MECHANICS OF MACHINERY

Dr. Mahmoud A. Mostafa

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Dr. Mahmoud A. Mostafa
University of Alexandria, Egypt

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Dedication

This text is dedicated to my recently born grand child Yahya Omar Mostafa

Contents

Foreword

It is not an easy task to write a foreword for a book authored by Professor Mostafa, who taught me the principles of machine dynamics and vibrations. In the course of his teaching and research career, Professor Mostafa has been the teacher and mentor to his students. He kept updating his methods of instruction according to the continuous developments in his field of expertise.

This book is the pinnacle of Professor Mostafa's contributions over the years. The chapters are based on his lectures at several universities in Egypt and the Middle East. The book covers both the graphical and analytical methods of the kinematics and dynamics of different types of mechanisms with low and high pairs. It presents new analytical approaches, which are helpful in the programming, and the kinematic and dynamic analysis of mechanisms and cams.

The book also presents new topics such as the analytical plot of cam contour, minimum cam size, and in-place balancing.

I am sure that both academia and the industry will benefit much from this book. The new topics, lucid language, and step-by-step examples are all assets to its success.

Professor Sohair F. Rezeka
Mechanical Engineering Department
Alexandria University, Egypt

Preface

This book is intended to serve as a reference for students in the mechanical field and practicing engineers, and is concerned with the analysis of machines.

This book discusses the kinematics and dynamics of mechanisms. It is intended as an informative guide to a more complete understanding of kinematics and its applications. It is hoped that the fundamental procedures covered here will transfer to problems the reader may encounter later.

This book was developed from an earlier version published in 1973. That early version was based on graphical analysis, which did not meet the requirements of modern developments. This version includes analytical analysis for all the topics. These analytical analyses makes it possible to use math software for fast, precise, and complete analysis.

Chapter 1 introduces several mechanisms to familiarize the reader with different motions and functions they can perform. Analytical analysis for the performance of the mechanism is also presented, which is adapted to use math software to facilitate the study of the performance of mechanisms.

Chapter 2 deals with the study of velocities and accelerations in the mechanism. This is a necessary step for the design of machines. The graphical method, which is based on vector equations, is presented and is applied to different mechanisms. The graphical method gives insight to the velocities and accelerations for members in the mechanism. Also, analytical analysis is presented and adapted for use with math software for an overall study of the mechanisms. One distinct feature of this book is the analysis of sliding links using a theory developed by the author. It is a replacement for Coriolis components, which are generally difficult to apply in most cases.

The subject of cams is presented in Chapter 3. For specified motion cams, the profile is obtained by graphical method. To obtain the contour analytically, equations in Cartesian coordinates, which was developed by the author, is presented. Special emphasis is directed toward the factors affecting the cam design, such as the pressure angle and the radius of curvature.

Chapters 4 through 6 are devoted to giving a realistic study of the geometry and kinematics of all types of gears. The study of gear reduction units is very important for machine application.

Chapter 7 is concerned with the study of force analysis in mechanisms. Force analysis is divided into three parts—static force analysis, friction force analysis, and dynamic force analysis. In this book, the traditional graphical method is used in addition to the analytical method. The analytical method lays down the foundation for using math software to perform the analysis. Programs using MathCAD are presented for complete analysis of all kinds of mechanisms, which include position analysis, velocities and acceleration analysis, and force analysis. This chapter also includes the study of the torque variation and the use of flywheels to reduce the speed variation.

Chapter 8 covers the study of balancing of machines. It explains how to balance rotating parts and reciprocating parts. In-place balancing of machines using vibration measurements is also presented.

Professor Mahmoud Mostafa
Mechanical Engineering Department
Faculty of Engineering
University of Alexandria
Alexandria, Egypt

Acknowledgments

The author wishes to express his gratitude to the coauthors, Professor Nomaan Moharem, late Professor Elssayyed Elbadawy, and late Professor Hasan Elhares, of the previous version who encouraged the idea of making a complete version, with updated materials, in the field of “Mechanics of Machinery.”

I would like to express my deep appreciation to my wife Suzy, my sons Hosam and Omar, and my daughter Gina for their support and their help and encouragement to finish this book. So, this book is also dedicated to them.

Also, I would like to thank my students of the Faculty of Engineering, University of Alexandria, for their help to rectify some materials in the book.

Finally, I would like to express my appreciation to the editorial staff of Taylor & Francis Group for their encouragement. Their comment that my book will be a very good addition to their list made me proud.

Author

Dr. Mahmoud Mostafa is currently a professor in the mechanical engineering department, Faculty of Engineering, University of Alexandria. He received his BSc degree in mechanical engineering from the University of Cairo. He also received his MSc and PhD degrees from Oregon State University. He held several positions such as research engineer in E.I. Du Pont Company, USA, June 1964; visiting professor in University of Riyadh, Saudi Arabia; and visiting professor in Beirut Arab University. Dr. Mostafa was the head of the mechatronics department at Alexandria High Institute of Engineering and Technology and is still supervising the department. His area of research is in the fields of mechanical vibrations, kinematics of machinery, and synthesis of mechanisms. He was a consultant for several corps in Egypt. He attended international conferences in the United States, Canada, Pakistan, and Egypt. He has several inventions, which incorporate certain mechanisms.

1 Mechanisms

1.1 DEFINITIONS

Machine: A machine, according to Reuleaux, is a combination of resistant bodies (rigid, elastic, or fluid) so arranged that by their means the mechanical forces in nature can be compelled to produce some effect or work accompanied by certain determinate motions. Figure 1.1 shows a cross section of a single-cylinder engine (or compressor). For an engine, a mixture of air and vapor of flammable fluid enters the cylinder, is ignited, pushes the piston, and the connecting rod, which in turn causes the crank to rotate. Thus, the engine transmits the gas force to be a torque on the crank. Also, it converts the reciprocating motion of the piston to a rotary motion for the crank. The function of the compressor is opposite to the engine.

Mechanism: A combination of bodies meant for transmitting, controlling, or constraining the relative motion between the bodies. If we look at a machine only from the point of view of motion, then it is a mechanism. Figure 1.2 shows the skeleton outline of an engine and is considered to be a mechanism.

Planar and spatial mechanisms: Mechanisms can be divided into planar mechanisms and spatial mechanisms according to the relative motion of rigid bodies. In planar mechanisms, all the relative motions of rigid bodies are in one plane or in parallel planes. If there is any relative motion between the bodies that is not in the same plane or in parallel planes, the mechanism is called a spatial mechanism. In other words, planar mechanisms are essentially two dimensional, whereas spatial mechanisms are three dimensional. This chapter covers only planar mechanisms.

Kinematics: Kinematics of mechanisms is concerned with the motion of the parts without considering the actual shape of the bodies or the forces in a machine. In other words, kinematics deals with the motion, velocity, and acceleration of the parts.

Kinetics: Kinetics deals with all the forces in a machine, including the forces resulting from the masses and the acceleration.

Dynamics: Dynamics is a combination of kinematics and kinetics.

Links: A link is defined as any part of a machine having motion relative to some other part. It must be capable of transmitting a force. There are three types of links:

1. Rigid links, which may transmit tension or compressive forces such as crank, connecting rod, and piston
2. Tension links, which transmit only tensile forces such as belts, ropes, and chains
3. Compression links, which transmit only compressive force, for example, the fluid in hydraulic jacks or the automobile braking system

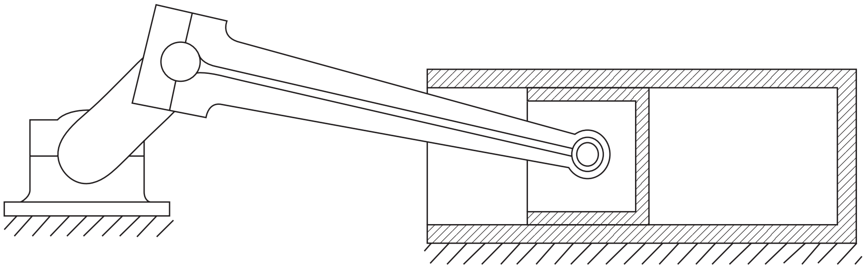


FIGURE 1.1 A cross section of a single-cylinder engine.

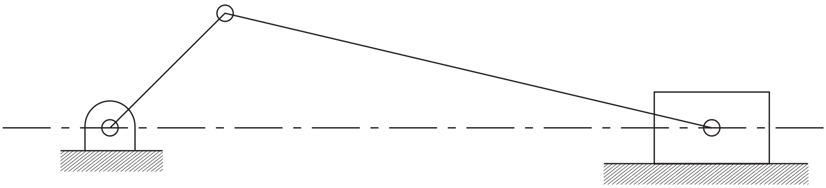


FIGURE 1.2 The skeleton outline of an engine.

Frame: The frame in a mechanism is considered as a fixed link. In a machine, the frame is considered as all the fixed bodies connected together by welding or bolting.

Pairs: A pair is a joint between the surfaces of two rigid bodies which keeps them in contact and to have relative motion. Pairs are divided into two types, that is, lower pairs and higher pairs.

Lower pairs: A joint between two bodies is defined as a lower pair when the contact between them is on a surface. There are two types of lower pairs in plane mechanisms: One is the revolute joint as the case of doors (Figure 1.3a). In the study of mechanisms, the revolute joint is represented as skeletons (Figure 1.3b and c). The other type of lower pairs is the prismatic (sliding) joints as in the case of drawers and the ram of the shaping machine. The skeleton outline of these joints is shown in Figure 1.4a and b.

Higher pairs: In higher pairs, the contact between two bodies is through a point as in the case of ball bearings or through a line as in roller bearings, gears, cams, and cam followers. The skeleton outline of this type of pairs is shown in Figure 1.5a. In fact, a higher pair can be considered as two lower pairs, that is, sliding and revolute pairs (Figure 1.5b).

Other types of joints:

Spherical joint: A ball and a socket represent a spherical joint (Figure 1.6). The shift stick in an automobile is an example for the spherical joint. The handle can move in all directions.

Screw joint: Bolts and nuts are examples of screw joints. Power screws and screw jacks are other examples (Figure 1.7).

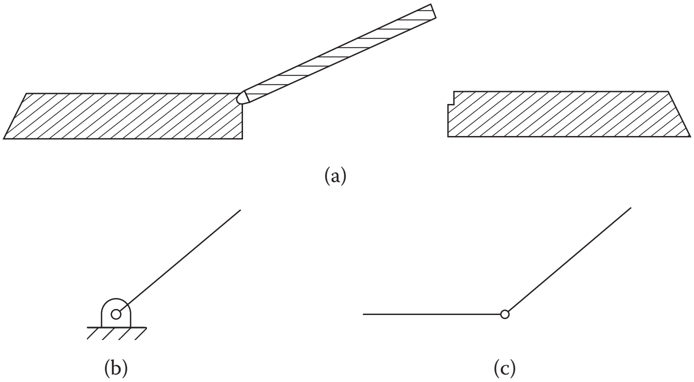


FIGURE 1.3 (a) Represents a door connection, (b) and (c) represent the revolute joints.

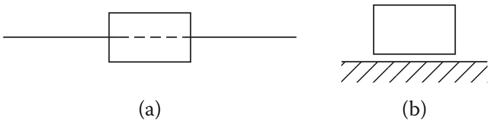


FIGURE 1.4 Representation of prismatic joint. (a) Link slides inside another (b) link slide on the surface of another.

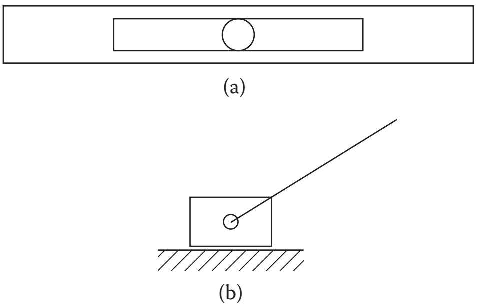


FIGURE 1.5 (a) Represents a higher pair joint, (b) represents the equivalent higher pair joint.

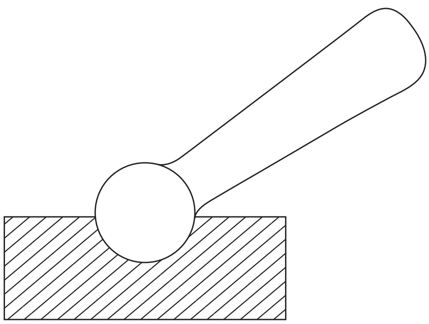


FIGURE 1.6 Spherical joint.

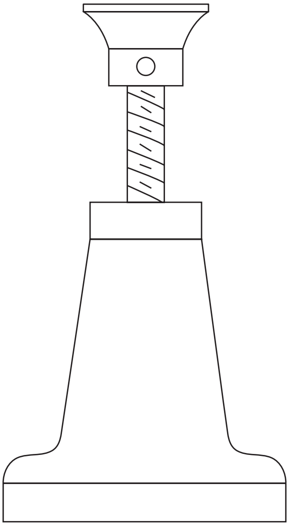


FIGURE 1.7 Screw joint.

Complete, incomplete, and successful constraints:

- Complete constraint determines in a definite direction the relative displacements between two links independently of the line of action of the impressed force, for example, a square bar sliding in a square hole.
- In the case of incomplete constraint, a little change in direction of the impressed force may alter the direction of the relative displacement, for example, a cylinder in a hole. The cylinder may rotate and slide inside the hole.
- In the case of successful constraint, an external force, for example the force of gravitation or a force applied to a spring or fluid, is impressed on an element to prevent motions other than the desired relative motion within the limits of the displacement. For example, the relative motion between the piston and the cylinder of an engine is not completely constrained. But the connecting rod between the piston and the crank prevents the piston from rotating inside the cylinder.

Kinematics chain: Kinematic chains are combinations of links and pairs without a fixed link. If one of the links is fixed, we get a mechanism. All links have at least two pairs. The relative motion between the links is completely constrained.

Kinematics analysis: Kinematics analysis is the investigation of an existing mechanism regarding its performance and motion and estimating the velocity and acceleration of its links.

Kinematics synthesis: It is the process of designing a mechanism to accomplish a desired task. It is involved with choosing the type and dimensions of the mechanism to achieve the required performance.

Degrees of freedom (DOFs): The number of DOFs of a system is defined as the number of independent relative motions among the rigid bodies of the system.

For example, for a revolute pair (Figure 1.3b and c), relative motion between the links is a rotational motion about the joint. So, the revolute pair has only one DOF. This applies to prismatic pairs also (Figure 1.4) for which the relative motion is sliding motion. For higher pairs (Figure 1.5a), the number of DOFs is two. This is because the joint allows both rotational and translational motions as demonstrated in Figure 1.5b. For a spherical joint (Figure 1.6), the motion is not restricted to certain directions. Thus, it has infinite DOFs.

If the number of DOFs of a chain is zero or negative, then it forms a structure, that is, there is no relative motion between the links.

The number of DOFs of a mechanism is also called the mobility of the device. Mobility of a device is the number of input parameters (usually pair variables) that must be independently controlled to bring the device into a particular position.

Important: The number of pairs at a joint is equal to the number of links connected to the joint subtracted by one.

1.2 DEGREES OF FREEDOM OF PLANAR MECHANISMS

The number of DOFs of a mechanism can be estimated by using Gruebler's equation, which is written in the following form:

$$\text{DOFs} = 3(n-1) - 2l - h$$

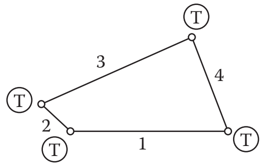
Where

DOF is the number of degrees of freedom in the mechanism.
n is the number of links including the fixed link.
l is number of lower pairs.
h is the number of higher pairs.

EXAMPLE 1.1

Find the number of DOFs for each of the following chains:

Figure (a)

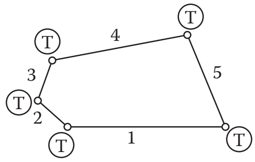


(a)

$n = 4$
$l = 4$
$h = 0$

$$\begin{aligned}\text{DOFs} &= 3\times(4-1) - 2\times 4 - 1\times 0\\ &= 1\end{aligned}$$

Figure (b)

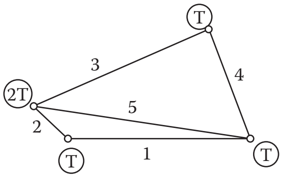


(b)

$n = 5$
$l = 5$
$h = 0$

$$\begin{aligned} DOF &= 3 \times (5-1) - 2 \times 5 - 1 \times 0 \\ &= 2 \end{aligned}$$

Figure (c)

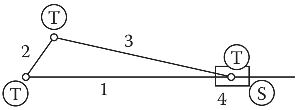


(c)

$n = 5$
$l = 6$
$h = 0$

$$\begin{aligned} DOF &= 3 \times (5-1) - 2 \times 6 - 1 \times 0 \\ &= 0 \end{aligned}$$

Figure (d)

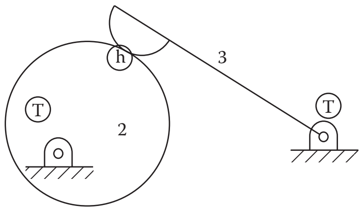


(d)

$n = 4$
$l = 4$
$h = 0$

$$\begin{aligned} DOF &= 3 \times (4-1) - 2 \times 4 - 1 \times 0 \\ &= 1 \end{aligned}$$

Figure (e)



(e)

$n = 3$
$l = 2$
$h = 1$

$$\begin{aligned} DOF &= 3 \times (3-1) - 2 \times 2 - 1 \times 1 \\ &= 1 \end{aligned}$$

EXAMPLE 1.2

Find the number of DOFs for the mechanism shown in Figure 1.8.

$n = 8$
$l = 10$
$h = 0$

$$\begin{aligned} DOF &= 3 \times (8-1) - 2 \times 10 - 1 \times 0 \\ &= 1 \end{aligned}$$

Note that,
T denotes a turning joint.
S denotes a sliding joint.

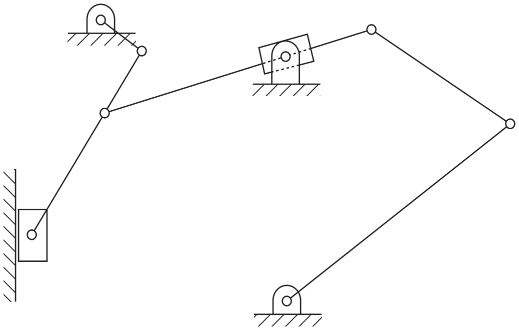


FIGURE 1.8 Examples for degrees of freedom.

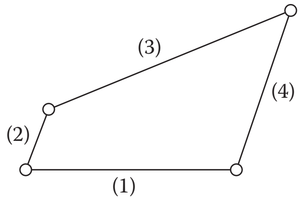


FIGURE 1.9 Four-link chain.

1.3 FOUR-REVOLUTE-PAIRS CHAIN

The four-bar linkage is the simplest of all chains. It is often used to construct very useful mechanisms. Consider a four-link chain with four revolute pairs (Figure 1.9). Consider that the links have different lengths, r_1, r_2, r_3, and r_4. Grashof's theorem states that a four-bar mechanism has at least one revolving link if

$$r_1 + r_2 < r_3 + r_4$$

Also, the three moving links rock if

$$r_1 + r_2 > r_3 + r_4$$

For this chain, if we fix one link at a time we obtain, in a general sense, several mechanisms that may be different in appearance and in the purposes for which they are used. Each mechanism is termed an inversion of the original kinematics chain.

1.3.1 Four-Bar Mechanism

A four-bar mechanism (Figure 1.10) is obtained by fixing link (1) in the four revolute chains shown in Figure 1.9. This mechanism transfers the rotary motion of one link to an oscillatory motion for another link or vice versa. The links of the four-bar mechanism are denoted as follows:

Link (1) is called the frame.
Link (2) is called the crank.
Link (3) is called the coupler.
Link (4) is called the rocker.

1.3.1.1 Performance of the Four-Bar Mechanism

Referring to Figure 1.10,

- Link (2) makes a complete revolution
- Link (4) oscillates through an angle β, called the rocking angle. These motions are assured if the following conditions are applied:
- According to the extreme right position,

$$r_2 + r_3 < r_1 + r_4$$

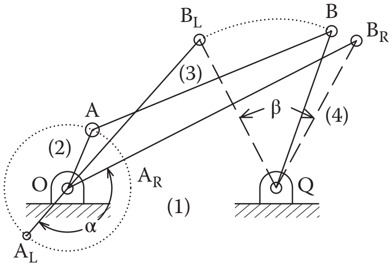


FIGURE 1.10 The four-bar mechanism.

- According to the extreme left position,

$$r_3 - r_2 < r_1 + r_4$$

From the previous two conditions, we can deduce that

$$r_3 < r_1 + r_4$$

- When the crank coincides with link OQ to the right,

$$r_1 - r_2 < r_3 + r_4$$

- When the crank is along link OQ to the left,

$$r_1 + r_2 < r_3 + r_4$$

From these two conditions we can deduce that

$$r_1 < r_3 + r_4$$

- The extreme right position, point B_R of the rocker, is when the coupler is along the crank and the crank is at point A_R.
- The extreme left position, point B_L of the rocker, is when the coupler coincides with the crank at point A_L.
- The crank rotates through an angle α when the rocker moves from the extreme left position to the extreme right position, assuming that the crank rotates counterclockwise.
- The crank rotates through an angle $2\pi - \alpha$ when the rocker moves from the extreme right position to the extreme left position.
- If α is not equal to π, the motion of the rocker is described as quick return motion. That is, the rocker moves faster when going from left to right than when going back. The ratio of the two angles, assuming the crank rotates with uniform speed, is called the time ratio, λ:

$$\lambda = \frac{2\pi - \alpha}{\alpha}$$

- The static driving force is transmitted from the crank to the rocker through the coupler. This force is either tension or compression. The angle between the rocker and the coupler (angle ABQ) is called the transmission angle. The torque transmitted to the rocker has a maximum value when this angle is $\pi/2$. When this angle deviates from $\pi/2$, the torque transmitted to the rocker decreases. It is advisable to keep the value of this angle as close to $\pi/2$ as possible.
- The motion of the mechanism is traced by the following steps (Figure 1.11):
  - Draw a circle with radius equal to the length of the crank and center at point O.
  - Divide the circle to an equal number of divisions. The more divisions the more accurate results.
  - Draw an arc of a circle with radius equal to the length of the rocker and center at point Q.
  - At each point A on the circle, line OA makes an angle θ with the horizontal position of the crank, draw an arc with radius equal to the length of the rocker to intersect the arc of the rocker at point B.
  - Measure the angle of line QB, that is, angle φ in Figure 1.11.

The relation between the output angle φ and the input angle θ is shown in Figure 1.12. The angles are measured from the horizontal datum.

1.3.1.2 Coupler Curves

A point on a coupler link traces a curve (Figure 1.13). Tracing is carried out by using the steps described as in Section 1.3.1.1 and then locating the position of point C at different locations. By changing the position of this point, we can obtain a vast number of curves that may be helpful in several mechanical engineering applications.

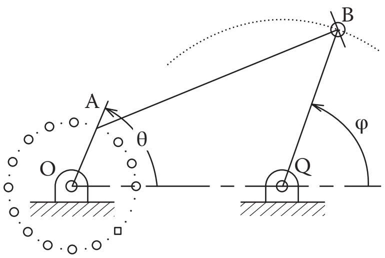


FIGURE 1.11 Tracing the four-bar mechanism.

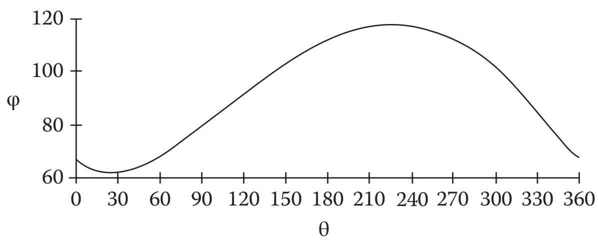


FIGURE 1.12 Relation between the rocker angle and the crank angle.

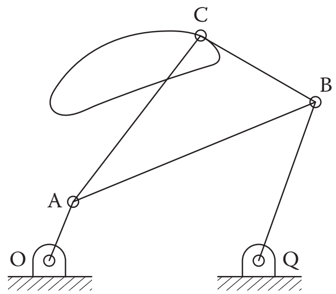


FIGURE 1.13 The coupler curve.

Important: The process of tracing a mechanism is tedious. However, the analytical method which will be explained in this chapter is much simpler and more powerful than tracing.

1.3.1.3 Synthesis of a Four-Bar Mechanism

It is interesting to design a four-bar mechanism to give a certain performance. This is explained in the following examples.

EXAMPLE 1.3

Design a four-bar mechanism such that the length of the crank (r_2) is 30 mm, the length of the fixed link (r_1) is 100 mm, the rocking angle (β) is 60°, and the time ratio (λ) is 1.

Solution

In Figure 1.10, at the extreme right position, line A_RB_R is along line OB_R. Also, line A_LB_L is along line OB_L. Since the time ratio (λ) is 1,

$$\lambda = \frac{2\pi - \alpha}{\alpha} = 1$$

$$\alpha = \pi = 180°$$

Since α = 180°, lines OB_L and OB_R coincide. Also, the distance B_LB_R is twice the length of the crank. We use this data and proceed with the following steps:

Procedure

1. Draw line B_LB_R with length = 60 mm.
2. Draw lines QB_L and QB_R, which are equal and make an angle 60°. Line QB_R is equal in length to the rocker.
3. Draw an arc of a circle of radius 100 mm (the length of the fixed link) to intersect line B_RB_L extended at point O. Point C is in the middle of line B_LB_R. Line OC represents the coupler.

Therefore, the lengths of links of the four-bar mechanism in Figure 1.14 are as follows:

$$r_1 = 100\,\text{mm},\ r_2 = 30\,\text{mm},\ r_3 = 85.4\,\text{mm},\ r_4 = 60\,\text{mm}.$$

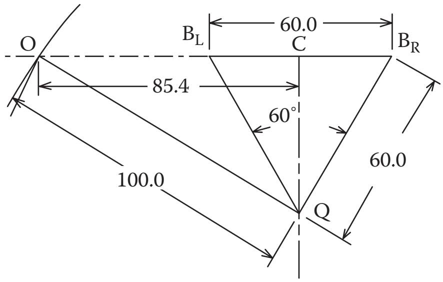


FIGURE 1.14 Graphical solution of Example 1.3.

EXAMPLE 1.4

Design a four-bar mechanism such that the length of the fixed link (r_1) is 80 mm, the length of the rocker (r_4) is 60 mm, the rocking angle (β) is 90°, and the time ratio (λ) is 1.4.

Solution

Since the time ratio (λ) is 1.4,

$$\lambda = \frac{2\pi - \alpha}{\alpha} = 1.4$$

$$\alpha = 150°$$

In Figure 1.10, the angle between lines OB_R and OB_L is equal to $180° - \alpha = 30°$.

Procedure

1. Draw lines QB_L and QB_R each of length 60 mm. Angle B_LQB_R is equal to 90°. These two lines represent the rocker at the two extreme positions as in Figure 1.15.
2. Draw lines $Q'B_L$ and $Q'B_R$ such that the angle $B_LQ'B_R$ is equal to 30°.
3. Draw a circle passing through points B_L, Q', and B_R. We should bear in mind that lines from points B_L and B_R to any point on this circle make an angle equal to 30°.
4. From point Q, draw an arc of a circle of radius 80 mm (the length of the fixed link) to intersect the circle at point O.

Line OB_R represents $r_3 + r_2 = 130.4$ mm
Line OB_L represents $r_3 - r_2 = 58.7$ mm
Therefore, the lengths of links are as follows:

$$r_1 = 80\,\text{mm},\ r_2 = 40.850\,\text{mm},\ r_3 = 89.550\,\text{mm, and } r_4 = 60\,\text{mm}.$$

Synthesis can also be performed for coupler curves by satisfying certain precision points [19] or for specific outputs [59], which is not within the scope of this chapter.

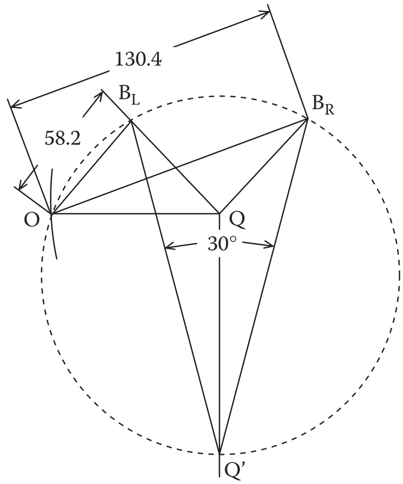


FIGURE 1.15 Graphical solution of Example 1.4.

1.3.2 Drag (Double Rotating) Link Mechanism

If the shortest link in a chain, link (2) in the chain shown in Figure 1.9, is fixed, we obtain a mechanism in which two links rotate continuously (Figure 1.16). This condition is ensured by satisfying the following conditions:

- When link (4) is horizontal to the right,

$$r_1 + r_4 < r_2 + r_3$$

- When link (2) is horizontal to the left,

$$r_1 + r_2 < r_3 + r_4$$

This mechanism is called double-crank mechanism or commonly named as drag link mechanism. It is usually used as a part of compound mechanisms to obtain certain performance, as will be explained later in Section 1.8.1.1.

1.3.3 Double-Rocker Mechanism

If link (2) in the chain shown in Figure 1.9, which is the shortest link, is used as a coupler and link (3) is fixed, we obtain a mechanism in which the other two links oscillate, as shown in Figure 1.17. This condition is ensured by satisfying the following conditions:

- When link (4) is at the extreme right position,

$$r_1 + r_4 < r_2 + r_3$$

- When link (2) is at the extreme left position,

$$r_1 + r_2 < r_3 + r_4$$

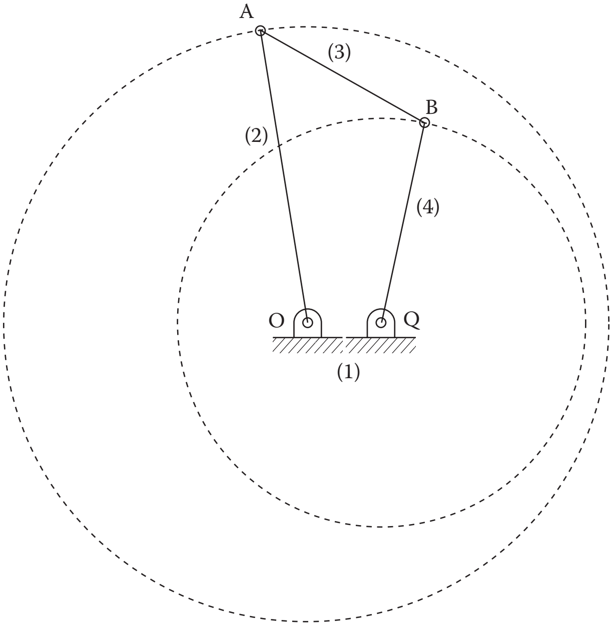


FIGURE 1.16 Drag link mechanism.

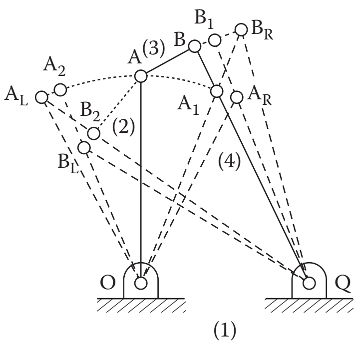


FIGURE 1.17 The double rocker mechanism.

1.3.3.1 Performance of the Double-Rocker Mechanism

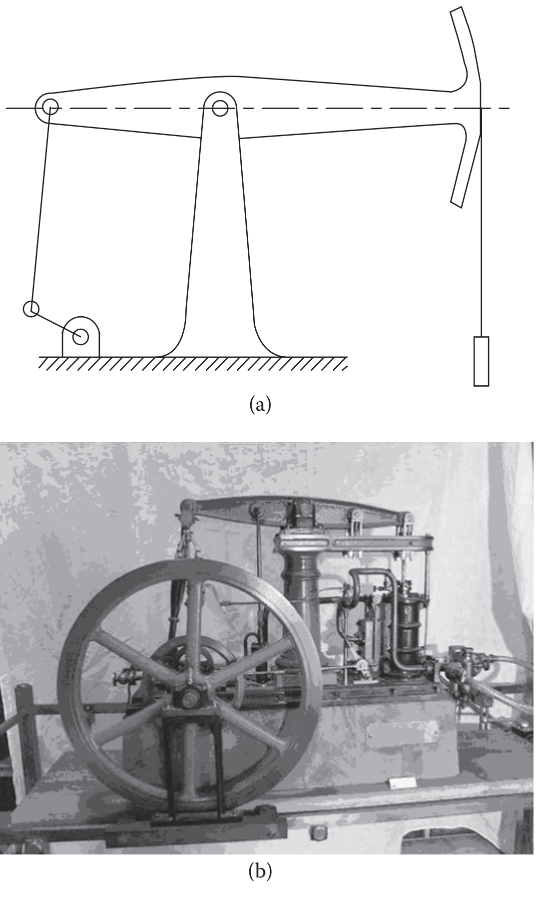
- We start the trace by moving link (4) to the right. It reaches its extreme right position when link (3) is along link (2). Point B becomes point B_R and point A becomes point A_1.
- From this position, link (2) keeps moving to the right while link (4) starts to move to the left. Link (2) reaches its extreme right position when link (3) coincides with link (4). At this position, point B becomes B_1 and point A becomes point A_R.

- From this position, link (4) keeps moving to the left dragging link (2) behind it. Link (4) reaches its extreme left position when link (3) coincides with link (2). At this position, point B becomes point B_L and point A becomes point A_2.
- From this position, link (4) moves to the right while link (2) keeps on moving to the left. Link (2) reaches it extreme left position when link (3) is along link (4). At this position, point B becomes point B_2 and point A becomes point A_L.

The motion is repeated as described.

1.3.4 Applications Based on Four-Bar Linkages

There are many practical applications that are based on four-bar linkages. Some of them are listed in Sections 1.3.4.1 through 1.3.4.3 and some others are listed later.



(a)

(b)

FIGURE 1.18 The beam engine. (a) Skeleton outline of a beam engine used in deep oil wells (b) photograph of the engine.

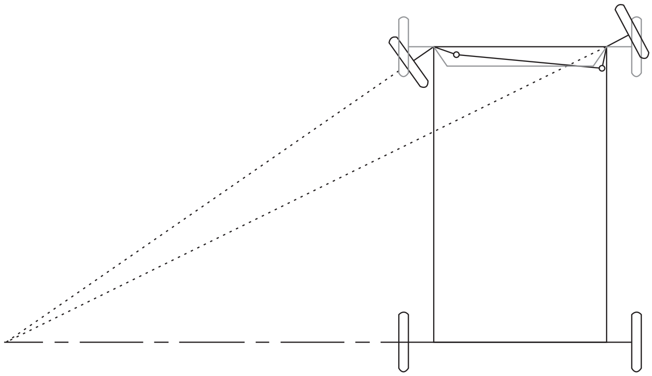


FIGURE 1.19 Ackermann steering mechanism.

1.3.4.1 Windshield Wiper of Automobiles

The oscillating motion of automobile wipers is achieved by using a four-bar mechanism.

1.3.4.2 Beam Engine

The skeleton outline of the beam engine used in deep oil wells is shown in Figure 1.18a. A photograph of the engine is shown in Figure 1.18b.

1.3.4.3 Automobile Steering Mechanism

This mechanism is essential for vehicles. During turns, if the steered wheels, usually the front wheels, of a vehicle are kept parallel, each wheel will have a different center of rotation. This will cause slip in the wheels, accelerating their damage. The correct situation is to make the whole vehicle rotate around one center only (Figure 1.19). This is accomplished by adjusting the angles of rotation of the front wheels. This is accomplished by using the Ackermann steering mechanism as shown in Figure 1.19.

1.4 SINGLE-SLIDER CHAIN

A single-slider chain is obtained by replacing one of the revolute pairs in the four-revolute-pairs, discussed in Section 1.3, by a prismatic pair, as shown in Figure 1.20. The prismatic pair is between links (1) and (4). We obtain different mechanisms if we fix one link at a time. Each mechanism is an inversion of the original single-slider chain.

1.4.1 Engine Mechanism

The engine mechanism (Figure 1.21) is obtained by fixing link (1) in the single-slider chain shown in Figure 1.20. This mechanism transfers the rotary motion of one link to a reciprocating motion for another link or vice versa.

Link (1) is called the frame.
Link (2) is called the crank; it has a length R.
Link (3) is called the connecting rod; it has a length L.
Link (4) is called the piston.

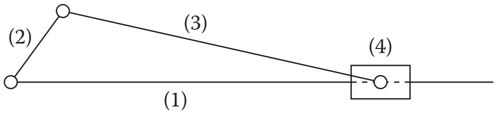


FIGURE 1.20 Single-slider chain.

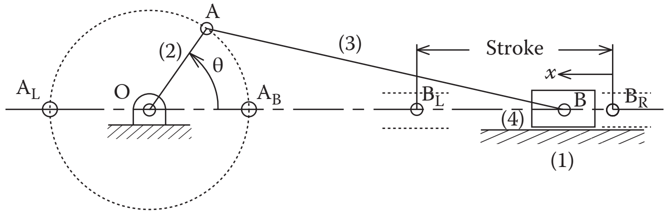


FIGURE 1.21 The engine mechanism.

1.4.1.1 Performance of the Engine Mechanism

- The crank, link (2), makes a complete revolution.
- The piston, link (4), has a reciprocating motion.
- The extreme right position of the piston, called top dead center, occurs when the connecting rod is along the crank. At this position, point B becomes point B_R and point A becomes point A_R.
- The extreme left position of the piston, called bottom dead center, occurs when the connecting rod coincides with the crank. At this position, point B becomes point B_L and point A becomes point A_L.
- The distance between the top dead center and the bottom dead center is called the stroke.
- In this configuration, the centerline of the piston passes through the center of rotation of the crank. In some designs, this line is shifted away from the center of rotation of the crank. In this case, the time taken by the piston to move from right to left is not the same as the time taken when it moves from left to right.
- The motion of the piston from the top dead center as the crank rotates through an angle θ is given as follows:

$$x = R(1-\cos\theta) + L\left[1-\sqrt{1-\left(\frac{R}{L}\sin\theta\right)^2}\right]$$

1.4.1.2 Radial Engine

The radial engine mechanism is used in automobile engines. It is available in multibanks either in line or radial (called V engine). Radial engines are used in aircraft engines where a group of cylinders are arranged in radial positions with the crank shaft (Figure 1.22a). The number of cylinders is usually an odd number. In some engines, the cylinder bank is fixed while the crank rotates. Some engines are available in which the crank is fixed while the cylinder bank rotates as in the motorcycle

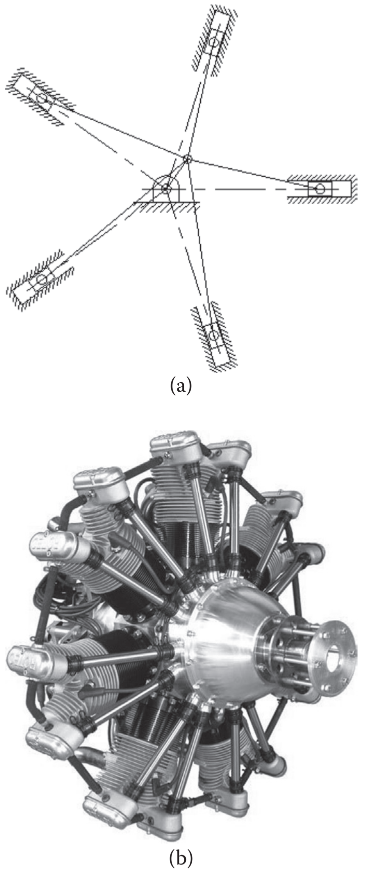


FIGURE 1.22 The radial engine. (a) Schematic view (b) photo for the radial engine.

engine shown in Figure 1.22b. This gives more mass moment of inertia for the rotating parts. The photograph is of Rotec's R2800-7 Cylinder 110 HP, courtesy of Rotec Engineering, Houston, Texas.

1.4.2 Quick Return Oscillating Link Mechanism

This mechanism is an inversion of the single-slider chain of Figure 1.21. Link (3) in the slider chain is fixed and is denoted as link (1) in the quick return oscillating mechanism. The sliding joint between links (3) and (4) in the chain is placed at the end of the crank, link (2), as shown in Figure 1.23. It is also the same as fixing link (2) and making it longer than link (3).

The extreme positions of the oscillating link (4) occur when it is tangent to the crank circle. The extreme positions of the crank are located at points A_R and A_L. Link (4) moves through an angle β between these two positions. When the crank moves from the extreme right position to the extreme left position, assuming clockwise rotation, it rotates through an angle α and returns back through an angle $2\pi - \alpha$:

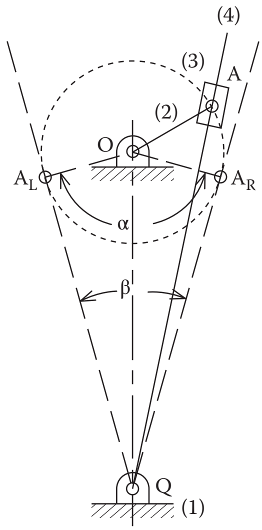


FIGURE 1.23 The quick return oscillating link mechanism.

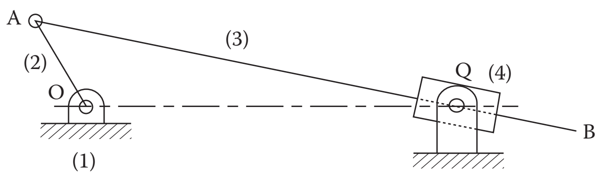


FIGURE 1.24 The tilting block mechanism.

$$\alpha = 2\cos^{-1}\frac{OA}{OQ}$$

Link (4) has a quick return motion with a time ratio, λ, given by

$$\lambda = \frac{2\pi - \alpha}{\alpha}$$

1.4.3 Oscillating (Tilting) Block Mechanism

This mechanism (Figure 1.24) is practically similar to the oscillating link mechanism described in Section 1.4.2. In this case, block (3) in Figure 1.23 is replaced by a link and link (4) is replaced by a block.

1.4.4 Double Rotating Link Mechanism

In this mechanism (Figure 1.25), the shortest link is fixed and the other links rotate continuously. It is usually used as a part of compound mechanisms to obtain certain performance as will be explained in Section 1.8.1.

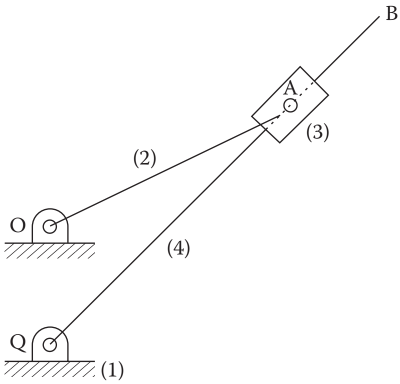


FIGURE 1.25 Double rotating link mechanism.

1.5 DOUBLE-SLIDER MECHANISMS

Some of the mechanisms using two sliders are presented in Sections 1.5.1 through 1.5.3.

1.5.1 Scotch Yoke Mechanism

This mechanism (Figure 1.26) transfers the rotary motion of the crank to a reciprocating motion for the yoke, link (4). The displacement of the yoke from the extreme right position is given by

$$x = R(1 - \cos\theta)$$

The function of this mechanism is similar to the engine mechanism presented in Section 1.4.1. The difference is that the motion of the yoke is pure harmonic motion, which is useful in many applications.

1.5.2 Ellipse Trammel

This mechanism (Figure 1.27) is used to trace an exact ellipse. It consists of a board, that is, link (1) in the figure, which has two perpendicular slots. Each slot has a slider, links (2) and (4), which slides along it. The two sliders are connected to link (3) by revolute pairs. Point P on link (3) traces an exact ellipse. Line AP represents the major axis and line BP represents the minor axis of the ellipse. For an angle θ, the coordinates of point P are given as follows:

$$x = \text{AP}\cos\theta$$
$$y = \text{BP}\sin\theta$$

This is the parametric equation of the ellipse.

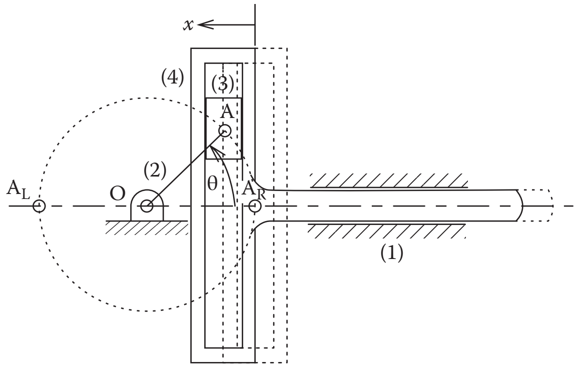


FIGURE 1.26 The Scotch yoke mechanism

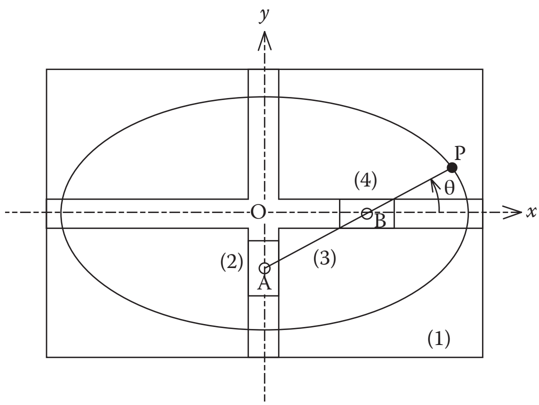


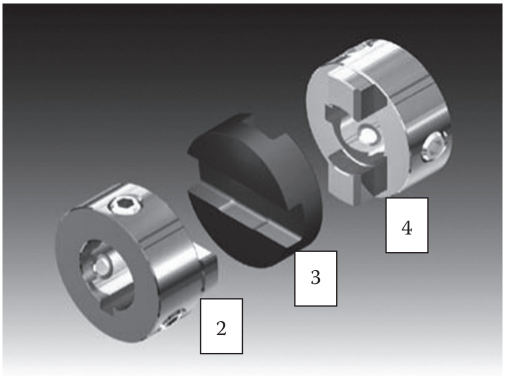
FIGURE 1.27 Ellipse trammel.

1.5.3 Oldham Coupling

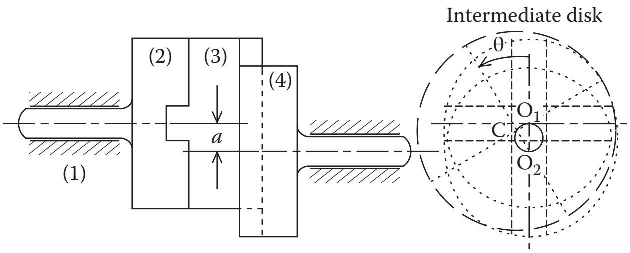
Oldham coupling transmits uniform angular speed between two parallel shafts whose axes do not coincide and who are a radial distance apart.

The parts of the mechanism are shown in Figure 1.28a. The assembled mechanism is shown in Figure 1.28b. It consists of a driving disk (2), intermediate disk (3), and a driven disk (4).

Both driving and driven disks have rectangular recesses, which are positioned perpendicular to each other. The intermediate disk has two perpendicular rectangular slots, one at each side, which engage the recesses of the driving and driven disks. The center of the intermediate disk is located at the intersection of the centerlines of the driving and driven recesses. When the recess of the driving disk rotates through an angle θ, the center of the intermediate disk is located at point C, which is the intersection of the two recesses. Therefore, this center rotates on a circle with diameter, O_1O_2, equal to a (notice that angle O_1CO_2 is 90°; Figure 1.28c). When the driving disk rotates through 90°, the center of the intermediate disks describes 180° on its path circle. Thus, this center rotates with twice the angular speed of the driving and driven shafts.



(a)



(b)

FIGURE 1.28 The Oldham coupling. (a) Photo of the parts (b) assembled drawing (the photograph is courtesy of Knoll).

1.6 MECHANISMS WITH HIGHER PAIRS

Higher pairs, as explained in Section 1.1, are pairs in which the contact between two bodies is through a point or a line. Some mechanisms with higher pairs are listed in Sections 1.6.1 through 1.6.3.

1.6.1 Cam Mechanisms

Cam mechanisms (Figure 1.29) are used to transmit motion from a machine element (cam) to another machine element (follower) through direct contact. The nature of the contact depends on the type of the follower tip.

1.6.2 Gears

Gears (Figure 1.30) are used to transmit positive motion between shafts, change the direction of motion, and change the speed of rotation.

1.6.3 Geneva Wheel

Geneva wheels (Figure 1.31) are used to transfer the rotary motion of a shaft to an intermittent motion for another shaft.