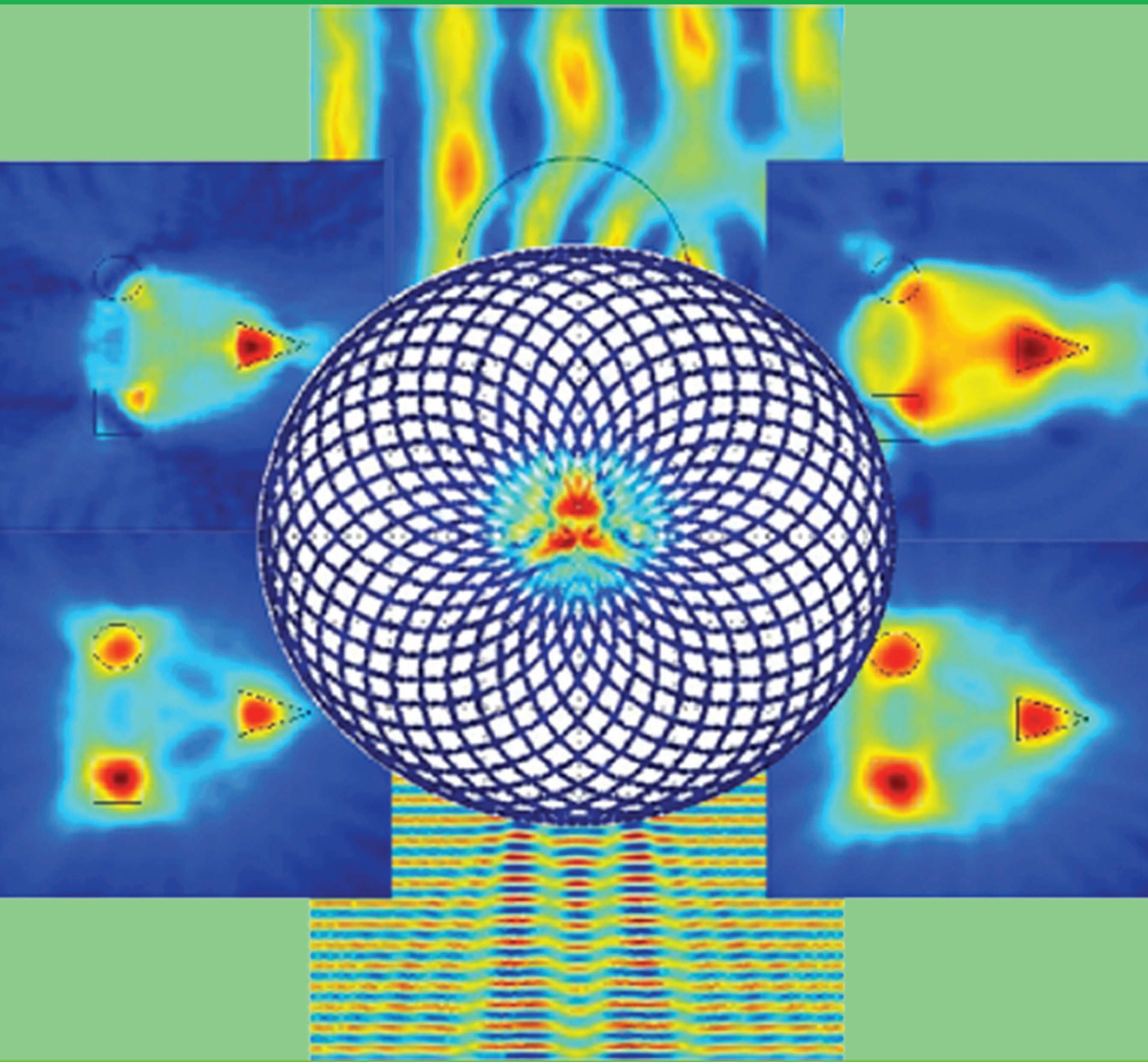
Introduction to Imaging from Scattered Fields



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This book is dedicated to my wife, Carolyn, and our family,
Howard, Aidan, Sam, Colleen, Grant, and Rosa.

Michael A. Fiddy

This book is dedicated to my wife, Julie, and
our family, Kayla, David, Matthew, Ruth, Sarah,
Susanna, Daniel, Aaron, Abbie, and Micaiah.

R. Shane Ritter

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Preface

The objective of this book is to present an overview of the challenging problem of determining information about an object from measurements of the field scattered from that object. This problem is a very old one, since, in a fundamental sense, most of what we perceive and learn about objects around us is a result of electromagnetic or acoustic waves impinging on, interacting with and scattering from those objects. The theoretical formalism of a scattering problem is increasingly complex, as the extent of the interactions increase between the fields with the object. The forward or direct problem generally demands a good model for the anticipated response of the object. Deducing information about the object generally demands knowledge of that model or that (acceptable) approximations can be made to simplify matters. Theoretical approaches to solving inverse problems have been widely studied and as a broad class of problems are known to suffer from concerns over lack of uniqueness and solution stability (ill-conditioning) but, despite modeling a physically well-defined problem, could also be formulated in a way that the very existence of a solution is questionable. In the specific context of inverse scattering theories and algorithms, we present in this text an overview of some of the more widely used approaches to recover information about objects. We consider both the assumptions made *a priori* about the object as well as the consequences of having to recover object information from limited numbers of noisy measurements of the scattered fields.

There is a wealth of literature dealing with scattering and inverse scattering methods for relatively simple structures embedded in a homogeneous background. We introduce the terminology and concepts early in the text and review some important inverse methods. When the scattering is assumed to be "weak," which we define in the text, inversion methods allow more straightforward inverse algorithms to be exploited. We highlight the consequences of the widespread practice of adopting such methods when they are not justified while recognizing their attractiveness from a practical implementation point of view. Assuming weak scattering allows many well-established techniques developed in Fourier-based signal and image processing to be incorporated. The weak scattering models facilitate a simple mapping of scattered field data onto a locus of points in the Fourier domain of the object of interest. More rigorous scattering methods that rely on iterative techniques or strong prior knowledge of the forward scattering model are often slow to implement and may not yield reliable information.

Over the last several years, we have been developing and improving an approach which, while governed by the usual limitations associated with inverse problems, retains many advantages in terms of implementing the weak scattering methods while addressing directly the multiple and strongly scattering phenomena that occur with most objects of interest. The approach is based on a nonlinear filtering step in the inverse algorithm, which requires some preprocessing of the measured data. We illustrate how one can use this algorithm in a very practical way, providing MATLAB® code to help quickly begin applying the approach to a wide variety of inverse scattering problems. We illustrate it using a number of two-dimensional electromagnetic scattering examples.

In later chapters of the book, we draw attention to some very important and often forgotten overarching constraints associated with exploiting inverse scattering algorithms. The inherent lack of uniqueness of a solution to an inverse problem when using finite data requires either implicitly or explicitly that a single solution be selected somehow. A figure of merit or cost function is needed to restore some confidence to the interpretation of the calculated image of the scattering object. The number of measurements made has an obvious and very significant effect on the quality and reliability of an object reconstruction. We explain how considerations of the number of degrees of freedom associated with any given scattering experiment can be found and how this dictates a minimum number of data that should be measured. We argue that estimating the properties of an object from scattered field measurements necessarily requires some prior estimate of the volume from which scattered field data are collected. The use of prior knowledge about the object or properties of the illuminating fields can be used for this purpose to good effect. We describe in detail what we refer to as the prior discrete Fourier transform or "PDFT" algorithm, which accomplishes this. The PDFT restores stability and improves estimates of the object even with severely limited data, provided it is sufficient to meet a criterion based on the number of degrees of freedom.

We have organized this book with graduate students and those practicing imaging from scattered fields in mind. This includes, for example, those working with medical, geophysical, defense, and industrial inspection inverse problems. It will be helpful for readers to have an understanding of basic electromagnetic principles, some background in calculus and Fourier analysis, and preferably familiarity with MATLAB (and possibly COMSOL®) in order to take advantage of the source code provided. The text is self-contained and gives the required background theory to be able to design improved experiments and process measured data more effectively, to recover for a strongly scattering object an estimate that is not perfect, but probably the best that one can hope for from limited scattered field data.

The authors would like to acknowledge their productive collaborations over the years on imaging and inverse scattering with Umer Shahid, Charlie Byrne, Markus Testorf, Bob McGahan, and Freeman Lin.

Supplementary materials including MATLAB code for exercises are available on the book's page at www.crcpress.com. Please visit the site, look up the book and click to the Downloads and Updates tab.

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Michael A. Fiddy received the PhD degree from the University of London in 1977 and was a research fellow in the Department of Electronic and Electrical Engineering at the University College London before becoming a faculty member at the London University (King's College) in 1979. He moved to the University of Massachusetts Lowell in 1987 where he was Head, Department of Electrical and Computer Engineering from 1994 until 2001. In January 2002, he was appointed the founding director of the newly created Center for Optoelectronics and Optical Communications at The University of North Carolina at Charlotte. He has been a visiting professor at the Institute of Optics, Rochester, NY; Mathematics Department, Catholic University, Washington, DC; Nanophotonics Laboratory, Nanyang Technical University, Singapore; and Department of Electrical and Computer Engineering, University of Christchurch, Christchurch, New Zealand. He has also been the Editor-in-Chief of the journal *Waves in Random and Complex Media* since 1996 and holds editorial positions with several other academic journals. He was the topical editor for signal and image processing for the *Journal of the Optical Society of America* from 1994 until 2001. He has chaired 20 conferences in his field, and is a fellow of the OSA, IOP, and SPIE. His current research interests are inverse problems related to super-resolution and meta-material design.

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List of Symbols

CHAPTER 1

e	Euler's constant (2.71828182845905)
$f(x,y)$	Inverse Fourier transform
$F(x,y)$	Fourier transform
k	Wave number ($=2\pi/\lambda$)
k_x, k_y	Spatial frequency
$\boldsymbol{r}_{\mathrm{inc}}$	Radius vector in direction of incident wave
$\boldsymbol{r}_{\mathrm{sct}}$	Radius vector in direction of the scattered wave
$\boldsymbol{R}$	Radius vector
$u(t,s)$	System response
V_{m}	Max or mean of $V(\boldsymbol{r})$
$V(\boldsymbol{r})$	Target in terms of $\boldsymbol{r}$
δ	Delta function
∇	Gradient operator
Λ	Wavelength
π	pi (3.14159265358979)
ϕ	Phase angle
Ψ_{s}	Scattered field
$\Psi_{\mathrm{s}}^{\mathrm{BA}}$	Scattered field in the Born approximation

CHAPTER 2

B	Magnetic induction
c_0	Speed of light in a vacuum ($= (\varepsilon_0\mu_0)^{1/2}$)
D	Electric displacement
$\boldsymbol{E}$	Electric field
$\boldsymbol{E}_0$	Electric field for incident wave
$\boldsymbol{E}_0(\omega)$	Complex electric amplitude
G	Green's function
H	Magnetic field
$\boldsymbol{H}_0(\omega)$	Complex magnetic amplitude
$\boldsymbol{I}$	Unit tensor
$\boldsymbol{J}$	Free current density
$\boldsymbol{J_c}$	Conduction current density
$\boldsymbol{J_s}$	Source current density
J_{su}	Source current density
M	Magnetization
n	Refractive index ($= (\varepsilon_r\mu_r)^{1/2}$)
$\hat{\boldsymbol{n}}$	Unit vector normal to the interface pointing from the input medium into the second medium
P	Polarization
$\boldsymbol{q}$	Dipole moment

V	Volume
ε_0	Electric permittivity
μ_0	Magnetic permeability
μ_r	Relative magnetic permeability $(1 + \chi_m)$
ρ	Free charge density
ρ_{su}	Surface charge density
χ_e	Electric susceptibility
χ_m	Magnetic susceptibility

CHAPTER 4

a	Measure of physical size of target
$f(k\hat{\boldsymbol{r}},\ k\hat{\boldsymbol{r}}_{\text{inc}})$	Scattering amplitude
$f^{\text{BA}}(k\hat{\boldsymbol{r}},\ k\hat{\boldsymbol{r}}_{\text{inc}})$	Scattering amplitude in the Born approximation
$G_0(\boldsymbol{r}, \boldsymbol{r}')$	Green's function in free space
$H_0^{(1)}$	Zero order Hankel function of the first kind
$\hat{\boldsymbol{r}}_{\text{inc}}$	Unit vector that specifies the direction of the incident field
ε_0	Permittivity of free space
$\varepsilon(\boldsymbol{r})$	Target permittivity as a function of radius
ϕ_{inc}	Angle of incident wave with the x-axis
ϕ_s	Angle of scattered wave with x-axis
$\Phi(\boldsymbol{r})$	Complex phase function
$\Phi_s(\boldsymbol{r})$	Complex phase function of the scattered field
$\Psi(\boldsymbol{r})$	Total field in terms of r
Ψ_s	Scattered field
Ψ_{inc}	Solution for the incident wave
Ω	Solid angle over H^2

CHAPTER 6

A_V	Target area
B_V	Target volume
n_{max}	Maximum index of refraction
$N_{\text{2-D}}$	Minimum degrees of freedom required in 2-D
$N_{\text{3-D}}$	Minimum degrees of freedom required in 3-D
Q	Mie measure of scattering cross section $(B_V n_{\text{ma}}/\lambda^2)$

CHAPTER 7

BIM	Born iterative method
CGM	Conjugate gradient method
D	Difference in simulated and measured fields
DBIM	Distorted Born iterative method
f_{PDFT}	PDFT estimator function
$p(r)$	Non-negative prior weighting function
P	Fourier transform of $p(r)$
RRE	Relative residual error
$V_{\text{BA}}^1(\boldsymbol{r})$	First estimate or starting point of the BIM

$X[V_{\mathrm{est}}(\boldsymbol{r})]$	Norm of discrepancy between simulated and measured scattered fields
ξ	PDFT weighted error
τ	Regularization constant
Ψ_s^{sim}	Simulated field
$\Psi_s^{\mathrm{measured}}$	Measured field

I

FUNDAMENTALS

One

Introduction to Inverse Scattering

1.1 INTRODUCTION

Considerable knowledge of the world around us is based on receiving and interpreting electromagnetic and acoustic waves. We extend the bandwidths and sensitivities of our senses by using instruments and collecting radiation from sources and scatterers of radiation. Active illumination or insonification of objects to probe and image their structures is an important tool in advancing our knowledge. However, we need to have a good physical model that describes the possible interactions of those waves with scattering objects. Constitutive parameters (such as permittivity, permeability, refractive index, impedance, etc.) that have spatially and temporally varying properties describe the scattering objects. Wave propagation and scattering characteristics are governed by the fundamental relationships between these properties and their effects on the components of the field, as governed, for example, by Maxwell's equations. In either the electromagnetic case or the acoustical case, we need to derive a wave equation, both in differential or integral form, with appropriate boundary conditions or coefficients, and then analytically or numerically solve that equation to find the field outside the object. This so-called "direct" problem, which assumes that the object parameters are known and scattered fields are to be determined, is itself a nontrivial exercise but well defined.

The complementary or "inverse" problem is much more difficult and is the focus of this book. Making measurements of the scattered field at various locations near or far from the object takes time and effort. One has to specify the incident field properties such as wavelength, polarization, and direction, and then, relative to these, measure the scattered field properties. The question immediately arises as to how many measurements does one need in order to recover the information one wants about the object being probed. Inverting the governing wave equation is, from a purely mathematical perspective, the so-called ill-posed problem. Such problems require that one formally establish the following:

1. Whether there is a solution at all.
2. Whether the solution, should it exist, is unique.
3. Whether a calculated solution is or is not ill conditioned.

In most practical situations, one only measures a finite number of data on the scattered field, and uniqueness is impossible. One can fit an infinite number of functions (i.e., images) to a finite data set. This lack of uniqueness requires that we either explicitly or implicitly adopt a uniqueness criterion such as minimum energy, maximum entropy or some other such criterion using which one can define a unique solution and hope that it has a physical meaning.

In some imaging applications, one cannot measure the scattered field itself, for example at very high frequencies. Above ~1 THz we do not have detectors fast enough to measure the fluctuating field and all we acquire is a time averaged quantity. In electromagnetic problems we assume this is proportional to the magnitude squared of the (electric) field, $|E|^2$. As we shall see, the information required to solve the inverse problem and calculate an image of the object demands that we solve another problem, namely that of estimating from $|E|$, the function $E = |E| \exp(i\phi)$ or solve the so-called phase-retrieval problem (ϕ denotes phase). This is also nontrivial and, without knowledge of the phase, the information we can recover about the object is severely limited and at best statistical in nature.

Most problematic is the inevitable presence of noise in our measured data. Inverse procedures, as we shall see in the coming chapters, are always ill conditioned. This means that one can expect small changes in the data as a result of noise to lead to very large differences in our images. The instability of inverse methods can be understood mathematically and remedied using the so-called regularization techniques. The price to be paid to control ill conditioning is a degradation of the image, for example, a loss of resolution. However, since we cannot guarantee a unique solution in practice, we have to accept further compromises in order to obtain an image we can have some confidence in.

From a practical standpoint, we hope to collect the minimal amount of data to provide the image quality needed for the task at hand. Maps of spatially varying contrast might suffice while, for other purposes, for example, in medical imaging, a quantitatively accurate map of a constitutive parameter such as impedance might be essential. Overarching all of these issues is the more important problem of the governing equation to be inverted being inherently nonlinear in nature. The scattered field for all but the weakest scattering objects depends on the complexity of the scattering processes that occur within the object itself. For inverse problems, for by very definition we do not know the structure of the object, we cannot know *a priori* the extent of multiple scattering that occurs within it. We can define what we mean by "weakly" scattering, and that assumption, while rarely valid in practice, does lead to a more tractable inversion method but one that still suffers from the questions of uniqueness and ill conditioning mentioned above. For more interesting, but strongly scattering objects, we need to address the nonlinear aspect of the inverse problem. We will describe methods that do this but emphasize now that there are, at the time of this writing, still no fast and reliable methods one can take off the shelf and use. Indeed, despite many decades of effort, inverse scattering methods remain very challenging and an active field of research. Methods we describe here have a range of applicability that limits their use to situations for which some prior knowledge about the object is available. This is certainly possible in some applications such as imaging a limb or probing a suitcase, and prior knowledge can play an important role in addressing the uniqueness question, as we shall see.

1.2 INVERSE SCATTERING PROBLEM OVERVIEW

The wavelength of the radiation used with respect to the scale of the features one wishes to image provides a convenient way to segregate inverse scattering problems. In the limit of the wavelength becoming relatively small, geometrical optics or ray-based approximations become reasonable. In the very high frequency limit, for example, when using x-rays, one can assume that the radiation emerging from an object has not been refracted at all, and the measured data are interpreted as a shadow of the attenuation in the object. The mathematics describing this is well established, dating back to Radon (1986). Johann Radon's original paper was published in 1917 (Radon, 1917). The Fourier transform plays an important role here and throughout this book (see Appendix A). The technique of computed tomography, which incorporates a Radon transform (Wolf, 1969), uses projection data which measures the line integral of an object parameter, for example, of $f(x,y)$ in the equation shown below, along straight lines (y-axis in this example). This enables the Fourier Slice Theorem to be used to build up information about $F(k_x,k_y)$ by rotating the illumination direction.

$$F(k_x,0) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{i(k_x x)}\,\mathrm{d}x\,\mathrm{d}y = \int_{-\infty}^{+\infty}\left\{\int_{-\infty}^{+\infty} f(x,y)\,\mathrm{d}y\right\}e^{i(k_x x)}\,\mathrm{d}x \tag{1.1}$$

where k_x and k_y are the spatial frequency variables that have units of reciprocal distance, that is, $k_x x$ is dimensionless. When object constitutive parameter fluctuations or inhomogeneities such as refractive index fluctuations in a semitransparent object are comparable in size to the interrogating wavelength, then scattering or diffraction effects become significant. As we shall see, Fourier data on the object are still obtainable in this situation provided the Born or Rytov approximations are valid. We will describe these approximations, which allow inversion algorithms to be formulated, and we will discuss in detail the criteria for their validity. When inverting Fourier data there is the question of how to make the best use of the limited set of noisy samples available. At optical frequencies, there is also an additional problem: that the phase of the scattered field may only be measured with difficulty. Some methods for phase retrieval are discussed in Appendix B.

Usually, approximations are employed to make the scattered fields (which can be expressed by Fredholm integral equations of the first kind) more tractable for numerical computation. The merits of the Born and the Rytov approximations, and more sophisticated techniques derived from them, have spawned a lot of controversy over the years. A principle cause for controversy is that these approximations are based on the interpretation given when strong inequalities are met, in order to simplify (or linearize) the governing equation. The physical interpretation of imposing these inequalities can be rather subjective. It is also problematic that sometimes these approximations appear to provide reasonably good images when one might not expect them to. This issue also illustrates one of the cautionary messages to be conveyed when working with inverse problems, which is that deliberate or inadvertent inverse crimes can be committed! These are crimes by which, because of the difficulty of acquiring real data from known objects with which to test an inversion method, the direct problem is solved to generate data. Occasionally approximations made in solving the direct problem are the very ones employed in the inverse method, thereby increasing the chances that the recovered image will look good. Consequently, we spend some time in this book describing the importance of understanding the nature of scattered field data used to validate imaging algorithms and suggest methods to generate such data. This of course is only necessary in the absence of real measured data from known targets, but despite the best efforts of many, real data sets are still few and far between. Data provided since the early 1990s by the US Air Force and the Institut Fresnel (Belkebir and Saillard, 2001, 2005) have done a tremendous service in providing high quality data from known objects, which provides a means for comparing different inverse scattering techniques and thereby improves them. For a scatterer of compact support (with d as the size of its largest dimension), the qualitative statement is usually made that the (first) Born approximation is valid only when the scatterer is "small" on the scale of the incident wavelength; this is discussed in more detail in Chapter 4.

The Born series solution to the integral equation of scattering is an infinite series which is traditionally defined as only valid when the criterion $kV_m d < 1$ is met. Here, k is the wavenumber $k = 2\pi/\lambda$ where λ is a measure of the wavelength *inside* the scattering object. This is obviously difficult to determine for an unknown object's constitutive parameter $V(\boldsymbol{r})$ that is being imaged. V_m is some measure of the maximum or mean value of $V(\boldsymbol{r})$ which is also unknown; consequently there is a temptation to apply the first Born approximation. This requires that $kV_m d \ll 1$ and, as we shall see, makes recovering an image computationally straightforward. Indeed it reduces the inverse scattering problem to one of a limited-data Fourier estimation problem. This is a problem on which there is much written, and it provides a comfort zone in which to work and process scattered field data, in the (vain) hope that images obtained when $kV_m d$ is not less than 1 still convey some meaningful information. The criterion for the validity of the Rytov approximation is equally vague, relying on the qualitative statement that spatial fluctuations in V be slow on the scale of the wavelength, but that the magnitude of the fluctuations of V need not necessarily be small or of low contrast. In other words, this physical interpretation of the validity of the Rytov approximation is based on the requirement that the absolute value of the rate of change of the complex phase of the scattered field within V be small compared with $k\nabla V$, where ∇ is the gradient operator. If this assumption is reasonable, one can formulate the inverse Rytov method as a limited-data Fourier estimation problem as well.

An interesting and important question to ask is what errors are introduced if one does adopt the Born or Rytov approximation. This is a very reasonable and insightful step to take and doing so has revealed classes of objects for which one can expect the approximations to do poorly or fail altogether. There is also much to be said for bringing to the inverse scattering problem a wealth of signal and image-processing knowledge that has been established over the years for dealing with limited data, especially limited Fourier data. By more carefully formulating the inverse problem in terms of these approximations and having a description for the errors and artifacts the "first Born approximate" image might possess, one can develop methods to postprocess those images to try to recover $V(\boldsymbol{r})$. This is the approach we have adopted and will describe in more detail in a later chapter.

These inverse scattering algorithms that have been developed over the years, often referred to as diffraction tomography algorithms, fall into two classes. Devaney (1983) and Pan and Kak (1983) have modified the filtered back-projection algorithm used in conventional tomography to give a filtered