# Developing Primary Mathematics Teaching

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## Reflecting on Practice with the Knowledge Quartet

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| Name | Career stage | Year group | Chapter (part of lesson) | Mathematical content of lesson or extract | Focus dimension/s of Knowledge Quartet | Video clip (approx. time) |
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PGCE: Postgraduate Certificate in Education – final placement of the one-year course

NQT: Newly Qualified Teacher – first year in post

ECT: Early Career Teacher – second or third year in post

ET: Experienced Teacher

# Acknowledgements

We acknowledge with thanks and gratitude the assistance of the many people who have contributed to the development and production of this book. In particular we thank Jane Warwick for her contribution to the research which is at the heart of the book, David Thwaites for expert help with video editing, and Jamie Turner for his imaginative work on the cover photographs. The book would never have come into being without the collaboration of the many teacher-participants whose lessons are featured in it. Their willingness to offer their practice for others to reflect on, warts and all, demonstrates their commitment to the improvement of mathematics teaching – their own, and that of the readers of this book.

# Introduction

This book differs, in some significant ways, from other books on primary mathematics teaching. A short explanation of who the book was written for, what it is intended to do, and how best to use it, may therefore be helpful.

The book is for primary mathematics teachers and those who support their professional development in schools, local authorities and universities. It will be especially useful to student teachers (sometimes called 'trainees' in England) and early-career teachers, for whom the information in the book is likely to be novel. But the heart of this book is not so much information, as a *process* of structured reflection whereby teachers – at any stage of their career – can take control of the development of their expertise in teaching mathematics. The reflective process is the outcome of five years of research at the University of Cambridge. It is built on a framework which enables teachers and teacher educators to engage critically with actual lessons and teaching episodes – their own, or others' – with the aim of learning from teaching-in-action. We call our framework *The Knowledge Quartet*: because it is in four parts, and because it is a way of building up professional knowledge for mathematics teaching. A full explanation is given in Chapter 2.

Theory and practice are interwoven throughout the book. In each chapter you will find:

- justification for the focus of the chapter;
- some exposition related to the topic, or some illuminating aspect of it;
- tasks – things for you to think about and discuss, where possible, with colleagues;
- descriptions of actual lessons, or episodes from lessons, for you to consider, discuss and analyse. Some of these episodes can be viewed as video clips on the book's companion website;
- our own reflections and analytical comments on these lessons and episodes.

The final chapter is devoted to accounts, tasks and analyses relating to a wide range of classroom scenarios, all taken from actual lessons taught by beginning or early-career teachers.

Each chapter can be read on its own, but you will gain most from the book if you read Chapters 1 and 2 first. Chapter 2, in particular, explains the *Knowledge Quartet*, the framework for reflection which features throughout Chapters 3 to 8.

Of course, the book can be read at home, or on the bus or the train. It is also recommended as a stimulus for group work and plenary discussion in teacher education settings and in continuing professional development. The *Knowledge Quartet* framework is also ideally suited for use in the improvement of teaching through lesson observation, both within initial teacher education and ongoing teaching development. In these various settings, the observer could be a mentor, tutor, colleague, subject coordinator or school manager of some kind. We would add, however, that we wrote this book so that teachers at different career stages might be encouraged and supported, and not as a means for them to be judged. The key to this positive, critical support is to allow time – if only 15 minutes – for post-lesson review and discussion, shared by teacher and observer, and structured by the *Knowledge Quartet*.

We hope that readers will find the book enjoyable, useful and informative: we welcome any feedback on the content, and on the ways that the book is being used.

Tim Rowland, Fay Turner, Anne Thwaites, Peter Huckstep.
Cambridge, 2008

# 1

# Inside Naomi's Classroom

**In this chapter you will read about:**

- **the focus of the book on teachers' knowledge;**
- **the distinction between mathematical content knowledge and generic knowledge;**
- **how teachers can develop knowledge for mathematics teaching;**
- **a particular lesson on subtraction taught by a student teacher.**

This book is about some of the things that teachers know, that help them to teach mathematics well. There will be some 'theory', but most of the book is rooted firmly in real classrooms, with some teachers and pupils who helped to make the book possible. In fact, we shall visit one of these classrooms very soon.

Teachers are very serious about their work, and constantly want to get better at what they do. This improvement comes about through a variety of influences. You might want to pause a moment to think what these influences include, and list a few of them.

One obvious possibility is 'experience'. We hope to get better at doing something simply by *doing* it. So we might imagine that our teaching of, say, mental addition strategies would be better in our second year of teaching than it was in the first, and so on. This may well be the case, although it is worth asking *why* it should, or what would help to make it more likely that it would. At the very least, you would need to be able to

recall what you learned from your last experience of teaching mental addition strategies – what seemed to work well, and what did not. Fortunately, we learn a lot from things that do not go well, because we want to avoid them happening again. The key to all this is what is usually called 'reflection' on practice. Teachers' open-mindedness and their desire to do a good job lead them to look for reasons for their actions in the classroom, and to analyse the educational consequences of those actions. Donald Schön's term 'reflective practitioner' (Schön, 1983) is often used to conjure up the notion of teachers as professionals who learn from their own actions – and those of others. Schön distinguished between two kinds of reflection. The first, reflection *on* action, refers to thinking back on our actions after the event. Most of this book is about that kind of reflection, and we promote the idea that it is most fruitful to reflect on action with a supportive colleague who observed you teaching mathematics. The second kind of reflection is what Schön called reflection *in* action, being a kind of monitoring and self-regulation of our actions even as we perform them. This is also something that we think about in this book, especially in Chapter 6. Because reflection *in* action is especially difficult, a supportive observer can also be helpful in drawing attention to opportunities or issues that the teacher may have missed, often because their attention was on something more urgent.

We should also point out, from the outset, that in observing and commenting on someone else teaching, the supportive observer stands to learn as much as, or more than, the one being observed. This book is witness to this claim. We could not have written it, and we would not have learned much of what we have to say in the book, without the benefit of a great deal of supportive observation of other teachers teaching mathematics. If we take any credit, it would be for our own efforts at reflection on other teachers' actions in the past, and on and in our own teaching more recently.

In this spirit, then, this book offers you the opportunity to 'observe' other teachers and to reflect on what they do. Your observation may be fairly direct, because some lesson excerpts can be watched as video clips. Others will be 'observed' as you read succinct accounts of them and read some verbatim transcript selections. The advantage of the transcripts is that you can easily revisit and dissect them if you wish. With few exceptions, these

teachers whom you will observe are relatively inexperienced, and their lessons are not offered as models for you to copy. You can read about why we videotaped these lessons in Chapter 2. Sometimes you will think that a teacher could, or should, have done something differently. As we have already said, you will learn something merely by thinking, and especially by making, that reflection explicit in discussion, or in a written note of some kind. Paradoxically, you would learn very little from commenting that 'it went well'.

In the UK, many graduate student teachers (sometimes called 'trainees') follow a one-year, full-time course leading to a Postgraduate Certificate in Education (PGCE) in a university education department. About half the year is spent teaching in a school under the guidance of a school-based mentor. All primary trainees are trained to be generalist teachers of the whole primary curriculum. The mathematics lessons featured in this book were filmed while the teachers were in their PGCE year or in the early stages of their teaching career. The index of teachers and lessons on pp. x–xiii summarises where each teacher's lesson occurs in the book along with the career stage of the teacher, an indication of the mathematical content, the part of the lesson and, where appropriate, the video clip number on the companion website.

In this chapter, you will observe a lesson on subtraction. The pupils, boys and girls, are in Year 1 (age 5–6 years). The teacher is Naomi, who was, at the time, a PGCE student in the third and final term of her course. For most of that term, she was on a teaching placement in a primary school. Naomi chose to specialise in early years education in her PGCE. In most of the UK, it is usual to study only three or four subjects at school between 16 and 18. At school, Naomi had specialised in mathematics, English, French and psychology. Relatively few primary PGCE students have undertaken such advanced study in mathematics. Following school, Naomi's undergraduate degree study had been in philosophy.

In this book, we will sometimes ask you to read a description of a lesson, or part of a lesson. Sometimes we will give verbatim transcripts of short lesson episodes. In the case of the lesson featured in this chapter, you can also view a video clip (Clip 1) on the companion website if you wish.

## Naomi's lesson

Naomi's classroom is bright and spacious, with a large, open, carpeted area. We can see around 20 young children in the class: there might be a few more off-camera. There is also a teaching assistant positioned among the children. The learning objectives stated in Naomi's lesson plan are: 'To understand subtraction as "difference". For more able pupils, to find small differences by counting on. Vocabulary – *difference, how many more than, take away*.' Naomi notes in her plan that they have learnt *how many more than*.

Naomi settles the class in a rectangular formation around the edge of the carpet in front of her, then the lesson begins with a seven-minute oral and mental starter designed to practise number bonds to 10. A 'number bond hat' is passed from child to child until Naomi claps her hands. The child wearing the hat is then given a number between 0 and 10, and expected to state how many more are needed to make 10. Naomi chooses the numbers in turn: her sequence of starting numbers is 8, 5, 7, 4, 10, 8, 2, 1, 7, 3. When she chooses 8 the second time, it is Bill's turn. Bill rapidly answers 'two'. Next it is Owen's turn:

Naomi: Owen. Two.

[12-second pause while Owen counts his fingers]

Naomi: I've got two. How many more to make ten?

Owen: [six seconds later] Eight.

Naomi: Good boy. [addressing the next child] One.

Child: [after 7 seconds of fluent finger counting] Nine.

Naomi: Good. Owen, what did you notice … what did you say makes ten?

Owen: Um … four …

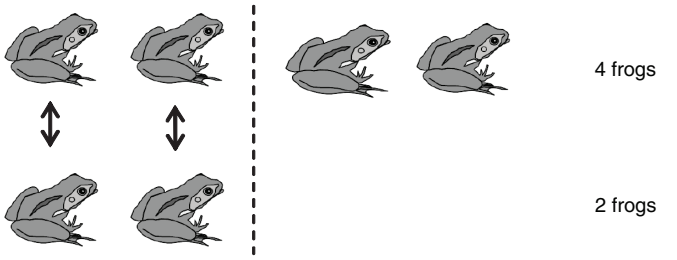
Naomi: You said two add eight. Bill, what did you say? I gave you eight.

Bill: [inaudible]

Naomi: eight and two, two and eight, it's the same thing.

Later, Naomi gives two numbers to the child with the number bond hat. The child must add them and say how many more are then needed to make 10.

The introduction to the main activity lasts nearly 20 minutes. Naomi wants to introduce them to the idea of subtraction as difference, and the language that goes with it. To start with, she sets up various difference problems, in the context of frogs in two ponds. Magnetic 'frogs' are lined up on a board, in




Note: the arrows and dotted line have been added for clarity.

**Figure 1.1** Naomi's representation of the frogs

two neat rows. In the first problem, Naomi says that her pond has four frogs, and her neighbour's pond has two, as shown in Figure 1.1.

Naomi: I went to my garden this weekend, and I've got a really nice pond in my garden, and when I looked I saw that I had … [Naomi tries to stick some 'frogs' on the board] … I don't think they're sticking. Let me get some Blu-tack. It's supposed to be magnetic, but it doesn't seem to be sticking. Right. I had four frogs, so I was really pleased about that, but then my neighbour came over. She's got some frogs as well, but she's only got two. How many more frogs have I got? Martin?

Martin: Two.

Naomi: Two. So what's the difference between my pond and her pond in the number of frogs? Jeffrey.

Jeffrey: Um, um when he had a frog you only had two frogs.

Naomi: What's the difference in number? […] this is my pond here, this line – that's what's in my pond, but this is what's in my neighbour's pond, Mr Brown's pond, he's got two. [Gender of neighbour has changed!] But I've got four, so, Martin said I've got two more than him. But we can say that another way. We can say the difference is two frogs. There's two. You can take these two and count on three, four, and I've got two extra.

Right, let's see who wants to be my helper.

A couple of minutes later, Naomi says:

Naomi: Morag's been sitting beautifully, oh no, Morag's been reading a poetry book. […] That should be on my desk, thank you. Put your hand up please, you know the rule. Yes Hugh?

Hugh: You could both have three, if you give one to your neighbour.

Naomi: I could, that's a very good point, Hugh. I'm not going to do that today though. I'm just going to talk about the difference. Morag, if you had a pond, how many frogs would you like in it?

Pairs of children are invited forward to choose numbers of frogs (e.g. 5, 4) and to place them on the board. The differences are then explained and discussed.

Before long, Naomi asks how these differences could be written as a 'take away sum'. With assistance, a girl, Zara, writes $5 - 4 = 1$. Later, Naomi shows how the difference between two numbers can be found by counting on from the smaller.

The children are then assigned their group tasks. The usual class practice is to group the children by 'ability' for mathematics. The actual numbers used in the difference problems are the same for each group, but the activity is differentiated by resource. One group (called the Whales), supported by a teaching assistant, has been given a worksheet on which drawings of cars, apples and the like are lined up on the page, as Naomi had done earlier with the frogs. Two further groups (Dolphins and Octopuses) have difference problems set in 'real life' scenarios, such as 'I have 8 sweets and you have 10 sweets'. These two groups are directed to use multilink plastic cubes to solve them, lining them up and pairing them, as Naomi had done with the 'frogs' in her demonstration. The remaining two groups have a similar problem sheet, but are directed to use the counting-on method to find the differences. Naomi works with individuals.

In the event, the children in the Dolphin and Octopus groups experience some difficulty working with the multilink. This is partly because 'lining up' requires some manual dexterity, and also because the children find more interesting (for them) things to do with the interlocking cubes. Naomi comes over to help the Dolphins. She emphasises putting eight cubes in a row, then ten. 'Then you can *see* what the difference is.' She demonstrates again, but none of the children seems to be copying her. Jared can be seen moving the multilink cubes around the table, apparently aimlessly. Another child says 'I don't know what to do'. Naomi moves away to give her attention to the Octopuses. In her absence from the table, one boy sets about building a tower with the cubes. Later, Naomi returns to the Dolphins, and tries once again to clarify the multilink method. She asks: 'What's the difference between seven and twelve?' Without looking up, the boy who is making the tower replies 'Don't ask me, I'm too busy building'. Naomi responds by saying 'Goodness me, let's put these away. I'll show you a different way to do it.' She collects up the multilink cubes into a tray, and takes the Dolphins and Octopuses back to the carpet, where she shows them the counting-up strategy for the difference between 8 and 10. 'You start with the lower number … you

start with the smallest number. Count on – show me your fist – nine, ten.' She then works through the fist three worksheet questions, doing them for the children, by counting up.

Finally, Naomi calls the class together on the carpet for an eight-minute plenary, in which she uses two large foam 1–6 dice to generate two numbers, asking the children for the difference each time. Their answers indicate that there is some confusion among the children about the meaning of 'difference'.

Naomi: Right, I'm going to roll the dice, and I want you to find the difference between the two numbers. Five and three. Now starting with the smaller number can you count up to see what the difference is. […] I can show it with the frogs as well. Jeffrey, can you have a go at working it out? The difference between three and five.

Jeffrey: Seven.

Naomi: No, we're starting with three …

Jeffrey: Eight.

Naomi: and counting up to five. What's the difference? It's like a take away sum. Stuart.

Stuart: Two.

Naomi: Excellent. Can you tell us how you worked it out? Come to the front. Owen stand up. Sit in your rows please. Right, Stuart just worked out the difference between three and five and said it was two. How did you work it out? Stuart.

Stuart: I held out three fingers and five and then there's two left.

Naomi: Ah, OK. That *does* work because you've got five fingers on your hands so if you've got five here and three you've got two left to make five. But I know an even *better* way to work it out. Does anybody know another way to work it out? Ayesha. No. Who knows another way to work it out? Leo.

Leo: Count in your head …

Naomi: Yeah, how did you count? What did you count in your head?

Leo: I thought of three …

Naomi: Jeffrey stand up, Hugh stand up!

Leo: Then I added two. But I still had two left.

Naomi: Right, started with, started with three, did you say, and then you counted on two, till you got five. Right, let's see what we get next.

Who can do this one for me? Three and six. Three and six. What's the difference between three and six? Jim.

The plenary continues in this way, and finishes with:

Naomi: What is the difference between four and six? So hold the number four in your head and count on. Four, five, six. What's the difference? Jared?

Jared: Uh, can't remember.

Naomi: The difference between four and six. Jeffrey?

Jeffrey: Two.

Naomi: Good boy. Right, the difference between two and four? What's the difference? So start with the smaller number, two and count up till you get to four. What's the difference?

The one-hour videotape tape ran out here, just before the conclusion of the lesson.

## Reflecting on Naomi's lesson

You should now have a good sense of what Naomi was trying to achieve in her lesson, how she intended to go about it, and how things turned out. You might feel that you 'know' Naomi a little, or someone like her. You might recognise some of the children in her class, in that they remind you of children that you have taught or seen in other classes.

At this point we would like you to do some thinking about Naomi's lesson. You can do this on your own. Better still, discuss it with a friend, colleague, another student or small group of students, according to your circumstances at the moment. Have ready a piece of paper to write on, a whiteboard, or a flipchart – whatever suits those circumstances. Think and talk about anything that came to your attention as you read the account of the lesson, and/or watched the video clips. Later in the book, we will ask you to focus on specific aspects of this and other lessons. For the moment, we leave it to you to make the choice. You might imagine that you are Naomi's friend, or her mentor, and that she is expecting you to offer her some comments on the lesson.

Once you have begun to think and talk about particular aspects of the lesson, make a note of what they are – write a brief statement of what it is that you are thinking about, and what people are saying.

Magnetic frogs. Like fridge magnets. Have seen something like this used before – magnetic numbers on a 1–100 square, but they kept falling off! Seems that Naomi had the same problem. Issue of resources that won't distract from intended purpose because they don't work. Naomi resorted to blu-tack. She could have checked the magnets before the lesson.

**Figure 1.2** A note about Naomi's lesson

For example, you might write something such as the notes in Figure 1.2. This is not meant to be a particularly good example of the notes that you might write. It isn't particularly bad, either. It's just an example of the kind of thing that you might discuss and how you could record it briefly.

You could spend a long time thinking about Naomi's lesson, but we suggest about 20–30 minutes.

Now group the issues that you've chosen to focus on into a small number of categories. The issues in each category will have something in common. What that 'something' is is entirely up to you. There are no right and wrong categories. Give a short name to each category. Don't spend too long on this. If you are in a class situation, and several pairs or groups are also doing this exercise, it will be valuable and interesting to compare the categories that different pairs or groups come up with.

Then make a note of anything that your reflections and discussion have particularly highlighted for you. Perhaps something you might not have noticed on your own. Perhaps something you think is a key issue for this topic, or for teaching generally. Perhaps something to keep in mind when you prepare a lesson, or when you teach a class, in the future. This could be at various possible levels – preparing or teaching any lesson, or a mathematics lesson, or a Year 1 lesson, or a lesson on subtraction, or …