ELECTROMAGNETIC DISTANCE MEASUREMENT

Electromagnetic distance measurement, by using light and microwaves for direct linear measurements and thus circumventing the need for traditional methods of triangulation, may well introduce a new era in surveying. This book brings together the work of forty-eight geodesists from twenty-five countries. They discuss various new EDM instruments—among them the Tellurometer, Geodimeter, and air- and satellite-borne systems—and investigate the complex sources of error. The book is therefore a unique and comprehensive source on the subject. UNESCO and R.I.C.S. have assisted financially in its production.

INTERNATIONAL ASSOCIATION OF GEODESY

ELECTROMAGNETIC DISTANCE MEASUREMENT

A SYMPOSIUM
HELD IN OXFORD
UNDER THE AUSPICES OF
SPECIAL STUDY GROUP NO. 19
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SYMPOSIUM EXHIBITION

Throughout the Symposium an exhibition of EDM instruments was held in the Department of Engineering Science with the following exhibitors:

Exhibit	*Exhibitor*
The Geodimeter model 6	AGA Signals Ltd, Beacon Works, Brentford, Middlesex
Laser Rangefinder	G. &. E. Bradley Ltd, Electral House, Neasden Lane, London NW 10
The Mekometer	Hilger & Watts Ltd, 98 St Pancras Way, Camden Road, London NW 1
Tellurometer models MRA 4, 101 and 3	Tellurometer (UK) Ltd, Survey House, Windmill Road, Sunbury-on-Thames
Distomat DI 50	Wild Heerbrugg Ltd, Switzerland
E.O.S. Telemeter	C.Z. Scientific Instruments Ltd, 93/97 New Cavendish St, London W 1
The Ordnance Survey Thermistor	Ordnance Survey, Leatherhead Road, Chessington, Surrey
Gallium Arsenide Modulated Light Source	Tellurometer (UK) Ltd, Survey House, Windmill Road, Sunbury-on-Thames
NPL Mekometer II	National Physical Laboratory, Teddington, Middlesex

FOREWORD

Special Study Group No. 19 of the International Association of Geodesy was set up as the result of a decision of the Eleventh General Assembly of the Association at Toronto in 1957. The original purpose of the Study Group was to investigate the new instruments for electromagnetic measurement of distances on the Earth's surface, which had recently made their appearance, and to make recommendations regarding their use for various geodetic purposes. The instruments at that time were the Geodimeter, invented by Dr Bergstrand of Sweden and first used in 1947, and the Tellurometer, invented by Dr Wadley of South Africa and first used in 1956. The Geodimeter making use of a modulated light beam has revolutionized base measurement. The Tellurometer, using microwaves in place of light waves, has enabled electromagnetic distance measurements to be performed in daylight, and has also extended the range over which it is possible to measure.

The impact of these instruments upon geodesy has been great. It is now possible to apply a rigorous control of scale to triangulation systems. The precise geodetic traverse has become a task rapid in execution. In all phases of survey work, valuable applications have been discovered and the task which faced the Study Group has not been one that could be quickly completed.

It has been necessary to study the principles, new to most geodesists, upon which these new instruments operate and to understand the nature and assess the magnitude of the errors that affect them. It was necessary to devise means for minimizing these errors both by better design of instruments and by improved operating techniques. The Study Group's function has been essentially to record and disseminate information about research and development carried out by many organizations and individuals.

There has been a continuous process of improvement. The Geodimeter has been re-designed, rendered more portable and more accurate, and modified to work in daylight. The Tellurometer now operates on the three-centimetre as well as on the original ten-centimetre wavelength. Electromagnetic distance measuring instruments are now being made by other manufacturers in a number of countries on both sides of the Atlantic. The study envisaged in 1958 has become more complex, both in the number of instruments to be studied and in the number of problems that have emerged. In particular, the importance of refraction and the accurate measurement of refractive index have been realized and much work has been done in this field and on the connected problems of micro-meteorology. Studies have been made of the phenomenon of ground reflection of microwaves, and light has been thrown on this problem, which at first seemed beyond detailed analysis.

Foreword

The application of electromagnetic techniques has been extended. At the Twelfth General Assembly of the IUGG in Helsinki (1960), the study of airborne systems such as HIRAN and AERODIST was added to the scope of the Study Group. The technique has now been extended to satellite geodesy, as for example in the SECOR and Doppler systems. Finally the laser has entered the field, with potential applications as a highly concentrated and penetrative light source and also, with its coherent beam, as a method for applying the technique of interferometry over much greater distances than has hitherto been possible.

The work of the Study Group has thus continued to expand, and the purpose of the Symposium was to throw light upon some of the many problems that have been revealed.

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OPENING CEREMONY

Professor A. Marussi opened the proceedings and said how much he regretted the absence of Brigadier G. Bomford, who was prevented by illness from attending. He explained that the Symposium was arranged by Special Study Group No. 19 of the International Association of Geodesy, the IAG being a section of the International Union of Geodesy and Geophysics. The IUGG was in turn part of the International Council of Scientific Unions. He welcomed to the Symposium Professor H. W. Thompson, President of ICSU. He then called upon Sir William Hayter, Warden of New College, and representing the Vice-Chancellor of the University, to open the Symposium.

Sir William Hayter welcomed the delegates to Oxford University and declared the Symposium open. Professor L. Asplund replied on behalf of the IAG.

The closing speech was made by Major-General R. C. A. Edge, President of Special Study Group No. 19, and a visit was made to the Instrument Exhibition which was also formally opened by Sir William Hayter.

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General Subjects and Reports

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EDM Research in Austria

K. RINNER
Technical University of Graz

This report deals with research work at the Technical University of Graz. It investigates the accuracy obtained with EDM instruments in the Lower Alps and discusses a device for reducing the influence of reflections. The geometric strength required for continental or world networks is considered and electromagnetic tacheometry and the geometry of radar pictures are discussed.

1. *Introduction*

The use of EDM techniques has proved a valuable help to Austrian land surveyors in performing their tasks, which are so important to the economic development of the country. In this way, it is quite easy to simplify and accelerate the tasks which are in most cases difficult to solve in a mountainous terrain, as far as the determination of the site of basic points is concerned. On the other hand, in view of the complicated measuring conditions which are always arising in the mountains, a thorough consideration of the influence of meteorological anomalies, of the reflections, and of the geometric and physical reductions is necessary. This is the reason why we observe a direct, practical interest in the results of EDM research in Austria, in addition to the general scientific interest arising from the well-known tradition of the country in the sphere of geodesy.

The research aspirations are being encouraged by the Austrian Surveying Commission and the Austrian Research Council. The work is being executed at the technical universities and by surveying authorities (Federal Office for Weights and Measures and Office for Geodesy). Unfortunately, the report on EDM research in Austria is still not coordinated and it is available only in two different parts. This report refers exclusively to the activities at the Technical University of Graz. The work at other institutes is being reserved for a special report.

2. *Research Programme*

The Technical University of Graz has instigated a series of investigations which are still under development:

(*a*) Determination of the relative obtainable accuracies for different measuring instruments and methods when using a test-net with sides up to 20 km, in mountains of medium height.

(*b*) Development of equipment for reducing the influence of reflections.

(*c*) Examination of the geometric strength of different net forms for the tasks of continental and world surveying, having the aim of evaluating optimal results with the minimum effort.

(*d*) Consideration of the possible developments for electromagnetic tacheometry and of the geometry of electromagnetic images of the terrain.

The results which we have now obtained are summarized in the following report. In addition to this report, special reference is made to a volume to be issued in the near future under the title *Die Entfernungsmessung mit elektromagnetischen Wellen und ihre geodätische Anwenung*, which in the present report appears as reference [1] (i.e. *Handbuch der Vermessungskunde*, by Jordan, Eggert and Kneissl, of which it is the sixth volume); further reference should be made to the reports, which are still in preparation, of the appropriate authorities. These reports will enumerate the results in detail.

3. *Examination of the Test-net of Graz*

The accuracy of distances fixed by way of electromagnetic waves is dependent on the instrument used, on the method of measuring, on the meteorological circumstances and, finally, on the land profile over which the survey is executed. Therefore, if we want to compare the accuracy of different instruments and processes, such a comparison can be executed—if we want to be exact—only over the same profiles and under similar meteorological circumstances. For this reason, it will be suitable to take into consideration test-nets with profiles of different length and topography, and possibly different meteorological conditions. Supposing we have to measure, within the net, both the directions and distances, then it is possible to study the propagation of errors with EDM equipment and the relations of weight for different quantities, under ascertained geometric conditions.

The test-net of Graz is located in mountains of medium size, at heights from 360 up to 1440 m. The net includes twenty-four sides, the lengths of which fluctuate between 0·7 and 18·2 km (Fig. 1). The total length of the net sides is about 200 km. An exact description of the points and profiles is available in the report submitted to the EDM course held in 1964 at Zürich[2], and, therefore, is omitted here. We want, however, to repeat that the net contains profiles having characteristic meteorological conditions and reflecting planes.

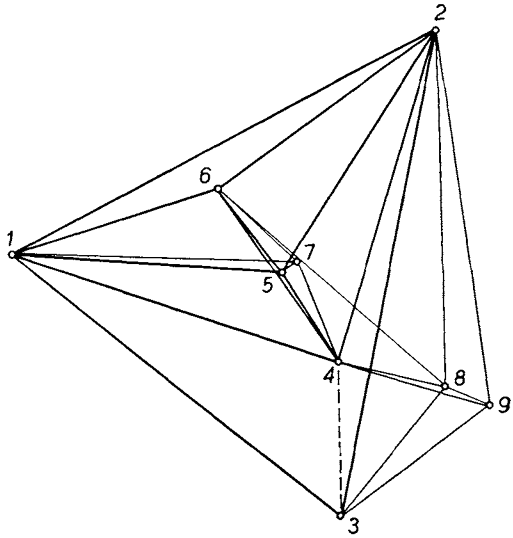

FIG. 1. The Graz test-net, 1:200 000.

1 Pleschkogel KT
2 Schöckl KT West
3 Buchkogel Pyr.
4 Plabutsch KT Bolzen
5 Hartbauer Rohr N.
6 Eggenberg Nagel TH
7 Harthopfer Rohr 1
8 Schlossberg A
9 TH Observ. Pf. SW.

Within the area of part of the net, which consisted of seven points, all directions were measured and, in this way, a joint adjustment of directions and distances could be executed. The sides of this net were measured during the period beginning in 1961 with different types of Geodimeter (G), Tellurometer (T), Electrotape (E) and a Distomat (D). In addition to this, some new types of instruments were tested while under development. The measurements were taken by working groups of the institutes which placed the instruments at our disposal and which were quite familiar with the measuring techniques, as follows:

1961: Bundesamt für Eich- und Vermessungswesen (Federal Office for Weights and Measures, and Office for Geodesy), Vienna (direction measurements with Geodimeter NASM-2A and 4B).
Technical University of Graz (Geodimeter NASM-4B).
Zentralanstalt für Meteorologie (Central Office for Meteorology), Vienna (meteorological measurements).

1962: Deutsches Geodätisches Forschungsinstitut (German Research Institute for Geodesy), Munich (Tellurometer MRA-1).
Technical University of Hanover (Electrotape DM-20).
Messrs Wild, Heerbrugg (Distomat).
Messrs Aga, Stockholm (Geodimeter NASM-4D).
Bundesamt für Eich- und Vermessungswesen (Federal Office for Weights and Measures, and Office for Geodesy), Vienna (Geodimeter NASM-2A and 4D).
Technical University of Graz (Geodimeter NASM-4B, 4D).
1963: Deutsches Geodätisches Forschungsinstitut (German Research Institute for Geodesy), Munich (Tellurometer MRA-1 and Electrotape DM-20).
Technical University of Graz (Geodimeter NASM-4D).
1964: Technical University of Munich (Tellurometer MRA-3).
Technical University of Graz (Tellurometer MRA-2).

During 1965, some measurements were expected to be executed with a Geodimeter 6 and Distomat (Ertel, Munich).

All of the measurements were reduced in the same manner and to the same points. The distances measured in this way are compared, as in Fig. 2, with the distances which were fixed by way of a Geodimeter

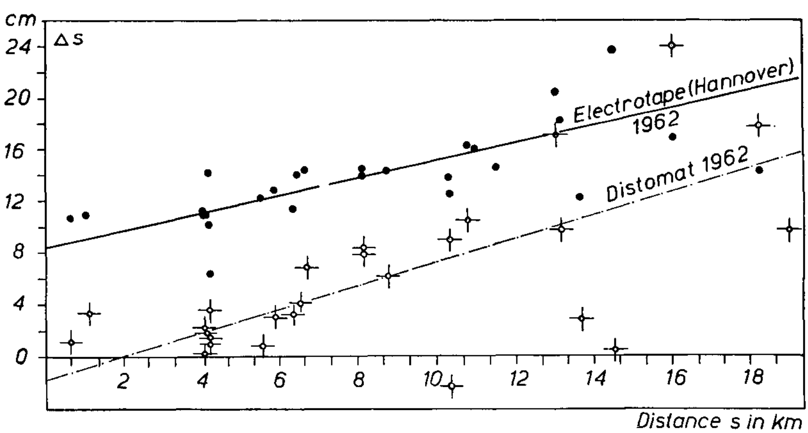

FIG. 2. Comparison of measurements by the Electrotape and the Distomat.

4D (Hg light) in 1963. For each of the instruments, we evaluated the coefficient of a function

$$\Delta s = a + bs$$

and the mean square error, in which a is the zero point constant, and b is the scale error.

The results which we obtained in this way point to the possibility of errors in an Electrotape DM-20. However, the observer who used this instrument denies this possibility, saying that extreme meteorological circumstances were believed to be the cause.[4] A report [2] describes the time needed for the measurements. From the comparison, which is in some way rather problematic, we obtained some good results with the Distomat within a very short time.

The parallel measurements executed in 1964 by a Tellurometer MRA-2, with a carrier wave of 10 cm (by the Technical University of Graz), and a Tellurometer MRA-3 (of the Technical University of Munich) are particularly interesting. A report on this work has been published.[3] Simultaneous measurement was possible with no disturbances while the measuring time was generally lower for the non-digitalized MRA-2 instrument.

A detailed report enumerating the results of parallel measurements[2] has been prepared, and also describes the experiment of using a helicopter for evaluating the average values of temperature and pressure by flying along the path of the waves during the measurements. The result of continuous measurements by use of light waves shows a correlation between the reduced lengths and the time of day, pointing to an incomplete perception or consideration of the meteorological influences. The measurements taken by helicopter show the possibility of measuring the average values of the refractive indices by flying along the ray path, and it is also a practical example of the inadmissibility of averaging meteorological data measured at the end points. The summary report[2] on the test-net of Graz will also include a detailed discussion.

The adjustment of the measured data was made by variation of co-ordinates with different weight assumptions for distances and directions.

For the directions, the following matrix was used:

$$R + v_R = (R) + o + \mathrm{d}R$$

where

$$\begin{aligned} R &= \text{measured direction} \\ (R) &= \text{preliminary orientation} \\ o &= \text{orientation constant} \\ v_R &= \text{direction residual} \end{aligned}$$

As two points of the net were kept as fixed points, i.e., the co-ordinates were taken from the Federal surveying net, we had to introduce a scale factor for distances.

Therefore, the matrix for the distances had the following form:

$$s + \lambda s + v_s = (s) + \mathrm{d}s$$

where

s = measured distances
(s) = distances from preliminary co-ordinates
λ = scale factor
v_s = residuals

In order to make the examination apply generally, real zero point constants were introduced, as we have to assume them when comparing the measurements with different instruments. A constant λ_o has, therefore, been added to the matrix in a second adjustment:

$$s + \lambda_o + \lambda s + v_s = (s) + ds$$

The weights of the measured data were enumerated from ascertained figures by a roster taken from different assumptions, and evaluated in such a way that all of the possibilities determined in practice were included. Even the most extreme events were taken into consideration.

For a joint adjustment of directions and distances, we have the following law:

$$\sum (pvv)_R + (pvv)_s = \min$$

in which we have to introduce the weights:

$$p_R = C/m_R^2, \qquad p_s = C/m_s^2$$

with the same constant C and the mean square errors in the same dimension as the residuals v. The adjustment was executed for two groups, each of four weight assumptions, which were taken from the following set-up:

$$p_R = 1, \qquad p_s' = C'/s^2, \qquad p_s'' = C'' \text{ (const)}$$

The determination of the dimensions was made in such a way that successive weights have a geometric difference of ten. That means, between the first and fourth weights we get the proportion of $1 : 10^{-3}$.

The pure direction adjustment was introduced by the formula of $p_s = 0$. The nine weight assumptions used for the adjustment are enumerated in Table 1.

TABLE 1

No.	mm/km	C'	No.	m_s (cm)	p_s
0	0	0			
1	1·0	200	5	±1·0	2
2	3·2	20	6	3·2	0·2
3	10·0	2	7	10·0	0·02
4	32·0	0·2	8	32·0	0·002

For the adjustment of the distances only, we use $p_R = o$, and introduce the weights shown in Table 2.

TABLE 2

No.	p_s
9	$10 : s$
10	$100 : (56+0{\cdot}2\, s^2)$
11	$100 : (56+2\, s^2)$
12	$10 : (4+0{\cdot}06\, s)^2$
13	$100 : (4+0{\cdot}6\, s)^2$
14	$10 : (4+0{\cdot}001\, s^2)^2$
15	$10 : (4+0{\cdot}01\, s^2)^2$
16	1
17	$100 : s^2$

With these assumptions, the figures which are already fixed vary in geometric differences of ten. Assumption No. 9 corresponds to an error proportional to the square root of s, No. 17 corresponds to an error increasing linearly with s, and assumption No. 16 corresponds to an error independent of the distance. All of the nine complete measurements of the net with the Geodimeter, Tellurometer, Electrotape and Distomat were adjusted for these seventeen weight assumptions with a zero point constant being introduced for each distance net. Summarizing all this, the net adjustment was executed with twenty-six different weight assumptions for each instrument. At every adjustment, we determined the unknown quantities: the co-ordinates dx, dy, the orientation constant o, the scale factor λ and the zero point constant λ_o and, in addition to these, the residuals v_R and v_s of the measured data and the mean square errors of the unknowns, as well as of the distances and directions after the adjustment.

In order to compare the results of the different adjustments, we introduced two average error factors for the net:

1. *An average point error:*

$$m_p = m_o \sqrt{(\bar{Q}_{pp})} \qquad \bar{Q}_{pp} = 1/n_p \sum (Q_{xx} + Q_{yy})$$

where

n_p = number of new points, and

Q_{ii} = weight coefficient.

2. *An average relative side error:*

$$\bar{\mu}s = m_o/\bar{s} \sqrt{(\bar{Q}_{ss})}; \qquad \bar{s} = 1/n_s \sum s; \qquad \bar{Q}_{ss} = 1/n_s \sum Q_{ss}$$

where n_s = number of sides.

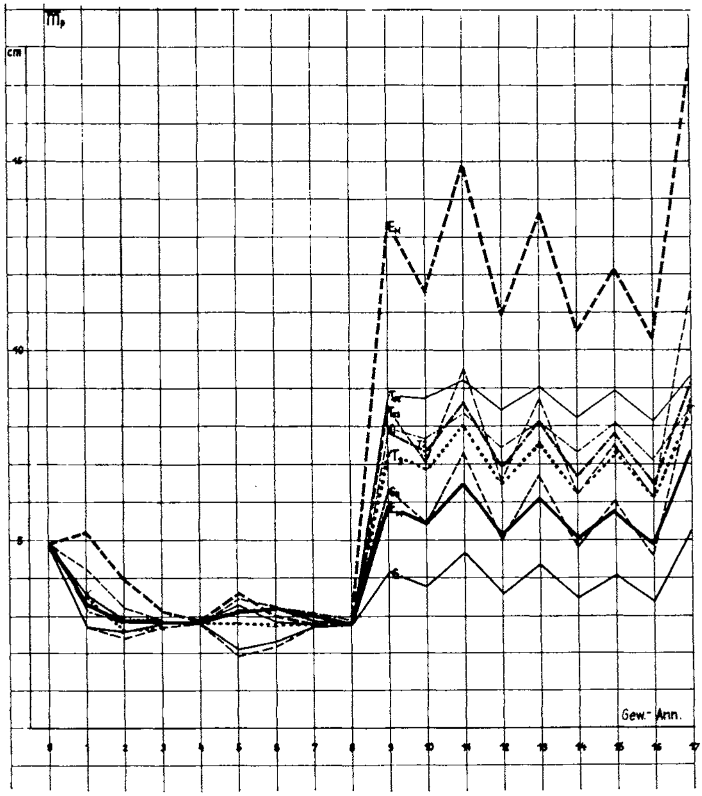

Fig. 3*a*.

For the parallel measurement with the instruments MRA-3 and MRA-2, which were affected similarly by reflection effects as a result of equal profiles and equal meteorological conditions, a special comparison was executed. The result is as follows:

1. In the combined adjustment, the pure direction net for all of the weight assumptions is inferior to the combined net with directions and distances. If we also use approximate distance measurements, the accuracy of the point determination can be increased. The combined net with directions and distances is also superior to the pure distance net (Fig. 3*d*).
2. Distance measurements by light waves led to the best results.
3. The weight assumptions, which are different by three factors of ten for the distances, cause a fluctuation of the average point errors

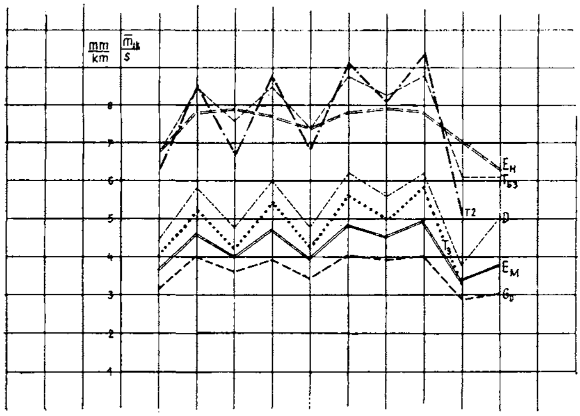

FIG. 3*b*.

only of approximately ± 2 cm, and of the relative distance error of $\pm 1{\cdot}5$ mm (Figs. 3*a*, *b*, *c*, *d*, 4*a*, *b*, *c*).

4. For the pure distance adjustment, without a zero point constant, there is a well-defined order of rank, according to the instruments. After the Geodimeter, there are two 3-cm carrier-wave instruments, then the 10-cm instruments and finally another 3-cm instrument (Figs. 3*b*, *c*, *d*).

5. The introduction of a zero point constant changes the result. Now, all of the 3-cm instruments rank after those using light waves and before the 10-cm instruments. We recognize a well-defined order of rank in accordance with the length of the carrier wave (Figs. 4*a*, *b*, *c*). In view of the fact that a determination of the constants of the Electrotape was executed before and after the measurement in Graz, a change of the zero point constant is less admissible. The supposition, however, that during the measurement quite extraordinary turbulent meteorological conditions existed is also rather unacceptable, since the anomalies which would be required for such an explanation were not evident. Therefore, the observation that the introduction of a zero point constant into the adjustment can considerably improve the result can only be accounted for by the combination of several unexplained circumstances. The elimination of a systematic or pseudo-systematic constant part is justified, even though no physical explanation can be given for it. It is a similar case to that of photogrammetric triangulation, in which the practically determined pseudo-systematic figure can also be proved by the theory of errors.

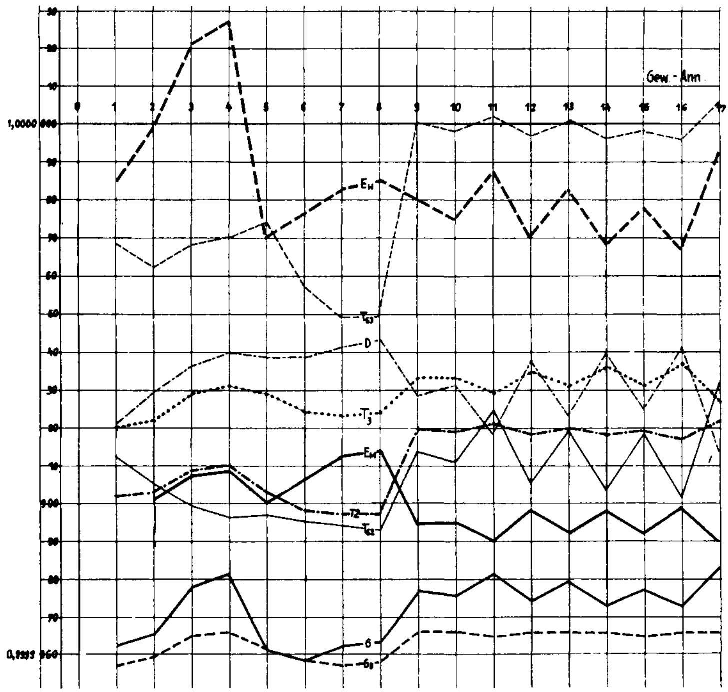

FIG. 3*c*.

6. Simultaneous measurements with 3-cm and 10-cm instruments show the superior accuracy of the former. These also statistically proved that the 3-cm instrument measurements are less influenced by ground reflections.

7. For the determination of points, distance nets supported by directional data are recommended. Weights of directions and distances can be introduced as constants, their ratio being in the range 1·0–0·2 depending on the instruments used (weight assumption No. 6).

8. It is intended to enlarge the net in the near future by adding sides of up to 50 km. Meteorological investigations as well as systematic analysis of the influence of ground reflections will be carried out.

9. The sides of the test-net at Graz have been measured with all available types of instrument so that standard values are therefore available. Moreover, care has been taken to create homogeneous conditions for the measurements and to obtain reliable meteorological

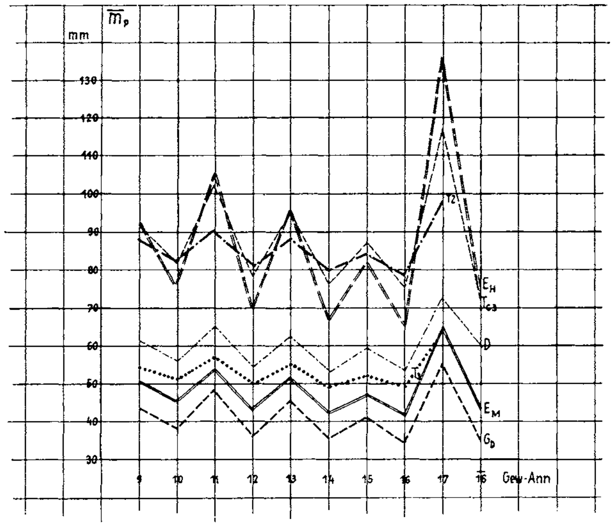

FIG. 3*d*.

data. A programme for an electronic computer (Univac 490) permits instantaneous evaluation of the measured results. Thus, the net is suitable for testing new instruments and for carrying out scientific research.

N.B. Firms and institutes are invited to make use of the test-net of the Technical University at Graz. At the same time we propose that a recommendation by the Symposium be issued to make this possibility known.

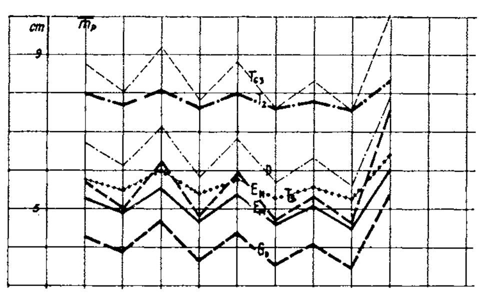

FIG. 4*a*.

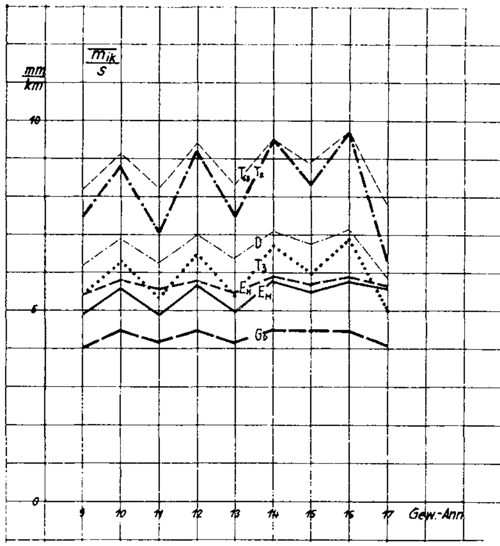

FIG. 4*b*.

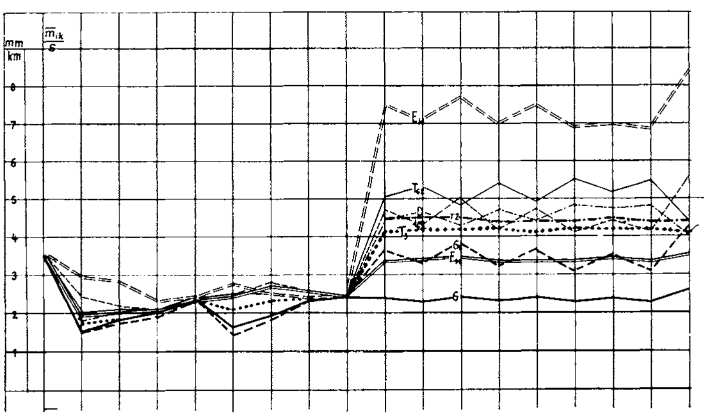

FIG. 4*c*.

4. *Device for Diminishing the Influence of Ground Reflections*

The well-known effect of reflections in EDM can theoretically be eliminated to a great extent if pulse-type signals are used. Because of the inferior accuracy of pulse distance measurements, this has been of no practical importance. However, it is now possible to combine the advantages of pulse and phase distance measuring procedures by using continuous waves with a fixed frequency relation. As such waves have the same transit time, but different phase differences, the influence of ground reflections can be reduced by a compensation method. A device which can be installed in every EDM instrument has been constructed by Prof. Benz (Graz) and an Austrian patent No. 225.753 (61) has been applied for.

5. *Analysis of the Geometric Strength of Networks with Distances*

At the present time, long distances can be measured accurately enough for geodetic purposes by microwave techniques only. Between terrestrial points, the range of such measurements is limited to about 100 km by the need for inter-visibility. Longer distances must either be subdivided or measured by the well-known method of line crossing. The measured spatial distances can be used to connect points on the Earth's surface and elevated points above them, thus constructing a polyhedron inscribed in or circumscribed about the Earth.

In addition to long distances, astronomically orientated directions along the edges of the polyhedron can be determined. This greatly increases the stability of the polyhedron and the positional accuracy of the points of intersection. Such combined networks are independent of the Earth's potential field and are therefore suitable as a basis for the geometric survey, not subject to hypotheses about continents and the Earth as a whole. By suitably combining the available measured values and a proper selection of network shapes, nets with optimum performance and minimum number of measured quantities can be determined. Similarly, studies for the detail nets for ordnance survey can be carried out.

At the Technical University of Graz, this problem is being studied within the frame of a research programme. Starting from the geometry of the method of line crossing for measuring long distances, the geometric strengths of various shapes of networks are systematically investigated, and proposals for their practical application worked out. This work is in progress, and some results are presented below.

5.1 *The Geometry of Line Crossing*

The method of line crossing, as used in SHORAN and similar systems, is well known. From a measuring instrument moving at constant height (aeroplane, rocket or satellite) crossing in a straight line and at approximately right angles to the line to be measured, the distances s_1, s_2 to the end points of the line can be measured. Then the minimum value of the sum of the distances (s_1+s_2) is determined as a function of the flight time; this value is reduced to an osculating sphere and thus can be derived the length of the geodetic line on the reference surface. Prerequisites for this method are knowledge of the altitudes of the end points of the line and of the measuring instrument, and the assumption that the minimum value of the sum of the distances as well as the line to be measured is located in a plane perpendicular to the osculating sphere. In this case, not only the geodetic distance L in the reference surface, but also the spatial distance D, can be determined from the rectangle defined by the centre O of the sphere, end points P_1, P_2, and the point M from which the minimum value of the sum of distances has been measured. Closer examination, however, discloses discrepancies of the selected model from reality, which impair the accuracy. First, the position of the minimum-value point M depends on the direction of the flight and on the altitudes of the points; only when the direction is perpendicular to the line, and with equal altitudes of both points, will it be located in the normal plane. In the general case, the point will be outside this plane, and the reduction will not yield the geodetic line on the surface or in space. The error formulae have been investigated, and part of the results will be contained in the handbook[1] to be published soon, and will not be treated here.

Accurate determination of the distance from the multitude of values measured during the crossing is possible in two ways: either by projecting the individual measurements on a reference surface, utilizing the altitudes of the measuring points and end points, or by additionally measuring directions.

As the length of the crossing line is only 1/10 to 1/20 of the distance to be measured, the reference surface along the line can be approximated by an osculating sphere. When the altitudes are known, the measured distance can be transferred to this sphere by radial lines. This leads to the two-dimensional spherical problem to determine, from a number of spherical distance sums, the geodetic line which can be solved with strict accuracy, independently of the course of the measuring instrument (as a minimum problem). From the geodetic line, through the chord of the sphere and introducing the altitudes, the spatial distance can be deduced. The only difference between the usual approximation method and the strict method is that the individual measurements are projected, instead of their minimum value. As this projection can be carried out by a computer, the additional work involved is unnecessary. Similarly, the spatial distance can be determined as a minimum value.

For this method, heights must be known, but they cannot always be measured with sufficient accuracy. Their measurement can be avoided if, from the end points of the line, the directions to the measuring instrument are astronomically determined simultaneously with the distances. This can be done, as is known, by photographing light flashes on the background of the fixed stars in the sky. In this case, the angle between the sides s_1, s_2 is determined for each measurement and, from each measurement, the spatial distance can be calculated.

By thus improving the line crossing method, it is possible to determine the chords of a polyhedron which is inscribed in the Earth. This method is an indirect one, whereas the direct method yields spatial distances between elevated points.

5.2 *Spatial Networks*

Spatial networks can be constructed from direct or indirect measurements. In the first case, some of the points of intersection (elevated targets) are moving, and only the terrestrial ones are fixed. The networks enclose the Earth. A network determined by indirect distances contains only fixed points of intersection, and is inscribed in the Earth. The former type of network is more simple to adjust, but contains more points of intersection, some of which are variable for each group of measurements. For indirectly measured distances, a separate determination of weights is necessary, but the networks formed by them are fixed, and there are fewer points of intersection.

In both cases, there are different types of network, with different performances. For determining these, the overdetermined shape is adjusted by variation of co-ordinates, noting that the inverse matrix contains the weight coefficients:

$$Nx - l = 0, \qquad x = N^{-1}l = Ql$$

The spatial error in the position of a point, independent of the selection of the system of co-ordinates, is calculated from the formula:

$$m_p = m_o \sqrt{(Q_{xx} + Q_{yy} + Q_{zz})}$$

in which m_o is the average error of unit weight. With observations of equal weight, m_o is the average error of an adjusted measurement, whereas the root depends only on the geometrical shape of the network. We define, therefore, the geometrical strength of the network at a point by the reciprocal:

$$L = 1/\sqrt{(Q_{xx} + Q_{yy} + Q_{zz})}$$

Thus a high strength of network corresponds to a small average point error, and vice versa.

The simplest shape of a spatial network is an arc section. The geometrical strength of sections determined from both four and twenty-six distance units, with symmetrically located basic points, has been numerically calculated for various altitudes and positions of the points. From the curves connecting equal point errors, it follows that, with the increase in the number of basic points and distances from four to twenty-six, the point error decreases with the square root. Selecting a unit distance of 500 km, the side length of the basic square is 1000 km, the altitude of the new point is assumed to be 2500 km; the dimensions of the rectangular base are 3250 × 1000 km. For an elevated point with x and y co-ordinates of 1500 km and with a distance error of $m = \pm 1$ m, a point error in the first case would be $\pm 5\cdot 2$ m and, in the second case, $\pm 1\cdot 6$ m. Analogous diagrams have been prepared for a point in a cubic spatial network of side length = 5 × length of the basic area.

As a second task, the transferring of four points at ground level has been investigated, first with four and then with twenty-six elevated points. This gives a reply to the question, whether it is more advantageous to use limited overdetermination coupled with high geometrical strength or a high degree of overdetermination with low strength of the network. The results show that it is an advantage to use a greater number of points, and confirm the use of SECOR for the adjustment of all satellite distance records.

Following the simple transferring of points, it is intended to investigate the repeated transferring in a strip circling the globe. For this case too, a mathematical model is first prepared and calculated. The introduction of suitable error distributions for the measured values makes it then possible to simulate the conditions prevalent in nature.

For investigating networks covering the whole Earth, various pyramidal icosahedrons are used as models. By lengths only, this polyhedron, well known in statics, is many times overdetermined. Adding directions for some of the edges further increases the stability or network strength. Finally, a combination of both types of measurements, advantageous from the viewpoint of error theory, will show a valuable way to achieve optimum performance with reduced measuring work.

Part of the study of basic types of spatial nets is Dr Killian's question as to networks solely determined by distance measurements to elevated points, while distances between terrestrial points are not measured at all. If n_t and n_h are the number of terrestrial and elevated points, respectively, the relation

$$n_t n_h - 3(n_t + n_h) + 6 = 0$$

applies to such systems. The only integral solutions are $n_t = 4$, $n_h = 6$ and $n_t = 6$, $n_h = 4$ and define the only possible network shape with ten points. If, in addition, s distances between terrestrial targets are measured, the relation is:

$$n_t n_h + s - 3(n_t + n_h) + 6 = 0$$

In this case, for every value of s, two solutions are obtained. These interesting basic shapes become indeterminate if the points of the network are located on surfaces of the second order. This fact must be kept in mind when planning networks for the transfer of points by distances using this very simple method.

For extensive networks, it is desirable to introduce a criterion for the quality of the network with regard to its geometrical performance. It is proposed to use, as a criterion, the average error m_p of the position of the point, calculated as the root mean square of the position errors for all new points:

$$m_p = \sqrt{[\sum(m_p^2)/n]} = m_o\sqrt{[\sum(Q_{xx}+Q_{yy}+Q_{zz})/n]}$$

where

n = number of new points
m_o = mean square error of a measurement

As another error criterion of a network, the mean distance error after adjustment can be introduced:

$$m_s = \sqrt{[\sum(m_s^2)/n]} = m_o\sqrt{[\sum(Q_{ss})/n]}$$

More suitable is a mean square relative distance error determined as follows:

$$\bar{\mu}s = (\bar{m}_s/s) = \sqrt{[\sum(m_s/s)^2/n]} = m_o\sqrt{[\sum(Q_{ss})/s^2]}$$

The same can also be calculated (as has been done for the verification net at Graz) from a different relation:

$$\bar{\mu} = \bar{m}_s/\bar{s}_s = m_o\sqrt{[n\sum(Q_{ss})/\sum s]}$$

but, for systematic reasons, the first relation is more appropriate.

6. *Electromagnetic Tacheometry and Image-forming*

Geodetic tacheometry measures spatial or planar co-ordinates of points of the terrain with relation to local polar systems. As the use of electromagnetic waves permits the measurement not only of distances but also differences, sums, or quotients of distances, it is possible to measure, in addition to polar co-ordinates, elliptic and hyperbolic co-ordinates as well as co-ordinates based on quotients, of the points on the terrain in local systems. These can then be either directly used for geodetic purposes or transformed into a uniform system. Geodetic purposes make it necessary to adopt co-ordinate systems other than those used at present, the co-ordinate lines being hyperbolae, ellipses or apollonic curves. These possibilities make it desirable to investigate systematically the geometric properties of these systems, in order to be able to use them for geodetic purposes in the case of possible developments of electromagnetic measuring systems.

This problem has been studied for plane, spherical and spheroidal curves, and the results reported[1] where conditions prevailing in space are also discussed. Further investigations are intended.

Worthy of special interest are the methods using quotients of distances, as—if propagation conditions are homogeneous—no absolute values are required, but only ratios determined by quotients of transit times.

It is well known that pictures can be obtained through the use of electromagnetic waves (radar). Owing to qualitative improvements, these pictures have gained importance for geodetic and cartographic purposes. In order to use them for geodetic purposes, it is necessary to know the laws of image-formation and of geometric distortion. Investigations in this field have been carried out, and their principles reported[1]. Further investigations, especially on the geometric effects of electromagnetic transit time distortion, are in progress.

Summary

A report is given on research programmes completed or in progress at the Technical University of Graz. An invitation to use the verification net in Graz is made. Some of the research results are reported in Vol. VI of the *Handbuch der Vermessungskunde*.[1]

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DISCUSSION

G. Jelstrup: What was the difference in temperature along the line compared with the mean of the two ends?

K. Rinner: The maximum difference was 6°, it being higher as measured by the helicopter.

R. C. A. Edge: Was humidity as well as temperature measured by the helicopter?

K. Rinner: No, only the air temperature.

R. C. A. Edge: Where on the helicopter was the thermometer carried?

K. Rinner: On the tail.

Electromagnetic Distance Measurements in Finland

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National Board of Survey
and
T. J. KUKKAMÄKI
Finnish Geodetic Institute
Presented by
PROFESSOR L. ASPLUND

The paper briefly describes the various orders of control nets in Finland, electronic distance measuring equipment having been used since 1963 for scale checks on the work done by traditional methods. First-order sides of average length 29·6 km gave a mean square difference of 1 : 475 000. Second order gave 1 : 280 000 and third order 1 : 130 000.

The first-order triangulation of the Finnish Geodetic Institute consists of triangle chains, which form loops of 500 to 800 km perimeter. The National Board of Survey has now started to fill the loops with nets, called main-order nets, with an average side length of 30 km, in which the angles are measured as well as the distances. After preparing experiments, the National Board of Survey started the electronic distance measurements in 1963 and now uses Tellurometer MRA-3 in main-order measurements.

Every distance is measured twice so that at both ends the instrument is used as the master instrument. This method is used in the first place in order to control the frequencies currently. In addition, the frequencies are calibrated at a Laboratory when there seems to be any reason to suspect a change, usually three times every field season. Most of the distances have been measured between triangulation towers rather high above the ground over wooded areas; perhaps this is the reason why only slight traces of reflections can be found in the results. Special attention has been paid to the determination of the refractive index. The air pressure has been measured with Thommen barometers which have been calibrated frequently. The temperature and the humidity have been determined with carefully calibrated Assmann-type psychrometers. The psychrometers whirled by hand have been rejected for their inaccuracies.

The accuracy of the measured distances has been investigated in different ways. In 1964, thirty-six first-order triangulation sides were measured in connection with the main-order network. Nineteen of these sides had already been adjusted and were then used for comparison. These sides were situated in different parts of the first-order net, but no remarkable systematic differences between different areas could be discerned. Instead, a large systematic difference between first-order triangulation and the corresponding tellurometer measurement was apparent, amounting to 1 : 164 000, the tellurometer distances being shorter. After the elimination of this systematic difference, the remaining mean square difference was ± 62 mm on sides with an average length of 29·6 km, or 1 : 475 000.

As another attempt to investigate the accuracy, an adjustment of a net may be mentioned. The main-order net of nineteen new points was adjusted independently of the first-order net. Then the mean square error of a distance of 30 km was ± 50 mm or 1 : 600 000.

The triangulations of the second order have now been carried out as a combination of triangulation and trilateration. The Tellurometers MRA-3 and, especially, MRA-2 have been applied. The adjustments of five 15-point blocks gave a relative accuracy of 1 : 280 000 on average. In the third-order nets, no towers were erected and so the observations were made on the ground. Mostly the angles were measured only at the point to be determined and, in addition, a couple of distances with Tellurometer MRA-2. The relative mean square error of a third-order point was about 1 : 130 000 on average. The density of second-order points was about 1 point per 100 km^2 and that of the third order 4–5 points per 100 km^2.

All the different steps of the computations have been made with electronic computers. The readings have been recorded in field books especially planned for the use of computers. The data are punched and then reduced and, finally, adjusted with an electronic computer.

The Finnish Geodetic Institute started its investigation into trilateration with experiments held in May–June 1965 on the Vihti enlargement net. The longest side of the net had the length of $29\,114{\cdot}1372 \pm 0{\cdot}0573$ m according to the triangulation. Two Tellurometer units MRA-3–Mk II were used as master and as remote instrument alternatively. All eleven sides of the enlargement net were measured and the results deviated from the side lengths of triangulation by ± 24 mm on average according to provisional computations. For the extrapolation of the humidity and of the temperature for the actual Tellurometer beam along the whole distance, these atmospheric factors were measured at two altitudes of 2 m and of 10–24 m above the ground at both ends and in the middle of the measured side. The periodic factor of the index error in the instruments was determined at a 432-m distance on the Nummela Standard Base by varying the measured distance during the whole 20 min. period of the A-reading.

DISCUSSION

There were no questions.

The Caithness Base Investigation

Presented by

M. R. RICHARDS

The Ordnance Survey of Great Britain

In 1964, the Ordnance Survey conducted a large-scale practical investigation into the determination of refractive index for microwaves. Stations for measuring meteorological conditions were established along the length of the Caithness Base (24·8 km), previously measured by invar in catenary. Precise measurements of barometric pressure, and of wet-bulb and dry-bulb temperatures at various altitudes, were made simultaneously with distance measurement of the base by Tellurometers MRA-3 and MRA-2. A Geodimeter NASM-4 was also employed. General weather conditions (rain, cloud cover, wind speed and direction and visibility) were also recorded. Observations were spread over two periods of ten days each, while over 900 distance measurements were made. The resulting data are being subjected to analysis. Anyone who wishes to carry out research into any aspect of the investigation may obtain copies of the field data from the Ordnance Survey. The paper sets out all the necessary background information, and includes samples of the available field data.

1. *General*

The Report of Special Study Group No. 19 of the IAG, for 1960–3, contained the recommendation that research into the measurement of refractive index should be energetically continued. The particular aspect of research with which this paper deals is the repeated measurement of a line under varying meteorological conditions. Many previous investigations have been carried out on this subject by numerous organizations. However, these have generally suffered from one or other of two serious shortcomings. Firstly, the line being measured might be of very accurately known length, but relatively short, over flat ground, and thus not typical of normal observing conditions. Alternatively, the line might be long, with elevated terminals and more typical of lines normally measured, but the length would not be known to better than about 10 parts per million (ppm); this is the order of accuracy that might be expected from geodetic triangulation.

It was felt that there would be considerable advantage in carrying out an investigation on a line that was relatively long, of accurately known length, and having elevated terminals that would give a more typical profile. In Caithness, Northern Scotland, the Ordnance Survey have such a line. It was measured in 1952 by invar tapes in catenary. It is 24·8 km long, and is thought to be the longest in the world. The estimated standard error of the measurement is ± 2 cm, or approximately 1 ppm. The terminals are 125 and 176 m respectively above mean sea level, and the intervening ground lies between 30 and 75 m above sea level. The whole length of the line is readily accessible.

The Ordnance Survey of Great Britain, jointly with the British Military Survey Service, considered that they were in a position to make a positive contribution to the research into refractive index measurement urged by Study Group No. 19. The opportunity was perhaps unique in the survey sphere, as few other organizations could provide the large manpower force required to carry out a large-scale investigation. It was therefore decided to mount a practical investigation, the main purpose of which would be to study, and provide data on, the behaviour of the lower layers of the atmosphere under different conditions. The effect of the meteorological variations on the derived refractive index would also be studied, in conjunction with a series of electronic distance measurements of the line. Provided that the necessary equipment could be obtained, it would not be difficult to measure meteorological conditions along the line, and at some distance above the ground. From this, some conclusions on the optimum conditions for observations might be obtained.

During the investigation, different types of EDM instrument would be used, and a comparison made of their performance. The 'ground swing' patterns obtained from microwave instruments could also be studied. Previous indications had been that the line might prove difficult to measure owing to abnormal and erratic swing characteristics, but it was hoped that the 3-cm instruments would overcome this. Finally, as a by-product of the very extensive observations that it was intended to make, a great deal of data would be produced which could be made available for others to use for research.

The investigation was staffed by a contingent of Military Survey personnel from 19 Topographic Squadron R.E. together with a party from the Ordnance Survey. The observations were split into two periods of ten days each in order to vary the conditions as widely as possible. The first was in July 1964 when it was expected that there would be fine summer weather. The second was in September–October 1964 when cooler more wintry conditions were expected. In the event the weather differed only slightly between each period; July was cold and wet, October surprisingly mild.

The observations were taken throughout the whole of each day. The twenty days of field work produced over 900 distance measurements of the line, recorded on a corresponding number of booking sheets. The meteorological data, having been abstracted and corrected for calibration, fill over 300 sheets approximately three times the size of this page. These data are available to any person (or any organization) wishing to carry out research into any aspect of the investigation. The only proviso is that the Ordnance Survey shall be provided with a copy of any results which may ensue.

The object of this paper is to set out in detail what was actually done. All relevant information will be given, together with samples of the available data. No conclusions have been drawn, but one aspect of the investigation forms the subject of a separate paper.

2. *Base Line*

The Caithness Base is the primary triangulation side between the stations Spital Hill and Warth Hill. The accepted spheroidal distance between terminals, as derived from the catenary measurements, is 24 828·000 m. Auxiliary stations, to accommodate the numerous EDM instruments at each terminal, were established for the July period as in Fig. 1 (see also Appendix A on p. 35).

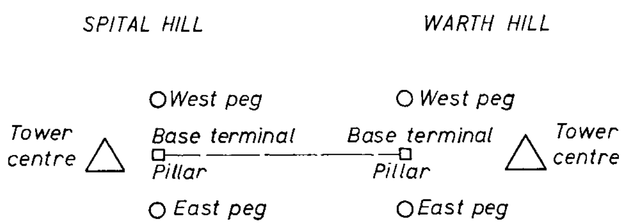

FIG. 1. Layout of the Caithness Base, showing auxiliary stations in July, 1964.

Stations described as East Peg and West Peg were established precisely at right angles to the line of sight, and 3 m from the base terminal. Stations, each consisting of a 30-ft (9-m) survey (Bilby) tower, were established immediately behind each base terminal and on the line of the base.

Auxiliary stations for the October visit were established in slightly different positions as in Fig. 2. The auxiliary stations were established in order to vary the distance measured slightly, so that any possible index error which was a function of the A reading could be recognized as a systematic error, and eliminated. In addition, on each Bilby tower a second position was established on the handrail so as to prevent reflections from the metal work.

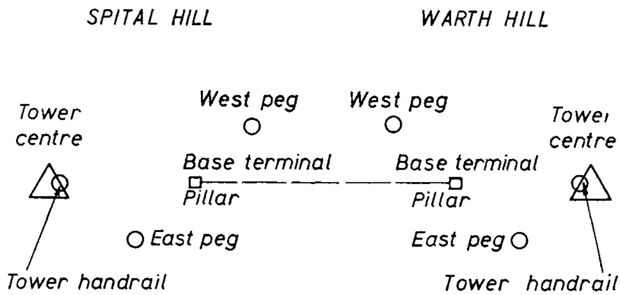

FIG. 2. Layout of the Caithness Base, showing auxiliary stations in October, 1964.

To reduce the measured distance to the spheroidal distance the following corrections must be made:

$$\text{Slope correction} = -\frac{\Delta h^2}{2S}$$

$$\text{Reduction to sea level} = -\frac{S.h_m}{R}$$

$$\text{Chord to arc correction} = +\frac{S^3}{43R^2}$$

where h_m is the mean height of the instruments above sea level, Δh is the difference in height of the two instruments, S the spheroidal distance, and R the radius of the Earth.

The corrections that should be applied to the measured air distances are set out in Table 1 and include corrections for the eccentricity of the auxiliary stations. Individual corrections may vary slightly from the tabulated values according to the height of instrument above datum, but variations will be insignificant.

3. *Instruments*

Two pairs of Tellurometer MRA-3 instruments and one pair of Tellurometer MRA-2 were available for each period.

MRA-3 instruments Nos. 308 and 311 were designated the standard instruments for the July period, and were used exclusively between

Table 1*a*. *Corrections to be applied to air distances, July* 1964

All corrections are in metres.

Type of measure	Correction for			Total correction	Correction for eccentricity at		Overall correction to air dist.
	Slope	MSL	Chord to arc		Warth	Spital	
Pillar to pillar	−0·0545	−0·5874	+0·0087	−0·633	Nil	Nil	−0·633
East Peg to East Peg	−0·0544	−0·5869	+0·0087	−0·633	Nil	Nil	−0·633
West Peg to West Peg	−0·0542	−0·5864	+0·0087	−0·632	Nil	Nil	−0·632
Tower centre to tower centre	−0·0550	−0·6191	+0·0087	−0·665	−3·895	−4·007	−8·567

Table 1*b*. *Corrections to be applied to air distances, October* 1964

All corrections are in metres.

Type of measure	Correction for			Total correction	Correction for eccentricity at		Overall correction to air dist.
	Slope	MSL	Chord to arc		Warth	Spital	
Pillar to pillar	−0·0544	−0·5874	+0·0087	−0·633	Nil	Nil	−0·633
East Peg to East Peg	−0·0545	−0·5873	+0·0087	−0·633	−1·477	−1·486	−3·596
West Peg to West Peg	−0·0534	−0·5862	+0·0087	−0·631	+1·516	+1.482	+2·367
Tower centre to tower centre	−0·0551	−0·6192	+0·0087	−0·666	−3·880	−4·004	−8·550
Tower handrail to tower handrail	−0·0554	−0·6200	+0·0087	−0·667	−2·923	−3·112	−6·702

the terminal pillars, making a measurement each hour (timed to start on the hour) throughout the period. Instruments Nos. 270 and 274 were used for a similar purpose during the October period.

MRA-3 instruments Nos. 270 and 274 alternated with MRA-2 instruments Nos. 773 and 774 making measurements from the offset positions during the first period. Nos. 308 and 311 changed with Nos. 270 and 274 in the second period so that the latter became the standard instruments. Measurements were made hourly with these instruments, timed to start at each half hour. Measurements were not permitted to overlap, i.e. the measurement with the standard instruments had to be completed (or abandoned), before the secondary pair took over, and vice versa.

The Tellurometers MRA-2 Nos. 773 and 774 were overhauled and had the crystal frequencies calibrated and adjusted by Tellurometer (UK) Ltd, before and after both periods of fieldwork, and crystal frequencies were certified as correct. The MRA-3 instruments were unfortunately available to be checked only after the second period. The actual deviations from standard of the A crystal frequencies found on the final check, and prior to any adjustment, are indicated in Table 2.

TABLE 2. *Modulation frequency errors (c/s)*
(as determined at final check)

	Instrument					
Crystal	MRA-2 773	MRA-2 774	MRA-3 270	MRA-3 274	MRA-3 308	MRA-3 311
Master A	−14	−7	−4	−8	0	+6
Remote A+	−4	−8	+5	−6	+11	0
A−	−10	−2	−2	−6	−2	0

It would appear probable that the maximum error introduced into any of the measurements would be from MRA-2 No. 773, of the order of 1·4 ppm, and hardly significant. Errors introduced by any of the other instruments should be still less significant, and negligible in the context of the investigation.

It was not possible, owing to shortage of time, to test the instruments for zero error before the experiment, although this is now being done. From available evidence, it seems unlikely that, as far as the Tellurometers MRA-2 are concerned, the zero error is significant on a measured length of 25 km. For the Tellurometers MRA-3, there is evidence that by measuring with these instruments alternately as master and remote from opposite ends of the line, as was done in this investigation, residual zero error is reduced to less than 1·5 cm (0·6 ppm) which is almost negligible[1].

A total of 893 Tellurometer measurements were completed.

Additionally, measurements were made with a Geodimeter NASM-4 (No. 266, with mercury-vapour lamp) from one or other of the auxiliary stations whenever visibility permitted. A total of fifty-seven measurements were made, forty during the first period and seventeen during the second. During the second period, vertical angles (zenith distances) were observed simultaneously with the measurements. The object of this was to make a reduction of the measured distance in the manner suggested by Saastamoinen.[2] There was a slight doubt about the accuracy of the Geodimeter because of asymmetry in the Kerr cell, produced by progressive oxidization of the nitrobenzine.

4. *Meteorological Instruments*

Preliminary investigations indicated that there was no suitable equipment available commercially for the remote measurement of temperatures. The necessary apparatus was specially made, based on a design developed by the Meteorological Office. The main components were thermistors mounted in silicone rubber inside stainless steel sheaths, with a temperature response time of a few seconds. They were wired to a galvanometer through a switch box, the latter incorporating balancing resistors; several thermistors could be read on one meter. The galvanometer scale was graduated in °C with an upper and lower scale range, each covering 20°C, and could be read to 0·05°C. Initially, some trouble was experienced with the temperature scales since they did not overlap; 10°C was the change-over point, and calibration was critical at this point. The meters were later modified to incorporate a 5°C overlap between scales, and this solved the initial problems.

Calibrations were carried out, using a water bath, both before and after each observing period. The calibration figures show remarkable consistency, indicating very little drift. It is considered that the field temperatures, having been corrected for calibration, can be relied on to be correct to ±0·2°C.

The thermistor probes were mounted in pairs in retaining brackets (see Appendix B). One had a textile sheath fed by a wick from a water bottle to act as a wet bulb. Each was housed centrally inside two concentrically mounted open-ended cylinders, which formed a shield against radiated heat. Each pair was aspirated by an electric fan mounted on the end of the protective cylinders, drawing 30 ft^3 of air per minute over the probes. To prevent possible heat transfer from the fan to the probes, 'Tufnol' insulating blocks were inserted between the base of the fan and the cylinders. A single unit thus performed the function of a wet- and dry-bulb psychrometer, with the advantage that the instrument could be placed at some distance from the recording station.

Measurements of atmospheric pressure were made with altimeters operated in pairs. Pressures were read in equivalent metres of altitude and have been converted directly to mm of mercury through the calibration values. The average difference between simultaneous readings on a pair of altimeters is less than 1·0 mm Hg; (1 mm error in pressure represents approximately 0·3 ppm error in the refractive index determination).

5. *Meteorological Observations*

Two meteorological stations were established at each terminal, and three more along the line of the base. In addition, standard meteorological measurements were taken by the Tellurometer observers using aspirated psychrometers and aneroid barometers. The position of all these stations is shown on the diagram in Appendix A. At each terminal a 50-ft (15-m) mast was erected, approximately 14° off line to avoid interference by reflections, such that the top of the mast was level with the line of sight between base terminals. Wet- and dry-bulb thermistor units were placed at the top, and at the 30-ft (9-m) level, of each mast. They were also placed at the top of each terminal tower, i.e. approximately 30 ft (9 m) above the base terminal, and on the tower, level with the instrument on the terminal pillar. At Annfield, approximately 8 km from Spital Hill a 103-ft (30-m) Bilby tower was erected. Wet- and dry-bulb thermistor units were established at the top, at the 60-ft (18-m) and 20-ft (6-m) levels on this tower. At Hillhead, the mid-point of the base line, and at Slickly, further 103-ft towers were erected. At each of these towers, a wet- and dry-bulb unit was attached to a rope and pulley, so that it could be raised and lowered at will to predetermined levels on the tower. All temperatures were read remotely on a meter at some convenient position at each location. Temperatures were read and recorded to 0·05°C.

At the base terminals, and at Annfield, the reading drill was the same. The observer recorded wet- and dry-bulb temperatures from each of his thermistors at five-minute intervals. He also recorded barometric pressures at ten-minute intervals. Owing to the shortage of equipment and man-power it was not possible to man the Hillhead and Slickly stations full time; the observer alternated between these two stations.

At Slickly, he read temperatures at every 20-ft (6-m) level [i.e. at 100, 80, 60, 40 and 20 ft (30, 24, 18, 12 and 6 m)] of the tower, twice in succession, commencing on the hour. He then moved to Hillhead to make similar observations on the half hour. Barometric pressures were not recorded at these stations.

In addition to these readings, each observer made a detailed note every half hour of the prevailing weather conditions, including wind speed and direction.

6. *Preliminary Reductions and Analysis*

6.1 *Distance Measurements—Tellurometer*

Each of the distance measurements has been reduced, on the field sheets, to an air distance. A sample booking sheet is shown in Appendix C, p. 37. Note that the times booked are British Summer Time (subtract one hour for Greenwich Mean Time). The standard meteorological observations taken at the instrument station, and given on the field sheets, have been used in this reduction. All air distances have been further reduced to the spheroidal distance between terminals using the corrections tabulated in Table 1. One surprising fact has emerged. The measurements made with Tellurometers MRA-3 between the terminal pillars are systematically shorter than those made with the same instruments between any other combination of auxiliary stations, no matter which pair of instruments is involved. The reason for this is not apparent, and investigation into it continues. It is not a function of the fixation of the auxiliary stations; these have been checked and are errorless. It will be noted that the same phenomenon does not appear to occur with Tellurometers MRA-2, although the pillar-to-pillar sample is too small for this to be said definitely. The relevant figures are given in Table 3.

Note that 133 measurements made from steel towers are not included in these mean figures as, despite being made in an atmosphere free from possible ground anomaly, these proved very difficult to observe. The results, particularly those from the MRA-2 Tellurometer, indicate gross swings from which no pattern emerges. They give widely divergent answers ranging from 24 826·850 to 24 829·232 m.

6·2 *Distance Measurements—Geodimeter*

All Geodimeter measurements were made between auxiliary stations and reduced to the terminal pillars. There are rather fewer of these measurements (fifty-seven only), but no more detailed analysis than that given below has yet been possible. The spheroidal distances obtained

TABLE 3. *Mean derived distances*

Stations	Period	Instruments	Numbers in mean	Derived distance (m)	S.D. of single observation (m)	S.D. of mean (m)
Pillar to pillar	1st	MRA-3 (308, 311)	237	24 827·958	±0·081	±0·005
	1st	MRA-2 (773, 774)	6	28·111	±0·168	±0·068
	2nd	MRA-3 (270, 274)	240	27·922	±0·081	±0·005
	Both	All instruments	483	27·942	±0·083	±0·004
Peg to peg	1st	MRA-3 (270, 274)	64	28·064	±0·124	±0·016
	1st	MRA-2 (773, 774)	92	28·080	±0·083	±0·009
	2nd	MRA-3 (308, 311)	87	28·026	±0·082	±0·009
	2nd	MRA-2 (773, 774)	34	28·068	±0·080	±0·014
	Both	All MRA-3	151	28·042	±0·102	±0·008
	Both	All MRA-2	126	28·077	±0·082	±0·007
	Both	All instruments	277	28·058	±0·094	±0·006
Overall mean figure			760	24 827·984	±0·087	±0·003

The accepted catenary measurement is 24 828·000 m.

(compare catenary 24 828·000 m) were:

First visit, mean of 40 measurements	24 828·097 m
S.D. of single measure	±0·026 m
S.D. of mean	±0·004 m
Second visit, mean of 17 measurements	24 828·029 m
S.D. of single measure	±0·039 m
S.D. of mean	±0·009 m

Applying Saastamoinen's[2] method to the results of the second period gives:

Mean of 17 measurements	24 828·006 m
S.D. of single measure	±0·054 m
S.D. of mean	±0·013 m

(Note that vertical angles were not observed during the first period.)

6.3 *Temperature and Pressure Observations*

All thermistor temperature measurements and barometric pressures have been corrected for calibration and abstracted on to sheets similar to the samples shown in Appendix C, p. 37. Abstraction has been done by stations with the sheet headed simply Spital, Annfield, Hillhead, Slickly, or Warth. The nominal altitude of the thermistor unit above ground level at the station is given. Where sheets include a column headed Meter Temperature this should be ignored. It was originally included because it was suspected that the calibration procedure had introduced an error by having probes at one temperature and the meter at another. This suspicion was subsequently disproved. Remarks are included where necessary on the occasions when the readings may be suspect. This happened from time to time when the wick for the wet bulb failed to draw up water, when electrical faults occurred, when the battery activating the meter became discharged, or on one occasion when a cow chewed the cable.

7. *Available Data*

Samples of the available data are attached. Those which can be supplied to interested research workers are as follows:

Tellurometer Field Booking Sheets (see Appendix C)

First period	439 sheets
Second period	455 sheets

Geodimeter Field Booking Sheets (see Appendix D)

First period	40 sheets
Second period	17 sheets