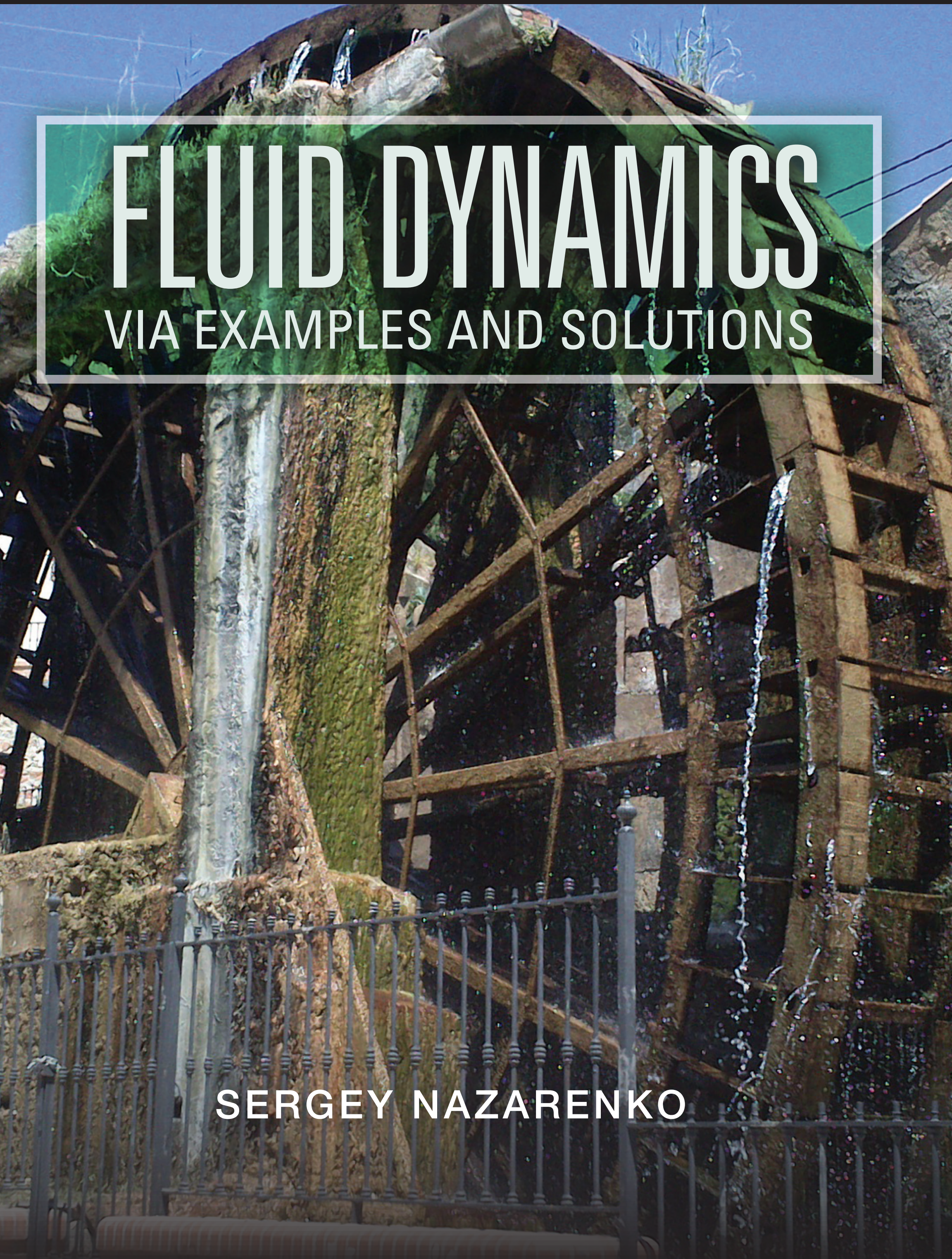

FLUID DYNAMICS

VIA EXAMPLES AND SOLUTIONS

SERGEY NAZARENKO

CRC Press
Taylor & Francis Group

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Preface

As an applied subject, fluid dynamics is best studied via considering specific examples and solving problems dealing with various phenomena and effects in fluids. This is well recognised in most Fluid Dynamics courses, which often have support classes devoted to considering physically motivated exercises. Also, such Fluid Dynamics courses are typically assessed via solution of specific problems more often than via reproducing mathematical proofs or general abstract constructions. However, original Fluid Dynamics problems are rather hard to invent, and the Fluid Dynamics lecturers are often "on their own" having to reinvent successful ideas, tricks and representative examples. The present book addresses these issues by systematically providing such ideas and model examples.

A distinct feature of the present book is that it is problem oriented. Of course, there are many wonderful fluid dynamics textbooks, classical and more recent, which contain exercises and examples, to name just a few: *Hydrodynamics* by H. Lamb [13], *Essentials of Fluid Dynamics* L. Prandtl [21], *An Introduction to Fluid Dynamics* by G.K. Batchelor [4], *Fluid Dynamics* by L.D. Landau and E.M. Lifshitz [14], *Prandtl-Essentials of Fluid Mechanics* by Oertel et al. [17], *Elementary Fluid Dynamics* by D.J. Acheson [1], *Fluid Mechanics* by P.K. Kundu and I.M. Cohen [12], *Physical Fluid Dynamics* by D.J. Tritton [28], *A First Course in Fluid Dynamics* by A.R. Paterson [19], *Elementary Fluid Mechanics* by T. Kambe [9], *Fluid Mechanics: A Short Course for Physicists* by G. Falkovich [6], *Fundamentals of Geophysical Fluid Dynamics* by J.C. McWilliams [16] and *Waves in Fluids* by J. Lighthill [15]. However, most of the existing books make an accent on the theory or expositions. There has been a clear lack of a text which would contain a sizeable set of example problems and detailed model solutions. The present book is intended to fill this gap by presenting a number of fluid dynamics problems organised in chapters dealing with several sub-areas, types of flows and applications. The problems form a "skeleton" of the book structure. Throughout this book, we include supplementary theoretical material when necessary, with an extended list of references for suggested further reading material at the end of each chapter. We also provide a complete set of model solutions.

The book is designed to be used in problem solving support classes and for exam revision in undergraduate and graduate fluid dynamics courses. Also, the book will aid lecturers by offering a pool of possible exam questions for such fluid dynamics courses. It is my hope that the book could be useful also to students and lecturers in related subjects, such as continuum mechanics, turbulence, ocean and atmospheric sciences, etc. More broadly, the provided set of example problems should help an effective hands-on study of fluid dynamics, within or outside of a university course, including an independent study by specialists in other scientific areas who would like to learn basics of fluid dynamics.

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Author Biography

Sergey Nazarenko's research is in the areas of fluid dynamics, turbulence and waves arising in different applications. This includes wave turbulence, magneto-hydrodynamic turbulence, superfluid turbulence, water waves, Rossby waves, vortices and jets in geophysical fluids, drift waves and zonal jets in plasmas, optical vortices and turbulence, turbulence in Bose-Einstein condensates.

Sergey Nazarenko has been working at the University of Warwick since 1996, where presently he holds a position of full professor. Prior to Warwick, Sergey Nazarenko worked as visiting assistant professor at the Department of Mathematics, University of Arizona in 1993–1996, postdoc at the Department of Mechanical and Aerospace Engineering, Rutgers University, New Jersey in 1992–1993, and researcher at the Landau Institute for Theoretical Physics, Moscow in 1991–1992.

Sergey Nazarenko wrote *Wave Turbulence* published by Springer in 2011. He was a co-editor of the books *Non-equilibrium Statistical Mechanics and Turbulence*, CUP 2008, and *Advances in Wave Turbulence*, World Scientific, 2013. Sergey Nazarenko has organised twelve international scientific conferences and workshops in fluid dynamics and turbulence.

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Chapter 1

Fluid equations and different regimes of fluid flows

1.1 Background theory

A fluid is a continuous medium whose state is characterised by its velocity field, $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$, pressure and density fields, $p = p(\mathbf{x}, t)$ and $\rho = \rho(\mathbf{x}, t)$ respectively, and possibly other relevant fields (e.g. temperature). Here, t is time and $\mathbf{x} \in R^d$ is the physical coordinate in the d-dimensional space, $d = 1, 2$ or 3. Respectively, $\mathbf{u} \in R^d$, although in some special flows the dimensions of $\mathbf{x}$ and $\mathbf{u}$ may be different from each other.

Most of the fluid dynamics results have been obtained starting from the *Navier-Stokes equations*. These equations have many variations depending on the forces acting on the fluid, as well as the properties of the fluid itself—compressibility, thermoconductivity, viscosity, density homogeneity/inhomogeneity, chemical composition, etc. We will consider a relatively small but important subset of idealised cases.

1.1.1 Incompressible flows

The *Navier-Stokes equation* for incompressible flow is given by

$$D_t\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{f}, \tag{1.1}$$

which is a momentum balance equation for fluid particles (a continuous medium version of Newton's second law). We introduced notation for the fluid particle acceleration,

$$D_t\mathbf{u} \equiv \partial_t\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u}. \tag{1.2}$$

Operator $D_t \equiv \partial_t + (\mathbf{u} \cdot \nabla)$ is a time derivative along the fluid particle trajectory—a Lagrangian time derivative. Possible external forces acting on the fluid (per unit mass) are denoted by term $\mathbf{f}$ in the right-hand side of equation (1.1); these could be gravity, electrostatic force, etc.

The momentum balance equation (1.1) has to be complemented by a mass balance equation, which for an incompressible fluid is

$$\nabla \cdot \mathbf{u} = 0, \tag{1.3}$$

and is usually called the *incompressibility condition.*

Note that fluids can be incompressible and yet the density of the fluid particles may vary in physical space. In this case we need an extra equation describing the conservation of ρ along the fluid particle trajectories,

$$D_t \rho = 0. \tag{1.4}$$

1.1.2 Inviscid flows

When the Reynolds number is large one can ignore viscosity (see problem 1.3.1), and the momentum balance equation (1.1) reduces to

$$D_t \mathbf{u} = -\frac{1}{\rho} \nabla p + \mathbf{f}, \tag{1.5}$$

which is known as the *Euler equation.* It is valid for both compressible and incompressible fluids. However, for compressible fluids, the mass balance equation is now different:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0. \tag{1.6}$$

(This equation remains the same in presence of viscosity). Also, since there is an extra unknown field, $\rho(\mathbf{x}, t)$, we need an extra evolution equation for the model to be complete. Generally, such an equation is provided by the energy balance relation. In particular, assuming that different fluid particles are thermally insulated from each other, one can write the additional equation in the form of conservation of *entropy* $S \equiv S(\mathbf{x}, t)$ along the fluid paths,

$$D_t S = 0. \tag{1.7}$$

For the polytropic gas model

$$S = C_v \ln \frac{p}{\rho^\gamma}, \tag{1.8}$$

where constants C_v and γ are called the specific heat constant and the adiabatic index respectively. Obviously, C_v drops out of the equation (1.8) and therefore it is irrelevant in this case. For monatomic ideal gas (e.g. helium, neon, argon) $\gamma = 5/3$; for diatomic gas $\gamma = 7/5$ (e.g. oxygen, nitrogen).

In the simplest case of *isentropic gas*, $S =$const, i.e.

$$p \propto \rho^\gamma. \tag{1.9}$$

In incompressible fluids the equation (1.7) implies conservation of temperature T along the fluid paths,

$$D_t T = 0. \tag{1.10}$$