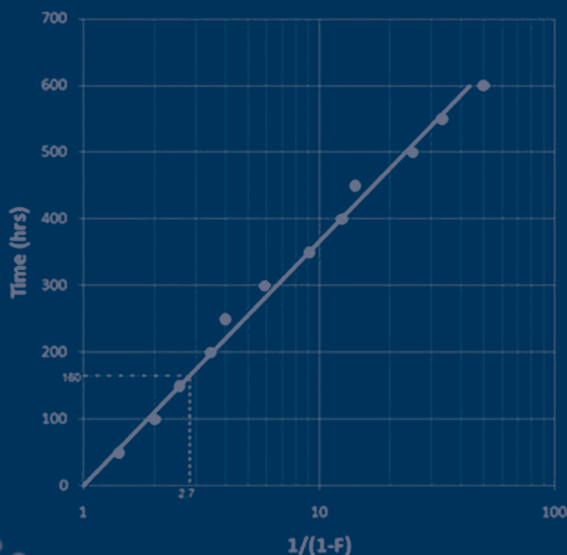
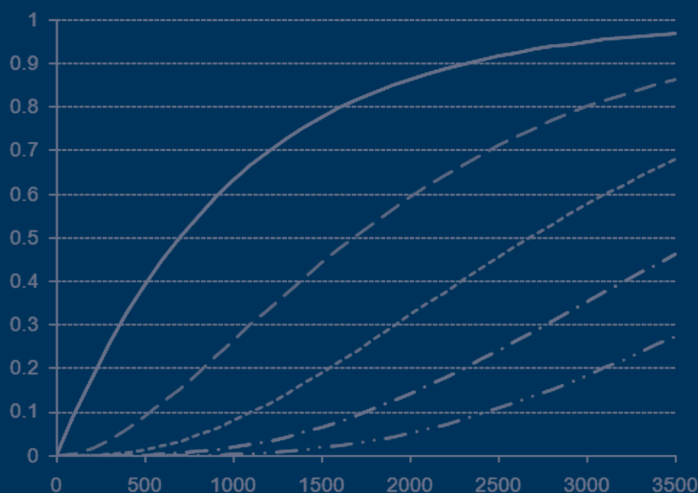


Applied Reliability

Third Edition

Paul A. Tobias

David C. Trindade



Applied Reliability

Third Edition



Taylor & Francis

Taylor & Francis Group

<http://taylorandfrancis.com>

Applied Reliability

Third Edition

Paul A. Tobias

David C. Trindade



CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group an **informa** business
A CHAPMAN & HALL BOOK

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

© 2012 by Taylor & Francis Group, LLC
CRC Press is an imprint of Taylor & Francis Group, an Informa business

No claim to original U.S. Government works

International Standard Book Number: 978-1-58488-466-8 (Hardback)

This book contains information obtained from authentic and highly regarded sources. Reasonable efforts have been made to publish reliable data and information, but the author and publisher cannot assume responsibility for the validity of all materials or the consequences of their use. The authors and publishers have attempted to trace the copyright holders of all material reproduced in this publication and apologize to copyright holders if permission to publish in this form has not been obtained. If any copyright material has not been acknowledged please write and let us know so we may rectify in any future reprint.

Except as permitted under U.S. Copyright Law, no part of this book may be reprinted, reproduced, transmitted, or utilized in any form by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying, microfilming, and recording, or in any information storage or retrieval system, without written permission from the publishers.

For permission to photocopy or use material electronically from this work, please access www.copyright.com (<http://www.copyright.com/>) or contact the Copyright Clearance Center, Inc. (CCC), 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. CCC is a not-for-profit organization that provides licenses and registration for a variety of users. For organizations that have been granted a photocopy license by the CCC, a separate system of payment has been arranged.

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation without intent to infringe.

Library of Congress Cataloging-in-Publication Data

Tobias, Paul A.
Applied reliability / Paul A. Tobias and David C. Trindade. -- 3rd ed.
p. cm.

Summary: "This popular book is an easy-to-use guide that addresses basic descriptive statistics, reliability concepts, the exponential distribution, the Weibull distribution, the lognormal distribution, reliability data plotting, acceleration models, life test data analysis and systems models, and much more. The third edition includes a new chapter on Bayesian reliability analysis and expanded, updated coverage of repairable system modeling. Taking a practical and example-oriented approach to reliability analysis, it also provides detailed illustrations of software implementation throughout using several widely available software packages. Software and other files are available for download at www.crcpress.com"-- Provided by publisher.

Summary: "It's been over 15 years since the publication of the 2nd edition of Applied Reliability. We continue to receive positive feedback from old users, and each year hundreds of engineers, quality specialists, and statisticians discover the book for the first time and become new fans. So why a 3rd edition? There are always new methods and techniques that update and improve upon older methods, but that was not the primary reason we felt the need to write a new edition. In the last 15 years, the ready availability of relatively inexpensive, powerful, statistical software has changed the way statisticians and engineers look at and analyze all kinds of data. Problems in reliability that were once difficult and time consuming for even experts now can be solved with a few well-chosen clicks of a mouse. Additionally, with the quantitative solution often comes a plethora of graphics that aid in understanding and presenting the results. All this power comes with a price, however. Software documentation has had difficulty keeping up with the enhanced functionality added to new releases, especially in specialized areas such as reliability analysis. Also, in some cases different well-known software packages use different methods and output different answers. An analyst needs to know how to use these programs effectively and which methods are the most highly recommended. This information is hard to find for industrial reliability problems"-- Provided by publisher.

Includes bibliographical references and index.

ISBN 978-1-58488-466-8 (hardback)

1. Reliability (Engineering) 2. Quality control--Statistical methods. I. Trindade, David C. II. Title.

TA169.T63 2011
620'.00452--dc22

2011011223

Visit the Taylor & Francis Web site at
<http://www.taylorandfrancis.com>

and the CRC Press Web site at
<http://www.crcpress.com>

Contents

| | |
|--|-----------|
| Preface..... | xiii |
| List of Figures | xv |
| List of Tables | xxvii |
| List of Examples | xxxi |
| | |
| 1. Basic Descriptive Statistics..... | 1 |
| 1.1 Populations and Samples..... | 1 |
| 1.2 Histograms and Frequency Functions..... | 2 |
| 1.3 Cumulative Frequency Function | 5 |
| 1.4 The Cumulative Distribution Function and the Probability Density Function | 6 |
| 1.5 Probability Concepts | 9 |
| 1.6 Random Variables | 16 |
| 1.7 Sample Estimates of Population Parameters..... | 16 |
| 1.8 How to Use Descriptive Statistics..... | 22 |
| 1.9 Data Simulation..... | 23 |
| 1.10 Summary | 25 |
| Appendix 1A..... | 26 |
| 1.1A Creating a Step Chart in a Spreadsheet..... | 26 |
| Problems..... | 27 |
| | |
| 2. Reliability Concepts..... | 29 |
| 2.1 Reliability Function | 29 |
| 2.2 Some Important Probabilities | 31 |
| 2.3 Hazard Function or Failure Rate | 32 |
| 2.4 Cumulative Hazard Function | 33 |
| 2.5 Average Failure Rate..... | 34 |
| 2.6 Units..... | 35 |
| 2.7 Bathtub Curve for Failure Rates | 36 |
| 2.8 Recurrence and Renewal Rates..... | 38 |
| 2.9 Mean Time to Failure and Residual Lifetime | 39 |
| 2.10 Types of Data | 41 |
| 2.10.1 Exact Times: Right-Censored Type I..... | 41 |
| 2.10.2 Exact Times: Right-Censored Type II..... | 42 |
| 2.10.3 Readout Time or Interval Data..... | 42 |
| 2.10.4 Multicensored Data | 42 |
| 2.10.5 Left-Censored Data..... | 43 |
| 2.10.6 Truncated Data..... | 43 |
| 2.11 Failure Mode Separation..... | 45 |
| 2.12 Summary | 45 |
| Problems..... | 46 |

| | |
|--|-----|
| 3. Exponential Distribution | 47 |
| 3.1 Exponential Distribution Basics | 47 |
| 3.2 The Mean Time to Fail for the Exponential | 51 |
| 3.3 The Exponential Lack of Memory Property | 52 |
| 3.4 Areas of Application for the Exponential | 53 |
| 3.5 Exponential Models with Duty Cycles and Failure on Demand | 55 |
| 3.6 Estimation of the Exponential Failure Rate λ | 56 |
| 3.7 Exponential Distribution Closure Property | 58 |
| 3.8 Testing Goodness of Fit: The Chi-Square Test | 59 |
| 3.9 Testing Goodness of Fit: Empirical Distribution Function Tests | 62 |
| 3.9.1 D -Statistics: Kolmogorov–Smirnov | 63 |
| 3.9.2 W^2 -Statistics: Cramer–von Mises | 64 |
| 3.9.3 A^2 -Statistics: Anderson–Darling | 64 |
| 3.10 Confidence Bounds for λ and the MTTF | 67 |
| 3.11 The Case of Zero Failures | 69 |
| 3.12 Planning Experiments Using the Exponential Distribution | 71 |
| 3.13 Simulating Exponential Random Variables | 75 |
| 3.14 The Two-Parameter Exponential Distribution | 76 |
| 3.15 Summary | 77 |
| Appendix 3A | 78 |
| 3.1A Test Planning via Spreadsheet Functions | 78 |
| Determining the Sample Size | 78 |
| Determining the Test Length Using Spreadsheet Functions | 80 |
| Determining the Number of Allowed Failures via Spreadsheet Functions | 81 |
| 3.2A EDF Goodness-of-Fit Tests Using Spreadsheets | 81 |
| KS Test | 81 |
| Problems | 84 |
| 4. Weibull Distribution | 87 |
| 4.1 Empirical Derivation of the Weibull Distribution | 87 |
| 4.1.1 Weibull Spreadsheet Calculations | 90 |
| 4.2 Properties of the Weibull Distribution | 90 |
| 4.3 Extreme Value Distribution Relationship | 95 |
| 4.4 Areas of Application | 96 |
| 4.5 Weibull Parameter Estimation: Maximum Likelihood Estimation Method | 98 |
| 4.6 Weibull Parameter Estimation: Linear Rectification | 110 |
| 4.7 Simulating Weibull Random Variables | 111 |
| 4.8 The Three-Parameter Weibull Distribution | 112 |
| 4.9 Goodness of Fit for the Weibull | 113 |
| 4.10 Summary | 113 |
| Appendix 4A | 114 |
| 4.1A Using a Spreadsheet to Obtain Weibull MLEs | 114 |
| 4.2A Using a Spreadsheet to Obtain Weibull MLEs for Truncated Data | 116 |
| 4.3A Spreadsheet Likelihood Profile Confidence Intervals for Weibull Parameters | 116 |
| Problems | 121 |

| | |
|--|-----|
| 5. Normal and Lognormal Distributions | 123 |
| 5.1 Normal Distribution Basics | 123 |
| 5.2 Applications of the Normal Distribution | 129 |
| 5.3 Central Limit Theorem | 130 |
| 5.4 Normal Distribution Parameter Estimation | 131 |
| 5.5 Simulating Normal Random Variables | 134 |
| 5.6 Lognormal Life Distribution | 135 |
| 5.7 Properties of the Lognormal Distribution | 136 |
| 5.8 Lognormal Distribution Areas of Application | 140 |
| 5.9 Lognormal Parameter Estimation | 141 |
| 5.10 Some Useful Lognormal Equations | 146 |
| 5.11 Simulating Lognormal Random Variables | 148 |
| 5.12 Summary | 148 |
| Appendix 5A | 149 |
| 5.1A Using a Spreadsheet to Obtain Lognormal MLEs | 149 |
| 5.2A Using a Spreadsheet to Obtain Lognormal MLEs for Interval Data | 150 |
| Problems | 151 |
| 6. Reliability Data Plotting | 153 |
| 6.1 Properties of Straight Lines | 153 |
| 6.2 Least Squares Fit (Regression Analysis) | 155 |
| 6.3 Rectification | 159 |
| 6.4 Probability Plotting for the Exponential Distribution | 161 |
| 6.4.1 Rectifying the Exponential Distribution | 162 |
| 6.4.2 Median Rank Estimates for Exact Failure Times | 163 |
| 6.4.3 Median Rank Plotting Positions | 164 |
| 6.4.4 Confidence Limits Based on Rank Estimates | 168 |
| 6.4.5 Readout (Grouped) Data | 171 |
| 6.4.6 Alternative Estimate of the Failure Rate and Mean Life | 172 |
| 6.4.7 Confidence Limits for Binomial Estimate for Readout Data | 172 |
| 6.5 Probability Plotting for the Weibull Distribution | 175 |
| 6.5.1 Weibull Plotting: Exact Failure Times | 176 |
| 6.5.2 Weibull Survival Analysis via JMP | 178 |
| 6.5.3 Weibull Survival Analysis via Minitab | 178 |
| 6.6 Probability Plotting for the Normal and Lognormal Distributions | 178 |
| 6.6.1 Normal Distribution | 178 |
| 6.6.2 Lognormal Distribution | 181 |
| 6.7 Simultaneous Confidence Bands | 184 |
| 6.8 Summary | 187 |
| Appendix 6A | 187 |
| 6.1A Order Statistics and Median Ranks | 187 |
| Problems | 191 |
| 7. Analysis of Multicensored Data | 193 |
| 7.1 Multicensored Data | 193 |
| 7.1.1 Kaplan–Meier Product Limit Estimation | 193 |

| | | |
|-----------|--|------------|
| 7.2 | Analysis of Interval (Readout) Data | 203 |
| 7.2.1 | Interval (Readout) Data Analysis in JMP and Minitab | 205 |
| 7.2.2 | Minitab Solution | 206 |
| 7.2.3 | JMP Solution | 206 |
| 7.3 | Life Table Data | 209 |
| 7.4 | Left-Truncated and Right-Censored Data | 213 |
| 7.5 | Left-Censored Data | 217 |
| 7.6 | Other Sampling Schemes (Arbitrary Censoring: Double and Overlapping Interval Censoring)—Peto–Turnbull Estimator | 220 |
| 7.6.1 | Current Status Data | 220 |
| 7.7 | Simultaneous Confidence Bands for the Failure Distribution (or Survival) Function | 223 |
| 7.7.1 | Hall–Wellner Confidence Bands | 224 |
| 7.7.2 | Nair Equal Precision Confidence Bands | 229 |
| 7.7.3 | Likelihood Ratio-Based Confidence Bands | 229 |
| 7.7.4 | Bootstrap Methods for Confidence Bands | 229 |
| 7.7.5 | Confidence Bands in Minitab and JMP | 230 |
| 7.8 | Cumulative Hazard Estimation for Exact Failure Times | 231 |
| 7.9 | Johnson Estimator | 233 |
| | Summary | 235 |
| | Appendix 7A | 235 |
| 7.1A | Obtaining Bootstrap Confidence Bands Using a Spreadsheet | 235 |
| | Problems | 239 |
| 8. | Physical Acceleration Models | 241 |
| 8.1 | Accelerated Testing Theory | 241 |
| 8.2 | Exponential Distribution Acceleration | 243 |
| 8.3 | Acceleration Factors for the Weibull Distribution | 244 |
| 8.4 | Likelihood Ratio Tests of Models | 256 |
| 8.5 | Confidence Intervals Using the LR Method | 258 |
| 8.6 | Lognormal Distribution Acceleration | 260 |
| 8.7 | Acceleration Models | 265 |
| 8.8 | Arrhenius Model | 266 |
| 8.9 | Estimating ΔH with More than Two Temperatures | 268 |
| 8.10 | Eyring Model | 273 |
| 8.11 | Other Acceleration Models | 279 |
| 8.12 | Acceleration and Burn-In | 281 |
| 8.13 | Life Test Experimental Design | 283 |
| 8.14 | Summary | 284 |
| | Appendix 8A | 285 |
| 8.1A | An Alternative JMP Input for Weibull Analysis of High-Stress Failure Data | 285 |
| 8.2A | Using a Spreadsheet for Weibull Analysis of High-Stress Failure Data | 287 |
| 8.3A | Using A Spreadsheet for MLE Confidence Bounds for Weibull Shape Parameter | 288 |
| 8.4A | Using a Spreadsheet for Lognormal Analysis of the High-Stress Failure Data Shown in Table 8.5 | 290 |

| | | |
|------------|--|------------|
| 8.5A | Using a Spreadsheet for MLE Confidence Bounds for the Lognormal Shape Parameter..... | 291 |
| 8.6A | Using a Spreadsheet for Arrhenius–Weibull Model..... | 293 |
| 8.7A | Using a Spreadsheet for MLEs for Arrhenius–Power Relationship Lognormal Model..... | 294 |
| 8.8A | Spreadsheet Templates for Weibull or Lognormal MLE Analysis | 296 |
| | Problems..... | 297 |
| 9. | Alternative Reliability Models | 301 |
| 9.1 | Step Stress Experiments..... | 301 |
| 9.2 | Degradation Models | 307 |
| 9.2.1 | Method 1 | 308 |
| 9.2.2 | Method 2 | 309 |
| 9.3 | Lifetime Regression Models..... | 313 |
| 9.4 | The Proportional Hazards Model..... | 320 |
| 9.4.1 | Proportional Hazards Model Assumption | 320 |
| 9.4.2 | Properties and Applications of the Proportional Hazards Model..... | 320 |
| 9.5 | Defect Subpopulation Models..... | 321 |
| 9.6 | Summary | 335 |
| | Appendix 9A..... | 335 |
| 9.1A | JMP Solution for Step Stress Data in Example 9.1 | 335 |
| 9.2A | Lifetime Regression Solution Using Excel | 336 |
| 9.3A | JMP Likelihood Formula for the Defect Model..... | 342 |
| 9.4A | JMP Likelihood Formulas for Example 9.7 Multistress Defect Model Example | 342 |
| | Problems..... | 342 |
| 10. | System Failure Modeling: Bottom-Up Approach | 345 |
| 10.1 | Series System Models | 345 |
| 10.2 | The Competing Risk Model (Independent Case)..... | 346 |
| 10.3 | Parallel or Redundant System Models..... | 348 |
| 10.4 | Standby Models and the Gamma Distribution | 350 |
| 10.5 | Complex Systems | 352 |
| 10.6 | System Modeling: Minimal Paths and Minimal Cuts..... | 356 |
| 10.7 | General Reliability Algorithms..... | 360 |
| 10.8 | Burn-In Models..... | 362 |
| 10.9 | The “Black Box” Approach: An Alternative to Bottom-Up Methods..... | 365 |
| 10.10 | Summary | 367 |
| | Problems..... | 367 |
| 11. | Quality Control in Reliability: Applications of Discrete Distributions..... | 369 |
| 11.1 | Sampling Plan Distributions..... | 369 |
| 11.1.1 | Permutations and Combinations..... | 370 |
| 11.1.2 | Permutations and Combinations via Spreadsheet Functions | 371 |
| 11.1.3 | The Binomial Distribution..... | 372 |
| 11.1.4 | Cumulative Binomial Distribution..... | 374 |
| 11.1.5 | Spreadsheet Function for the Binomial Distribution | 375 |
| 11.1.6 | Relation of Binomial Distribution to Beta Distribution | 376 |

| | | |
|------------|---|------------|
| 11.2 | Nonparametric Estimates Used with the Binomial Distribution | 377 |
| 11.3 | Confidence Limits for the Binomial Distribution..... | 377 |
| 11.4 | Normal Approximation for Binomial Distribution | 379 |
| 11.5 | Confidence Intervals Based on Binomial Hypothesis Tests | 380 |
| 11.6 | Simulating Binomial Random Variables | 382 |
| 11.7 | Geometric Distribution | 384 |
| 11.8 | Negative Binomial Distribution..... | 385 |
| 11.9 | Hypergeometric Distribution and Fisher's Exact Test..... | 386 |
| 11.9.1 | Hypergeometric Distribution | 386 |
| 11.9.2 | Fisher's Exact Test | 387 |
| 11.9.3 | Fisher's Exact Test in JMP and Minitab | 389 |
| 11.10 | Poisson Distribution | 391 |
| 11.11 | Types of Sampling..... | 393 |
| 11.11.1 | Risks..... | 394 |
| 11.11.2 | Operating Characteristic Curve..... | 395 |
| 11.11.3 | Binomial Calculations | 395 |
| 11.11.4 | Examples of Operating Characteristic Curves | 396 |
| 11.12 | Generating a Sampling Plan..... | 400 |
| 11.12.1 | LTPD Sampling Plans..... | 402 |
| 11.13 | Minimum Sample Size Plans | 406 |
| 11.14 | Nearly Minimum Sampling Plans..... | 406 |
| 11.15 | Relating an OC Curve to Lot Failure Rates..... | 407 |
| 11.16 | Statistical Process Control Charting for Reliability | 410 |
| 11.17 | Summary | 414 |
| | Problems..... | 414 |
| 12. | Repairable Systems Part I: Nonparametric Analysis and | |
| | Renewal Processes..... | 417 |
| 12.1 | Repairable versus Nonrepairable Systems..... | 417 |
| 12.2 | Graphical Analysis of a Renewal Process..... | 419 |
| 12.3 | Analysis of a Sample of Repairable Systems..... | 424 |
| 12.3.1 | Solution Using Spreadsheet Methods..... | 428 |
| 12.4 | Confidence Limits for the Mean Cumulative Function (Exact Age Data) | 430 |
| 12.4.1 | True Confidence Limits..... | 430 |
| 12.5 | Nonparametric Comparison of Two MCF Curves | 435 |
| 12.6 | Renewal Processes | 440 |
| 12.7 | Homogeneous Poisson Process..... | 441 |
| 12.7.1 | Distribution of Repair Times for HPP | 442 |
| 12.8 | MTBF and MTTF for a Renewal Process | 446 |
| 12.9 | MTTF and MTBF Two-Sample Comparisons | 450 |
| 12.10 | Availability..... | 453 |
| 12.11 | Renewal Rates..... | 455 |
| 12.12 | Simulation of Renewal Processes | 456 |
| 12.13 | Superposition of Renewal Processes | 457 |
| 12.14 | CDF Estimation from Renewal Data (Unidentified Replacement) | 458 |
| 12.15 | Summary | 462 |
| | Appendix 12A..... | 462 |
| 12.1A | True Confidence Limits for the MCF | 462 |
| 12.2A | Cox F-Test for Comparing Two Exponential Means | 465 |

| | |
|---|------------|
| 12.3A Alternative Approach for Estimating CDF Using the Fundamental Renewal Equation | 466 |
| Problems | 469 |
| 13. Repairable Systems Part II: Nonrenewal Processes | 471 |
| 13.1 Graphical Analysis of Nonrenewal Processes | 471 |
| 13.2 Two Models for a Nonrenewal Process | 474 |
| 13.3 Testing for Trends and Randomness..... | 477 |
| 13.3.1 Other Graphical Tools | 478 |
| 13.4 Laplace Test for Trend | 480 |
| 13.5 Reverse Arrangement Test..... | 482 |
| 13.6 Combining Data from Several Tests..... | 486 |
| 13.7 Nonhomogeneous Poisson Processes | 488 |
| 13.8 Models for the Intensity Function of an NHPP..... | 489 |
| 13.8.1 Power Relation Model | 489 |
| 13.8.2 Exponential Model..... | 496 |
| 13.9 Rate of Occurrence of Failures | 499 |
| 13.10 Reliability Growth Models | 500 |
| 13.11 Simulation of Stochastic Processes | 512 |
| 13.12 Summary | 515 |
| Problems..... | 515 |
| 14. Bayesian Reliability Evaluation | 517 |
| 14.1 Classical versus Bayesian Analysis | 517 |
| 14.1.1 Bayes' Formula, Prior and Posterior Distribution Models, and Conjugate Priors | 518 |
| 14.1.2 Bayes' Approach for Analysis of Exponential Lifetimes | 519 |
| 14.2 Classical versus Bayes System Reliability | 522 |
| 14.2.1 Classical Paradigm for HPP System Reliability Evaluation | 522 |
| 14.2.2 Bayesian Paradigm for HPP System Reliability Evaluation | 522 |
| 14.2.3 Advantages and Disadvantages of Using Bayes' Methodology | 522 |
| 14.3 Bayesian System MTBF Evaluations..... | 523 |
| 14.3.1 Calculating Prior Parameters Using the 50/95 Method | 524 |
| 14.3.2 Calculating the Test Time Needed to Confirm an MTBF Objective | 526 |
| 14.4 Bayesian Estimation of the Binomial p | 529 |
| 14.5 The Normal/Normal Conjugate Prior | 532 |
| 14.6 Informative and Noninformative Priors | 533 |
| 14.7 A Survey of More Advanced Bayesian Methods..... | 536 |
| 14.8 Summary | 537 |
| Appendix 14A..... | 538 |
| 14.1A Gamma and Chi-Square Distribution Relationships | 538 |
| Problems..... | 538 |
| Answers to Selected Exercises | 541 |
| References | 551 |
| Index | 557 |



Taylor & Francis

Taylor & Francis Group

<http://taylorandfrancis.com>

Preface

It has been more than 15 years since the publication of the second edition of *Applied Reliability*. We continue to receive positive feedback from old users, and each year, hundreds of engineers, quality specialists, and statisticians discover the book for the first time and become new fans. So, why a third edition? There are always new methods and techniques that update and improve upon older methods, but that was not the primary reason we felt the need to write a new edition. In the past 15 years, the ready availability of relatively inexpensive, powerful, statistical software has changed the way statisticians and engineers look at and analyze all kinds of data. Problems in reliability that were once difficult and time consuming for even experts can now be solved with a few well-chosen clicks of a mouse. Additionally, with the quantitative solution often comes a plethora of graphics that aid in understanding and presenting the results.

All this power comes with a price, however. Software documentation has had difficulty keeping up with the enhanced functionality added to new releases, especially in specialized areas such as reliability analysis. Also, in some cases, different well-known software packages use different methods and output different answers. An analyst needs to know how to use these programs effectively and which methods are the most highly recommended. This information is hard to find for industrial reliability problems.

The third edition of *Applied Reliability* was written to fulfill this software documentation need for reliability analysts. We chose two popular software packages that are well maintained, supported, and frequently updated: Minitab and SAS JMP. Minitab is popular in universities and JMP is widely used within leading high-technology companies. Both packages have extensive capabilities for reliability analysis and graphics that improve with every new release.

In addition, we included solutions using spreadsheet programs such as Microsoft Excel and Oracle OpenOffice Calc. With a little formula programming, spreadsheet functions can solve even very difficult reliability problems. Spreadsheet methods cannot easily produce custom, specialized reliability graphics, however, and are included primarily because they are so widely available and surprisingly powerful.

Unfortunately, producing detailed examples using software has many pitfalls. We would generate graphics of screenshots and describe how to obtain specific platforms and run analyses only to have a new release of either JMP or Minitab come out, which looked and operated somewhat differently. Even spreadsheet mechanics change with new releases. We frequently had to go back and redo problem solutions to remain current with updates.

Finally, we realized that our readers would inevitably see panels and screens coming from later releases of these software packages that might differ slightly from the screenshots shown in our text. However, it is likely that the basic methods and approaches will remain the same for a long time. Many of the suggestions we made to software developers based on methods described in the second edition are now a part of these packages or will be in future releases. Two examples are the very useful defect model (incorporated in JMP release 9) and the ability to input negative frequencies when analyzing truncated data (already in JMP 8).

We stated in the preface to the second edition: "Our goal remains that the text be application oriented, with numerous practical examples and graphical illustrations." Statements of theory and useful equations are essential building blocks, but what the industrial

reliability analyst needs to know is how to apply these building blocks to numerically solve typical problems. The new edition has more than 150 worked-out examples, many done with both JMP and Minitab and even spreadsheet programs. Along with these examples, there are nearly 300 figures, and hundreds of exercises and additional problems at the end of each chapter. We also took the opportunity to add new material throughout. Sometimes, this new material increased the level of difficulty, and we chose to put this material in appendices at the end of several chapters.

Since many of the examples, exercises, and problems use lengthy spreadsheets or worksheets of failure data, we have many of these files on the publisher's website for the book. These data sets, in Excel, JMP, or Minitab format, can be accessed via the "Downloads & Updates" tab on the book's web page at <http://www.crcpress.com/product/isbn/9781584884668>. Data sets are organized by book chapter and given a name either mentioned in the text or based on the number of the example, exercise, or problem to which they relate. There is also a directory containing Excel templates that can be used to find maximum likelihood solutions for Weibull and lognormal multistress, life test, or field data. There are even templates incorporating the defect model or for testing equal slopes or equal parameters across several cells of data.

Another powerful software package not used in the text deserves mention: SPLUS, with the addition of Bill Meeker's SPLIDA (SPLUS Life Data Analysis) downloadable front end, which offers graphics and analysis capabilities that can also be used successfully on many of the data sets in the third edition.

Finally, we gratefully acknowledge the comments and suggestions made by our colleagues who provided feedback on the sections of the second edition and/or reviewed draft copies of many prepublication chapters of the third edition. In particular, we appreciate the comprehensive suggestions and critiques offered by Wayne Nelson, Doug Montgomery, Judy Koslov, Bill Heavlin, Ed Russell, Ken Stephens, Leon Lopez, and the many users of the text.

List of Figures

| | | |
|-------------|--|----|
| Figure 1.1 | Histogram of fuse breakdown measurements..... | 4 |
| Figure 1.2 | Plot of PDF cumulative frequency function | 5 |
| Figure 1.3 | Plot of PDF for the exponential distribution | 6 |
| Figure 1.4 | CDF for exponential distribution | 7 |
| Figure 1.5 | EDF for fuse data | 8 |
| Figure 1.6 | Minitab histogram of fuse data | 9 |
| Figure 1.7 | The uniform PDF | 20 |
| Figure 1.8 | The CDF for the uniform distribution..... | 20 |
| Figure 1.9 | Beta density functions | 22 |
| Figure 1.10 | Mean and Sigma do not tell us enough. These four distributions have the same mean and standard deviation..... | 23 |
| Figure 1.11 | Simulating ordered random variables..... | 25 |
| Figure 1.1A | Spreadsheet table for experiment..... | 26 |
| Figure 1.2A | Derived spreadsheet table for step chart..... | 26 |
| Figure 1.3A | Step chart | 27 |
| Figure 2.1 | Cumulative distribution function | 30 |
| Figure 2.2 | Bathtub curve for failure rates | 36 |
| Figure 2.3 | Example of component failure data | 37 |
| Figure 2.4 | Readout data..... | 42 |
| Figure 3.1 | The exponential distribution failure rate $h(t)$ | 48 |
| Figure 3.2 | Histogram of memory chip failure data | 51 |
| Figure 3.3 | Piecewise approximation of actual failure rate..... | 54 |
| Figure 3.4 | Memory chip data histogram compared to $f(t)$ shape | 58 |
| Figure 3.5 | Illustration of D_n^+ and D_n^- statistics for KS test | 63 |
| Figure 3.6 | JMP histogram of test data | 65 |
| Figure 3.7 | Empirical distribution function plot and KSL D -statistics..... | 65 |
| Figure 3.8 | Minitab exponential analysis of failure times..... | 66 |
| Figure 3.9 | JMP exponential analysis of failure times | 66 |
| Figure 3.10 | Spreadsheet columns for evaluating the product $r \times k_{r1-\alpha}$ | 74 |

| | | |
|--------------------|--|-----|
| Figure 3.1A | Spreadsheet entries to determine sample size | 79 |
| Figure 3.2A | Spreadsheet entries to determine test length | 80 |
| Figure 3.3A | Spreadsheet entries to determine number of failures allowed | 81 |
| Figure 3.4A | Spreadsheet entries for KS goodness-of-fit test | 82 |
| Figure 3.5A | Empirical distribution function and exponential CDF model (mean time to fail = 100) | 83 |
| Figure 4.1 | Weibull CDF | 91 |
| Figure 4.2 | Weibull PDF | 91 |
| Figure 4.3 | Weibull failure rate (hazard rate) | 92 |
| Figure 4.4 | JMP data table for exact times, censored data analysis | 103 |
| Figure 4.5 | Inputs for JMP Fit Parametric Survival analysis—exact times | 104 |
| Figure 4.6 | JMP Weibull MLEs for Exercise 4.5 exact times, censored data | 104 |
| Figure 4.7 | JMP data table for interval data from Exercise 4.5 | 105 |
| Figure 4.8 | Inputs for JMP Fit Parametric Survival analysis—interval data | 105 |
| Figure 4.9 | JMP Weibull MLEs for Exercise 4.5 interval data | 106 |
| Figure 4.10 | JMP data table for exact times treated as interval data | 106 |
| Figure 4.11 | Minitab analysis for exact times, right-censored data analysis | 107 |
| Figure 4.12 | Minitab analysis inputs for interval data | 108 |
| Figure 4.1A | Genweibest spreadsheet with interval data from Example 4.5 | 114 |
| Figure 4.2A | Solver entries for MLE analysis of the filled-in Genweibest spreadsheet | 115 |
| Figure 4.3A | Genweibest spreadsheet after solver finds MLEs | 116 |
| Figure 4.4A | Genweibest solution for MLEs based on truncated data | 117 |
| Figure 4.5A | First iteration—the starting point is the MLE solution spreadsheet | 118 |
| Figure 4.6A | Solver run (first iteration) | 118 |
| Figure 4.7A | Second iteration run of Goal Seek | 119 |
| Figure 4.8A | Second iteration run of Solver | 119 |
| Figure 4.9A | Start of fourth iteration confirming convergence when there is no change | 120 |
| Figure 5.1 | The normal distribution PDF | 124 |
| Figure 5.2 | The normal distribution CDF | 124 |
| Figure 5.3 | Plot of data from Table 5.2 | 133 |
| Figure 5.4 | Relationship of lognormal distribution to normal distribution | 136 |

| | | |
|--------------------|--|-----|
| Figure 5.5 | The lognormal distribution PDF | 137 |
| Figure 5.6 | The lognormal distribution CDF..... | 138 |
| Figure 5.7 | The lognormal distribution failure rate | 139 |
| Figure 5.8 | Minitab inputs and output for Table 5.5 data | 143 |
| Figure 5.9 | JMP inputs and output for Table 5.5 data..... | 144 |
| Figure 5.10 | Minitab input and output screens for lognormal interval data | 145 |
| Figure 5.11 | JMP input and output screens for interval data..... | 146 |
| Figure 5.1A | Genlnest.xls after converging to MLEs for Table 5.5 data | 150 |
| Figure 5.2A | Excel solution for interval data to obtain lognormal MLEs | 151 |
| Figure 6.1 | Straight line plot..... | 154 |
| Figure 6.2 | Regression line example | 155 |
| Figure 6.3 | JMP regression example | 158 |
| Figure 6.4 | Minitab regression example..... | 159 |
| Figure 6.5 | Ideal gas law plot | 160 |
| Figure 6.6 | Ideal gas law plot using rectification | 160 |
| Figure 6.7 | Exponential probability plot of Table 6.2 data, exact times, median ranks..... | 166 |
| Figure 6.8 | Minitab probability plot of Table 6.2 data | 166 |
| Figure 6.9 | Exponential model fit to data, exact times, LS MTTF estimate..... | 168 |
| Figure 6.10 | Exponential probability plot, exact times, 90% confidence limits on transformed CDF | 169 |
| Figure 6.11 | Exponential probability plot, exact times, 90% approximate confidence limits on failure time quantiles | 170 |
| Figure 6.12 | Exponential probability plot, readout data | 172 |
| Figure 6.13 | Exponential probability plot of readout data with approximate 90% pointwise confidence limits on time t | 174 |
| Figure 6.14 | Exponential CDF plot of readout data with approximate 90% pointwise confidence limits on time t | 174 |
| Figure 6.15 | Weibull probability plot, exact times | 177 |
| Figure 6.16 | JMP output Weibull model analysis, exact times | 179 |
| Figure 6.17 | Minitab output Weibull model analysis, exact times..... | 180 |
| Figure 6.18 | Lognormal probability plot, exact times, $n = 600$ | 183 |
| Figure 6.19 | Extrapolation to T_{50} in lognormal probability plot | 184 |
| Figure 6.20 | EDF plot with 90% confidence level band..... | 186 |

| | | |
|--------------------|---|-----|
| Figure 7.1 | Eight units on stress: six failures and two censored (units 2 and 4) | 194 |
| Figure 7.2 | Nonparametric survival and CDF curves..... | 195 |
| Figure 7.3 | CDF and two-sided 95% confidence limits..... | 198 |
| Figure 7.4 | JMP dialog box for exact data example | 199 |
| Figure 7.5 | JMP output for exact data example..... | 200 |
| Figure 7.6 | JMP save estimates table for exact data example..... | 200 |
| Figure 7.7 | Minitab dialog box for exact data example..... | 201 |
| Figure 7.8 | Minitab summary output for exact data example | 202 |
| Figure 7.9 | Minitab graph for exact data example..... | 203 |
| Figure 7.10 | Minitab spreadsheet for readout example, censoring at beginning of interval | 206 |
| Figure 7.11 | Minitab dialog boxes for readout data example..... | 206 |
| Figure 7.12 | Minitab output for readout example, actuarial estimate..... | 207 |
| Figure 7.13 | JMP dialog box for readout example | 208 |
| Figure 7.14 | JMP output for readout example, assuming censoring at beginning and end of interval..... | 208 |
| Figure 7.15 | Minitab worksheet (partial) for Table 7.1 data | 211 |
| Figure 7.16 | Minitab output (partial) actuarial table and failure plot | 212 |
| Figure 7.17 | Number at risk | 216 |
| Figure 7.18 | Cumulative failure distribution..... | 216 |
| Figure 7.19 | Plot of CDF estimate versus time for left-censored data | 219 |
| Figure 7.20 | JMP data table and output for left-censored data in Table 7.14..... | 219 |
| Figure 7.21 | Minitab output and graph for left-censored data in Table 7.14 | 220 |
| Figure 7.22 | Disk drive data CDF plot..... | 222 |
| Figure 7.23 | Current status data table and analysis output in JMP | 222 |
| Figure 7.24 | Current status data table and analysis output in Minitab..... | 223 |
| Figure 7.25 | Spreadsheet showing the calculations for determining the Hall–Wellner confidence bands for the first 18 observations in Table 7.20..... | 227 |
| Figure 7.26 | Kaplan–Meier $F(t)$ estimate and Hall–Wellner 90% confidence bands: linear (H-W), log (H-W LT), and logit (H-W LG) transformations..... | 228 |
| Figure 7.27 | JMP output showing Nair 95% EP confidence bands | 231 |
| Figure 7.1A | Partial table for capturing “max” of bootstrap runs | 237 |

| | | |
|--------------------|---|-----|
| Figure 7.2A | Dialog box for creating a data table | 237 |
| Figure 7.3A | One-way data table with varying max values..... | 237 |
| Figure 7.4A | Bootstrap 90% confidence bands | 238 |
| Figure 7.5A | CDF estimate and 90% bootstrap confidence bands | 238 |
| Figure 8.1 | Worksheet for the data in Table 8.2..... | 246 |
| Figure 8.2 | Weibull probability plot in worksheet..... | 247 |
| Figure 8.3 | Worksheet using indicator variables | 248 |
| Figure 8.4 | Portion of data analysis summary output | 249 |
| Figure 8.5 | Minitab output for LS analysis of the data in Table 8.2 | 249 |
| Figure 8.6 | Minitab Weibull plot of the data in Table 8.2..... | 250 |
| Figure 8.7 | Partial Minitab worksheet for the data in Table 8.2..... | 250 |
| Figure 8.8 | Minitab Weibull plot of the data in Table 8.2: equal slopes..... | 251 |
| Figure 8.9 | JMP-7 data table for analysis of the data in Table 8.2 | 252 |
| Figure 8.10 | JMP-7 reliability/survival screen inputs | 252 |
| Figure 8.11 | JMP–Weibull plot and MLEs for the data cells of Table 8.2..... | 253 |
| Figure 8.12 | JMP data table for common-slope analysis of the data in Table 8.2 using indicator variables | 254 |
| Figure 8.13 | JMP fit parametric survival model screen inputs for common-slope analysis of the data in Table 8.2..... | 254 |
| Figure 8.14 | JMP fit parametric survival model screen outputs for common-slope analysis of the data in Table 8.2..... | 255 |
| Figure 8.15 | Region for finding lognormal parameter confidence limits..... | 258 |
| Figure 8.16 | JMP output for Weibull parameter likelihood confidence limits for data in Table 8.2 | 259 |
| Figure 8.17 | Lognormal plot—cumulative percent failure data from Table 8.5 | 262 |
| Figure 8.18 | Lognormal plot (common slope) | 262 |
| Figure 8.19 | JMP analysis for calculating same-slope MLEs | 264 |
| Figure 8.20 | JMP analysis results with same slope..... | 265 |
| Figure 8.21 | Arrhenius plot using LS estimates..... | 270 |
| Figure 8.22 | Minitab inputs for Arrhenius–Weibull fit..... | 270 |
| Figure 8.23 | Minitab Arrhenius–Weibull analysis output..... | 271 |
| Figure 8.24 | JMP Arrhenius–Weibull analysis entry screens | 272 |
| Figure 8.25 | JMP Arrhenius–Weibull analysis results | 272 |

| | | |
|---------------------|--|-----|
| Figure 8.26 | JMP worksheet for Arrhenius–power relationship–lognormal model | 275 |
| Figure 8.27 | JMP inputs for Arrhenius–power relationship–lognormal model | 276 |
| Figure 8.28 | JMP results for Arrhenius–power relationship–lognormal model fit | 276 |
| Figure 8.29 | JMP dialog to estimate survival probabilities for Arrhenius–power relationship–lognormal model fit | 277 |
| Figure 8.30 | JMP survival analysis at possible use conditions | 277 |
| Figure 8.31 | Minitab inputs for Arrhenius power law analysis | 278 |
| Figure 8.32 | Minitab plots for Arrhenius power law analysis | 278 |
| Figure 8.33 | Minitab results of accelerated life-test analysis | 279 |
| Figure 8.34 | Minitab results of accelerated life-test predictions..... | 279 |
| Figure 8.35 | Minitab input box for accelerated life-test prediction..... | 280 |
| Figure 8.1A | JMP inputs for analysis of the data in Table 8.2 | 286 |
| Figure 8.2A | Analysis results for the data in Table 8.2 (MLEs assuming equal slopes) | 286 |
| Figure 8.3A | Excel spreadsheet for calculating MLEs of individual cells..... | 287 |
| Figure 8.4A | Excel spreadsheet for calculating MLEs assuming a common shape..... | 288 |
| Figure 8.5A | Using Goal Seek confidence bound calculations | 289 |
| Figure 8.6A | Solver example for confidence limit calculations..... | 290 |
| Figure 8.7A | Excel spreadsheet for calculating MLEs of individual cells..... | 291 |
| Figure 8.8A | Excel spreadsheet for calculating MLEs of same-sigma cells | 292 |
| Figure 8.9A | Spreadsheet confidence bound calculation for common sigma | 292 |
| Figure 8.10A | Solver screen for confidence bound calculation..... | 293 |
| Figure 8.11A | Excel spreadsheet fit for Arrhenius–Weibull model..... | 293 |
| Figure 8.12A | Spreadsheet calculation of Arrhenius–power relationship model parameter estimates | 294 |
| Figure 8.13A | Spreadsheet for calculating use CDFs and confidence bounds | 295 |
| Figure 8.14A | Goal Seek and Solver inputs for calculating profile likelihood limits..... | 296 |
| Figure 9.1 | Arrhenius step stress data schematic | 302 |
| Figure 9.2 | Plot of step stress data for $\Delta H = 0.5, 0.86$, and 1.0 | 303 |
| Figure 9.3 | Spreadsheet for calculating step stress interval widths under Arrhenius acceleration | 304 |
| Figure 9.4 | Spreadsheet for calculating step stress Arrhenius lognormal MLEs..... | 305 |

| | | |
|--------------------|---|-----|
| Figure 9.5 | Spreadsheet for calculating step stress Arrhenius/power law Weibull MLEs..... | 307 |
| Figure 9.6 | Degradation data extrapolation to failure times..... | 309 |
| Figure 9.7 | Projected degradation failure times, 105°C..... | 312 |
| Figure 9.8 | Projected degradation failure times, 125°C..... | 312 |
| Figure 9.9 | JMP spreadsheet for the plant/process field reliability data..... | 315 |
| Figure 9.10 | Fit parametric survival screen for plant/process reliability data..... | 316 |
| Figure 9.11 | JMP analysis results for plant/process reliability data..... | 317 |
| Figure 9.12 | Minitab spreadsheet for the plant/process field reliability data..... | 318 |
| Figure 9.13 | Minitab regression with life data screen for plant/process data..... | 318 |
| Figure 9.14 | Minitab regression with life data output for plant/process data..... | 319 |
| Figure 9.15 | Lognormal probability plot of 15 out of 100 | 323 |
| Figure 9.16 | Lognormal probability plot of 15 out of 18 | 323 |
| Figure 9.17 | JMP data table for defect model analysis of the Example 9.5 data | 325 |
| Figure 9.18 | JMP nonlinear analysis entry screen..... | 326 |
| Figure 9.19 | JMP nonlinear analysis platform control screen..... | 326 |
| Figure 9.20 | MLEs for Example 9.5 defect model data | 327 |
| Figure 9.21 | Before and after panels for $P = 1$ for the nonlinear analysis | 328 |
| Figure 9.22 | Excel spreadsheet for MLE fitting of lognormal defect model data..... | 329 |
| Figure 9.23 | Defect model, multistress cell data | 330 |
| Figure 9.24 | JMP MLEs for one cell Weibull defect analysis..... | 332 |
| Figure 9.25 | Excel MLE fitting of defect model data inputted as truncated data | 334 |
| Figure 9.26 | JMP table showing defect model data inputted as truncated data | 334 |
| Figure 9.27 | JMP fitting of defect model data inputted as truncated data..... | 335 |
| Figure 9.1A | JMP data table for analysis of Example 9.2 Arrhenius step stress data | 336 |
| Figure 9.2A | Formula for Weibull, one cell, defect model..... | 337 |
| Figure 9.3A | Formula for Weibull, three cells, defect model | 338 |
| Figure 9.4A | Formula for Weibull, effective delta temperature acceleration, defect model | 338 |
| Figure 9.5A | EXCEL spreadsheet for the plant process field reliability data | 339 |
| Figure 9.6A | Solver screen for the plant/process field reliability data..... | 340 |
| Figure 9.7A | JMP negative log-likelihood column formula for the lognormal defect model..... | 340 |

| | | |
|---------------------|--|-----|
| Figure 9.8A | Formula for Weibull, one cell, defect model..... | 341 |
| Figure 9.9A | Formula for Weibull, three cells, defect model | 341 |
| Figure 9.10A | Formula for Weibull, effective delta temperature acceleration, defect model | 342 |
| Figure 10.1 | Five-component system diagram | 354 |
| Figure 10.2 | Reduced five-component system diagram..... | 354 |
| Figure 10.3 | Fully reduced five-component system diagram | 354 |
| Figure 10.4 | Six-component system diagrams..... | 355 |
| Figure 10.5 | Backup components | 355 |
| Figure 10.6 | Backup components with switch..... | 355 |
| Figure 10.7 | Equivalent diagram of system with working switch | 356 |
| Figure 10.8 | Bridge structure system diagram..... | 357 |
| Figure 10.9 | Equivalent to bridge structure system diagram | 358 |
| Figure 10.10 | Minimal cut analysis of bridge structure diagram | 359 |
| Figure 10.11 | Example 10.10 system diagram | 359 |
| Figure 10.12 | General reliability algorithm failure rate example | 361 |
| Figure 10.13 | Failure rate before and after burn-in | 364 |
| Figure 11.1 | Binomial distribution..... | 374 |
| Figure 11.2 | CDF for binomial distribution..... | 375 |
| Figure 11.3 | Binomial data analysis in JMP..... | 381 |
| Figure 11.4 | JMP binomial confidence interval calculation | 381 |
| Figure 11.5 | Binomial CDF $n = 4, p = 0.5$ | 383 |
| Figure 11.6 | Input for Fisher's exact in Minitab | 390 |
| Figure 11.7 | Output for Fisher's exact in Minitab..... | 390 |
| Figure 11.8 | Operating characteristic curve | 397 |
| Figure 11.9 | Operating characteristic curves for different acceptance numbers | 397 |
| Figure 11.10 | Operating characteristic curves for different sample sizes..... | 398 |
| Figure 11.11 | AOQ curve with AOQL | 399 |
| Figure 11.12 | Spreadsheet set-up for determining acceptance sampling plan..... | 401 |
| Figure 11.13 | LTPD versus sample size for different acceptance values..... | 404 |
| Figure 11.14 | Three-sigma control chart for binomial proportions..... | 411 |
| Figure 11.15 | Cumulative count control..... | 412 |
| Figure 12.1 | Dot plot of repair pattern..... | 419 |

| | | |
|---------------------|--|-----|
| Figure 12.2 | Cumulative plot | 420 |
| Figure 12.3 | Interarrival times versus system age | 421 |
| Figure 12.4 | Recurrence rate versus system age | 421 |
| Figure 12.5 | Lognormal probability plot..... | 422 |
| Figure 12.6 | CDF model fit versus observed..... | 423 |
| Figure 12.7 | Event plot of repair histories for five systems | 425 |
| Figure 12.8 | Repair history (cumulative plots) for five systems | 426 |
| Figure 12.9 | Repair history for two systems..... | 426 |
| Figure 12.10 | Repair history for five systems | 427 |
| Figure 12.11 | Mean cumulative repair function | 428 |
| Figure 12.12 | Spreadsheet method for estimating the MCF | 429 |
| Figure 12.13 | Spreadsheet method for estimating the MCF and naive confidence limits..... | 431 |
| Figure 12.14 | MCF and 95% naive confidence limits..... | 431 |
| Figure 12.15 | JMP data table for recurrence analysis..... | 432 |
| Figure 12.16 | JMP dialog box for recurrence analysis | 432 |
| Figure 12.17 | JMP output for recurrence analysis | 433 |
| Figure 12.18 | Minitab data worksheet for repairable system analysis | 434 |
| Figure 12.19 | Minitab dialog boxes for repairable system analysis | 434 |
| Figure 12.20 | Minitab output for analysis of five repairable systems..... | 435 |
| Figure 12.21 | MCF comparison between East and West Coast locations..... | 437 |
| Figure 12.22 | MCFs for East and West Coast locations..... | 437 |
| Figure 12.23 | MCF differences between East and West Coast locations..... | 438 |
| Figure 12.24 | JMP plot of MCF difference between East and West Coast locations | 439 |
| Figure 12.25 | Partial spreadsheet for time to k th repair | 443 |
| Figure 12.26 | Gamma distribution CDF for time to k th repair, MTBF = 1000 hours | 444 |
| Figure 12.27 | Spreadsheet example for spare parts determination..... | 445 |
| Figure 12.28 | Alternating renewal process | 453 |
| Figure 12.29 | Markov two-state model..... | 454 |
| Figure 12.30 | Partial spreadsheet for 10 HPP systems with MTBF = 1000 | 456 |
| Figure 12.31 | Cumulative plots of 10 simulated HPP systems with MTBF = 1000 (censored at 10,000 hours)..... | 457 |

| | | |
|---------------------|---|-----|
| Figure 12.32 | Superposition of renewal processes for system of three components | 458 |
| Figure 12.33 | System of c components viewed as a superposition of renewal processes | 459 |
| Figure 12.1A | Spreadsheet setup for variance estimates..... | 463 |
| Figure 12.2A | Calculations for variance estimates | 464 |
| Figure 12.3A | MCF variance, standard error, and confidence limits..... | 465 |
| Figure 12.4A | Possible outcomes for time differences in renewal estimation..... | 467 |
| Figure 13.1 | Dot plot of repair pattern..... | 472 |
| Figure 13.2 | Cumulative plot (improving trend)..... | 472 |
| Figure 13.3 | Interarrival times versus system age (improving trend) | 473 |
| Figure 13.4 | Dot plot of repair pattern..... | 473 |
| Figure 13.5 | Cumulative plot (degrading trend) | 473 |
| Figure 13.6 | Interarrival times versus system age (degrading trend)..... | 474 |
| Figure 13.7 | Power law model rectification | 475 |
| Figure 13.8 | Exponential model rectification | 476 |
| Figure 13.9 | Exponential model fit..... | 476 |
| Figure 13.10 | Average repair rates versus time (renewal data)..... | 479 |
| Figure 13.11 | Average repair rates versus time (improving)..... | 479 |
| Figure 13.12 | Average repair rates versus time (degrading) | 479 |
| Figure 13.13 | Cumulative plot of repair data..... | 485 |
| Figure 13.14 | Interarrival times versus system age | 486 |
| Figure 13.15 | Cumulative plot of MLE model fit to system data | 495 |
| Figure 13.16 | Spreadsheet setup for applying SOLVER routine (MLE parameters)..... | 495 |
| Figure 13.17 | Spreadsheet showing SOLVER results | 495 |
| Figure 13.18 | Cumulative plot of MLE model fit to system data | 496 |
| Figure 13.19 | Cumulative plot of HPP and NHPP models fit to system data..... | 499 |
| Figure 13.20 | Duane plot of cumulative MTBF versus cumulative time with least squares line..... | 503 |
| Figure 13.21 | Duane plot with modified MLE lines..... | 509 |
| Figure 13.22 | Duane plot of software cumulative MTBF estimates..... | 511 |
| Figure 13.23 | Excel trendline dialog box..... | 512 |
| Figure 14.1 | Bayesian gamma prior and posterior from Example 14.1..... | 522 |

| | | |
|--------------------|---|-----|
| Figure 14.2 | Calling up Goal Seek..... | 525 |
| Figure 14.3 | Using Goal Seek to find the gamma prior a parameter a | 525 |
| Figure 14.4 | Calculating the gamma prior b parameter b | 526 |
| Figure 14.5 | Bayesian beta prior and posterior from Example 14.6..... | 531 |
| Figure 14.6 | Prior and posterior densities from Example 14.7 | 535 |



Taylor & Francis

Taylor & Francis Group

<http://taylorandfrancis.com>

List of Tables

| | | |
|-------------------|--|-----|
| Table 1.1 | Sample Data on 100 Fuses | 3 |
| Table 1.2 | Frequency Table of Fuse Data | 3 |
| Table 1.3 | Cumulative Frequency Function for Fuse Data | 5 |
| Table 1.4 | Possible Outcomes for Drives | 13 |
| Table 1.5 | Properties of Distributions Used in Reliability Studies..... | 19 |
| Table 3.1 | Equivalent Failure Rates in Different Units..... | 48 |
| Table 3.2 | Sample Data of Equivalent Month of Memory Chip Failure | 50 |
| Table 3.3 | Frequency Table of Memory Chip Data | 50 |
| Table 3.4 | Chi-Square Goodness-of-Fit Worksheet for the Memory Chip Data | 61 |
| Table 3.5 | Spreadsheet Functions for k -Factors for Confidence Limits on the Exponential Failure Rate..... | 69 |
| Table 3.6 | Exponential Zero Failure Estimates..... | 70 |
| Table 3.7 | Summary of Exponential Distribution Properties | 78 |
| Table 3.1A | Percentage Points for Modified Kolmogorov D^* -Statistics for $F(t)$ Known | 83 |
| Table 3.2A | Percentage Points for Modified Kolmogorov D^* -Statistics (Mean Unknown)..... | 84 |
| Table 4.1 | Solution to Example 4.1 | 89 |
| Table 4.2 | Weibull Distribution Properties | 92 |
| Table 4.3 | Weibull Formulas Summary | 94 |
| Table 4.4 | 32 Field Failure Times from 101 Burned-In Components | 109 |
| Table 4.1A | Adjustment Constants for L for Computing Likelihood Profile Intervals..... | 117 |
| Table 5.1 | Standard Normal CDF Values | 126 |
| Table 5.2 | Example 5.3 Worksheet | 133 |
| Table 5.3 | Results of Simulation Example (1000 Iterations per C_{pk})..... | 135 |
| Table 5.4 | Lognormal Formulas and Properties | 139 |
| Table 5.5 | Life Test Failure Data (20 Units on Test) | 150 |
| Table 6.1 | LINEST Output | 157 |
| Table 6.2 | Failure Times of 20 Components under Normal Operating Conditions (Time in Hours) | 164 |

| | | |
|-------------------|---|-----|
| Table 6.3 | Probability Plot Values, Exponential Distribution, Exact Times ($n = 20$)..... | 165 |
| Table 6.4 | 90% Confidence Interval Estimates, Exponential Distribution, Exact Failure Times ($n = 20$)..... | 169 |
| Table 6.5 | Probability Plotting Values, Exponential Distribution, Readout Data ($n = 100$)..... | 171 |
| Table 6.6 | Readout Data ($n = 100$), 90% Pointwise Confidence Limits | 173 |
| Table 6.7 | Weibull Example, Exact Times ($n = 20$)..... | 176 |
| Table 6.8 | Lognormal Example, Exact Times ($n = 600$)..... | 183 |
| Table 6.9 | Percentage Points for Modified Kolmogorov D^* -Statistics | 185 |
| Table 6.10 | Failure Times with EDF and 90% Confidence Band Limits | 186 |
| Table 7.1 | Product Limit Estimated Survival Probabilities | 195 |
| Table 7.2 | Variance and Standard Error Estimates | 196 |
| Table 7.3 | Two-Sided 95% Confidence Limits..... | 198 |
| Table 7.4 | JMP Data Table for Exact Data Example | 199 |
| Table 7.5 | Minitab Worksheet for Exact Data Example..... | 201 |
| Table 7.6 | Summary of Readout (Interval) Data | 204 |
| Table 7.7 | Joint Risk and Product Limit Estimates for Readout (Interval) Data with Losses Occurring Randomly | 205 |
| Table 7.8 | JMP Data Tables for Readout Example, Censoring Occurring at Beginning and at End of Interval..... | 207 |
| Table 7.9 | Table of CDF Estimates for Readout Example, Random Censoring within Intervals | 208 |
| Table 7.10 | Survival Data from Six-Week Reliability Study | 210 |
| Table 7.11 | Life Table (Actuarial) Estimation of Failure Probabilities | 210 |
| Table 7.12 | Table of Stress Results for 20 Units..... | 214 |
| Table 7.13 | Partial Table of Ordered Ages of Entry, Failure, or Censored to Determine Number at Risk | 215 |
| Table 7.14 | Table of Observed Times to Failure | 217 |
| Table 7.15 | Analysis of Left-Censored Data | 218 |
| Table 7.16 | Analysis of Left-Censored Data | 218 |
| Table 7.17 | Disk Drive Data..... | 221 |
| Table 7.18 | Percentiles of Distribution of Kolmogorov $\frac{d_{N,1-\alpha}}{\sqrt{N}}$ Statistics | 225 |
| Table 7.19 | Critical Values of $d_{N,1-\alpha}$ for H-W Confidence Bands When $K_N(t_{\max}) < 0.75$ | 225 |

| | | |
|-------------------|--|-----|
| Table 7.20 | Failure and Censor Times for Primary Mechanism ($N = 50$) | 226 |
| Table 7.21 | Hall–Wellner 90% Confidence Bands—Untransformed and with Log and Logit Transformations..... | 228 |
| Table 7.22 | Cumulative Hazard Calculation | 232 |
| Table 7.23 | Cumulative Hazard Calculation for Exact Failure Times Example | 233 |
| Table 7.24 | Multicensored Results..... | 234 |
| Table 7.25 | Possible Outcomes for $n = 5$, FSFSE, Assuming Possible Eventual Suspension Failures | 234 |
| Table 7.26 | Mean Order Numbers and Median Ranks..... | 234 |
| Table 7.27 | Mean Order Numbers Using Johnson Formula..... | 235 |
| Table 7.1A | Original Data, CDF Estimate, Standard Error, and Hall–Wellner Terms..... | 236 |
| Table 7.2A | One Bootstrap Run of Data in Table 7.1A | 236 |
| Table 8.1 | General Linear Acceleration Relationships..... | 242 |
| Table 8.2 | Weibull Temperature–Stress Failure Data | 246 |
| Table 8.3 | Weibull Least Square Parameter Estimates | 247 |
| Table 8.4 | Experimental Design Matrix..... | 261 |
| Table 8.5 | Lognormal Stress-Failure Data..... | 261 |
| Table 8.6 | Lognormal Stress Cell Parameter Estimates | 263 |
| Table 8.7 | Summary of Arrhenius–Weibull Data Analysis | 273 |
| Table 8.1A | Spreadsheet Templates..... | 297 |
| Table 9.1 | Arrhenius Step Stress Example | 303 |
| Table 9.2 | Degradation Data..... | 311 |
| Table 9.3 | Summary of Shipment and Failure Data | 314 |
| Table 9.4 | Negative Log-Likelihood Values for Different Models..... | 332 |
| Table 9.5 | Step Stress Data for Problem 9.1 | 343 |
| Table 11.1 | Binomial Cumulative Distribution Function: $n = 4$, $p = .5$ | 382 |
| Table 11.2 | Cumulative Probability..... | 384 |
| Table 11.3 | Contingency Table 1 for Fisher’s Exact Test | 388 |
| Table 11.4 | Contingency Table 2 for Fisher’s Exact Test | 388 |
| Table 11.5 | JMP Data Table..... | 389 |
| Table 11.6 | Fisher’s Exact Test Results | 389 |
| Table 11.7 | Matrix of Possible Choices..... | 394 |
| Table 11.8 | Binomial Probability Calculations for Sample of Size $n = 50$ and $p = 0.02$ | 396 |

| | | |
|--------------------|--|-----|
| Table 11.9 | Probability of Three or Less Failures in Sample of Size $n = 50$ for Various Lot Percent Defective Values | 396 |
| Table 11.10 | LTPD Sampling Plans | 403 |
| Table 11.11 | LTPD Evaluation | 405 |
| Table 11.12 | Spreadsheet for Nearly Minimum Sampling Plans..... | 407 |
| Table 11.13 | Minimum Sample Sizes for Zero Rejects at Various Probabilities | 413 |
| Table 12.1 | Repair Age Histories (Hours) | 425 |
| Table 12.2 | Repair Histories for Four Machines..... | 429 |
| Table 12.3 | Repair Histories for Services at Two Different Data Centers | 436 |
| Table 12.4 | Repair Histories for Two Locations..... | 439 |
| Table 12.5 | One-Sided Lower Confidence Bound Factors for the MTBF (Failure-Censored Data) | 447 |
| Table 12.6 | One-Sided Lower Confidence Bound Factors for the MTBF (Time-Censored Data)..... | 448 |
| Table 12.7 | One-Sided Upper Confidence Bound Factors for the MTBF (Failure of Time-Censored Data)..... | 448 |
| Table 12.8 | Test Length Guide | 449 |
| Table 12.9 | Failure Times in Hours | 451 |
| Table 12.10 | Different Availability Levels..... | 454 |
| Table 12.1A | Repair Histories for Five Systems | 463 |
| Table 13.1 | Probability of R Reversals by Chance for $n = 4$ | 484 |
| Table 13.2 | Critical Values of R_{nr} , % of the Number of Reversals for the Reverse Arrangement Test | 484 |
| Table 13.3 | Steps for Fisher's Composite Test | 487 |
| Table 13.4 | Critical Values for Goodness-of-Fit Test..... | 492 |
| Table 13.5 | Repair History in Hours (Simulated Data: $a = 0.25$ and $b = 0.50$)..... | 492 |
| Table 13.6 | Transformed Repair Times..... | 494 |
| Table 13.7 | R_1 and R_2 Values to Multiply MTBF Estimate and Obtain Confidence Bounds (Test Ends at n th Fail) | 506 |
| Table 13.8 | P_1 and P_2 Values to Multiply MTBF Estimate and Obtain Confidence Bounds (Test Ends at Time T) | 507 |
| Table 13.9 | Results of Software Evaluation Testing..... | 510 |
| Table 14.1 | Bayesian Paradigm: Advantages and Disadvantages | 523 |
| Table 14.2 | Beta Distribution Parameters..... | 531 |

List of Examples

| | | |
|---------------------|--|----|
| Example 1.1 | Automobile Fuse Data..... | 1 |
| Example 1.2 | Conditional Probabilities..... | 12 |
| Example 1.3 | Total Probabilities..... | 14 |
| Example 1.4 | Bayes' Rule..... | 14 |
| Example 1.5 | Bayes' Rule Applied to Misclassified Items..... | 15 |
| Example 1.6 | Probability Expression for CDF..... | 16 |
| Example 1.7 | The Uniform Distribution..... | 20 |
| Example 1.8 | The Beta Distribution..... | 21 |
| Example 1.9 | Data Simulation..... | 24 |
| Example 1.10 | Data Simulation..... | 25 |
| Example 2.1 | Life Distribution Calculations..... | 30 |
| Example 2.2 | System Reliability..... | 32 |
| Example 2.3 | Failure Rate Calculations..... | 35 |
| Example 2.4 | Estimating the CDF, Reliability Function, and AFR..... | 37 |
| Example 2.5 | Residual MTTF(T_0) Calculation..... | 39 |
| Example 2.6 | Multicensored Experimental Data..... | 44 |
| Example 2.7 | Multicensored Field Failure Data..... | 44 |
| Example 2.8 | Left-Truncated Data..... | 44 |
| Example 2.9 | Left- and Right-Censored Data..... | 44 |
| Example 3.1 | Exponential Probabilities..... | 48 |
| Example 3.2 | Constant Failure Rate..... | 49 |
| Example 3.3 | Exponential Data..... | 49 |
| Example 3.4 | Mean Time to Fail..... | 52 |
| Example 3.5 | Piecewise Exponential Approximation..... | 54 |
| Example 3.6 | Failure Rate and MTTF..... | 57 |
| Example 3.7 | Chi-Square Goodness of Fit..... | 61 |
| Example 3.8 | Goodness-of-Fit Tests Based on EDF Statistics..... | 64 |
| Example 3.9 | Confidence Bounds for λ | 69 |
| Example 3.10 | Zero Failures Estimation..... | 70 |

| | | |
|---------------------|---|-----|
| Example 3.11 | Confidence Bounds on MTTF | 70 |
| Example 3.12 | Choosing Sample Sizes | 72 |
| Example 3.13 | Choosing the Test Times..... | 73 |
| Example 3.14 | Choosing Pass/Fail Criteria | 73 |
| Example 3.15 | Minimum Sample Sizes | 74 |
| Example 3.16 | Minimum Test Times | 74 |
| Example 3.17 | Simulating Exponential Data | 76 |
| Example 3.18 | Fitting a Two-Parameter Exponential Model to Data..... | 76 |
| Example 3.1A | Determining the Sample Size Using Goal Seek (Example 3.12 revisited) | 79 |
| Example 3.2A | Choosing the Test Times..... | 80 |
| Example 3.3A | Choosing Pass/Fail Criteria | 81 |
| Example 3.4A | KS Test | 82 |
| Example 4.1 | Weibull Properties | 89 |
| Example 4.2 | Weibull Closure Property..... | 93 |
| Example 4.3 | Rayleigh Radial Error..... | 97 |
| Example 4.4 | MLE for the Exponential..... | 99 |
| Example 4.5 | Weibull MLE Parameter Estimation | 102 |
| Example 4.6 | Weibull MLE Parameter Estimation: Left-Truncated Data..... | 109 |
| Example 5.1 | Normal Distribution Calculations..... | 125 |
| Example 5.2 | Root-Mean-Square Example | 128 |
| Example 5.3 | Censored Normal Data | 132 |
| Example 5.4 | Simulation of C_{pk} Distribution | 134 |
| Example 5.5 | Lognormal Properties | 137 |
| Example 5.6 | Lognormal MLEs and Likelihood Profile Confidence Limits: Exact Times of Failure..... | 142 |
| Example 5.7 | Lognormal MLEs and Likelihood Profile Confidence Limits: Interval Data..... | 144 |
| Example 5.8 | Lognormal Calculations | 147 |
| Example 6.1 | Linear Equations..... | 154 |
| Example 6.2 | Regression Line | 156 |
| Example 6.3 | Linear Rectification..... | 160 |
| Example 6.4 | Probability Plots for Exponential Distribution..... | 164 |
| Example 6.5 | Weibull Probability Plotting: Exact Times | 176 |

| | | |
|---------------------|---|-----|
| Example 6.6 | Lognormal Probability Plot..... | 182 |
| Example 6.7 | EDF and Simultaneous Confidence Bounds Calculation..... | 185 |
| Example 6.1A | Order Statistics for Exponential Distribution..... | 189 |
| Example 6.2A | Confidence Limits on Order Statistics for Exponential Distribution..... | 190 |
| Example 7.1 | Kaplan–Meier Product Limit Estimates for Exact Failure Time Data..... | 194 |
| Example 7.2 | Actuarial Life Table Estimation | 209 |
| Example 7.3 | Left-Truncated Data | 214 |
| Example 7.4 | Left-Censored Data..... | 217 |
| Example 7.5 | Current Status Data | 221 |
| Example 7.6 | Estimating $F(t)$ with H-W Confidence Bounds..... | 226 |
| Example 7.7 | Cumulative Hazard Plotting..... | 232 |
| Example 7.1A | Bootstrap Confidence Interval Calculation..... | 235 |
| Example 8.1 | Acceleration Factors for Exponential Distribution | 244 |
| Example 8.2 | Weibull Analysis of High-Stress Failure Data..... | 245 |
| Example 8.3 | Weibull Likelihood Equal-Shapes Test..... | 257 |
| Example 8.4 | Confidence-Bound Calculation for a Common Weibull Slope | 259 |
| Example 8.5 | Lognormal Stress-Failure Data..... | 260 |
| Example 8.6 | Calculation of Acceleration Factor Given ΔH | 268 |
| Example 8.7 | Estimating ΔH from Two Temperature Stress Cells of Data | 268 |
| Example 8.8 | Arrhenius Model Analysis Using Both Regression and MLE Methods | 269 |
| Example 8.9 | MLE Analysis of the Six-Stress Cells Given in Example 8.5..... | 274 |
| Example 8.10 | Calculating Needed Burn-In Time | 282 |
| Example 8.11 | Life Test Experimental Design..... | 284 |
| Example 8.1A | Weibull Likelihood Equal-Shapes Test..... | 287 |
| Example 8.2A | Confidence Bound Calculation for a Common Weibull Slope..... | 288 |
| Example 9.1 | An Arrhenius Step Stress Experiment..... | 302 |
| Example 9.2 | An Arrhenius, Power Law Step Stress Experiment..... | 306 |
| Example 9.3 | Degradation Data Analysis | 311 |
| Example 9.4 | Lifetime Regression Used to Estimate the Reliability Effects of Vintage and Plant of Manufacture and Their Significance | 314 |

| | | |
|----------------------|---|-----|
| Example 9.5 | Defect Model..... | 322 |
| Example 9.6 | Maximum Likelihood Estimation for the Defect Model | 324 |
| Example 9.7 | Multistress Defect Model Example | 329 |
| Example 9.8 | Defect Model Data Treated as Truncated Data..... | 333 |
| Example 9.1A | JMP's Nonlinear Modeling Platform | 335 |
| Example 10.1 | Series Systems | 346 |
| Example 10.2 | Bottom-Up Calculations | 347 |
| Example 10.3 | Redundancy Improvement..... | 348 |
| Example 10.4 | Maximizing Reliability Using Redundancy | 349 |
| Example 10.5 | Standby Model | 351 |
| Example 10.6 | Expected Lifetime of k -Out-of- n System of Independent Exponentially Distributed Components | 353 |
| Example 10.7 | Complex System Reduction (Five Components) | 354 |
| Example 10.8 | Complex System Reduction (Six Components)..... | 354 |
| Example 10.9 | Minimal Path Analysis | 357 |
| Example 10.10 | Minimal Cut Set Analysis..... | 358 |
| Example 10.11 | Minimal Path Analysis When " k -Out-of- n " Blocks Are Present..... | 359 |
| Example 10.12 | General Reliability Algorithm | 361 |
| Example 10.13 | Burn-In Model | 363 |
| Example 10.14 | Black Box Testing I..... | 366 |
| Example 10.15 | Black Box Testing II..... | 366 |
| Example 11.1 | Binomial Calculations | 373 |
| Example 11.2 | Binomial pmf..... | 374 |
| Example 11.3 | Shortcomings of the Normal Approximation..... | 379 |
| Example 11.4 | Score Confidence Intervals | 380 |
| Example 11.5 | Simulation of System Reliability..... | 383 |
| Example 11.6 | Geometric Distribution | 384 |
| Example 11.7 | Negative Binomial Distribution..... | 385 |
| Example 11.8 | Hypergeometric Distribution | 386 |
| Example 11.9 | Poisson Distribution | 391 |
| Example 11.10 | Confidence Limits for Expected Value of a Poisson Distribution | 393 |

| | | |
|----------------------|--|-----|
| Example 11.11 | Poisson Confidence Limits | 393 |
| Example 11.12 | Sampling Plan for Accelerated Stress, Weibull Distribution | 409 |
| Example 11.13 | Cumulative Count Control Charts for Low PPM..... | 411 |
| Example 12.1 | The Mean Cumulative Function..... | 427 |
| Example 12.2 | Naive Confidence Limits for the MCF | 430 |
| Example 12.3 | Correct Approximate Confidence Limits for the MCF..... | 430 |
| Example 12.4 | Comparison of MCFs for Servers at Two Different Datacenters..... | 436 |
| Example 12.5 | HPP Probability Estimates | 441 |
| Example 12.6 | HPP Estimates in Terms of the MTBF..... | 442 |
| Example 12.7 | Time to k th Repair for HPP Process | 443 |
| Example 12.8 | Spare Parts for an HPP | 444 |
| Example 12.9 | Memoryless Property of the Poisson Process | 445 |
| Example 12.10 | Confidence Bounds on the Population MTBF for an HPP | 449 |
| Example 12.11 | Test Length Guide for an HPP | 449 |
| Example 12.12 | Likelihood Ratio Test for Comparison of Two Exponential MTTFs (Nonrepairable Components)..... | 451 |
| Example 12.13 | Likelihood Ratio Test for Comparison of Two HPP MTBFs (Repairable Systems) | 452 |
| Example 12.14 | Simulation of 10 Time-Censored HPPs..... | 456 |
| Example 12.15 | Renewal Data Calculation of CDF | 461 |
| Example 12.1A | The Cox F-Test | 466 |
| Example 12.2A | Renewal Data Calculation of CDF | 468 |
| Example 13.1 | Laplace Test for Trend versus a Poisson Process..... | 481 |
| Example 13.2 | Reverse Arrangement Test..... | 485 |
| Example 13.3 | Fisher's Composite Test..... | 487 |
| Example 13.4 | Nonhomogeneous Poisson Process..... | 488 |
| Example 13.5 | NHPP with Power Relation Intensity | 491 |
| Example 13.6 | NHPP with Exponential Intensity Model | 498 |
| Example 13.7 | Duane Reliability Growth Estimation | 502 |
| Example 13.8 | Confidence Bounds and Modified MLEs | 505 |
| Example 13.9 | Power Relationship Model Reliability Growth | 508 |
| Example 13.10 | Software Reliability Improvement | 510 |
| Example 13.11 | Simulating an NHPP with Power Relation Intensity | 513 |

| | | |
|----------------------|--|-----|
| Example 13.12 | Simulating the First Six Repair Times for NHPP with Specified Power Relation Model | 514 |
| Example 14.1 | Lower MTBF Bounds Using a Bayesian Gamma Prior | 521 |
| Example 14.2 | Calculating Prior Parameters Using the 50/95 Method | 525 |
| Example 14.3 | Calculating a Bayesian Test Time | 526 |
| Example 14.4 | A Minimum Bayesian Testing Time Calculation | 527 |
| Example 14.5 | Using Engineering Judgment to Arrive at Bayesian Prior Parameters | 528 |
| Example 14.6 | MTBF Estimate after Test Is Run | 529 |
| Example 14.7 | Bayesian Estimation and Credibility Intervals for p | 530 |
| Example 14.8 | Bayesian Estimation and Credibility Intervals for the Lognormal T_{50} | 532 |
| Example 14.9 | Using an Improper Noninformative Prior for Exponential Fail Times | 534 |

1

Basic Descriptive Statistics

One of the most useful skills that a reliability specialist can develop is the ability to convert a mass (mess?) of data into a form suitable for meaningful analysis. Raw numbers by themselves are not useful; what is needed is a distillation of the data into information.

In this chapter, we discuss several important concepts and techniques from the field of descriptive statistics. These methods are used to extract a relevant summary from collected data. The goal is to describe and understand the random variability that exists in all measurements of real world phenomena and experimental data. These concepts and techniques are basic and are applied to reliability data throughout the book.

The topics we cover include populations and samples; frequency functions, histograms, and cumulative frequency functions; the population cumulative distribution function (CDF) and probability density function (PDF); elementary probability concepts, random variables, population parameters, and sample estimates; theoretical population shape models; and data simulation.

1.1 Populations and Samples

Statistics is concerned with variability, and it is a fact of nature that variation exists. No matter how carefully a process is run, an experiment is executed, or a measurement is taken, there will be differences in repeatability due to the inability of any individual or system to completely control all possible influences. If the variability is excessive, the study or process is described as lacking control. If, on the other hand, the variability appears reasonable, we accept it and continue to operate. How do we visualize variability in order to understand if we have a controlled situation?

Consider the following example:

EXAMPLE 1.1 AUTOMOBILE FUSE DATA

A manufacturer of automobile fuses produces lots containing 100,000 fuses rated at 5A. Thus, the fuses are supposed to open in a circuit if the current through the fuse exceeds 5A. Since a fuse protects other elements from possibly damaging electrical overload, it is very important that fuses function properly. How can the manufacturer be assured that the fuses do indeed operate correctly and that there is no excessive variability?

Obviously, he cannot test all fuses to the rated limit since that act would destroy the product he wishes to sell. However, he can sample a small quantity of fuses (say, 100 or 200) and test them to destruction to measure the opening point of each fuse. From the sample data, he could then *infer* what the behavior of the entire group would be if all fuses were tested.

In statistical terms, the entire set or collection of measurements of interest (e.g., the blowing values of all fuses) define a *population*. A population is the entire set or collection of measurements of interest.

Note that a population may be finite, as in the case of a fuse lot, or it may be infinite, as occurs in a manufacturing process where the population could be all product of a specific type that has been or could ever be produced in a fabricating area.

The *sample* (e.g., the 100 or 200 fuses tested to destruction) is a subset of data taken from the population. A sample is a subset of data from the population. The objective in taking a sample is to make inferences about the population.

Note that reliability data commonly exists in one of two forms. In *variables data*, the actual measurement of interest is continuous, such as time in minutes, length in inches, or temperature in degrees Celsius. In *attributes data*, the measurements are quantified into discrete categories such as pass or fail, go or no go, in spec or out of spec, small or medium or large, and so on. Attributes data includes counts, proportions, and percentages. Although both types of data are discussed in this text, applications and analysis of attributes data is treated extensively in [Chapter 11](#).

In the fuse data example, we record variables data, but we could also transform the same results into attributes data by stating whether a fuse opened before or after the 5A rating. Similarly, in reliability work one can measure the actual failure time of an item (variables data) or record the number of items failing before a fixed time (attributes data). Both types of data occur frequently in reliability studies. In [Chapter 3, Sections 3.10–3.12](#), we will discuss such topics as choosing a sample size, drawing a sample randomly, and the “confidence” in the data from a sample. For now, however, let’s assume that the sample has been properly drawn and consider what to do with the data in order to present an informative picture.

1.2 Histograms and Frequency Functions

In stating that a sample has been randomly drawn, we imply that each measurement or data point in the population has an equal chance or probability of being selected for the sample. If this requirement is not fulfilled, the sample may be “biased” and correct inferences about the population might not be possible.

What information does the manufacturer expect to obtain from the sample measurements of 100 fuses? First, the data should cluster about the rated value of 5A. Second, the spread in the data (variability) should not be large, because the manufacturer realizes that serious problems could result for the users if some fuses blow at too high a value. Similarly, fuses opening at too low a level could cause needless repairs or generate unnecessary concerns.

The reliability analyst randomly samples 100 fuses and records the data shown in [Table 1.1](#). It is easy to determine the high and low values from the sample data and see that the measurements cluster roughly about the number 5. Yet, there is still difficulty in grasping the full information contained in this set of data.

Let’s apply the following procedure:

1. Find the *range* of the data by subtracting the lowest from the highest value. For this set, the range is $5.46 - 4.43 = 1.03$.
2. Divide the range into ten or so equally spaced intervals such that readings can be uniquely classified into each cell. Here, the cell width is $1.03/10 \approx 0.10$, and we choose the starting point to be 4.395, a convenient value below the minimum of the data and carried out one digit more precise than the data to avoid any confusion in assigning readings to individual cells. Note that the terms “bin,” “class,”

or “cell” are used interchangeably in the literature and also by statistical software programs to denote one of these equally spaced intervals.

3. Increment the starting point by multiples of the cell width until the maximum value is exceeded. Thus, since the maximum value is 5.46, we generate the numbers 4.395, 4.495, 4.595, 4.695, 4.795, 4.895, 4.995, 5.095, 5.195, 5.295, 5.395, and 5.495. These values represent the endpoints or boundaries of each cell, effectively dividing the range of the data into equally spaced class intervals covering all the data points.
4. Construct a *frequency table* as shown in [Table 1.2](#), which gives the number of times a measurement falls inside a class interval.
5. Make a graphical representation of the data by sketching vertical bars centered at the midpoints of the class cells with bar heights proportionate to the number of values falling in that class. This graphical representation shown in [Figure 1.1](#) is called a *histogram*.

A histogram is a graphical representation in bar chart form of a frequency table or frequency distribution. The vertical axis in a histogram may represent the actual count in a cell, or it may state the percentage of observations of the total sample that occur in a cell. Also, the range here is divided by the number 10 to generate a cell width, but any convenient number

TABLE 1.1

Sample Data on 100 Fuses

| | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|
| 4.64 | 4.95 | 5.25 | 5.21 | 4.90 | 4.67 | 4.97 | 4.92 | 4.87 | 5.11 |
| 4.98 | 4.93 | 4.72 | 5.07 | 4.80 | 4.98 | 4.66 | 4.43 | 4.78 | 4.53 |
| 4.73 | 5.37 | 4.81 | 5.19 | 4.77 | 4.79 | 5.08 | 5.07 | 4.65 | 5.39 |
| 5.21 | 5.11 | 5.15 | 5.28 | 5.20 | 4.73 | 5.32 | 4.79 | 5.10 | 4.94 |
| 5.06 | 4.69 | 5.14 | 4.83 | 4.78 | 4.72 | 5.21 | 5.02 | 4.89 | 5.19 |
| 5.04 | 5.04 | 4.78 | 4.96 | 4.94 | 5.24 | 5.22 | 5.00 | 4.60 | 4.88 |
| 5.03 | 5.05 | 4.94 | 5.02 | 4.43 | 4.91 | 4.84 | 4.75 | 4.88 | 4.79 |
| 5.46 | 5.12 | 5.12 | 4.85 | 5.05 | 5.26 | 5.01 | 4.64 | 4.86 | 4.73 |
| 5.01 | 4.94 | 5.02 | 5.16 | 4.88 | 5.10 | 4.80 | 5.10 | 5.20 | 5.11 |
| 4.77 | 4.58 | 5.18 | 5.03 | 5.10 | 4.67 | 5.21 | 4.73 | 4.88 | 4.80 |

TABLE 1.2

Frequency Table of Fuse Data

| Cell Boundaries | Number in Cell |
|-----------------|----------------|
| 4.395–4.495 | 2 |
| 4.495–4.595 | 2 |
| 4.595–4.695 | 8 |
| 4.695–4.795 | 15 |
| 4.795–4.895 | 14 |
| 4.895–4.995 | 13 |
| 4.995–5.095 | 16 |
| 5.095–5.195 | 15 |
| 5.195–5.295 | 11 |
| 5.295–5.395 | 3 |
| 5.395–5.495 | 1 |
| Total count | 100 |

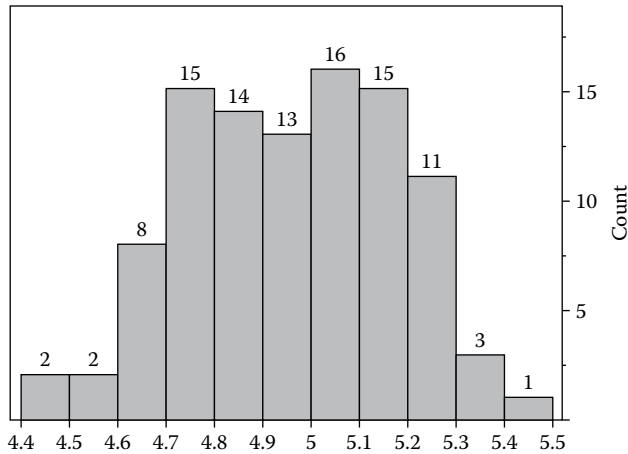


FIGURE 1.1
Histogram of fuse breakdown measurements.

(usually between 8 and 20) may be used. Too small a number may not reveal the shape of the data and too large a number can result in many empty cells and a flat-appearing distribution. Sometimes, a few tries are required to arrive at a suitable choice.

There is a useful Excel spreadsheet function called **FREQUENCY** that will generate a frequency table such as that shown in [Table 1.2](#). Say the 100 fuse breakpoints are entered into column A, cells A1–A100, and the 12 cell boundaries, starting with 4.395 and ending with 5.495, are entered into column B, cells B1–B12. Next, we highlight (**click and drag**) into an empty column, say C, 13 blank rows in cells C1–C13. Then we type in the function `=FREQUENCY(A1:A100,B1:B12)`. The expression is evaluated as a matrix operation by pressing the keys **Ctrl+Shift+Enter** together instead of pressing just **Enter** alone. This action produces the [Table 1.2](#) frequencies in rows C2–C12 (C1 contains counts up to 4.395 and C13 contains counts after 5.495). The **FREQUENCY** function in OpenOffice software works the same way, except that a semicolon is used between arguments instead of a comma.

EXERCISE 1.1

Use Excel or OpenOffice to generate the frequencies given in [Table 1.2](#), using the [Table 1.1](#) sample data and the same interval endpoints as used in [Table 1.2](#).

In summary, the histogram provides us with a picture of the data from which we can intuitively see the center of the distribution, the spread, and the shape. The shape is important because we usually have an underlying idea or model as to how the entire population should look. The sample shape either confirms this expectation or gives us reason to question our assumptions. In particular, a shape that is symmetric about the center, with most of the observations in the central region, might reflect data from certain symmetric distributions such as the normal or Gaussian distribution. Alternatively, a nonsymmetric appearance would imply the existence of data points spaced farther from the center in one direction than in the other, which could lead to the consideration of a distribution, such as a Weibull or lognormal.

For the data presented in Example 1.1, we note that the distribution appears reasonably symmetric. Hence, based on the histogram and the way the ends of the distribution taper off, the manufacturer believes that values much greater or much less than about 10% of the central target are not likely to occur. This variability is accepted as reasonable.

1.3 Cumulative Frequency Function

There is another way of representing the data that can be very useful. By reference to [Table 1.2](#), let us accumulate the number of observations less than or equal to each upper cell boundary as shown in [Table 1.3](#). This representation of the data is called a *cumulative frequency function*.

The graphical rendering of the cumulative frequency function is shown in [Figure 1.2](#). Note that the cumulative frequency distribution is never decreasing—it starts at zero and reaches the total sample size. It is often convenient to represent the cumulative count in terms of a fraction or percentage of the total sample size used. In that case, the cumulative

TABLE 1.3

Cumulative Frequency Function for Fuse Data

| Upper Cell Boundary (UCB) | Number of Observations Less than or Equal to UCB |
|---------------------------|---|
| 4.495 | 2 |
| 4.595 | 4 |
| 4.695 | 12 |
| 4.795 | 27 |
| 4.895 | 41 |
| 4.995 | 54 |
| 5.095 | 70 |
| 5.195 | 85 |
| 5.295 | 96 |
| 5.395 | 99 |
| 5.495 | 100 |

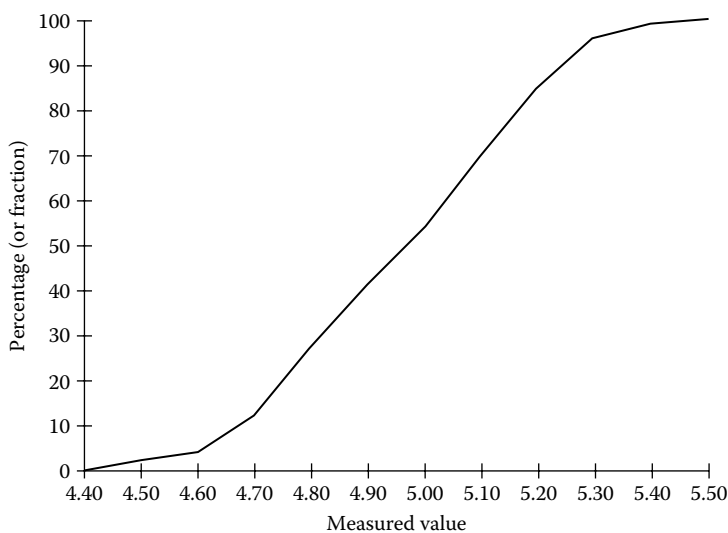


FIGURE 1.2

Plot of cumulative frequency function.

frequency function ranges from 0 to 1.00 in fractional representation or 0% to 100% in percentage notation. In this text, we often employ the percentage form.

Table 1.3 and Figure 1.2 show that the cumulative frequency curve is obtained by summing the frequency function count values. This summation process will be generalized by integration when we discuss the population concepts underlying the frequency function and the cumulative frequency function in Section 1.4.

1.4 The Cumulative Distribution Function and the Probability Density Function

The frequency distribution and the cumulative frequency distribution are calculated from sample measurements. Since the samples are drawn from a population, what can we state about this population? The typical procedure is to assume a mathematical formula that provides a theoretical model for describing the way the population values are distributed. The sample histograms and the cumulative frequency functions are the estimates of these population models.

The model corresponding to the frequency distribution is the PDF denoted by $f(x)$, where x is any value of interest. The PDF may be interpreted in the following way: $f(x)dx$ is the fraction of the population values occurring in the interval dx . In reliability work, we often have the failure time t as the variable of interest. Therefore, $f(t)dt$ is the fraction of failure times of the population occurring in the interval dt . A very simple example for $f(t)$ is the exponential distribution, given by the equation

$$f(t) = \lambda e^{-\lambda t}, \quad 0 \leq t < \infty$$

where λ is a constant. The plot of $f(t)$ is shown in Figure 1.3. The exponential distribution is a widely applied model in reliability studies and forms the basis of Chapter 3.

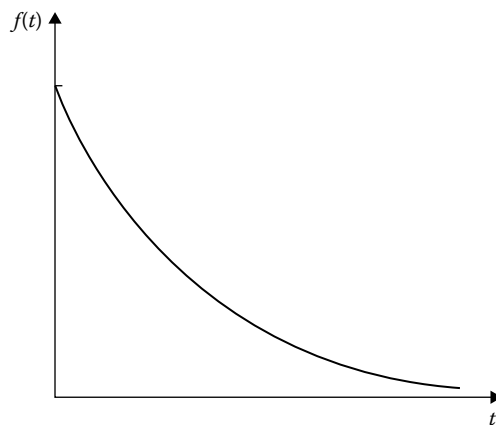


FIGURE 1.3

Plot of PDF for the exponential distribution.

The cumulative frequency distribution similarly corresponds to a population model called the CDF and is denoted by $F(x)$. The CDF is related to the PDF via the relationship

$$F(x) = \int_{-\infty}^x f(y) dy$$

where y is the dummy variable of integration. $F(x)$ may be interpreted as the fraction of values less than or equal to x in the population. Alternatively, $F(x)$ gives the probability of a value less than or equal to x occurring in a single random draw from the population described by $F(x)$. Since in reliability work we usually deal with failure times, t , which are nonnegative, the CDF for population failure times is related to the PDF by

$$F(t) = \int_0^t f(y) dy$$

For the exponential distribution,

$$F(t) = \int_0^t \lambda e^{-\lambda y} dy = [-e^{-\lambda y}]_0^t = 1 - e^{-\lambda t}$$

The CDF for the exponential distribution is plotted in [Figure 1.4](#).

When we calculated the cumulative frequency function in the fuse example, we worked with grouped data (i.e., data classified by cells). However, another estimate of the population CDF can be generated by ordering the individual measurements from smallest to largest, and then plotting the successive fractions

$$\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \dots, \frac{n}{n}$$

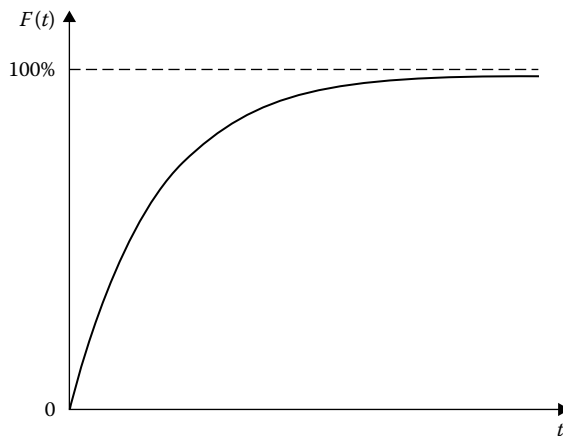


FIGURE 1.4

CDF for exponential distribution.

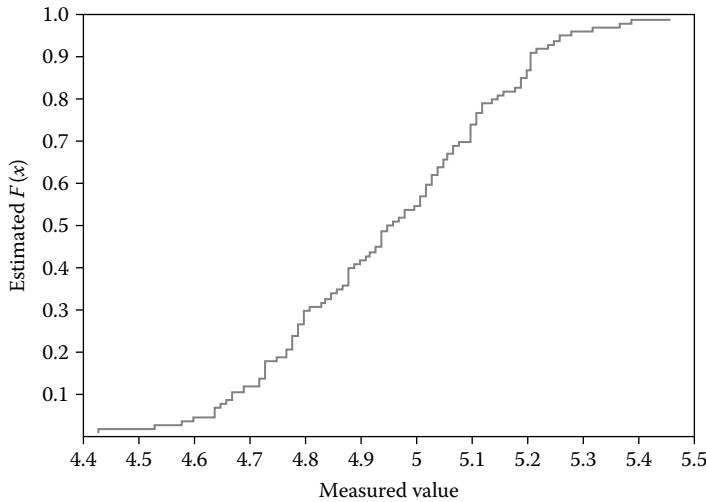


FIGURE 1.5
EDF for fuse data.

versus the ordered data points. Such a representation is called the *empirical distribution function* (EDF) and is shown in Figure 1.5 for the data from the fuse example. Note that the EDF steps up by $1/n$ at each data point and remains constant until the next point. The advantage of using the EDF instead of grouping the data is obviously that all data points are pictured; the disadvantage is that more computational effort is involved. However, spreadsheet software can easily perform the calculations and plotting. See Appendix 1A for a method to create step charts using spreadsheet plots. Since $F(x)$ is a probability, all the rules and formulas for manipulating probabilities can be used when working with CDFs. Some of these basic rules are described in Section 1.5.

EXERCISE 1.2

For the data in Table 1.1, construct a frequency table using 4.395 as the starting point and 0.2 as the interval width. Create a histogram of this frequency table. Compare it to Figure 1.1.

EXERCISE 1.3

Using the results from Exercise 1.2, construct a cumulative frequency table and create a plot of the cumulative frequency function. How does it compare to Figure 1.2?

EXERCISE 1.4

Take columns 2, 5, and 8 (left to right) from Table 1.1, for a total of 30 data points. Assume a random sample, arrange the points in order from smallest to largest, and plot the EDF. Compare to Figure 1.5.

EXERCISE 1.5

The histogram shown in Figure 1.1 was generated using JMP software. Use the JMP **Analyze, Distribution** platform and the data set shown in Table 1.1 (FuseData.jmp at the “Downloads & Updates” tab of the web page for this book at <http://www.crcpress.com/product/isbn/9781584884668>) to generate a histogram. The result may

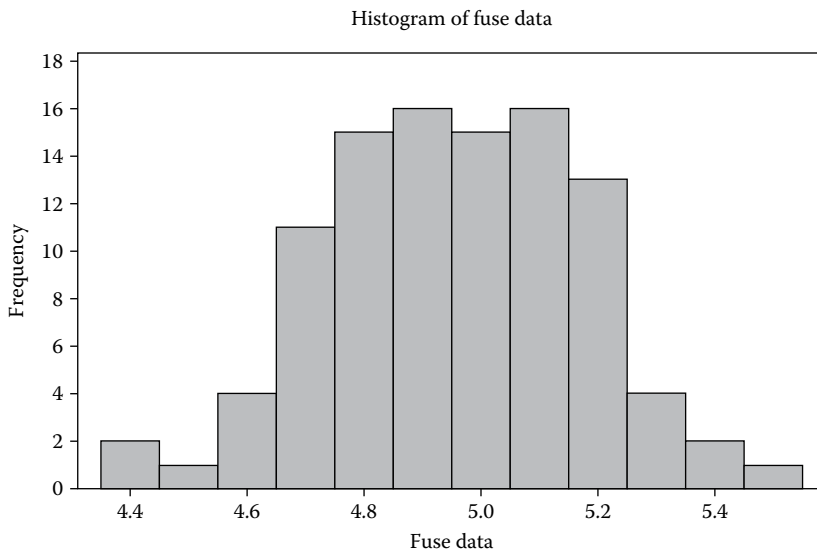


FIGURE 1.6
Minitab histogram of fuse data.

be somewhat different based on a JMP default choice for the number of bins. If so, use the “hand” cursor to change the number of bins until the same histogram as shown in [Figure 1.1](#) is obtained. Next, use the **histogram** spreadsheet function of Excel (in **Tools, Data Analysis**) to obtain a histogram. The [Table 1.1](#) data set for spreadsheet use is FuseData.xls. In order to get the exact same graph shown in [Figure 1.1](#), input bin numbers starting with 4.495 and increasing in steps of 0.1 to 5.495. Finally, use Minitab to get the histogram shown in [Figure 1.6](#) (one must change the default number of bins from 12 to 11).

NOTE: There is nothing “wrong” or “misleading” with obtaining dissimilar histograms from different software programs. It is up to the analyst to vary the bin numbers and locations to obtain one of many reasonable (although varying) views of the data. For example, the default output histogram from Minitab is shown in [Figure 1.6](#).

1.5 Probability Concepts

In the classical sense, the term *probability* can be thought of as the expected relative frequency of occurrence of a specific event in a very large collection of possible outcomes. For example, if we toss a balanced coin many times, we expect the number of occurrences of the event “heads” to comprise approximately half of the number of outcomes. Thus, we say the probability of heads on a single toss is 0.5, 50%, or 50–50. It is typical to express probabilities either as a fraction between 0 and 1 or as a percentage between 0% and 100%.

There are two very useful relations often invoked in probability theory. These rules relate to the occurrence of two or more events. In electrical engineering terms, we are defining “and” and “or” relations. The first rule states that if $P(A)$ is the probability of event

A occurring and $P(B)$ is the probability of event B occurring, then the probability of events A and B occurring simultaneously, denoted $P(AB)$, is

$$P(AB) = P(A)P(B|A)$$

or

$$P(AB) = P(B)P(A|B)$$

where $P(A|B)$ designates the “conditional” probability of A given that event B has occurred.

Let’s explain conditional probability further. We imply by the terminology that one event may be affected by the occurrence of another event. For example, suppose we ask what the probability is of getting two black cards in a row in successive draws from a well-shuffled deck of cards, without replacing the first card drawn. Obviously, the probability of the first card being a black card (call this event A) is

$$P(A) = \frac{\text{favorable outcomes}}{\text{total outcomes}} = \frac{26}{52} = \frac{1}{2}$$

The probability of the second card being a black card (event B) changes depending on whether or not the first card drawn is a black card. If yes, then the probability of the second card being a black card is

$$P(B|A) = \frac{25}{51}$$

Therefore, the probability of two successive black cards is

$$\begin{aligned} P(AB) &= P(B)P(A|B) \\ &= \frac{1}{2} \frac{25}{51} \\ &= \frac{25}{102} \end{aligned}$$

Two events, A and B , are said to be *independent* if the occurrence of one does not affect the probability of the other occurrence. The formal definition states that two events A and B are independent *if and only if*

$$P(AB) = P(A)P(B)$$

This expression is sometimes referred to as the *multiplication rule* for the probability of independent events occurring simultaneously. In general, the probability of independent events occurring is just the product of the individual probabilities of each event. For example, in the card situation, replacing the first card drawn and reshuffling the deck will make event B independent of event A . Thus, the probability of two successive black cards, with replacement and reshuffling between draws, is

$$\begin{aligned} P(AB) &= P(A)P(B) = \frac{26}{52} \frac{26}{52} \\ &= \frac{1}{4} \end{aligned}$$

Similarly, the probability of simultaneously getting a 6 on one roll of a die and an ace in one draw from a deck of cards, apparently independent events, is

$$\begin{aligned} P(AB) &= \frac{1}{6} \frac{4}{52} \\ &= \frac{1}{78} \end{aligned}$$

The extension of these conditional probability principles to three or more events is possible. For example, the rule for the joint probability of three events, A, B and C, is

$$P(ABC) = P(A)P(B|A)P(C|AB)$$

For independent events, the formula becomes

$$P(ABC) = P(A)P(B)P(C)$$

The second important probability formula relates to the situation in which either of two events, A or B, may occur. The expression for this “union” is

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

If the events are independent, then the relation becomes

$$P(A \cup B) = P(A) + P(B) - P(A)P(B)$$

The last term in the above expressions corrects for double counting of the same outcomes. For example, what is the probability of getting either an ace (event A) or a black card (event B) in one draw from a deck of cards? The events are independent (see Exercise 1.6.), and therefore

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A)P(B) \\ &= \frac{4}{52} + \frac{26}{52} - \frac{4}{52} \frac{26}{52} \\ &= \frac{14}{52} = \frac{7}{13} \end{aligned}$$

Note that the term $P(A)P(B)$ subtracts out the probability for black aces. This probability has already been counted twice, once in the $P(A)$ term and once in the $P(B)$ term.

When events A and B are mutually exclusive or disjoint, that is, both events cannot occur simultaneously, then $P(AB) = 0$, and

$$P(A \cup B) = P(A) + P(B)$$

Furthermore, if both events are exhaustive in that at least one of them must occur when an experiment is run, then

$$P(A \cup B) = P(A) + P(B) = 1$$

Thus, event A is the complement of event B . Event B can be viewed as the nonoccurrence of A and designated as event \bar{A} . Hence, the probability of occurrence of any event is equal to one minus the probability of occurrence of its complementary event. This *complement rule* has important applications in reliability work because a component may either fail (event A) or survive (event \bar{A}), resulting in

$$P(\text{Failure}) = 1 - P(\text{Survival})$$

As another example, we note that the event “at least one occurrence” and the event “zero occurrences” are mutually exclusive and exhaustive events. Therefore, the probability of at least one occurrence is equal to $1 - \text{probability of no occurrences}$.

An extension to three or more events is also possible. For three events A , B , and C , the formula is

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC)$$

For independent events, the relation becomes

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A)P(B) \\ &\quad - P(B)P(C) - P(A)P(C) + P(A)P(B)P(C) \end{aligned}$$

For mutually exclusive, exhaustive events, we have

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) = 1$$

For four events, we begin by adding the four single-event probabilities. Then, we subtract the six possible probabilities of two events occurring simultaneously. Next, we add back in the four possible probabilities of three events occurring simultaneously. Finally, we subtract the probability of all four events occurring simultaneously. This “in and out” procedure works for any number of events, and the total number of terms in the final expression when there are n events will be $2^n - 1$.

EXAMPLE 1.2 CONDITIONAL PROBABILITIES

A tricky word problem that appears often in many forms can be stated as follows: A computer hack visits the surplus store and sees two similar hard drives displayed. The sign says, “Specially Reduced: 50–50 Chance of Working.” He asks the dealer whether the hard drives operate properly. The dealer replies “at least one of them is working.” What is the probability that both drives are functional? Does the probability change if the dealer says “the one on the left works”?

SOLUTION

The first question asks for the probability that both drives work, given that at least one is working, that is, $P(\text{both work} \mid \text{at least one works})$. Let A be the event “both drives work” and let B be the event “at least one drive works.” We want the $P(A|B)$. From our conditional probability formula, we can rewrite the expression as follows:

$$P(A|B) = \frac{P(AB)}{P(B)}$$

TABLE 1.4

Possible Outcomes for Drives

| No Dealer Information | | At Least One Works | | Left Drive Works | |
|--|-------------|--------------------|-------------|------------------|-------------|
| Left Drive | Right Drive | Left Drive | Right Drive | Left Drive | Right Drive |
| W | W | W | W | W | W |
| W | N | W | N | W | N |
| N | W | N | W | | |
| N | N | | | | |
| <i>Probability that both drives work</i> | | | | | |
| 1/4 | | 1/3 | | 1/2 | |

Now $P(AB)$ is the probability that both drives work (event A) and at least one drive works (event B). This joint event is actually the same as the probability of event A alone since event A includes event B ; that is, if both drives work, then at least one works. Therefore, $P(AB) = P(A) = ([0.5][0.5]) = 0.25$, assuming the drives are independent. Since the event B “at least one drive works” and the event “both drives are not working” are mutually exclusive and exhaustive events, the denominator $P(B) = P(\text{at least one works}) = 1 - P(\text{both not working}) = 1 - (0.5)(0.5) = 0.75$. Hence, the desired probability is $P(A|B) = (0.25)/(0.75) = 1/3$.

This result surprises many individuals who incorrectly assume that the conditional probability of two working drives given at least one works should be $1/2$, instead of the correct answer $1/3$, since they reason that the other disk drive is equally likely to work or not work. However, the sample space of possible outcomes is listed in [Table 1.4](#).

With no dealer information, there are four equally likely outcomes: (work, work), (work, not work), (not work, work), and (not work, not work), for the left and right drives, respectively. Thus, the probability is only $1/4$ that both drives work. When we are told that at least one drive works, we eliminate the outcome (not work, not work). Therefore, we have only three equally likely outcomes remaining: (work, work), (work, not work), and (not work, work). Consequently, the probability that both drives work has increased from $1/4$ to $1/3$ with the added data. Alternatively, the probability that at least one of the drives does not work has decreased from $3/4$ to $2/3$.

On the other hand, if the dealer points out the working drive (maybe he did not have the time to test both drives), the probability that both drives work does change. Let event A be “both drives work” and C be “the left drive works.” Now, $P(A|C) = (0.5)/(1) = 0.5$. In this case, there are only two possible outcomes (work, work) and (work, not work), where the first position indicates the left drive, and only one outcome of the two has both drives working.

For a set of events, E_1, E_2, \dots, E_k , that are mutually exclusive and exhaustive, another useful relationship, sometimes called the *law of total probabilities*, applies. Any event A can be written as follows:

$$P(A) = \sum_{j=1}^k P(A|E_j)P(E_j)$$

In words, $P(A)$ is the weighted average of conditional probabilities, each weighted by the probability of the event on which it is conditioned. This expression is often easier to calculate than $P(A)$ directly.

EXAMPLE 1.3 TOTAL PROBABILITIES

A computer manufacturer purchases equivalent microprocessor components from three different distributors. The assembly of each computer utilizes one microprocessor, randomly chosen from an in-house inventory. Typically, the inventory consists of 30% of this component type from distributor *A*, 50% from distributor *B*, and 20% from distributor *C*. Historical records show that components from distributors *A* and *C* are twice as likely to cause a system failure as those from distributor *B*. The probability of system failure with component *B* is 0.5%. What is the probability that a computer system will experience failure?

SOLUTION

Since there are three distributors and we randomly chose a component from one of the distributors, we have three mutually exclusive and exhaustive events. The theorem of total probability is the basis for the solution:

$$\begin{aligned} P(\text{failure}) &= P(\text{failure}|\text{distributor } A)P(\text{distributor } A) + P(\text{failure}|\text{distributor } B)P(\text{distributor } B) \\ &\quad + P(\text{failure}|\text{distributor } C)P(\text{distributor } C) \\ &= 2(.005)(.3) + (.005)(.5) + 2(.005)(.2) \\ &= .0075 \text{ or } .75\% \end{aligned}$$

A final key probability formula, known as Bayes' rule, allows us to "invert" conditional probabilities, that is, determine which one of the conditioning events E_j is likely to have occurred, given that event *A* has occurred. Again, for a set of mutually exclusive and exhaustive events, E_1, E_2, \dots, E_k , Bayes' rule states that

$$P(E_j|A) = \frac{P(A|E_j)P(E_j)}{\sum_{j=1}^k P(A|E_j)P(E_j)}$$

Note that by the law of total probabilities, the denominator of this expression is just $P(A)$.

EXAMPLE 1.4 BAYES' RULE

The probability that a batch of incoming material from any supplier is rejected is 0.1. Typically, material from supplier S_1 is rejected 8% of the time, from supplier S_2 , 15%, and S_3 , 10%. We know that 50% of the incoming material comes from S_1 , 20% from supplier S_2 , and 30% from S_3 . Given that the latest lot of incoming material is rejected, what is the probability the supplier is S_1 ?

SOLUTION

Let *A* denote the event that the batch is rejected. Then, by Bayes' rule,

$$P(S_1|A) = \frac{P(A|S_1)P(S_1)}{P(A)} = \frac{(0.08)(0.5)}{(0.08)(0.5) + (0.15)(0.2) + (0.1)(0.3)} = 0.4$$

In this example, the starting (i.e., before we know the batch is rejected) probability of the event S_1 is 0.5. This knowledge is sometimes referred to as the "a priori" probability of S_1 . After the batch rejection, Bayes' rule allows us to calculate the new (conditional) probability of S_1 as .4. The result is sometimes called the *a posteriori* probability of S_1 .

EXAMPLE 1.5 BAYES' RULE APPLIED TO MISCLASSIFIED ITEMS

Assume we perform a test on a component to check for a specific defect. Historically, 1% of the components have this defect. Based on a detailed analysis of previous results, 95% of the components with the defect are detected, but 8% of the components without the defect are wrongly categorized as defective. If a component is classified as defective, what is the probability that the component actually has the defect? What is the probability that a component with a negative test has the defect?

SOLUTION

We first solve this problem by an approach using simple average calculations. Consider a test of 2000 components. On the average, 1%, or 20, of the total components will have the defect. Of those with the defect, 95%, or 19, will be caught. However, of the 1980 without the defect, 8%, or 158, will have a false positive and be called *defective*. Therefore, 19 out of the $19 + 158 = 177$ classified as defective will have the defect, and the probability of actually having the defect and a positive test result is $19/177 = 11\%$. This result shows that it may be a good idea to do a more extensive retest of rejected components and recover from the false positives. Also, the probability of having the defect and a negative test result is $1/(1000 - 177) = 1/1823 = .055\%$, which is about $1/18$ of the prior probability. Next, we get the same result using the Bayes' rule formula. Let A denote a positive test and E_1 denote a defective unit. Also, let E_2 represent a unit with no defect. We want

$$\begin{aligned} P(E_1|A) &= P(E_1A)/P(A) = P(A|E_1)P(E_1)/[P(A|E_1)P(E_1) + P(A|E_2)P(E_2)] \\ &= .95(.01)/[.95(.01) + .08(.99)] = .0095/.0887 = 11\% \end{aligned}$$

Let B = negative result. The probability of having the defect E_1 given a negative result of B is

$$\begin{aligned} P(E_1|B) &= P(E_1B)/P(B) = P(B|E_1)P(E_1)/[P(B|E_1)P(E_1) + P(B|E_2)P(E_2)] \\ &= .05(.01)/[.05(.01) + .92(.99)] = .0005/.9113 = .055\% \end{aligned}$$

EXERCISE 1.6

From a well-shuffled deck of cards, let drawing an ace event A and let drawing a black card be event B . Determine $P(AB)$, the probability of getting a black ace in a single draw, and show that events A and B are independent.

EXERCISE 1.7

Three assembly plants produce the same parts. Plant A produces 25% of the volume and has a shipment defect rate of 1%. Plant B produces 30% of the volume and ships 1.2% defectives. Plant C produces the remainder and ships 0.6% defectives. Given that a component picked at random from the warehouse stocked by these plants is defective, what are the probabilities that it was manufactured by plant A or B or C?

EXERCISE 1.8

An electronic card contains three components: A, B, and C. Component A has a probability of .02 of failing in 3 years. Component B has a probability of .01 of failing in 3 years and component C has a probability of .10 of failing in 3 years. What is the probability that the card survives 3 years without failing? What assumptions were made for this calculation?

1.6 Random Variables

In reliability studies, the outcome of an experiment may be numerical (e.g., time to failure of a component) or the result may be other than numerical (e.g., type of failure mode associated with a nonfunctional device). In either case, analysis is made possible by assigning a number to every point in the space of all possible outcomes—called the *sample space*. Examples of assigning numbers are as follows: the time to failure is assigned the elapsed hours of operation, and the failure mode may be assigned a category number 1, 2, and so on. Any rule for assigning a number creates a random variable. A random variable is a function for assigning real numbers to points in the sample space.

The practice is to denote the random variable by a capital letter (X, Y, Z , etc.) and the realization of the random variable (i.e., the real number or piece of sample data) by the lower case letter (x, y, z , etc.). Since this definition appears a bit abstract, let us consider a simple example using a single die with six faces, each face having one to six dots. The experiment consists of rolling the die and observing the upside face. The random variable is denoted X , and it assigns numbers matching the number of dots on the side facing up. Thus, $(X = x)$ is an event in the sample space, and $X = 6$ refers to the realization where the face with six dots is the side up. It is also common to refer to the probability of an event occurring using the notation $P(X = x)$. In this example, we assume all six possible outcomes are equally likely (fair die), and therefore, $P(X = x) = 1/6$ for $x = 1, 2, 3, 4, 5$, or 6 .

EXAMPLE 1.6 PROBABILITY EXPRESSION FOR CDF

The CDF $F(x)$ can be defined as $F(x) = P(X \leq x)$, that is, $F(x)$ is the probability that the random variable X has a value less than or equal to x . Similarly, the survival function can be defined as $S(x) = 1 - F(x) = P(X > x)$.

1.7 Sample Estimates of Population Parameters

We have discussed descriptive techniques such as histograms to represent observations. However, in order to complement the visual impression given by the frequency histogram, we often employ numerical descriptive measures called *parameters* for a population and *statistics* for a sample. These measures summarize the data in a population or sample and also permit quantitative statistical analysis. In this way, the concepts of central tendency, spread, shape, symmetry, and so on take on quantifiable meanings.

For example, we state that the frequency distribution is centered about a given value. This central tendency can be expressed in several ways. One simple method is just to cite the most frequently occurring value, called the *mode*. For grouped data, the mode is the midpoint of the interval with the highest frequency. For the fuse data in [Table 1.1](#), the mode is 5.05.

Another procedure involves selecting the *median*, that is, the value that effectively divides the data in half. For individual readings, the n data points are first ranked in order, from smallest to largest, and the median is chosen according to the following algorithm: the middle value if n is odd, and the average of the two middle values if n is even.

Alternatively, for data that has already been grouped or binned ([Table 1.2](#)), the median occurs in the interval for which the cumulative frequency distribution registers 50%; that is, a vertical line through the median divides the histogram into two equal areas. For

grouped data with n points, to get the median, one first determines the number of observations in the class containing the middle measurement $n/2$ and the number of observations in the class to get to that measurement. For example, for the fuse data in [Table 1.3](#), $n = 100$, and the middle value is the 50th point, which occurs in the class marked 4.895 to 4.995 (width 0.1). There are 41 data points before the interval and 13 points in this class. We must count 9/13 of the interval width to get to the median. Hence, the median is

$$4.895 + \left(\frac{9}{13}\right) \times 0.1 = 4.964$$

(In reliability work, it is common terminology to refer to the median as the T_{50} value for time to 50% failures.)

The most common measure of central tendency, however, is called the arithmetic *mean* or average. The sample mean is simply the sum of the observations divided by the number of observations. Thus, the mean, denoted by \bar{X} , of n readings is given by the statistic

$$\begin{aligned}\bar{X} &= \frac{X_1 + X_2 + X_3 + \cdots + X_n}{n} \\ &= \frac{\sum_{i=1}^n X_i}{n}\end{aligned}$$

This expression is called a *statistic* because its value depends on the sample measurements. Thus, the sample mean will change with each sample drawn, which is another instance of the variability of the real world. In contrast, the population mean depends on the entire set of measurements, and thus it is a fixed quantity, which we call a *parameter*. The sample mean \bar{X} estimates the population mean μ . We also mention here a notation common in statistics and reliability work. A parameter estimate is commonly denoted by a caret (^) over the parameter symbol. Thus, $\hat{\mu}$ is an estimate of the population mean μ and here $\hat{\mu} = \bar{X}$.

For a discrete (i.e., countable) population, the mean is just the summation over all discrete values where each value x_i is weighted by the probability of its occurrence p_i :

$$\mu = \sum_i x_i p_i$$

For a continuous population, the mean parameter is expressed in terms of the PDF model as

$$\mu = \int_{-\infty}^{\infty} x f(x) dx$$

For reliability work involving time, the population mean is

$$\mu = \int_0^{\infty} t f(t) dt$$

An alternate expression for the mean of a lifetime distribution is sometimes easier to evaluate. The form of the equation, when a finite mean exists, is

$$\mu = \int_0^{\infty} [1 - F(t)] dt$$

(See Feller 1968, page 148, for a proof.)

A common practice in statistics is to refer to the mean for both discrete and continuous random variables as the *expected value* of the random variable and use the notation $E(X) = \mu$ or $E(T) = \mu$. We occasionally use this terminology in this text.

Knowing the center of the distribution is not enough; we are also concerned about the spread of the data. The simplest concept for variability is the *range*, the difference between the highest and lowest readings. However, the range does not have very convenient statistical properties, and therefore, another measure of dispersion is more frequently used. This numerical measure of variation is called the *variance*. The variance has certain statistical properties that make it very useful for analysis and theoretical work. The variance of a random variable X is defined as the expected value of $(X - \mu)^2$, that is, $V(x) = E[(X - \mu)^2]$. An alternative formula is $V(x) = E[X^2] - \mu^2$. For continuous data, the population variance for common reliability analysis involving time is

$$V(t) = \sigma^2 = \int_0^{\infty} (t - \mu)^2 dt$$

In engineering terms, we see that the variance is the expected value of the second moment about the mean.

The square root of the variance is called the *standard deviation*. The standard deviation is expressed in the same units as the observations. The sample standard deviation is denoted by s and the formula is

$$s = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}}$$

Since \bar{X} is used in the formula rather than the population mean, statistical theory shows that dividing by $n - 1$ gives a better (i.e., unbiased) estimate of the population variance (denoted by $\hat{\sigma}^2 = s^2$) than just dividing by n . Alternatively, we may state that one degree of freedom has been taken to estimate the population mean μ using \bar{X} .

We have defined numerical measures of central tendency (\bar{X}, μ) and dispersion (s^2, σ^2). It is also valuable to have a measure of symmetry about the center and a measure of how peaked the data is over the central region. These measures are called *skewness* and *kurtosis*, and are respectively defined as expected values of the third and fourth moments about the mean, that is,

$$\text{skewness: } \mu_3 = E[(X - \mu)^3]; \quad \text{kurtosis: } \mu_4 = E[(X - \mu)^4]$$

Symmetric distributions have skewness equal to zero. A unimodal (i.e., single peak) distribution with an extended right “tail” will have positive skewness and will be referred to as skewed right; skewed left implies a negative skewness and a corresponding extended left tail. For example, the exponential distribution in Figure 1.3 is skewed right. Kurtosis, on the other hand, indicates the relative flatness of the distribution or how “heavy” the tails are.

Both measures are usually expressed in relative (i.e., independent of the scale of measurement) terms by dividing μ^3 by σ^3 and μ^4 by σ^4 . The kurtosis estimate is also offset by an amount that goes to three as the sample size increases so that data from a normal population has a kurtosis of approximately zero. Sample estimates are calculated using the formulas

$$\text{Skewness estimate} = \frac{n}{(n-1)(n-2)} \left(\frac{\sum_{i=1}^n (x_i - \bar{x})}{s} \right)^3$$

$$\text{Kurtosis estimate} = \frac{n(n-1)}{(n-1)(n-2)(n-3)} \left(\frac{\sum_{i=1}^n (x_i - \bar{x})}{s} \right)^4 - 3 \frac{(n-1)^2}{(n-2)(n-3)}$$

These formulas are used by the spreadsheet SKEW and KURT functions and also by Minitab and JMP in their descriptive statistics platforms.

These various measures allow us to check the validity of the assumed model. Ott (1977) shows applications to the normal distribution. Table 1.5 contains a listing of properties of distributions frequently used in reliability studies.

The important statistical concept involved in sample estimates of population parameters (e.g., mean, variance, etc.) is that the population parameters are fixed quantities, and we infer what they are from the sample data. For example, the fixed constant θ in the exponential model $F(t) = 1 - e^{-t/\theta}$, where $\theta = 1/\lambda$, can be shown to be the mean of the distribution of failure times for an exponential population. The sample quantities, on the other hand, are random statistics that may change with each sample drawn from the population.

TABLE 1.5

Properties of Distributions Used in Reliability Studies

| | Uniform | Normal | Weibull | Exponential | Lognormal | Rayleigh (Weibull with Shape Parameter 2 and Linear Failure Rate) | Extreme Value |
|---------------------------------------|---------------|------------|-------------|-------------------|-------------|---|---------------|
| Symmetric | Yes | Yes | No | No | No | No | No |
| Bell-shaped | No | Yes | No | No | No | No | No |
| Skewed | No | No | Yes (right) | Yes (right) | Yes (right) | Yes (right) | Yes (left) |
| | Skew = 0 | Skew = 0 | | Skew = 2 | | Skew = 0.63 | Skew = -1.14 |
| Kurtosis | -1.8 | 0 | | 6 | | 0.26 | 2.4 |
| Log data is symmetric and bell-shaped | No | No | No | No | Yes | No | No |
| Cumulative distribution shape | Straight line | "S" shaped | | Exponential curve | | | |

EXAMPLE 1.7 THE UNIFORM DISTRIBUTION

The uniform distribution is a continuous distribution with PDF for the random variable T given by

$$f(t) = \frac{1}{\theta_2 - \theta_1}, \quad \theta_1 \leq t \leq \theta_2$$

and zero elsewhere, where θ_1 and θ_2 are the parameters specifying the range of T . The rectangular shape of this distribution is shown in Figure 1.7.

We note that $f(t)$ is constant between θ_1 and θ_2 . The CDF of T , denoted by $F(t)$, for the uniform case is given by

$$F(t) = \frac{t - \theta_1}{\theta_2 - \theta_1}$$

Thus, $F(t)$ is linear in t in the range $\theta_1 \leq t \leq \theta_2$, as shown in Figure 1.8.

EXERCISE 1.9

Show that the uniform distribution has expected value $E(t) = (\theta_1 + \theta_2)/2$ and variance

$$V(t) = \frac{(\theta_2 - \theta_1)^2}{12}$$

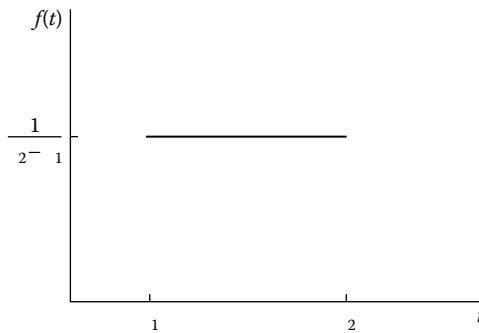


FIGURE 1.7
The uniform PDF.

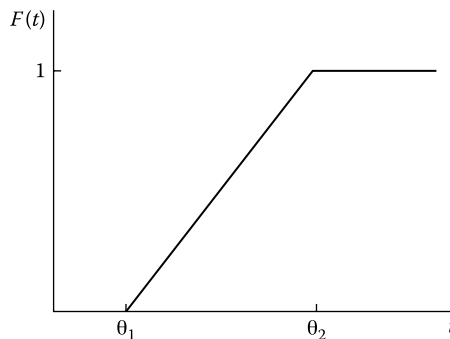


FIGURE 1.8
The CDF for the uniform distribution.

EXERCISE 1.10

The uniform distribution defined on the unit interval $[0,1]$ is a popular and useful model—so much so that the name uniform distribution is often taken to refer to this special case. Find $f(u)$, $F(u)$, $E(u)$, and $V(u)$ for this distribution.

EXERCISE 1.11

Let $F(t) = 1 - (1+t)^{-1}$, $0 \leq t \leq \infty$. This is a legitimate CDF that goes from 0 to 1 continuously as t goes from 0 to ∞ . Find the PDF and the T_{50} for this distribution. Try to calculate the mean. (Hint: Use either integration by parts or the alternate formula given in the text for calculating the mean.)

EXAMPLE 1.8 THE BETA DISTRIBUTION

The (standard) beta distribution, like the uniform distribution discussed in Exercise 1.10, is also defined on the unit interval $[0,1]$. However, it is a far more flexible distribution and even includes the uniform distribution as a special case. Its flexibility is one of the reasons it is an excellent choice for modeling numbers between 0 and 1, such as probabilities or proportions.

For a random variable X having a beta distribution with parameters $a > 0$ and $b > 0$, the PDF in the unit interval is given by

$$f(x) = \frac{x^{a-1}(1-x)^{b-1}}{B(a,b)}$$

where $B(a,b)$, in terms of gamma functions (see the discussion in [Chapter 4](#) after Exercise 4.6), is

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

For a and b integers,

$$B(a,b) = \frac{(a-1)!(b-1)!}{(a+b-1)!}$$

where we use the factorial notation $a!$ to represent $a(a-1)(a-2)\cdots 1$. Note that when $a = b = 1$, the beta distribution is the same as the uniform distribution on $[0,1]$.

The CDF of the beta distribution is commonly called the *incomplete beta function*. For any $0 < x < 1$, the incomplete beta function $F(x)$ is given by

$$F(x) = \int_0^x \frac{y^{a-1}(1-y)^{b-1}}{B(a,b)} dy = I_x(a,b)$$

Like the normal distribution (covered in [Chapter 5](#)), this integral cannot be written in closed form. Tables of the incomplete beta function are available (Pearson 1968). However, both Excel and OpenOffice provide the worksheet function BETADIST for the beta CDF. The arguments are x , a , b , respectively. In Excel, for example, =BETADIST(0.5,2,3) returns the result 0.6875.

[Figure 1.9](#) shows a variety of beta density functions for different values of the parameters a and b . The incomplete beta function $I_x(a,b)$ is closely related to the binomial distribution, a key distribution used in quality control and other sampling applications. This important relationship will be covered in [Chapter 10](#).

EXERCISE 1.12

Find the expected value (mean) for a random variable having a beta density function defined on the unit interval, with parameters a and b . What relationship must these parameters have in order for the mean to be located in the center of the interval (or $\mu = 0.5$)?

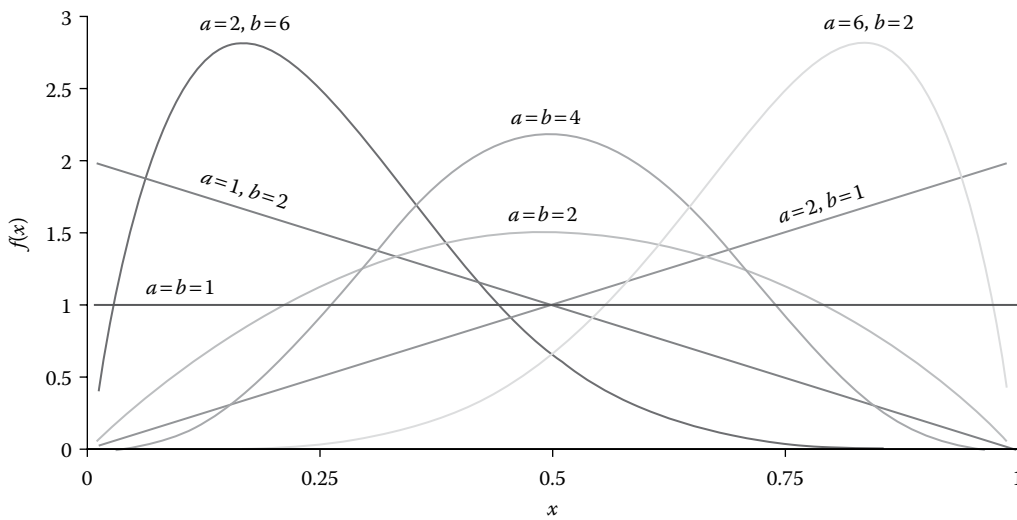


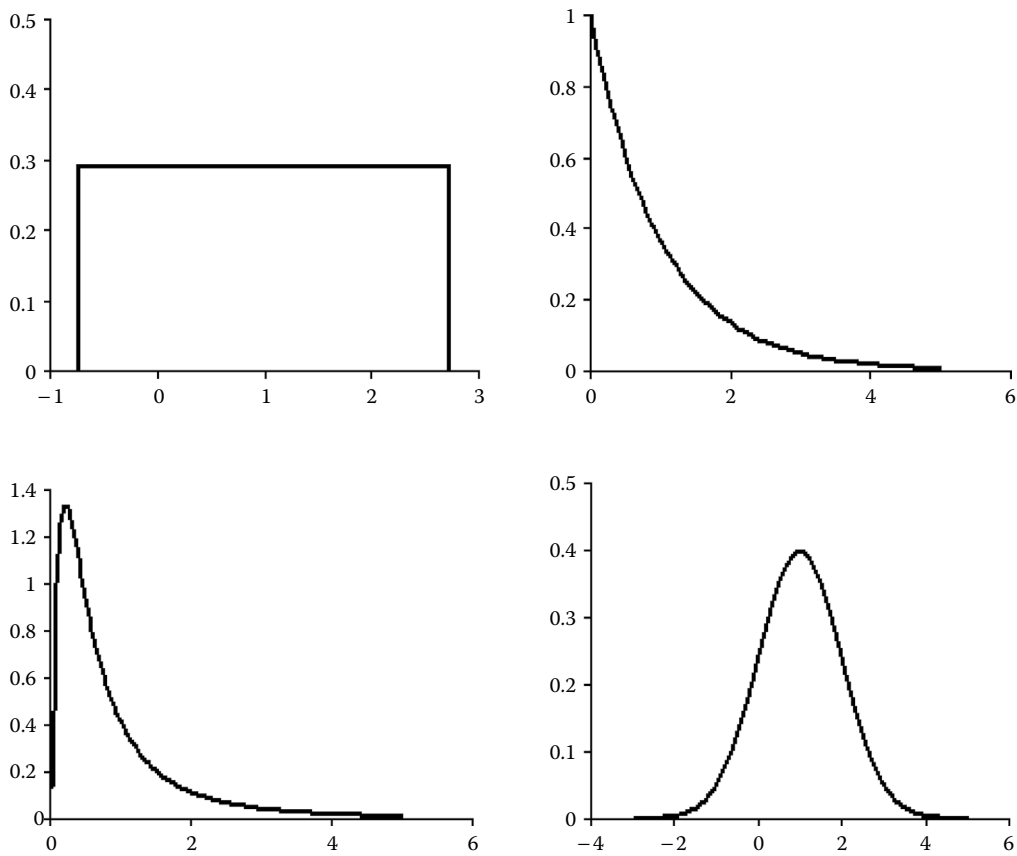
FIGURE 1.9
Beta density functions.

1.8 How to Use Descriptive Statistics

At this point, it is important to emphasize some considerations for the analyst. No matter what summary tools or computer programs are available, the researcher should always “look” at the data, preferably in several ways. For example, many data sets can have the same mean and standard deviation and still be very different—and that difference may be of critical significance (see [Figure 1.10](#) for an illustration of this effect).

Generally, the analyst will start out with an underlying model in his mind based on the type of data, where the observations came from, previous experience, familiarity with probability models, and so on. However, after obtaining the data, it is necessary that the analyst go through a verification stage before he blindly plunges ahead with his model. This requirement is where the tools of descriptive statistics are very useful. Indeed, in many cases we utilize descriptive statistics to help us choose an appropriate model right at the start of our studies. Other useful graphical techniques include Boxplots, dot plots, stem and leaf plots, 3D plots, and so on (see Chambers et al. 1983 for further information on graphical analysis).

In this text, we focus on several key continuous distributions that are most applicable to reliability analysis: the exponential, Weibull, normal, and lognormal distributions. By learning what these distributions should look like, we can develop a yardstick to measure our data for appropriateness to some model. Graphics (frequency histograms, cumulative frequency curves) and summary values (mean, median, variance, skewness, etc.) are the means by which the characteristics of distributions are understood. In [Chapter 6](#), we shall introduce other valuable descriptive procedures such as probability plotting.

**FIGURE 1.10**

Mean and Sigma do not tell us enough. These four distributions have the same mean and standard deviation.

1.9 Data Simulation

Many different PDFs (and CDFs) exist, and reliability studies are often concerned with determining what model is most appropriate for the analysis. In reliability work, one may wish to simulate data from various distributions in order to do the following:

1. Determine percentiles of complicated distributions that are functions of common distributions.
2. Evaluate the effectiveness of different techniques and procedures for analyzing sample data.
3. Test the potential effectiveness of various experimental designs and sample size selections.
4. Illustrate statistical concepts, especially to understand the effects of variability in data.

Computer programs that will generate random variables from almost any desired distribution are available. However, there is a simple and general technique that allows us to produce what are called *pseudorandom numbers* from many common distributions. (The term *pseudorandom* is used because a specific computer algorithm generates the numbers to be as nearly random as possible.) To begin, we need a good table of random numbers or we can use a spreadsheet function to generate random numbers.

For simplicity, we consider only distribution functions $F(x)$ that are continuous and map one-to-one onto the unit interval $(0,1)$, that is, $0 \leq F(x) \leq 1$. This class includes all the life distributions discussed in this text. Let $F(x) = u$. Then, we can define an inverse function $F^{-1}(u) = x$ that provides the specific percentile corresponding to the CDF value in the unit interval. For example, given $F(x) = .5$, then $F^{-1}(.5) =$ the median, which is the 50th percentile. F and its inverse have the following properties: $F(F^{-1}(u)) = u$ and $F^{-1}(F(x)) = x$.

To generate a random sample x_1, x_2, \dots, x_n from $F(x)$, first generate a random sample u_1, u_2, \dots, u_n from the uniform distribution defined on $[0,1]$. This procedure is done with random numbers. For example, if a five-digit random number is obtained from a table or a spreadsheet, divide the number by 100,000 to obtain a pseudorandom number from the uniform distribution. (The spreadsheet function `RAND()` provides random numbers directly in the unit interval.) Next, set $x_1 = F^{-1}(u_1)$, $x_2 = F^{-1}(u_2)$, \dots , $x_n = F^{-1}(u_n)$. It is easy to show that the sample of x 's is distributed according to the $F(x)$ distribution. (See the hint to Problem 1.4 at the end of this chapter.)

EXAMPLE 1.9 DATA SIMULATION

Let $F(t)$ be the distribution given in Exercise 1.11. Generate a sample of five random times from this distribution.

SOLUTION

We obtain F^{-1} by solving for t in $F(t) = u = 1 - (1+t)^{-1}$ to get $t = u/(1-u) = F^{-1}(u)$. Next, we use a random number generator via a spreadsheet function to obtain the uniform distribution sample (0.880, 0.114, 0.137, 0.545, 0.749). Transforming each of these by F^{-1} gives the values $t_1 = 0.880 \div (1 - 0.880) = 7.333$, $t_2 = 0.129$, $t_3 = 0.159$, $t_4 = 1.198$, and $t_5 = 2.984$. The sample $(t_1, t_2, t_3, t_4, t_5)$ is the desired random sample from F .

In a typical reliability experiment, n units are placed on stress, and the exact times to failure are recorded. The successive failure times naturally occur in increasing order, that is, the first failure time is less than the second failure time, the second is less than the third, and so on. This property of ordered failure times is a key characteristic of reliability work. In contrast, consider selecting n individuals randomly and measuring, for example, their weight or height. The successive observations will not necessarily occur in increasing order. Consequently, in simulating random variables for reliability studies, one would like the values arranged in increasing order. For a single set of simulated observations, one could do a manual sort using the spreadsheet **sort** routine available under the menu item **Data**. However, for repeated simulations (involved in Monte Carlo studies), a non-manual procedure is desirable. In Excel (and OpenOffice), the spreadsheet function `PERCENTILE(array, k)` can be used. This function returns the k th percentile (where $0 \leq k \leq 1$) of values in the range defined by the array. The trick is to choose the k values to be the $(i - 1)$ multiples of $1/(n - 1)$, where $i = 1, 2, \dots, n$ is the failure count and n is the sample size. We illustrate the procedure in the Example 1.10.