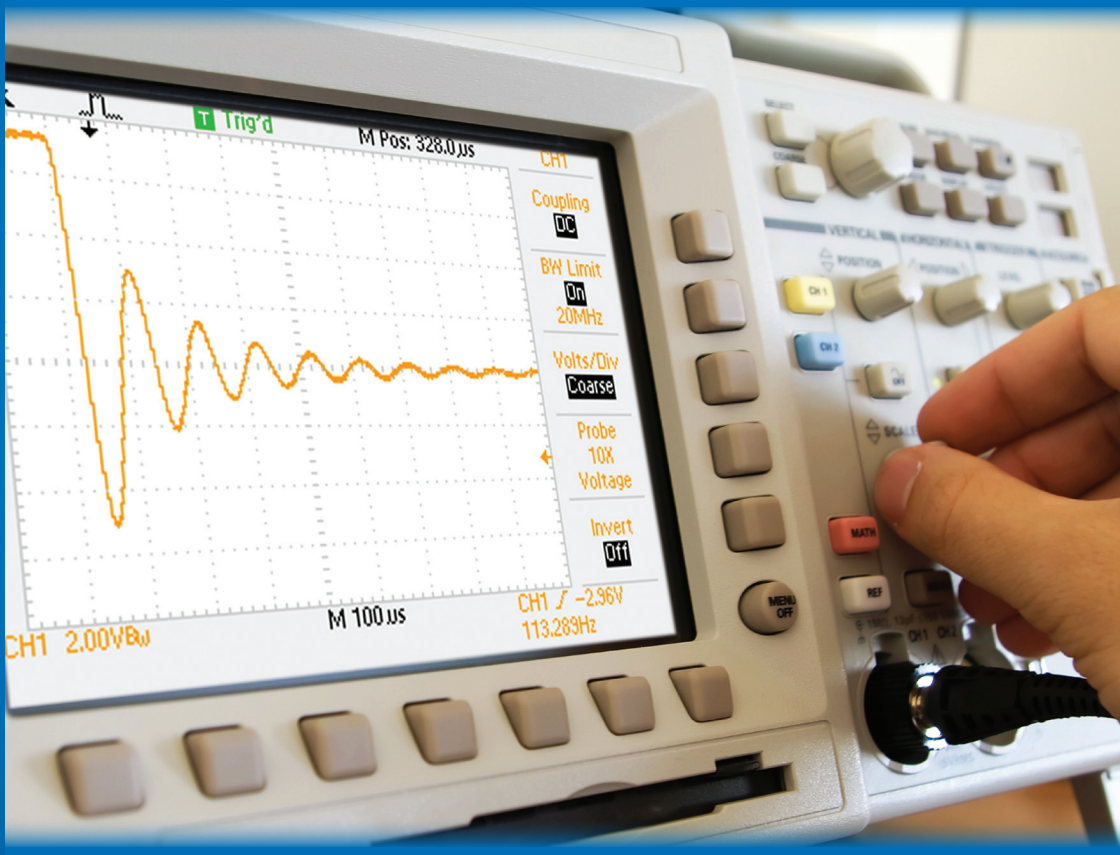
Numerical and Analytical Methods with MATLAB®

for Electrical Engineers



William Bober • Andrew Stevens

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Preface

I have been teaching two courses in computer applications for engineers at Florida Atlantic University (FAU) for many years. The first course is usually taken in the student's sophomore year; the second course is usually taken in the student's junior or senior year. Both computer classes are run as lecture-laboratory courses, and the MATLAB® software program is used in both courses. To familiarize students with engineering-type problems, approximately six or seven projects are assigned during the semester. Students have, depending on the difficulty of the project, either one week or two weeks to complete each project. I believe that the best source for students to complete the assigned projects in either course is this textbook.

This book has its origin in a previous textbook, *Numerical and Analytical Methods with MATLAB®* by William Bober, Chi-Tay Tsai, and Oren Masory, also published by CRC Press. The previous book was primarily oriented toward mechanical engineering students. I and Jonathan Plant of CRC Press envisioned that a similar text would fill a need in electrical engineering curricula; as a result, I enlisted Dr. Andrew Stevens to replace the projects in the existing textbook with those oriented toward electrical engineering students. This new textbook retains the philosophy of teaching that exists in the original textbook.

The advantage of using the MATLAB software program over other packages is that it contains built-in functions that numerically solve systems of linear equations, systems of ordinary differential equations, roots of transcendental equations, integrals, statistical problems, optimization problems, signal-processing problems, and many other types of problems encountered in engineering. A student version of the MATLAB program is available at a reasonable cost. However, to students, these built-in functions are essentially black boxes. By combining a textbook on MATLAB with basic numerical and analytical analysis (although I am sure that MATLAB uses more sophisticated numerical techniques than are described in these textbooks), the mystery of what these black boxes might contain is somewhat alleviated. The text contains many sample MATLAB programs that should provide guidance to the student on completing the assigned projects. Many of the projects in this book are non-trivial and, I believe, will be good training for a graduating engineer entering industry or in an advanced degree program.

Furthermore, I believe that there is enough material in this textbook for two courses, especially if the courses are run as lecture-laboratory courses. The advantage of running these courses (especially the first course) as a lecture-laboratory course is that the instructor is in the computer laboratory to help the students debug their programs. This includes the sample programs as well as the projects.

The common core of the book is the introduction to the MATLAB programming environment in Chapter 2. Then, depending on the individual curriculum (but typically sophomore year), a course might proceed with mathematical techniques for matrix algebra, root finding, integration, and differential equations (Chapters 3 through 6). A more advanced course (perhaps in junior or senior year) might include transform techniques (Chapters 7 and 8) and advanced topics in curve fitting and optimization (Chapters 9 and 10). MATLAB's graphical design environment, Simulink®, is introduced in Chapter 11 and could be relocated elsewhere in the syllabus.

We have tried to make each chapter stand alone so that each may be rearranged based on the preference of the instructor. In many cases, we have used the resistor-inductor-capacitor (RLC) circuit as an example, and we have put the basic derivation of this circuit in Appendix A to minimize the chapter dependencies and facilitate reordering. In all cases, we have attempted to provide illustrative examples using modern topics in electrical engineering.

All chapters (except for Chapter 1) contain projects, and some also contain several exercises that are less difficult than the projects and might be assigned prior to a project assignment. All projects require the student to write a computer program, most requiring the use of MATLAB built-in functions and solvers.

Additional materials, including down loadable copies of all examples in the textbook, are available from the CRC press website:

http://www.crcpress.com/product/isbn/9781439854297

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The MathWorks, Inc.
3 Apple Hill Drive
Natick, MA 01760-2098 USA
Tel: 508-647-7000
Fax: 508-647-7001
E-mail: info@mathworks.com
Web: www.mathworks.com

William Bober
Florida Atlantic University
Department of Civil Engineering

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We also wish to express our deep gratitude to Selma Bober, Theresa Stevens, and Emma Stevens for tolerating the many hours we spent on the preparation of the manuscript, the time that otherwise would have been devoted to our families.

About the Authors

William Bober, Ph.D., received his B.S. degree in civil engineering from the City College of New York (CCNY), his M.S. degree in engineering science from Pratt Institute, and his Ph.D. degree in engineering science and aerospace engineering from Purdue University. At Purdue University, he was on a Ford Foundation Fellowship; he was assigned to teach one engineering course each semester. After receiving his Ph.D., he went to work as an associate engineering physicist in the Applied Mechanics Department at Cornell Aeronautical Laboratory in Buffalo, New York. After leaving Cornell Labs, he was employed as an associate professor in the Department of Mechanical Engineering at the Rochester Institute of Technology (RIT) for the following twelve years. After leaving RIT, he obtained employment at Florida Atlantic University (FAU) in the Department of Mechanical Engineering. More recently, he transferred to the Department of Civil Engineering at FAU. While at RIT, he was the principal author of a textbook, *Fluid Mechanics*, published by John Wiley & Sons. He has written several papers for *The International Journal of Mechanical Engineering Education* (IJMEE) and more recently coauthored a textbook, *Numerical and Analytical Methods with MATLAB®*.

Andrew Stevens, Ph.D., P.E., received his bachelor's degree from Massachusetts Institute of Technology, his master's degree from the University of Pennsylvania, and his doctorate from Columbia University, all in electrical engineering. He did his Ph.D. thesis work at IBM Research in the area of integrated circuit design for high-speed optical networks. While at Columbia, he lectured a course in the core undergraduate curriculum and won the IEEE Solid-State Circuits Fellowship. He has held R&D positions at AT&T Bell Laboratories in the development of T-carrier multiplexer systems and at Argonne National Laboratory in the design of radiation-hardened integrated circuits for colliding beam detectors. Since 2001, he has been president of Electrical Science, an engineering consulting firm specializing in electrical hardware and software. He has published articles in several scientific journals and holds three patents in the areas of analog circuit design and computer user interfaces.

Chapter 1

Numerical Methods for Electrical Engineers

1.1 Introduction

All disciplines of science and engineering use numerical methods for the analysis of complex problems. However, electrical engineering particularly lends itself to computational solutions due to the highly mathematical nature of the field and its close relationship with computer science. It makes sense that engineers who design high-speed computers would also use the computers themselves to aid in the design (a process known as bootstrapping). In fact, the entire field of computer-aided design (CAD) is dedicated to the creation and improvement of software tools to enable the implementation of highly complex designs.

This book describes various methods and techniques for numerically solving a variety of common electrical engineering applications, including circuit design, electromagnetic field theory, and signal processing. Classical engineering curricula teach a variety of methods for solving these problems using techniques such as linear algebra, differential equations, transforms, vector calculus, and the like. However, in many cases, the search for a closed-form solution leads to extreme complexity, which can cause us to lose physical insight into the problem. In solving these same problems numerically, we will often revert to fundamental physical relations, such as the differential relationship between capacitor current and voltage or the electric field of a point charge. Often, a problem that seems intractable when solved symbolically can become trivial when solved numerically. And sometimes, the simpler numerical solution can be elusive because we are so used to thinking in terms of advanced calculus (a classic case of not being able to see the forest but for the trees).

1.2 Engineering Goals

Some fundamental goals in engineering include

- Design new products or improve existing ones
- Improve manufacturing efficiency
- Minimize cost, power consumption, and nonreturnable engineering (NRE) cost
- Maximize yield and return on investment (ROI)
- Minimize time to market

The engineer will frequently use the laws of physics and mathematics to achieve these goals.

Many electrical engineering processes involve expensive manufacturing steps that are both delicate and time consuming. For example, the fabrication of integrated circuits can involve thousands of manufacturing steps, including wafer preparation, mask creation, photolithography, diffusion and implantation, dicing, testing, packaging, and more. These steps can take weeks or months to perform at substantial expense and in clean rooms. Any design mistakes require repeating the process; thus, it is our job as designers to model and simulate designs as much as possible in advance of manufacture to eliminate flaws and minimize the iterations necessary to produce the final product.

Using integrated circuit design as an example, we might use computers for the

a. Design stage: Solve mathematical models of physical phenomena (e.g., predicting the behavior of PN junctions)
b. Testing stage: Store and analyze experimental data (e.g., comparing the laboratory-measured actual behavior of PN junctions to the prediction)
c. Manufacturing stage: Controlling machine operations to fabricate and test silicon wafers and dice

1.3 Programming Numerical Solutions

Physical phenomena are always described by a set of governing equations, and numerical methods can be used to solve the set of governing equations even in the absence of a closed-form solution. Numerical methods invariably involve the computer, and the computer performs arithmetic operations on discrete numbers in a defined sequence of steps. The sequence of steps is defined in the program. A useful solution is obtained if

a. The mathematical model accurately represents the physical phenomena; that is, the model has the correct governing equations.
b. The numerical method is accurate.
c. The numerical method is programmed correctly.

This text is mainly concerned with items (b) and (c).

The advantage of using the computer is that it can carry out many calculations in a fraction of a second; at the time of this writing, computer speeds are measured in teraflops (trillions of floating point operations per second). However, to leverage this power, we need to write a set of instructions, that is, a program. For the problems of interest in this book, the digital computer is only capable of performing arithmetic, logical, and graphical operations. Therefore, arithmetic procedures must be developed for solving differential equations, evaluating integrals, determining roots of an equation, solving a system of linear equations, and so on. The arithmetic procedure usually involves a set of algebraic equations. A computer solution for such problems involves developing a computer program that defines a step-by-step procedure for obtaining an answer to the problem of interest. The method of solution is called an *algorithm*. Depending on the particular problem, we might write our own algorithm, or as we shall see, we can also use the algorithms built into a package like MATLAB® to perform well-known algorithms such as the Runge-Kutta method for solving a set of ordinary differential equations or use Simpson's rule for evaluating an integral.

1.4 Why MATLAB?

MATLAB was originally written by Dr. Cleve Moler at University of New Mexico in the 1970s and was commercialized by MathWorks in the 1980s. It is a general-purpose numerical package that allows complex equations to be solved efficiently and subsequently generate tabular or graphical output. While there are many numerical packages available to electrical engineers, many are highly focused toward a particular application (e.g., SPICE for modeling electronic circuits). Also, MATLAB is not to be confused with CAD software for schematic capture, layout, or physical design, although this software often integrates with an accompanying numerical package.

Originally, MATLAB was a command-line program that ran on MS-DOS and UNIX hosts. As computers have evolved, so has MATLAB, and modern editions of the program run in windowed environments. As of the time of this writing, MATLAB R2011b runs natively on Microsoft Windows, Apple MacOS, and Linux. In this text, we assume that you are running MATLAB on your local machine in a Microsoft Windows environment. It should be straightforward for non-Windows users to translate the usage descriptions to their preferred environment. In any case, these differences are largely limited to the cosmetics and presentation of the program and not the MATLAB commands themselves. All versions of MATLAB (on any platform) use the same command set, and the Command Window on all platforms should behave identically.

MATLAB is offered with accompanying "toolboxes" at additional cost to the user. A wide variety of toolboxes are available in fields such as control systems, image processing, radio frequency (RF) design, signal processing, and more. However, in this text, we largely focus on fundamental numerical concepts and limit ourselves to basic MATLAB functionality without requiring the purchase of any additional toolboxes.

1.5 The MATLAB Programming Language

There are many methodologies for computer programming, but the tasks at hand boil down to the following:

a. Study the problem to be programmed.
b. List the algebraic equations to be used in the program based on the known physical phenomena and geometries of the problem.
c. Create a general design for the program flow and algorithms, perhaps by creating a flowchart or by writing high-level pseudocode to outline the main program modules.
d. Carry out a sample calculation by hand to prove the algorithm.
e. Write the program using the list of algebraic equations and the outline.
f. Debug the program by running it and fixing any syntax errors.
g. Test the program by running it using parameters with a known (or intuitive) solution.
h. Iterate over these steps to refine and further debug the algorithm and program flow.
i. If necessary, revise the program to obtain faster performance.

Experienced programmers might omit some of these steps (or do them in their head), but the overall process resembles any engineering project: design, create a prototype, test, and iterate the process until a satisfactory product is achieved.

MATLAB may be considered a programming or scripting language unto itself, but like every programming language, it has the following core components:

a. Data types, i.e., formats for storing numbers and text in the program (e.g., integers, double-precision floating point, strings, vectors, matrices)
b. Operators (e.g., commands for addition, multiplication, cosine, log)
c. Control flow directives for making decisions and performing iterative operations (e.g., `if`, `while`, `switch`)
d. Input/output ("I/O") commands for receiving input from a user or file and for generating output to a file or the screen (e.g., `fprintf`, `fscanf`, `plot`, `stem`, `surf`)

MATLAB borrows many constructs from other languages. For example, the `while`, `switch`, and `fprintf` commands are from the C programming language (or its descendents C++, Java, and Perl). However, there are some fundamental differences as well. For example, MATLAB stores *functions* (known in other languages as "subroutines") in separate files. The first entry in a *vector* (known in most other languages as an *array*) is indexed by the number 1 and not 0. However, the biggest difference is that all MATLAB variables are vectors, thus providing the ability to manipulate large amounts of data with a terse syntax and allowing for the

solution of complicated problems in just a few lines of code. In addition, because MATLAB is normally run interactively, it is also rich in presentation functions to display sophisticated plots and graphs.

1.6 Conventions in This Book

We use the following typographical conventions in this text:

- All input/output to and from MATLAB are in `typewriter` font.
- In cases where you are typing directly into the computer, the typed text is displayed in **`bold`**.

We illustrate this in the following example, for which we use MATLAB to find the value $x = \sin\frac{\pi}{4}$:

```
>> x = sin(pi/4)
x =
   0.7071
```

In this case, >> represents MATLAB's prompt, **`x=sin(pi/4)`** represents text typed into the MATLAB command window, and `x = 0.7071` represents MATLAB's response.

1.7 Example Programs

The example programs in this book may be downloaded from the publisher's Web site at http://www.crcpress.com/product/isbn/9781439854297. Students may then run the example programs on their own computer and see the results. It also may be beneficial for students to type in a few of the sample programs (along with some inevitable syntax and typographical errors), thereby giving them the opportunity to see how MATLAB responds to program errors and subsequently learn what they need to do to fix the problem.

Chapter 2

MATLAB Fundamentals

2.1 Introduction

MATLAB® is a software program for numeric computation, data analysis, and graphics. One advantage that MATLAB has for engineers over programming languages such as C or C++ is that the MATLAB program includes functions that numerically solve large systems of linear equations, systems of ordinary differential equations, roots of transcendental equations, integrals, statistical problems, optimization problems, control systems problems, and many other types of problems encountered in engineering. MATLAB also offers toolboxes (which must be purchased separately) that are designed to solve problems in specialized areas.

In this chapter, the following items are covered:

- The MATLAB desktop environment
- Constructing a script (also called a program) in MATLAB
- MATLAB fundamentals and basic commands, including `clear`, `clc`, colon operator, arithmetic operators, trigonometric functions, logarithmic and exponential functions, and other useful functions such as `max`, `min`, and `length`
- Input/output in MATLAB, including the `input` and `fprintf` statements
- MATLAB program flow, including `for` loops, `while` loops, `if` and `elseif` statements, and the `switch` group statement
- MATLAB function files and anonymous functions
- MATLAB graphics, including the `plot` and `subplot` commands

- Working with matrices
- Working with functions of a vector
- Working with characters and strings
- Interpolation and MATLAB's `interp1` function
- MATLAB's `textscan` function
- Exporting MATLAB data to other software, such as Microsoft Excel
- Debugging a program

Many example scripts are included throughout the chapter to illustrate these various topics.

2.2 The MATLAB Windows

Under Microsoft Windows, MATLAB may be started via the Start menu or clicking on the MATLAB icon on the desktop. On startup, a new window will open containing the MATLAB "desktop" (not to be confused with the Windows desktop), and one or more MATLAB windows will open within the desktop (see Figure 2.1 for the

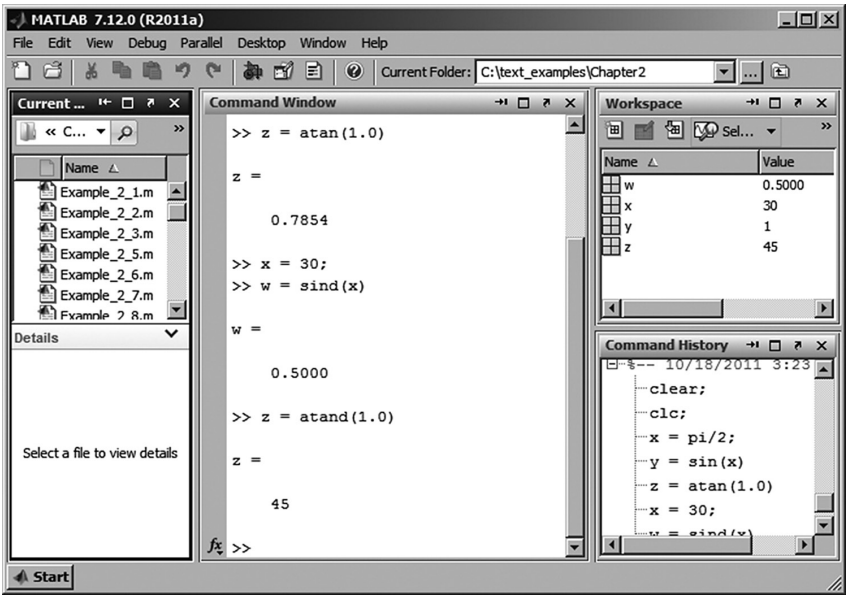

Figure 2.1 MATLAB desktop windows. (From MATLAB, with permission.)

default configuration). The main windows are the Command window, Command History, Current Folder, and Workspace. You can customize the MATLAB windows that appear on startup by opening the *Desktop* menu and checking (or unchecking) the windows that you wish to appear on the MATLAB desktop. Figure 2.1 shows the Command window (in the center), the Current Folder (on the left), the Workspace (on the top right), the Command History (on the bottom right), and the Current Folder box (in the icon toolbar, second from the top, just above the Command window). These windows and the current folder box are summarized as follows:

- *Command Window:* In the Command window you can enter commands and data, make calculations, and print results. You can write a program in the Command window and execute the program. However, writing a program directly into the Command window is discouraged because it will not be saved, and if an error is made, the entire program must be retyped. By using the up arrow (↑) key on your keyboard, the previous command can be retrieved (and edited) for reexecution.
- *Command History Window:* This window lists a history of the commands that you have executed in the Command window.
- *Current Folder Box:* This box lists the active Current Folder (also called the Current Directory in older versions of MATLAB). **To run a MATLAB script (program), the script of interest needs to be in the folder listed in this box.** By clicking on the down arrow within the box, a drop-down menu will appear that contains names of folders that you have previously used. This will allow you to select the folder in which the script of interest resides (see Figure 2.2). If the folder containing the script of interest is not listed in the drop-down menu, you can click on the adjacent little box containing three dots, which allows you to browse for the folder containing the program of interest (see Figure 2.3).
- *Current Folder Window* (on the left): This window lists all the files in the Current Folder. By double clicking on a file in this window, the file will open within MATLAB.
- *Script Window* (also called the Editor window in older MATLAB versions): To open this window, use the *File* menu at the top of the MATLAB desktop and choose *New* and then *Script* (or in older versions of MATLAB, click on *New M-File*) (see Figure 2.4). The Script Window may be used to create, edit, and execute MATLAB scripts. Scripts are then saved as *M-Files*. These files have the extension *.m*, such as *circuit.m*. To execute the program, you can click the *Save and Run* icon (the green arrow) in the Script window or return to the Command window and type in the name of the program (without the *.m* extension).

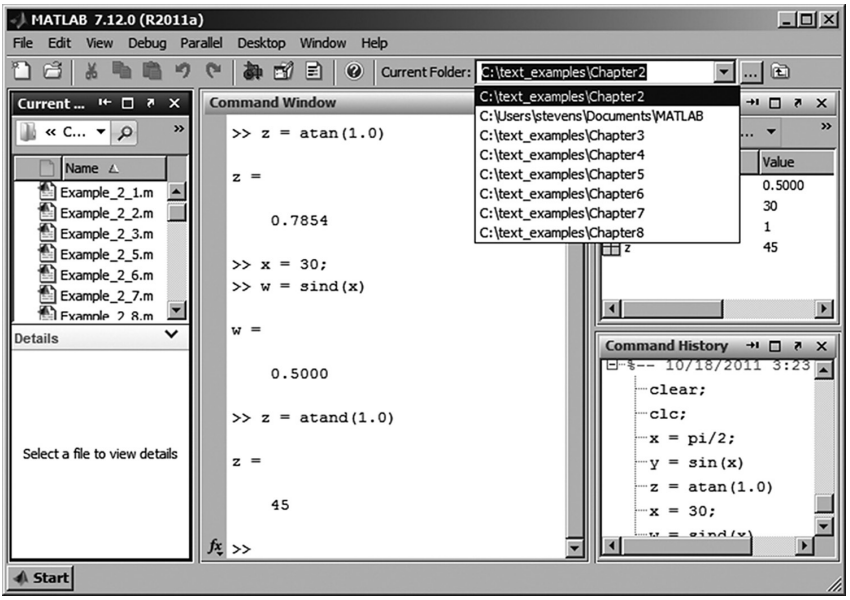

Figure 2.2 Drop-down menu in the Current Folder window. (From MATLAB, with permission.)

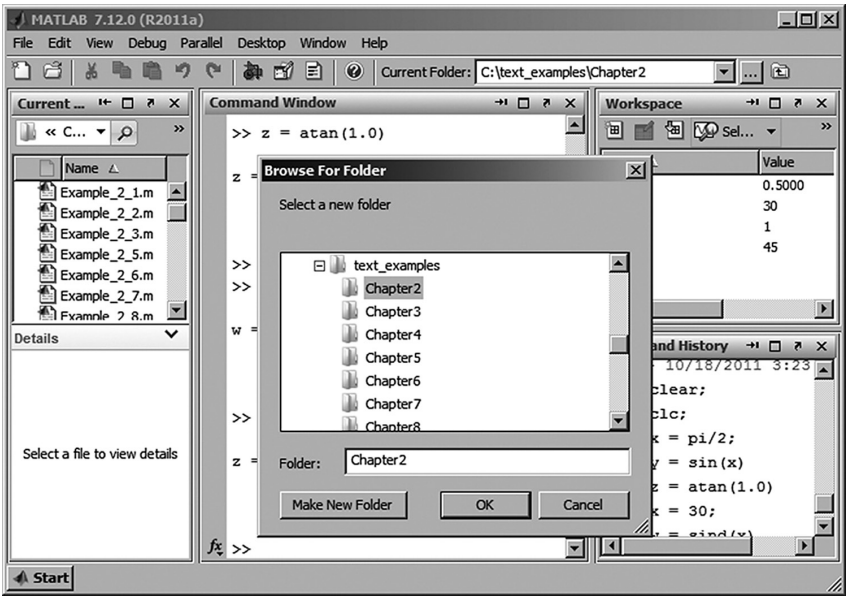

Figure 2.3 Drop-down menu for selecting folder containing program of interest. (From MATLAB, with permission.)

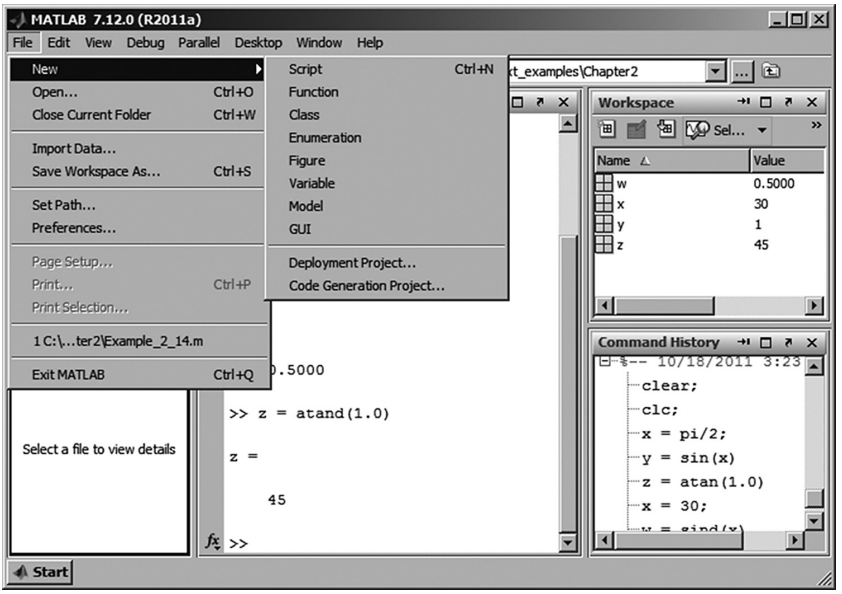

Figure 2.4 Opening up the Script window. (From MATLAB, with permission.)

2.3 Constructing a Program in MATLAB

This list summarizes the steps for writing your first MATLAB program:

1. Start the MATLAB desktop via the Windows Start menu or by double clicking on the MATLAB icon on the desktop.
2. Click on *File-New-Script.* This brings up a new Script window.
3. Type your script into the Script window.
4. Save the script by clicking on the *Save* icon in the icon toolbar or clicking on *File* in the menu bar and selecting *Save* in the drop down menu. In the dialog box that appears, select the folder where the script is to reside and type in a file name of your own choosing. It is best to use a folder that contains only your own MATLAB scripts.
5. Before you can run your script, you need to go to the Current Folder box at the top of the MATLAB desktop, clicking on the down arrow and in the drop down menu, selecting (or browsing to) the folder that contains your new script.
6. You may run your script from the Script window by clicking on the *Save and Run* green arrow in the icon toolbar (see Figure 2.5) or alternatively, from the

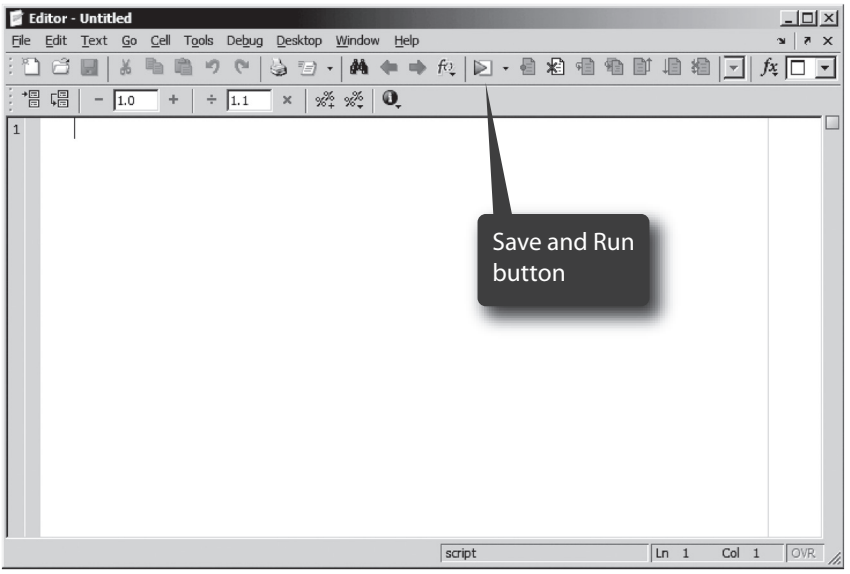

Figure 2.5 Save and run button in the script window. (From MATLAB, with permission.)

command window by typing the script name (without the *.m* extension) after the MATLAB prompt (>>). For example, if the program has been saved as *circuit.m,* then type `circuit` after the MATLAB prompt (>>), as shown below:

```
>> circuit
```

If you need additional help getting started, you can click on *Help* in the menu bar in the MATLAB window and then select *Product Help* from the drop-down menu. This will bring up the help window as shown in Figure 2.6. By clicking on the little '+' box next to the MATLAB listing in the left column, you will get additional help topics as shown in Figure 2.7. Once you select one of the help topics, the help information will be in the right-hand window. You can also type in a topic in the search window to obtain information on that topic.

2.4 MATLAB Fundamentals

- Variable names
  - must start with a letter.

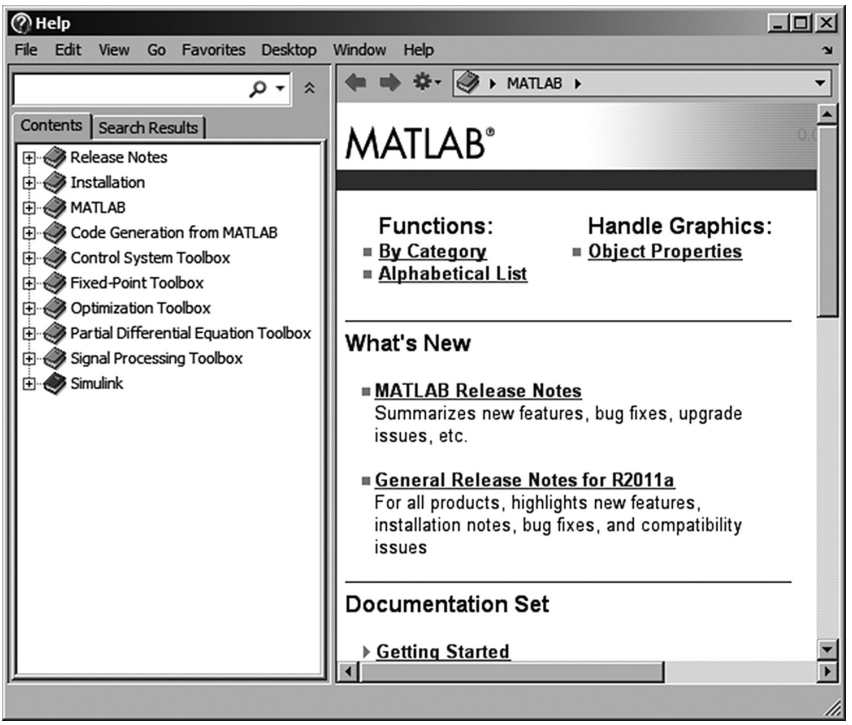

Figure 2.6 Product help window. (From MATLAB, with permission.)

- can contain letters, digits and the under score character.
- can be of any length, but must be unique within the first 19 characters.

Note: Do not use a variable name that is the same name as the name of a file, a MATLAB function, or a self written function.

- MATLAB command names and variable names are case sensitive. Use lower case letters for commands.
- Semicolons are usually placed after variable definitions and program statements when you do not want the command echoed to the screen. In the absence of a semicolon, the defined variable appears on the screen. For example, if you entered the following assignment in the command window:

```
>> A = [3 4 7 6]
```

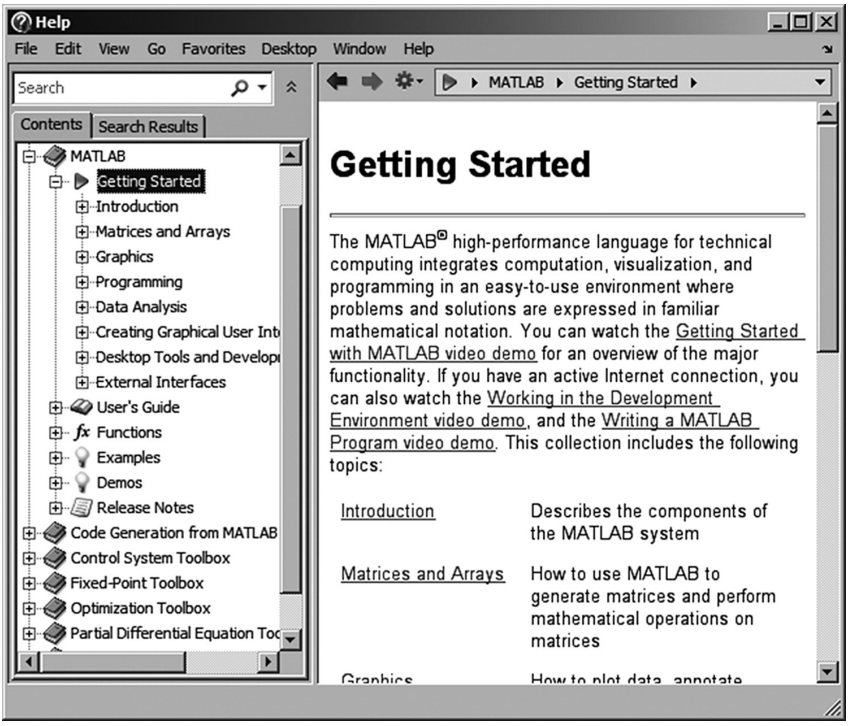

Figure 2.7 Getting started in product help window. (From MATLAB, with permission.)

In the command window, you would see

```
A =
   3  4  7  6
>>
```

Alternatively, if you add the semicolon, then your command is executed but there is nothing printed to the screen, and the prompt immediately appears for you to enter your next command:

```
>> A = [3 4 7 6];
>>
```

- Percent sign (%) is used for a comment line.
- A separate Graphics window opens to display plots and graphs.
- There are several commands for clearing windows, clearing the work space and stopping a running program.

`clc`	clears the Command window
`clf`	clears the Graphics window
`clear`	removes all variables and data from the workspace
Ctl-C	aborts a program that may be running in an infinite loop

- the `quit` or `exit` commands terminate MATLAB.
- the `save` command saves variables or data in the workspace of the current directory. The file name containing the data will have *.mat* extension.
- User-defined functions (also called *self-written* functions) are also saved as M-files.
- Scripts and functions are saved as ASCII text files. Thus, they may be written either in the built-in Script window, Notepad, or any word processor (saved as a text file).
- The basic data structure in MATLAB is a matrix.
- A matrix is surrounded by brackets and may have an arbitrary number of rows and columns; for example, the matrix $A = \begin{matrix} 1 & 3 \\ 6 & 5 \end{matrix}$ may be entered into MATLAB as

```
>> A = [1 3 <enter>
        6 5]; <enter>
```

or

```
>> A = [1 3 ; 6 5]; <enter>
```

where the semicolon within the brackets indicates the start of a new row within the matrix.

- A matrix of one row and one column is a scalar; for example:

```
>> A = [3.5];
```

Alternatively, MATLAB also accepts `A = 3.5` (without brackets) as a scalar.

- A matrix consisting of one row and several columns or one column and several rows is considered a vector; for example:

```
>> A = [2 3 6 5]  (row vector)

>> A = [2
        3
        6
        5]  (column vector)
```

A matrix can be defined by including a second matrix as one of the elements; example:

```
>> B = [1.5 3.1];
>> C = [4.0 B];   (thus C = [4.0 1.5 3.1])
```

- A specific element of matrix C can be selected by writing

  >> **a = C(2);** (thus $a = 1.5$)

  If you wish to select the last element in a vector, you can write

  >> **a = c(end);** (thus $a = 3.1$)

- The colon operator (:) may be used to create a new matrix from an existing matrix; for example:

$$\text{if} \quad A = \begin{matrix} 5 & 7 & 10 \\ 2 & 5 & 2 \\ 1 & 3 & 1 \end{matrix}$$

$$\text{then} \quad \texttt{x = A(:,1)} \text{ gives} \quad x = \begin{matrix} 5 \\ 2 \\ 1 \end{matrix}$$

The colon in the expression `A(:,1)` implies all the rows in matrix *A*, and the 1 implies column 1.

$$\texttt{x = A(:,2:3)} \text{ gives} \quad x = \begin{matrix} 7 & 10 \\ 5 & 2 \\ 3 & 1 \end{matrix}$$

The first colon in the expression `A(:,2:3)` implies all the rows in *A*, and the 2:3 implies columns 2 and 3.

We can also write

`y = A(1,:),` which gives $y = [5 \;\; 7 \;\; 10]$

The 1 implies the first row, and the colon implies all the columns.

- A colon can also be used to generate a series of numbers. The format is:

  n = starting value : step size : final value. If omitted, the default step size is 1. For example:

  `n = 1:8` gives $n = [1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8]$.

  To increment in steps of 2, use

  `n = 1:2:7` gives $n = [1 \quad 3 \quad 5 \quad 7]$

  These types of expressions are often used in a `for` loop, which is discussed later in this chapter.

- Arithmetic operators:

+	addition
-	subtraction
*	multiplication
/	division
^	exponentiation

- To display a variable value, just type the variable name without the semicolon, and the variable will appear on the screen.

  Examples (try typing these statements into the Command window):

```
clc;
x = 5;
y = 10;
z = x + y
w = x - y
z = y/x
z = x*y
z = x^2
```

  Note that in the arithmetic statement `z = x + y`, the values for `x` and `y` were assigned in the two prior lines. In general, all variables on the right-hand side of an arithmetic statement must be assigned a value before they are used.

- Special values:

`pi`	π
`i` or `j`	$\sqrt{-1}$
`inf`	∞
`ans`	the last computed unassigned result to an expression typed in the Command window

Examples (try typing these statements in the command window):

```
x = pi;
z = x/0  (gives inf)
```

- Trigonometric functions:

`sin`	sine
`sinh`	hyperbolic sine
`asin`	inverse sine
`asinh`	inverse hyperbolic sine
`cos`	cosine
`cosh`	hyperbolic cosine
`acos`	inverse cosine
`acosh`	inverse hyperbolic cosine
`tan`	tangent
`tanh`	hyperbolic tangent
`atan`	inverse tangent
`atan2`	four-quadrant inverse tangent
`atanh`	inverse hyperbolic tangent
`sec`	secant
`sech`	hyperbolic secant
`asec`	inverse secant
`asech`	inverse hyperbolic secant
`csc`	cosecant
`csch`	hyperbolic cosecant
`acsc`	inverse cosecant
`acsch`	inverse hyperbolic cosecant
`cot`	cotangent
`coth`	hyperbolic cotangent
`acot`	inverse cotangent
`acoth`	inverse hyperbolic cotangent

The arguments of these trigonometric functions are in radians. However, the arguments can be made in degrees if a "`d`" is placed after the function name, such as `sind(x)`.

Examples (try typing these statements into the Command window):

```
clc;
x = pi/2;
y = sin(x)
```

```
z = atan(1.0)
x = 30;
w = sind(x)
z = atand(1.0)
```

■ Exponential, square root, and error functions:

`exp`	exponential
`log`	natural logarithm
`log10`	common (base 10) logarithm
`sqrt`	square root
`erf`	error function

Examples (try typing these statements into the Command window):

```
clc;
x = 2.5;
y = exp(x)
z = log(y)
w = sqrt(x)
```

■ Complex numbers:

Complex numbers may be written in two forms: Cartesian, such as `z = x + yj;` or polar, such as `z = r * exp(j*theta)`. Note that we use `j` for $\sqrt{-1}$ throughout this text. However, MATLAB allows the use of `i` for $\sqrt{-1}$ as well. Note: `i` and `j` are also legal MATLAB variable names which are often used within loops. To avoid confusion, programs which involve complex numbers should not use `i` or `j` as variable names.

`abs`	absolute value (magnitude)
`angle`	phase angle (in radians)
`conj`	complex conjugate
`imag`	complex imaginary part
`real`	complex real part

Examples (try typing these statements into the Command window):

```
clc;
z1 = 1 + j;
z2 = 2 * exp(j * pi/6)
y = abs(z1)
```

```
w = real(z2)
v = imag(z1)
phi = angle(z1)
```

- Other useful functions:

`length(X)`	Gives the number of elements in the vector *X*.
`size(X)`	Gives the size (number of rows and the number of columns) of matrix *X*.
`sum(X)`	For vectors, `sum(X)` gives the sum of the elements in *X*. For matrices, gives a row vector containing the sum of the elements in each column of the matrix.
`max(X)`	For vectors, `max(X)` gives the maximum element in *X*. For matrices, `max(X)` gives a row vector containing the maximum in each column of the matrix. If *X* is a column vector, it gives the maximum value of *X*.
`min(X)`	Same as `max(X)` except it gives the minimum element in *X*.
`sort(X)`	For vectors, `sort(X)` sorts the elements of *X* in ascending order. For matrices, `sort(X)` sorts each column in the matrix in ascending order.
`factorial(n)`	$n! = 1 \times 2 \times 3 \times \cdots \times n$
`mod(x,y)`	modulo operator, gives the remainder resulting from the division of *x* by *y*. For example, `mod(13,5) = 3`, that is, $13 \div 5$ gives 2 plus remainder of 3 (the 2 is discarded). As another example, `mod(n,2)` gives zero if *n* is an even integer and one if *n* is an odd integer.

Examples (try typing these statements into the Command window):

```
clc;
A = [2 15 6 18];
length(A)
y = max(A)
z = sum(A)
A = [2 15 6 18; 15 10 8 4; 10 6 12 3];
x = max(A)
y = sum(A)
size(A)
mod(21,2)
mod(20,2)
```

- A list of the complete set of elementary math functions can be obtained by typing **`help elfun`** in the Command window.

- Sometimes, it is necessary to preallocate a matrix of a given size. This can be done by defining a matrix of all zeros or ones; for example:

```
                    0   0   0
A = zeros(3) =      0   0   0
                    0   0   0

                        0   0
B = zeros(3,2) =        0   0
                        0   0

                    1   1   1
C = ones(3) =       1   1   1
                    1   1   1

                        1   1   1
D = ones(2,3) =
                        1   1   1
```

The function to generate the identity matrix (main diagonal of ones; all other elements are zero) is `eye`; example:

```
                    1   0   0
I = eye(3) =        0   1   0
                    0   0   1
```

2.5 MATLAB Input/Output

- If you wish to have your program pause to accept input from the keyboard, use the `input` function; for example, to enter a 2 by 3 matrix, use

```
Z = input('Enter values for Z in brackets \n');
```

then type in:

```
[5.1  6.3  2.5;  3.1  4.2  1.3]
```

Thus,

$$Z = \begin{matrix} 5.1 & 6.3 & 2.5 \\ 3.1 & 4.2 & 1.3 \end{matrix}$$

Note that the argument to `input()` is a character string enclosed by single quotation marks, which will be printed to the screen. The `\n` (newline) tells MATLAB to move the cursor to the next line. Alternatively, `\t` (tab) tells MATLAB to move the cursor several spaces along the same line.

If you wish to enter text data to `input`, you need to enclose the text with single quotation marks. However, you can avoid this requirement by entering a second argument of `'s'` to `input` as shown in the following statement:

```
response = input('Plot function? (y/n):\n', 's');
```

In this case, the user can respond with either a `y` or `n` (without single quotation marks).

Examples (try typing these statements into the Command window):

```
z = input ('Enter a 2x3 matrix of your choosing\n')
name=input('Enter name enclosed by single quote marks:')
response = input('Plot function? (y/n):\n', 's');
```

- The `disp` command prints *just* the contents of a matrix or alphanumeric information; for example (assuming that matrix `X` has already been entered in the Command window):

```
>>    x = [3.6 7.1]; disp(X); disp('volt');
```

The following will be displayed on the screen:

```
      3.6000 7.1000
      volt
>>
```

- The `fprintf` command prints formatted text to the screen or to a file; for example:

```
>> I = 2.2;
>> fprintf ('The current is %f amps \n', I);
```