applied loads and gravity forces, and all reactive forces. The reactive forces are supplied by the ground, walls, pins, rollers, cables, or other means. The sign of the reaction may not be known, but it can be assigned arbitrarily or guessed. If, after the static equilibrium analysis, the sign of the reactive force is positive, the initial direction is correct; if it is negative, the direction is opposite to that initially guessed.

Example 2.5: Equilibrium of a Suspended Sphere

Given: A steel sphere, shown in Fig. 2.6a, has a mass of 10 kg and hangs from two wires. A spring attached to the bottom of the sphere applies a downward force of 150 N.

Find: The forces acting on the two wires. Also, draw a free-body diagram showing the forces acting on the sphere.

Solution: Figure 2.6b shows the free-body diagram of the forces acting on the sphere. Summation of the vertical forces gives

$$2P \cos 60^\circ - m_a g - 150 = 0.$$

or

$$P = \frac{(10)(9.807) + 150}{2 \cos 60^\circ} = 248.1 \text{ N}.$$

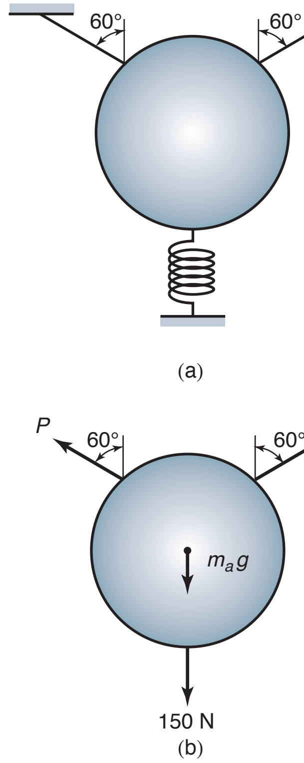


Figure 2.6: Sphere and applied forces. (a) Sphere supported with wires from top and spring at bottom; (b) free-body diagram of forces acting on sphere.

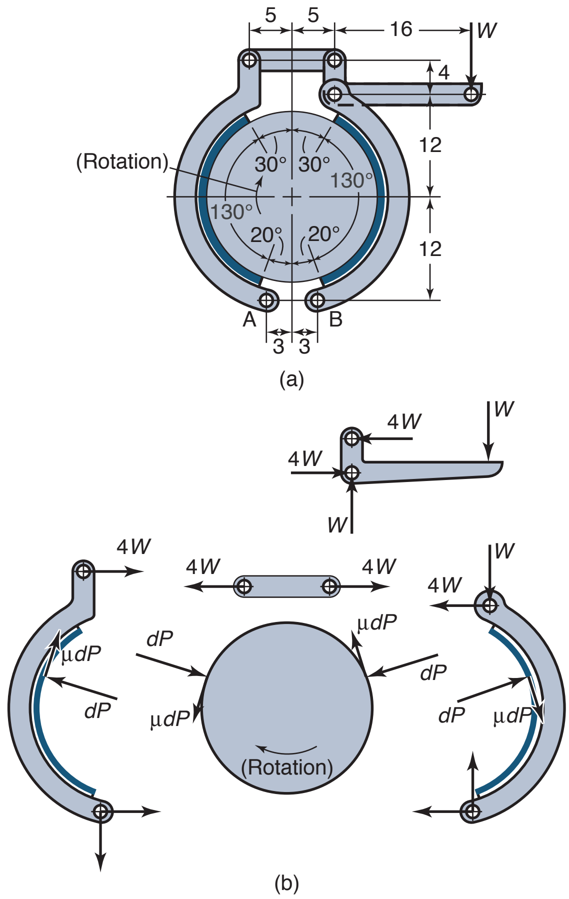


Figure 2.7: External rim brake and applied forces, considered in Example 2.6. (a) External rim brake; (b) external rim brake with forces acting on each part. (Linear dimensions are in millimeters.)

Example 2.6: Free-Body Diagram of an External Rim Brake

Given: The external rim brake shown in Fig. 2.7a.

Find: Draw a free-body diagram of each component of the system.

Solution: Figure 2.7b shows each brake component as well as the forces acting on them. The static equilibrium of each component must be preserved, and the friction force acts opposite to the direction of motion on the drum and in the direction of motion on both shoes. The $4W$ value in Fig. 2.7b was obtained from the moment equilibrium of the lever. Details of brakes are considered in Chapter 18, but in this chapter it is important to be able to draw the free-body diagram of each component.

2.7 Supported Beams

A **beam** is a structural member designed to support loading applied perpendicular to its longitudinal axis. In general, beams are long, often straight bars having a constant cross-section. Often, they are classified by how they are supported. Three major types of support are shown in Fig. 2.8:

1. A **simply supported beam** (Fig. 2.8a) is pinned at one end and roller-supported at the other.

2. A **cantilevered beam** or **cantilever** (Fig. 2.8b) is fixed at one end and free at the other.

3. An **overhanging beam** (Fig. 2.8c) has one or both of its ends freely extending past its supports.

Two major parameters used in evaluating beams are strength and deflection, as discussed in Chapter 5. Shear and bending are the two primary modes of beam loading. However, if the height of the beam is large relative to its width, elastic instability can become important and the beam can twist under loading (see *unstable equilibrium* in Section 9.2.3).

2.8 Shear and Moment Diagrams

Designing a beam on the basis of strength requires first finding its maximum shear and moment. This section describes three common and powerful approaches for developing shear and moment diagrams. Usually, any of these methods will be sufficient to analyze any statically determinate beam, so the casual reader may wish to emphasize one method and then continue to the remaining sections.

2.8.1 Method of Sections

One way to obtain shear and moment diagrams is to apply equilibrium to sections of the beam taken at convenient locations. This allows expression of the transverse shear force, V, and the moment, M, as functions of an arbitrary position, x, along the beam's axis. These shear and moment functions can then be plotted as shear and moment diagrams from which the maximum values of V and M can be obtained.

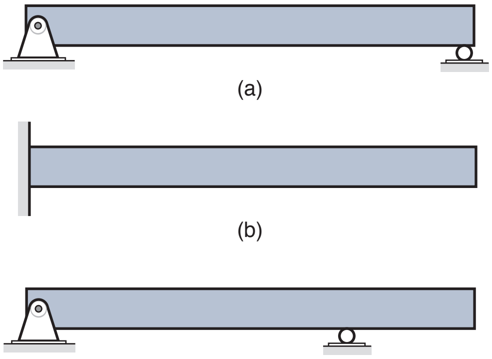


Figure 2.8: Three types of beam support. (a) Simply supported; (b) cantilevered; (c) overhanging.

Design Procedure 2.2: Drawing Shear and Moment Diagrams by the Method of Sections

The procedure for drawing shear and moment diagrams by the method of sections is as follows:

1. Draw a free-body diagram and determine all the support reactions. Resolve the forces into components acting perpendicular and parallel to the beam's axis.

2. Choose a position, x, between the origin and the length of the beam, l, thus dividing the beam into two segments. The origin is chosen at the beam's left end to ensure that any x chosen will be positive.

3. Draw a free-body diagram of the two segments and use the equilibrium equations to determine the transverse shear force, V, and the moment, M.

4. Plot the shear and moment functions versus x. Note the location of the maximum moment. Generally, it is convenient to show the shear and moment diagrams directly below the free-body diagram of the beam.

5. Additional sections can be taken as necessary to fully quantify the shear and moment diagrams.

Example 2.7: Shear and Moment Diagrams by Method of Sections

Given: The bar shown in Fig. 2.9a.

Find: Draw the shear and moment diagrams.

Solution: For $0 \le x < l/2$, the free-body diagram of the bar section is as shown in Fig. 2.9b. The unknowns V and M are positive. Applying the equilibrium equations gives

$$\sum P_y = 0 \rightarrow V = -\frac{P}{2}, \qquad (a)$$

$$\sum M_z = 0 \rightarrow M = \frac{P}{2}x. \qquad (b)$$

For $l/2 \le x < l$, the free-body diagram is shown in Fig. 2.9c. Again, V and M are shown in the positive direction.

$$\sum P_y = 0 \rightarrow \frac{P}{2} - P + V = 0, \quad \text{or} \quad V = P/2. \qquad (c)$$

$$\sum M_z = 0 \rightarrow M + P\left(x - \frac{l}{2}\right) - \frac{P}{2}x = 0.$$

Therefore,

$$M = \frac{P}{2}(l - x). \qquad (d)$$

The shear and moment diagrams in Fig. 2.9d can be obtained directly from Eqs. (a) to (d).

2.8.2 Direct Integration

Note that if $q(x)$ is the load intensity function in the y-direction, the transverse shear force is

$$V(x) = -\int_{-\infty}^{x} q(x)\,dx, \qquad (2.4)$$

and the bending moment is

$$M(x) = -\int_{-\infty}^{x} V(x)\,dx = \int_{-\infty}^{x}\int_{-\infty}^{x} q(x)\,dx\,dx. \qquad (2.5)$$

For simple loading cases, direct integration is often the most straightforward method of producing shear and moment diagrams. Since the integral of a curve is its area, graphically producing a shear or moment diagram follows directly from the loading. The only complication arises from point loadings and their use in developing a shear diagram. With concentrated loadings, the shear diagram will take a "jump" equal in magnitude to the applied load. The sign convention used for moment diagrams is important; recall that the sign convention described in Fig. 2.3b is used in this textbook.

Example 2.8: Shear and Moment Diagrams by Direct Integration

Given: The beam shown in Fig. 2.10a. From static equilibrium, it can be shown that R_A = 12 kN and R_B = 4 kN in the directions shown.

Find: The shear and moment diagrams by direct integration. Determine the location and magnitude of the largest shear force and moment.

Solution: The shear diagram will be constructed first. Consider the loads on the beam and work from left to right to construct the shear diagram. The following steps are followed to construct the shear diagram:

1. At the left end (at $x = 0$), there is a downward acting force. As discussed above, this means that the shear diagram will see a jump in its value at $x = 0$. From Eq. (2.4), a downward acting load leads to an upward acting shear force (that is, its sign is opposite to the loading). Thus, the diagram jumps upward by a magnitude of 4 kN.

2. Moving to the right, this value is unchanged until $x = 2$ m, where a 12 kN concentrated load acts upward. This results in a downward jump as shown.

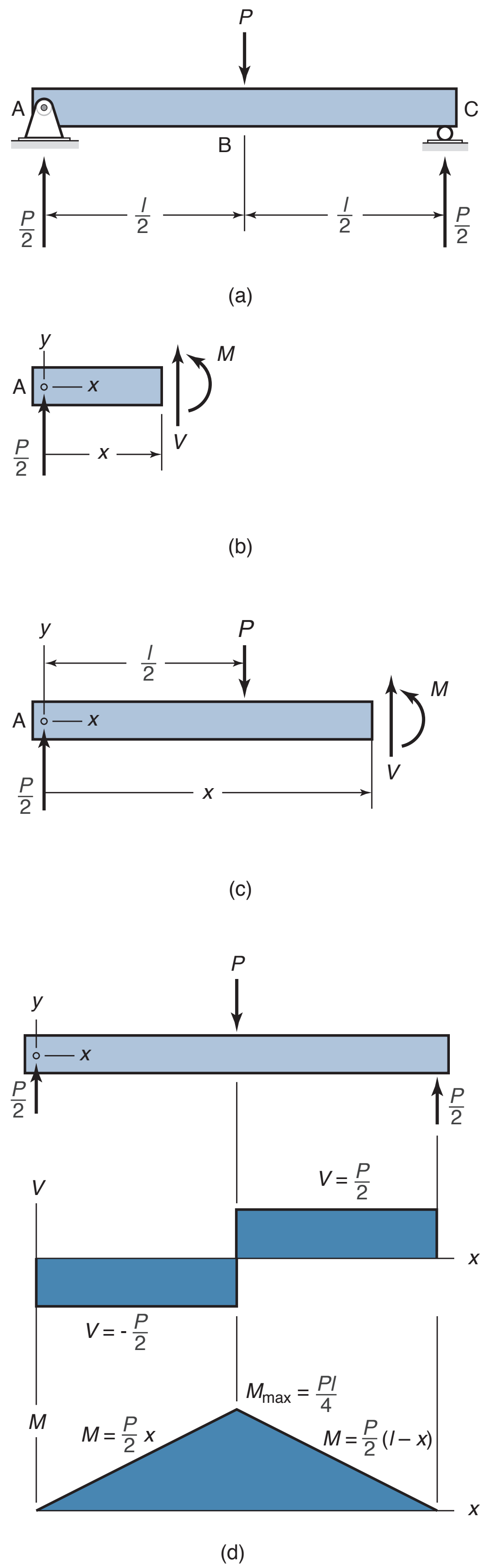


Figure 2.9: Simply supported beam. (a) Midlength load and reactions; (b) free-body diagram for $0 < x < l/2$; (c) free-body diagram for $l/2 \leq x < l$; (d) shear and moment diagrams.

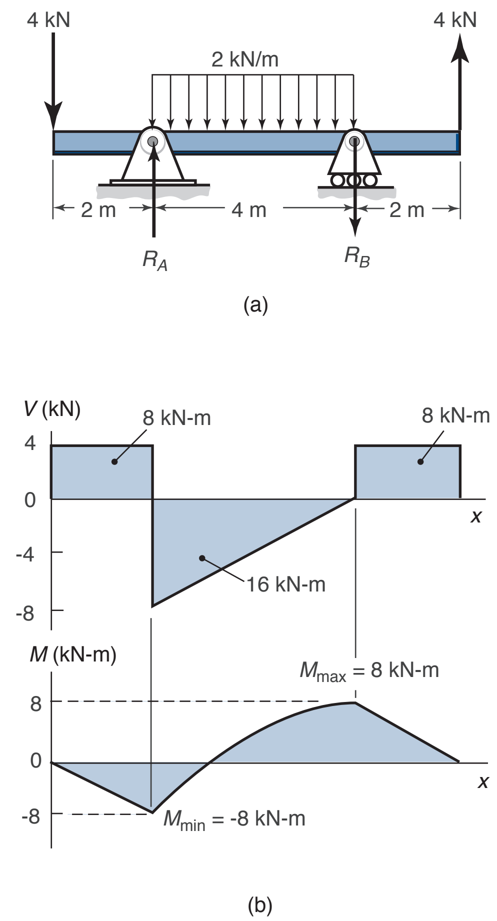


Figure 2.10: Beam for Example 2.8. (a) Applied loads and reactions; (b) shear diagram with areas indicated, and moment diagram with maximum and minimum values indicated.

3. The constant distributed loading to the right of $x = 2$ m will result in a shear force that changes linearly with respect to x. From Eq. (2.4), the magnitude of the total change is the integral of the applied load, or just its area. Thus, the total change due to the 2kN/m distributed load from $x = 2$ to $x = 6$ is 8 kN, and since the distributed load acts downward, this change is upward in the shear diagram because of the sign convention used in Eq. (2.4). Therefore, the value of the shear force at $x = 6$ is $(-8 \text{ kN}) + 8 \text{ kN} = 0$. The line from $x = 2$ to $x = 6$ is shown.

4. At $x = 6$, there is a concentrated force associated with the downwards acting force R_B, so there is an upward jump of 4 kN.

5. At $x = 8$, the upward acting force leads to a downward jump of 4 kN, returning the shear to zero.

The bending moment is obtained from repeated application of Eq. (2.5). However, note that the integral of the shear force is the area under the shear force curve. The shear diagram just developed consists of rectangles and triangles, where the area is calculated from geometry. The areas have been indicated in the shear diagram. For example, the shear diagram up to $x = 2$ consists of a rectangle with a height of $V = 4$ kN and a base of $x = 2$ m. Thus, its area is 8 kN-m.

The moment diagram is then constructed using the following steps.

1. At a starting value of $M = 0$ at $x = 0$, the diagram will be constructed from left to right. From $x = 0$ to $x = 2$ m, the value of the shear diagram is positive and constant. Integrating this curve results in a linear profile. Since the shear diagram is positive, the moment that results must be negative according to Eq. (2.5), and at $x = 2$ m, the value is 8 kN-m. This linear profile is shown in the figure.

2. From $x = 2$ m to $x = 6$ m, the shear diagram is linear with respect to x, so that the moment diagram will be quadratic. At $x = 6$ m, it is known that the moment will have a value of 8 kN-m by summing the areas of the shear diagram segments. The slope of the moment curve is equal to the value of the shear curve, as seen by taking the derivative of Eq. (2.5). Thus, the slope is initially large and at $x = 6$ it is zero.

3. From $x = 6$ m to $x = 8$ m, the moment diagram has a linear profile and ends at $M = 0$. This can be seen by summing the areas in the shear diagram, remembering that areas below the abscissa are considered negative.

The shear and moment diagrams are shown in Fig. 2.10b. It can be seen that the largest magnitude of shear stress is at $x = 2$ m and has a value of $|V|_{\max} = 8$ kN. The largest magnitude of bending moment is $|M|_{\max} = 8$ kN-m.

2.8.3 Singularity Functions

If the loading is simple, the method for obtaining shear and moment diagrams described in Sections 2.8.1 or 2.8.2 can be used. Often, however, this is not the situation. For more complex loading, methods such as **singularity functions** can be used. A singularity function in terms of a variable, x, is written as

$$f_n(x) = \langle x - a \rangle^n. \tag{2.6}$$

where n is any integer (positive or negative) including zero, and a is a reference location on a beam. Singularity functions are denoted by using angular brackets. The advantage of using a singularity function is that it permits writing an analytical expression directly for the transverse shear and moment over a range of discontinuities.

Table 2.2 shows six singularity and load intensity functions along with corresponding graphs and expressions. Note in particular the inverse ramp example. A unit step is constructed beginning at $x = a$, and the ramp beginning at $x = a$ is subtracted. To have the negative ramp discontinued at $x = a + b$, a positive ramp beginning at this point is constructed; the summation results in the desired loading.

Design Procedure 2.3: Singularity Functions

Some general rules relating to singularity functions are:

1. If $n > 0$ and the expression inside the angular brackets is positive (i.e., $x \geq a$), then $f_n(x) = (x - a)^n$. Note that the angular brackets to the right of the equal sign in Eq. (2.6) are now parentheses.

2. If $n > 0$ and the expression inside the angular brackets is negative (i.e., $x < a$), then $f_n(x) = 0$.

3. If $n < 0$, then $f_n(x) = 0$.

4. If $n = 0$, then $f_n(x) = 1$ when $x \geq a$ and $f_n(x) = 0$ when $x < a$.

5. If $n \geq 0$, the integration rule is

$$\int_{-\infty}^{x} \langle x - a \rangle^n = \frac{\langle x - a \rangle^{n+1}}{n+1}.$$

Note that this is the same as if there were parentheses instead of angular brackets.

6. If $n < 0$, the integration rule is

$$\int_{-\infty}^{x} \langle x - a \rangle^n dx = \langle x - a \rangle^{n+1}.$$

7. When $n \geq 1$, then

$$\frac{d}{dx}\langle x - a \rangle^n = n\langle x - a \rangle^{n-1}.$$

Design Procedure 2.4: Shear and Moment Diagrams by Singularity Functions

The procedure for drawing the shear and moment diagrams by making use of singularity functions is as follows:

1. Draw a free-body diagram with all the applied distributed and concentrated loads acting on the beam, and determine all support reactions. Resolve the forces into components acting perpendicular and parallel to the beam's axis.

2. Write an expression for the load intensity function $q(x)$ that describes all the singularities acting on the beam. Use Table 2.2 as a reference, and make sure to "turn off" singularity functions for distributed loads and the like that do not extend across the full length of the beam.

3. Integrate the negative load intensity function over the beam length to get the shear force. Integrate the negative shear force distribution over the beam length to get the moment, in accordance with Eqs. (2.4) and (2.5).

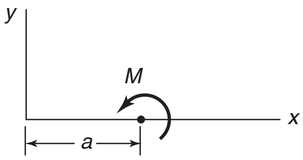
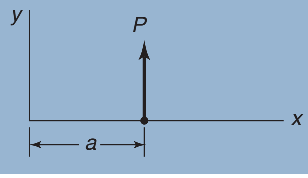
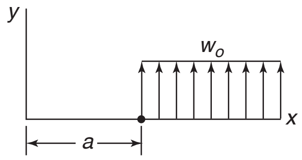
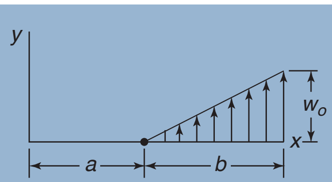
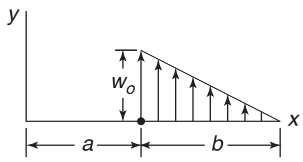
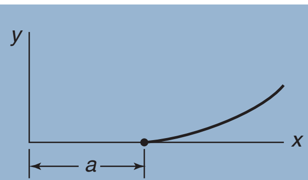
4. Draw shear and moment diagrams from the expressions developed.

Example 2.9: Shear and Moment Diagrams Using Singularity Functions

Given: The same conditions as in Example 2.7.

Find: Draw the shear and moment diagrams by using a singularity function for a concentrated force located midway on the beam.

Table 2.2: Singularity and load intensity functions with corresponding graphs and expressions.

Singularity	**Graph of $q(x)$**	**Expression for $q(x)$**
Concentrated moment		$q(x) = M\langle x-a\rangle^{-2}$
Concentrated force		$q(x) = P\langle x-a\rangle^{-1}$
Unit step		$q(x) = w_o\langle x-a\rangle^{0}$
Ramp		$q(x) = \frac{w_o}{b}\langle x-a\rangle^{1}$
Inverse ramp		$q(x) = w_o\langle x-a\rangle^{0} - \frac{w_o}{b}\langle x-a\rangle^{1}$
Parabolic shape		$q(x) = \langle x-a\rangle^{2}$

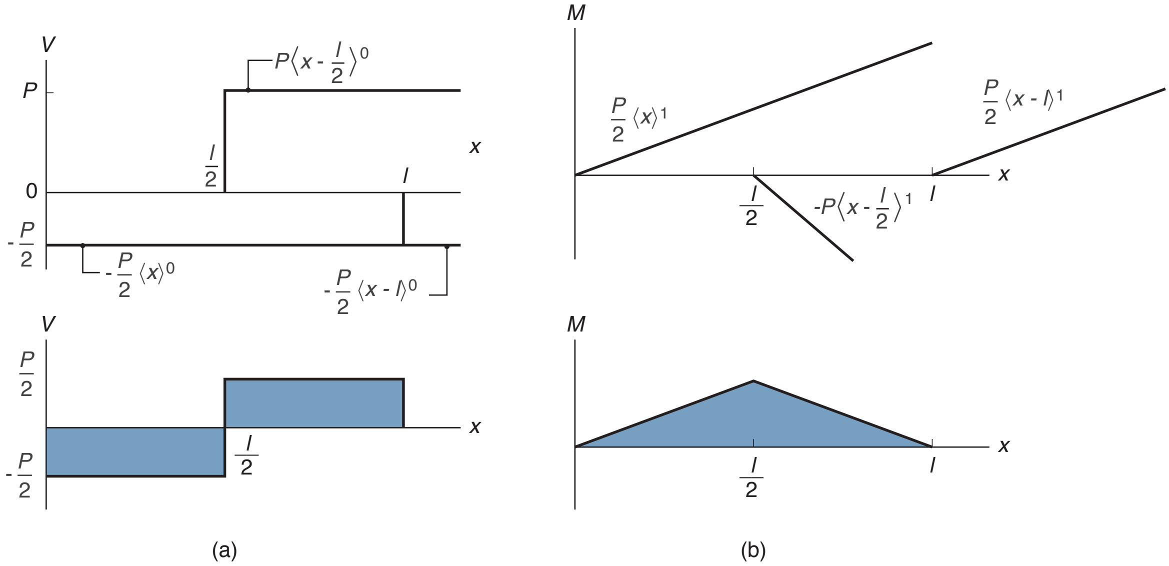


Figure 2.11: (a) Shear and (b) moment diagrams for Example 2.9.

Solution: The load intensity function for the simply supported beam shown in Fig. 2.9a is

$$q(x) = \frac{P}{2}\langle x\rangle^{-1} - P\left\langle x - \frac{l}{2}\right\rangle^{-1} + \frac{P}{2}\langle x - l\rangle^{-1}$$

The shear expression is

$$V(x) = -\int_{-\infty}^{x}\left[\frac{P}{2}\langle x\rangle^{-1} - P\left\langle x - \frac{l}{2}\right\rangle^{-1} + \frac{P}{2}\langle x - l\rangle^{-1}\right]dx$$

or

$$V(x) = -\frac{P}{2}\langle x\rangle^{0} + P\left\langle x - \frac{l}{2}\right\rangle^{0} - \frac{P}{2}\langle x - l\rangle^{0}$$

Figure 2.11a shows the resulting shear diagrams. The diagram at the top shows individual shear, and the diagram below shows the composite of these shear components. The moment expression is

$$M(x) = -\int_{-\infty}^{x}\left[-\frac{P}{2}\langle x\rangle^{0} + P\left\langle x - \frac{l}{2}\right\rangle^{0} - \frac{P}{2}\langle x - l\rangle^{0}\right]dx$$

or

$$M(x) = \frac{P}{2}\langle x\rangle^{1} - P\left\langle x - \frac{l}{2}\right\rangle^{1} + \frac{P}{2}\langle x - l\rangle^{1}$$

Figure 2.11b shows the moment diagrams. The diagram at the top shows individual moments; the diagram at the bottom is the composite moment diagram. The slope of M_2 is twice that of M_1 and M_3, which are equal. The resulting shear and moment diagrams are the same as those found in Example 2.7.

Example 2.10: Shear and Moment Expressions Using Singularity Functions

Given: A simply supported beam shown in Fig. 2.12a where $P_1 = 8$ kN, $P_2 = 5$ kN, $w_o = 4$ kN/m, and $l = 12$ m.

Find: The shear and moment expressions as well as their corresponding diagrams while using singularity functions.

Solution: The first task is to solve for the reactions at $x = 0$ and $x = l$. The force representation is shown in Fig. 2.12b. Note that w_o is defined as the load per unit length for the central part of the beam. In Fig. 2.12b it can be seen that the unit step w_o over a length of $l/2$ produces a resultant force of $w_o l/2$ and that the positive ramp over the length of $l/4$ can be represented by a resultant vector of

$$w_o\left(\frac{l}{4}\right)\left(\frac{1}{2}\right) \qquad \text{or} \qquad \frac{w_o l}{8}$$

Also, note that the resultant vector acts at

$$x = \left(\frac{2}{3}\right)\left(\frac{l}{4}\right) = \frac{l}{6}$$

From force equilibrium

$$0 = R_1 + P_1 + P_2 + R_2 - \frac{w_o l}{2} - \frac{w_o l}{8} \qquad (a)$$

$$R_1 + R_2 = -P_1 - P_2 + \frac{5 w_o l}{8} \qquad (b)$$

Making use of moment equilibrium and the moment of the triangular section load gives

$$\frac{(P_1 + 2P_2)l}{4} - \frac{w_o l^2}{4} - \frac{w_o l}{8}\left(\frac{l}{6}\right) + R_2 l = 0$$

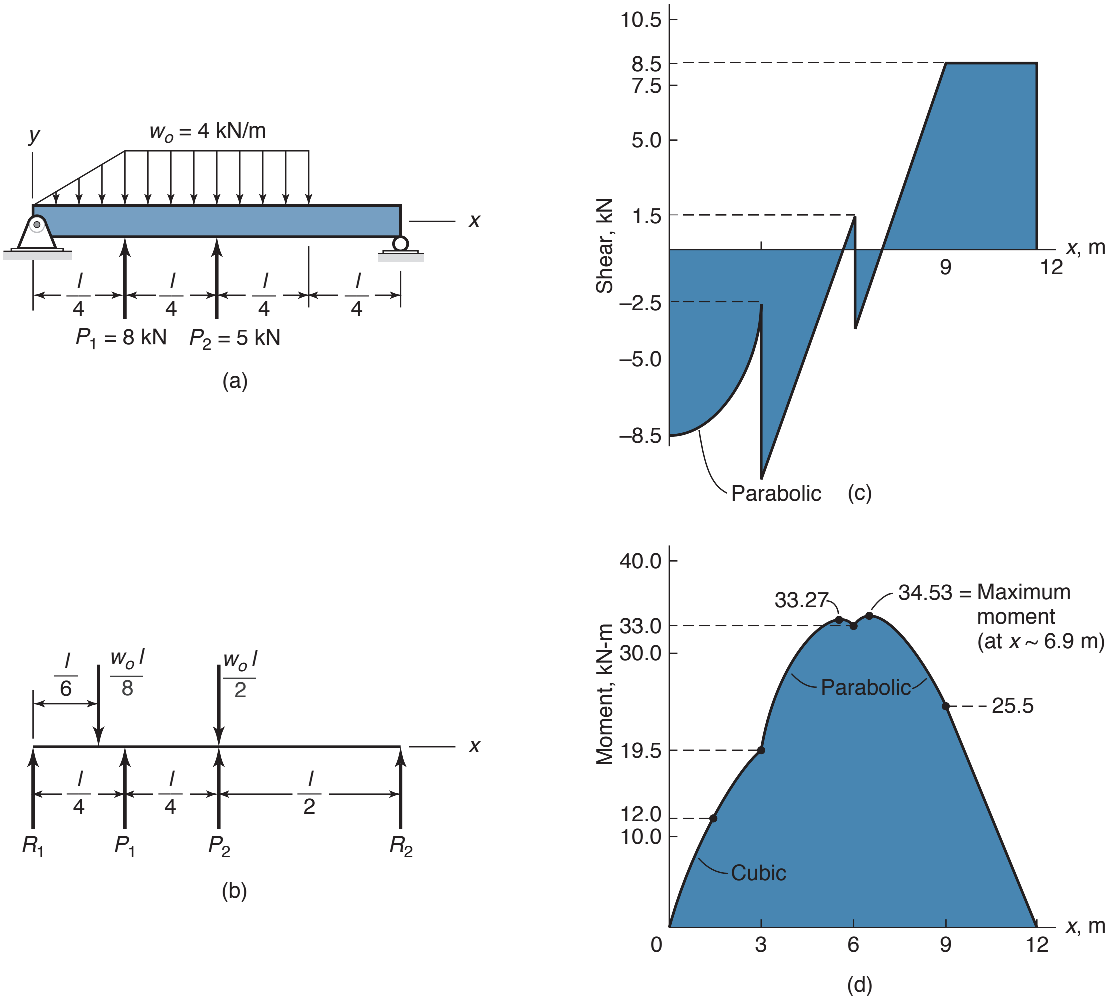


Figure 2.12: Simply supported beam examined in Example 2.10. (a) Forces acting on beam when P_1 = 8 kN, P_2 = 5 kN; w_o = 4 kN/m; l = 12 m; (b) free-body diagram showing resulting forces; (c) shear and (d) moment diagrams.

or

$$R_2 = \frac{13w_o l}{48} - \frac{P_1 + 2P_2}{4} \qquad (c)$$

Substituting Eq. (*c*) into Eq. (*b*) gives

$$R_1 = -\frac{3P_1}{4} - \frac{P_2}{2} + \frac{17w_o l}{48} \qquad (d)$$

Substituting the given values for P_1, P_2, w_o, and l gives

$$R_1 = 8.5 \text{ kN} \qquad \text{and} \qquad R_2 = 8.5 \text{ kN} \qquad (e)$$

The load intensity function can be written as

$$\begin{aligned} q(x) &= R_1\langle x\rangle^{-1} - \frac{w_o}{l/4}\langle x\rangle^1 + \frac{w_o}{l/4}\left\langle x - \frac{l}{4}\right\rangle^1 \\ &+P_1\left\langle x - \frac{l}{4}\right\rangle^{-1} + P_2\left\langle x - \frac{l}{2}\right\rangle^{-1} \\ &+w_o\left\langle x - \frac{3l}{4}\right\rangle^0 + R_2\langle x - l\rangle^{-1} \end{aligned}$$

Note that a unit step beginning at $l/4$ is created by initiating a ramp at $x = 0$ acting in the negative direction and summing it with another ramp starting at $x = l/4$ acting in the positive direction, since the slopes of the ramps are the same. The second and third terms on the right side of the load intensity function produce this effect. The sixth term on the right side of the equation turns off the unit step. Integrating the load intensity function gives the shear force as

$$\begin{aligned} V(x) &= -R_1\langle x\rangle^0 + \frac{2w_o}{l}\langle x\rangle^2 - \frac{2w_o}{l}\left\langle x - \frac{l}{4}\right\rangle^2 \\ &-P_1\left\langle x - \frac{l}{4}\right\rangle^0 - P_2\left\langle x - \frac{l}{2}\right\rangle^0 \\ &-w_o\left\langle x - \frac{3l}{4}\right\rangle^1 - R_2\langle x - l\rangle^0 \end{aligned}$$

Integrating the shear force gives the moment, and substituting the values for w_o and l gives

$$\begin{aligned} M(x) &= 8.5\langle x\rangle^1 + \frac{2}{9}\langle x\rangle^3 - \frac{2}{9}\langle x - 3\rangle^3 + 8\langle x - 3\rangle^1 \\ &+5\langle x - 6\rangle^1 + 2\langle x - 9\rangle^2 + 8.5\langle x - 12\rangle^1 \end{aligned}$$

The shear and moment diagrams are shown in Fig. 2.12c and d, respectively.

2.9 Stress

One of the fundamental problems in engineering is determining the effect of a loading environment on a part. This determination is an essential part of the design process; one cannot choose a dimension or a material without first understanding the intensity of force inside the component being analyzed. **Stress** is the term used to define the intensity and direction of the internal forces acting at a given point. Strength, on the other hand, is a property of a material and will be covered in later chapters.

For normal loading on a load-carrying member in which the external load is uniformly distributed over a cross-section, the magnitude of the average normal stress can be calculated from

$$\sigma_{\text{avg}} = \frac{\text{Average force}}{\text{Cross-sectional area}} = \frac{P}{A}. \tag{2.7}$$

Thus, the unit of stress is force per unit area. Consider a small area ΔA on the cross section, and let ΔP represent the internal forces acting on this small area. The average intensity of the internal forces transmitted by the area ΔA is obtained by dividing ΔP by ΔA. If the internal forces transmitted across the section are assumed to be continuously distributed, the area ΔA can be made increasingly smaller and will approach a point on the surface in the limit. The corresponding force ΔP will also become increasingly smaller. The stress at the point on the cross section to which ΔA converges is

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta P}{\Delta A} = \frac{dP}{dA}. \tag{2.8}$$

The stress at a point acting on a specific plane is a vector and thus has a magnitude and a direction. Its direction is the limiting direction ΔP as area ΔA approaches zero. Similarly, the shear stress can be defined in a specific plane. Thus, a stress must be defined with respect to a direction.

Example 2.11: Stress in Beam Supports

Given: As shown in Fig. 2.13a, a 3-m-long beam is supported at the left end by a 6-mm-diameter steel wire and at the right end by a 10-mm-diameter steel cylinder. The bar carries a mass $m_{a1} = 200$ kg and the bar's mass is $m_{a2} = 50$ kg.

Find: Determine the stresses in the wire and in the cylinder.

Solution: The wire and cylinder areas are $A_B = 28.27\text{ mm}^2$ and $A_C = 78.54\text{ mm}^2$. Figure 2.13b shows a free-body diagram of the forces acting on the bar. Moment equilibrium about point C gives

$$3R_{\text{B}} = 2(200)(9.81) + 1.5(50)(9.81) = 4660\text{ N}$$

or $R_{\text{B}} = 1553$ N. From force equilibrium,

$$R_{\text{B}} - m_{a1}g - m_{a2}g + R_{\text{C}} = 0$$

$$R_{\text{C}} = g\,(m_{a1} + m_{a2}) - R_{\text{B}} = 9.81(200 + 50) - 1553 = 900\text{ N}$$

The stresses at points B and C are, from Eq. (2.7),

$$\sigma_{\text{B}} = \frac{R_{\text{B}}}{A_{\text{B}}} = \frac{1553}{28.27} = 54.93\text{ N/mm}^2 = 54.93\text{ MPa}$$

$$\sigma_{\text{C}} = -\frac{R_{\text{C}}}{A_{\text{C}}} = -\frac{900}{78.54} = -11.46\text{ N/mm}^2 = -11.46\text{ MPa}$$

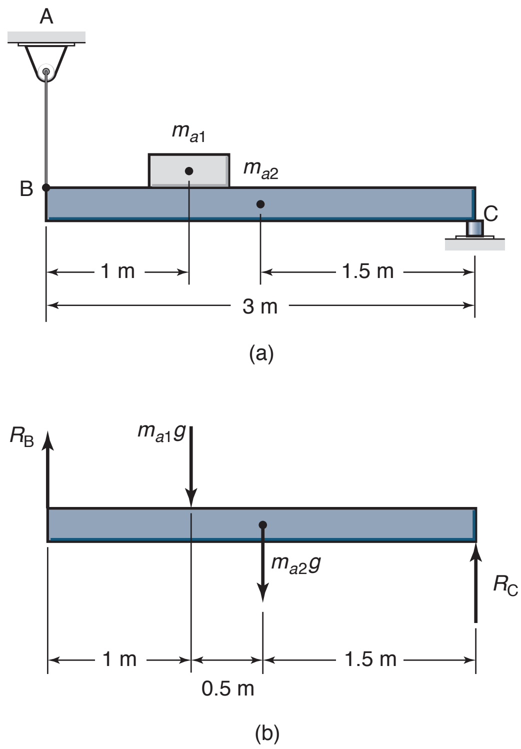


Figure 2.13: Figures used in Example 2.11. (a) Load assembly drawing; (b) free-body diagram.

2.10 Stress Element

Figure 2.14 shows a stress element with the origin of stress placed inside the element. Across each of the mutually perpendicular surfaces there are three stresses, yielding a total of nine stress components. Of the three stresses acting on a given surface, the normal stress is denoted by σ and the shear stress by τ. A normal stress will receive a subscript indicating the direction in which the stress acts (e.g., σ_x). A shear stress requires two subscripts, the first to indicate the plane of the stress and the second to indicate its direction (e.g., τ_{xy}). The **sign convention for normal stress** distinguishes positive for tension and negative for compression. A positive shear stress points in the positive direction of the coordinate axis denoted by the second subscript if it acts on a surface with an outward normal in the positive direction. The **sign convention for shear stress** is directly associated with the coordinate directions. If both the normal from the surface and the shear are in the positive direction or if both are in the negative direction, the shear stress is positive. Any other combinations of the normal and the direction of shear will produce a negative shear stress. The surface stresses of an element have the following relationships:

1. The normal and shear stress components acting on opposite sides of an element must be equal in magnitude but opposite in direction.

2. Moment equilibrium requires that the shear stresses be symmetric, implying that the subscripts can be reversed in order, or

$$\tau_{xy} = \tau_{yx}, \qquad \tau_{xz} = \tau_{zx}, \qquad \tau_{yz} = \tau_{zy}, \tag{2.9}$$

thus reducing the nine different stresses acting on the

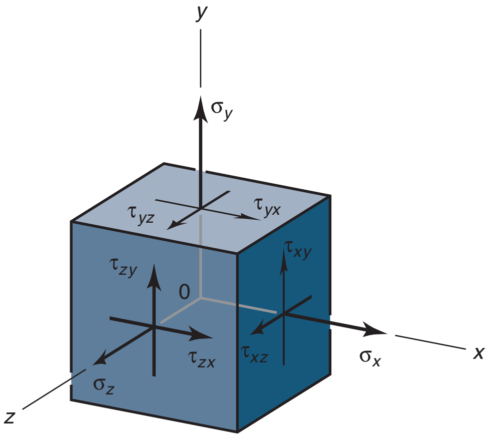


Figure 2.14: Stress element showing general state of three-dimensional stress with origin placed in center of element.

element to six: three normal stresses σ_x, σ_y, σ_z and three shear stresses τ_{xy}, τ_{yz}, τ_{xz}.

The general laws of stress transformation, given in Appendix B (Section B.1), enable the determination of stresses acting on any new orthogonal coordinate system.

Example 2.12: Stresses in Stress Element

Given: The stress element shown in Fig. 2.14 is put into a pressure vessel and pressurized to 10 MPa. An additional shear stress of 5 MPa acting on the bottom surface is directed in the positive x-direction.

Find: Are the stresses positive or negative?

Solution: The normal stress here is thus $\sigma = -10$ MPa. A positive pressure results in a negative normal stress by definition; since the element is loaded in compression by the pressure, it has a negative value. For the shear stress, τ_{zx}, a positive shear stress is directed in the positive coordinate direction when the normal to the surface is directed in the positive coordinate direction. A shear stress acting on a surface with the normal in the negative coordinate direction is positive when the stress is directed in the negative coordinate direction. The shear stress in this problem acts on a surface with the normal in the negative y-direction, but the stress is directed in the positive x-direction. Thus, the shear is negative.

2.11 Stress Tensor

In engineering practice, it is common to encounter scalar quantities, those that have numerical value. Vectors, such as force, have a magnitude as well as a direction. Stress requires six quantities for its definition; thus, stress is a *tensor*. From the stress element of Fig. 2.14 and Eq. (2.9) the stress tensor is

$$\mathbf{S} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix}, \tag{2.10}$$

which is a symmetrical tensor. A property of a symmetrical tensor is that there exists an equivalent tensor with an orthogonal set of axes 1, 2, and 3 (called *principal axes*) with respect to which the tensor elements are all zero except for those in the principal diagonal; thus,

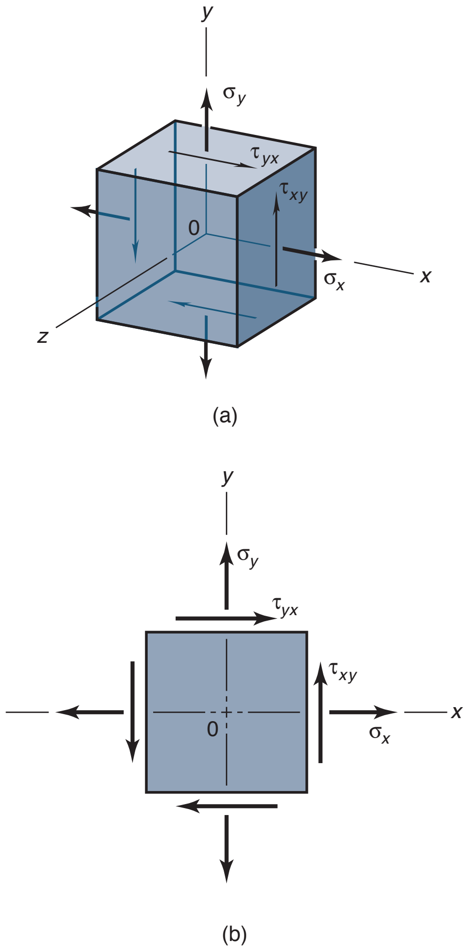


Figure 2.15: Stress element showing two-dimensional state of stress. (a) Three-dimensional view; (b) plane view.

$$\mathbf{S}' = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}, \tag{2.11}$$

where σ_1, σ_2, and σ_3 are principal stresses and will be discussed further below. Note that no shear stresses occur in Eq. (2.11).

2.12 Plane Stress

Many cases of stress analysis can be simplified to the case of **plane stress**, where the stresses all occur inside one plane. This is a common and valuable simplification, as the third direction can thus be neglected, and all stresses on the stress element act on two pairs of faces rather than three, as shown in Fig. 2.15. This two-dimensional stress state is sometimes called **biaxial** or **plane stress**.

In comparing the two views of the plane stress element shown in Fig. 2.15, note that all stresses shown in Fig. 2.15b act on surfaces perpendicular to the paper, with the paper being designated as either the x-y plane or the z plane. The stresses shown in Fig. 2.15 all have positive values in accordance with the conventions presented in Section 2.10.

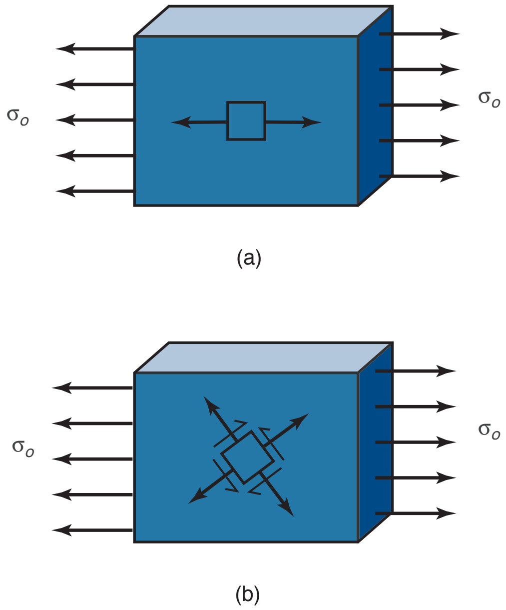


Figure 2.16: Illustration of equivalent stress states. (a) Stress element oriented in the direction of applied stress; (b) stress element oriented in different (arbitrary) direction.

The magnitude of stress depends greatly on the coordinate system orientation. For example, consider the stress element shown in Fig. 2.16a. When a uniform stress is applied to the element, the stress state is clearly $\sigma_x = \sigma_o$, $\sigma_y = 0$, and $\tau_{xy} = 0$. However, if the original orientation of the element were as shown in Fig. 2.16b, this would no longer be the case, and all stress components in the plane would be nonzero. A profound question can be raised at this point: How does the material know the difference between these stress states? The answer is that there is no difference between the stress states of Fig. 2.16, so that they are *equivalent*. Obviously, it is of great importance to be able to transform stresses from one orientation to another, and the resultant stress transformation equations will be of great use throughout the remainder of the text.

Consider if, instead of the stresses acting as shown in Fig. 2.15b, they act in an oblique plane at angle ϕ as shown in Fig. 2.17. The stresses σ_x, σ_y, and τ_{xy} can then be determined in terms of the stresses on an inclined surface whose normal stress makes an angle ϕ with the x-axis.

Note from Fig. 2.17 that if the area of the inclined surface is A (length of the surface times the thickness into the paper), the area of the horizontal side of the triangular element will be $A \sin\phi$, and the area of the vertical side, $A\cos\phi$. From force equilibrium

$$\begin{aligned}\sigma_\phi A &= \tau_{xy}\sin\phi A\cos\phi + \tau_{yx}\cos\phi A\sin\phi \\ &\quad +\sigma_x\cos\phi A\cos\phi + \sigma_y \sin\phi A \sin\phi.\end{aligned}$$

This reduces to

$$\sigma_\phi = 2\tau_{xy}\sin\phi\cos\phi + \sigma_x\cos^2\phi + \sigma_y\sin^2\phi. \qquad (2.12)$$

By using trigonometric identities for the double angle, Eq. (2.12) can be written as

$$\sigma_\phi = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi. \qquad (2.13)$$

Similarly, from force equilibrium, the shear stress in the oblique plane can be expressed as

$$\tau_\phi = \tau_{xy}\cos 2\phi - \frac{\sigma_x-\sigma_y}{2}\sin 2\phi. \qquad (2.14)$$

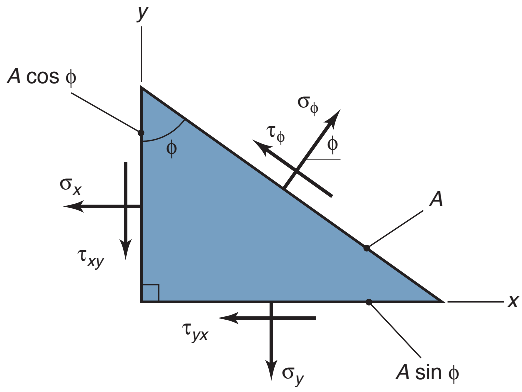


Figure 2.17: Stresses in an oblique plane at an angle ϕ.

Equations (2.13) and (2.14) have maximum and minimum values that are of particular interest in stress analysis. The angle ϕ_σ, which gives the extreme value of σ_ϕ, can be determined by differentiating σ_ϕ with respect to ϕ and setting the result equal to zero, giving

$$\frac{d\sigma_\phi}{d\phi} = -(\sigma_x-\sigma_y)\sin 2\phi_\sigma + 2\tau_{xy}\cos 2\phi_\sigma = 0$$

or

$$\tan 2\phi_\sigma = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}. \qquad (2.15)$$

where ϕ_σ is the angle where normal stress is extreme. Equation (2.15) has two roots, 180° apart, and for the double-angle nature of the left side of Eq. (2.15) this suggests roots of ϕ_σ being 90° apart. One of these roots corresponds to the maximum value of normal stress, the other to the minimum value.

Substituting Eq. (2.15) into Eqs. (2.13) and (2.14) gives the following after some algebraic manipulation:

$$\sigma_1, \sigma_2 = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\tau_{xy}^2 + \frac{(\sigma_x-\sigma_y)^2}{4}}, \qquad (2.16)$$

$$\tau_{\phi_\sigma} = 0. \qquad (2.17)$$

At this stress element orientation, where the normal stresses are extreme, the shear stress is zero. The axes that define this orientation are called the **principal axes**, and the normal stresses from Eq. (2.16) are called the **principal normal stresses**. Principal stresses are given numerical subscripts to differentiate them from stresses at any other orientation. A common convention is to order the principal stresses according to

$$\sigma_1 \geq \sigma_2 \geq \sigma_3. \qquad (2.18)$$

In plane stress, one of the principal stresses is always zero.

Another orientation of interest is the one where the shear stress takes an extreme value. Differentiating Eq. (2.14) with respect to ϕ and solving for τ gives the orientation ϕ_τ, with resulting extreme shear stress of

$$\tau_{\max}, \tau_{\min} = \tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (2.19)$$

and

$$\sigma_{\phi_\tau} = \frac{\sigma_x+\sigma_y}{2}. \qquad (2.20)$$

The shear stresses from Eq. (2.19) are called **principal shear stresses**. Thus, on the stress element oriented to achieve a maximum shear stress, the normal stresses on the two faces are equal. Also, it can be shown that

$$|\phi_\tau - \phi_\sigma| = \frac{\pi}{4}. \tag{2.21}$$

In summary, for a plane stress situation where σ_x, σ_y, and τ_{xy} are known, the normal and shear stresses σ_ϕ and τ_ϕ can be determined for any oblique plane at angle ϕ from Eqs. (2.13) and (2.14). Also, the principal normal and shear stresses σ_1, σ_2, τ_1, and τ_2 can be determined from Eqs. (2.16) and (2.19).

If the principal normal stresses σ_1 and σ_2 are known, the normal and shear stresses at any oblique plane at angle ϕ can be determined from the following equations:

$$\sigma_\phi = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\phi \tag{2.22}$$

$$\tau_\phi = \frac{\sigma_1 - \sigma_2}{2}\sin 2\phi. \tag{2.23}$$

In Eq. (2.23) a second subscript is not needed because τ_ϕ represents a shear stress acting on any oblique plane at angle ϕ as shown in Fig. 2.17.

Example 2.13: Stress Transformation

Given: A thin, square steel plate is oriented in the x- and y-directions. A tensile stress, σ, acts on the four sides. Thus, $\sigma_x = \sigma_y = \sigma$.

Find: The normal and shear stresses acting on the diagonal of the plate.

Solution: From Eq. (2.13),

$$\sigma_\phi = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi$$

Thus,

$$\sigma_{45^\circ} = \frac{\sigma + \sigma}{2} + \frac{\sigma - \sigma}{2}\cos 90^\circ + \tau_{xy}\sin 90^\circ = \sigma$$

Similarly, from Eq. (2.14)

$$\tau_\phi = \tau_{xy}\cos 2\phi - \frac{\sigma_x - \sigma_y}{2}\sin 2\phi$$

Therefore,

$$\tau_{45^\circ} = \tau_{xy}\cos 90^\circ - \frac{\sigma - \sigma}{2}\sin 90^\circ = 0$$

2.13 Mohr's Circle

Mohr's circle for a triaxial state of stress at a point was first constructed in 1914 by a German engineer, Otto Mohr, who noted that Eqs. (2.13) and (2.14) define a circle in a σ-τ plane. This circle is used extensively as a convenient method of graphically visualizing the state of stress acting in different planes passing through a given point. The approach used in this text is first to apply Mohr's circle to a two-dimensional stress state; a three-dimensional stress state is discussed in Section 2.14. Indeed, Mohr's circle is most useful for stress visualization in plane stress situations.

Figure 2.18 shows a typical Mohr's circle diagram. A number of observations can be made:

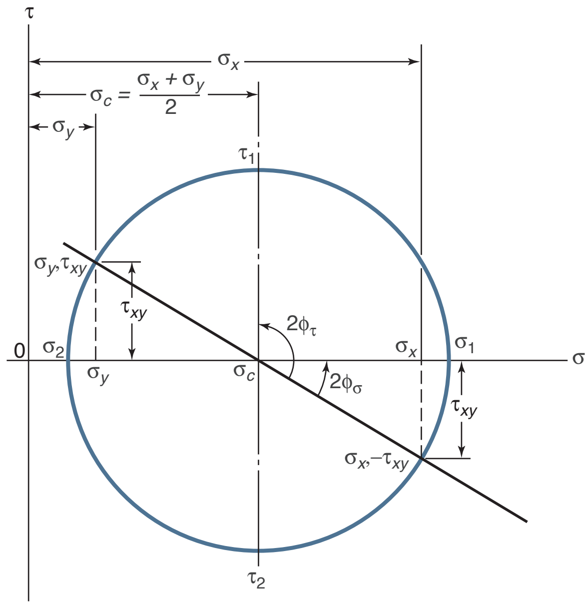


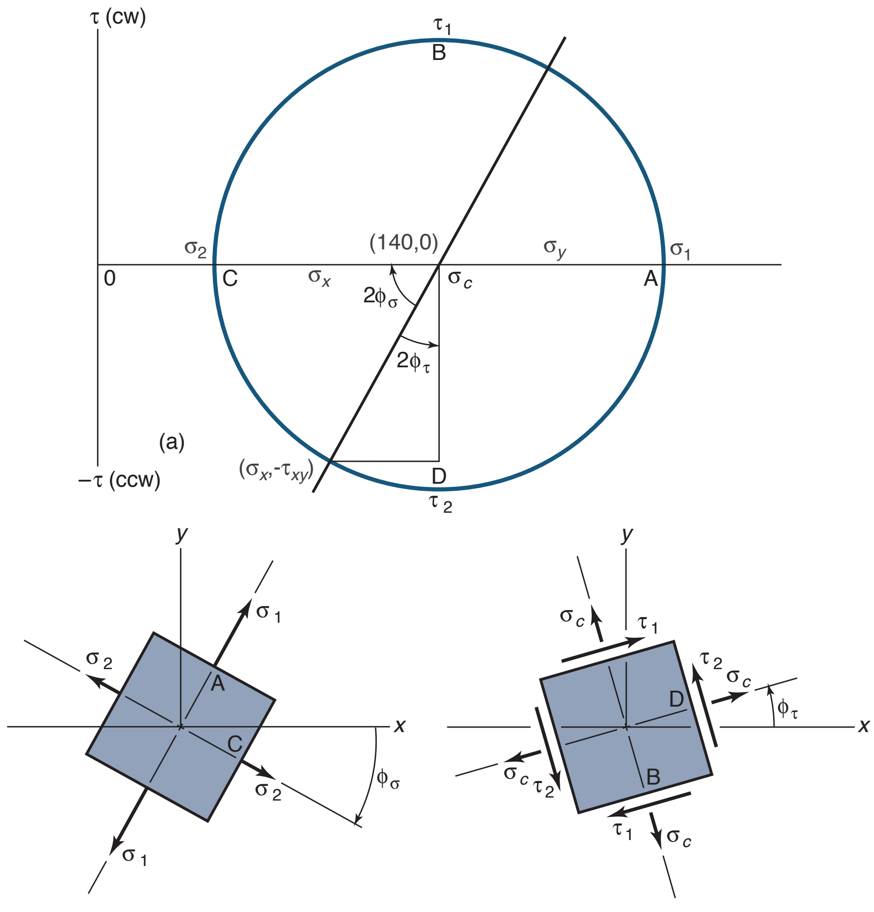
Figure 2.18: Mohr's circle diagram of Eqs. (2.13) and (2.14).

1. Normal stresses are plotted along the abscissa (x-axis), and shear stresses are plotted on the ordinate (y-axis).
2. The circle defines all stress states that are equivalent.
3. The biaxial stress state for any direction can be scaled directly from the circle.
4. The principal normal stresses (i.e., the extreme values of normal stress) are at the locations where the circle intercepts the x-axis.
5. The maximum shear stress equals the radius of the circle.
6. A rotation from a reference stress state in the real plane of ϕ corresponds to a rotation of 2ϕ from the reference points in the Mohr's circle plane.

Design Procedure 2.5: Mohr's Circle

The steps in constructing and using Mohr's circle in two dimensions are as follows:

1. Calculate the plane stress state for any x-y coordinate system so that σ_x, σ_y, and τ_{xy} are known.
2. The center of the Mohr's circle can be placed at
$$\left(\frac{\sigma_x + \sigma_y}{2}, 0\right). \tag{2.24}$$
3. Two points diametrically opposite to each other on the circle correspond to the points $(\sigma_x, -\tau_{xy})$ and (σ_y, τ_{xy}). Using the center and either point allows one to draw the circle.
4. The radius of the circle can be calculated from stress transformation equations or through geometry by using the center and one point on the circle. For example, the radius is the distance between points $(\sigma_x, -\tau_{xy})$ and the center, which directly leads to



(b) (c)

Figure 2.19: Results from Example 2.14. (a) Mohr's circle diagram; (b) stress element for principal normal stress shown in x-y coordinates; (c) stress element for principal shear stresses shown in x-y coordinates.

$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}. \tag{2.25}$$

5. The principal stresses have the values $\sigma_{1,2}$ = center $\pm$ radius.

6. The maximum shear stress equals the radius.

7. The principal axes can be found by calculating the angle between the x-axis in the Mohr's circle plane and the point $(\sigma_x, -\tau_{xy})$. The principal axes in the real plane are rotated one-half this angle in the same direction relative to the x-axis in the real plane.

8. The stresses in an orientation rotated ϕ from the x-axis in the real plane can be read by traversing an arc of 2ϕ in the same direction on the Mohr's circle from the reference points $(\sigma_x, -\tau_{xy})$ and (σ_y, τ_{xy}). The new points on the circle correspond to the new stresses $(\sigma_{x'}, -\tau_{x'y'})$ and $(\sigma_{y'}, \tau_{x'y'})$, respectively.

Example 2.14: Mohr's Circle

Given: The plane stresses σ_x = 90 MPa, σ_y = 190 MPa, and τ_{xy} = 80 MPa.

Find: Draw the Mohr's circle and find the principal normal and shear stresses in the x-y plane. Determine the stress state when the axes are rotated 15° counterclockwise.

Solution: This solution will demonstrate the eight-step approach given in Design Procedure 2.5, with the first step already done in the problem statement. Step 2 advises to calculate the center of the circle and place it at $(\sigma_c, 0)$, where

$$\sigma_c = \frac{\sigma_x + \sigma_y}{2} = \frac{(90 + 190)}{2} = 140 \text{ MPa}.$$

According to Step 3, either point $(\sigma_x, -\tau_{xy})$ or (σ_y, τ_{xy}) can be used to draw the circle. This has been done with the point $(\sigma_x, -\tau_{xy})$ = (90 MPa, −80 MPa) to draw the circle as shown in Fig. 2.19. From Step 4 and from the triangle defined by the x-axis and the point $(\sigma_x, -\tau_{xy})$, the radius can calculated as

$$r = \sqrt{(90 - 140)^2 + (-80)^2} = 94.3 \text{ MPa}.$$

From Step 5 the principal stresses have the values $\sigma_{1,2}$ = 140 ± 94.3, or σ_1 = 234.3 MPa and σ_2 = 45.7 MPa. From Step 6, the maximum shear stress equals the radius, or $\tau_{\max}$ = 94.3 MPa. The principal stress orientation can be determined, if desired, from trigonometry. In the Mohr's circle plane (Fig. 2.19a), the point $(\sigma_x, -\tau_{xy})$ makes an angle of $2\phi = \tan^{-1}(80/50) = 58^\circ$ with the x-axis. To reach the point on the x-axis, an arc of this angle is needed in the clockwise direction on the Mohr's circle. Thus, the principal plane is $\phi = 29^\circ$ clockwise from the x-axis. Finally, the stresses at an angle of 15° can be obtained From Eqs. (2.13) and (2.14) using

$$\sigma_{y'} = 140 + (94.3)\cos 28^\circ = 223.2 \text{ MPa},$$

$$\tau_{x'y'} = (94.3)\sin 28^\circ = 44.3 \text{ MPa}.$$

Figure 2.19b shows an element of the principal normal stresses as well as the appropriate value of ϕ_σ. Figure 2.19c shows an element of the principal shear stresses as well as the appropriate value of ϕ_τ. The stress at the center of the Mohr's circle diagram is also represented in Fig. 2.19c along with the principal shear stresses.

2.14 Three-Dimensional Stresses

The general laws of strain transformation, given in Appendix B, enable the determination of strains acting on any orthogonal coordinate system. Considering the general situation shown in Fig. 2.14, the stress element has six faces, implying that there are three principal directions and three principal stresses σ_1, σ_2, and σ_3. Six stress components (σ_x, σ_y, σ_z, τ_{xy}, τ_{xz}, and τ_{yz}) are required to specify a general state of stress in three dimensions, in contrast to the three stress components (σ_x, σ_y, and τ_{xy}) that were used for two-dimensional (plane or biaxial) stress. Determining the principal stresses for a three-dimensional situation is much more difficult. The process involves finding the three roots to the cubic equation

$$\begin{aligned} 0 \quad = \quad & \sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\,\sigma^2 + \\ & \left(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2\right)\sigma \\ & - \left(\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2\right). \end{aligned} \tag{2.26}$$

In most design situations many of the stress components are zero, greatly simplifying evaluation of this equation.

If the principal orientation of an element associated with a three-dimensional stress state, as well as the principal stresses, is known, this condition is called **triaxial stress**. Figure 2.20 shows a Mohr's circle for a triaxial stress state. It consists of three circles, two externally tangent and inscribed within the third circle. The principal shear stresses shown in Fig. 2.20 are determined from

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2}, \quad \tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2}, \quad \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2}. \tag{2.27}$$

The principal normal stresses must be ordered as described in Eq. (2.18). From Eq. (2.27), the maximum principal shear stress is $\tau_{1/3}$.

A Mohr's circle can be generated for triaxial stress states, but this is often unnecessary. In most circumstances it is not necessary to know the orientations of the principal stresses; it is sufficient to know their values. Thus, Eq. (2.26) is usually all that is needed.

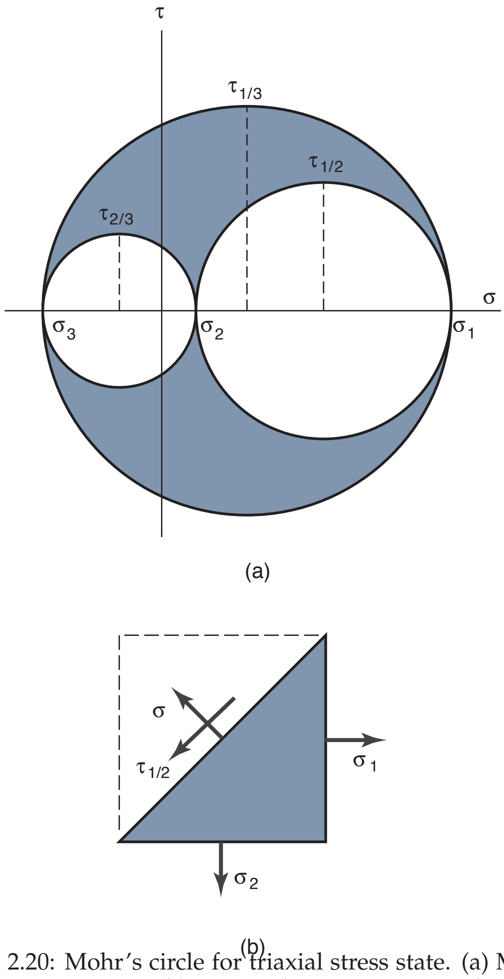


Figure 2.20: Mohr's circle for triaxial stress state. (a) Mohr's circle representation; (b) principal stresses on two planes.

Example 2.15: Three-Dimensional Mohr's Circle

Given: Assume that the principal normal stresses obtained in Example 2.14 are the same for triaxial consideration with $\sigma_3 = 0$. That is, $\sigma_1 = 234.3$ MPa, $\sigma_2 = 45.7$ MPa, and $\sigma_3 = 0$.

Find:

(*a*) Determine the principal shear stresses for a triaxial stress state and draw the appropriate Mohr's circle diagram.

(*b*) If the shear stress τ_{xy} is changed from 80 to 160 MPa, show how the Mohr's circles for the biaxial and triaxial stress states change.

Solution:

(*a*) From Eq. (2.27), the principal shear stresses in a triaxial stress state are

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} = \frac{234.3 - 45.7}{2} = 94.3 \text{ MPa},$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} = \frac{45.7}{2} = 22.85 \text{ MPa},$$

$$\tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2} = \frac{(23.43 - 0)}{2} = 117.15 \text{ MPa}.$$

Figure 2.21a shows the appropriate Mohr's circle diagram for the triaxial stress state.

(*b*) If the shear stress in Example 2.14 is doubled (τ_{xy} = 160 MPa instead of 80 MPa), Eq. (2.19) gives

$$\begin{aligned}\tau_1, \tau_2 &= \pm\sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} \\ &= \pm\sqrt{160^2 + \left(\frac{90-190}{2}\right)^2}\ \text{MPa} \\ &= \pm 167.6\ \text{MPa}.\end{aligned}$$

The principal normal stresses for the biaxial stress state are

$$\sigma_1 = \sigma_c + \tau_1 = 140 + 167.6 = 307.6\ \text{MPa},$$

$$\sigma_2 = \sigma_c - \tau_2 = 140 - 167.6 = -27.6\ \text{MPa}.$$

Figure 2.21b shows the resultant Mohr's circle diagram for the biaxial stress state. In a triaxial stress state that is ordered σ_1 = 307.6 MPa, σ_2 = 0, and σ_3 = -27.6 MPa, from Eq. (2.27) the principal shear stresses can be written as

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} = \frac{307.6 - 0}{2}\ \text{MPa} = 153.8\ \text{MPa},$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} = \frac{0 + 27.6}{2}\ \text{MPa} = 13.8\ \text{MPa},$$

$$\tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2} = \frac{307.6 + 27.6)}{2}\ \text{MPa} = 167.6\ \text{MPa}.$$

Figure 2.21c shows the Mohr's circle diagram for the triaxial stress state. From Fig. 2.21b and c, the maximum shear stress in the biaxial stress state, τ_1, is equivalent to $\tau_{1/3}$, the maximum shear stress in the triaxial stress state. However, comparing Figs. 2.19a and 2.21a shows that the maximum shear stress in the plane (or biaxial) stress state is not equal to that in the triaxial stress state. Furthermore, the maximum triaxial stress is larger than the maximum biaxial stress. Thus, if σ_1 and σ_2 have the same sign in the biaxial stress state, the triaxial maximum stress $\tau_{1/3}$ must be used for design considerations. However, if σ_1 and σ_2 have opposite signs in the biaxial stress state, the maximum biaxial and triaxial shear stresses will be the same and either one can be used in the analysis.

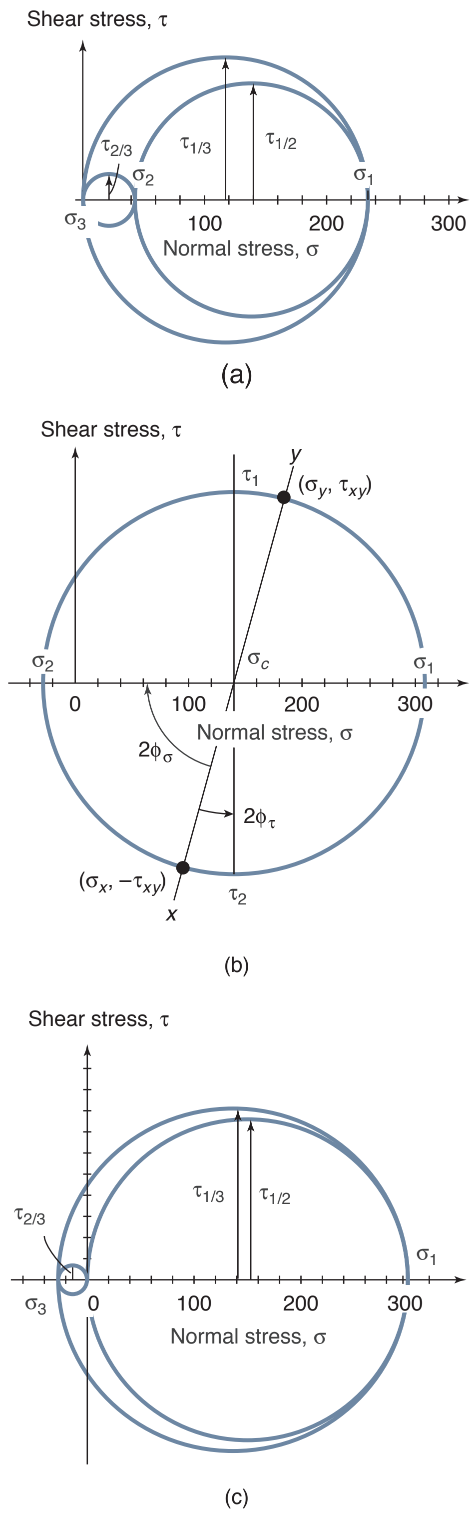


Figure 2.21: Mohr's circle diagrams for Example 2.15. (a) Triaxial stress state when σ_1 = 234.3 MPa, σ_2 = 457 MPa and σ_3 = 0; (b) biaxial stress state when σ_1 = 307.6 MPa and σ_2 = −27.6 MPa; (c) triaxial stress state when σ_1 = 307.6 MPa, σ_2 = 0, and σ_3 = −27.6 MPa.

2.15 Octahedral Stresses

Sometimes it is advantageous to represent the stresses on an octahedral stress element rather than on a conventional cubic element of principal stresses. Figure 2.22 shows the orientation of one of the eight octahedral planes that are associated with a given stress state. Each **octahedral plane** cuts across a corner of a principal element, so that the eight planes together form an octahedron (Fig. 2.22). The following characteristics of the stresses on a octahedral plane should be noted:

1. Identical normal stresses act on all eight planes. Thus, the normal stresses tend to compress or enlarge the octahedron but do not distort it.

2. Identical shear stresses act on all eight planes. Thus, the shear stresses tend to distort the octahedron without changing its volume.

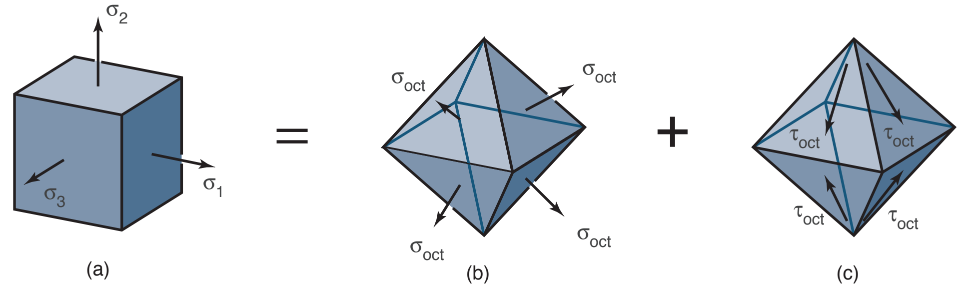


Figure 2.22: Stresses acting on octahedral planes. (a) General state of stress; (b) normal stress; (c) octahedral shear stress.

The fact that the normal and shear stresses are the same for the eight planes is a powerful tool in failure analysis.

The normal octahedral stress can be expressed in terms of the principal normal stresses, or the stresses in the x, y, z coordinates, as

$$\sigma_{\text{oct}} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3}, \tag{2.28}$$

$$\begin{aligned} \tau_{\text{oct}} &= \frac{1}{3}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2} \\ &= \frac{2}{3}\left[\tau_{1/2}^2 + \tau_{2/3}^2 + \tau_{1/3}^2\right]^{1/2}. \end{aligned} \tag{2.29}$$

In terms of octahedral normal stresses,

$$\begin{aligned} 9\tau_{\text{oct}}^2 &= (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 \\ &\quad +6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right). \end{aligned} \tag{2.30}$$

Example 2.16: Octahedral Stresses

Given: Consider the stress state from Example 2.15, where $\sigma_1 = 234.5$ MPa, $\sigma_2 = 45.7$ MPa, and $\sigma_3 = 0$.

Find: Determine the octahedral stresses.

Solution: The normal and octahedral stress can be written as

$$\sigma_{\text{oct}} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{(234.3 + 45.7 + 0)}{3} = 93.3 \text{ MPa}.$$

The shear octahedral stress from Eq. (2.29) can be written as

$$\begin{aligned} \tau_{\text{oct}} &= \frac{2}{3}\left[\tau_{1/2}^2 + \tau_{2/3}^2 + \tau_{1/3}^2\right]^{1/2} \\ &= \frac{2}{3}\left[94.3^2 + 22.9^2 + 117.2^2\right]^{1/2} \\ &= 101.4 \text{ MPa}. \end{aligned}$$

2.16 Strain

Strain is defined as the displacement per length produced in a solid as the result of stress. In designing a machine element, not only must the design be adequate when considering the stress relative to the strength, but it must also be ensured that the displacements and/or deformations are not excessive and are within design constraints. Depending on the application, these deformations may be either highly visible or practically unnoticeable.

Just as the direction and intensity of the stress at any given point are important with respect to a specific plane passing through that point, the same is true for strain. Thus, just as for stress, strain is a tensor. Also, just as there are normal and shear stresses, so too there are normal and shear strains. **Normal strain**, designated by the symbol ϵ, is used to describe a measure of the elongation or contraction of a linear segment of an element in which stress is applied. The average normal strain is

$$\epsilon_{\text{avg}} = \frac{\delta}{l} = \frac{\text{Average elongation}}{\text{Original length}}. \tag{2.31}$$

Note that strain is dimensionless. Furthermore, the strain at a point is

$$\epsilon = \lim_{\Delta l \to 0} \frac{\Delta\delta_{\text{avg}}}{\Delta l} = \frac{d\delta_{\text{avg}}}{dl}. \tag{2.32}$$

Figure 2.23 shows the strain on a cubic element subjected to uniform tension in the x-direction. The element elongates in the x-direction while simultaneously contracting in the y- and z-directions, a phenomenon known as the **Poisson effect**, and discussed further in Section 3.5.2. From Eq. (2.32), the normal strain components can be written as

$$\epsilon_x = \lim_{x \to 0} \frac{\delta_x}{x}, \quad \epsilon_y = \lim_{y \to 0} \frac{\delta_y}{y}, \quad \epsilon_z = \lim_{z \to 0} \frac{\delta_z}{z}. \tag{2.33}$$

Figure 2.24 shows the shear strain of a cubic element due to shear stress in both a three-dimensional view and a two-dimensional (or plane) view. The **shear strain**, designated by γ, is used to measure angular distortion (the change in angle between two lines that are orthogonal in the undeformed state). The shear strain as shown in Fig. 2.24 is defined as

$$\gamma_{yx} = \lim_{y \to 0} \frac{\delta_x}{y} = \tan\theta_{yx} \approx \theta_{yx}, \tag{2.34}$$

where θ_{yx} is the angle representing deviation from initial right angle. Note that a small angle approximation has been used for θ_{yx} in Eq. (2.34).

The subscripts used to define the shear strains are like those used to define the shear stresses in Section 2.10. The first subscript designates the coordinate direction perpendicular to the plane in which the strain acts, and the second subscript designates the coordinate direction in which the strain acts. For example, γ_{yx} is the strain resulting from taking adjacent planes perpendicular to the y-axis and displacing them relative to each other in the x-direction. The sign conventions for strain follow directly from those developed for stress. A positive stress produces a positive strain and a negative stress produces a negative strain. The shear strain shown in Fig. 2.24 and described in Eq. (2.34) is positive. The strain

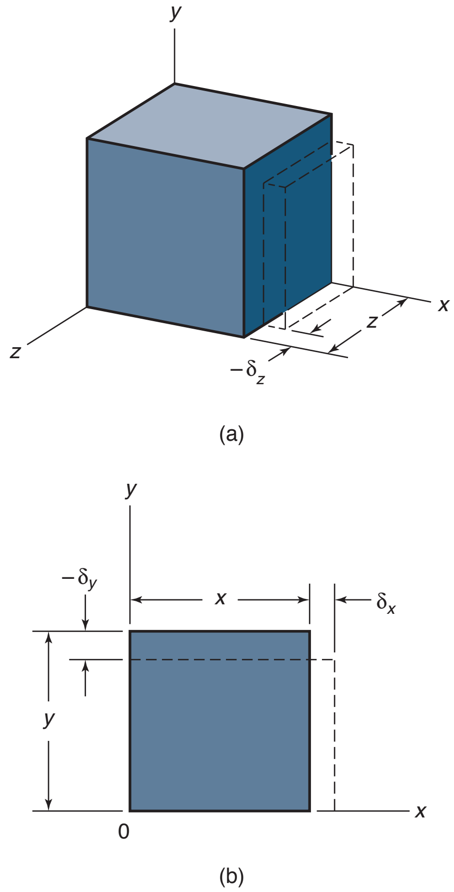


Figure 2.23: Normal strain of cubic element subjected to uniform tension in x-direction. (a) Three-dimensional view; (b) two-dimensional (or plane) view.

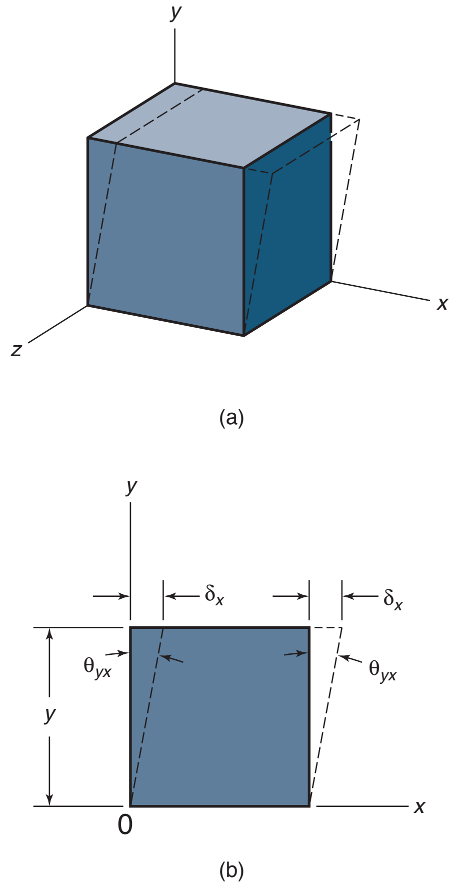


Figure 2.24: Shear strain of cubic element subjected to shear stress. (a) Three-dimensional view; (b) two-dimensional (or plane) view.

of the cubic element thus contains three normal strains and six shear strains, just as found for stresses. Similarly, symmetry reduces the number of shear strain elements from six to three.

Example 2.17: Calculation of Strain

Given: A 300-mm-long circular aluminum bar with a 50-mm diameter is subjected to a 125-kN axial load. The axial elongation is 0.2768 mm and the diameter is decreased by 0.01522 mm.

Find: The transverse and axial strains in the bar.

Solution: The axial strain is

$$\epsilon_a = \frac{\delta}{l} = \frac{0.2768}{300} = 9.227 \times 10^{-4}$$

The transverse strain is

$$\epsilon_t = \frac{\delta_t}{d} = \frac{-0.01522}{50} = -3.044 \times 10^{-4}$$

The sign for the transverse strain is negative because the diameter decreased after the bar was loaded. The axial strain is positive because the axial length increased after loading. Note that strain has no dimension, although it is commonly reported in units of m/m or μm/m.

2.17 Strain Tensor

For strains within the elastic range, the equations relating normal and shear strains with the orientation of the cutting plane are analogous to the corresponding equations for stress given in Eq. (2.10). Thus, the state of strain can be written as a tensor:

$$\mathbf{T} = \begin{pmatrix} \epsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \epsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \epsilon_z \end{pmatrix} \qquad (2.35)$$

In comparing Eq. (2.35) with Eq. (2.10) note that ϵ_x, ϵ_y, and ϵ_z are analogous to σ_x, σ_y, and σ_z, respectively, but it is half of the shear strain, $\gamma_{xy}/2$, $\gamma_{yz}/2$, $\gamma_{zx}/2$ that is analogous to τ_{xy}, τ_{yz}, and τ_{zx}, respectively.

2.18 Plane Strain

Instead of the six strains for the complete strain tensor, in plane strain the components ϵ_z, γ_{xz}, and γ_{yz} are zero. Thus, only two normal strain components, ϵ_x and ϵ_y, and one shear strain component, γ_{xy}, are considered. Figure 2.25 shows the deformation of an element caused by each of the three strains considered in plane strain. The normal strain components ϵ_x and ϵ_y, shown in Fig. 2.25a and b, are produced by changes in element length in the x- and y-directions, respectively. The shear strain γ_{xy}, shown in Fig. 2.25c, is produced by the relative rotation of two adjacent sides of the element. Figure 2.25c also helps to explain the physical significance that τ is analogous to $\gamma/2$ rather than to γ. Each side of an element changes in slope by an angle $\gamma/2$ when subjected to pure shear.

The following sign convention is to be used for strains:

1. Normal strains ϵ_x and ϵ_y are positive if they cause elongation along the x- and y-axes, respectively. In Fig. 2.25a and b, ϵ_x and ϵ_y are positive.

2. Shear strain γ_{xy} is positive when the interior angle of a strain element (A0B in Fig. 2.25c) becomes smaller than $90°$.

The principal strains, planes, and directions are directly analogous to those found earlier for principal stresses. The principal normal strains in the x-y plane, the maximum shear strain in the x-y plane, and the orientation of the principal axes relative to the x- and y-axes are

$$\epsilon_1, \epsilon_2 = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{1}{2}\gamma_{xy}\right)^2 + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2}, \tag{2.36}$$

$$\gamma_{\max} = \pm 2\sqrt{\left(\frac{1}{2}\gamma_{xy}\right)^2 + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2}, \tag{2.37}$$

$$2\phi = \tan^{-1}\left(\frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}\right). \tag{2.38}$$

From here there are two important problem classes:

1. If the principal strains are known and it is desired to find the strains acting at a plane oriented at angle ϕ from the principal direction, the equations are

$$\epsilon_\phi = \frac{\epsilon_1 + \epsilon_2}{2} + \frac{\epsilon_1 - \epsilon_2}{2}\cos 2\phi, \tag{2.39}$$

$$\gamma_\phi = (\epsilon_1 - \epsilon_2)\sin 2\phi. \tag{2.40}$$

   In Eq. (2.40), γ_ϕ represents a shear strain acting on the ϕ plane and directed $90°$ from the ϕ-axis. Just as for stress, the second subscript is omitted for convenience and no ambiguity results. A Mohr's circle diagram can also be used to represent the state of strain.

2. The second problem of interest is the case where a normal strain component has been measured in three different but specified directions and it is desired to obtain the strains ϵ_x, ϵ_y, and γ_{xy} from these readings. In this case the equation

$$\epsilon_\theta = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta \tag{2.41}$$

   is of great assistance. Here, ϵ_θ is the measured strain in the direction rotated θ counterclockwise from the x-axis, and ϵ_x, ϵ_y, and γ_{xy} are the desired strains. Thus, measuring a strain in three different directions gives three equations for the three unknown strains and is sufficient for their quantification. Strain gages are often provided in groups of three, called *rosettes*, for such purposes.

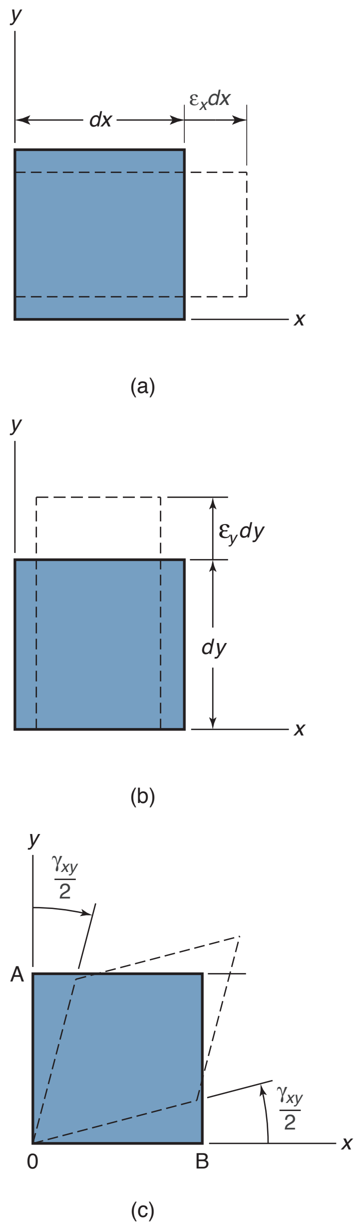


Figure 2.25: Graphical depiction of plane strain element. (a) Normal strain ϵ_x; (b) normal strain ϵ_y; and (c) shear strain γ_{xy}.

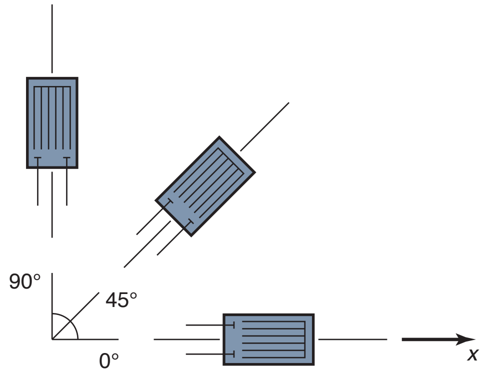


Figure 2.26: Strain gage rosette used in Example 2.18.

Example 2.18: Strain Gage Rosette

Given: A $0^\circ - 45^\circ - 90^\circ$ strain gage rosette as shown in Fig. 2.25 is attached to a structure with the 0° gage placed along a reference (x) axis. Upon loading, the strain in the 0° direction reads +50 μm/m, the strain gage in the 45° direction reads -27 μm/m, and the gage in the 90° direction reads 0.

Find: The strains ϵ_x, ϵ_y, and γ_{xy}.

Solution: Equation (2.41) can be applied three times to obtain three equations:

$$\epsilon_{0^\circ} = 50\ \mu\text{m/m} = \epsilon_x \cos^2(0^\circ) + \epsilon_y \sin^2(0^\circ) + \gamma_{xy} \sin 0^\circ \cos 0^\circ$$

or $\epsilon_x = 50\ \mu$m/m,

$$\epsilon_{90^\circ} = 0 = \epsilon_x \cos^2(90^\circ) + \epsilon_y \sin^2(90^\circ) + \gamma_{xy} \sin 90^\circ \cos 90^\circ$$

or $\epsilon_y = 0$, and

$$\epsilon_{45^\circ} = \epsilon_x \cos^2(45^\circ) + \epsilon_y \sin^2(45^\circ) + \gamma_{xy} \sin 45^\circ \cos 45^\circ$$

or $\gamma_{xy} = -27 - 50 - 0 = -77\ \mu$m/m.

2.19 Summary

This chapter described how load, stress, and strain affect the design of machine elements. If the proper type of machine element has been selected, a potential cause of failure is the design stress exceeding the strength of the machine element. Therefore, it is important to evaluate the stress, strain, and strength of the machine element at the critical section. To do so first requires a determination of load in all its forms. The applied load on a machine element was described with respect to time, the area over which load is applied, and the location and method of application. Furthermore, the importance of support reactions, application of static force and moment equilibrium, and proper use of free-body diagrams were investigated.

The chapter then focused on shear and moment diagrams applied to a beam. Singularity functions introduced by concentrated moment, concentrated force, unit step, ramp, inverse ramp, and parabolic shape were considered. Various combinations of these singularity functions can exist within a beam. Integrating the load intensity function for the various beam singularity functions over the beam length establishes the shear force. Integrating the shear force over the beam length determines the moment. From these analytical expressions the shear and moment diagrams can be readily constructed.

Stress defines the intensity and direction of the internal forces at a particular point and acting on a given plane. The stresses acting on an element have normal and shear components. Across each mutually perpendicular surface there are two shear stresses and one normal stress, yielding a total of nine stresses (three normal stresses and six shear stresses). The sign conventions for both the normal and shear stresses were presented. The nine stress components may be regarded as the components of a second-order Cartesian tensor. It was found that the stress tensor is symmetrical, implying that the tensor can be written with zero shear stress and the principal normal stresses along its diagonal.

In many engineering applications, stress analysis assumes that a surface is free of stress or that the stress in one plane is small relative to the stresses in the other two planes. The two-dimensional stress situation is called the biaxial (or plane) stress state and can be expressed in terms of two normal stresses and one shear stress, for example σ_x, σ_y, and τ_{xy}. That the stresses can be expressed in any oblique plane is important in deriving and applying Mohr's circle for a biaxial stress.

The concepts of strain and deflection were also investigated, since these are often design constraints. Just as with stress, strain is a tensor, and transformation equations and Mohr's circle are equally applicable to strain analysis. The concept of strain gage rosettes was introduced as a method to obtain plane strains.

Key Words

beam structural member designed to support loads perpendicular to its longitudinal axis

bending load load applied transversely to longitudinal axis of member

biaxial or plane stress condition where one surface is comparatively free of stress

cantilevered beam support where one end is fixed and the other end is free

combined load combination of two or more previously defined loads

concentrated load load applied to small nonconformal area

critical section section where largest internal stress occurs

cyclic load load varying throughout a cycle

distributed load load distributed over entire area

free-body diagram sketch of part showing all acting forces

impact load load rapidly applied

loads force, moment, or torque applied to a mechanism or structure

Mohr's circle method used to graphically visualize state of stress acting in different planes passing through a given point

normal load load passing through centroid of resisting section

normal strain elongation or contraction of linear segment of element in which stress is applied

overhanging beam support where one or both ends freely extend past support

principal normal stresses combination of applied normal and shear stresses that produces maximum principal normal stress or minimum principal normal stress, with a third principal stress between or equivalent to the extremes

principal shear stresses combination of applied normal and shear stresses that produces maximum principal shear stress or minimum principal shear stress

shear load load collinear with transverse shear force

shear strain measure of angular distortion in which shear stress is applied

sign convention for normal strain positive if elongation is in direction of positive axes

sign conversion for normal stress positive for tension and negative for compression

sign convention for shear strain positive if interior angle becomes smaller after shear stress is applied

sign convention for shear stress positive if both normal from surface and shear are in positive or negative direction; negative for any other combination

simply supported beam support where one end is pinned and the other is roller-supported

singularity functions functions used to evaluate shear and moment diagrams, especially when discontinuities, such as concentrated load or moment, exist

static load load gradually applied and equilibrium reached in a short time

strain dimensionless displacement produced in solid as a result of stress

stress intensity and direction of internal force acting at a given point on a particular plane

sustained load a load that is constant over a long time

symmetrical tensor condition where principal normal stresses exist while all other tensor elements are zero

torsion load a load that results in twisting deformation

triaxial stress stress where all surfaces are considered

uniaxial stress condition where two perpendicular surfaces are comparatively free of stress

Summary of Equations

Force equilibrium: $\sum P_x = 0, \sum P_y = 0, P_z = 0$
Moment equilibrium: $\sum M_x = 0, \sum M_y = 0, \sum M_z = 0$
Transverse shear in beams: $V(x) = -\int\limits_{-\infty}^{x} q(x)dx$

Bending moment in beams: $M(x) = -\int\limits_{-\infty}^{x} V(x)dx$

Principal stresses in plane stress:

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\tau_{xy}^2 + \frac{(\sigma_x - \sigma_y)^2}{4}}$$

Mohr's circle

Center: $\left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$

Radius: $r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Octahedral Stresses:

Normal: $\sigma_{\text{oct}} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$

Shear:

$$\begin{aligned} \tau_{\text{oct}} &= \frac{1}{3}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2} \\ &= \frac{2}{3}\left[\tau_{1/2}^2 + \tau_{2/3}^2 + \tau_{1/3}^2\right]^{1/2} \end{aligned}$$

Principal strains in plane strain:

$$\epsilon_1, \epsilon_2 = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{1}{2}\gamma_{xy}\right)^2 + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2}$$

Recommended Readings

Beer, F.P., Johnson, E.R., DeWolf, J., and Mazurek, D. (2011) *Mechanics of Materials*, 6th ed., McGraw-Hill.
Craig, R.R. (2011) *Mechanics of Materials*, 3rd ed., Wiley.
Hibbeler, R.C. (2010) *Mechanics of Materials*, 8th ed. Prentice-Hall, Upper Saddle River.
Popov, E.P. (1968) *Introduction to Mechanics of Solids*, Prentice-Hall.
Popov, E.P. (1999) *Engineering Mechanics of Solids*, 2nd ed., Prentice-Hall.
Riley, W.F., Sturges, L.D., and Morris, D.H. (2006) *Mechanics of Materials*, 6th ed., Wiley.
Shames, I.H., and Pitarresi, J.M. (2000) *Introduction to Solid Mechanics*, 3rd ed., Prentice-Hall.
Ugural, A.C. (2007) *Mechanics of Materials*, Wiley.

Questions

2.1 What is a concentrated load? What is a distributed load?

2.2 What kind of reaction occurs with a roller support? What occurs with a pin?

2.3 Define *static equilibrium*.

2.4 What is a simply supported beam? What is a cantilever?

2.5 Why are singularity functions useful?

2.6 Under what conditions does a singularity function *not* equal zero?

2.7 Define the terms *stress* and *strain*.

2.8 What is a tensor?

2.9 Define *normal stress* and *shear stress*.

2.10 What is Mohr's circle?

2.11 What is a principal stress?

2.12 What are the units for stress? What are the units for strain?

2.13 What are octahedral stresses?

2.14 What is elongation?

2.15 What is a rosette?

Qualitative Problems

2.16 Give three examples of (a) static loads; (b) sustained loads; (c) impact loads; and (d) cyclic loads.

2.17 Explain the sign convention for shear forces.

2.18 Explain the common sign conventions for bending moments. Which is used in this book?

2.19 Without the use of equations, explain a methodology for producing shear and moment diagrams.

2.20 Give two examples of scalars, vectors, and tensors.

2.21 Explain the difference between plane stress and plain strain. Give an example of each.

2.22 Without the use of equations, qualitatively determine the bending moment diagram for a bookshelf.

2.23 Explain why $\tau_{xy} = \tau_{yx}$.

2.24 Define and give two examples of (a) uniaxial stress state; (b) biaxial stress state; and (c) triaxial stress state.

2.25 Sketch and describe the characteristics of a three-dimensional Mohr's circle.

2.26 What are the similarities and differences between deformation and strain?

2.27 The text stated that $0°–45°–90°$ strain gage rosettes are common. Explain why.

2.28 Draw a free body diagram of a book on a table.

2.29 If the three principal stresses are determined to be 100 MPa, -50 MPa and 75 MPa, which is σ_2?

2.30 Derive Eq. (2.16).

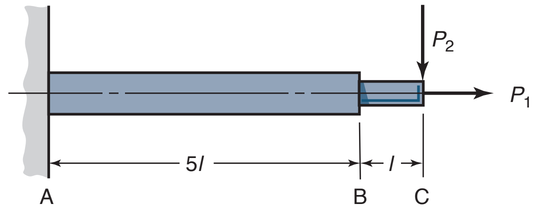
Quantitative Problems

2.31 The stepped shaft A-B-C shown in Sketch a is loaded with the forces P_1 and/or P_2. Note that P_1 gives a tensile stress σ in B-C and $\sigma/4$ in A-B and that P_2 gives a bending stress σ at B and 1.5σ at A. What is the critical section

(a) If only P_1 is applied?

(b) If only P_2 is applied?

(c) If both P_1 and P_2 are applied?



Sketch a, used in Problems 2.31 and 2.32.

2.32 The stepped shaft in Sketch a has loads P_1 and P_2. Find the load classification if P_1's variation is sinusoidal and P_2 is the load from a weight

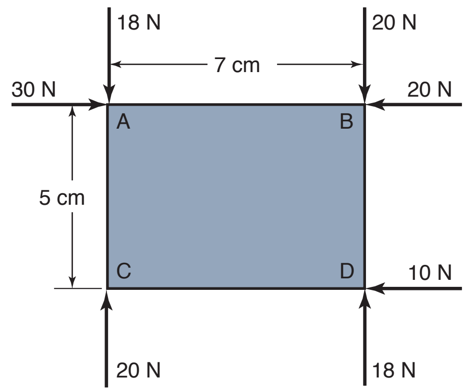
(a) If only P_1 is applied

(b) If only P_2 is applied

(c) If both P_1 and P_2 are applied

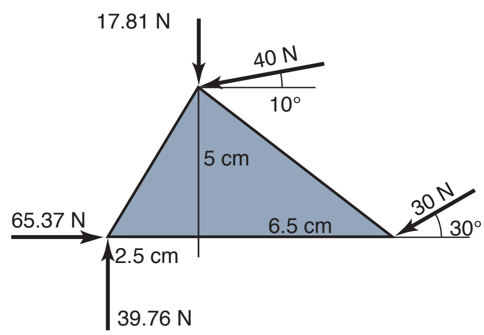
2.33 A bar hangs freely from a frictionless hinge. A horizontal force P is applied at the bottom of the bar until it inclines $45°$ from the vertical direction. Calculate the horizontal and vertical components of the force on the hinge if the acceleration due to gravity is g, the bar has a constant cross section along its length, and the total mass is m_a. *Ans.* $R_x = \frac{1}{2}m_a g$, $R_y = m_a g$.

2.34 Sketch b shows the forces acting on a rectangle. Is the rectangle in equilibrium? *Ans.* No.



Sketch b, used in Problem 2.34

2.35 Sketch c shows the forces acting on a triangle. Is the triangle in equilibrium? *Ans.* Yes.



Sketch c, used in Problem 2.35